C.P. No. 925

C.P. No. 925

MINISTRY OF AVIATION

AERONAUTICAL RESEARCH COUNCIL

CURRENT PAPERS

Experimental Investigation of the Positions of the Leading-Edge Vortices above Slender Delta Wings with Various Rhombic Cross-Sections in Subsonic Conical Flow

by

D. L. I. Kirkpatrick and J. D. Field

LONDON: HER MAJESTY'S STATIONERY OFFICE

1967

PRICE 6*s* 6*d* NET

U.D.C. No. 532.527 : 533.693.3 : 533.694.25 : 533.692.2 : 533.6.011.32/34

C.P. No.925*
March 1966

EXPERIMENTAL INVESTIGATION OF THE POSITIONS OF THE LEADING-EDGE VORTICES ABOVE SLENDER DELTA WINGS WITH VARIOUS RHOMBIC CROSS-SECTIONS IN SUBSONIC CONICAL FLOW

by

D. L. I. Kirkpatrick

and

J. D. Field

SUMMARY

The positions of the centres of the leading-edge vortices above four slender wings have been located at several angles of incidence using a Kiel tube probe, and measured using a mutual inductance method of distance measurement. This method was developed for use in this experiment to provide accurate information about the position of a probe in the flow field by measuring the mutual inductances between a coil on the probe and two coils fixed in the wing. The method enabled changes in the probe's position of 0·002 inch to be detected and measured without touching the probe itself or disturbing the flow field round it, and it is capable of being developed to give even greater accuracy.

The variation with incidence of the positions of the centres of the vortices above four slender wings whose rhombic cross-sections have leading-edge angles of $\pi/6$, $\pi/3$, $\pi/2$ and $2\pi/3$ have been determined and are presented in this report. The results show how the development with incidence of the leading-edge vortices above such wings is affected by the size of the edge angle of the wings' cross-sections.

---

*Replaces R.A.E. Tech. Report No.66068 - A.R.C.28153

CONTENTS

1 INTRODUCTION

A characteristic and dominant feature of the flow field round a slender wing at incidence is the pair of vortices which is formed above such a wing by the rolling-up of vortex sheets originating at the leading edges. Despite many theoretical and experimental studies, the structure, properties and development of these vortices are not yet fully understood and there exists no reliable method of predicting either the development of the vortices above a slender wing or their effect on its characteristics. However, Maskell has produced a similarity theory* which enables the development of the conical flow field round a whole class of slender wings to be predicted, provided that the development of the field round one member of the class has been determined. The theory postulates that close to the leading edge of a slender wing at small incidence the velocity field is dependent in form on the edge angle of the wing's cross-section and on a generalized incidence parameter, and in magnitude on a scaling factor. The scaling factor and the incidence parameter are both functions of the geometrical incidence, the aspect ratio and the cross-sectional shape of the wing, and both can be calculated once the conformal transformation of the wing's cross-section into a circle is known. If the theory is correct, it is possible to relate the characteristic features, such as the variation with incidence of the position of the leading-edge vortex, of the conical flow fields round two wings with the same edge angle but different cross-sectional shapes.

A programme of tests is in progress at the R.A.E. at Farnborough particularly to test the validity of this theory, and also to study more generally the development of the vortices above a slender wing and their effect on its characteristics in subsonic conical flow. To do this, new experimental techniques have had to be evolved to measure accurately the positions of the leading-edge vortices and the normal forces associated with them. The techniques used to measure the normal forces and the results obtained from tests on a series of four slender wings with rhombic cross-sections have been discussed in an earlier report[2]. The present report describes the apparatus and method used to measure the positions of the centres of the leading-edge vortices (sections 2 and 3) and presents in the results of tests on the same series of wings (section 4) whose rhombic cross-sections are shown in Fig.1. Other series of wings with different dihedral angles are being tested, using the techniques described herein and in Ref.2, to determine the validity of the similarity theory, and the results of

---

*The ideas from which this theory was developed were first presented[1] at the 10th International Congress of Applied Mechanics at Stresa in 1960.

these tests will be the subject of further reports. All the wings are designated by the edge angle of their cross-sections and the dihedral of the bisector of the edge angle, e.g. $\frac{\pi}{6}/0$ refers to a wing of edge angle $\frac{\pi}{6}$ and zero dihedral. The wings were made of glasscloth and araldite as described in Ref.2 and were tested at a wind speed of 120 fps in the 4 ft × 3 ft low-turbulence wind-tunnel at Farnborough.

2 LOCATION OF THE LEADING-EDGE VORTEX

2.1 Pitot probe

The structure and properties of the leading-edge vortices formed above a slender wing at incidence have been closely studied by Earnshaw[3] and others, and it has been found that the centres of these vortices are characterised by low total pressure, low static pressure and high velocity. Earnshaw used a Conrad 5-tube yawmeter for his investigations and demonstrated that its readings yielded comprehensive and reliable information about the central region of the vortex. However, because the purpose of this experiment was to locate rather than to study the vortex centre and because the wings used were considerably smaller than Earnshaw's, it was decided that the yawmeter might introduce unacceptably large disturbances into the flow, and that it would be better to use a smaller, simpler probe. An attempt was made to locate the position of the centre of the vortex by using a pitot tube, 1 mm in diameter and aligned parallel to the wing's axis, to search for the position of minimum total pressure. However, it was found that because of the high circumferential velocities close to the vortex centre the values of total pressure measured by the pitot tube were spuriously low and completely obscured the total pressure drop at the vortex centre. It was therefore necessary to use a pitot tube which was insensitive to yaw, i.e. a shrouded pitot tube, whose development is outlined below.

If the angle between the axis of an ordinary pitot tube and the direction of the airflow exceeds a few degrees, the total pressure measured by the tube will be in error by an amount dependent on the ratio of the bore of the tube to its external diameter and on the geometry of its tip[4]. However, the influence of yaw on the total pressure measured can be greatly reduced by fitting to the tube a venturii attachment first proposed by Kiel[5] in 1935. Since then his design has been improved[6,7] and the total pressure measured by the latest types is almost completely insensitive to angles of yaw of up to 60 deg. In this experiment it was not practical to use the best available design because of the difficulties in making a shrouded pitot tube, or Kiel tube, sufficiently small, so a rather simpler design, shown in Fig.2(a), was used. The calibration given in Fig.2(b) showed that the total pressure measured by the tube of this design was virtually unaffected by angles of yaw of up to 35 deg.

The pitot pressures were measured on a sloping alcohol manometer whose inclination was adjusted at each incidence to maximise the manometer reading of the total pressure drop at the vortex centre.

2.2 Traversing apparatus

The traversing apparatus used to move the Kiel tube through the vortex was mounted on the sting bar behind the wing; this is better than mounting it on one of the wind-tunnel walls because, in the latter case, both the apparatus itself and the necessary movement of the probe relative to the mounting would be much larger. Fig.3 shows the traversing apparatus mounted in position behind wing $\frac{\pi}{3}/0$. From the square strut fixed to the sting bar, a long shaft parallel to the wing's centre line extended forward past the mid-chord point of the wing. On the end of this shaft, and fitted perpendicular to it, was a short strut on the outer end of which the Kiel tube was mounted. This strut and the long shaft could be rotated by an electric motor to alter the position of the probe while the wind-tunnel was running; if the desired position could not be attained in this way the wind-tunnel was stopped and the distance of the shaft from the centre line of the wing and the orientation of the square strut were adjusted manually.

The shaft was driven through reduction gearing by a series D.C. electric motor designed to run at 5000 rpm with an input voltage of 24 volts. When the motor was running at this speed the gearing reduced the rotational speed of the shaft to 0·35 rpm. It was found that the motor would operate satisfactorily with reduced input voltage, so for fine adjustment of the Kiel tube's position the input voltage was reduced to give a shaft rotational speed of less than 0·10 rpm corresponding to a probe speed of about 0·04 in/sec. The motor was controlled by a morse key in series with the input voltage supply and it was therefore possible to run the motor for very short periods of time (i.e. fractions of a second) and hence achieve precise adjustment of the pitot probe's position.

The lengths of the shaft and the probe were determined by the necessity that the tip of the Kiel tube probe should be in a plane where the flow was conical and where the scale of the vortex was as large as possible. It had already been established[2] that the flow over the front half of each of the wings was conical* and laminar at the wind speed of 120 fps used for these tests, and hence that the positions of the leading-edge vortices above the front halves of the different wings could usefully be compared. The shaft and the probe were therefore made so that the tip of the probe moved in a plane located 45% of the wing's root chord aft of its apex.

2.3 Vortex-locating technique

The approximate position of the vortex was found by inserting a filament of a feather into the wind-tunnel to provide a starting point from the search for the vortex centre using the Kiel tube probe. The technique used to locate the vortices is described below and is illustrated in Fig.4 which shows a typical set of results. The position of the motor mounting on the square strut was first adjusted so that, when the shaft was rotated, the probe would pass through the estimated position of the vortex. After the speed of the airstream in the wind-tunnel had been adjusted, the shaft was rotated so that the pitot probe passed through the vortex and stopped when the probe was measuring its minimum value of total pressure. This total pressure was recorded and the probe's position measured as described in section 3. Then the motor mounting was moved about 0·01 inch along the square strut so that the searching probe followed a slightly different track when the shaft was rotated, and the same process was repeated. When the values of the minimum total pressure on three or more tracks had been found and their positions measured, the position of the vortex centre could be estimated provided that the tracks passed on different sides of the vortex centre and that the vortex's total pressure contours were not excessively asymmetric. The measured values of the minimum total pressure on each of the tracks were plotted against the coordinates, along and perpendicular to the wing's surface, of the point at which the pressure was measured. On each of the graphs a straight line was drawn between the points corresponding to the largest and smallest values of total pressure and a second line was drawn from the other point perpendicular to the total pressure axis to intersect the first line. The mid-point of this second line has then approximately the same coordinate as the vortex centre. At each incidence this process was used to find the vortex centre's coordinates from the measured positions of the minimum values of total pressure on three or more tracks. It is clear that, if the tracks are sufficiently close together, the position of the vortex centre can be estimated with an accuracy

---

*In earlier tests to investigate the aerodynamic forces associated with conical flow, the force on only the front half of a wing was measured; the front half was mounted on a strain-gauge balance and separated from the rear part by a narrow spanwise gap. During the tests described in this report the gap was sealed with tape.

only slightly less than that of the measurement of the position of the minimum total pressure on the different tracks.

When using a pitot probe to investigate the flow in a vortex, the possibility that the probe disturbs the vortex and consequently yields misleading results cannot entirely be discounted. However, the results of pitot probe investigations have been verified by Cox[8] and Earnshaw[9] using different techniques, neither of which introduce obstruction into the flow. This suggests that only negligible errors are introduced into the results discussed in section 4 below by probe-induced disturbances of the flow in the vortex.

3 MEASUREMENT OF THE PROBE'S POSITION

When using a probe to investigate the flow round a wing, it is important to know the location of the probe relative to the wing, but it is generally difficult to determine this accurately. Hitherto, the deflections of the probe due to the aerodynamic loads on them have often been neglected, or minimised by using stiff wing mountings and traversing apparatus. The latter course may lead to unacceptable disturbance of the flow, while the former may appreciably reduce the accuracy of the results. To avoid both these undesirable courses, it was decided that in this experiment the position of the probe relative to the wing should be measured while the wind-tunnel was running. A method of measuring the probe's position by using the phenomenon of mutual inductance was therefore developed and its more important features are described below.

