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The Calculation of the Shape of an Electric Arc Discharge Rotating in an Annular Gap under the Influence of a Non-Uniform Longitudinal Applied Magnetic Field

by

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THE CALCULATION OF THE SHAPE OF AN ELECTRIC ARC DISCHARGE ROTATING IN AN ANNULAR GAP UNDER THE INFLUENCE OF A NON-UNIFORM LONGITUDINAL APPLIED MAGNETIC FIELD

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SUMMARY

This Paper explores the possible shapes of an electric arc discharge rotating in an annular gap under the influence of a non-uniform longitudinal magnetic field.

It is shown that there are two families of possible arc shapes, all dependent on the form of the applied magnetic field. The first extend outwards from a radius r_1 where the arc is radial and the second extend inwards from radius r_1.

The first family can be subdivided into three types. Arcs which are S-shaped, extending only to a second radius where they are again radial; arcs which may extend out to infinity in an involute-like form; and arcs which are involute-like near to r_1 but terminate as a circle at a radius where the magnetic field is zero.

The second family can be subdivided into two types. Arcs which are S-shaped, extending only to a smaller radius where they are again radial; and arcs which terminate as a circle at a radius where the magnetic field is zero. The circle may have zero radius in the limiting case where the field is zero at the centre.

It is shown that under certain precise conditions arcs may extend from one family through radius r_1 to the other family forming one long uniformly rotating arc.

Finally, the possible marriage of an arc from the first family with one from the second family to form a mutually compatible arc pair is demonstrated. In this case the arcs rotate in opposite directions.

Replaces R.A.E. Technical Note No. 2965 - ARC 26358

CONTENTS

ILLUSTRATIONS

ILLUSTRATIONS (Contd.)

SYMBOLS

a a constant defined in equation (12)

B the magnetic field strength

b a constant defined in equation (12)

$F(r)$ a function of r used to designate B

I the arc current

K a constant defined in equation (4)

m a constant defined in equation (4)

n a constant defined in equation (12)

r the radius

u the velocity of an arc element

α an arc parameter. $\alpha \equiv \dfrac{B_o}{B_o + b\, r_1^n}$, para 3.3

θ an angle defined in Fig.1

ϕ an angle defined in Fig.1

ω the apparent angular velocity of the arc

Suffices

o indicates a quantity at centre

1 indicates a quantity where the arc is radial

t indicates a particular value of α such that $\alpha = \alpha_t = \dfrac{n-2}{n}$ and the curve of $\left\{\alpha_t + \left(1 - \alpha_t\right)\left(\dfrac{r}{r_1}\right)^n\right\}$ is tangent to $\left(\dfrac{r}{r_1}\right)^2$ at $\left(\dfrac{r}{r_1}\right) = 1$

1 INTRODUCTION

A recent paper by V.W.Adams[1] gives a theoretical model for an electric arc, from which the shape of an arc in an annular gap rotating under the influence of a uniform axial magnetic field is deduced. The shape in this case is an involute. Experimental evidence is given to show that when the gap between the electrodes is sufficiently large, the arc does frequently take-up this form with the central electrode the base circle of the involute.

The purpose of this Paperis to determine the arc shape when the magnetic field is not uniform. It is assumed that the applied axial magnetic field B is of the form

$$B = a + b r^n ,$$

where a, b and n are constants and r is the radius. (The arc is assumed to stay in the one plane where there is no radial field.) More complicated expressions for the magnetic field could be used and the method is applicable to all fields which are known functions of the radius. It can, therefore, be used to deduce the arc shape in any particular experiment.

2 THE ARC EQUATION

Assume that the arc appears to rotate uniformly at an angular velocity ω about a point O (Fig.1) and that this apparent rotation is produced by the instantaneous motion of each part of the arc at velocity u in a direction always normal to the current and to the magnetic field. Physically this means that we regard as small the inertia of the arc and the rate of change of momentum of each section of arc, as its length changes, compared to the electro-magnetic force applied to the arc.

Consider point P on the arc at distance r from O where the current I is in the direction shown in Fig.1(a). (Fig.1(b) is an alternative case. The following discussion applies equally to both figures.) Then at P, u is normal to I and $r\,\omega$ is normal to OP. The angle between u and $r\,\omega$ is ϕ where

$$\cos \phi = \frac{u}{r\omega} \tag{1}$$

and

$$\tan \phi = r \frac{d\theta}{dr} . \tag{2}$$

Assume that the longitudinal magnetic field

$$B = F(r) , \tag{3}$$

that in the plane of the arc there is no other component and that at all times

$$u = K B^m \tag{4}$$

where K and m are constants. Then from (1) and (4)

$$r\omega \cos \phi = K B^m = K \{F(r)\}^m . \tag{5}$$

If at $r = r_1$, $\phi = 0$ then

$$r_1 \omega = K \{F(r_1)\}^m \tag{6}$$

and from (5)

$$\cos \phi = \frac{r_1}{r} \left\{ \frac{F(r)}{F(r_1)} \right\}^m \tag{7}$$

from which it follows that

$$\tan \phi = \pm \left[\frac{r^2 \left\{ F(r_1) \right\}^{2m} - r_1^2 \left\{ F(r) \right\}^{2m}}{r_1^2 \{F(r)\}^{2m}} \right]^{\frac{1}{2}} \tag{8}$$

and from (2)

$$\frac{d\theta}{dr} = \pm \frac{1}{r} \left[\frac{r^2 \{F(r_1)\}^{2m} - r_1^2 \{F(r)\}^{2m}}{r_1^2 \{F(r)\}^{2m}} \right]^{\frac{1}{2}} . \tag{9}$$

The arc shape in terms of θ as a function of r can be obtained by integrating this equation. However there will be limitations on the possible values of r, r_1 and $F(r)$ such that $d\theta/dr$ is real. Furthermore having met the condition that $d\theta/dr$ is real there will still be two solutions as indicated by the ± signs. The positive sign is illustrated by Fig.1(a) and the negative sign by Fig.1(b). Some physical reasoning will be needed to deduce which sign, or whether both are admissible, in any particular case. This is discussed further in para 3.2 where it is concluded that the positive sign is most probable when r/r_1 is greater than one and the negative sign when r/r_1 is less than one.

2.1 Simplification

Adams[1] reports that for his experimental observations at atmospheric pressure with long arcs

$$u \propto B^{0.6} I^{0.3} .$$

It will clearly make further consideration of equation (9) much easier if we assume that, for constant current

$$u \propto B^{0 \cdot 5}$$

i.e.

$$m = 0 \cdot 5 \, .$$

This is the same assumption as made by Adams in deducing that the arc should be of involute shape when in a constant magnetic field and is consistent with the very crude idea that the arc behaves as a solid body of constant cross-section and drag coefficient.

When m = 0·5 equation (9) may be rewritten as

$$\frac{d\theta}{dr} = \pm \frac{1}{r} \left\{ \left(\frac{r}{r_1}\right)^2 \frac{F(r_1)}{F(r)} - 1 \right\}^{\frac{1}{2}} \tag{9a}$$

and this approximation is followed throughout the discussion below.

3 ARC SHAPES

3.1 Constant longitudinal applied magnetic field

If B is constant, $\frac{F(r_1)}{F(r)} = 1$ and from equation (9a)

$$d\theta = \pm \left\{ \frac{r^2}{r_1^2} - 1 \right\}^{\frac{1}{2}} \frac{dr}{r} = \pm \left\{ \frac{r \, dr}{r_1 (r^2 - r_1^2)^{\frac{1}{2}}} - \frac{r_1 \, dr}{r (r^2 - r_1^2)^{\frac{1}{2}}} \right\} .$$

If $\theta = 0$ when $r = r_1$ we have a solution when $\frac{r}{r_1}$ greater than 1 given by

$$\theta = \pm \left[\frac{(r^2 - r_1^2)^{\frac{1}{2}}}{r_1} - \cos^{-1} \frac{r_1}{r} \right] \tag{10}$$

This is the equation of either a forward or backward facing involute with a base circle of radius r_1 as given by Adams.

