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"Zero Rate of Climb Speed" as a Low Speed Limitation for the Stall-Free Aircraft

by

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"ZERO RATE OF CLIMB SPEED" AS A LOW SPEED LIMITATION FOR THE STALL-FREE AIRCRAFT

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SUMMARY

An attempt is made to assess the significance to flight safety of a condition at low speeds where drag exceeds the available engine thrust. For the low aspect ratio aircraft and especially with slender wing designs having practically no stall, the "zero rate of climb speed" defined by the condition may constitute the lowest limit of the practical speed range. Methods are suggested to assess the necessary margins to protect aircraft of this general class against the accidental, possibly catastrophic, loss of performance below this speed.

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*Replaces R.A.E. Technical Report No.66144 - A.R.C. 28297

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1 INTRODUCTION

The low speed potential of the classical aircraft with its high aspect ratio wing is sharply limited by the stall. In non-manoeuvring flight the stall defines a speed below which flight cannot be sustained and furthermore it is a condition from which recovery is only possible with a substantial loss of height even if the pilot can avoid entry into a spin. These unique characteristics made the stall a compulsive datum from which to define speed and manoeuvring margins to provide safety in low speed flying. It is not surprising therefore that in the formulation of airworthiness requirements the stalling speed v_s plays a dominant role.

However, as wing aspect ratio is decreased the stall is delayed to larger angles of incidence and in the extreme case of a slender wing, say if the aspect ratio is below 2, it would occur at an incidence which is completely outside the practical flight range. With such aircraft low speed flying must be limited by considerations other than the stall, such as for instance general deterioration of lateral or longitudinal control, pitch up tendencies or difficulties in speed holding in flight below minimum drag speed.

With the possible exception of pitch up, if and when it occurs, these characteristics will deteriorate fairly gradually with decreasing speed and it is generally impossible, certainly in the design stage, to use them for the definition of a sharply and uniquely determined absolute low speed limit. As an alternative it has been proposed that the manufacturer shall select operational speeds, such as for instance the initial climb out speed, and demonstrate that at these speeds and at speeds a specified number of knots below these, the aircraft satisfies certain standards of controllability. This procedure appears generally satisfactory, since the use of auto-stabilisation is permitted to achieve these handling characteristics - provided it is designed to an acceptable standard of reliability - and these regulations give the designer good scope to exploit the low speed potential of a design.

More recently, however, in an attempt to formulate requirements for the supersonic transport aircraft, the British and French airworthiness authorities have suggested new minimum speeds to take the place of the stalling speed of the conventional aircraft in the take off performance requirements. These are the "zero rate of climb speeds", defined as the lowest speeds at which with either full power or with one engine inoperative level flight can just be maintained. Fig.1 shows how these conditions are defined by the thrust and drag-characteristics of an aircraft.

For the legislator these speeds are clearly attractive as they share at least two of the outstanding features of the stalling speed:

(i) they are sharply defined and can be established with adequate precision in flight

(ii) they define speeds below which level flight cannot be maintained and they therefore set a limit to the practical speed range.

It should be noted, however, that whereas the stall is uniquely determined by a purely aerodynamic phenomenon and is usually preceded by symptoms indicating to the pilot the imminence of flow breakdown, no such warning will signal the approach to zero rate of climb speed, which is entirely governed by performance and thus a direct function of weight, drag, configuration, and parameters influencing engine power such as air temperature and altitude.

As the term zero rate of climb speed implies, it considers the ability of an aircraft to maintain level flight. It may be more appropriate for take off for instance, to consider instead the speed at which a given minimum climb gradient can be maintained, rather than level flight. Such a speed would of course be higher than zero rate of climb speed, as the available propulsive force is now reduced to $\left(\frac{T}{W} - \sin\gamma\right)$. However these are matters for the certification authorities to consider and will not be further discussed here. We shall only investigate the nature of the hazard - in relation to the flight condition to be maintained - when an aircraft drops below this minimum speed and the nature of the protection afforded by speed margins.

Quite clearly it is of interest to know whether zero rate of climb speed has a practical significance to flight safety similar to that of the stalling speed of conventional aircraft. Equally it is necessary to consider if the speed margins required to protect an aircraft against the consequences of a stall would be also appropriate with respect to zero rate of climb speed.

The drag characteristics of the low aspect ratio wing and thus of the typical high speed aircraft of the foreseeable future, are such as to bring the speed below which drag is greater than the available engine thrust near to the speed range considered for take off. In certain cases this condition might even be relevant in the landing phase. The establishment of realistic safety margins against the occurrence of catastrophic fall off in speed is therefore an important task.

The airworthiness authorities argue that since the existing speed margins have been effective in preventing stalls in civil operations, similar margins will safeguard aircraft equally well against the exceedence of zero rate of climb speed. On the other hand such a requirement may appear unduly severe as a safeguard against an essentially less severe phenomenon. The present paper is written as a first contribution towards a possible resolution of this problem although, and this must be clearly stated, it considers only some aspects of the problems, which may well require study on a broader basis.

In particular the height loss in recovering from flight below zero rate of climb speed (V_o) is calculated and also the probability of falling below V_o is compared with the probability of stalling a conventional aeroplane.

2 RECOVERY FROM FLIGHT BELOW ZERO RATE OF CLIMB SPEED

The first question which arises with an aircraft limited by zero rate of climb speed rather than by the stall is the nature of the hazard presented to the pilot in the region below this speed.

In the case of the stall we require that the motion of the aircraft entering the stall shall be essentially symmetric and that recovery shall require not more than a specified loss of height.

As zero rate of climb speed is not necessarily associated with a change of flow behaviour on the aircraft, control difficulties need not be expected, although they may of course appear in about the same speed regime for separate reasons. The only flying hazard directly associated with the phenomenon under discussion is loss of performance. This is perhaps best expressed as the height loss incurred in recovery to a condition from which level flight can be maintained.

This recovery manoeuvre can of course be demonstrated in flight, where one would expect the height loss to increase progressively with the amount the speed has been allowed to drop initially below zero rate of climb speed. The certification authorities are interested mainly in two specific zero rate of climb speeds, that with full engine power and one with full power on all but one engine. These are of special interest for take off.