When the electric current passing through a coil changes, the magnetic field of the coil alters. If another coil is located in the magnetic field so that some of the magnetic lines of force pass through it, a voltage will be induced across its terminals. If the two coils are parallel and remain abreast, then the alternating voltage across the terminals of one of them, induced by a steady alternating current of constant frequency passing through the other, is a function only of the perpendicular distance between the coils. The position of a coil in two-dimensional space can therefore be found by passing an alternating current through it and measuring the alternating voltages thus induced in two other coils whose positions are known.

In the tests described in this report, a coil of copper wire was wound on the Kiel tube probe and two other coils were fixed in the wing as shown in Fig.5 so that the position of the probe could be found by measuring the voltages induced in the two fixed coils. During the development of the measuring system its performance was considerably improved, as described in the Appendix, and it proved possible to detect and measure changes in the position of the probe of as little as 0·002 inch, i.e. 0·0007 of the local semi-span of the wing, when it was operating at its maximum distance of 3 inches from the wing's surface. In the course of the tests on the wings whose cross-sections are shown in Fig.1, it was found by monitoring the voltages in the fixed coils that the aerodynamic deflection of the probe relative to the wing was less than 0·005 inch, i.e. negligibly small. It was therefore not essential to use the mutual inductance method to measure the actual position of the probe so, to avoid the need for a tedious calibration of the mutual inductance of the coils, the mutual inductance method was used only to ascertain that the aerodynamic deflection of the probe relative to the wing was always negligibly small and the probe's position was measured as described below after the wind-tunnel had been stopped. More information on the mutual inductance method of distance measurement is given in the Appendix, which includes a summary of the relevant theory, a description of the coils and circuits used and some comments on the possible improvements and future applications of the method.

Even after the wind-tunnel had been stopped, the accurate measurement of the position of the Kiel tube probe was not completely straightforward because the tip of the probe was very fragile and might easily be damaged if it were touched by a metal measuring device such as a rule or calipers. Consequently a simple measuring technique was developed to avoid the danger of damaging the probe and this technique, which is described below, proved to be safe, quick and accurate. A metal framework supporting a diaphram of tracing paper was set on the wing's surface, perpendicular to it and to the vertical plane of symmetry of the wing, and moved until the tracing paper touched the tip of the Kiel tube as shown in Fig.6. The paper was then pricked with a needle so that the hole corresponded with the centre of the Kiel tube. Throughout the measuring process the mutual inductances of the coils were checked to ensure that the probe's position was not altered by contact with the tracing paper. The framework was then taken out of the wind-tunnel and the position of the hole in the tracing paper was measured as accurately as possible.

A check on the accuracy of this method of locating a point of minimum total pressure and measuring its position is provided by the fact that the positions measured should lie on a straight line through the vortex centre. An error in locating a point or measuring its position is immediately apparent and the errant position can be checked. It is unlikely that the cumulative error in the estimated position of the vortex centre exceeds 0·01 inch i.e. 0·003 of the local semi-span.

4 DISCUSSION OF RESULTS

The positions of the leading-edge vortices above each of the wings were located and measured over a range of incidence whose upper bound was formed by the need to avoid significant vortex interaction or wind-tunnel wall constraint and whose lower bound was reached when the characteristic total pressure drop at the vortex centre was obscured by the total pressure loss in the wing's boundary layer. This lower bound was found to occur at lower incidences for wings with small leading-edge angles than for wings with large ones, e.g. it was possible to define the vortex centre above wing $\frac{\pi}{6}/0$ at an incidence of 3 deg. but not possible to locate that above wing $\frac{2\pi}{3}/0$ below 10 deg. Below this lower bound it is virtually impossible to define the path traced by the vortex, but a close approximation to the path at very low incidences may be obtained by drawing a curve through the leading edge and the vortex centre positions measured at higher incidences.

The positions of the vortex centres above each wing were plotted in Fig.7 relative to the bisector of the edge angle and the line perpendicular to it through the leading-edge. The non-dimensional coordinates, ξ and η, of the position of the vortex centre relative to those axes are the same as those used by Maskell[1] in his similarity theory for leading-edge vortices. Fig.7 shows that, as the edge angle of the cross-section decreases, the path traced by the leading-edge vortex as the wing's incidence is varied tends to become more curved, i.e. farther from the wing's surface at high incidences and more tangential to it at low incidences.

Slender body theory suggests that the positions of the centres of leading-edge vortices above slender wings of different aspect-ratio can best be related by plotting the positions against α/K, where K is the cotangent of the leading-edge sweep angle. The positions of the vortex centres above the wings tested at chosen values of α/K were found by interpolation and plotted in Fig.8. It is clear that as the edge angle is reduced from π while α/K is kept constant the leading-edge vortex moves in a curved path, initially perpendicular to the line bisecting the edge angle and then curving inboard. The vortex attains its maximum distance from the bisecting line when the edge angle approximately equals $\pi/3$.

Unfortunately, among the many recent investigations of the flow field above slender wings, it was not possible to find any results directly comparable with those described above. The wings used for the other tests were either non-conical or the vortex centres' positions were measured in the region of non-conical flow near the trailing edge. However, the results of an investigation by Fink and Taylor[10] can be used to give an approximate indication of the positions of the leading-edge vortices above a slender conical wing whose rhombic cross-section has an edge angle of 4·75 degrees. The wing used for this investigation was of aspect-ratio 0·705; its upper surface was flat and its lower surface had a constant chamfer parallel to the leading-edge. The cross-sectional shape of the wing in the plane where the vortex positions were measured is shown in Fig.9 (section a) but, since most of the wing forward of this plane has a triangular cross-section, it is not unreasonable to assume that the vortex positions measured are very similar to those associated with the flow field round a conical wing of triangular cross-section (section b). Maskell's similarity theory[1] suggests that a change of a few degrees in the anhedral of the edge angle's bisector would not greatly affect the positions of the vortices relative to the ξ, η axes, and therefore the positions of the vortices formed at a given incidence above wings with the cross-sections shown in Fig.9 (sections b and c) are very similar. So, by interpolation from the results plotted in Fig.9, it is interesting to obtain approximate positions of the vortices formed above a wing 0·026 $\pi/0$ and to plot them in Fig.8. The vortex positions thus obtained lie outside the pattern of the vortex positions found by the methods presented in this report, but tend to support the conclusions which can be drawn from it.

5 CONCLUDING REMARKS

An accurate method of measuring the positions of the centres of the leading-edge vortices which are an important feature of the flow round sharp-edged slender wings has been developed. This method has been used to determine the effect of the leading-edge angle of a wing with a rhombic cross-section on the positions of the vortices above it. The results show how the shape of the path traced by the vortex centre as the incidence of a wing is increased depends on the size of the wing's leading-edge angle and how, at a given value of incidence, the vortex moves inboard as the size of the leading-edge angle is reduced.

Appendix

THE MUTUAL INDUCTANCE METHOD OF DISTANCE MEASUREMENT

(1) Mutual inductance

The method of finding the distance between a pair of coils by measuring their mutual inductance, and the ways in which the efficiency of the method can be improved, are best studied by examining the performance of an imperfectly coupled transformer whose equivalent circuit is shown in Fig.10. The equations relating the voltage, current and impedence of the two circuits are

$$\left.\begin{aligned} V_1 &= i_1(r_1 + R_1 + j\omega L_1) - i_2\, j\omega M \\ V_2 &= i_2\, r_2 = -\, i_2(R_2 + j\omega L_2) + i_1\, j\omega M \end{aligned}\right\} \qquad (1)$$

where V_1, r_1 are the open-circuit voltage (at frequency $\omega/2\pi$) and the internal resistance of the oscillator, r_2, V_2 are the internal resistance of the voltmeter and the voltage across it, R_1, R_2, L_1 and L_2 are the resistances and self inductances of the two coils, i_1, i_2 are the currents in the coils and M is their mutual inductance. Hence

$$\frac{V_1}{V_2} = \frac{(r_1 + R_1 + j\omega L_1)(r_2 + R_2 + j\omega L) + \omega^2 M^2}{j\omega M\, r_2} \quad . \qquad (2)$$

To obtain the greatest possible output voltage, and thus the greatest sensitivity, the resistances and reactances of the coils should be chosen to *minimise* V_1/V_2. *If, as is generally the case, a high-input-impedance voltmeter* is used, then r_2 may be considered to be infinite and equation (2) may be written

$$\frac{V_1}{V_2} = \frac{r_1 + R_1 + j\omega L_1}{j\omega M} \quad . \qquad (3)$$

Since M is equal to $k\sqrt{L_1 L_2}$, where k is a coupling factor dependent on the relative position and orientation of the coils, it is obvious that V_1/V_2 can be decreased by increasing k or L_2. Also

$$\left|\frac{V_1}{V_2}\right|^2 = \frac{(r_1 + R_1)^2 + \omega^2 L_1^2}{\omega^2 k^2 L_1 L_2}$$

therefore

$$\frac{d}{dL_1}\left|\frac{V_1}{V_2}\right|^2 = \frac{1}{k^2 L_2}\left\{1 - \left(\frac{r_1 + R_1}{\omega L_1}\right)^2\right\}$$

therefore

$$\frac{d^2}{dL_1^2}\left|\frac{V_1}{V_2}\right|^2 = + \frac{2(r_1 + R_1)^2}{\omega^2 k^2 L_2 L_1^3} \quad . \tag{4}$$

Hence the voltage transfer of the circuit is greatest when $\omega L_1 = r_1 + R_1$ and there is no advantage in making ωL_1 larger than this value.

The results of this simple analysis should not be applied indiscriminately, since it depends on the not unreasonable assumption that r_2 is very large compared with both $|R_2 + j\omega L_2|$ and ωM. A more rigorous and complex analysis was not considered to be justified because the design of the coils and circuits for future experiments is more likely to be determined by the conditions of the experiment and the availability of electrical equipment than by exact optimization of the circuit.

To determine whether the mutual inductance method can be used effectively in any particular experiment, it is useful to estimate the coupling factor k of the coils using the formula

$$k = F f \sqrt{a_1 a_2} \tag{5}$$

where F and f are functions of the distance between the coils and their radii a_1 and a_2. The functions F and f have been calculated and tabulated for a wide range of coil separations and sizes[11].

(2) Coils and circuit

Since the above analysis showed that it was more important to have a high self inductance in the receiving (secondary circuit) coils, it was decided to put the emitting (primary circuit) coil on the Kiel tube probe (see Fig.11) rather than inside the wing because the cross-sectional area and, consequently the self inductance of this coil is restricted by the need to avoid excessive disturbance of the flow. The emitting coil was formed by winding about 500 turns of 38 swg copper wire round the probe; the choice of this gauge of wire was determined by the necessity of having a sufficient number of turns to give adequate coupling between the emitting coil on the probe and the receiving coils in the wing, without having the wire so thin that heating effects become significant or so thick that the coil becomes large enough to disturb the flow at the tip of the probe. The Kiel tube had been made of a silver steel of low permeability before the mutual inductance method was proposed and it was found that the performance of the emitting coil was considerably improved by increasing the effective permeability of the core by wrapping some steel foil 0·001 inch thick round the Kiel tube before winding the coil on it. In the course of the experiment it was found necessary to ensure a good connection between the Kiel tube probe and earth because the intermittent connection across the bearings of the shaft led to an irregular variation of the voltage induced in a receiving coil as the probe was moved.

The two receiving coils fixed inside the wing needed to have a high impedance yet be small enough to fit inside it; two suitable coils were obtained by dismantling a discarded relay. These coils were mounted on a fibreglass framework which was designed to fit on to blocks of fibreglass fixed in each of the wings. During the tests it was found that the performance of the system was improved if the alternating induced voltages in the receiving coils were measured successively and the coil whose voltage was not being measured was short-circuited.