3.2 Magnetic field varies with radius but zero at the centre

If $B = b \, r^n$, $F(r_1)/F(r) = r_1^n / r^n$ and from equation (9a)

$$d\theta = \pm \left\{ \frac{r^{2-n}}{r_1^{2-n}} - 1 \right\}^{\frac{1}{2}} \frac{dr}{r} .$$

Clearly $n = 2$ is a critical value. When n is less than 2, r/r_1 must be greater than 1 and when n is greater than 2, r/r_1 must be less than 1. In both cases the integration can be performed by substitution and putting $\theta = 0$ when $r = r_1$, we get

$$\theta = \pm \frac{2}{2-n}\left[\left\{\left(\frac{r}{r_1}\right)^{2-n} - 1\right\}^{\frac{1}{2}} - \cos^{-1}\left(\frac{r}{r_1}\right)^{\frac{n-2}{2}}\right] . \qquad (11)$$

We thus obtain two families of arc shapes as illustrated in Figs.2 and 3, corresponding to n less than 2 or greater than 2 respectively. The first family reach outwards from the radius r_1 where the arc is radial and the second family reach inwards towards the centre. In the particular case considered here, where the field at the centre is zero, the first family are "involute-like" and reach out to infinity whilst the second family reach into the centre finally tending to circles of zero radius.

It should be noted that in Figs.2 and 3, the arcs have been drawn so that the section of arc at radius r_1 leads in the direction of rotation. That is the positive sign has been chosen in equation (11) when n is less than 2 and the negative sign when n is greater than 2. This appears to be the most probable interpretation from considerations of stability and from considering the development of the arc shape with time. There is of course experimental evidence to support this for arcs of the first family.

The radius r_1 does not necessarily represent an electrode. It is simply the radius of a circle where $\phi = 0$. When n is less than 2, the radius of the inner electrode can be equal to or greater than r_1 and the radius of the second electrode must be greater than r_1. Similarly when n is greater than 2, the radius of the outer electrode can be equal to or less than r_1 and the radius of the second electrode must be less than r_1.

3.3 Magnetic field varies with radius and finite at the centre

For our most general case assume

$$B = a + b\, r^n \qquad (12)$$

where a, b and n are constants.

If $B = B_o$ when $r = r_o$ and $B = B_1$ when $r = r_1$ then

$$\frac{B}{B_1} = \frac{B_o}{B_o + b\, r_1^n} + \frac{b\, r_1^n}{B_o + b\, r_1^n}\left(\frac{r}{r_1}\right)^n \equiv \alpha + (1-\alpha)\left(\frac{r}{r_1}\right)^n$$

where $\alpha = \dfrac{B_o}{B_o + b\, r_1^n} = \dfrac{B_o}{B_1}$ and $b = \dfrac{B_o(1-\alpha)}{\alpha\, r_1^n}$.

We then have that $\dfrac{F(r)}{F(r_1)} = \dfrac{B}{B_1} = \alpha + (1-\alpha)\left(\dfrac{r}{r_1}\right)^n$ and from equation (9a)

$$\frac{d\theta}{dr} = \pm \frac{1}{r}\left[\left(\frac{r}{r_1}\right)^2 \left\{\alpha + (1-\alpha)\left(\frac{r}{r_1}\right)^n\right\}^{-1} - 1\right]^{\frac{1}{2}}$$

or

$$\frac{d\theta}{d\left(\frac{r}{r_1}\right)} = \pm\left[\left\{\alpha + (1-\alpha)\left(\frac{r}{r_1}\right)^n\right\}^{-1} - \left(\frac{r_1}{r}\right)^2\right]^{\frac{1}{2}} . \qquad (13)$$

A general solution to equation (13) has not been found. However we can proceed by selecting appropriate values of n and finding the range of α where solutions exist. Particular values of α can then be chosen and arc shapes determined by simple graphical integration.

For $\dfrac{d\theta}{d\left(\frac{r}{r_1}\right)}$ to be real $\left\{\alpha + (1-\alpha)\left(\dfrac{r}{r_1}\right)^n\right\}^{-1}$ must be greater than or equal to $\left(\dfrac{r_1}{r}\right)^2$ and because $\left(\dfrac{r_1}{r}\right)^2$ is always positive the following equation always holds when solutions are possible.

$$0 \leqslant \alpha + (1-\alpha)\left(\frac{r}{r_1}\right)^n \leqslant \left(\frac{r}{r_1}\right)^2 \qquad (14)$$

or

$$0 \leqslant \frac{B}{B_1} \leqslant \left(\frac{r}{r_1}\right)^2 . \qquad (14a)$$

Note that when $\alpha + (1-\alpha)\left(\dfrac{r}{r_1}\right)^n = 0$ then $B = 0$,

when $\alpha = 1$ then $B = B_1$, i.e. the field is constant (para 3.1)

and when $\alpha = 0$ then $B_o = 0$, i.e. $B = b\, r^n$ (para 3.2).

In Figs.4 to 8 curves of $\alpha + (1 - \alpha)\left(\frac{r}{r_1}\right)^n$ and $\left(\frac{r}{r_1}\right)^2$ are plotted against $\left(\frac{r}{r_1}\right)$, each figure applying to a particular value of n. Equation (14) indicates that solutions to equation (13) exist when a value of $\alpha + (1 - \alpha)\left(\frac{r}{r_1}\right)^n$ plots above the horizontal axis and to the right of the $\left(\frac{r}{r_1}\right)^2$ curve. That is electric arcs which are radial at r_1 and have an apparent speed of rotation proportional to $B_1^{0\cdot5}/r_1$ can exist in regions where $\frac{B}{B_1}$ lies between 0 and $\left(\frac{r}{r_1}\right)^2$. It should be noted that this does not mean that there are magnetic fields wherein no arc can exist but that with a particular magnetic field there are limits on the possible values of r_1.

Figs.4 to 8 will be more easily understood if it is first appreciated that when n is negative and $\left(\frac{r}{r_1}\right) = 0$, $\alpha + (1 - \alpha)\left(\frac{r}{r_1}\right)^n = \pm\infty$ according to whether α is less than or greater than 1. However when n is positive and $\left(\frac{r}{r_1}\right) = 0$ then $\alpha + (1 - \alpha)\left(\frac{r}{r_1}\right)^n = \alpha$.

We again have two families of arcs, those which extend outwards from r_1 and those which reach inwards from r_1 but this time the dependence does not rely entirely upon the value of n and there are variations in each family.

The first family contains arcs which extend outwards to infinity in an involute-like form (e.g. $n = -1$ and $\alpha = 2$, $n = 0\cdot5$ and $\alpha = 0\cdot2$, $n = 1$ and $\alpha = -0\cdot5$, $n = 2$ and $\alpha = 0\cdot2$). These cannot exist when n is greater than 2. Next there are arcs which extend outwards in a similar manner but become circular at a finite radius where the magnetic field becomes zero, (e.g. $n = -1$ and $\alpha < 1$, $n = 0\cdot5$ and $\alpha > 1$, $n = 1$ and $\alpha > 1$, $n = 2$ and $\alpha > 1$, $n = 3$ and $\alpha > 1$). Lastly there are arcs which extend outwards from radius r_1 in an 'S' shape to a second radius where, as at r_1, they are again radial. These arcs can only exist when n is greater than 2 and α is between α_t and 1 (α_t is defined below). Note that when $n = 2$ and $\alpha = 0$ the arc is radial at all radii.

Consider next the second family which reach inwards from radius r_1. Here we may distinguish two sub-types of arc. Firstly, those which reach inwards from r_1 to a second radius where the magnetic field is zero and the arc becomes circular. These include the limiting case when n is equal or greater than 2 and $\alpha = 0$ when the inner radius becomes zero. (e.g. $n = -1$ and $\alpha > \alpha_t$, $n = 0\cdot5$ and $\alpha < \alpha_t$, $n = 1$ and $\alpha < \alpha_t$, $n = 2$ and $\alpha < \alpha_t$, $n = 3$ and $\alpha < 0$). Second in this family are the arcs which run inwards from radius r_1 in an 'S' shape to a second radius where they again become radial. These arcs only exist when n is greater than 2 and α lies between α_t and 0.

