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A Survey of the Mechanics of Uniaxial Creep Deformation of Metals

By

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S u m m a r y

The time hardening, life fraction and strain hardening hypotheses variously proposed to describe uniaxial creep deformation are considered and compared with experimental evidence. It is concluded that the strain hardening hypothesis is the most nearly correct, the experimental strains usually being within 50% of those predicted.

Further, rheological and modified strain hardening hypotheses, proposed to overcome the limitations associated with strain hardening, are briefly surveyed and it is concluded that the latter is at present the most likely to be successful.

*Replaces Bristol College of Science and Technology Report No.84 - A.R.C. 28058

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NOTATION

σ σ'	Axial stress; equivalent axial stress
σ	Flow stress
χ	Function of stress
T	Absolute temperature
T_m	Melting temperature of material
η	Function of temperature
t; t_e	Time, effective time
$\phi = \eta t$	Pseudo-time
$\phi_e = \eta t_e$	Effective pseudo time
τ	Retardation time
ε	Total axial strain
ε_e	Elastic strain
ε_c	Creep strain
ε_a	Anelastic strain
ε_p $\varepsilon_{p'}$	Plastic strain; equivalent plastic strain
u	Function of creep strain
$J(t)$	Anelastic response for unit stress
ζ	Memory function
q	Material structure parameter
C; β; κ	Constants in the basic creep equation
a; b	Constants

1.0 INTRODUCTION

This survey of the mechanics of creep deformation avoids explicitly questions of the mechanics of fracture and of deformation under multiaxial stressing. Further as technological interest focuses primarily, for metals, on complex polycrystalline materials, which are likely to defy indefinitely anything but phenomenological description on a macroscopic scale, the discussion of possible microscopic mechanisms(Ref. 1 to 7) ,is restricted to the provision of conceptual models. The principal concern in the discussion is with the 'correctness' of the theories judged by their universal application to metals and economy of hypothesis, rather than by their analytical convenience. To this end the extensive literature is briefly surveyed and the principal theories are critically compared.

An analytic model of creep behaviour is necessary for the anticipation of deformation under varied,periodic and random load/ temperature environments and also for the anticipation of the behaviour in cases where the loading is a function of the deformation response. Further a reasonable analytic model is necessary in the formulation of suitable laboratory test programmes; Conrad (Ref. 2a) gives a survey of the conventional and differential tests variously used in the investigations leading to the conclusions surveyed here.

In several cases,for convenience,reference has been made to the surveys of standard texts rather than to the original work, although the original author is acknowledged.

2.0 CONSTANT LOAD/TEMPERATURE CREEP

Freudenthal (Ref. 6) points out that inelastic deformation varies from deformation where the thermal activation is small and most of the energy is supplied by the stress (athermal inelasticity) to deformation where the stress is low and most of the energy of activation is thermal (thermal inelasticity). Further he shows that inelastic deformation is characteristically time dependent and thus points out that creep is not a particular type of inelastic deformation but rather the response to a particular type of loading.

Thus in this survey it is convenient to assume that the basic features of inelastic deformation are embodied in creep response and to write the total strain

$$\varepsilon = \varepsilon_e + \varepsilon_c \qquad (1)$$

where ε_e is the elastic component of strain and ε_c is the inelastic (creep) component of strain. Further it is sometimes convenient to visualize the inelastic strain as being composed of a recoverable (anelastic) component ε_a and a non-recoverable (plastic) component ε_p, that is

$$\varepsilon_c = \varepsilon_a + \varepsilon_p \qquad (2)$$

The shape of the creep curve, that is the variation of the creep strain, ε_c, with time, is regulated initially by the ease of dislocation movement and the rate at which the strain introduces barriers to dislocation movement. Later the variation with time is regulated by a balance between the rate of introduction of barriers and the rate at which they are overcome by thermal action (thermal recovery), this leading eventually to tertiary creep when the rate of removal exceeds the rate of introduction (Ref. 2a, 5a.). However tertiary creep may be due in addition to progressive fracture through intercrystalline cracking or local necking at large values of strain (Refs.3,5b,6,8,9), both of which are beyond the scope of this survey; thus little attention is attached to tertiary creep in the following discussion.

A further influence on the shape of the creep curve is the discontinuous creep strain which is sometimes observed (analogous to the discontinuous yielding of mild steel). This behaviour is considered to be due to diffusion processes and as such a metallurgical rather than mechanical phenomenon and thus is not considered in the formulation of the basic response but as a modification (section 4.2.2.).

Several authors (Ref. 4,7,10) have given extensive surveys of expressions which have been variously proposed to represent the time, temperature and stress dependence of the creep response. However for the purpose of this survey it is convenient to adopt a single general expression for the total inelastic strain with which to demonstrate the various features of deformation, Thus we assume that the creep strain may be represented by Graham's general phenomenological expression (Ref 4)

$$\varepsilon_c = \Sigma_i c_i \sigma^{\beta_i} \phi_i^{\kappa_i} \qquad (3)$$

where C_i, β_i, κ_i = constants,

$\phi_i = \eta_i t$ = time-temperature parameter or pseudo time,

$\eta_i = f_i(T)$ = non-dimensional function of temperature.

The temperature function may be taken as that due to Dorn (Ref 4,5a), that is

$$\eta_i \equiv \eta \propto e^{-K/T} \qquad (4)$$

common to all the component terms of Eq.(3) and where the constant, K, varies with temperature but is assumed to be insensitive to both stress and creep strain. However, there are known limitations to this expression (Ref. 1, 2a) as the constant K is in general a function of stress and creep strain as well as temperature. The Larson Miller parameter (Ref.1) partially meets the requirement for stress dependence but makes the application of Eq (3) difficult.

Graham (Ref 4,11) proposes an alternative which is the use of a number of temperature functions η_i, in Eq. (3), this overcomes the stress and strain dependence of the temperature/time relationship through the changing dominance of the different terms. The form he suggests is

$$\eta_i \propto (T_i - T)^{-20}, \qquad \text{——————} \qquad (5)$$

where the T_i's are experimentally determined constants associated with various metallurgical reactions, this appears * to remove the temperature limitations associated with the application of an expression such as Eq.(4).

Returning to Eq(3), it is first of all noted that the expression allows virtually any stress or time dependence which might be described by the power, exponential or hyperbolic functions variously proposed, to be represented if sufficient terms are employed. The question of time dependence at low temperatures ($T < 0{\cdot}25$ Tm) requires further consideration as it is conventional to split the inelastic strain into an 'instantaneous' plastic component and a logarithmic creep component ($\varepsilon_C = c \log t$). However Crussard (Ref 12) concludes as a result of extensive differential creep analyses that logarithmic creep curves are seldom if ever observed, the creep curves being almost invariably best represented by a power function of time. Further if the time dependence of the total inelastic strain is investigated (i.e. including the 'instantaneous' plastic strain) it is the authors experience that this may also be represented by power function expressions such as Eq(3), with values of κ considerably less than unity. Thus it is concluded that it should be possible to represent the total inelastic (creep) strain, at constant temperature after 'instantaneous' loading to a condition of constant stress, by an expression of the form of Eq.(3).

For particular ranges of stress, temperature and time a limited number of the terms in Eq.(3) would dominate, with the result that for analytic purposes it may be more convenient to use simpler restricted expressions. It should be remembered that the components of the expression do not necessarily represent different creep mechanisms.

* Private communication with Mr.A.Graham.

3.0 EQUATION OF STATE

In order that material behaviour under varying stress and temperature may be anticipated an equation of state for the material is conjectured. This may be defined by the expression

$$\Phi \ (\dot{\varepsilon}_c , \ \sigma, \ T \ , \ q \ ) \ = \ 0 \qquad \text{——————— (6)}$$

where q is a parameter defining the state of the material structure. Differentiation of Eq.(3) yields for the creep rate

$$\dot{\varepsilon}_c = \Sigma_i C_i \sigma^{\beta_i} n_i^{\kappa_i} \kappa_i t_e^{\kappa_i - 1} \qquad \text{—————(7)}$$

which suggests that the influence of the state of the material structure be introduced through an effective time 't_e'.

3.1 General Expressions.

3.1.1. Time hardening (Fig 1.)

This hypothesis suggests that the material structure is only governed by the time under stress ($q \equiv t$). Therefore the creep rate is given by Eq.(7) with $t_e = t$. Rabotnov (Ref.13) points out that any fundamental equation which retains the time explicitly is contradictory, as a physical law must be invariant relative to a time origin. For example consider the effect upon the creep rate of an indefinitely long period of preload at vanishingly small stress.

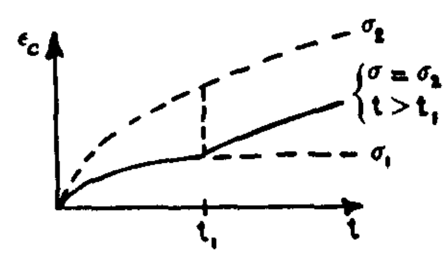


Fig.1. Time hardening hypothesis

3.1.2 Life fraction (Fig.2)

This hypothesis suggests that the material structure is only governed by the *fraction of the* material 'life' expended. In this context 'life' is defined as the time to the fracture strain or more generally, any convenient creep strain, ϵ_o, thus......

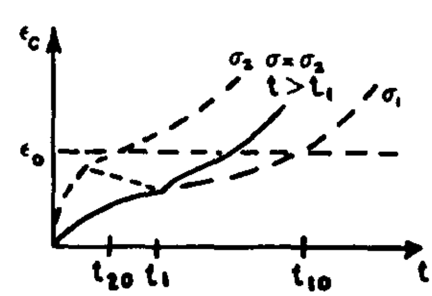


Fig.2. Life fraction hypothesis

$$q \equiv \sum_{r=1} \frac{t_r}{t_{ro}} \qquad (8)$$

where t_{ro} = time to ε_o at σ_r, T_r
t_r = time spent at σ_r, T_r

and q = 1, represents the material 'life'.

