C.P. No. 839

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CURRENT PAPERS

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by

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LONDON: HER MAJESTY'S STATIONERY OFFICE

1966

PRICE 7*s* 6*d* NET

U.D.C. No. 533.6.048.1 : 531.71

C.P. No. 839

August 1964

THE INTERPRETATION OF STRAIN MEASUREMENTS FOR FLIGHT LOAD DETERMINATION

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SUMMARY

The procedures of N.A.C.A. Report No.1178 for the interpretation of measured flight strains as structural loads are not entirely satisfactory for applications to delta or slender-body configurations. Problems arise from the severe non-linearities in the gauge response with the position of the calibrating load and from the need to support the aircraft representatively during the ground calibrations. These difficulties are overcome if distributed load data, obtained either directly or by superposition, are used in place of individual load data. In contrast to the original N.A.C.A. Report, the procedure will then establish directly the reliability of any particular flight load measurement. The modified technique is illustrated by an application to the Lightning fin in laboratory tests.

Replaces R.A.E. Tech. Note No. Structures 363 - ARC 26659

CONTENTS

TABLES

TABLES (Contd.)

ILLUSTRATIONS

1 INTRODUCTION

Flight load measurements on military and civil aircraft in the interests of safety and structural efficiency have been standard practice in the U.S.A. for many years and it has been usual to deduce the net flight loads from structural strain measurements. Similar strain measurements have also been made on many British aircraft but their interpretation as flight loads has not been based on a statistical method commonly used in America. This method[1], developed by Skopinski et al of the N.A.C.A. in the late 1940's, permits the rapid processing of the flight data. With the possibility that flight load measurement may become an essential part of the clearance procedure for British aircraft it was thought worthwhile to review the American technique. It was apparent that the method of N.A.C.A. Report No.1178, herein referred to as the N.A.C.A. method but not implied to be the current N.A.S.A. method, was satisfactory for the medium and high-aspect ratio aircraft for which it was developed but it had some deficiencies when applied to modern low-aspect ratio aircraft.

The N.A.C.A. method is based on the fact that, although the stress in a structural member is not necessarily a simple function of the three loading parameters, M the bending moment, V the shear and T the torque, it is often possible to combine the responses of selected strain gauges to provide a measure of each parameter. The selected gauges and their combination coefficients are chosen by statistical methods from a sample comprised of the gauge responses due to the successive application of a single load at various points on the structure. The appraisal of the N.A.C.A. technique for applications to some current and future aircraft showed that difficulties could arise from:

(a) The need to support the aircraft during the application of each calibration load. The choice of support position influences the stress distributions and hence also the gauge responses induced by the load. The satisfactory application of a statistical treatment requires that the sample data should be consistent with flight conditions and it is essential that the ground calibration load should produce the same gauge responses as those due to a flight load of the same magnitude acting at the same position. It is relatively easy to satisfy these conditions on an aircraft with high-aspect ratio wings and long fuselage, but the choice of supports for the delta and slender body configurations presents many difficulties because of the integrated wing and fuselage construction.

(b) The multiplicity of the load paths which result from the more redundant structure. With such designs the internal loads are not completely diffused into the structure for some distance from the applied external load. Thus, although the gauge response is directly proportional to the magnitude of the load, the responses are not linearly related to the chordwise or spanwise positions of a load of constant magnitude. These non-linearities are important because they influence the accuracy with which a combination of gauges can be fitted to the calibration data. If the external flight loads in the vicinity of the gauges make small contributions to the total external load, as in the case of a high-aspect ratio wing, then the poor diffusion of the local loads is comparatively unimportant. However the preponderance of non-linear responses in a multi-spar structure of low-aspect ratio, (see Figs.3 and 4) from the individual calibration loads adversely affects the confidence in an estimate by the combination of gauges.

It can be shown that both these difficulties are overcome by changing from a statistical sample based on individual loads[1] to a sample based on distributed loads more representative of those which occur in flight. Furthermore this latter sample gives the statistical data necessary for assessing the accuracy of any particular load measurement in flight.

This Note discusses these problems and then illustrates the proposed procedure by an application to a Lightning fin on which laboratory tests were conducted at the R.A.E.

2 COMMENTS ON THE N.A.C.A. METHOD

The statistical approach adopted by the N.A.C.A. was a valuable contribution to solving the problem of the interpretation of flight strain measurements. The alternative method of comparing the measured strains with either those estimated by calculations or those measured by similar gauges at similar positions on the strength test specimen introduce errors arising from differences in material characteristics and dimensions of the two structures and from variations in strain gauge sensitivities. Furthermore, in many practical cases the stressing or test information will not be for the appropriate flight conditions.

To overcome these difficulties, Skopinski et al[1] used standard statistical methods to interpret the responses of selected gauges attached across a section of wing or tailplane as net bending moments, shears and torques at that section. They make use of calibrations of the gauge installations in which individual loads are applied successively at a number of stations on the structure. The gauges are usually installed to measure the shear and bending strains in the spars and these quantities are dependent on the position of the calibration load in a simple or complex manner according to the detail design of the structure.

It is now postulated that the responses μ_1, μ_2, ... of the gauges G_1, G_2, ... at different locations on the section can be combined such that, for example,

$$M = \beta_{11} \mu_1 + \beta_{12} \mu_2 + \beta_{13} \mu_3 \qquad (1)$$

where β_{11}, β_{12}, β_{13} etc are constants to be obtained from the calibration data.

In general the gauges G_1,G_2 ... are attached to the more important load paths at the section and the individual load calibrations must be sufficiently extensive to represent the constituents of the flight distributions. Thus it can be expected that there will be more than sufficient equations to solve for the constants β_{11}, β_{12} etc and these quantities are then determined by a least squares procedure as described in the Appendix.