To measure accurately the voltage induced across the receiving coils requires that the signal be amplified and rectified, preferably in that order. It was found that a standard A.C. valve voltmeter was more suitable than a digital direct-current voltmeter with a separate rectifier in series. To increase the accuracy with which small changes in the voltages across the receiving coils could be measured, this voltage was balanced against a standard voltage supplied by an appropriate number of batteries, and the difference between the voltages measured on a high-sensitivity scale of the voltmeter. This type of voltmeter is constructed so that it is subject to error if either the induced voltage or the balancing voltage exceed a certain critical value; so it was necessary, when the emitting and receiving coils were close together, to insert a resistance in series with the receiving coils. A multichange switch in the circuit served simultaneously to connect one of the receiving coils to the voltmeter, to connect the appropriate balancing voltage from the batteries and to short-circuit the other receiving coil. The output voltage of the oscillator was set at a frequency of 1·84 kc/s and was monitored throughout the experiment.

(3) Improvement and future application

While the experimental accuracy and operating range of the present circuit and coils are adequate for the location of vortices, there are several ways in which the performance of the system can be improved if necessary. A significant increase in the operating efficiency can be achieved if, for example, capacitors of capacity $C = \frac{L}{R^2 + \omega^2 L^2}$ are connected in parallel across both the primary and secondary coils. Using more sophisticated oscillator and voltage measuring equipment, the accuracy which can be achieved is limited only by the experimental conditions.

Besides measuring the position of pressure probes etc. in conditions where the probe position needs to be measured with unusual accuracy and aerodynamic deflections may be appreciable, there are numerous other cases in which the mutual inductance method of distance measurement offers a distinct improvement over methods currently in use. The method can be used to measure the gap between components of a model under test when the exact width of the gap has a critical influence on the flow, e.g. in slot and flap configurations yielding high-lift, to detect extremely small distortion of models during experiments, or to provide accurate information on the position of an oscillating body. The mutual inductance method has the inherent disadvantage that its performance is significantly reduced in the presence of any conducting material due to eddy current generation. This limits its usefulness, in the field of wind-tunnel model testing, to cases where the models are made of an insulating material such as wood or fibreglass. Furthermore the accuracy of the method can be reduced by fluctuations in the power of neighbouring electrical equipment which must be switched off while the induced voltages are being measured. Despite these disadvantages, the essential simplicity and accuracy of the mutual inductance method of distance measurement may well entail its future application in many fields of scientific investigations.

Table 1

Vortex positions (measured)

Wing	K	α_{unc}	α	Distance perpendicular to surface	Distance along surface	ξ	η
$\frac{\pi}{6}/0$	0·2	3	3·1	0·144	0·540	0·1576	0·0908
		5	5·1	0·295	0·755	0·2126	0·1562
		10	10·3	0·634	1·038	0·2731	0·2868
		15	15·5	0·917	1·233	0·3105	0·3923
$\frac{\pi}{3}/0$	0·2	5	5·1	0·236	0·688	0·155	0·178
		10	10·3	0·495	1·069	0·220	0·312
		15	15·4	0·752	1·349	0·257	0·429
$\frac{\pi}{2}/0$	0·2	5	5·1	0·165	0·482	0·0727	0·1486
		10	10·2	0·306	0·868	0·1288	0·2692
		15	15·3	0·496	1·227	0·1677	0·3951
$\frac{2\pi}{3}/0$	0·115	10	10·1	0·169	0·673	0·107	0·375
		15	15·2	0·225	1·010	0·174	0·554

Table 2

Variation of vortex positions at constant values of α/k

Wing	K	$\frac{\alpha}{K}$	ξ	η
$\frac{\pi}{6}/0$	0·2	0·25	0·147	0·085
		0·50	0·222	0·172
		0·75	0·257	0·242
		1·00	0·280	0·308
		1·25	0·302	0·268
$\frac{\pi}{3}/0$	0·2	0·25	0·103	0·106
		0·50	0·164	0·195
		0·75	0·202	0·271
		1·00	0·229	0·339
		1·25	0·250	0·404
$\frac{\pi}{2}/0$	0·2	0·25	0·041	0·089
		0·50	0·080	0·161
		0·75	0·114	0·232
		1·00	0·139	0·30
		1·25	0·160	0·369
$\frac{2\pi}{3}/0$	0·115	0·25	0·019	0·061
		0·50	0·037	0·120
		0·75	0·052	0·180
		1·00	0·071	0·243
		1·25	0·090	0·305
0·026 π/0 (Ref.10)	0·176	0·25	0·215	0·071
		0·50	0·27	0·132
		0·75	0·294	0·183
		1·00	0·311	0·235
		1·25	0·325	0·285

SYMBOLS

A	aspect ratio
a, a'	radii of coils
C	capacity
F, f	coupling factors
H	total pressure
i	current
K	cot Λ
L	self inductance
M	mutual inductance
q	free stream dynamic pressure
R	resistance
r_1	oscillators internal resistance
r_2	voltmeters internal resistance
s	semi-span
V_1	open-circuit oscillator voltage
V_2	voltage measured by voltmeter
y	spanwise distance from wings plane of symmetry
z	vertical distance perpendicular to the wings flat upper surface
α	incidence of wing's axis of symmetry
Λ	leading edge sweep angle
ξ, η	coordinates, non-dimensionalized with respect to the local semi-span, measured parallel to and perpendicular to the edge angle bisector
ω	2π (oscillator frequency)

REFERENCES

No.	Author	Title, etc
1	E. C. Maskell	Similarity laws governing the initial growth of leading-edge vortex sheets in conical flow past sharp-edged slender bodies. Xth International Congress of Applied Mechanics, Stresa, 1960
2	D. L. I. Kirkpatrick	Investigation of the normal force characteristics of slender delta wings of various rhombic cross-sections in subsonic conical flow. A.R.C. C.P.922
3	P. B. Earnshaw	An experimental investigation of the structure of a leading-edge vortex. A.R.C. R. & M. 3281. 1962
4	R. C. Pankhurst D. W. Holder	Wind-tunnel technique C.4, p 180. Pitman, 1952
5	G. Kiel	Total head meter with small sensitivity to yaw. N.A.C.A. T.M.775. 1935
6	M. Moffat	Report on Kiel probes. Report No. PWA-576, Pratt and Whitney Aircraft, October 17th, 1945
7	W. R. Russel W. Gracey W. Letko P. G. Fournier	Wind-tunnel investigation of six shielded total-pressure tubes at high angles of attack. Subsonic speeds. N.A.C.A. TN 2530. November 1951
8	A. P. Cox	Measurements of the velocity at the vortex centre on an A.R.1 delta wing by means of smoke observations. A.R.C. C.P. No. 511 1960
9	P. B. Earnshaw	The electric-spark technique applied to the measurement of velocity in a leading-edge vortex. R.A.E. TN Aero 2947.A.R.C.25894 February 1964
10	P. T. Fink J. Taylor	Some low speed experiments with 20 degree delta wings. A.R.C.R. & M. 3489 September 1955
11	F. W. Grover	Inductance calculations. D. van Nostrand Co. Inc., N.Y. 1946

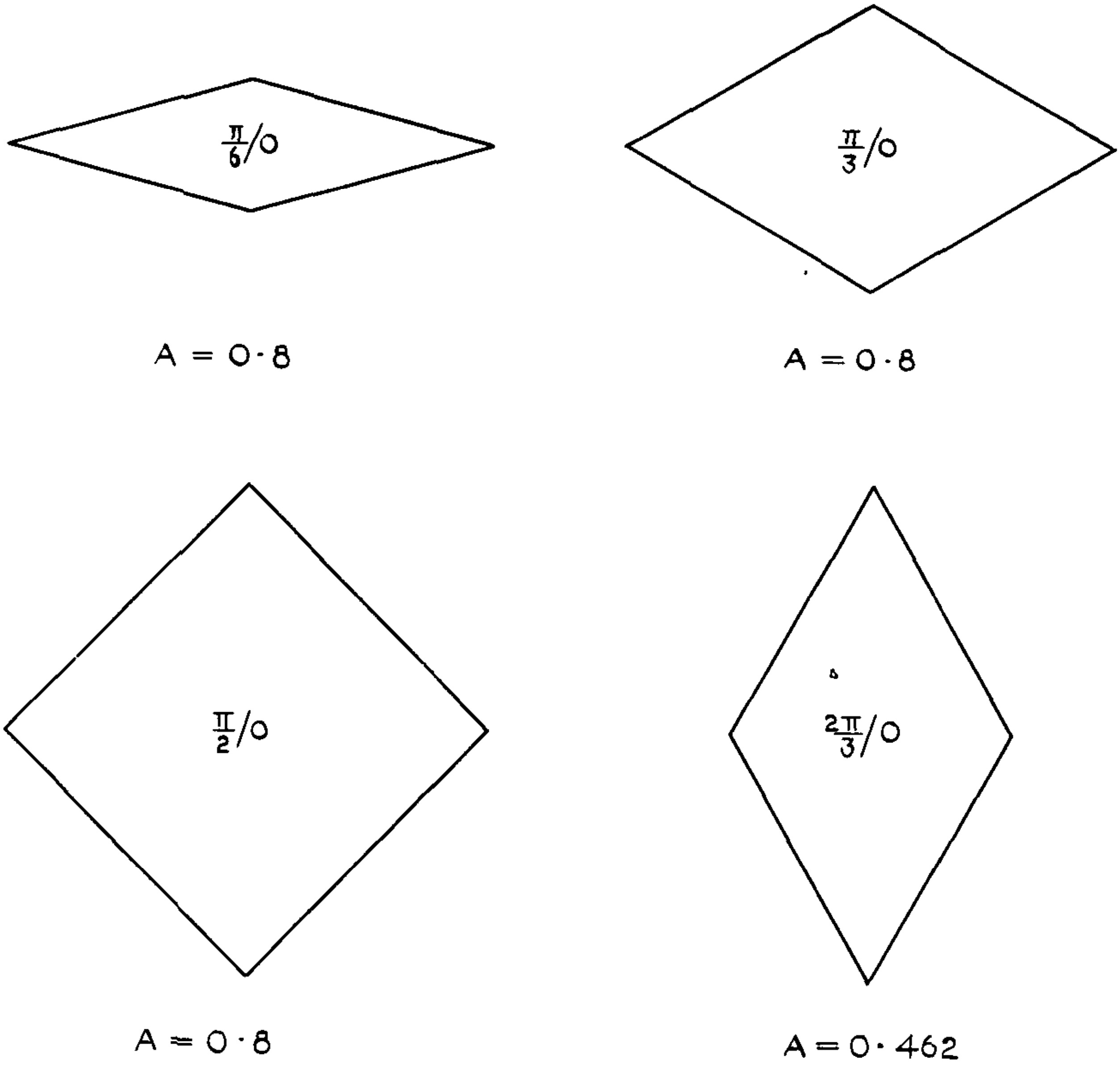
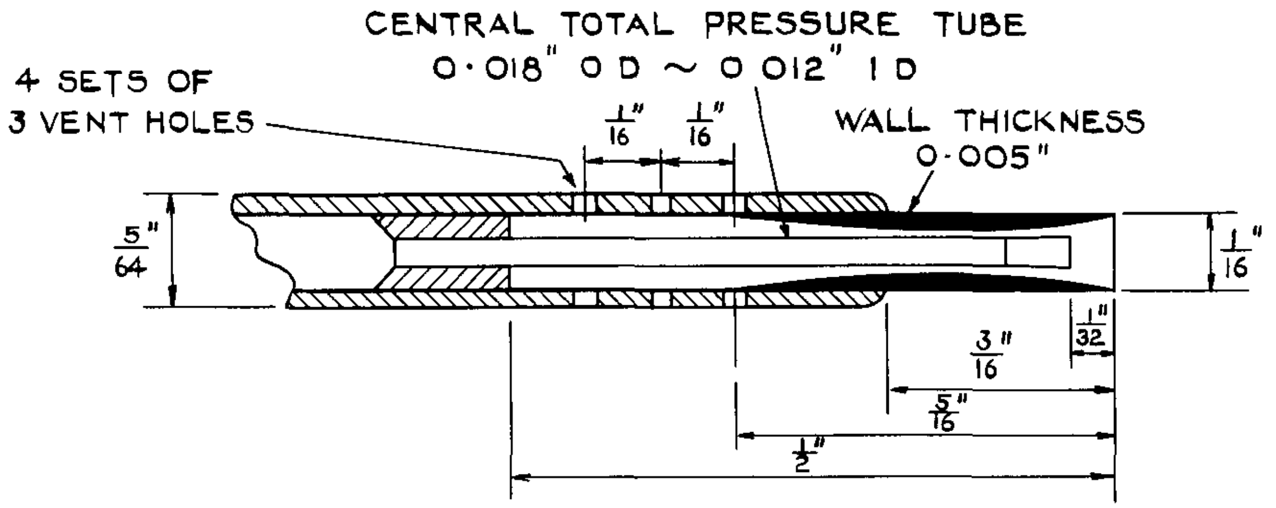