Thus the effective time for conditions σ_s, T_s in Eq.(7) is given by

$$t_{se} = t_{so} \sum_{r=1}^{s} \frac{t_r}{t_{ro}} \qquad (9)$$

A fundamental objection to the life fraction predictions defined above is that in general they depend on the value of ε_o selected.

3.1.3 Strain hardening (Fig.3)

This hypothesis suggests that the material structure is uniquely defined by the creep strain accumulated that is $q \equiv \varepsilon_c$. Thus the effective time in Eq.(7) is given by Eq.(3) that is......

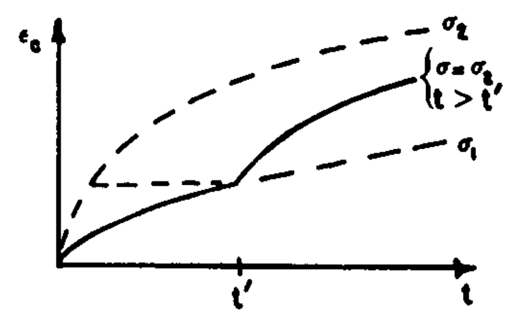


Fig.3 Strain hardening hypothesis.

$$\varepsilon_c = \Sigma_i C_i . \sigma^{\beta_i} . \eta_i^{\kappa_i} . t_e^{\kappa_i} \qquad (10)$$

where ε_c is the current accumulated creep strain. This corresponds with the isotropic strain hardening concept of 'instantaneous' plasticity theory. The strain hardening hypothesis is supported by metallographic and X-ray evidence of Dorn (Ref.2c), i.e. the metallurgical structure is substantially the same for the same creep strain.

3.2 Particular Expressions.

This section considers the properties of the various theories for certain restricted creep expressions.

3.2.1. For the temperature function, η, common to all creep components it may be shown that both strain hardening and life fraction give identical predictions for temperature variations. The creep rate expression Eq.(7) reduces to

$$\dot{\varepsilon}_c . \eta^{-1} = \Sigma_i C_i . \sigma^{\beta_i} . \kappa_i . \phi_e^{\kappa_i - 1} \qquad (11)$$

where the effective pseudo-time is given by

$$\phi_e = \int_0^t \eta \; dt. \qquad (12)$$

The expressions for effective pseudo-time under variable stress conditions reducing to

$$\phi_{se} = \phi_{so} \sum_{r=1}^{s} \frac{\phi r}{\phi ro} \qquad (13)$$

for the life fraction hypothesis and

$$\varepsilon_c = \sum_i C_i \; \sigma^{\beta_i} \; \phi_e^{\kappa_i} \qquad (14)$$

for the strain hardening hypothesis.

3.2.2. *In some cases in addition to a common temperature function*, the creep response, Eq.(3), may be split into the product of a stress function and a time/temperature function such that

$$\varepsilon_c = \Sigma_i a_i \sigma^{\beta i} \Sigma_j b_j \phi^{\kappa_j} \qquad (15)$$

where $a_i b_j = c_{ij}$. Thus in this case the creep rate expression Eq.(11) reduces to

$$\dot{\varepsilon}_c \eta^{-1} = \Sigma_i a_i \sigma^{\beta i} \Sigma_j b_j \kappa_j \phi_e^{\kappa_j - 1} \qquad (16)$$

This formulation has considerable structural advantages in that the stress dependence is separated if used with $\phi_e \equiv \phi$ and may be termed *pseudo-time hardening* (*c.f. section* 3.1.1). *Time hardening, life* fraction and strain hardening predictions are compared in Fig.4, for a fictitious material obeying Eq.(15), for two simple stress histories at constant temperature.

3.2.3 Further simplification results in those cases where the creep response may be represented with only a single time/temperature term. That is Eq.(15) reduces to

$$\varepsilon_c = \left\{ \Sigma_i C_i \sigma^{\beta_i} \right\} \phi_e^{\kappa} \qquad (17)$$

and the creep rate becomes

$$\dot{\varepsilon}_c \eta^{-1} = \left\{ \Sigma_i c_i \sigma^{\beta_i} \right\} . \kappa . \phi_e^{\kappa - 1} \qquad (18)$$

for which it may be shown that life fraction and strain hardening predictions are identical for stress variation as well as temperature variation (this also applying to time hardening predictions when $\kappa = 1$).

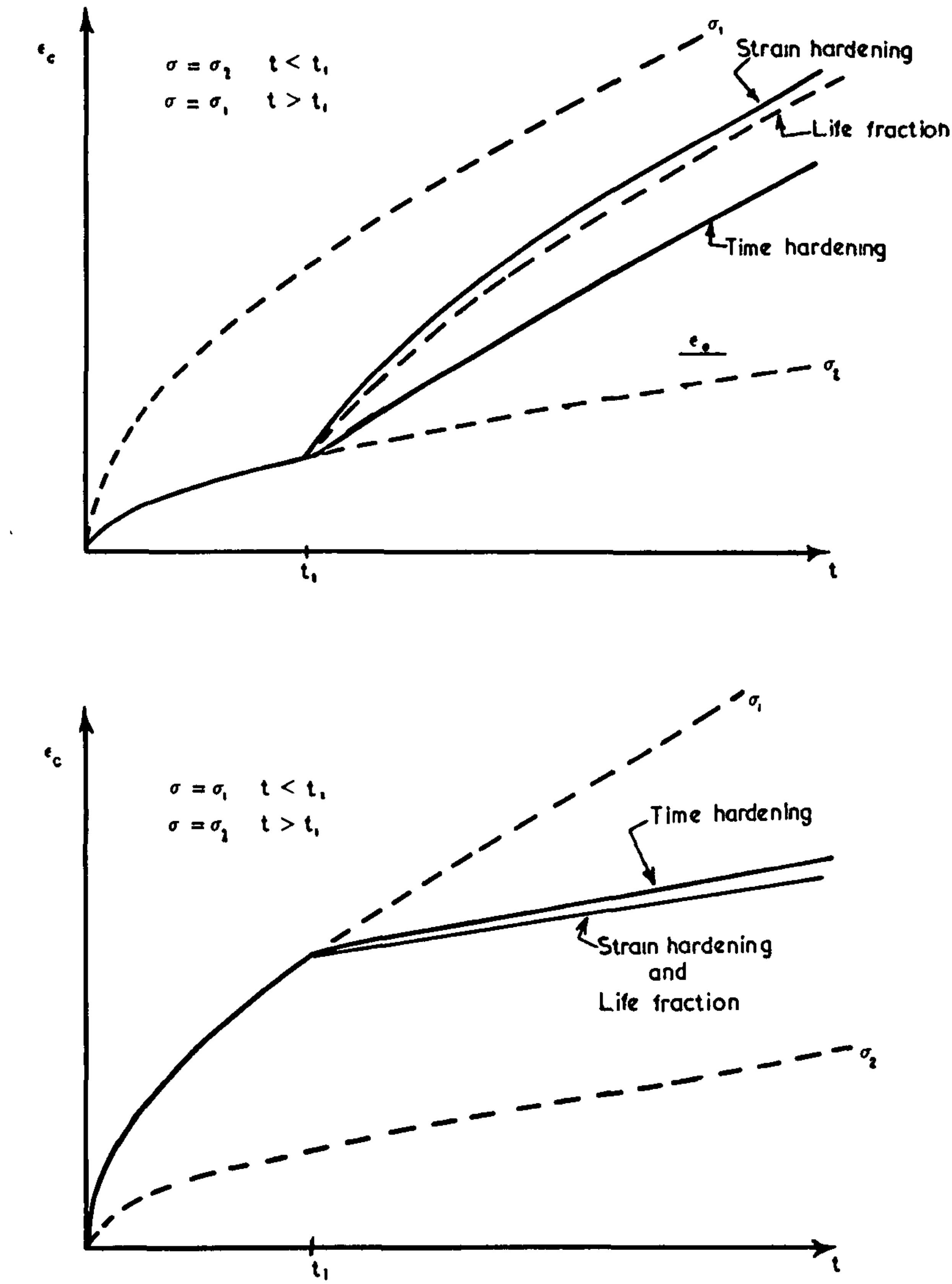


Fig.4. Comparisons of the various equation of state hypotheses for a material with a creep response functionally of the form $\epsilon_c = a(\sigma)b(t)$.

It should also be noted that in this case the response by either theory is commutative, that is for a given programme of loading the resulting creep is independent of the sequence. Substitution for the effective psuedo-time in Eq.(18) from Eq.(17) yields,

$$\dot{\epsilon}_c.\eta^{-1} = \left\{ \sum_i c_i.\sigma^{\beta_i} \right\}^{1/\kappa} . \kappa . \epsilon_c^{\frac{\kappa-1}{\kappa}} \qquad (19)$$

However an alternative and equally useful formulation may be derived directly from Eq.(17) as

$$\epsilon c = \left\{ \int_0^{\phi} (\Sigma_i c_i . \sigma^{\beta_i})^{1/\kappa} d\phi \right\}^{\kappa} . \qquad (20)$$

3.2.4. Finally consider simplification to a Nutting expression of the form (Ref. 4),

$$\epsilon_c = C . \sigma^{\beta} . \phi^{\kappa} . \qquad (21)$$

Then for strain hardending we have, from Eq.(19)

$$\dot{\epsilon}_c . \eta^{-1} = \kappa . C^{1/\kappa} . \sigma^{\beta/\kappa} . \epsilon_c^{\frac{\kappa-1}{\kappa}} \qquad (22)$$

which may be written

$$\sigma = D.(\dot{\epsilon}_c \eta^{-1})^{\kappa/\beta} . \epsilon_c^{1-\kappa/\beta} \qquad (23)$$

where $D = \left[\kappa c^{1/\kappa}\right]^{-\kappa/\beta}$

This latter formulation is adopted by Lubahn and Felgar(Ref. 3) the exponent κ/β being the rate sensitivity and the exponent $\frac{1-\kappa}{\beta}$ being the strain hardening rate (n.b. it reduces to the instantaneous plasticity concept for κ/β approaching zero). In practice the constants and exponents in Eq.(23) vary with stress, time and temperature, this variation is supposedly accounted for in the general case above,Eq.(7), by the large number of free constants. However it should be noted that this interpretation of Eq.(23) is an alternative to the formulation of Eq.(7) and forms a useful basis for subsequent discussion. It should also be noted that it indicates that the constant stress creep curve shape is dictated, as previously noted, by a balance between the rate sensitivity and the rate of strain hardening of the material.