Inspection of Figs.4 to 6 where n is less than 2 shows that where an arc runs outwards from radius r_1 there frequently exists for the same values of n and α a region between two radii, both less than r_1, where a second arc can exist. This arc will be circular at its inner end and radial at its outer end and therefore belongs to the second family of arc shapes. These pairs of arcs occur when the chosen values of n and α are such that two arcs rotating at the same angular velocity are possible. (e.g. $n = -1$ and $\alpha = 2$, $n = 1$ and $\alpha = -0{\cdot}5$.)

A similar pair of arcs, each with the same angular velocity, can also exist when the main arc runs inwards from r_1. In this case when n is less than 2 a second arc may extend from a radius greater than r_1, where it is radial, outwards to infinity. (e.g. $n = -1$ and $\alpha = 5$, $n = 1$ and $\alpha = -1{\cdot}5$.)

There is one value of $\alpha \equiv \alpha_t = \frac{n-2}{n}$ where, when n is less than or equal to 2, an arc can extend from a radius less than r_1 where it is circular, through r_1 and out to infinity. This is really a limiting case of the arc pairs discussed above. (e.g. $n = -1$ and $\alpha_t = 3$, $n = 0{\cdot}5$ and $\alpha_t = -3$, $n = 1$ and $\alpha_t = -1$, $n = 2$ and $\alpha_t = 0$.)

A number of calculated arc shapes are illustrated in Fig.9. It should be noted that those arcs in the first family marked $n = 1$ and $\alpha = 0{\cdot}8$, $n = 2$ and $\alpha = 0{\cdot}8$, $n = 1$ and $\alpha = 0{\cdot}2$, $n = 1$ and $\alpha = -1$, $n = 2$ and $\alpha = 0{\cdot}2$ are all of the type which extend outwards in an involute-like form to infinity. The arcs marked $n = 3$ and $\alpha = 0{\cdot}8$, $n = 3$ and $\alpha = 0{\cdot}5$ are both 'S'-shaped and have a finite outer-radius. Fig.10 (see para 5) illustrates an arc extending outwards to a finite radius where it becomes circular.

The second family of arcs are illustrated in Fig.9 by the two sub-types $n = 2$ and $\alpha = -0{\cdot}1$ which runs inwards to a circle and $n = 3$ and $\alpha = 0{\cdot}2$ which is 'S'-shaped.

4 ARCS COMMON TO EACH FAMILY

An 'S'-shaped arc of the first family which extends outwards from r_1 to a second radius r_2, where it is again radial, is equally an arc of the second family extending inwards from r_2 to a second radius r_1, where it is again radial. This can most easily be demonstrated numerically.

Consider the arc obtained when $n = 3$ and $\alpha = 0{\cdot}5$. If this arc extends from an inner radius r_1 to an outer radius r_2, then the value of r_2/r_1 can be obtained by solving the equation

$$\alpha_1 + (1 - \alpha_1)\left(\frac{r_2}{r_1}\right)^n = \left(\frac{r_2}{r_1}\right)^2 \qquad (15)$$

when $n = 3$ and $\alpha_1 = 0{\cdot}5$. Ignoring the negative root, this leads to

$$\frac{r_2}{r_1} = 1 \quad \text{or} \quad 1{\cdot}618$$

and

$$B_2 = B_o + B_o \left(\frac{r_2}{r_1}\right)^3 = B_o(1 + 1{\cdot}618^3) \ .$$

Therefore

$$\alpha_2 \equiv \frac{B_o}{B_2} = 0{\cdot}19 \ .$$

Inspection of Fig.8 will reveal that an arc where $\alpha = 0{\cdot}19$ and $n = 3$ is an arc of the second family running inwards to a second radius where it is again radial. We thus see that an 'S'-shaped arc may be regarded as belonging to both families.

5 MULTIPLE ARCS

We have already seen one double arc obtained in the limiting case where $\alpha = \alpha_t$ and n is less than or equal to 2. Such an arc is formed from one arc of each family meeting at a radius where both are radial. This is illustrated in Fig.9 for the case where $n = 1$ and $\alpha = -1$. In this case the two halves of the arc rotate at the same angular velocity in the same direction.

A second type of double arc appears to be possible if an arc of the first family which terminates at a circle where the magnetic field is zero can be joined with an arc of the second family which terminates at the same circle.

Consider two arcs, an inner one extending from radius r_1 (where $\phi = 0$) outwards to r_2 where $B = 0$ and an outer arc extending inwards from r_3 (where $\phi = 0$) to r_2. Assume that for both arcs $n = 2$ and for the first $\alpha_1 = 1{\cdot}2$, then

$$b = \frac{B_o\,(1 - \alpha_1)}{\alpha_1\, r_1^2} = -\frac{B_o}{6\, r_1^2}$$

and

$$B = B_o \left\{1 - \frac{1}{6}\left(\frac{r}{r_1}\right)^2\right\} \ .$$

But we require $B_2 = 0$ at $r = r_2$ so that

$$\frac{r_2}{r_1} = 6^{\frac{1}{2}} \approx 2{\cdot}45 \quad .$$

Consider now the outer second arc

$$b = \frac{B_o (1 - \alpha_3)}{\alpha_3 r_3^2} = - \frac{B_o}{6 r_1^2}$$

so that

$$\left(\frac{r_3}{r_1}\right)^2 = \frac{6(\alpha_3 - 1)}{\alpha_3}$$

We may choose any suitable value of α_3. i.e. α_3 less than zero as shown in Fig.7. If we assume $\alpha_3 = -0{\cdot}1$, then

$$\frac{r_3}{r_1} = 66^{\frac{1}{2}} \approx 8{\cdot}12 \quad \text{and} \quad \frac{r_3}{r_2} \approx 3{\cdot}32 \quad .$$

These two arcs are illustrated in Fig.10. They rotate in opposite directions. Whether they could be made to operate in practice without a solid conducting ring at r_2 is not known. However the idea is important as showing a system which might be designed so as to impart zero nett swirl to the air surrounding the arc and also as a possible way of producing a toroid of plasma.

These ideas can be extended further. For example, if a pair of arcs is constructed as above but with n less than 2 then a value of α for the outer arc can be chosen such that $\alpha_3 = \alpha_t$ and the outer arc will then extend out to infinity in a involute-like form.

6 CONCLUDING REMARKS

Using a very simple model of an electric arc we have deduced the shape of an arc in an annular gap under the action of a non-uniform longitudinal magnetic field. This model has previously been demonstrated to predict the correct shape when the magnetic field is uniform[1]. However this is not as significant as it might at first appear, because when the magnetic field is uniform the predicted shape is an involute and all parts of the arc then travel at constant speed along straight lines. Under these circumstances it is reasonable to assume that the arc will be nearly uniform in all its properties from end to end. Any non-uniformity would have to be primarily connected with the varying rate of growth per unit length of the arc.

When the magnetic field changes with radius each part of the arc moves along a curved path at a varying speed. This must result in an arc which is not uniform along its length. Furthermore the above analysis assumes that all parts of the arc are at any instant travelling in a direction and at a velocity appropriate to the local value of the magnetic field, thus ignoring the inertia of the arc.

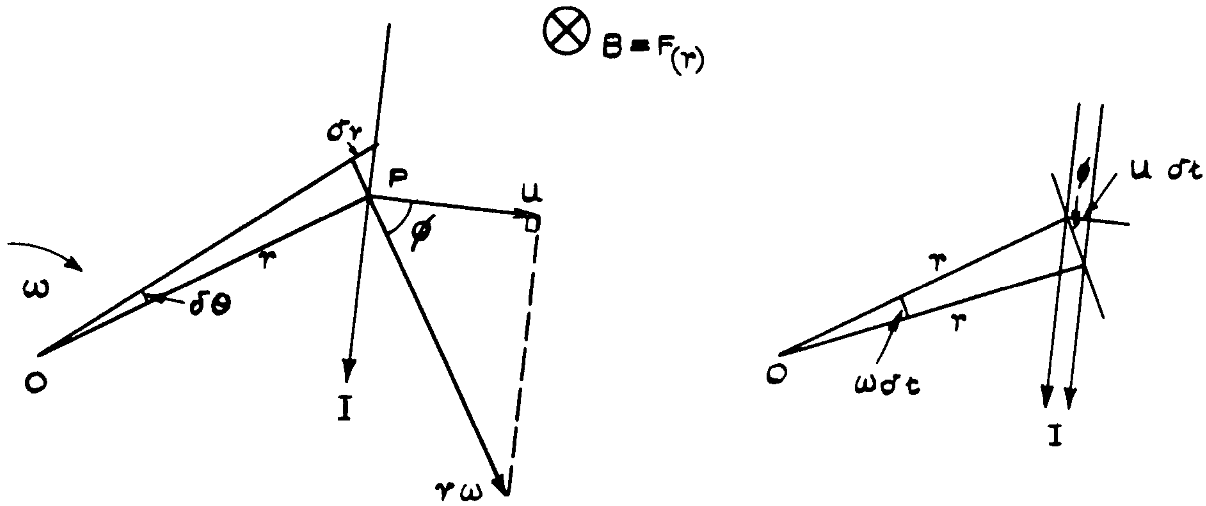
Experiments are now in hand to explore the arc shapes with a non-uniform magnetic field. There are, as always, practical difficulties and severe limitations on the fields which can be produced. However the results cannot fail to throw more light on the validity of the above assumptions and increase our understanding of arc behaviour.