FIG. 1 CROSS—SECTIONAL SHAPES OF WINGS TESTED



VENT AREA ≏ INLET AREA

FIG.2 (a) CROSS—SECTION OF THE KIEL TUBE

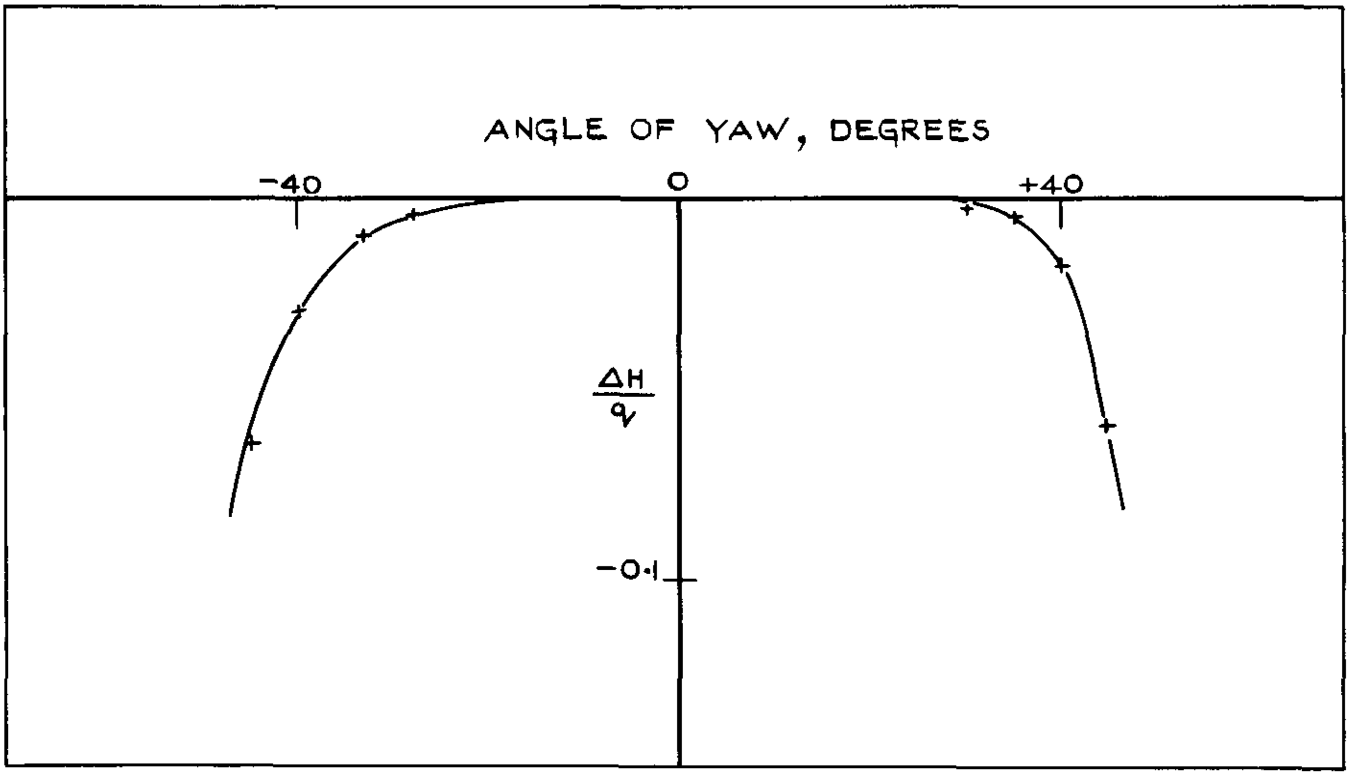


FIG. 2 (b) CALIBRATION CURVE FOR THE KIEL TUBE

FIG. 2 KIEL TUBE DESIGN AND PERFORMANCE

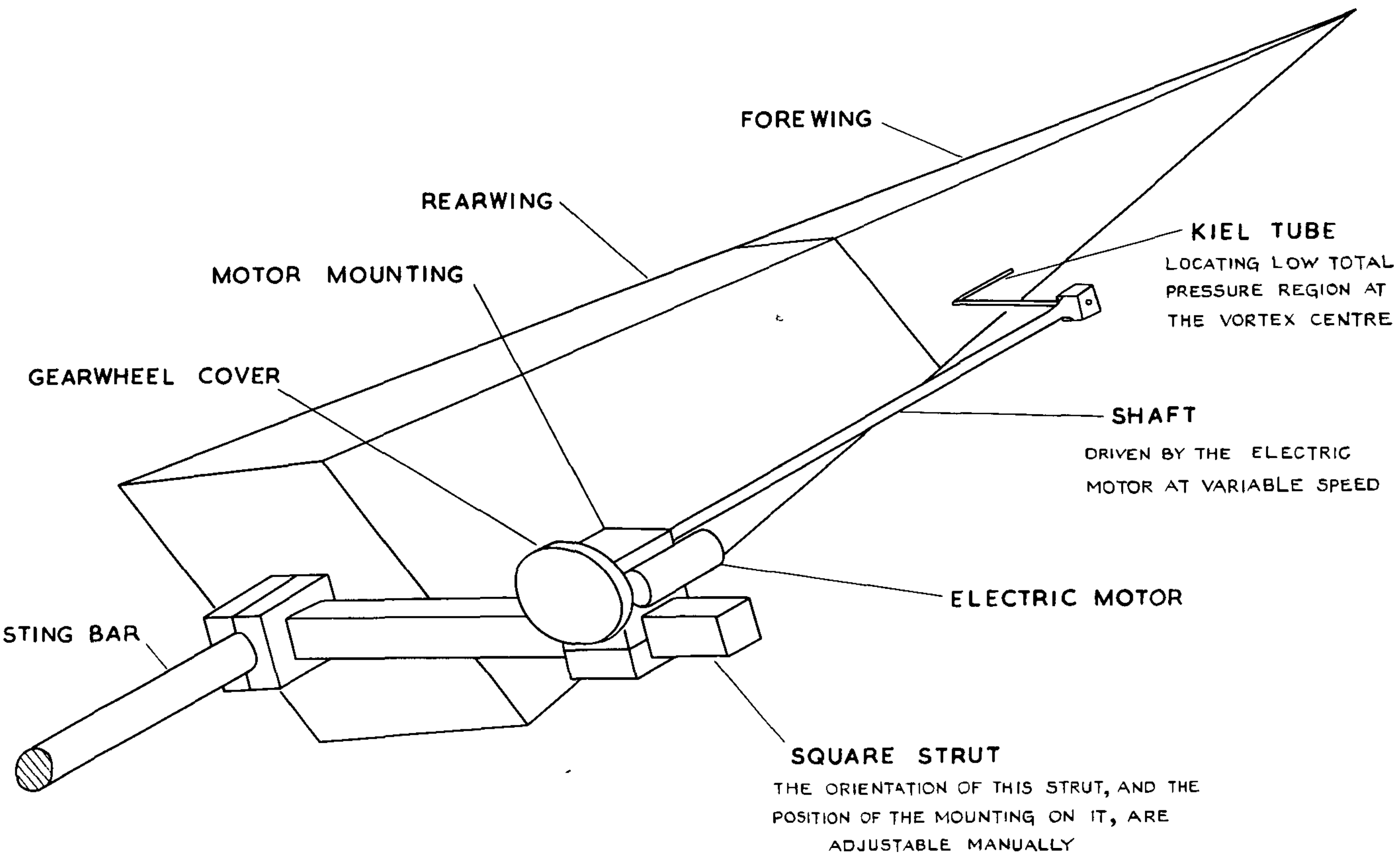


FIG. 3 VORTEX LOCATING APPARATUS