Lubahn and Felgar (Ref 3) point out that it is instructive to consider the 'strength' at a particular stage in the deformation process as the flow stress which would prevail following a change to standard creep rate and temperature conditions.

Thus if these standard conditions are defined as $\varepsilon_c \eta^{-1} = 1$ then from Eq.(23) the flow stress, σ^*, is given by

$$\sigma^* = D \cdot \varepsilon_c^{1-K/\beta} \qquad (24)$$

and substituting in Eq.(22) gives

$$\dot{\varepsilon}_c \cdot \eta^{-1} = \left(\frac{\sigma}{\sigma^*}\right)^{\beta/K} \qquad (25)$$

This expression demonstrates the essential nature of creep strain also, in that the $\dot{\varepsilon}_c \eta^{-1}$ is fixed by the rate sensitivity and the ratio of the applied stress to the flow stress.

For a typical strain hardening expression Eq. (23), the variation σ vs ε_c and $\dot{\varepsilon}_c \cdot \eta^{-1}$ vs ε_c for various constant values of $\dot{\varepsilon}_c \cdot \eta^{-1}$ and σ are shown in Fig's 5 and 6 respectively. Fig.5 represents the behaviour in a constant creep strain rate/temperature tensile test and also indicates the behaviour for an abrupt change in the conditions of the test (ε_c = const.). Fig 6 represents the behaviour in a constant stress creep test and in this case indicates the behaviour for an abrupt change of stress (ε_c = const.). These schematic representations of the consequences of the strain hardening hypothesis are quite general when η is common to all creep components, Eq. (11), and are useful in the discussion of the limitations of strain hardening.

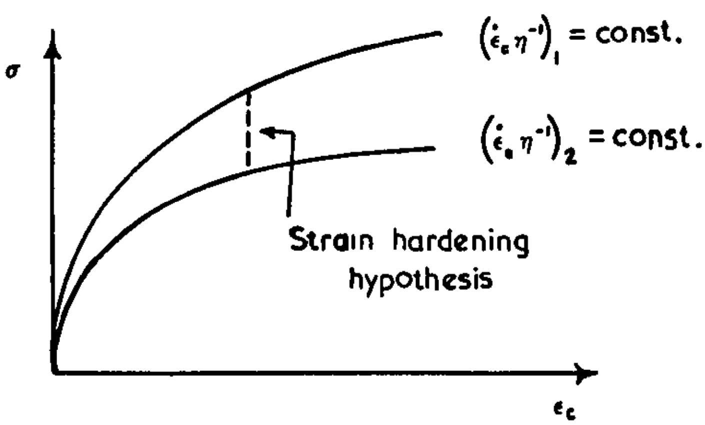


Fig.5 Tensile Tests

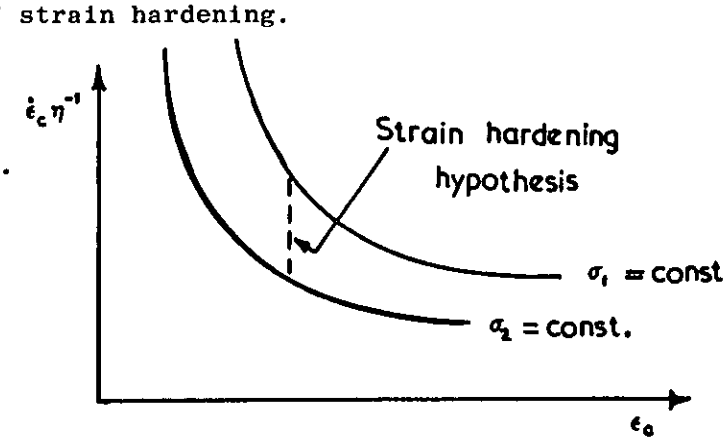


Fig.6 Creep Tests

3.3. Environmental Test Comparisons.

This section gives a short survey of the magnitudes of error to be anticipated as a result of direct application of the various equations of state to environmental situations. There are not many comparisons of deformation experiments with life fraction predictions, therefore some rupture comparisons are included to this effect, although strictly outside the scope of this report.

Further, where not explicitly mentioned the time hardening predictions are less accurate than the strain hardening predictions and in many cases considerably less accurate, the general conclusion being that environmental tests support the strain hardening hypothesis in preference to the others.

3.3.1. Relaxation.

Finnie and Heller (Ref.1) give an extensive review of the relaxation experiments and predictions of Johnson, Davis and Roberts and conclude that strain hardening gives better predictions than time hardening, the error being less than 5% in stress. They further point out that if creep recovery is important and is included it raises the predicted stress.

3.3.2. Intermittent Temperature Creep.

Some materials show as much as 40% increase in creep and reduced life although others show no effect (Ref.2c). Both time and strain hardening predict that there should be no effect. Thermal stresses resulting from material inhomogenity are mentioned as a possible source of deviation.

3.3.3. Intermittent Load Creep.

Some materials show as much as 40% increase in creep when the time under zero load is large compared with that under load, (due to thermal recovery at zero load) (Ref. 2c, 8), Other materials show deceleration of creep and rupture due to metallurgical changes during the rest period, yet others show no effect as would be anticipated from the strain hardening hypothesis (Ref. 2c).

3.3.4. Intermittent Load and Temperature Creep.

Some materials show an increase in creep and reduction in life, although others show little effect even when the load and temperature variations are out of phase (Ref.2c, 4). As above, both time and strain hardening hypotheses predict that there should be no effect, thermal stresses resulting from material inhomogenity again are mentioned as a possible source of deviation.

3.3.5. Variable Temperature Creep.

Avery (Ref.2c) found that very large effects may be produced by very small cyclic temperature variations on metallurgically unstable materials.

For cyclic temperatures Danials (Ref.2c) found deviations of ± 50% in creep strain and Miller (Ref.2c,4) factors of 2·5 on life with respect to strain hardening predictions. However Taira (Ref.8) found variations of only ± 10% in creep strain and 5% on stress for a given life in comparison with strain hardening predictions. It should be noted that these observations also apply to life fraction predictions as a common value of η was used. More accurate predictions would be anticipated for the use of the Graham and Walles parameter Eq.(5).

For general temperature variations Dorn found very good prediction of creep strain vs time both for high purity Al.(Ref.2c) and Nickel (Ref.4) during steady state creep by application of his time/temperature parameter, Eq(4), the strain and time hardening predictions being identical in this case. Kennedy (Ref.4) observes that the use of this time/temperature parameter is limited over extended ranges of time and temperature by changes in substructure and variations of the creep activation energy with temperature, although these objections might be removed by the use of Graham and Walles parameter Eq.(5), as previously noted.

Several workers have investigated the effects of overtemperature cycles (Ref.2c, 4) creep strain being increased and life reduced by thermal recovery at the higher temperature. However metallurgical changes may result from the overtemperature cycles, promoted by creep strain, and giving increases or decreases in life; multiple overtemperatures have greater effect than single overtemperatures of the same total duration.

For rapid temperature rise at constant stress, it would appear (Ref.2c) that very good agreement between theory and practice may be anticipated, with application of creep strain rate expressions (pseudo-time).

3.3.6. Variable Load Creep.

Taira (Ref.8) shows that the rupture life for mild steel 24S-T4 aluminium alloy and 13% Cr steel subjected to sinusoidal stress variation about a mean stress may be anticipated to within ± 5% on stress using a life fraction theory, Randall (Ref.14) finds that the life fraction theory predicts the rupture life of carbon steel to within ± 15%. The load increment/decrement and cyclic load tests of Berkovits (Ref.15) on aluminium alloy show that life fraction predicts rupture life to within ± 10%, with the exception of tests with a descending load sequence when as much as tenfold increases in life were observed. Similar behaviour was observed by Caughey and Hoyt (Ref.2c) in overload tests on Inconel where repeated overloads gave a reduction of life of 30% with respect to the life fraction predictions, while a single overload of the same total duration early in the history gave a twofold increase in life.

The sinusoidal variation tests of Taira (Ref.8) and the incremental/cyclic tests of Ohji and Marin (Ref.16), both on mild steel, show agreement with strain hardening predictions to within ± 10% on strain, that is, to within the scatter of the creep data. Further Ohji and Marin found good agreement with constant strain rate tensile test predictions based on strain hardening (within the range of applicability of the creep data). However a series of their tests with extended periods at the lowest stress showed increases in the creep strain of up to 50% greater than predicted by strain hardening attributed to thermal recovery occurring in the periods at the lowest stress.

The incremental/cyclic tests of Ohji and Marin (Ref.16) on 12% Cr steel indicated variation of creep strain of - 25% to +50% with respect to strain hardening predictions, although closer agreement was found for small stress ranges, this behaviour was attributed to the metallurgically unstable structure of the material as tested. The cyclic tests of Berkovits (Ref.15) on aluminium alloy indicated creep strains.typically only 50% of those predicted by strain hardening theory, the deviation being attributed to the neglected effect of creep recovery.