This procedure of fitting an expression of the type shown in equation (1) is included in regression analysis[2] commonly used in statistics. The form used in this application is called a linear regression and there are standard computer programmes for calculating the coefficients of the regression and its standard error.

In many applications it is found that some gauges have very similar responses and consequently the accuracy of prediction can be improved by the elimination of the "redundant" gauges. It is also possible for the contributions, $\beta\mu$, of particular gauges to be small and the loss of accuracy is negligible when these "irrelevant" gauges are omitted from the regression. Thus it can be expected that the final regressions for each loading parameter will utilise different gauges. It must be appreciated that the standard error only reflects the probable accuracy of the regression in predicting a particular calibration load of the sample from its associated gauge responses. It can be expected, from small sample theory, that about 2 out of 3 of the calibration loads will be estimated within ± the standard error and there is a 99% probability that the estimate will be within ± 3 times the standard error. Any justification for the application of this regression to distributed loadings must be sought from an application of the principle of superposition. However the necessary conditions of linearity and elasticity will be satisfied by most modern structures in the practical range of flight loads. The gauge responses from the distributed loadings are therefore the algebraic sums of the responses from the constituent loads, each of which can be estimated by the regression. Thus the statistical sample must be representative of all the constituent loads that can produce responses from the gauges. It is necessary therefore to include loads inboard of the measuring section because the gauges respond to the self-equilibrating systems of loads generated by the inboard loads. However these responses must be associated with zero inputs of M, V and T because the regressions are chosen to estimate M, V and T due to the loads outboard of the section. Under these conditions the standard error, expressed as a percentage, loses much of its significance as an indication of the accuracy obtainable from the regression. The N.A.C.A. Report did not use the standard error other than for the selection of the gauges, and the accuracy in the general application was implied by the satisfactory prediction, i.e. within ±5%, of one or more distributed loadings which were applied to the tailplanes in the two quoted examples.

It might be concluded from the above considerations that the N.A.C.A. method caters automatically for distributed loadings over a limitless range of centres of pressure. Certainly the regressions are the same regardless of the sign of the loads, but the probable accuracies of their predictions would vary for each distribution and their establishment in any particular case would present many formidable problems.

The N.A.C.A. procedure also recommended the electrical combination of gauge stations prior to the fitting of the regression. This has the advantage of reducing the number of recording channels and the subsequent processing of the flight data, but its adoption increases the number of gauge installations because a particular gauge station can be used only in the electrical circuit appropriate to one of the three parameters. The combining of selected gauges reduces the possibility of redundant gauges and the non-linearities in the data sample because the higher-than-average response of one gauge is balanced by the corresponding lower-than-average response(s) of the other gauge(s) to the same calibration load.

3 CHOICE OF SUPPORTS DURING THE CALIBRATION

It is implicit in the statistical treatment of the N.A.C.A. method that the calibration load should produce the same responses at the gauge stations as those due to a flight load of the same magnitude acting at the same positions. Thus if the sample is comprised of individual loads it is essential that the supports for the calibrations should not influence the stress distributions at the gauges. This is easy to specify but difficult to obtain in practice because there are a very limited number of strengthened positions for ground supports and their provision is not dictated by the flight condition. Although the number of potential support positions, such as undercarriages, jacking points etc, is about the same for the delta and slender body configurations as for the older high-aspect ratio aircraft, owing to the more compact layout and structural design a larger proportion of the structure will be affected by the diffusion of concentrated loads reacted at these points.

In general it is desirable to estimate the critical loads as accurately as possible rather than all flight loads at a lower accuracy. This is best effected by using regressions fitted to distributed load samples, each of which caters for a range of uncertainty in the expected flight distribution. The direct application of distributed loads which are representative of a flight condition overcomes the problem of supports. However the acquisition of the sample data would be most expensive because the different distributions must include all those investigated in flight and their number must exceed the number of gauges in the regression. Thus for practical and economic reasons it seems essential to retain the convenience of the individual load calibrations and to assemble the distributed load data by superposition. The calibrations provide the responses μ_{rs} of each gauge G_s for unit load at x_r, y_r. Then for any system of loads P_r at x_r, y_r, total response of

$$G_s = \sum_{r=1}^{r=n} P_r \mu_{rs}$$

is independent of the support positions if

$$\sum_{r=1}^{n} P_r = 0$$

and the other conditions of equilibrium are satisfied.

These conditions are automatically satisfied by the aerodynamic and inertial loads in the flight condition and consequently the responses of the gauges for the sample can be evaluated from the expected flight loads and the individual load calibrations obtained under any arbitrary support condition.

This implies that the calibration must be very comprehensive. However in any practical application there will be supports which can reduce the extent of the calibration. Their availability will depend on the particular structure and the scope of the flight investigations. As one is concerned with the interpretation of total responses the criterion for the inclusion of any particular calibration is the relative contribution made to the total response. Thus it becomes unnecessary to calibrate for loads in areas where either the expected local flight load is small or the gauge responses under the particular conditions of support are expected to be small.

4 PROPOSED METHOD USING DISTRIBUTED LOAD DATA

A change from an individual load sample to a distributed load sample for the statistics automatically reduces the non-linearities in the data. The higher-than-average response(s) of a particular gauge to some load(s) is balanced by the lower-than-average responses of the same gauge to loads at different positions and the regressions fitted to such data will have smaller standard errors. If the distributed load sample is typical of the distributions to be measured in flight the standard error can be used directly to assess the reliability of any subsequent estimation by the regression. One obvious criterion of the typicality of the sample is whether the centre of pressure of the measured loads falls within the range of the sample. Any extrapolation to higher stress levels than those induced by the individual load calibrations would be justified by test or other technical knowledge.

The following section illustrates the N.A.C.A. and proposed methods by an application to the Lightning fin. It is acknowledged that the choice of support presented no difficulties in the laboratory experiment but the "non-linearities" encountered in a low-aspect ratio multi-spar construction were present.