One may have to consider, however, also a condition where the aircraft is operated with partial power, e.g. in the approach. If in such a condition due to gross mishandling, speed is allowed to drop to a certain value still above the zero rate of climb speed appropriate to full power, imbalance of drag over thrust can reduce speed at so fast a rate that as a consequence drag increases faster than it is possible for the engines to increase thrust. This phenomenon would then mean that in this situation a speed exists slightly above zero rate of climb speed proper, from which recovery without loss of height is impossible.

Returning now to the basic case with engines operating at full power, it is clear that once speed has dropped below zero rate of climb speed recovery must involve loss of height. Some broad indications of the recovery penalties can be obtained from simple energy considerations.

If V_o is zero rate of climb speed and ΔV is an increment in speed with respect to V_o we get $V = V_o + \Delta V$. The minimum height ΔH_1 required for the aircraft to recover from a negative diversion in speed ΔV to V_o can be calculated by considering the exchange of potential and kinetic energy as

$$\Delta H_1 = \frac{1}{2g} (2 V_o \Delta V + \Delta V^2) . \qquad (1)$$

For negative values of ΔV, ΔH_1 will be negative, i.e. a loss in height. For a representative range of values for V_o and ΔV this function has been plotted in Fig.2.

This energy equation, however, presents an optimistic picture, as it does not allow for loss of energy during the recovery manoeuvre. In fact during this manoeuvre the aircraft flies below zero rate of climb speed where drag is in excess of thrust. Consequently the work done against this excess drag during the duration t_R of the recovery manoeuvre,

$$\int_0^{t_R} (D-T) V d t$$

requires the expenditure of additional potential energy

$$\Delta H_2 m g = - \int_0^{t_R} \frac{(D-T)}{W} m g V d t . \qquad (2)$$

Thus the total height loss for recovery from $(V_o + \Delta V_o)$ to V_o is obtained by adding equations (1) and (2).

$$\Delta H = \Delta H_1 + \Delta H_2 = \frac{1}{2g}(2V_o\,\Delta V + \Delta V^2) - \int_o^{t_R} \frac{D - T}{W} V\,d\,t. \qquad (3)$$

The terms under the integral are of course functions of time i.e. function of the nature of the recovery manoeuvre. A proper solution would therefore require the specification of pilots control. As here only a very crude estimate of the orders of magnitude is required, it is proposed to evaluate the integral by making suitably simple assumptions. The drag versus speed characteristics are assumed linear within the range of interest as

$$\Delta\left(\frac{T - D}{W}\right) = \Delta V \frac{K}{V_o}$$

where K is a constant. It is further assumed that V(t) can be replaced by the mean speed $V = (V_o + \frac{\Delta V}{2})$ = const and that ΔV varies symmetrically during the recovery so that it can be represented again by a mean $\Delta V_m = \frac{\Delta V}{2}$ if ΔV is the initial speed error.

Finally the duration of the recovery manoeuvre can be related to the mean vertical velocity $\dot{H}_m$ and the total recovery height ΔH as

$$t_R = \frac{\Delta H}{\dot{H}_m} . \qquad (4)$$

Thus we obtain

$$\Delta H = \frac{1}{2G} \frac{2V_o\,\Delta V + \Delta V^2}{1 + \frac{K}{2\dot{H}_m}\left(\Delta V + \frac{\Delta V^2}{2V_o}\right)} . \qquad (5)$$

This expression has been computed to cover a representative range of conditions with the results shown in Fig.3. Two values of $K = d\,(\frac{T - D}{W})/d(\frac{V}{V_o})$ are considered, K = 0·25 as more representative of a high aspect ratio wing and K = 0·5 as more appropriate for a slender wing. It is seen that the height loss resulting from this more realistic analysis is generally much greater than that predicted in Fig.2. It is also seen that the height loss can be minimised by using the fastest possible recovery technique, i.e. by using a large mean descent rate $\dot{H}_m$ during the manoeuvre. It should be noted that these sums do not consider the practicability of the implied manoeuvre, e.g. no account is taken of the pitch response characteristics of the aircraft. However, this can be roughly assessed by reference to the given values of the manoeuvre duration t_R. To exclude some obviously impracticable cases, those requiring completion within less than 4 seconds, the corresponding portions of the curves in Fig.3 are shown as dashed lines.

The results given in Fig.3 represent a theoretical minimum height loss. In practice variations of piloting technique, turbulence etc. are bound to lead to less favourable recoveries. It should also be noted that to be effective recovery must return speed to a value above zero rate of climb and this requires an additional expenditure of height.

Taking all this into account it is clear that substantial height will be lost in recovering an aircraft from a deviation below zero rate of climb speed and that this condition must be treated with almost as much caution as stalling speed. However, zero rate of climb speed is only a serious hazard in flight close to the ground, i.e. immediately after lift off, during landings and overshoots. Also, as Fig.1 shows the more restricting condition on a multi-engined aircraft is V_O with an engine failed and this situation is itself a rare event.

3 THE POTENTIAL RISK OF STALLING

An aircraft will stall if a certain critical incidence is exceeded. Assuming that this stalling incidence is independent of speed and also ignoring the fact that in dynamic manoeuvres the stall may be delayed to higher values of incidence, the margin in normal acceleration available in level flight is

$$\Delta n_S = \left(\frac{V}{V_S}\right)^2 - 1. \qquad (6)$$

In other words an aircraft will stall when flying at a given speed V, if due to a combination of pilot induced manoeuvre and vertical gusts the associated Δn_S (Fig.4) is reached or exceeded. If one can specify

(i) the statistical distribution of speed errors occuring operationally in relation to a target speed V_T and

(ii) the statistical probability of reaching or exceeding a given level of manoeuvre and gust induced normal acceleration,

and if one assumes that these two distributions are independent, (i.e. that the pilot will not simply cancel the gust induced loads by acting as a gust alleviator) the total probability of stalling anywhere over the full speed range say during the approach or some other specific flight phase, can be calculated. Evidently neither are these assumptions absolutely true nor can we specify the statistical distribution of the two relevant quantities V and Δn with any degree of confidence at least at the extremes constituting accident exposure. However, even using rather arbitrary assumptions, it is thought that such analysis may be able to reveal some typical qualitative trends.