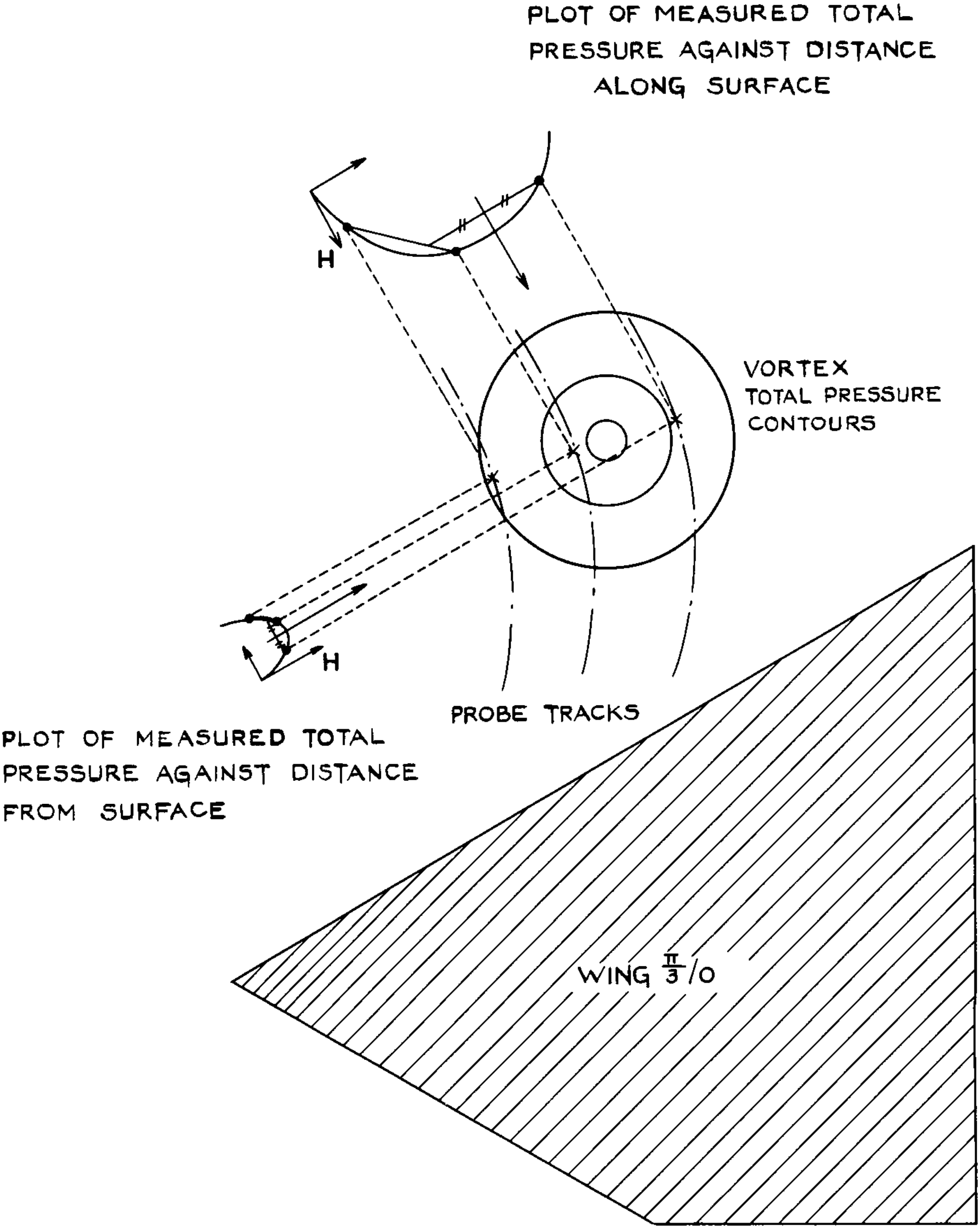


FIG. 4 THE ESTIMATION OF THE POSITION OF THE VORTEX CENTRE FROM TOTAL PRESSURE MEASUREMENTS

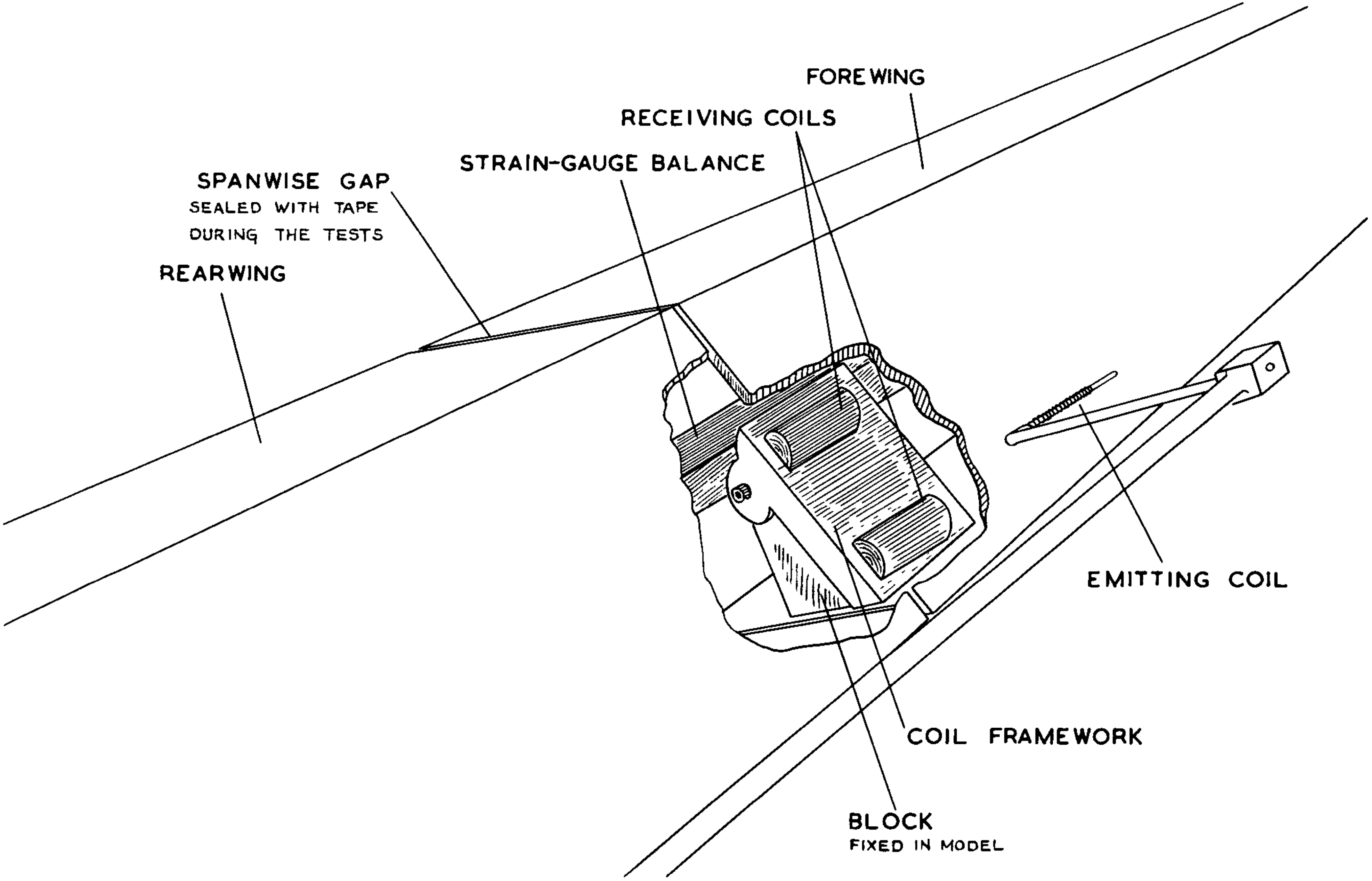


FIG. 5 POSITION MEASURING APPARATUS

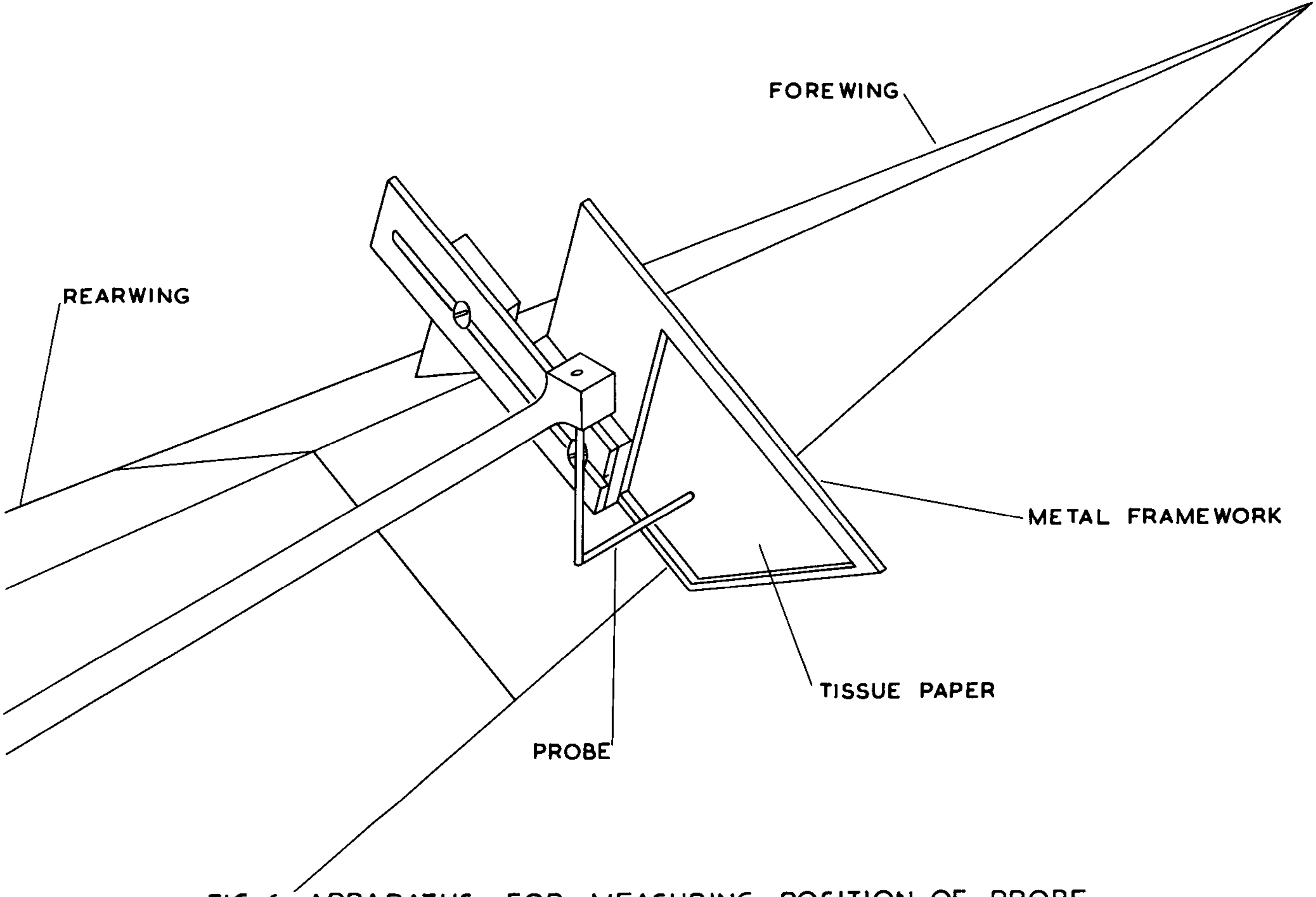


FIG. 6 APPARATUS FOR MEASURING POSITION OF PROBE

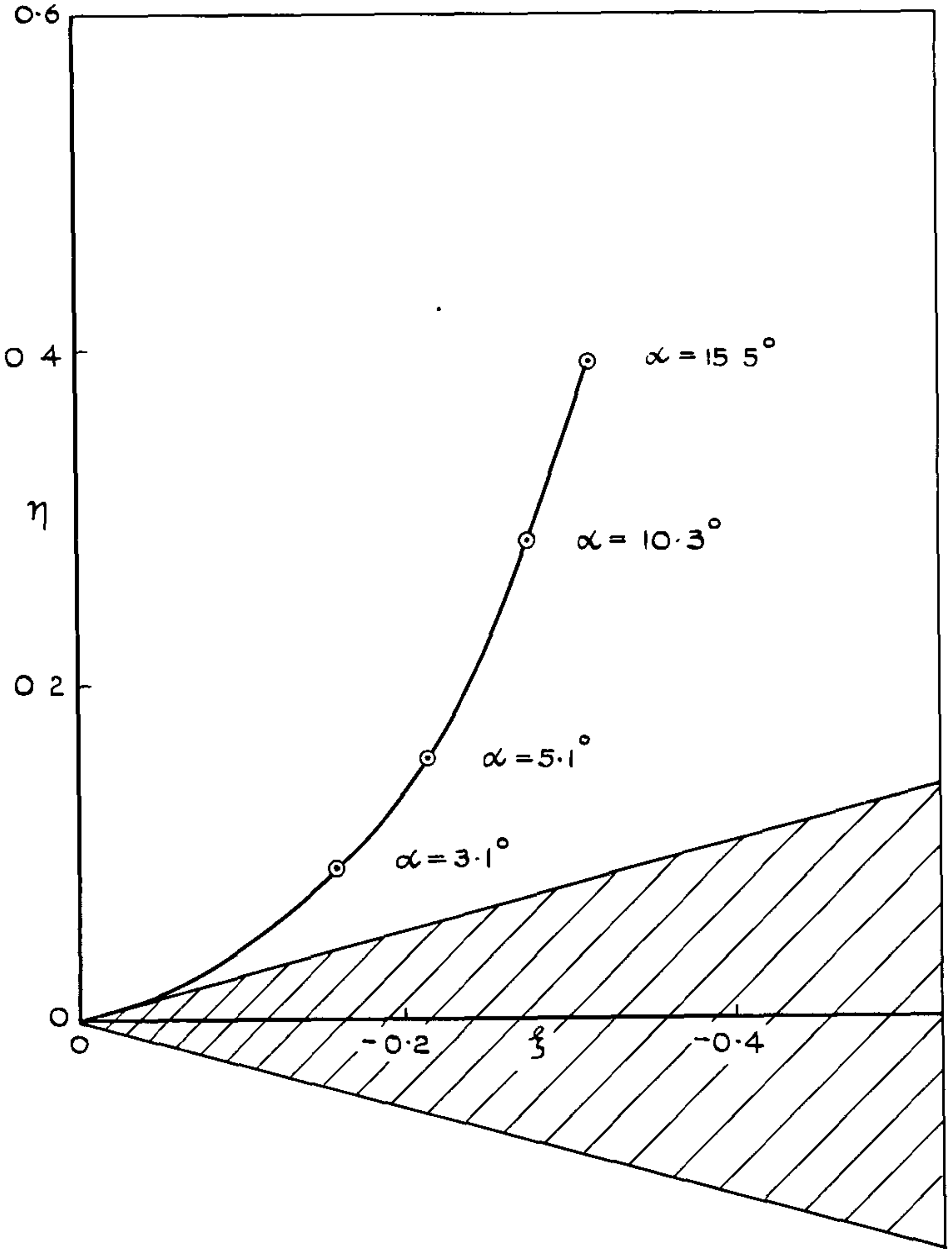