The stability of an arc shape is an important problem to the heater designer and, in this connection, the arcs which terminate at radii where both ends of the arc are radial are interesting. In this case if electrodes are placed at the appropriate radii, then the arc shape would appear to be fixed in contrast to the involute-like arc which is only restricted by the necessity that the inner electrode must be at a radius equal to or greater than r_1.

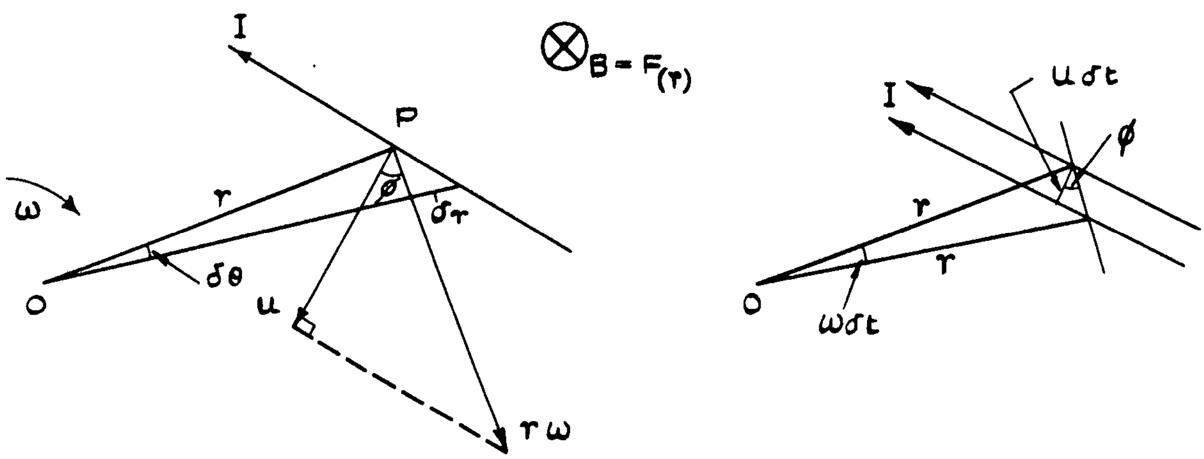
With a non-uniform magnetic field the analysis shows that it is possible to place the electrodes in a position so that the arc cannot be normal to either electrode surface. The effect this would have on arc stability is not obvious.

REFERENCE

No.	Author	Title, etc.
1	Adams, V.W.	The influence of gas streams at magnetic fields on electric discharges. Part 2. The shape of an arc rotating round an annular gap. A.R.C. C.P. 743, September 1963.



(a)



(b)

FIG.1. (a & b) ARC NOTATION

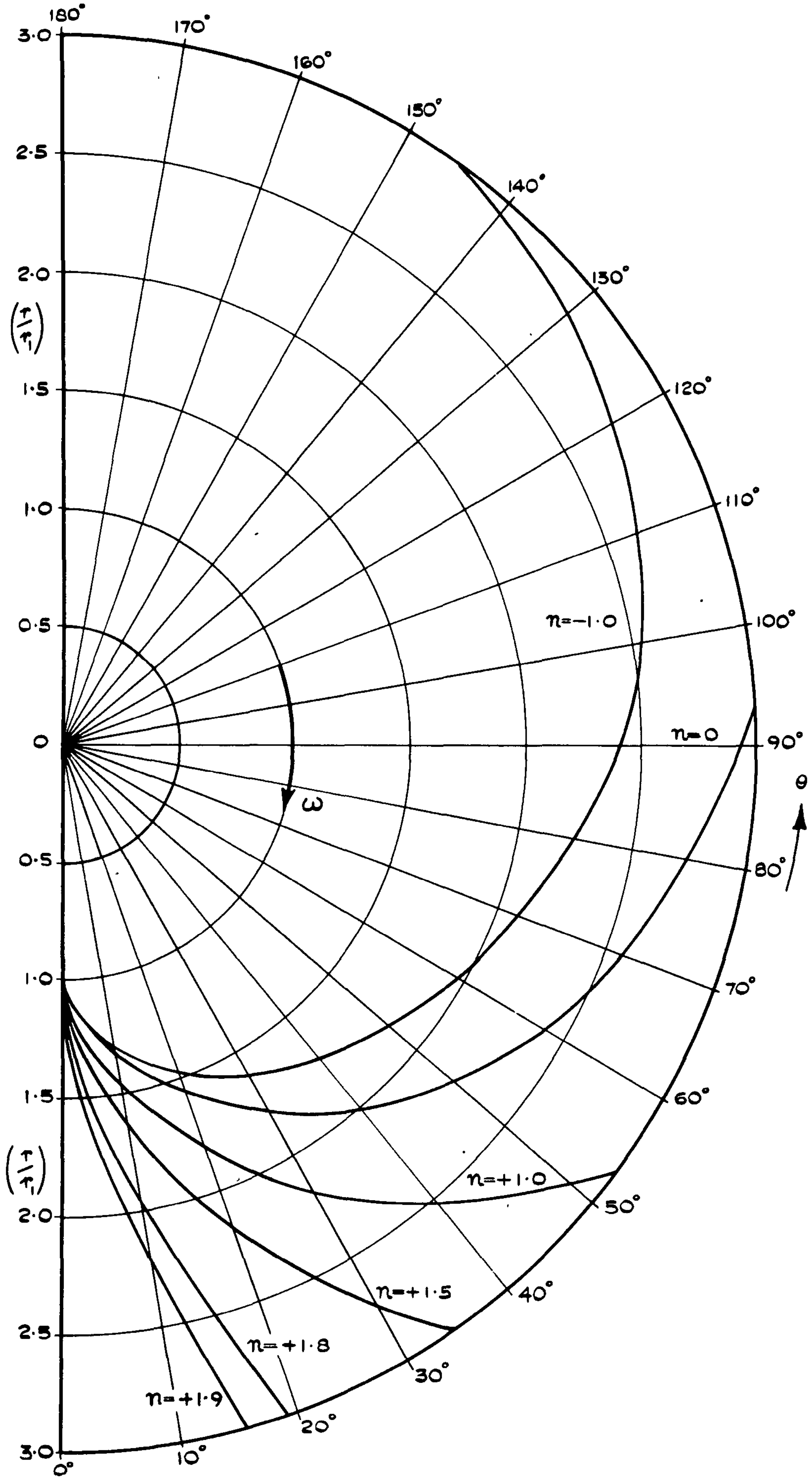


FIG. 2. ARC SHAPES WHEN $B = b\, r^n$ & $n < 2$.