4 THE POTENTIAL RISK OF CATASTROPHIC LOSS OF PERFORMANCE

When an aircraft flies below minimum drag speed, the available margin of thrust over drag will decrease progressively with decreasing speed until a point is reached at which level flight can just be sustained with the available engine power (Fig.1). Below this speed (V_0) the excess drag will lead to a speed divergence from which recovery is only possible by increasing the rate of descent, i.e. by losing height. It has been shown earlier in Section 2 that at least during approach and landing, flight in this regime is not permissible. However, a short term drop below this critical speed, the zero rate of climb speed, due to a brief exposure to a tail gust, will not necessarily lead to speed divergence, neither will speed divergence be of necessity fatal at the final stages of an approach provided that the aircraft has enough elevator power and tail clearance for a safe touch down at the required large incidence.

Although these conditions may well be claimed to provide further relief the results of the analysis in Section 2 suggest that any drop of speed below zero rate of climb speed close to the ground is almost as impermissible as is reaching the stall.

The probability of experiencing an impermissible speed divergence can thus conservatively be stated to be equivalent to the probability of flying at a speed $V \leqslant V_0$ when in close proximity to the ground.

5 CALCULATIONS AND RESULTS

As was said before the prediction by statistical analysis of the potential exposure to such flying hazards as the stall or speed divergence, leans heavily on a series of assumptions. For instance the awareness of the existence of these hazards will obviously restrain the pilot, although it will not affect the gust-induced variations. There will also be a strong correlation between the occurrence of gust induced speed variations and normal acceleration peaks, but here the corresponding probability distributions of speed and normal acceleration are treated as independent.

The principal variables are assumed to have a normal distribution. In detail the following assumptions are made:

(i) Speed variability is described by a normal distribution round a chosen mean speed V_m. This mean speed represents a speed target selected by the pilot and available statistical data suggests that in practice it lies above the corresponding target speed (V_T) recommended in the flight manuals. Two values are considered $V_m/V_S = 1{\cdot}35$ corresponding to a 30% speed margin and $V_m/V_S = 1{\cdot}25$ corresponding to a 20% margin. In both cases the pilot is assumed to allow himself a 5% margin above the recommended target speed. The statistical variability of speed is described by the standard deviation; two values are considered, $\sigma_V = 0{\cdot}07\ V_S$ and $\sigma_V = 0{\cdot}05\ V_S$, to represent conditions typical for flight in severe and moderate turbulence respectively.

(ii) Normal acceleration is assumed to contain two independent contributions, one resulting from piloted manoeuvres, the other being gust induced. Considering lift off or approach as relevant flight phases it is necessary to account for the required pull up in the flare and for this reason the mean normal acceleration is taken as 1·05g or 1·03g respectively, the incremental variations being normally distributed about this mean.

Pilot induced manoeuvring is assumed to produce normal accelerations proportional to V^2, i.e. the pilot is assumed to use approximately constant control movement irrespective of speed.* Gust induced normal accelerations are varied in proportion to V_1 as corresponds to real conditions.

In order to cover a plausible range three distinct "acceleration environments" are considered.

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* It is not suggested that this relationship is a correct representation of real flying practice or even a particularly plausible one. Other laws may be more appropriate but for the present purpose such subtleties are irrelevant.

Flight environment	Standard deviation of Pilot induced "g" σ_p	 Gust induced "g" σ_G	Mean "g" n_m
Smooth	$0{\cdot}03\left(\frac{V}{V_S}\right)^2 g$	$0{\cdot}02\left(\frac{V}{V_S}\right) g$	1·03g
Moderate	$0{\cdot}04\left(\frac{V}{V_S}\right)^2 g$	$0{\cdot}04\left(\frac{V}{V_S}\right) g$	1·05g
Severe	$0{\cdot}06\left(\frac{V}{V_S}\right)^2 g$	$0{\cdot}05\left(\frac{V}{V_S}\right) g$	1·05g

The sum of the two contributions to the normal acceleration is again a normal distribution with a standard deviation of $\sigma_n = \sqrt{\sigma_p^2 + \sigma_G^2}$. This quantity is plotted against V/V_S for the three cases considered in Fig.5.

The probability of experiencing a stall is now determined by the combination of the probabilities of finding oneself at a given speed and at the same time of experiencing a normal acceleration equal to or greater than that defining the stall at that speed. Mathematically this means multiplying the probability of flying at a given speed (as defined by the assumed variability σ_V) by the probability of at each speed exceeding the corresponding stalling incidence (plotted in Fig.6). This product is a distribution function of the probability of stalling against speed. Examples of such distribution functions for the case of a moderate flight environment are shown in Fig.7. It can be seen that that maximum stalling risk is associated with speeds well above the stalling speed. Integrating these functions from $0 < V < \infty$ gives then the total probability of stalling for an aircraft flying in the particular environment. These integrals were computed for all combinations of the above listed assumptions and are presented in Table 1 and plotted in Fig.8.

In the case of an aircraft being limited by zero rate of climb speed rather than by a stall, the only danger to be considered is that of speed falling below V_o. The probability of this happening is then simply the integral between $0 < V < V_o$ of the appropriate speed distribution function. This integral defines the probability of encountering a speed divergence from which recovery is only possible at the expense of height. Substituting V_o

for V_S used as a datum speed in the stall analysis, these integrals have been evaluated for the same conditions as treated above. The results are also given in Table 1 and Fig.8. To make the comparison between the stalling case and the performance limited aircraft absolutely fair, the zero rate of climb speed should be computed not for the 1g condition but for the mean "g" > 1·0 assumed for the stall analysis.

The probabilities derived by this procedure and shown in Fig.8 appear rather high in the more adverse conditions, in fact in some cases they appear to be in obvious contradiction to the known very low incidence of stalling observed in real life. Before any interpretation is attempted of these results, the following reservations must be noted:

(i) The calculations do not claim to predict absolute figures, they are clearly based on a series of arbitrary assumptions, which are not derived from operational statistics. Nevertheless they should be adequate to allow relative comparison between the various cases.

(ii) The quoted probabilities refer to particular environmental conditions (weather). To obtain from these an overall probability of experiencing the specific incident considered, one would have to multiply the results again with the probability for these environment conditions to exist. This will obviously give total probabilities much lower than those shown in Fig.8.