FIG 7(a) POSITIONS OF VORTEX CENTRE ABOVE WING $\frac{\pi}{6}/0$

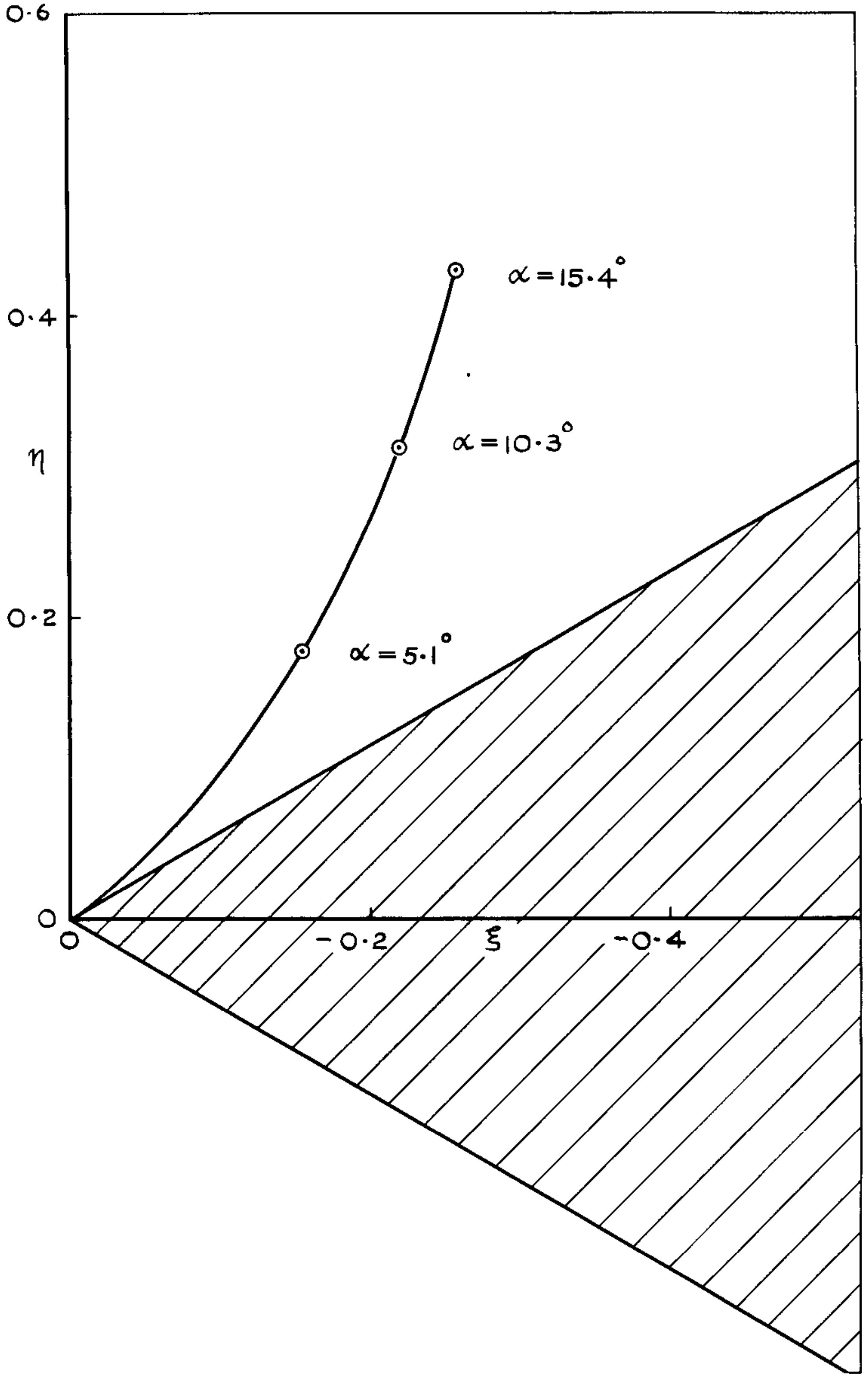


FIG. 7 (b) POSITIONS OF VORTEX CENTRES ABOVE WING $\frac{\pi}{3}/0$

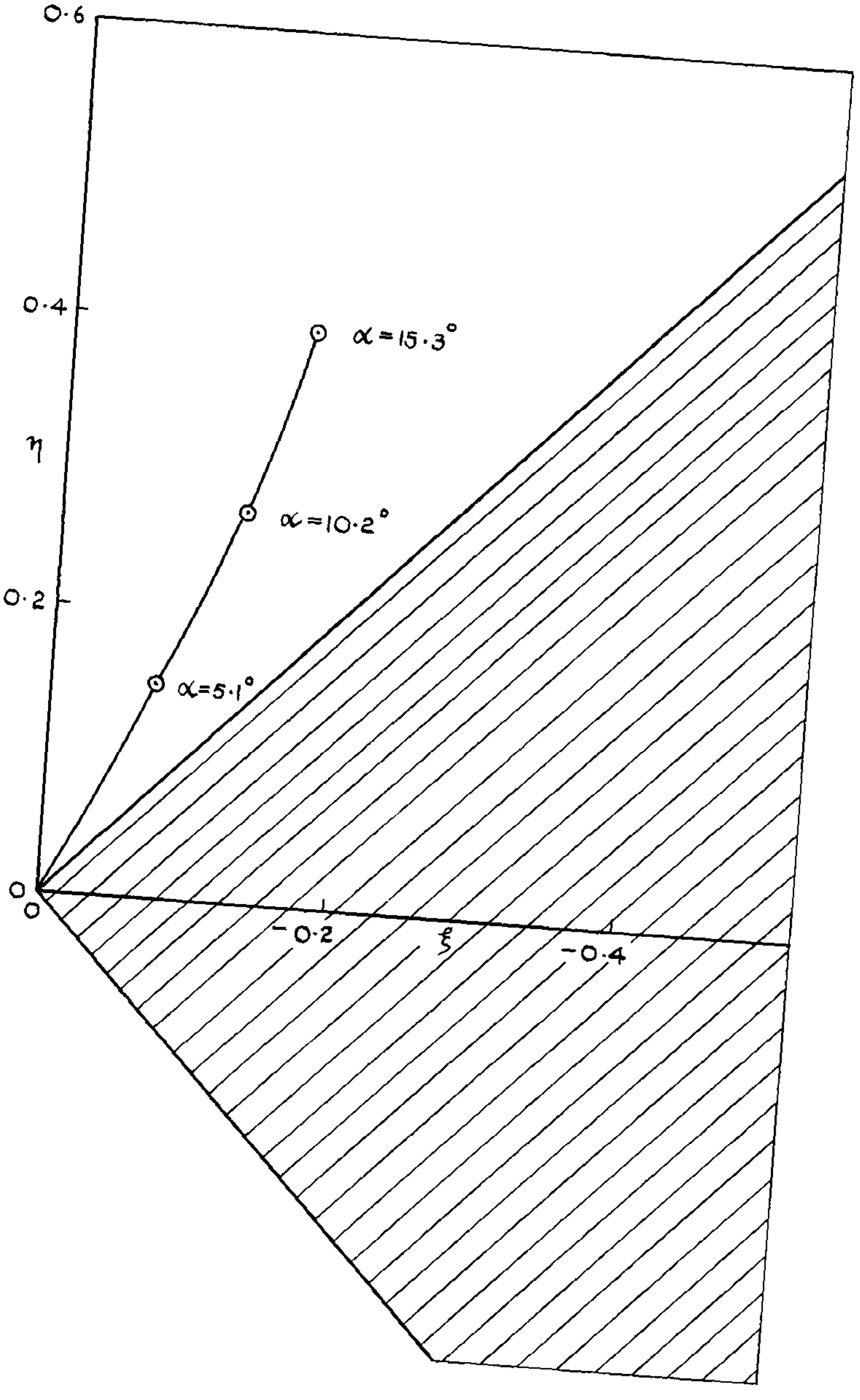


FIG 7(c) POSITIONS OF VORTEX CENTRES ABOVE WING $\frac{\pi}{2}/0$

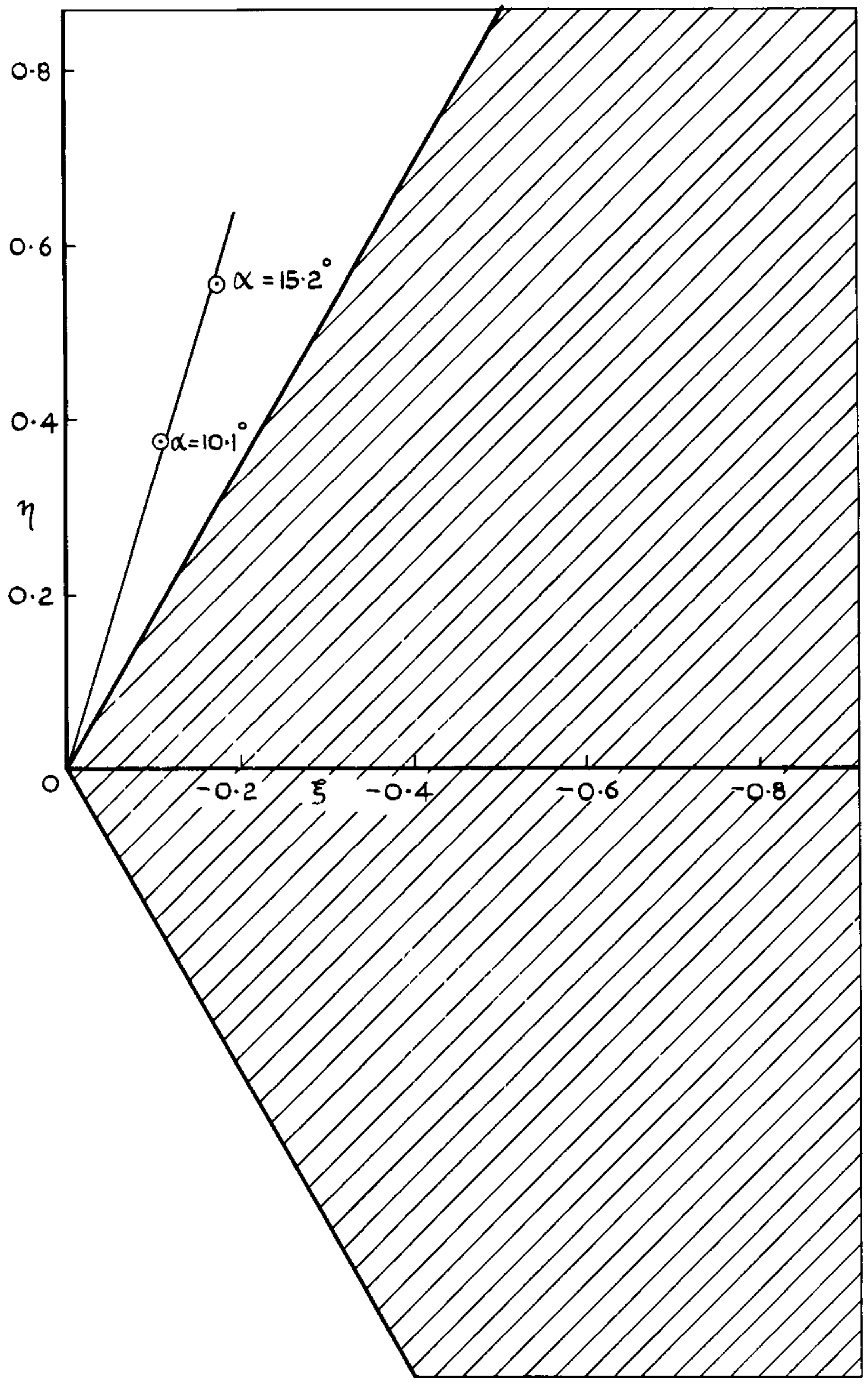