5 APPLICATION OF METHODS TO A LIGHTNING FIN

5.1 Description of specimen and strain gauge installations

The fin was a 5 spar structure (Figs.1 and 2); the spars were mounted vertically from fuselage frames and ribs ran horizontally between the front and rear shear walls which completed the main structure. A leading edge structure, of 16 S.W.G. skin and ribs normal to the front shear wall, was attached to the front shear wall. The main skin was 12 S.W.G.

For the laboratory programme the fin was bolted to a rear fuselage specimen in order to ensure a representative mounting. The fin and fuselage were rotated through 90° to ease the task of loading.

British Thermostat SE/A/2 200 Ω strain gauges were bonded with Araldite strain gauge cement at a section outboard of rib 1. Calibration loads of 1000 lb were applied in increments at each of the stations shown in Fig.2. The gauges were arranged to respond either to shear or to bending strains in the spars and the responses of the half-bridge installations were recorded by the R.A.E. Strain Recorder. The accuracies of measurement are estimated to be within ±1 digit (1000 digits = 0·5% $\Delta R/R$) for the electrical strains and within ±10 lb for the loads. It may be noticed in the subsequent tables that the electrical strains are given in fractions of a digit, this arises from the use of a Deuce Programme to provide the electrical strain for a nominal load of 1000 lb at each loading point from the slope of gauge response against load obtained by a least squares method.

5.2 Experimental results

Table 1 lists the responses, in digits, of the gauges at the section for 1000 lb load applied separately at each of the calibration points identified in Fig.2. Zero inputs (i.e. M = V = T = 0) indicate that the load was applied inboard of the section.

Table 2 lists the responses of the same gauges obtained by superposition of selected data from Table 1. The combinations of the individual loads used to obtain the "distributed load" responses are also given in Table 2. The inputs, M, V and T, given in the table were calculated from the individual loads outboard of the section and the resultant centres of pressure of these loads lie within the shaded area shown in Fig.2. The responses and inputs were scaled to V = 1000 lb.

Table 3 lists the gauge responses obtained by the superposition of the individual loads shown in Table 4 and a comparison is made with the mean responses obtained from two direct applications of the same distribution.

5.3 Regressions for the determination of M, V, and T

5.3.1 Individual load calibration method

The non-linear gauge responses with the position of the load are illustrated in Figs.3 and 4. The responses of A_7, a conventional shear gauge arrangement on Spar 5, are plotted as influence coefficients for a load of 1000 lb applied at each point. Similarly the responses of A_5, a "bending gauge" on the same spar, are plotted in Fig.4, but in this case the response is due to a single load which has been scaled to produce a bending moment of 10^5 lb in at Section XX. The figures show the higher-than-average responses for bending moments and shears induced by loads adjacent to the gauge station and demonstrate, at least for multi-spar construction of low-aspect ratio, the futility of attempting to deduce an accurate estimate of unknown loads from a single gauge calibrated by a single load. Thus it is necessary to follow the N.A.C.A. procedure and combine several gauges in order that the higher-than-average responses of one gauge are balanced by the lower-than-average responses of other gauges to the same calibration loads. The selection of the appropriate gauges from the mass of calibration data in Table 1 presents many difficulties and an attempt was made to sort the gauges by fitting linear surfaces to the individual responses of each gauge. Thus typically

$$\text{response of } A_7 = 19 + 0{\cdot}35x + 0{\cdot}43y$$

$$\text{response of } A_5 = 6{\cdot}3 + 0{\cdot}85x + 0{\cdot}01y$$

where x and y are the distances from the reference axes in Fig.2. In each expression the sensitivities of the gauge to shear, bending moment and torque are indicated by the constant, x coefficient and y coefficient respectively. Redundant gauges tend to have linearly related coefficients and to select gauges for a shear regression it is preferable to use gauges which tend to have, in combination, net zero coefficients of x and y. However little success was obtained and this was attributed to the poor fits obtained by the linear surfaces. To select possible combinations by fitting quadratic surfaces to the responses of each gauge is a most laborious task.

It was decided therefore to adopt the alternative procedure of fitting a regression containing a large number of the gauges to the data and to discard successively those gauges shown to be either irrelevant or redundant. A Mercury computer programme provided the information for this purpose and a typical product of the computation is shown in the Appendix.

Table 5 summarises the regressions obtained for the estimation of the bending moment (M) at Section XX and the regression coefficients based on individual load calibrations are given in column (b). Thus

$$M \times 10^{-3} = 0{\cdot}958A_8 - 0{\cdot}037D_7 + 0{\cdot}001B_2 - 0{\cdot}072A_9 + 0{\cdot}002D_6 + 0{\cdot}082B_4$$

or $$= 0{\cdot}961A_8 - 0{\cdot}037D_7 - 0{\cdot}072A_9 + 0{\cdot}002D_6 + 0{\cdot}082B_4$$

or $$= 0{\cdot}982A_8 - 0{\cdot}037D_7 - 0{\cdot}074A_9 + 0{\cdot}050B_4$$

or $$= - 0{\cdot}398D_7 - 0{\cdot}006A_9 + 0{\cdot}608D_6 + 0{\cdot}468A_5$$

or $$= 0{\cdot}998A_8 - 0{\cdot}029D_7 - 0{\cdot}077A_9 + 0{\cdot}011A_5$$

etc.

Each of these regressions was used to estimate the distributed load detailed in Table 4 and the contributions made by each gauge are listed in column (e) of Table 5. The percentage error of the prediction is compared with the percentage standard error of the regression. Similarly Tables 6 and 7 summarise the regressions for the estimation of shear and torque. Many of these regressions use the bending moment (M) as one of the independent variables and this is justified only if the estimation of M is well established by its own regression.