(iii) The cases evaluated for a mean speed $V_m = 1{\cdot}25\ V_S$ appropriate to an assumed recommended target speed of 20% above stalling speed, are not realistic when applied to the stalling case itself, since such low margins are not permitted by airworthiness regulation. These are only shown as a basis for comparison with the zero rate of climb speed case.

6 DISCUSSION

The results of these calculations show of course the expected trends, namely that the probability - other things being equal - of stalling an aircraft due to a combination of low speed and the application of normal acceleration is much greater than the probability of flying at $V < V_S$ alone. Also it is seen that a reduction of the speed margin of the selected target operational speed by 10% V_S increases the vulnerability to either hazard by a factor of between 10^2 to 10^4.

If one compares the potential danger of stalling an aircraft with that of just falling below $V_S = V_O$, it is seen that - except for the extreme case with small speed variability ($\sigma_V = 0{\cdot}05\ V_S$) and in a smooth environment - for instance a 25% margin offers better protection against speed divergence below V_o than does the 35% margin against stalling.

It would appear that, if present margins provide adequate stall protection, zero rate of climb speed could be equally protected by a margin 10% smaller than these. It can be argued that when V_o is only critical after an engine failure which itself is a rare event, the overall probability of dropping below V_o is substantially reduced and an even smaller V_o - margin might be acceptable.

7 GENERAL CONSIDERATIONS

The above process of attempting to arrive at a safety margin against a novel flight hazard by statistical analysis is based essentially on the customary approach of applying past experience to a new design. In this approach the operative word is always: "other things being equal". The condition that other things are equal is of course not automatically satisfied and further regulations are needed to ensure handling characteristics which make the avoidance of a given hazardous condition equally easy.

With the conventional aircraft, for instance, safety from stalling rests not only on the choice of suitable low speed margins but moreover on the aircraft possessing clearly distinguishable stall warning, either natural or synthetic. If zero rate of climb speed or indeed any other phenomenon replaces the stall as the primary low speed hazard it may be necessary to warn the pilot of the approach to this condition by an equivalent warning. Physically this may well take the form of the well proven stick-shaker. If take-off directors are used the warning may be incorporated into this instrument.

The sensing signal, however, is rather complex as zero rate of climb speed is a function of aircraft weight as well as those parameters affecting engine power such as air temperature and altitude. Frequently zero rate of climb speed will only be a practical hazard during take off and in this case a simple precomputed setting may be adequate.

Another aspect affecting the aircraft limited by zero rate of climb is the fact that in the approach to this hazard it will in addition be flying below minimum drag speed. It is generally recognised that in this regime speed control is more difficult and there appears to be a possibility therefore, that speed errors become larger and more frequent and as a consequence the potential exposure to zero rate of climb speed is sharply increased. However, in practice, this is not necessarily so and it is essential to consider the two primary low speed regimes separately.

(i) In take off full power is used at least until a height is reached at which zero rate of climb speed ceases to be a hazard. In this configuration the aircraft flies with excess power, usually converted into rate of climb and the speed instability associated with flight below minimum drag speed does not arise, unless the pilot attempts to fly a constant climb gradient.

(ii) In the approach where the constraint to the glide path can lead to temporary speed divergence, zero rate of climb speed is usually well outside the practical range of even gross speed errors. In any case it is now generally accepted that aircraft approaching below minimum drag speed require automatic throttle control and this should in fact lead to improved speed holding when compared with conventional aircraft. In this case there is, however, still one aspect that needs consideration. If the authority of the automatic throttle control is too limited, from time to time speed may fall below the operating range of the system and from then on, if the pilot is unaware of this situation speed will be lost rather rapidly. Since automatic throttle control may develop a habit in pilots to pay less attention to speed, it is imperative to ensure that the autothrottle is not only reliable but also has enough authority to cope with all likely extremes. It may be advisable to warn the pilot of a condition when the autothrottle applies all the thrust under its command for more than a brief moment.

8 CONCLUSIONS

The high induced drag of the modern low aspect ratio aircraft is capable of generating a condition at very low speed below which drag exceeds the available engine thrust so that level flight cannot be maintained. If down to this speed no stall or other prohibitive control problem arises, this "zero rate of climb speed" may then constitute the extreme limit of safe flight and operational speeds must be chosen to provide adequate margins against the accidental exposure to irrecoverable loss of performance below this speed.

Statistical considerations indicate however, that such margins could be significantly lower than those required to protect more conventional aircraft against the stall. It may be desirable, nevertheless, to provide the aircraft limited by zero rate of climb speed with a warning device similar to those developed for stall protection.

<u>Table 1</u>

Standard deviation of speed σ_V	Standard deviation of		Mean manoeuvre g n_m	Probability with $V_m = 1{\cdot}35\ V_S$		Probability with $V_m = 1{\cdot}25\ V_S$	
	Manoeuvre g σ_P	Gust g σ_G		of being at $V \leqslant V_S = V_o$	of stalling	of being at $V \leqslant V_S = V_o$	of stalling
$0{\cdot}07\ V_S$	$0{\cdot}06\ (V/V_S)^2$	$0{\cdot}05\ V/V_S$	1·05	$0{\cdot}2 \times 10^{-6}$	440×10^{-6}	15×10^{-6}	31000×10^{-6}
	$0{\cdot}04\ (V/V_S)^2$	$0{\cdot}04\ V/V_S$	1·05		110×10^{-6}		15000×10^{-6}
	$0{\cdot}03\ (V/V_S)^2$	$0{\cdot}02\ V/V_S$	1·03		12×10^{-6}		6700×10^{-6}
$0{\cdot}05\ V_S$	$0{\cdot}06\ (V/V_S)^2$	$0{\cdot}05\ V/V_S$	1·05	$0{\cdot}000003 \times 10^{-6}$	13×10^{-6}	$0{\cdot}2 \times 10^{-6}$	3600×10^{-6}
	$0{\cdot}04\ (V/V_S)^2$	$0{\cdot}04\ V/V_S$	1·05		$0{\cdot}16 \times 10^{-6}$		630×10^{-6}
	$0{\cdot}03\ (V/V_S)^2$	$0{\cdot}02\ V/V_S$	1·03		$0{\cdot}00004 \times 10^{-6}$		39×10^{-6}