FIG. 7(d) POSITIONS OF VORTEX CENTRE ABOVE WING $\frac{2\pi}{3}/0$

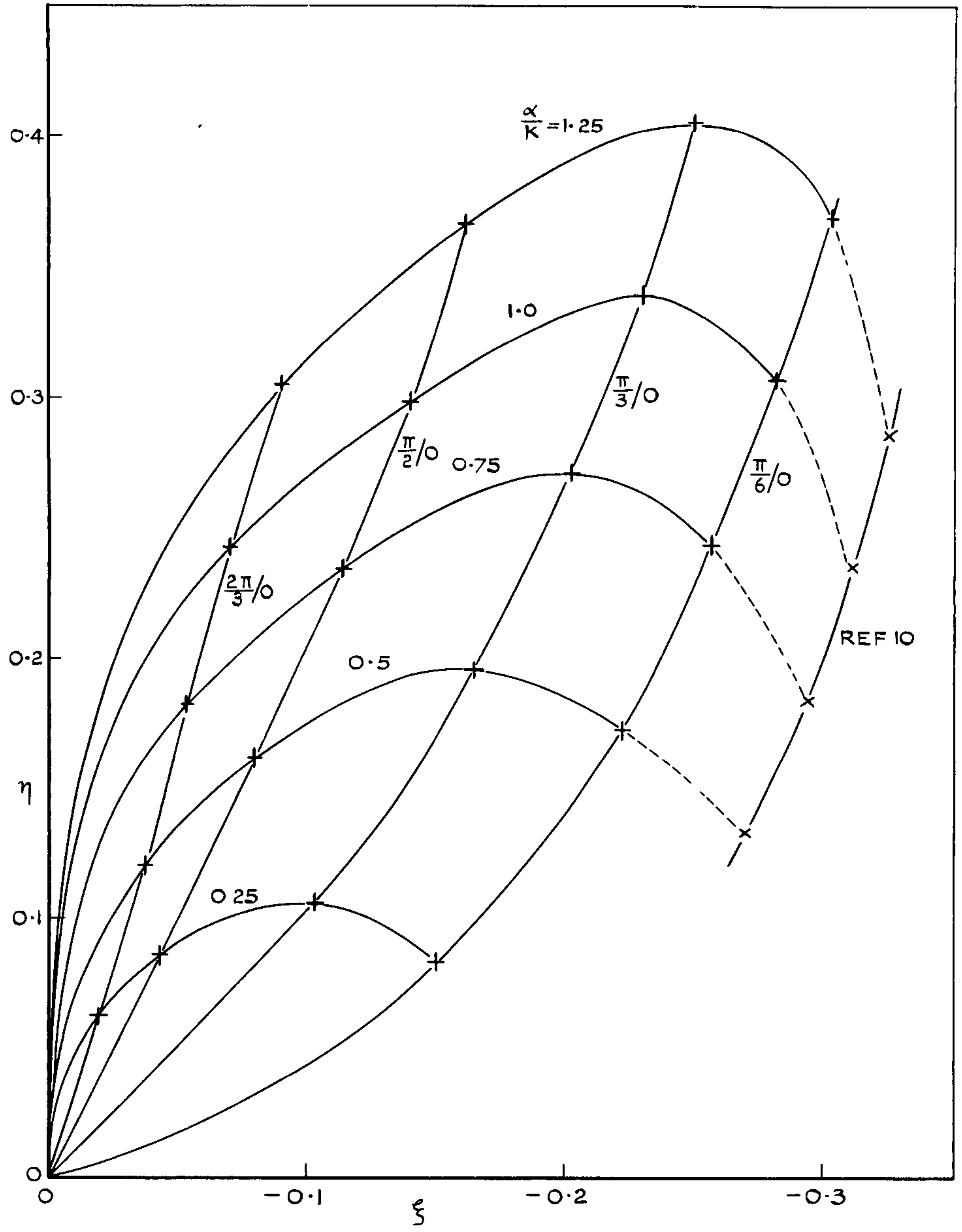
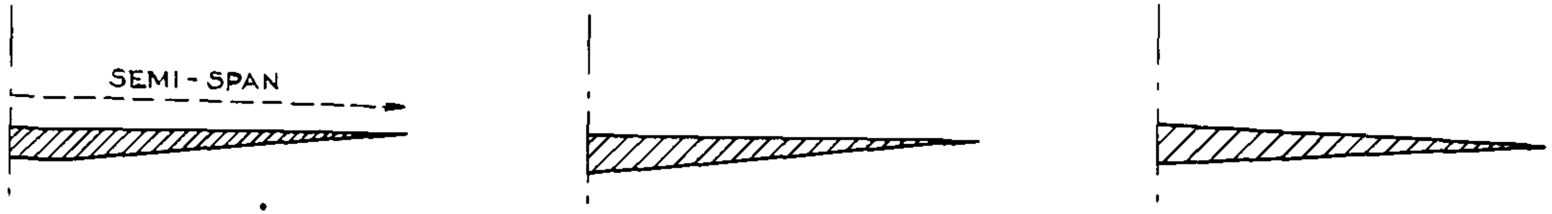


FIG. 8 POSITIONS OF THE VORTEX CENTRE AT CONSTANT α/κ



CROSS-SECTION a

CROSS-SECTION b

CROSS-SECTION c

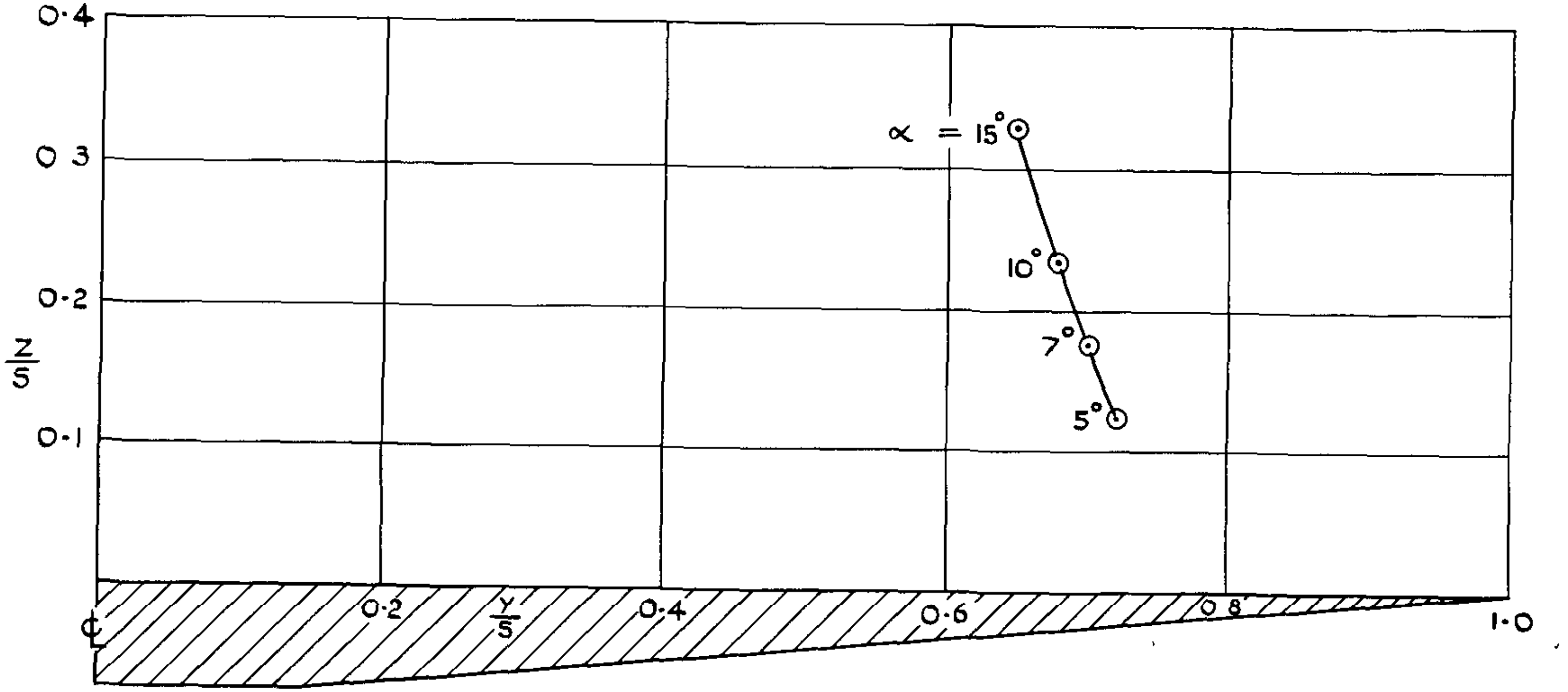


FIG. 9 VARIATION OF VORTEX POSITION WITH INCIDENCE (REF. 10)