Incidentally, in each of the regressions for M, V and T the responses have been scaled to unit values of bending moment, shear and torque respectively for the outboard loads. This was done to allow the standard error to be expressed as a percentage rather than an absolute quantity. Such a procedure has more relevance for the regressions based on distributed loads; the difficulty for those based on individual loads arises from the lack of significance of the standard error as an indicator of the probable accuracy when the regression is used to forecast a distributed load. The best agreements between known and estimated distributed loadings in Tables 5, 6 and 7, i.e. within 1% for bending moment, 4·2% for shear and 1·5% for torque, are better than would be expected from the quoted standard errors and could be fortuitous. The performance of the regression A_8 A_9 D_7 B_4 in predicting the bending moment from the responses in Table 2, indicated a standard error of 5·8% which compares favourably with the 3·2% obtained directly from the data. Its evaluation involved additional computation and it would seem more logical to use the data in Table 2 directly as in Section 5.3.2.

No attempt was made to use the technique of partial combination as discussed and applied in the N.A.C.A. Report. This technique has practical disadvantages and does not overcome the difficulties of establishing the reliability of a forecast of a distributed load by a regression based on individual load data.

5.3.2 Distributed load calibration method

The data for the distributed load sample are given in Table 2 and were obtained by superposition from the data in Table 1. The justification for this procedure was based on the knowledge that the structural components would not buckle at the stress levels used for the subsequent loading shown in Table 3. A check of the gauge responses for that particular loading and those obtained by superposition is included in Table 3 and agreement within ±2% was obtained except for the lower responses where reading accuracy was significant.

The inspection of Table 2 shows that the non-linearities in the response data were reduced - partly as a result of combining high and low responses and partly as a consequence of restricting the range of c.p. position. The coefficients of the regressions fitted to these data are given in columns (c) of Tables 5, 6 and 7, and in columns (f) are the contributions made by the selected gauges of each regression in the estimation of the directly applied distributed loading. The accuracy of the estimates varied for the different regressions and it was not always the case that the regressions with the smallest standard error predicted this particular distribution with the highest accuracy. However there were regressions which, 2 times out of 3, would predict any one of the statistical sample to within ±1·5% for shear, (regression A_7, A_9, B_2, C_5, D_1, D_6, M), within ±3% for bending moment (A_8, A_9, B_4, D_6, D_7) and ±2% for torque (A_7, A_9, B_2, C_5, D_2, D_6 M). For higher levels of confidence, say 99 times out of 100, these accuracies would be worsened to ±4.5%, ±9% and ±6% respectively.

5.4 Influence of the sample size

The extent of the individual load calibrations must be governed to some degree by the redundancy of the structure and, in either method, must allow a satisfactory synthesis of the expected flight loadings. In the case of the N.A.C.A. method the synthesis is made subsequent to the regression analysis and consequently there is little scope for any reduction in the size of the matrix. For the modified method there is a lower limit in the number of distributions to be used; there must be at least as many independent calibration data as there are gauge stations in the regression and it is advisable to have an excess in order to prevent ill-conditioning.

To illustrate the influence of sample size Table 8 lists in columns (b) and (c) the coefficients of regressions using the same gauges for samples of 29 and 10 distributed load data - the latter sample comprised the distributions marked with an asterisk in Table 2. It will be noted that the regression coefficients differ for the two samples but the standard errors for shear and bending moment are satisfactorily small. In the case of the torque regression there was an indication of redundancy and the omission of gauge A_9 improved the fit and the standard error. These standard errors indicate how each regression fits its own sample and it is more realistic to compare the accuracies of the two regressions for the same sample; i.e. the standard errors have been calculated for the applications of the 10 member regressions to the estimation of the 29 member sample. On this basis of comparison, the accuracy changed from 1·4% to 4·1% for shear, from 2·9% to 3·9% for bending moment and from 2·1% to 10% for torque when the sample size was changed from 29 to 10 distributions.

It should be appreciated that the statistical procedure does not lead automatically to the "best solution"; at any stage a decision can be made that the probable accuracy is adequate for the intended purpose but confidence in any estimate must depend on initial assumptions such as the extent of the calibration as well as on statistical theory.

6 CONCLUSIONS

A review of the N.A.C.A. technique for flight load measurement has shown that its application to future aircraft may not be entirely straightforward. The difficulties arise from the non-linearities in the strain gauge responses to the individual calibration loads and from the need to support the aircraft during their application. The former can be severe if the design incorporates multi-spar construction of low-aspect ratio and the choice of supports for delta and slender-body configurations may not be obvious or practical to fulfil the condition that the individual loads of the ground calibration must be reacted in a manner similar to that of the flight condition.

It is shown that the problem of the supports can be overcome by determining the combinations of gauge responses, which interpret the flight measurements, from a sample of the gauge responses to distributed loads instead of to individual loads as recommended in N.A.C.A. Report No.1178. It is necessary in the general case for the distributed load systems to be in equilibrium. The statistical method of analysing flight strains requires a large number of calibrations and it is suggested that these can be obtained by the superposition of individual load data instead of the direct application of distributed loads. This procedure would retain the convenience of individual load calibration and under certain conditions need not satisfy the general requirement that the distribution of loads should be in equilibrium.

The change to a sample of distributed load data automatically reduces the non-linearities in the gauge responses and consequently acceptable standard errors for the combinations of gauge responses should be obtained for multi-spar structures of low-aspect ratio. Furthermore, in contrast to the N.A.C.A. procedure, these standard errors can be used directly to assess the reliability of a particular load measurement and this is important if the measured flight load is to be used for the structural clearance of the aircraft.

Laboratory tests on a Lightning fin, which admittedly did not introduce any difficulties of supporting the structure representatively, showed that the modified method interpreted the strain measurements at a chordwise section as net bending moments, torques and shears within ±3% at the 67% confidence level.