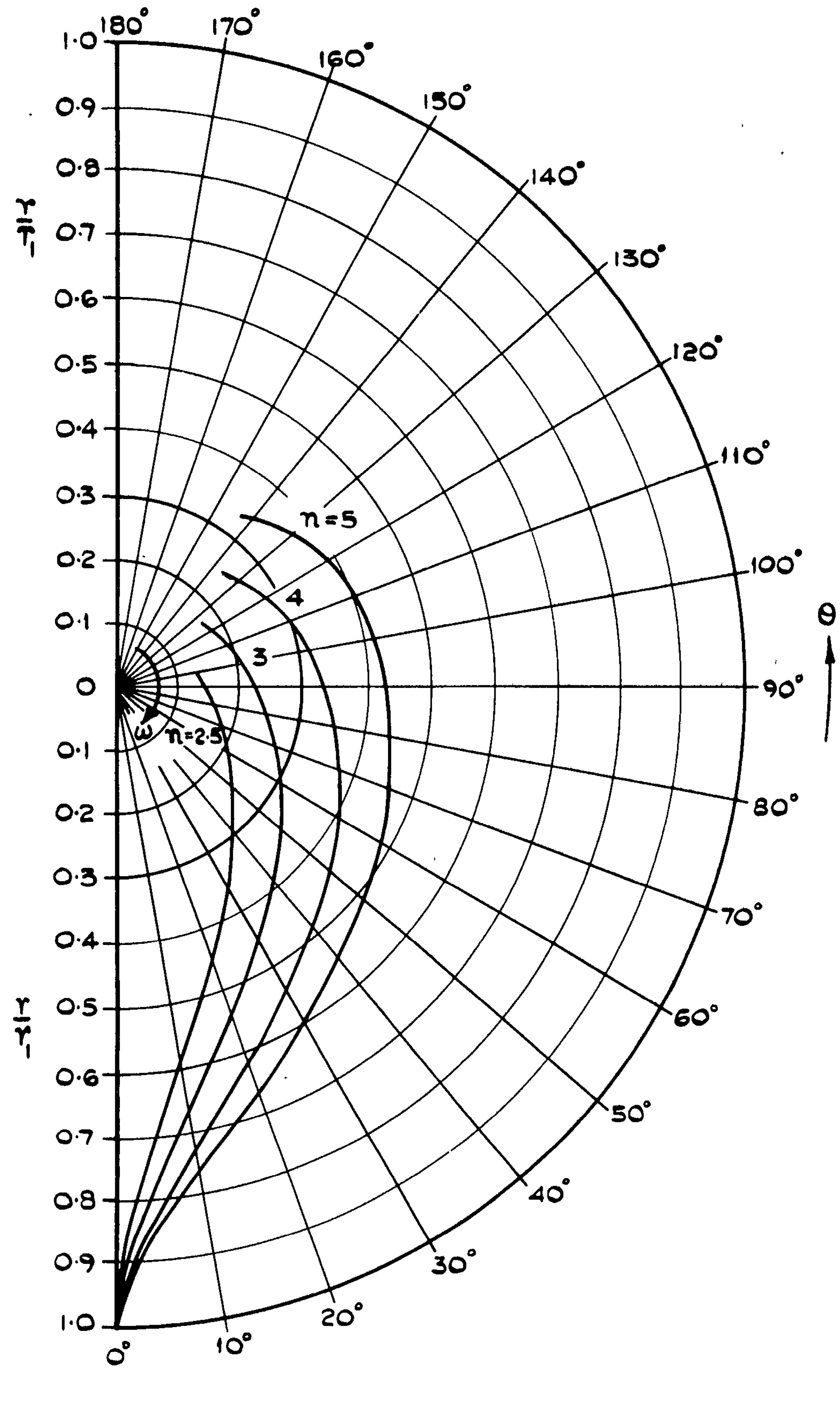


FIG. 3. ARC SHAPES WHEN $B = b\,r^{n}$ AND $n > 2$.

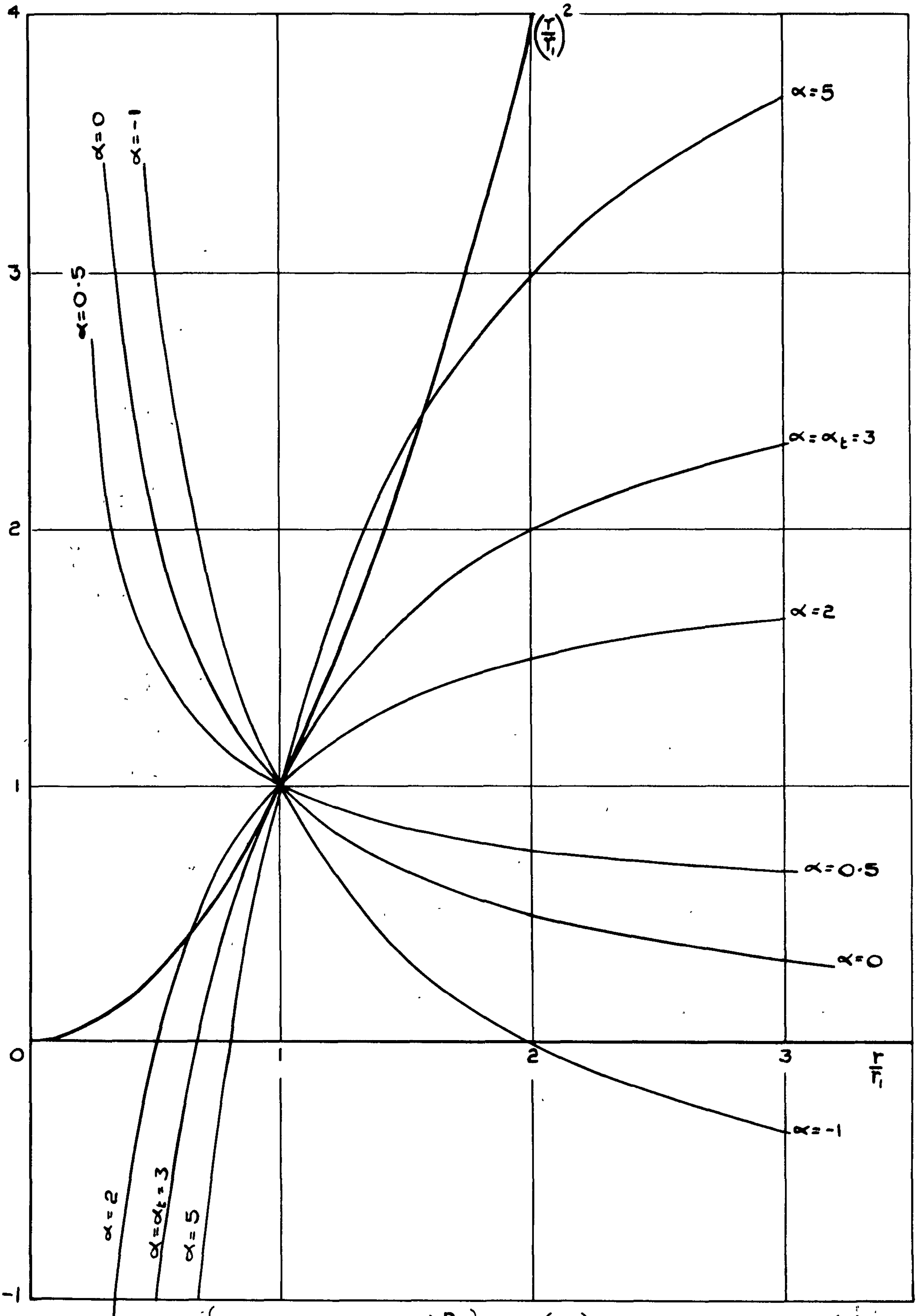


FIG. 4. $\left\{\alpha + (1-\alpha)\left(\frac{r}{r_1}\right)^n\right\}$ v. $\left(\frac{r}{r_1}\right)$ FOR $n = -1$