SYMBOLS

D	drag
$K =$	$d\left(\frac{T-D}{W}\right) \Big/ d\left(\frac{V}{V_o}\right)$
H	height
ΔH	recovery height
m	aircraft mass
$n(g)$	normal acceleration
n_m	mean normal acceleration
T	thrust
t_R	duration of manoeuvre
V	speed
V_o	zero rate of climb speed
V_m	mean speed
V_S	stalling speed
ΔV	incremental speed
ΔV_o	initial speed excursion from either V_S or V_o
σ_G	rms gust induced normal acceleration
σ_n	rms normal acceleration
σ_P	rms pilot induced normal acceleration
σ_V	rms speed variability

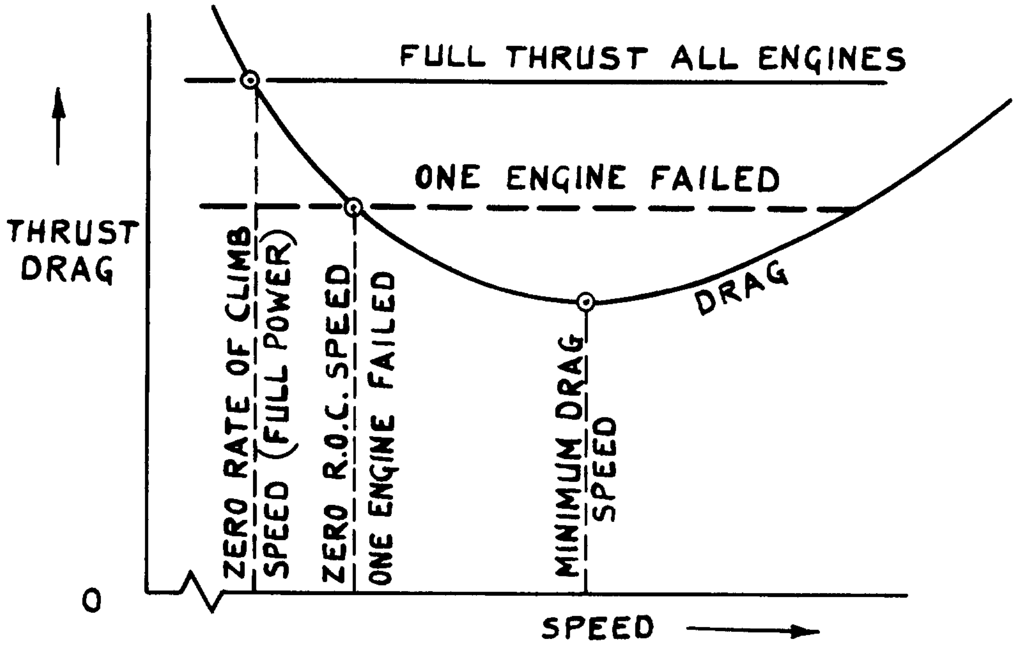

FIG. I DRAG AND THRUST VERSUS SPEED AND THE DEFINITION OF ZERO RATE OF CLIMB SPEED

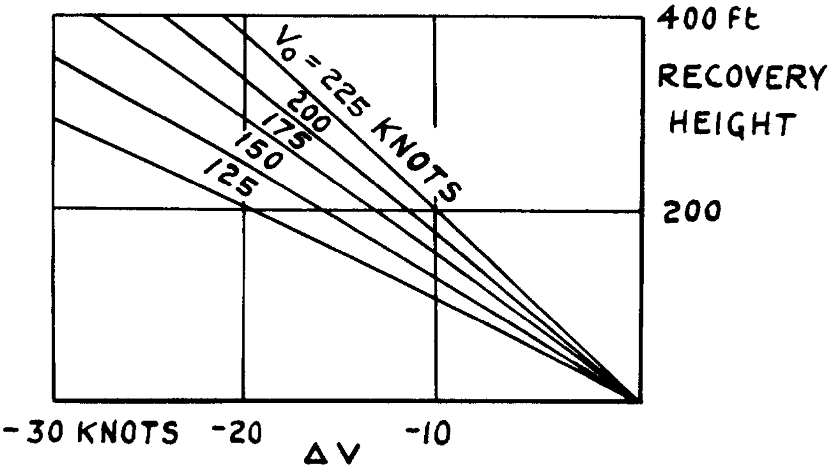

FIG. 2 POTENTIAL HEIGHT LOSS FOR RECOVERY FROM FLIGHT ΔV KNOTS BELOW ZERO RATE OF CLIMB SPEED V_0 ASSUMING NO ENERGY LOSSES