SYMBOLS

L	general symbol for bending moment, shear or torque
M	bending moment lb in
T	torque lb in
V	shear lb
β	coefficient in load equation
b	estimated coefficient in load equation
c	covariance of bridges
ε	residual, difference between calculated and applied load
i	row index
j	column index
n	number of loadings
μ	non-dimensional bridge response
S	standard error of estimate on the sample
s.e.	standard error of individual forecast

SYMBOLS (Contd.)

q	number of bridges	
v	variance of bridge	
x	distance from torque reference line	in
y	distance outboard from strain gauge section	in

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APPENDIX

METHOD OF ANALYSIS

The relationship between the expected values of one dependent variable and the observed values of a number of independent variables can be expressed in the form of a regression equation.

Once the regression equation has been established it may be used to derive estimates of the dependent variable.

In the application of regression analysis to flight load measurement the equations relating the response of the strain gauge bridges (μ) and the three loads, bending moment (M), shear (V), and torque (T) are required.

The relationship between each of these and the responses can be expressed by a multiple linear regression equation of the form

$$L = \beta_1 \mu_1 + \beta_2 \mu_2 + \beta_3 \mu_3 + \dots \beta_n \mu_n \qquad (1)$$

where L = appropriate values of M, V or T.

Thus the equations for n calibration loads at different locations may be written in the form

$$\left.\begin{aligned}
L_1 &= \beta_1 \mu_{11} + \beta_2 \mu_{12} + \beta_3 \mu_{13} + \dots + \beta_j \mu_{1j} \dots + \beta_q \mu_{1q} \\
L_2 &= \beta_1 \mu_{21} + \\
&\vdots \\
L_i &= \beta_1 \mu_{i1} + \qquad\qquad + \beta_j \mu_{ij} \dots + \beta_q \mu_{iq} \\
&\vdots \\
L_n &= \beta_1 \mu_{n1} + \qquad\qquad + \beta_j \mu_{nj} \dots + \beta_q \mu_{nq}
\end{aligned}\right\} \qquad (2)$$

$j = 1, 2 \dots\dots q$

$i = 1, 2 \dots\dots n$

When n = q the equations (2) can be solved for $\beta_1 \; \beta_2 \dots \beta_n$ directly. This however may not lead to a reliable estimate of the load. The number of calibration loads is therefore chosen to be greater than the total number of bridges. The coefficients are estimated by the least squares method.

If the estimated coefficients are represented by $b_1, b_2 \ldots b_q$ then the general form of equations(2) is now written

$$L_i = b_1 \mu_{i1} + b_2 \mu_{i2} \ldots + b_j \mu_{ij} \ldots + b_q \mu_{iq} + \varepsilon_i \qquad (3)$$

where ε is the error or residual in the estimation of L.

The sum of squares of these residuals is required to be a minimum.

Consider for simplicity the solution for three bridge coefficients and n loadings.

The sum of squares of residuals to be minimised is

$$\sum_1^n \varepsilon_i^2 = \sum_1^n [L_i - (b_1 \mu_{i1} + b_2 \mu_{i2} + b_3 \mu_{i3})]^2 \qquad (4)$$

i = 1,2 ,,, n.

The necessary conditions are that:

$$\frac{\partial}{\partial b_1} \sum_1^n \varepsilon_i^2 = 0, \quad \frac{\partial}{\partial b_2} \sum_1^n \varepsilon_i^2 = 0, \quad \frac{\partial}{\partial b_3} \sum_1^n \varepsilon_i^2 = 0 \qquad (5)$$

and the following three simultaneous equations are obtained which are solved for the coefficients b_1, b_2, b_3.

$$\left.\begin{aligned}
\sum_1^n (\mu_{i1})^2 b_1 + \sum_1^n (\mu_{i1}\mu_{i2}) b_2 + \sum_1^n (\mu_{i1}\mu_{i3}) b_3 &= \sum_1^n L_i \mu_{i1} \\
\sum_1^n (\mu_{i1}\mu_{i2}) b_1 + \sum_1^n (\mu_{i2})^2 b_2 + \sum_1^n (\mu_{i2}\mu_{i3}) b_3 &= \sum_1^n L_i \mu_{i2} \\
\sum_1^n (\mu_{i1}\mu_{i3}) b_1 + \sum_1^n (\mu_{i2}\mu_{i3}) b_2 + \sum_1^n (\mu_{i3})^2 b_3 &= \sum_1^n L_i \mu_{i3}
\end{aligned}\right\} \quad (6)$$

If equations (6) are expanded it will be seen that μ and L are always associated in product terms, e.g. $(\mu_{i1})^2$, $(\mu_{i1}\ \mu_{i2})$, $(\mu_{i1}\ \mu_{i3})$ and $(L_i\ \mu_{i1})$ and since changing the sign of L_i (i.e. loading direction reversed) automatically changes the signs of μ_{i1}, μ_{i2}, μ_{i3} and has no effect on the above equations; thus b_1, b_2 and b_3 are the same as before.

If the responses of any one bridge are linearly related to the response of any other bridge then equations (6) become ill-conditioned and this is indicated by large standard errors.

The selection of strain gauge bridges is made by starting with an equation containing all possible bridges and rejecting successively the bridge having the lowest value of coefficient divided by its own standard error.

The best set is reached when the further rejection of a bridge does not reduce the standard error of the estimate.

The smaller the standard error the more accurate the estimate is likely to be

$$\text{standard error of estimate} = \sqrt{\frac{\text{residual sums of squares}}{\text{degrees of freedom}}}$$

and the general form for q independent variables is:

$$\sqrt{\frac{\sum_{1}^{n} [L_i - (b_1\ \mu_{i1} + b_2\ \mu_{i2} + \dots\ b_q\ \mu_{iq})]^2}{n - q}}\ .$$

The general multiple regression calculation was performed by a Mercury digital computer using the O.U.C.L. programme Stat/11.