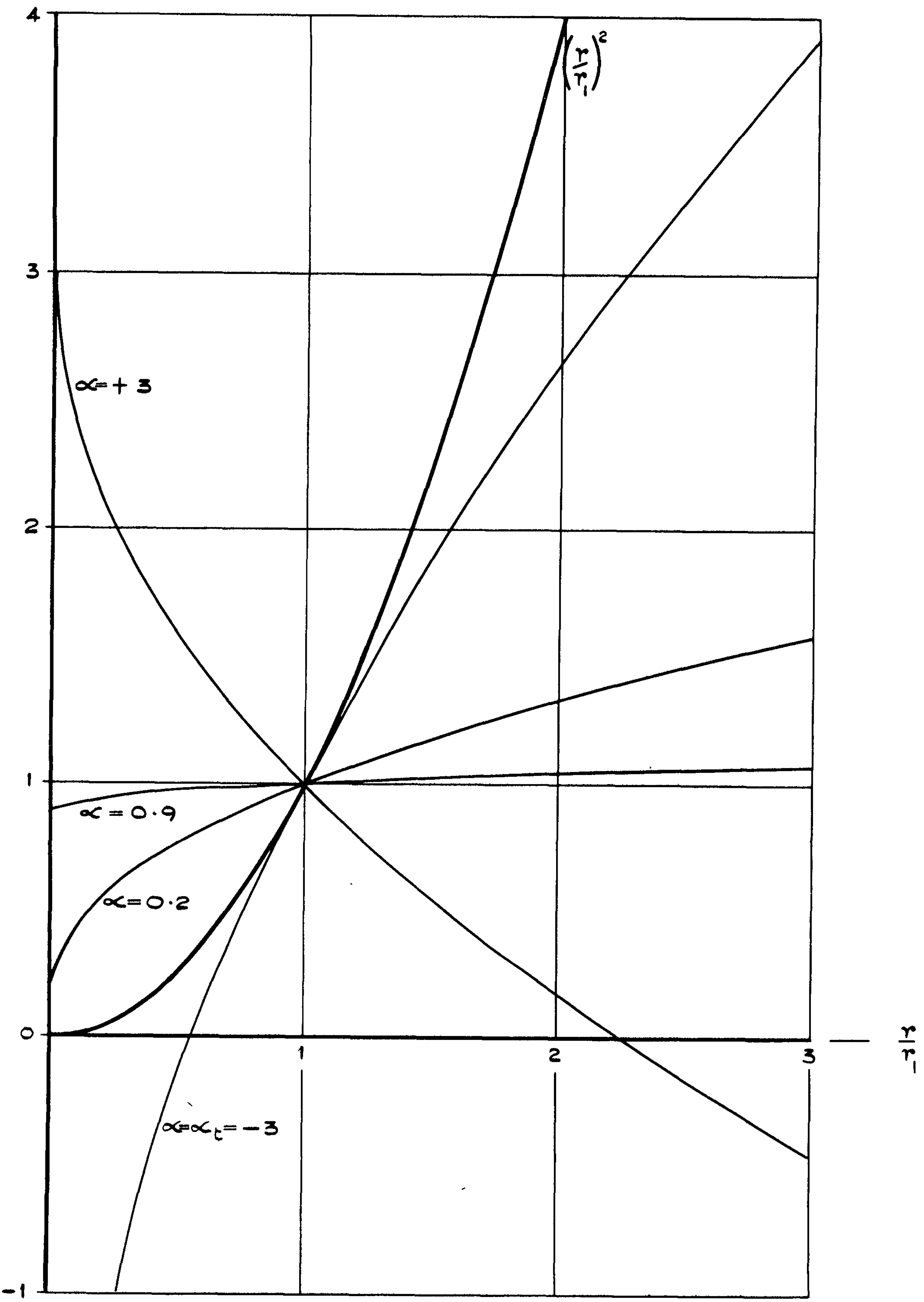


FIG. 5. $\left\{\alpha + (1-\alpha)\left(\frac{r}{r_1}\right)^n\right\}$ v. $\left(\frac{r}{r_1}\right)$ FOR $n = 0 \cdot 5$.

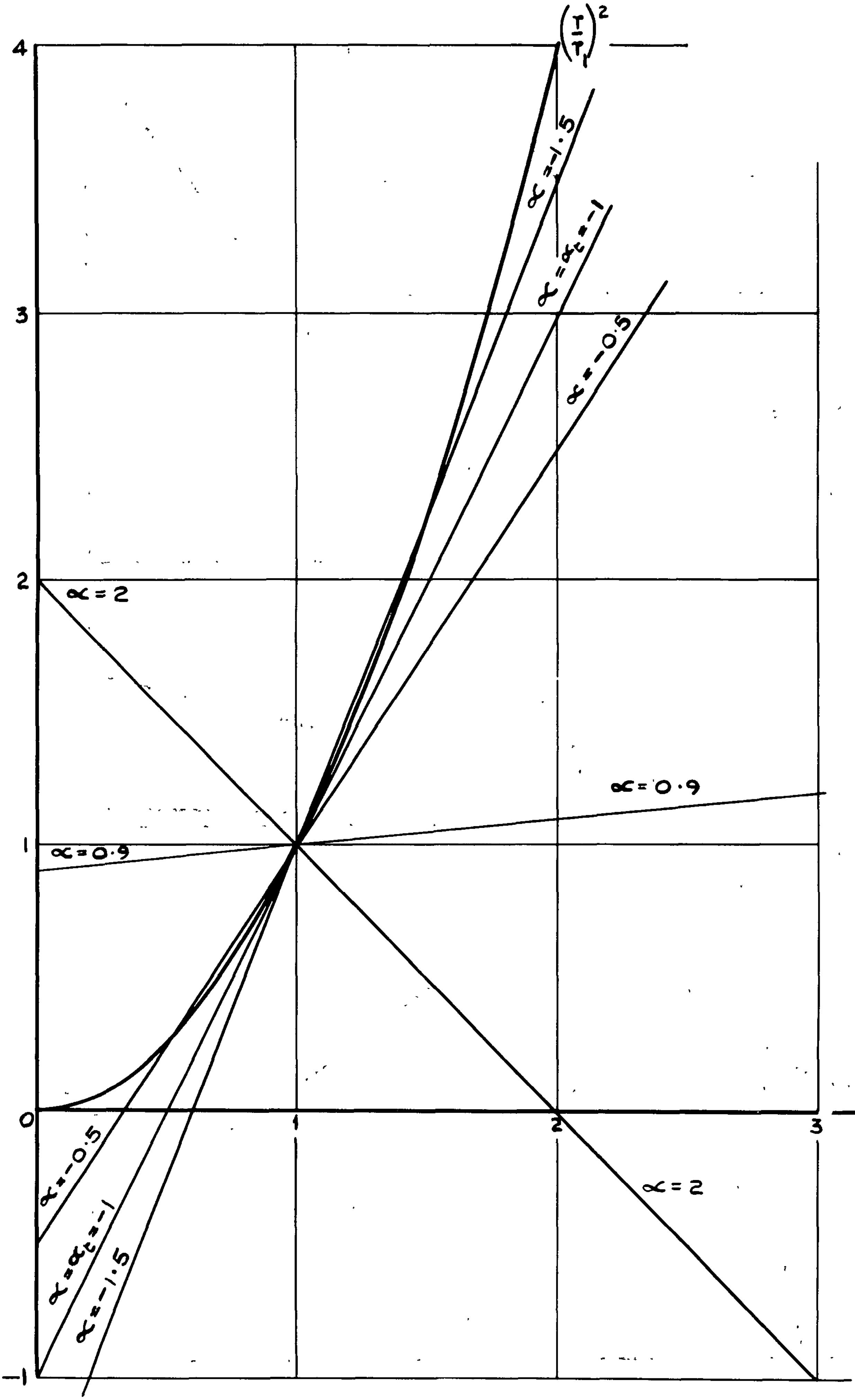


FIG 6 $\left\{\alpha + (1-\alpha)\left(\frac{r}{r_1}\right)^n\right\}$ v. $\left(\frac{r}{r_1}\right)$ FOR $n = 1$.