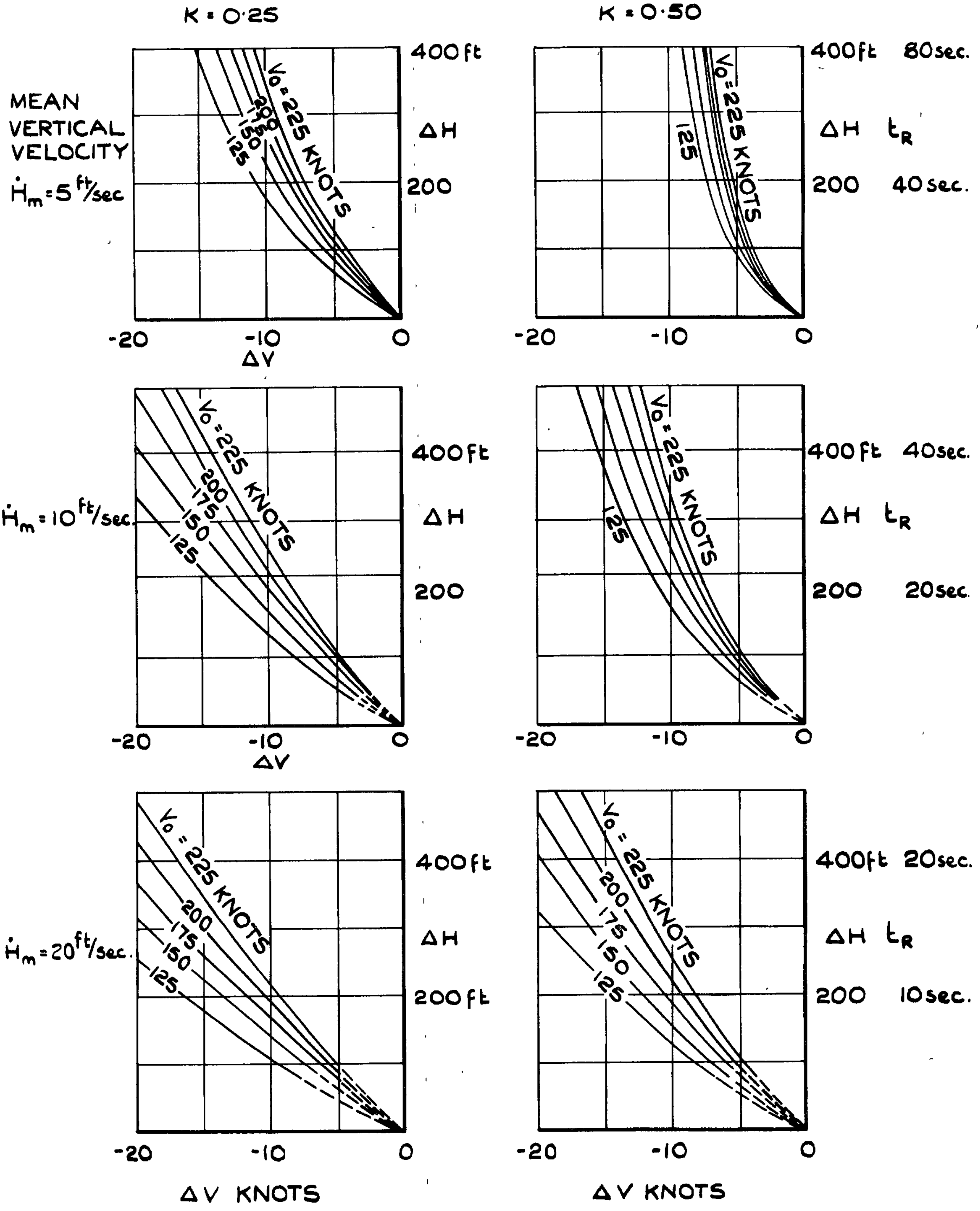

FIG. 3. HEIGHT ΔH AND TIME t_R LOST IN RECOVERY FROM SPEED ΔV KNOTS BELOW ZERO RATE OF CLIMB SPEED V_0 FOR A RANGE OF AIRCRAFT AND RECOVERY MANOEUVRES