A typical set of results is tabulated below and it corresponds to the estimation of bending moment from A_8, A_9, B_4, D_7 using combined load data (see Table 5).

It is possible to deduce from the standard error of estimate the variances of the b's and the covariances of the pairs of b's. The computer programme provides these values, as well as the standard error ($\sqrt{\text{variance}}$) of each b. The ratio of the coefficient to its standard error will determine the relevance of a particular b, being small for irrelevant coefficients.

	Mean values	
Bending moment	1.000000	2
Bridge A_8	1.229552	2
Bridge D_7	-6.790000	1
Bridge A_9	3.069621	2
Bridge B_4	3.583793	1

	Sums of squares		Degrees of freedom	Mean square (sums of squares / degrees of freedom)		Variance ratio
Total	2.900000	5	28	1.03571	4	
Residual	2.551600	2	25	1.02064	1	
Regression	2.897448	5	4	7.24362	4	7097.14

	Coefficients and covariances		Variance		Standard error ($\sqrt{\text{variance}}$)		Coefficient / standard error
Coefficient of A_8	6.197572	-1	1.10279	-2	1.05014	-1	5.902
Covariance of A_8 and D_7	5.97585	-3					
Covariance of A_8 and A_9	-1.49211	-3					
Covariance of A_8 and B_4	-1.36721	-2					
Coefficient of D_7	-5.258621	-1	6.49925	-3	8.06179	-2	-6.523
Covariance of D_7 and A_9	-3.26202	-4					
Covariance of D_7 and B_4	-5.44623	-3					
Coefficient of A_9	-7.990783	-2	2.82071	-4	1.67950	-2	-4.758
Covariance of A_9 and B_4	2.07733	-3					
Coefficient of B_4	3.496938	-1	1.88299	-1	1.37222	-1	2.548

Thus $M \times 10^{-3} = 0{\cdot}62\, A_8 - 0{\cdot}526\, D_7 - 0{\cdot}08\, A_9 + 0{\cdot}35\, B_4$ and the 10^{-3} factor must be inserted because the dependent variable was scaled before computation.

NOTE: The single figure columns contain the power of 10 by which the preceding columns are to be multiplied.

(1) The standard error of sample (S) = $\pm\, (10{\cdot}2064)^{\frac{1}{2}} = \pm\, 3{\cdot}2$ and expressed as a percentage of the mean value of the bending moment = ± 3·2%.

(2) The standard error of an individual forecast (s.e.).
When the responses of an individual forecast are scaled by the ratio of the mean of the sample and the individual forecast it is then possible to use

$$\text{s.e.} = \pm \left\{ S^2 \left(1 + \frac{1}{n}\right) + \Sigma v_i (\mu_i - \bar{\mu}_i)^2 + \Sigma c_{ij} (\mu_i - \bar{\mu}_i)\,(\mu_j - \bar{\mu}_j) \right\}^{\frac{1}{2}}$$

where

$$i \neq j, \quad c_{ij} = c_{ji} \quad \text{and } i \text{ and } j = 1, 2, \ldots q .$$

For the particular distributed load in Table 4 s.e. = ± 4·5% to be compared with the actual error of 1·1%.