The experiments of Morrow and Halford (Ref.17) indicate that for lead under repeated stress reversal there is a large and continued decrease in creep resistance after the first few reversals resulting from pronounced work softening (approximately 10 fold increase in creep rate after 100 cycles).

3.3.7. Random Load and Temperature Variations.

Giemza (Ref.18) reports experiments on 7075-T6 aluminium alloy in a random stress-temperature-time environment. He found that specimens taken from widely separated regions in the same sheet of material and subjected to identical histories had responses differing by large amounts, from 20 to 100%. The response differed from that anticipated using strain hardening and the Larson Miller time/temperature parameter (Ref.1) by amounts ranging from 20 to several hundred percent, these large deviations being attributed to the effects of material aging and overaging. When these effects were corrected for, by allowing the reference temperature for creep prediction to change with time, the response was within 10% of theory.

3.4. Limitations of the Equation of State.

3.4.1. Time Hardening and Life Fraction Hypotheses.

Fundamental objections to these hypotheses are mentioned in section 3.1, and the limitations indicated by environmental tests are noted in section 3.3.

The life fraction hypothesis sees application principally because it is particularly suited to periodic and random loading/ temperature rupture life calculations. The modification of Eq.(8) to read

$$A = \sum_{r=1} \frac{t_r}{t_{ro}} \qquad (26)$$

where A is a constant determined by periodic load experiments, has been suggested (Ref.19). However it would appear (section 3.2) that the accuracy of the life fraction hypothesis may be attributed to its approximation to the strain hardening hypothesis, in which case the selected strain, ϵ_o , might be more profitably varied in accordance with periodic load results to optimize the predictions using Eq.(8).

In structural applications involving stress redistribution under steady load, structural constraints result in there being little difference between time and strain hardening predictions (Ref.20). Thus as the time hardening hypothesis is frequently much the simpler of the two analytically it sees considerable application, in spite of the virtually universal agreement that it is less accurate.

[n.b. If the formulation used is that of Eq.(16), this pseudo-time-hardening corresponds with strain hardening for varying temperature conditions.]

3.4.2. Strain Hardening Hypothesis.

Experiments of Dorn et al (Ref. 2a, b, c & 3) show conclusively that the temperature of prior straining influences the mechanical behaviour, i.e. strain hardening depends on the history of temperature as well as the strain. Further tests by Dorn et al (Ref.2a) show that for $\dot{\epsilon}_c \eta^{-1}$ constant throughout there is a unique relationship $\sigma = f(\epsilon_c)$. This implies a rate history effect as well as the temperature history effect,confirmed by differential tests which removed the effect of stress on the deformation process. Lubahn and Felgar (Ref.3) conclude as a result of comparative creep and relaxation tests that a rate history effect is some times but not always present and as a result of comparative creep and constant strain rate tensile tests that a rate history effect is usually present. Graham (Ref.21) rejects strain hardening as a result of a comparison of creep and constant stress rate tensile tests, observing that the deficiencies are not removed by restricting interest to the small strains of engineering interest.

For metallurgically stable materials these temperature/strain rate history effects do not arise when the 'flow stress' is independent of $\dot{\epsilon}_c \eta^{-1}$ (Ref.2a). That is, strain hardening is satisfactory both at low temperature ($T < 0{\cdot}25$ Tm)/high $\dot{\epsilon}_c$ where there is no thermal recovery(experiments of Wyatt (Ref 4))and at high temperature ($T > .5$ Tm) /low $\dot{\epsilon}_c$ where thermal recovery is complete.

Thus at intermediate temperatures (0·25 Tm $<$ T $<$ 0·5 Tm) it appears that these rate and temperature history effects result from thermal recovery. The limitations of thermal recovery in environmental tests having been noted in section 3.3. It was also noted in that section that apart from metallurgical instability, the strain hardening hypothesis was further limited by creep recovery on unloading and relative softening on reversed loading. This relative softening on reversed loading also constitutes a major limitation to the 'instantaneous' plasticity concept (Bauschinger effect). These limitations are considered in greater detail in section 4.

It is convenient at this stage to consider another basic limitation of the strain hardening concept as developed in section 3.1.3, to which end we return temporarily to the 'instantaneous plastic'/'creep' division of inelastic strain. Rabotnov (Ref.9) observes that prior 'instantaneous plastic' strain of less than 1% has no strain hardening effect with respect to the 'creep' strain, while Bugakov (Ref.22) observes that materials may show either zero or marked strain hardening effect in this respect. However, Kennedy (Ref.4) states that the strain hardening effect is roughly in proportion, although the 'instantaneous plastic' and 'creep' strains are microscopically different in appearance. This latter view appears to be implicit in the work of many authors (Refs. 2a,3,5a,8) and is supported on balance by Bugakov.

Rabotnov (Ref.9) suggests that deformation predictions should be based on the separate strain hardening of the 'instantaneous plastic' and 'creep' components of strain.

To account for the situations where there is some cross coupling between the component strains, ε_{c_j} , he further suggests that the stress, σ , be replaced by $\sigma' = \sigma + \int b_j d\sigma$ where b_j is a function of σ , ε_{c_j} and T.

4.0 HEREDITARY THEORIES

In section 2 limitations of the basic creep expression arising from discontinuous creep were mentioned and in section 3 the limitations of the strain hardening hypothesis arising from recovery of creep strain, thermal/mechanical recovery of structure and metallurgical instability were noted. These limitations suggest the need for a hereditary theory, that is a theory which takes account of the stress/temperature history other than through the value of the current creep strain.

Two methods appear to be available to avoid these limitations, firstly as the strain hardening hypothesis is in many applications not seriously in error there is the possibility of modification and secondly there is the possibility of a purely rheological theory. However, it is convenient to consider creep recovery initially under the heading 'anelastic creep', then to consider the modifications of strain hardening and lastly the rheological theories.

It should be noted that the various creep mechanisms to be described, are phenomenological in character and do not necessarily correspond with separate microscopic mechanisms.

4.1 Anelastic Creep

Early work suggested that for creep divided into 'transient' and 'steady state' components the transient component was completely recoverable and at a rate nearly identical with that of the previous extention (Ref.2c). However subsequent evidence (Ref.1, 23) showed that more generally only a fraction of such 'transient' creep was recoverable. Systematic investigation of Lubahn (Ref.2b, c & 3) using creep recovery tests indicated that the creep recovery was linearly related to the stress decrement with the limitation of the magnitude of the prior creep strain.

Further he found that the creep strain might be divided into a recoverable (ϵ_a) and a plastic (ϵ_p) component, Eq. (2), with the plastic component, only, dictated by strain hardening. The plastic component of creep (ε_p) in general has a parabolic variation with time, thus explaining the unsatisfactory nature of the 'transient'/'steady-state' division. These conclusions are also supported by the work of Kennedy (Ref. 4,24) and Rabotnov (Ref. 9 & 13) and suggest that there is a plastic creep 'limit' (Ref. 2b, 3) below which all creep is recoverable. It should also be observed that the creep recovery is balanced by a further loss on reloading (Ref. 2c) although this may occur at a slower rate (Ref. 4).

This inelastic behaviour is termed anelastic (Ref. 3,6) as the recovery of creep strain results from creep across inhomogeneous regions (e.g. grain boundaries, slip bands and twin interfaces), due to elastic stresses stored in these regions as a result of prior deformation (Ref. 2a, c & 4). As Zener (Ref. 2a) maintains that grain boundary slipping is fully recoverable it is sometimes helpful to regard the recoverable creep component as a separate mechanism which will be termed anelastic creep, ε_a. Below the stress level for plastic deformation, in common with other anelastic effects e.g. internal damping etc. , the response is linearly viscoelastic (Ref. 3). Thus it obeys the differential equation

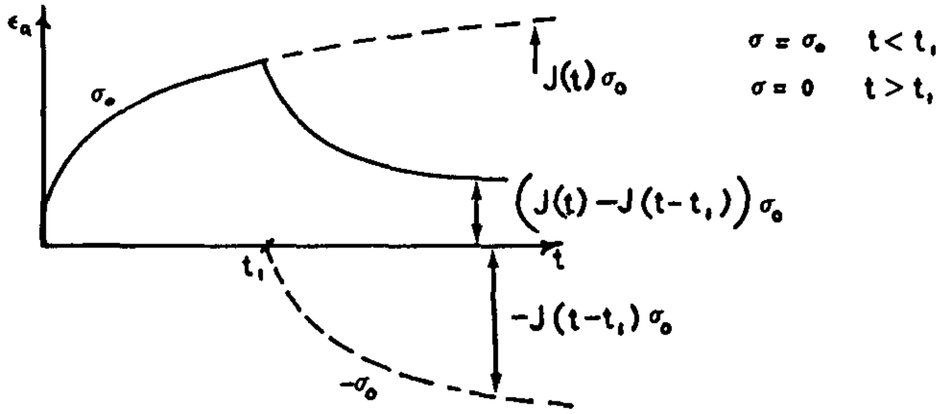
$$\sum_{r=o}^{m} a_r \; (t,T).\frac{d^r}{dt^r} \; \sigma = \sum_{s=o}^{n} b_s \; (t,T).\frac{d^s}{dt^s} \; \varepsilon_a \qquad (27)$$

and in particular the Boltzmann superposition principle (Ref. 4, 6, 10)

$$\varepsilon_a = \int_{-\infty}^{t} J \; (t-t_1) \, . \, \frac{d\sigma}{dt_1} \, dt_1 \qquad (28)$$

where J(t) is the creep response for unit stress. Eq.(28) indicates that the response is proportional to stress and the sum of the effects due to the stress changes taken individually, Fig. 7.