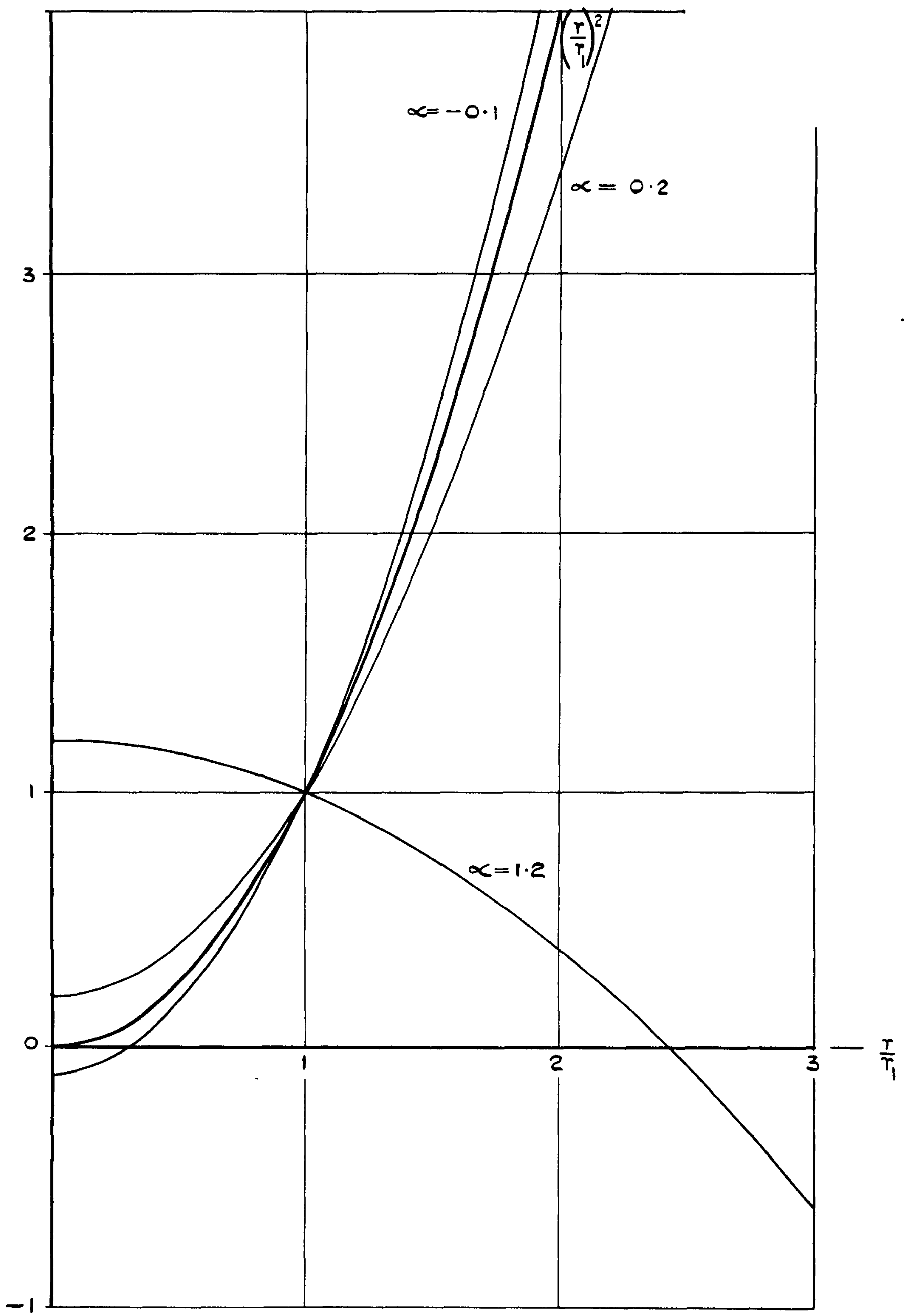


FIG. 7. $\left\{\alpha + (1-\alpha)\left(\frac{r}{r_1}\right)^n\right\}$ v. $\left(\frac{r}{r_1}\right)$ FOR $n = 2$.

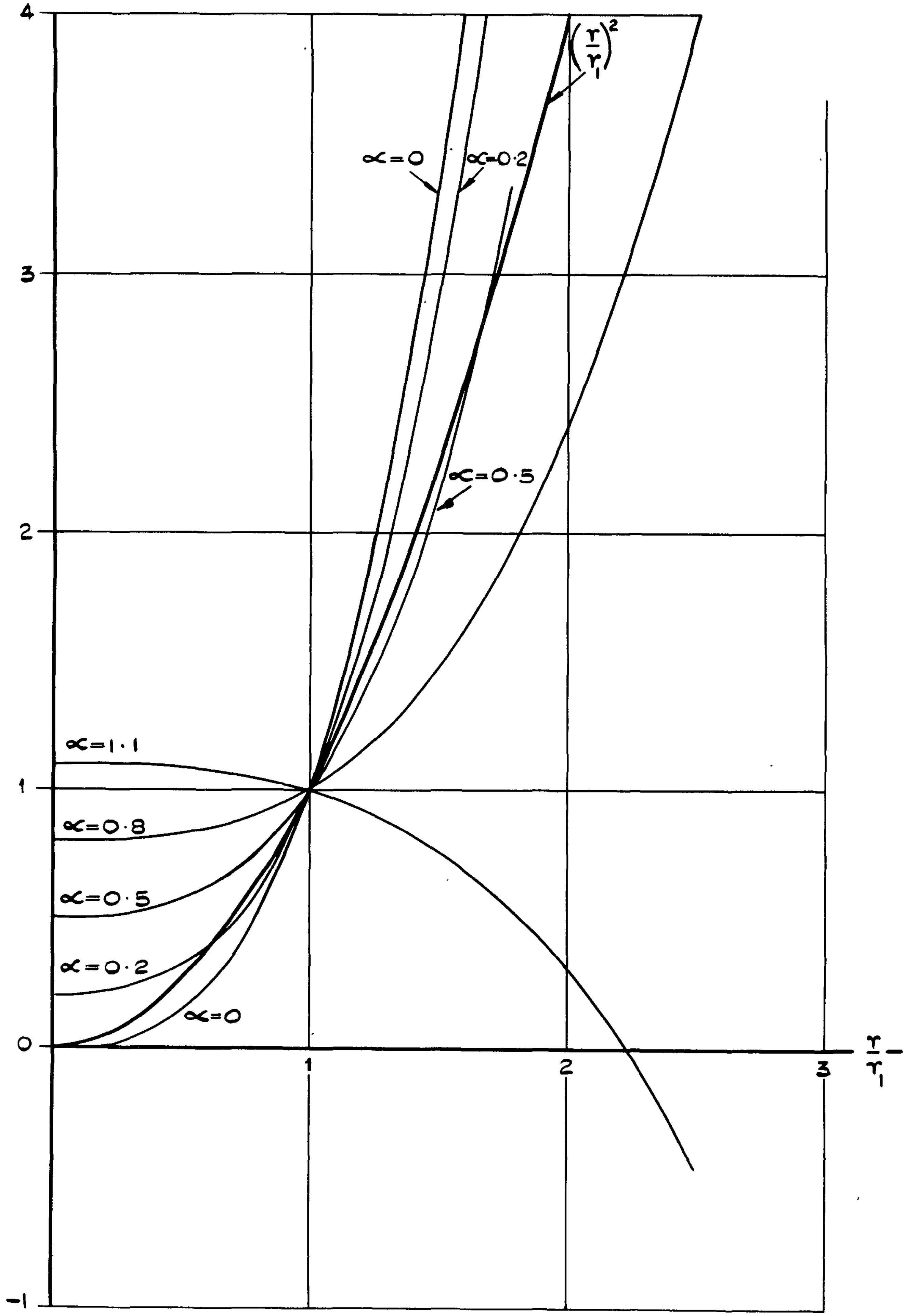


FIG.8. $\left\{\alpha + (1-\alpha)\left(\frac{r}{r_1}\right)^n\right\}$ v. $\left(\frac{r}{r_1}\right)$ FOR $n = 3$.