TABLE 1

Strain gauge responses at Section XX for 1000 lb load at loading pads

Pad No.	*$(M)\times10^{-3}$	(V)	*$(T)\times10^{-3}$	A_5	A_8	B_4	C_4	C_{12}	D_1	D_3	D_5	D_7	A_7	A_9	B_2	B_3	C_5	C_{13}	D_2	D_4	D_6
00	80·3	1000	138·3	73·9	81·2	53·4	58·0	86·7	29·0	21·5	32·8	-34·8	76·2	70·1	71·4	61·9	46·3	-143·3	57·8	89·7	28·6
01	58·8	1000	135·0	57·1	60·7	37·0	42·0	62·9	20·9	18·5	26·3	-24·7	102·2	67·6	125·9	74·6	34·4	-119·8	48·8	72·5	21·4
02	50·8	1000	133·3	53·2	54·7	32·5	37·6	52·8	19·5	18·6	23·2	-22·4	112·3	67·4	145·1	79·8	32·7	-106·6	40·9	63·9	20·9
03	32·8	1000	130·3	38·4	33·7	23·4	27·2	32·3	11·4	15·4	13·2	-15·5	130·9	67·1	184·1	90·0	39·0	- 87·5	33·7	45·9	7·5
04	15·6	1000	126·3	22·5	18·8	4·0	16·1	17·9	4·0	10·1	6·5	-11·5	125·5	52·8	296·2	97·4	22·7	- 61·4	20·7	30·2	10·1
05	0·0	0	0·0	- 2·7	4·0	-35·0	4·9	3·0	0·0	9·0	3·1	- 2·5	35·5	27·4	-99·0	-31·3	12·5	- 25·7	7·3	13·2	8·1
11	58·8	1000	111·8	58·1	63·6	27·3	48·7	64·5	16·5	13·4	32·2	-31·6	99·6	84·9	53·2	34·1	56·9	- 8·4	36·6	67·3	39·4
22	50·8	1000	102·3	52·9	56·3	20·6	46·1	55·7	9·8	-14·0	32·9	-32·0	105·1	90·9	38·7	24·1	63·7	- 58·9	28·6	56·0	43·2
23	32·8	1000	102·3	44·9	40·1	12·0	32·3	37·4	5·7	-10·9	22·5	-20·3	151·3	92·8	70·5	30·3	57·0	- 49·5	22·5	41·4	37·2
24	15·5	1000	102·3	45·4	21·3	4·8	17·5	16·3	0·0	- 9·7	15·1	-12·3	243·8	76·3	65·6	29·9	38·9	- 35·0	11·8	25·1	24·6
25	0·0	0	0·0	-13·5	0·0	-4·5	6·4	1·9	- 7·3	- 4·4	6·1	- 4·9	-67·1	12·3	8·5	10·4	14·8	- 10·9	3·2	9·6	8·6
33	38·8	1000	87·5	40·7	48·7	13·1	43·8	45·0	- 2·7	-24·2	25·0	-31·9	94·9	116·2	18·2	10·2	86·9	- 23·8	19·6	44·5	54·9
34	15·5	1000	87·5	20·0	32·5	3·8	21·3	12·3	- 3·8	-20·0	15·0	-13·8	83·8	232·5	22·5	12·5	61·3	- 40·0	7·5	23·8	31·3
35	0·0	0	0·0	1·5	-6·5	0·0	5·0	-10·5	- 9·5	- 9·5	6·5	- 5·0	10·5	-55·0	7·0	5·5	12·5	- 9·0	1·5	8·0	8·0
43	25·5	1000	72·0	25·0	30·0	0·0	36·8	24·6	-10·0	-42·0	27·0	-31·0	59·6	93·5	0·0	0·0	143·3	0·0	7·6	35·0	94·0
44	15·5	1000	72·0	20·0	23·0	5·0	31·0	14·0	- 9·0	-34·0	20·0	-16·0	48·0	79·0	4·0	0·0	229·0	4·0	1·2	20·0	46·0
45	0·0	0	0·0	6·5	5·0	0·0	0·0	0·0	-18·0	-12·5	5·0	- 4·0	15·5	15·5	1·5	1·5	-43·0	0·0	-1·5	4·0	- 1·5
51	80·2	1000	149·8	76·0	82·0	56·0	56·0	82·0	30·0	36·0	32·0	-36·0	78·0	56·0	120·0	82·0	38·0	-158·0	66·0	92·0	20·0
52	89·2	1000	141·5	80·0	92·0	58·0	68·0	94·0	30·0	26·0	32·0	-40·0	68·0	70·0	60·0	66·0	46·0	-156·0	68·0	96·0	30·0
53	90·6	1000	148·9	88·0	88·0	62·0	68·0	96·0	34·0	34·0	32·0	-38·0	70·0	62·0	82·0	76·0	36·0	-172·0	70·0	102·0	22·0
54	10·3	1000	53·8	13·0	14·0	0·0	14·0	11·0	-21·0	-124·0	19·0	- 4·0	28·0	43·0	-10·0	- 4·0	66·0	10·0	-18·0	19·0	323·0
55	0·0	0	0·0	8·0	8·0	0·0	5·0	0·0	-24·0	- 29·0	-13·0	20·0	13·0	20·0	6·0	0·0	16·0	0·0	0·0	15·0	-95·0
60	82·2	1000	130·0	74·3	83·8	50·0	61·4	87·4	25·6	11·6	36·1	37·6	72·8	74·1	40·0	44·0	50·2	-133·8	57·6	90·6	34·4
61	63·3	1000	107·3	61·6	69·6	28·6	52·0	68·9	13·0	- 8·1	35·1	-35·4	90·3	87·1	28·7	26·0	62·9	- 77·8	38·6	71·5	45·7
62	55·8	1000	98·3	65·2	74·5	27·6	59·0	73·3	11·2	-18·6	43·3	-39·8	115·6	111·9	24·3	21·2	81·8	- 62·5	35·3	73·7	59·2
63	29·8	1000	68·0	28·1	33·3	4·7	37·9	30·8	- 9·2	-43·4	35·2	-34·3	57·3	90·8	-10·1	- 5·1	120·8	8·8	0·0	36·6	91·9
64	15·5	1000	49·5	13·8	16·3	0·0	17·5	15·0	-15·0	-87·5	20·0	-16·3	28·8	50·0	-12·5	-10·0	72·5	17·5	-51·3	5·0	243·8
65	0·0	0	0·0	7·5	8·8	0·0	7·5	- 3·8	45·0	41·3	2·5	0·0	12·5	22·5	- 6·3	0·0	31·3	15·0	-103·8	- 28·8	85·0
66	44·1	1000	83·0	41·4	50·0	0·0	45·7	4·6	0·0	-27·1	37·1	-34·3	87·1	110·0	0·0	0·0	87·1	-18·6	17·1	-205·0	60·0

* See Fig.2 for reference axes.

TABLE 2

Superposed strain gauge responses at Section XX and combinations of loading pads for combined loads