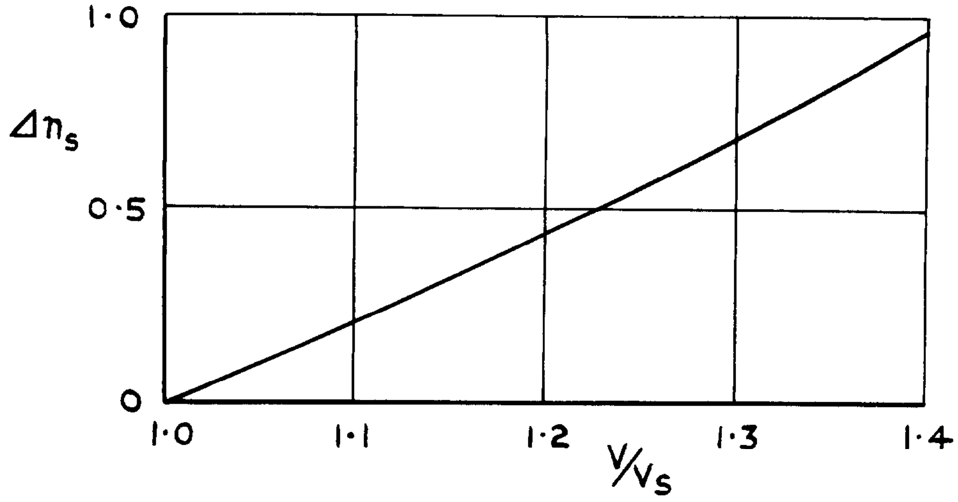

FIG.4 NORMAL ACCELERATION MARGIN AVAILABLE ABOVE STALLING SPEED V_s

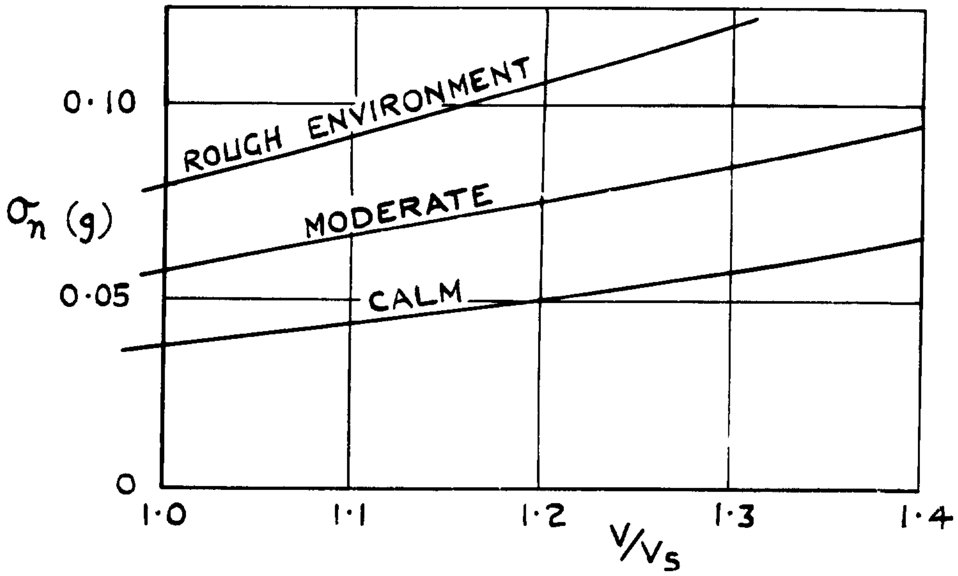

FIG. 5 ASSUMED STANDARD DEVIATION OF NORMAL ACCELERATION AS A FUNCTION OF SPEED

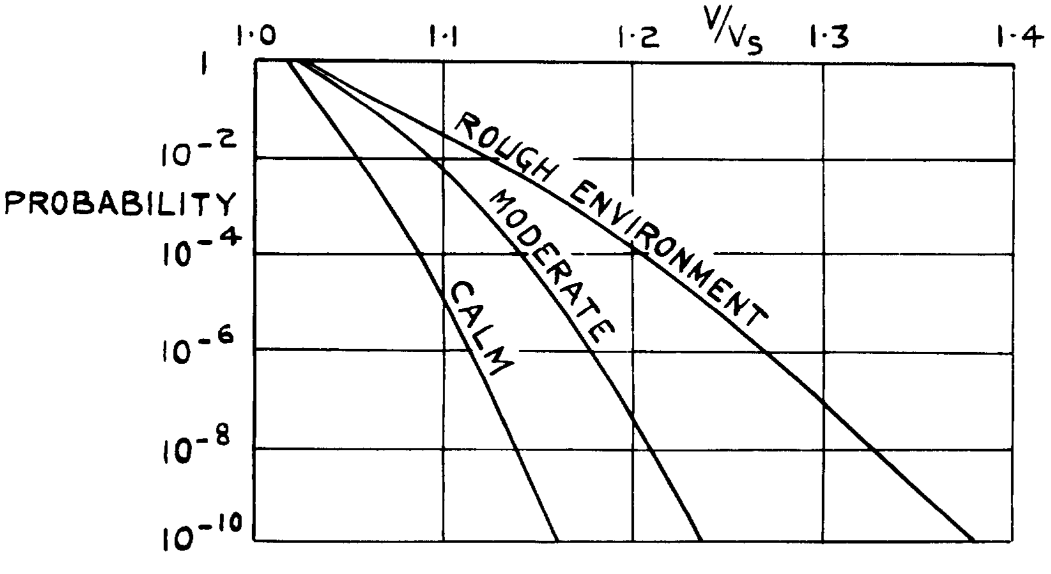

FIG.6 PROBABILITY OF EXCEEDING STALLING INCIDENCE WHEN FLYING AT A GIVEN SPEED V AS A FUNCTION OF V/ V_s AND OF THE SEVERITY OF FLIGHT ENVIRONMENT

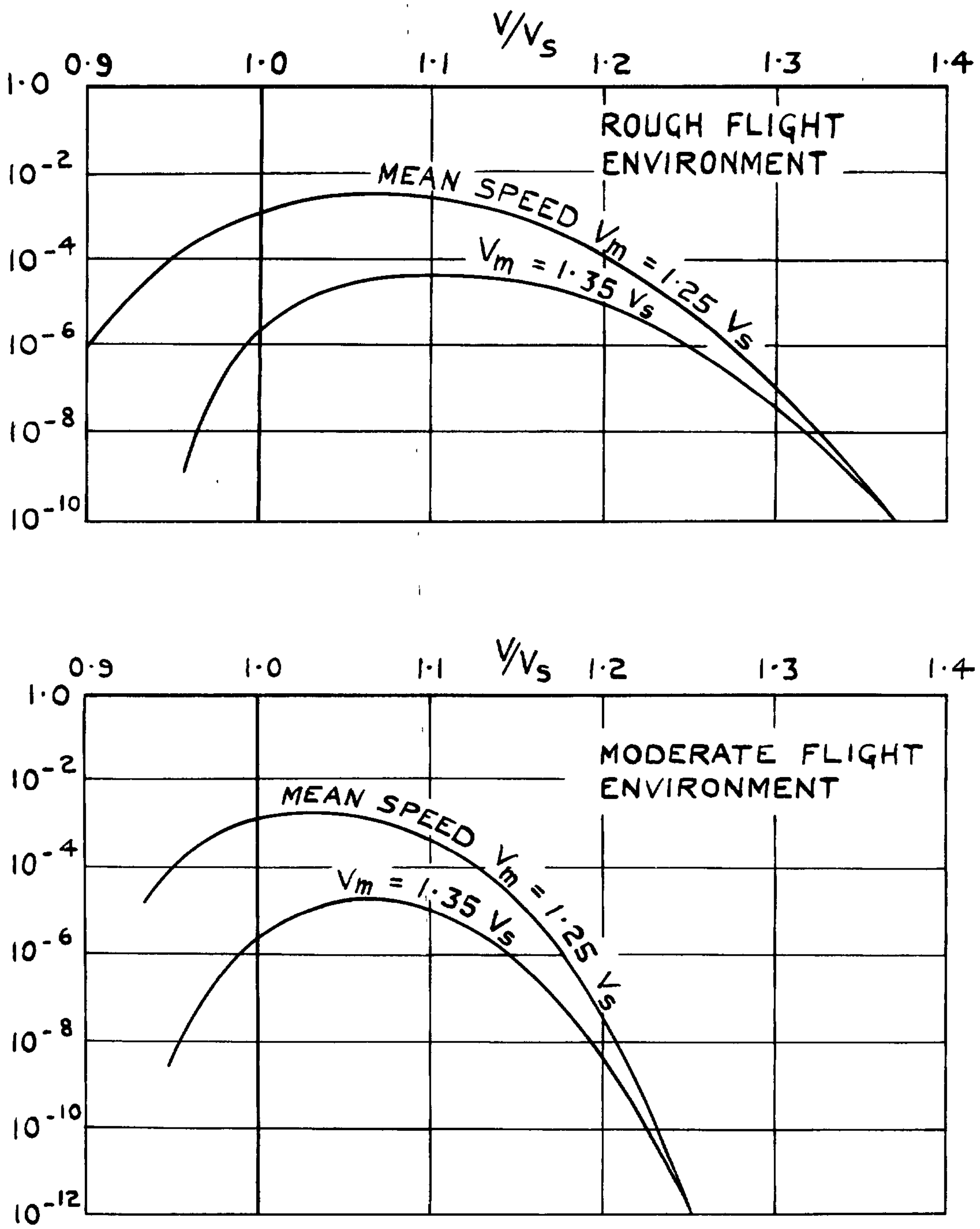

FIG. 7 TYPICAL EXAMPLES FOR PROBABILITY DISTRIBUTION FUNCTIONS OF g – STALLING OVER THE SPEED RANGE
SPEED VARIABILITY $\sigma_v = 0{\cdot}07\ V_s$ NORMALLY DISTRIBUTED ABOUT V_m