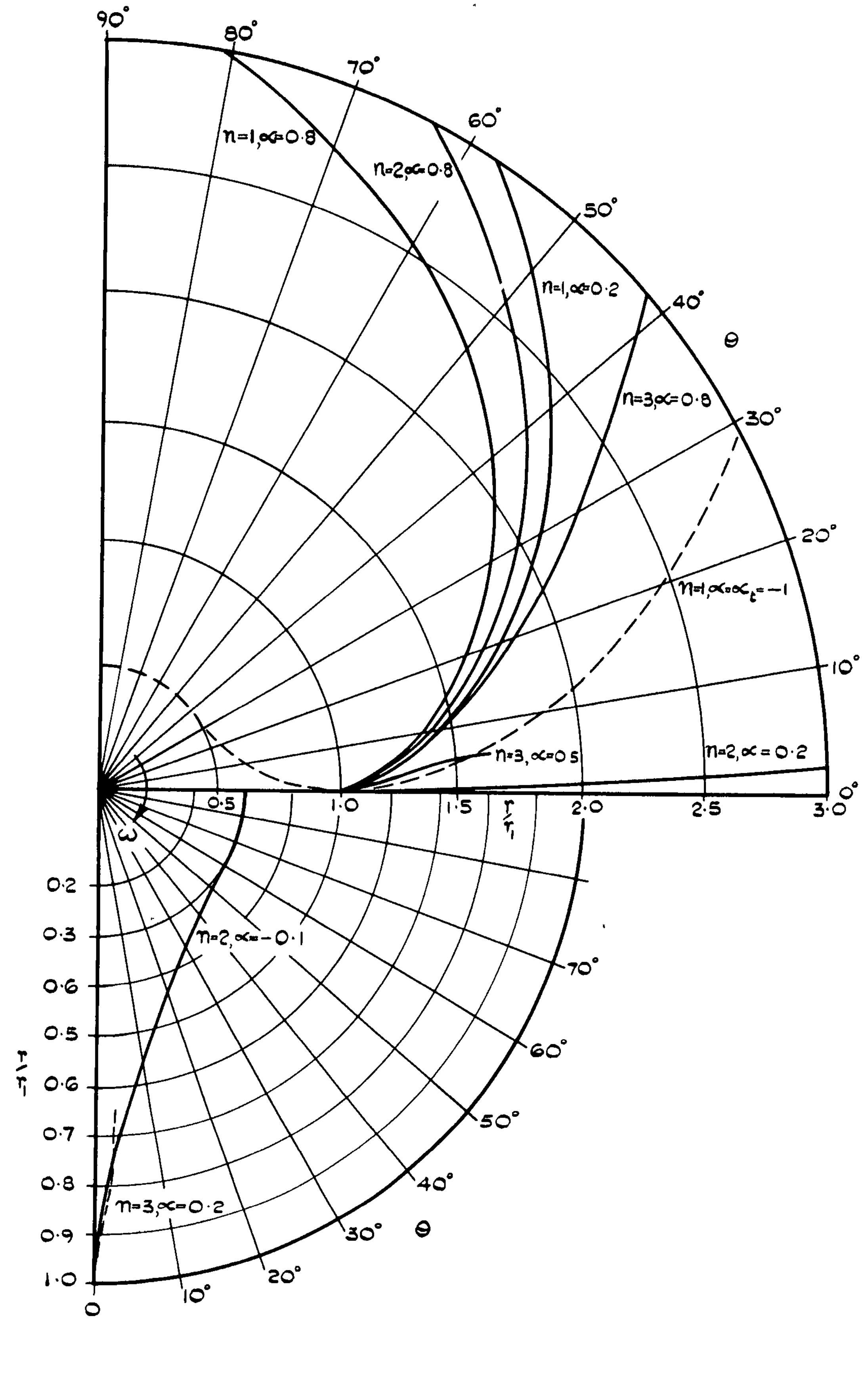
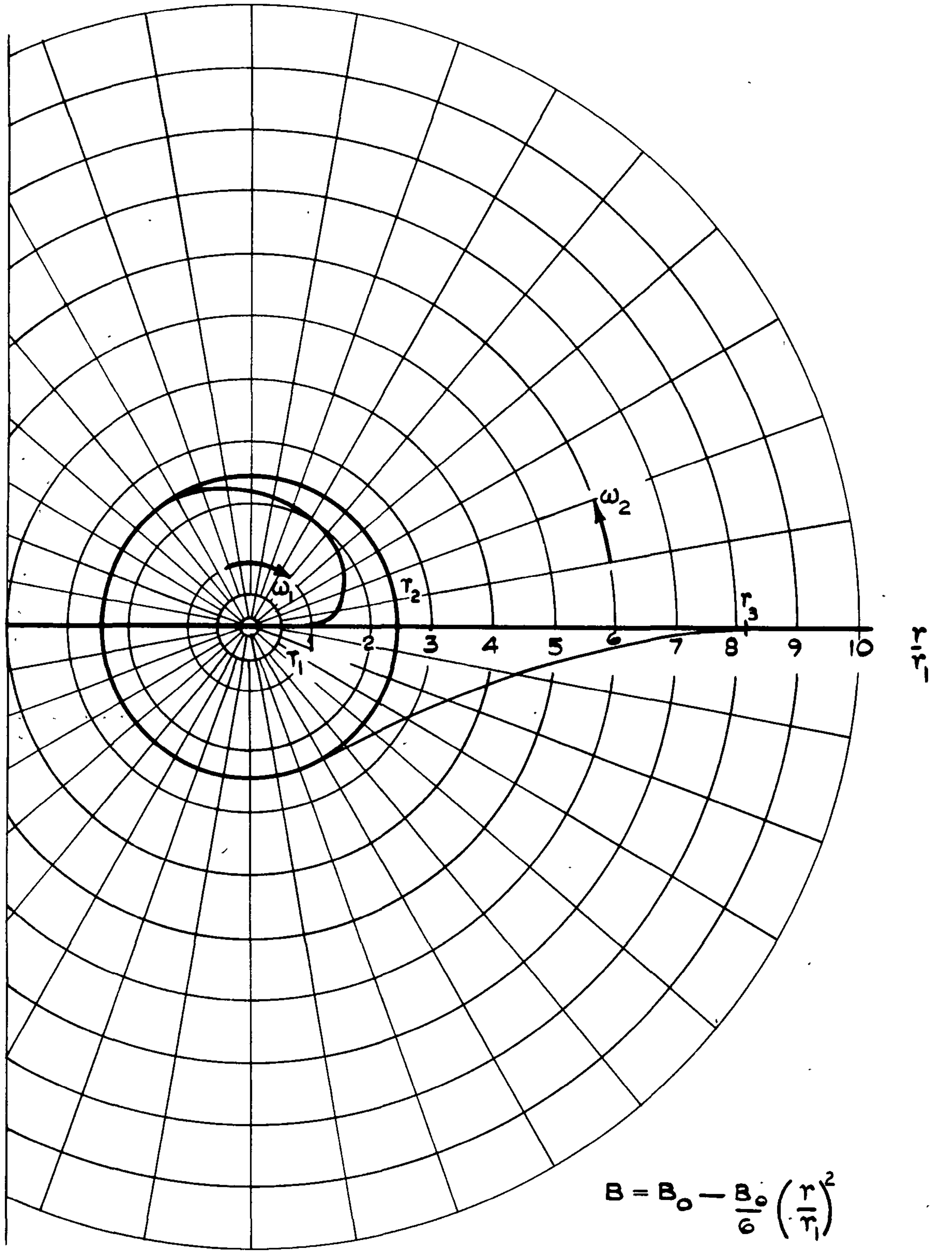


FIG. 9. ARC SHAPES WHEN $\frac{B}{B_1} = \alpha + (1-\alpha)\left(\frac{r}{r_1}\right)^n$ OR $B = a + b\,r^n$.



$$B = B_0 - \frac{B_0}{6}\left(\frac{r}{r_1}\right)^2$$

$n = 2, \ \alpha_1 = 1 \cdot 2, \ \alpha_3 = -0 \cdot 1$

$\omega_1 / \omega_3 = 2 \cdot 3$

FIG.10. ILLUSTRATION OF POSSIBLE DOUBLE ARC.

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It is shown that there are two families of possible arc shapes, all dependent on the form of the applied magnetic field. The first extend outwards from a radius r_1 where the arc is radial and the second extend inwards from radius r_1.

The first family can be subdivided into three types. Arcs which are S-shaped, extending only to a second radius where they are again radial;

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arcs which may extend out to infinity in an involute-like form; and arcs which are involute-like near to r_1 but terminate as a circle at a radius where the magnetic field is zero.

The second family can be subdivided into two types. Arcs which are S-shaped, extending only to a smaller radius where they are again radial; and arcs which terminate as a circle at a radius where the magnetic field is zero. The circle may have zero radius in the limiting case where the field is zero at the centre.

It is shown that under certain precise conditions arcs may extend from one family through radius r_1 to the other family forming one long uniformly rotating arc.

Finally, the possible marriage of an arc from the first family with one from the second family to form a mutually compatible arc pair is demonstrated. In this case the arcs rotate in opposite directions.

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