Fig. 7. Boltzmann superposition



Viscoelastic theory has developed historically from the representation of real solids by simple models such as the Kelvin-Voigt element for which the creep response is given by

$$J(t) = J(1 - e^{-t/\tau}) \qquad (29)$$

where τ is the retardation time. However, to cover the anelastic effects of real metals, and anelastic creep in particular, it has proved necessary to introduce the concept of a spectrum of retardation times, that is

$$J(t) = \sum J_i (1 - e^{-t/\tau_i}) \qquad (30)$$

where τ_i is the i^{th} retardation time (Ref. 1, 2b, 2c, 3, 4, 6, 10)

It will be noted that the evaluation of the constants J_i and the retardation times τ_i in practice involves no more than curve fitting using the creep recovery curve (Ref. 3, 10) Further, as Crussard (Ref. 12) offers objections to the spectrum of response concept in general there seems little reason to restrict the analysis to the concept. Several authors (Ref. 4, 11, 24) have noted recovery curves which might be represented adequately by assuming

$$\varepsilon_a = c\,.\,\sigma\,.t^{\kappa}$$

$$\therefore\ J(t) = ct^{\kappa} \qquad (31)$$

Thus the Boltzmann superposition expression, Eq. (28) reduces in this case to

$$\varepsilon_a = C\int_{-\infty}^{t} (t - t_1)^{\kappa} . \frac{d\sigma}{dt_1}\, dt_1, \qquad (32)$$

the effect of temperature being introduced through pseudo-times $\phi = \int^{t} \eta dt$ and $\phi_1 = \int^{t_1} \eta dt$ (Ref. 10)

Thus, to summarize, the total creep strain is visualized as composed of anelastic (recoverable) and plastic (non-recoverable) creep components, Eq.(2)

$$\varepsilon_c = \varepsilon_a + \varepsilon_p.$$

Under variable stress/temperature conditions the anelastic component, ε_a, is given by Eq. (32) and the plastic component is given by strain hardening, section 3.1.3.

4.2 Strain Hardening Modifications

If the recovery of creep strain limitations, of the standard strain hardening hypothesis may be removed by the introduction of anelastic and plastic creep components as suggested in the previous section, then the other limitations can possibly be dealt with by modification of the strain hardening hypothesis applied to this plastic creep component. It should be observed that the majority of authors suggesting strain hardening modifications have not made this anelastic/plastic division and thus, as is subsequently noted, it is frequently uncertain to what extent their observations might be alternatively interpreted.

Rabotonov (Ref. 9) suggests a comprehensive theory to cover possible modifications of the equation of state hypothesis, elaborating Eq. (6) to include a series of parameters q_i , defining the state of the material structure, that is

$$\Phi\ (\dot{\varepsilon}_c, \sigma\ , T,\ q_1 \ldots q_n) = 0 \qquad \text{———} \quad (33)$$

The parameters q_i , are defined incrementally as

$$dq_i = a_i d\varepsilon_c + b_i d\sigma + c_i dT + d_i dt \qquad \text{———} \quad (34)$$

where the constants a_i, b_i, c_i & d_i are functions of ε_c, σ, T, t and q_s in general. However in practice it is convenient to consider the various expressions proposed for Eq. (34) as modifications of the strain hardening hypothesis

$$q_1 = \varepsilon_c,\ q_2 \ldots \ldots \ldots .q_n \equiv 0 .$$

Thus the modifications suggested to account for the various limitations may be detailed under the headings recovery (loss of strain hardening), strain aging (discontinuous creep) and metallurgical instability. With the probable exception of 'mechanical recovery' each of these limitations result from metallurgical reactions.

4.2.1. Recovery

Recovery is defined as the loss of strain hardening due to any cause and obviously can only occur in plastically strained material. Two forms are rather arbitrarily distinguished, namely thermal recovery and mechanical recovery.

Thermal Recovery:-

Thermal recovery refers to the overcoming of barriers to dislocation movement by thermal activation below the re-crystallization temperature, 2/3 T_m (say), leading in some cases to polygonization of the plastically strained crystals to form stable subcrystals. Further at sufficiently high temperature, and for extensive plastic strain, recrystallization results in loss of strain hardening and may be viewed for convenience as the limiting case of thermal recovery.

Thermal recovery may occur statically during periods of zero or reduced stress (Ref.2a, 2c, 3, 8) the tendency being for no significant recovery for $T < 0.25\ T_m$, transient recovery (Fig. 8) for $0.25\ T_m < T < 0.5\ T_m$ and complete recovery (Fig. 9) for $T > 0.5\ T_m$. However thermal recovery may also occur dynamically during creep deformation for $T > 0.25\ T_m$, being greatest for over-temperature cycles and high working temperatures (Ref. 2c). In certain circumstances thermal recovery only occurs dynamically (Ref 2a, 6) requiring the progress of plastic strain for its activation and resulting in work softening.

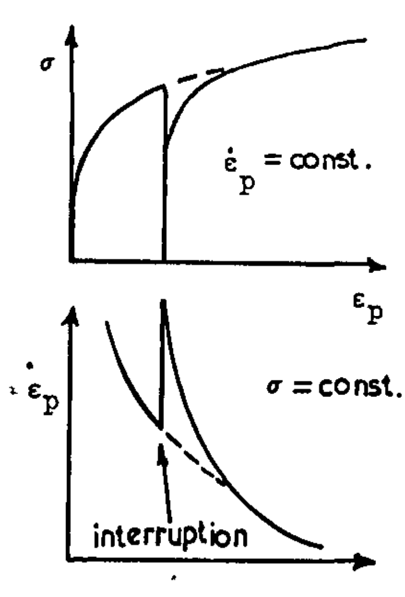


Fig.8. Thermal recovery $0.25\ T_m < T < 0.5\ T_m$.

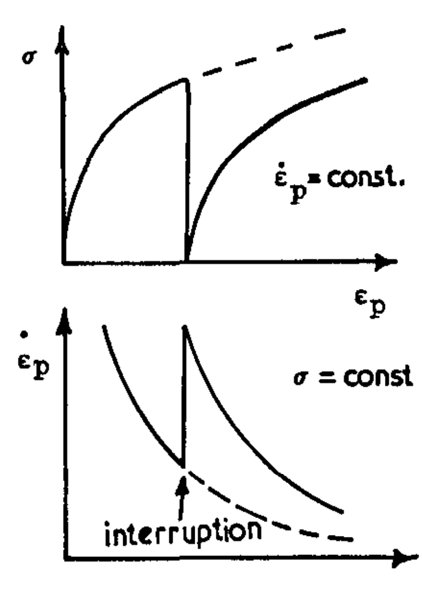


Fig.9. Thermal recovery $T > 0.5\ T_m$

It will be observed as mentioned in section 2, that the progress of thermal recovery which accompanies creep deformation will regulate the shape of the plastic creep curve through the balance with strain hardening, leading to seoondary and tertiary creep stages (Ref. 2a, 3). This account of secondary and tertiary creep is supported by the observation that the effect of static recovery is less marked in secondary creep (Ref. 4) and that intermittent loading may hasten tertiary creep (Ref. 8). Thus several authors have suggested (Ref. 2a, 3, 8) that the plastic creep curve may be regarded advantageously as a basic creep response (evaluated by differential tests (Ref 2a, 3)) modified by structural changes due to both strain hardening and thermal recovery (evaluated by tensile and creep tests respectively (Ref. 8)).

The basic creep response modified by strain hardening was considered in section 3.2.4. and represented by either of the Equations (22) or (25). Thus it is the further thermal recovery modification to account for the secondary and tertiary stages, together with the effect of interruption which requires consideration Lubahn and Felgar (Ref. 3) show that the effect of thermal recovery may be regarded as a reduction of the plastic strain available to cause strain hardening, that is an equivalent plastic strain. Conrad (Ref. 2a) maintains that the structure is uniquely defined by the flow stress prevailing and therefore, as above, by an equivalent plastic strain, Eq. (24). The use of the equivalent plastic strain formulation Eq. (22) or the flow stress formulation Eq. (25), is therefore seen to be a matter of convenience. It should be remembered that the coefficients and exponents in these equations vary with stress, temperature and time to given them their generality.

The character of the thermal recovery is such that it may be anticipated that the rate of reduction of strain hardening is proportional to the plastic creep strain and dependent on both time and temperature. Both Rabotnov (Ref 9) and Lubahn & Felgar (Ref. 3) adopt an equivalent plastic strain $q_1 \equiv \varepsilon_p'$ and define it incrementally as

$$\delta\varepsilon_p' = \delta\varepsilon_p + \frac{\partial\varepsilon_p'}{\partial t}\,\delta t \qquad (35)$$

where $\frac{\partial\varepsilon_p'}{\partial t}$ is a function of the equivalent plastic strain. Further more Lubahn suggests the explicit form

$$\delta\varepsilon_p' = \delta\varepsilon_p - H\,\varepsilon_p'\,\delta t \qquad (36)$$

where H is the thermal recovery rate, which gives zero strain hardening for H approaching infinity. Conrad (Ref 2a) adopts the flow stress formulation, $q_1 \equiv \sigma^*$, and proposed that it be represented by

$$\sigma^* = \int_0^{\varepsilon_p} h(T,\dot{\varepsilon}_p)\,d\varepsilon_p \qquad (37)$$

where $h\left\{\equiv\frac{d\sigma^*}{d\varepsilon_p}\right\}$ the strain hardening rate is a function of the temperature and the plastic strain rate. Taira (Ref. 8) also uses a flow stress formulation defining the flow stress in incremental form as

$$\delta\sigma^* = \frac{\partial\sigma^*}{\partial\varepsilon_p}\,\delta\varepsilon_p + \frac{\partial\sigma^*}{\partial t}\,\delta t \qquad (38)$$

where $\frac{\partial \sigma^*}{\partial \varepsilon_p}$ is the coefficient of strain hardening and $\frac{\partial \sigma^*}{\partial t}$ is the coefficient of thermal recovery, thus

$$h \equiv \frac{d\sigma^*}{d\varepsilon_p} = \frac{\partial \sigma^*}{\partial \varepsilon_p} + \frac{\partial \sigma^*}{\partial t} \cdot \frac{1}{\dot{\varepsilon}_p}$$

and

$$\sigma^* = \int_0^{\varepsilon_p} h(\sigma, \sigma^*, \dot{\varepsilon}_p)\, d\varepsilon_p \qquad (39)$$

It should be noted that Taira defines the flow stress, σ^*, as the critical stress at which the material deforms without the aid of additional activation energy, i.e. not as defined in section 3.2.4. Thus the explicit expressions which he suggests cannot be readily translated into the present formulation. Taira (Ref. 8) also gives a comparison of thermal recovery predictions with experimental results under periodic stress, the strains being only 10% in error compared with 25% error for strain hardening predictions. It will be observed that the flow-stress thermal-recovery expressions considered above are merely alternative formulations to those using the equivalent plastic strain.