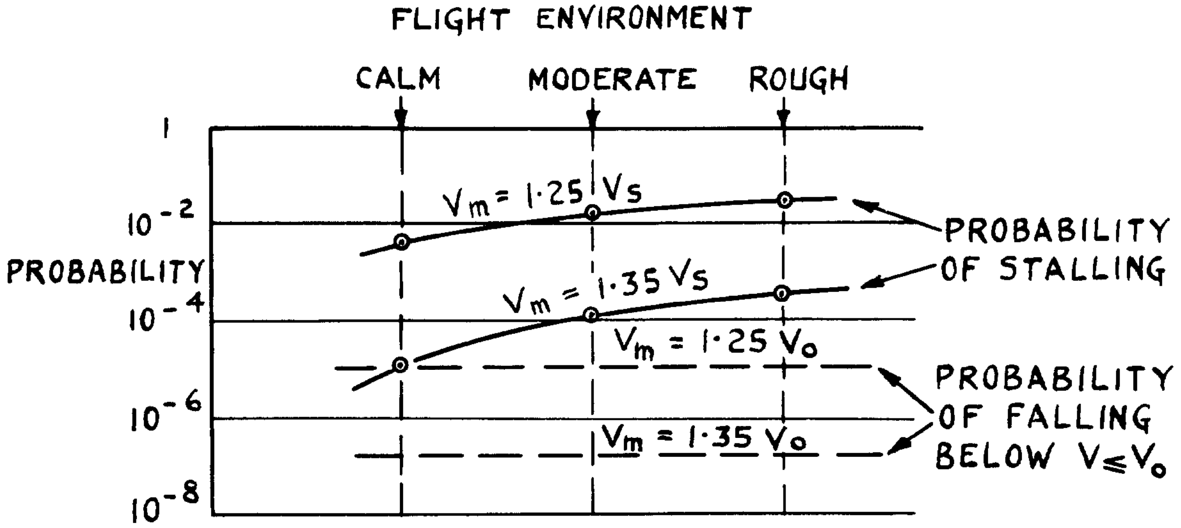

FIG. 8 (a) $\sigma_V = 0{\cdot}07\ V_s\ (V_o)$

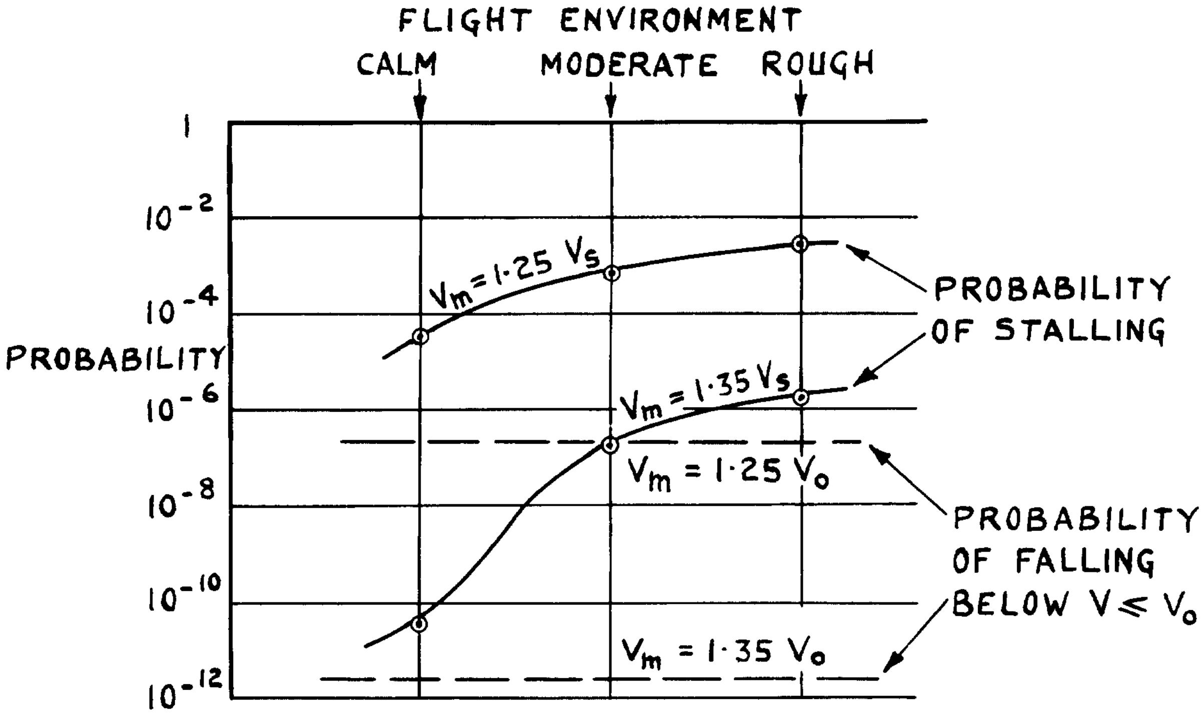

FIG. 8 (b) $\sigma_V = 0{\cdot}05\ V_s\ (V_o)$

FIG. 8 COMPARISON OF PROBABILITY OF STALLING AGAINST PROBABILITY OF DROPPING BELOW V_O FOR A RANGE OF ASSUMED FLIGHT ENVIRONMENTS

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"ZERO RATE OF CLIMB SPEED" AS A LOW SPEED LIMITATION
FOR THE STALL-FEEE AIRCRAFT

An attempt is made to assess the significance to flight safety of a condition at low speeds where drag exceeds the available engine thrust. For the low aspect ratio aircraft and especially with slender wing designs having practically no stall, the "zero rate of climb speed" defined by the condition may constitute the lowest limit of the practical speed range. Methods are suggested to assess the necessary margins to protect aircraft of this general class against the accidental, possibly catastrophic, loss of performance below this speed.

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