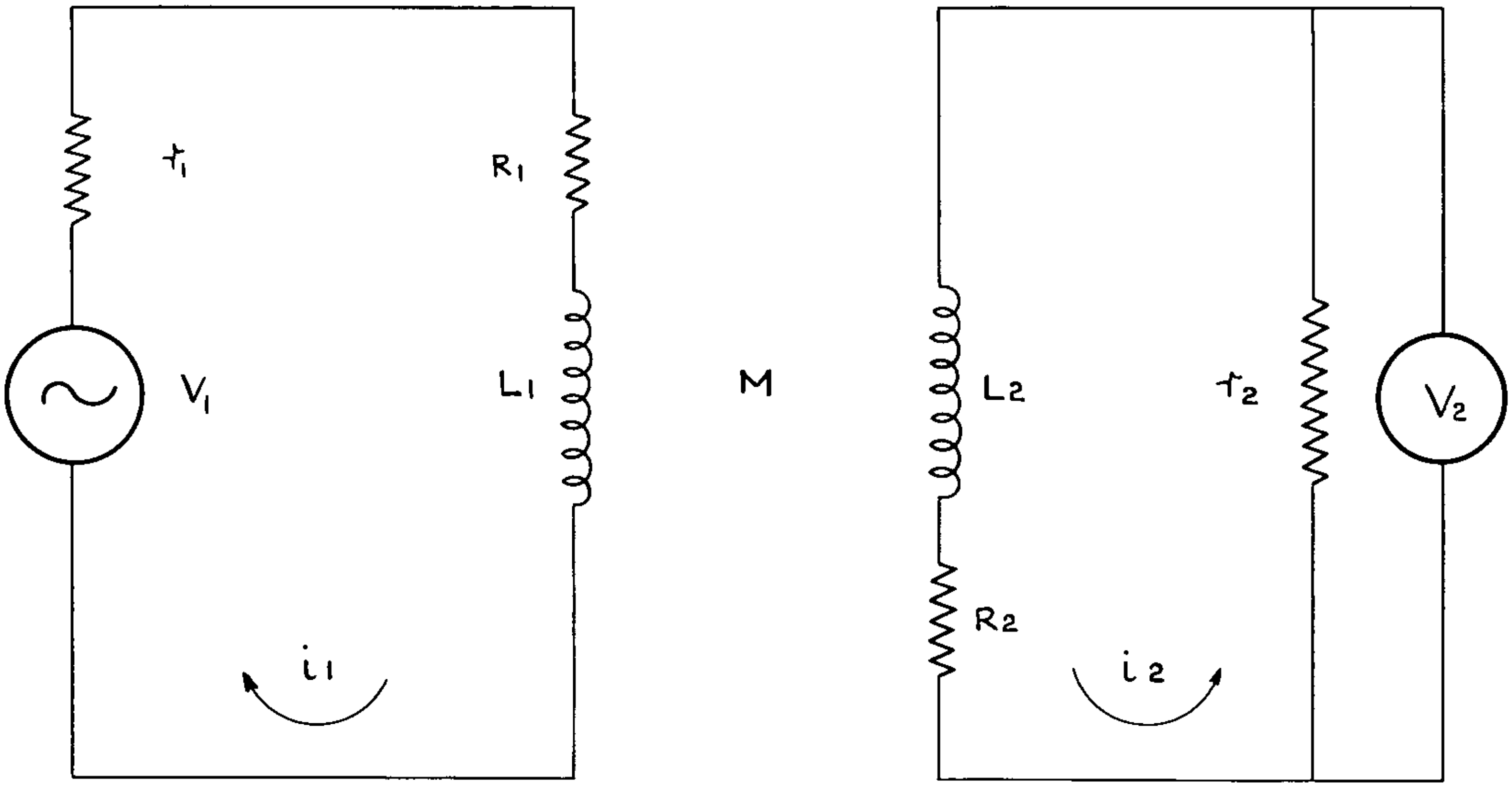


FIG.10 EQIVALENT CIRCUIT OF POSITION MEASURING APPARATUS

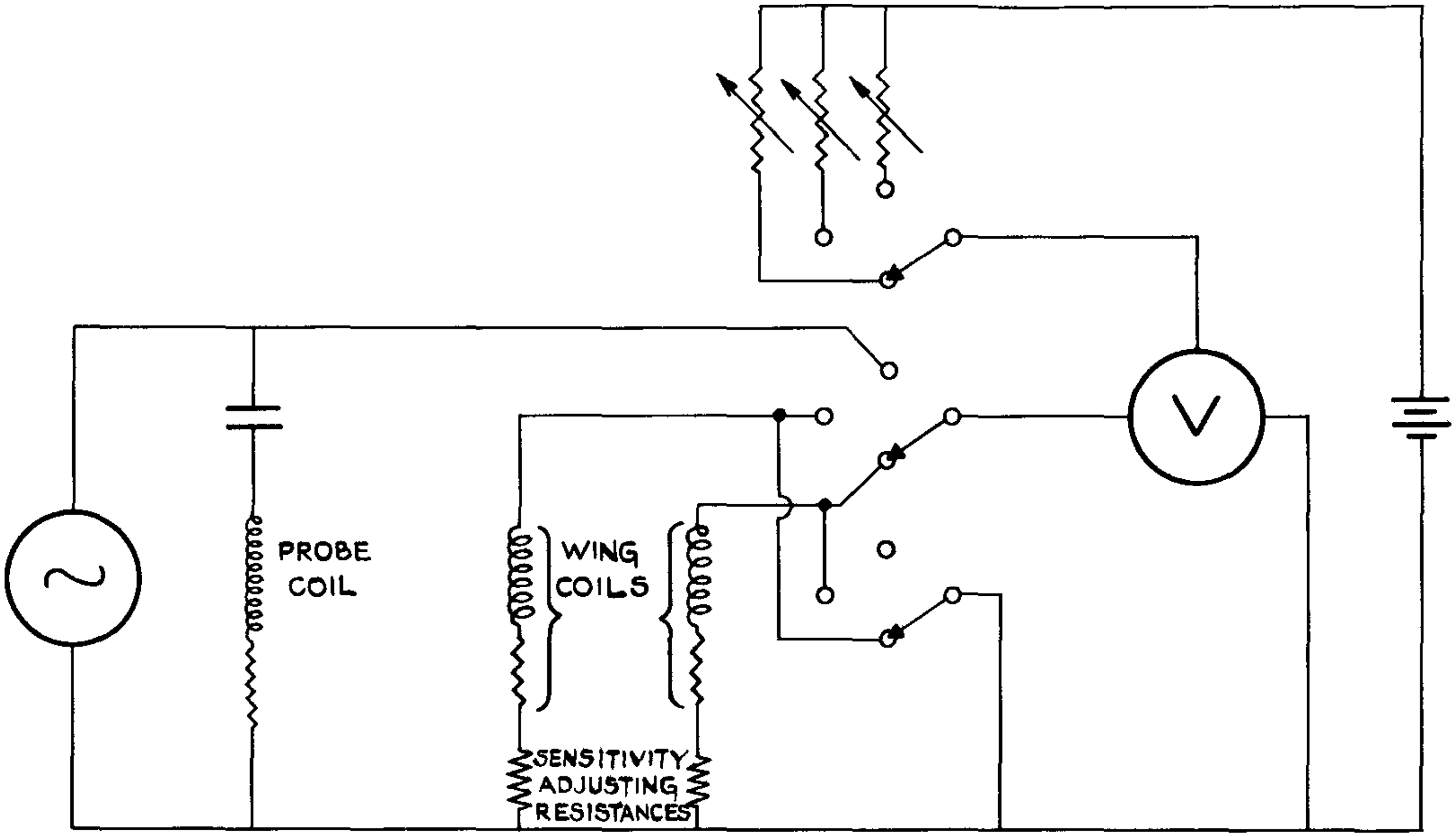


FIG.11 CIRCUIT DIAGRAM OF POSITION MEASURING APPARATUS

Printed in England for Her Majesty's Stationery Office by
the Royal Aircraft Establishment, Farnborough, Dd.125875 K.4

A.R.C. C.P. No.925
March 1966 532.527 :
Kirkpatrick, D. L. I. 533.693.3 :
Field, J. D. 533.694.25 :
EXPERIMENTAL INVESTIGATION OF THE POSITIONS OF THE 533.692.2 :
LEADING-EDGE VORTICES ABOVE SLENDER DELTA WINGS WITH 533.6.011.32/34
VARIOUS RHOMBIC CROSS-SECTIONS IN SUBSONIC CONICAL FLOW

The positions of the centres of the leading-edge vortices above four slender wings have been located at several angles of incidence using a Kiel tube probe, and measured using a mutual inductance method of distance measurement. This method was developed for use in this experiment to provide accurate information about the position of a probe in the flow field by measuring the mutual inductances between a coil on the probe and two coils fixed in the wing. The method enabled changes in the probe's position of 0.002 inch to be detected and measured without touching the

(Over)

A.R.C. C.P. No.925
March 1966 532.527 :
Kirkpatrick, D. L. I. 533.693.3 :
Field, J. D. 533.694.25 :
EXPERIMENTAL INVESTIGATION OF THE POSITIONS OF THE 533.692.2 :
LEADING-EDGE VORTICES ABOVE SLENDER DELTA WINGS WITH 533.6.011.32/34
VARIOUS RHOMBIC CROSS-SECTIONS IN SUBSONIC CONICAL FLOW

The positions of the centres of the leading-edge vortices above four slender wings have been located at several angles of incidence using a Kiel tube probe, and measured using a mutual inductance method of distance measurement. This method was developed for use in this experiment to provide accurate information about the position of a probe in the flow field by measuring the mutual inductances between a coil on the probe and two coils fixed in the wing. The method enabled changes in the probe's position of 0.002 inch to be detected and measured without touching the

(Over)

A.R.C. C.P. No.925
March 1966 532.527 :
Kirkpatrick, D. L. I. 533.693.3 :
Field, J. D. 533.694.25 :
EXPERIMENTAL INVESTIGATION OF THE POSITIONS OF THE 533.692.2 :
LEADING-EDGE VORTICES ABOVE SLENDER DELTA WINGS WITH 533.6.011.32/34
VARIOUS RHOMBIC CROSS-SECTIONS IN SUBSONIC CONICAL FLOW

The positions of the centres of the leading-edge vortices above four slender wings have been located at several angles of incidence using a Kiel tube probe, and measured using a mutual inductance method of distance measurement. This method was developed for use in this experiment to provide accurate information about the position of a probe in the flow field by measuring the mutual inductances between a coil on the probe and two coils fixed in the wing. The method enabled changes in the probe's position of 0.002 inch to be detected and measured without touching the

(Over)

DETACHABLE ABSTRACT CARDS

probe itself or disturbing the flow field round it, and it is capable of being developed to give even greater accuracy.

The variation with incidence of the positions of the centres of the vortices above four slender wings whose rhombic cross-sections have leading-edge angles of $\pi/6$, $\pi/3$, $\pi/2$ and $2\pi/3$ have been determined and are presented in this report. The results show how the development with incidence of the leading-edge vortices above such wings is affected by the size of the edge angle of the wings' cross-sections.

probe itself or disturbing the flow field round it, and it is capable of being developed to give even greater accuracy.

The variation with incidence of the positions of the centres of the vortices above four slender wings whose rhombic cross-sections have leading-edge angles of $\pi/6$, $\pi/3$, $\pi/2$ and $2\pi/3$ have been determined and are presented in this report. The results show how the development with incidence of the leading-edge vortices above such wings is affected by the size of the edge angle of the wings' cross-sections.

probe itself or disturbing the flow field round it, and it is capable of being developed to give even greater accuracy.

The variation with incidence of the positions of the centres of the vortices above four slender wings whose rhombic cross-sections have leading-edge angles of $\pi/6$, $\pi/3$, $\pi/2$ and $2\pi/3$ have been determined and are presented in this report. The results show how the development with incidence of the leading-edge vortices above such wings is affected by the size of the edge angle of the wings' cross-sections.

C.P. No. 925

Published by
HER MAJESTY'S STATIONERY OFFICE
To be purchased from
49 High Holborn, London w.c.1
423 Oxford Street, London w.1
13A Castle Street, Edinburgh 2
109 St. Mary Street, Cardiff
Brazennose Street, Manchester 2
50 Fairfax Street, Bristol 1
35 Smallbrook, Ringway, Birmingham 5
80 Chichester Street, Belfast 1
or through any bookseller

C.P. No. 925

S.O. CODE No. 23-9017-25