However an alternative concept has been used with considerable success by Kennedy (Ref. 4, 25) in application to lead. This is based on the hypothesis that during static recovery a certain fraction of the material completely recovers its original structure. Thus on reloading this fraction creeps as though virgin material while the remainder continues to creep unaffected. The general applicability of this technique is uncertain, since to the author's knowledge, it has not been applied to metals of technological interest.

Mechanical Recovery:-

The term mechanical recovery is introduced to refer to permanent or transient loss of strain hardening resulting from stress variations (e.g. relative softening on reversed stressing). Several mechanisms have been suggested to account for these various transient and permanent effects.

Manson (Ref. 2c) suggests that transient plastic deformation is governed by slip band nucleation and critical size growth, thus a load increase causes increased creep rate (relative softening) due to subcritical slip bands becoming critical and a load decrease gives nothing until further nucleation, this behaviour resulting in an excess net effect.

Namestnikov and Rabotnov (Ref 26) visualize a sudden increase in stress to cause relative softening connected with breakdown of obstacles and freeing of dislocations, but anticipate no relative hardening due to decrease of stress. They suggest the use of an equivalent stress

$$q_2 \equiv \sigma'$$

$$\sigma' = \sigma + a\int \varepsilon_c \, d\sigma \qquad \text{————— (40)}$$

to account for this relative softening on load increase, the constant, a, being evaluated to fit load increment tests. Bugakov and Vakulenko (Ref. 34) suggest on the basis of thermodynamic reasoning that the effect of sudden stress variations may be anticipated by the use of an equivalent plastic strain

$$\sigma \, \varepsilon'_p = \int_o^{\varepsilon_p} \sigma \, d\varepsilon_p \qquad \text{————— (41)}$$

which Bugakov (Ref. 22) shows, at least partially, to include expressions with the form of Eq. (40). This expression appears to give a better fit with experimental data than strain hardening for both load increments and decrements (Ref. 9).

Taira (Ref. 27) considers that the influence of load variation on creep behaviour is connected with the processes of thermal recovery and strain hardening. It is not clear to what extent thermal recovery was an important factor in the tests supporting the above mechanisms.

However thermal recovery as discussed in the first part of this section can account for relative softening with load increments (due to the greater time to a given strain at the lower stress) and relative hardening with load decrements, if thermal recovery is important at the temperatures considered. Taira (Ref. 27) suggests additional recovery mechanisms to account for the overestimates of the effect of abrupt stress variations, resulting from these considerations alone. He interprets strain hardening as the back pressure of piled up dislocations and thus following an abrupt change of stress he anticipates a climbing process of piled up dislocations from the active slip plane and a time dependent recoverable 'spring back' of piled up dislocations These mechanisms result in an incremental hardening for stress increases and an incremental softening plus a transient recovery for stress decreases Taira (Ref. 27) suggests that these transient effects may be anticipated from load decrement tests by the introduction of an abrupt change in the flow stress, $\delta\sigma^*$, and a transient change in the rate of recovery $\left\{\frac{\partial\sigma^*}{\partial t}\right\}_t$ (this latter term being neglected for load increments) into the equation for thermal recovery, Eq. (38). The increment $\delta\sigma^*$ is taken as constant with time and proportional to the stress increment, the transient $\left\{\frac{\partial\sigma^*}{\partial t}\right\}_t$ diminishing to zero with time. This form of transient recovery (Fig. 10) is also observed by Dorn and Mote (Ref. 5a) who conclude that the final structure is dictated by the final stress only, which is consistent with the present theory(Ref. 28)

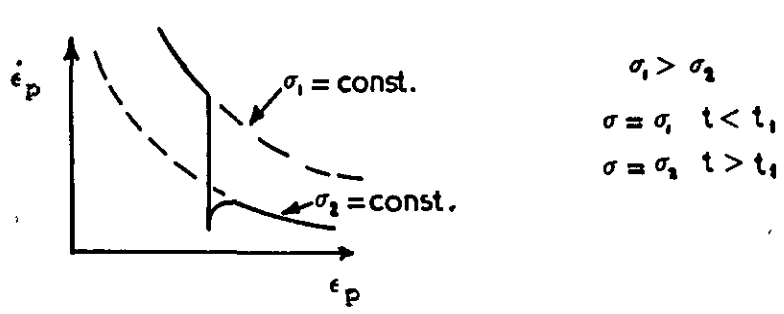


Fig 10 Load decrement, transient recovery.

The time dependent, recoverable 'spring back' mechanism of Taira, mentioned above, describes an anelastic behaviour. It is uncertain whether this behaviour observed by Taira might not be described by the separation of anelastic and plastic creep as proposed in section 4.1.

Likewise it is not certain to what extent the transient behaviours of Manson, and Namestnikov and Rabotnov might also be accounted for by a combination of anelastic and thermal recovery mechanisms.

However Lubahn and Felgar (Ref. 3) report tests in which both anelastic and recovery effects are small and observe transient softening for a stress decrement, Fig. 11. As a result of comparison of interrupted creep and tensile tests, Fig. 7, in which the transient effect dies out in a similar strain increment (Ref. 3), they conclude that these are manifestations of a Bauschinger effect. This conclusion is further supported by differential strain rate tests in which stress decrements show a higher rate sensitivity than stress increments. For stress reversal the existence of a Bauschinger effect would suggest considerable relative softening, as observed in the stress reversal tests of Morrow and Halford (Ref. 7), in which there was a 10 fold increase in creep rate.

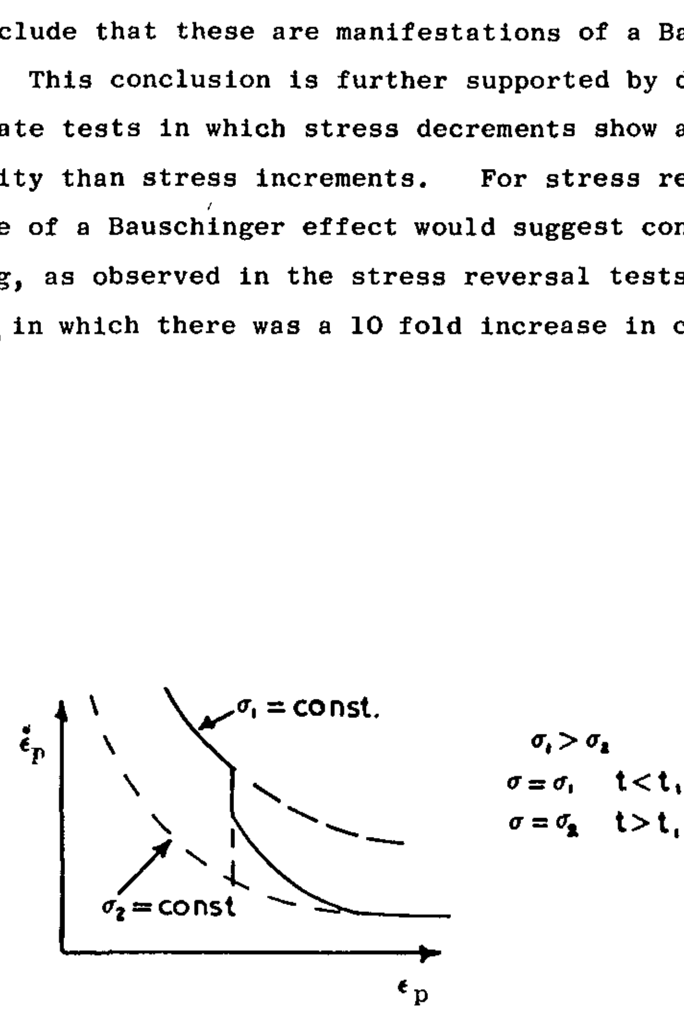


Fig. 11. Load decrement, Bauschinger effect.

4.2.2. Strain Aging

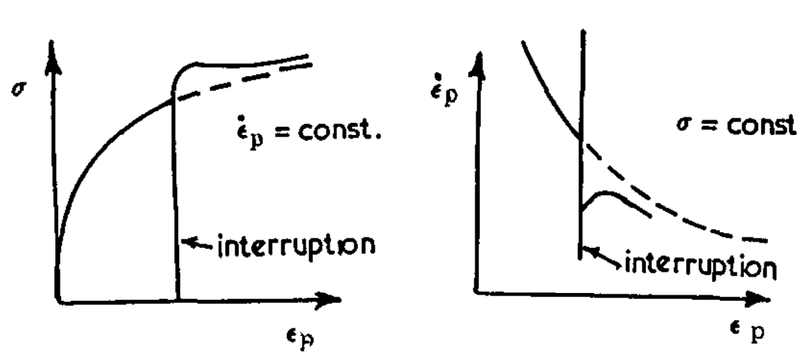
An important exception to the mechanics of creep deformation as discussed thus far, arises as a result of the diffusion of various foreign particles in the crystal lattice to dislocation centres (Ref. 6), thus blocking dislocation movements and resulting in the discontinuous yield/creep frequently observed. Such behaviour may arise either from the prior history of the material or from periods of aging (after plastic strain) at the temperature for this diffusion, the result being an increase in the flow stress and a yield point tendency coupled with the possibility of discontinuous flow and Lüder band formation (Fig. 12). Further for strain proceeding at the temperature for this diffusion the progress may be discontinuous and may result in the formation of Lüder's bands; for such conditions the rate sensitivity obtained from constant strain rate tests is fictitiously low. This metallurgical effect which depends on time and temperature and which will not proceed in the absence of plastic flow is termed strain aging (Ref. 3,4).

Lubahn and Felgar (Ref. 3) consider the magnitude of the strain aging strengthening which accompanies plastic creep. They conclude that it is probably in direct proportion to the prior strain increment (although independent of the magnitude of total prior strain) and dependent on the time lapse from the increment. They thus suggest the use of an equivalent stress.

$$\sigma' = \sigma - b \int_0^{\varepsilon_p} (t - t_1)^a d\varepsilon_{p1} \qquad \text{(42)}$$

where a and b are experimentally determined constants and t_1 is the time of the increment $\delta\varepsilon_{p1}$.

Fig 12 Strain aging effects in tensile and creep tests.



4.2.3. Metallurgical Instability

Thermal recovery and strain aging are examples of metallurgical reactions which will not occur in the absence of plastic flow. However other reactions such as aging/overaging, secondary hardening and reprecipitation, which can occur as a result of thermal action alone, are probably only slightly affected by plastic flow (Ref. 3). Thus, for working temperatures and lives for which these reactions are slight, their influence on the creep deformation may be ignored. For conditions where the reactions are considerable there may be either a loss or an increase of creep strength (Ref. 2c) and Lubahn and Felgar (Ref. 3) suggest that the deformation behaviour may be anticipated by using the basic equations with the constants, C_i, functionally similar to the hardness/time relationship of the material.

The other metallurgical reactions such as intergranular oxidation, corrosion, embrittlement etc., are considered to be of less importance from the deformation point of view, although they may be important with regard to fracture (Ref. 2c).

4.3 Rheological Theories

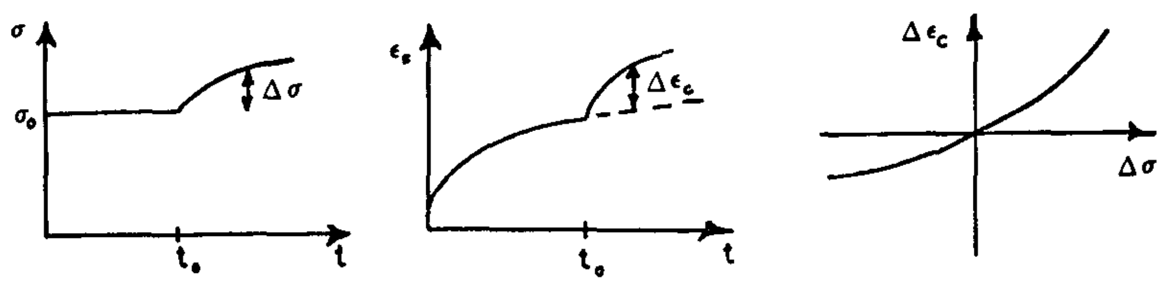
The considerable success of viscoelastic theories (previously outlined in section 4.1) in the rheological description of polymers, coupled with knowledge of anelastic behaviour in metals, has suggested their more general consideration. Further the rheological behaviour of a very wide range of materials is very similar (σ=const) (Ref. 10), suggesting that a phenomenological description might be successful where a mechanistic description is too complex. However it is observed that the majority of metals can be described only by nonlinear stress dependence, which rules out the direct application of linear viscoelasticity, this adding considerably to the difficulty of applying memory principles.

We consider first an attempt by Onat and Wang (Ref. 29) to apply linear viscoelasticity to the creep strain increment, $\Delta \varepsilon_c(t)$, following a small stress increment, $\Delta \sigma(t)$, for $(t-t_o)/t_o \ll 1$ (Fig. 13). They found that $\Delta \varepsilon_c$ was only linearly related to $\Delta\sigma$ for stress increments less than 4% and decrements less than 2%, the non-linearity being non-symmetric as shown schematically in Fig. 13. However within these limitations there was a strong indication of the validity of Boltzmann superposition i.e.

$$\Delta \varepsilon_c(t) = \int_{t_o}^{t} J(t - t_1) \frac{d\,\Delta\sigma}{dt_1}\, dt_1 \qquad (43)$$

where J(t) is the response to unit stress increment.

Fig. 13 Incremental creep response to a stress increment.



Several authors have introduced a generalised Boltzmann superposition principle by assuming a linear relation between some function of creep strain, u, and the history of a stress function, χ, weighed by a suitable memory function, ζ, (i.e. instead of between creep strain and the stress history directly). This may be written in general terms as the superposition integral

$$u(t) = \int_0^t \zeta (t - t_1) . \frac{d}{dt_1} \left[\chi (t_1)\right] dt_1 . \qquad (44)$$

In the comparison of the various forms of this superposition integral it is convenient to adopt a single component of Eq. (3)

$$\varepsilon_c = C \sigma^{\beta} . t^{\kappa}$$

i.e. the Nutting equation (Ref. 10, 21).

Johnson et al (Ref. 30) propose a form of Eq. (44) with $u \equiv \varepsilon_c$, $\zeta \equiv t^{\kappa}$ such that

$$\varepsilon_c = C \sum_{r=o}^{m} (t - t_r)^{\kappa} . (\Delta \sigma r)^{\beta} . \qquad (45)$$

This response is indicated in Fig. 14 where it will be observed that it considerably underestimates the behaviour for load increments (n.b. strain hardening usually underestimates the response to load increments). However in the complex-stress creep tests under changing load of Johnson (Ref. 30) the predictions are on the average no worse than those from strain hardening although tending to underestimate rather than overestimate.

Graham (Ref. 11, 21) proposes a form of Eq. (44) with $u \equiv \varepsilon_c$, $\chi \equiv \sigma^{\beta}$ and $\zeta \equiv t^{\kappa}$ such that Eq. (44) becomes

$$\varepsilon_c = C \int_0^t (t - t_1)^{\kappa} . \frac{d}{dt_1} (\sigma^{\beta}) \, dt_1 \qquad (46)$$

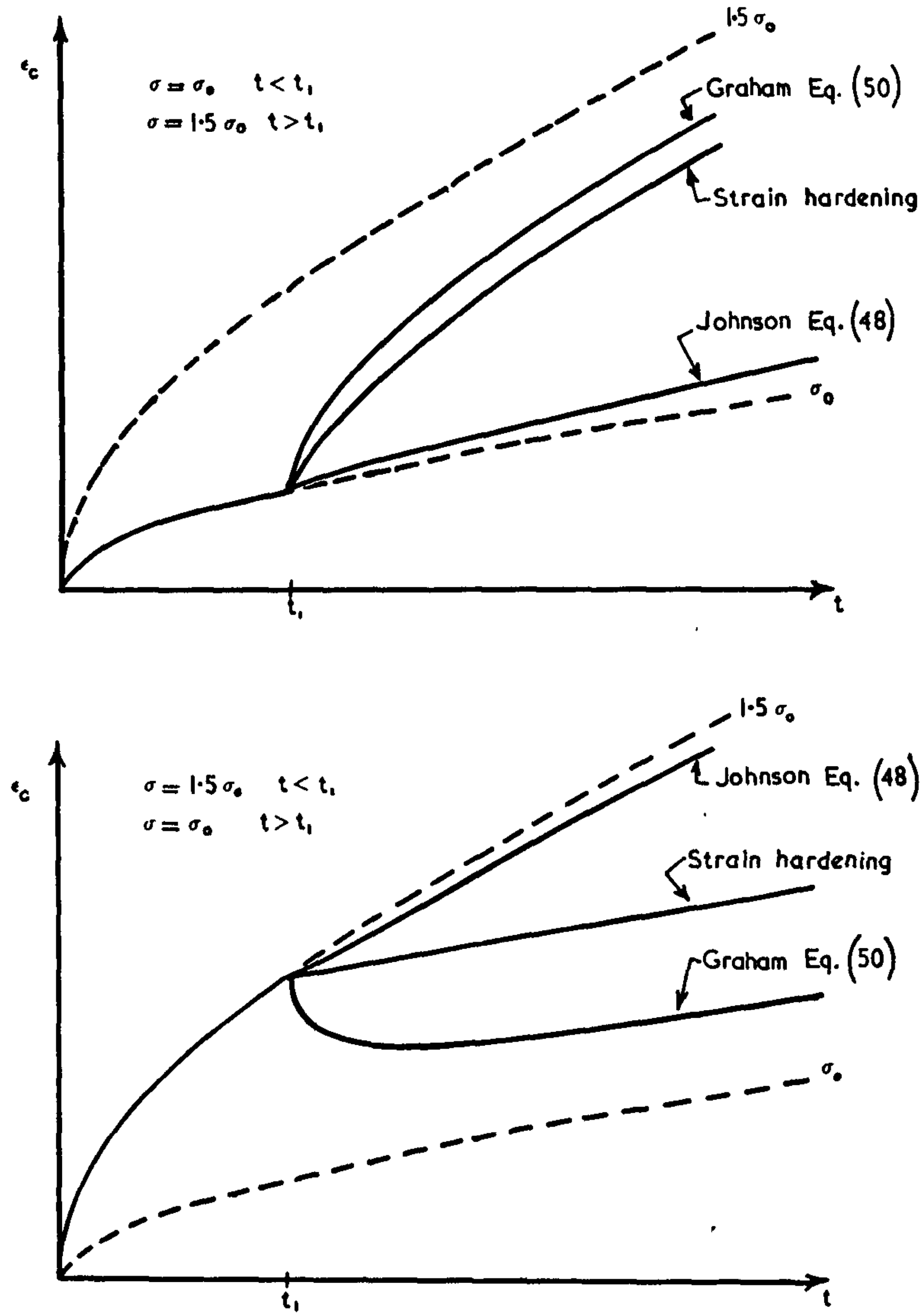


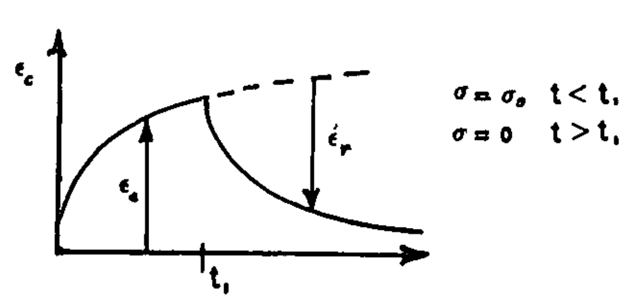
Fig. 14 Comparison of the strain hardening hypothesis and various rheological theories for a material with a creep response functionally of the form $\varepsilon_c = C\,\sigma^{2\cdot 7} \cdot b(t)$

which expression is also used by Arutyunyan and Rozovskii (Ref. 32). Leaderman (Ref. 10, 31) originally proposed and used an expression essentially of the form of Eq. (46) to deal with non-linear behaviour of fibrous materials. The response according to this equation is also illustrated in Fig. 14. It will be observed that the response to a stress increment is greater than that predicted by strain hardening (as frequently observed) and it would appear that the response in a constant-stress-rate tensile test is in better agreement than the corresponding strain hardening predictions (Ref.11). Graham suggests (Ref.11) the extension of Eq.(46) to account for further creep components, Eq.(3),

$$\varepsilon_c = \sum_i C_i \int_0^t (t - t_1)^{\kappa_i} \cdot \frac{d}{dt_1} (\sigma^{\beta_i}) \, dt_1 \qquad (47)$$

which, following unloading predicts complete creep recovery for $\kappa_i < 1$, no creep for $\kappa_i = 1$, and an endless increase of creep for $\kappa_i > 1$, In view of the observations on fractional creep recovery and the linear dependence of creep recovery on stress decrement (section 4.1) it would appear that the expression requires modification in this respect. Further for cyclic stress variations, $\kappa_i < 1$, the expression predicts a hysteresis loop and a kind of Bauschinger effect (Ref. 21). However the predictions are apparently inaccurate*, presumably due to the previously observed reasons. Ward and Onat (Ref. 31) in tests on polypropylene observe that for non-linear materials, creep (ε_c) and recovery (ε_r) response may be different (Fig. 15), thus offering a further objections to Eq. (46) which predicts identical response.

Fig. 15 Creep recovery (Graham theory).



* Private Communications with Mr. A. Graham.

Rabotnov (Ref. 13) proposes a form of Eq. (44) with $u \equiv \varepsilon_c^{1/\beta}$, $\chi \equiv \sigma$ and $\zeta \equiv t^{\kappa/\beta}$ such that

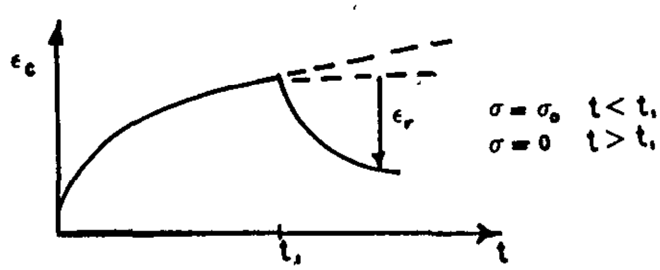
$$\varepsilon_c^{1/\beta} = C^{1/\beta} \int_0^t (t - t_1)^{\kappa/\beta} \cdot \frac{d\sigma}{dt_1} \, dt_1 \qquad (48)$$

for stress increasing. For a stress decrement at 't_o', he suggests a modification to the left hand side of Eq. (48) such that the subsequent creep is governed by the equation

$$D(\varepsilon_c - \varepsilon_{co}) + \varepsilon_{co}^{1/\beta} = C^{1/\beta} \int_0^t (t - t_1)^{\kappa/\beta} \cdot \frac{d\sigma}{dt_1} \, dt_1 \qquad (49)$$

where D is a constant. This results in the creep recovery ε_r (Fig. 16) being linearly dependent on the stress decrement. Namestnikov and Rabotnov (Ref. 32) conclude that this theory does not give an exact quantitative description of creep for stress decrements and that strain hardening is in general more accurate.

Fig. 16 Creep recovery (Rabotnov theory).



Returning to the application of memory principles to non-linear rheological problems generally, Hilton (Ref. 10) observes that the basic viscoelastic equations in either operator form

$$\sum_{r=o}^{m} a_r \cdot \frac{d^r}{dt^r} \sigma = \sum_{s=o}^{n} b_s \cdot \frac{d^s}{dt^s} \varepsilon_c \qquad (50)$$

or integral form

$$\varepsilon_c = \int_{-\infty}^{t} J(t, t_1) \frac{d\sigma}{dt_1} \, dt_1 \qquad (51)$$

can be made to represent non-linear behaviour if a_r, b_s and J are expressed as functions of stress, creep strain and their derivatives, although no application in this form is know to the author.

Graham (Ref. 33) shows that operator expressions of the form

$$\sum_{r=0}^{m} a_r \frac{d^r}{dt^r} u = \chi (t), \qquad (52)$$

where u & χ might be functions of creep strain and stress respectively and the coefficients a_r may be variable, can be written quite generally as

$$u(t) = \int_{o1}^{t} v (t,t_1) A(t_1) dt_1 . \qquad (53)$$

An alternative and potentially more useful formulation is given by

$$u(t) = \int_{o1}^{t} \zeta_1(t - t_1) \ldots\ldots \int_{on}^{t_{n-1}} \zeta_n (t_{n-1} - t_n) \chi (t_n) dt_n \ldots dt_1 \qquad (54)$$

which is seen to amount a succession of superposition integrals of the form of Eq. (44) and might overcome the limitations of Eq. (46) previously noted.

Ward and Onat (Ref. 31) used with some success the expression

$$\varepsilon_c (t) = \int_{-\infty}^{t} J_1(t-t_1) \frac{d\sigma}{dt_1} dt_1 + \ldots\ldots + \int_{-\infty}^{t} \ldots \int_{-\infty}^{t} J_n(t-t_1, \ldots\ldots t-t_n) \frac{d\sigma}{dt_1} \ldots\ldots \frac{d\sigma}{dt_n} dt_1 \ldots dt_n \qquad (55)$$

developed by Fréchet (as an extension of linear viscoelasticity to the non-linear domain) in the analysis of their tests, requiring the first and third terms to describe the non-linear behaviour.

To take account of temperature histories, Graham (Ref. 11) and Hilton (Ref. 10) suggest the use of pseudo-time, ϕ , where

$$\phi = \int_{o}^{t} \eta \quad dt_1 \qquad (56)$$

in place of time, t, in the various superposition integrals (n.b. as used in the strain hardening theory).

5.0. CONCLUSIONS.

Basic creep expressions and hardening theories have been briefly surveyed and it is concluded that the strain hardening hypothesis correctly describes the uniaxial creep deformation, except within the range of certain limitations. These limitations are found to be of two types. Firstly, within certain temperature ranges, metallurgical reactions such as thermal recovery, recrystallization, strain aging and various strain independent effects, result in the deformation depending on the creep rate/temperature history. Secondly, as a result of certain stress histories, deviations due to various deformation phenomena arise, such as the particular strain hardening effect of 'instantaneous' plastic strain, recovery of creep strain and mechanical recovery of structure.

Environmental results indicate that creep deformation predictions based on the strain hardening hypothesis, even in the range of these limitations, are not usually in error by more than ± 50%. This is with the exception of the case of stress reversal when the usual Bauschinger effect limitation may result in a marked increase in the creep rate. It should also be noted that for some materials inhomogenity may also be of the order of ± 50% on the creep strain.

Modifications to overcome the limitations of the strain hardening hypothesis have been considered, the conclusion being that if the prime interest is in the net effect rather than transient deformation such modifications are possible. The general impression is that the modification most likely to be successful is a division into an anelestic creep component obeying Boltzmann superposition, and a plastic component obeying strain hardening (with the mechanical recovery and metallurgical reaction limitations dealt with by equivalent plastic strains, equivalent stresses and variable coefficients). The major limitation of this modified strain hardening theory would appear to be the mechanical recovery resulting from stress reversal and stress increments/decrements generally. However the value of this theory is twofold.

Firstly, it can assist in anticipation of the importance of the various limitations in a particular case and in the selection of tests for their investigation. Secondly, it allows approximate estimations of the creep deformation in situations where the limitations are important.

Rheological theories, although attractive in principle, appear to be less accurate at present than the direct application of the strain hardening hypothesis, especially when the latter is modified as suggested.

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