NO.	†(M) x 10^{-3}	(V)	†(T) x 10^{-3}	A_5	A_8	B_4	C_4	C_{12}	D_1	D_3	D_5	D_7	A_7	A_9	B_2	B_3	C_5	C_{13}	D_2	D_4	D_6	Combinations of loading pads
1*	33.0	1000	100.9	35.5	38.5	15.2	33.9	35.7	- 1.1	-20.4	19.0	-18.8	86.0	80.1	92.4	41.5	83.5	- 50.7	15.4	31.1	48.8	00 03 04 23 25 33 44 55 64
2	39.1	1000	106.4	44.3	44.9	15.5	36.3	48.2	4.4	-11.6	25.0	-25.1	115.4	78.7	79.5	41.8	53.2	- 65.2	21.6	50.3	61.9	01 03 04 05 24 45 52 62 63 64
3	39.4	1000	110.7	47.2	45.5	17.5	37.1	34.8	1.8	1.6	24.1	-23.7	125.2	94.0	90.2	44.4	55.4	- 67.0	14.6	14.5	42.8	02 04 22 23 24 34 35 53 65 66
4	36.7	1000	108.5	43.4	42.5	15.0	35.1	37.5	6.6	- 4.2	22.9	-21.8	124.7	93.6	63.7	37.8	74.1	- 68.9	26.6	48.4	31.9	00 02 03 05 22 23 24 34 35 44
5*	44.5	1000	101.6	45.4	49.7	17.9	41.2	45.4	4.3	-21.1	27.5	-26.5	90.3	79.4	56.7	32.4	69.3	- 60.8	22.9	37.7	68.7	00 01 02 03 04 05 11 22 33 35 43 54 55 60 61 62 63 64 66
6	42.4	1000	106.8	42.5	48.6	20.2	35.1	45.0	5.1	- 8.6	26.7	-26.3	82.9	96.7	77.9	40.6	72.2	- 66.3	27.3	53.2	39.1	01 02 04 22 25 34 44 45 60 61 63
7	38.8	1000	90.0	42.5	47.5	14.1	40.8	37.6	-5.4	-24.4	28.5	-25.7	89.1	105.7	20.9	15.3	93.0	- 39.2	4.6	16.3	67.9	22 24 25 34 44 55 60 61 62 63 64 65 66
8	43.7	1000	106.5	45.2	48.6	18.4	40.6	43.2	5.3	10.7	24.2	-25.0	95.9	82.1	64.1	37.0	76.9	- 67.8	26.4	32.2	47.1	00 03 04 05 23 24 25 33 43 44 51 52 55 60 64 66
9	46.6	1000	104.7	50.0	54.9	17.1	45.6	51.6	-0.4	- 8.1	28.5	-27.9	103.1	93.4	54.0	30.3	80.7	- 63.9	16.7	52.8	49.6	00 04 05 22 23 43 55 60 62 63 65
10*	34.2	1000	89.0	37.8	40.0	11.7	36.2	27.7	- 0.5	-34.0	23.6	-19.7	86.0	83.5	28.6	18.4	100.9	- 37.3	8.0	-14.0	63.3	00 24 25 44 55 64 66
11	55.2	1000	118.0	54.4	58.1	29.0	47.1	58.9	9.9	- 3.3	24.6	-25.7	79.1	61.8	76.5	49.7	75.9	-100.6	34.5	63.0	46.0	00 04 05 35 44 51 52 53 55 64
12*	23.6	1000	90.3	38.6	37.5	7.7	31.7	24.2	-19.4	-19.9	18.4	-15.7	132.6	131.0	26.4	16.9	95.5	- 25.9	-8.5	23.0	36.5	23 24 33 34 44 45 55 65
13	23.0	1000	91.4	31.0	29.1	8.8	25.3	22.6	- 6.1	-39.9	20.2	-15.6	100.7	80.7	70.1	30.3	68.7	- 35.9	9.3	33.6	103.6	02 04 22 24 34 35 44 45 54 54
14	20.5	1000	82.5	28.2	29.7	5.6	28.2	21.7	- 9.3	-39.1	21.1	-17.6	92.3	106.7	26.4	13.0	100.6	- 20.2	6.8	29.6	81.5	23 24 25 33 34 44 44 45 54
15	18.2	1000	72.2	25.0	24.6	2.6	26.6	16.2	-14.5	-57.0	20.7	-16.1	81.9	90.0	10.4	4.1	100.9	- 6.2	-5.7	22.5	109.7	24 34 35 43 44 54 55 63 64
16*	19.8	1000	79.5	26.3	25.3	4.8	24.4	19.3	-10.3	-52.1	21.5	-17.0	86.0	79.9	43.7	16.4	78.9	-17.0	-1.9	27.4	127.0	04 22 24 34 35 44 45 54 54 63 64
17	23.4	1000	82.5	31.9	31.2	4.8	30.4	23.0	-10.8	-44.5	20.6	-18.5	106.4	101.9	25.7	12.1	89.3	-21.7	8.6	31.0	81.7	23 24 33 34 35 43 43 54 55
18	27.8	1000	89.4	34.6	36.7	7.8	33.6	29.6	- 5.0	-24.5	24.0	-23.8	105.0	115.6	32.8	17.0	93.2	-30.6	14.4	37.1	48.3	22 23 24 25 33 34 43 44 45
19*	51.5	1000	112.3	50.8	54.6	25.4	45.9	52.8	7.4	- 8.2	24.9	-26.4	76.6	65.7	66.9	43.4	84.4	-88.0	31.1	59.5	52.0	00 04 05 35 43 44 51 52 53 55 64
20	48.0	1000	112.6	48.7	51.7	23.2	41.9	48.4	10.4	- 2.4	25.6	-26.9	94.9	77.8	73.8	44.5	68.0	-80.0	31.4	42.0	68.3	00 01 02 03 04 05 23 24 25 33 43 44 51 52 53 60 64 66
21	45.6	1000	112.7	51.0	51.1	21.7	40.7	43.1	5.7	0.9	26.1	-25.9	117.4	91.1	81.5	43.4	55.0	-75.0	21.1	27.5	41.6	02 04 11 22 23 24 34 35 53 60 65 66
22*	33.0	1000	100.9	35.5	38.5	15.2	33.9	35.7	- 1.1	-20.5	19.0	-18.7	85.9	80.0	92.4	41.5	83.5	-50.8	15.3	38.8	48.9	00 03 04 23 25 33 44 55 64
23	33.5	1000	95.9	36.6	39.1	13.8	34.9	32.4	- 2.9	-26.1	20.9	-19.1	85.9	81.5	65.8	31.9	90.7	-45.1	12.3	16.7	54.8	00 00 03 04 23 24 25 25 33 44 44 55 55 64 64 66
24*	35.5	1000	96.9	36.8	41.1	11.8	36.5	23.8	- 0.9	-21.9	23.0	-22.2	86.2	86.7	71.8	32.2	84.3	-43.3	15.7	-15.4	51.3	00 03 04 23 25 33 44 55 64 66 66
25	32.8	1000	92.7	46.3	48.1	13.4	39.5	38.3	-10.6	-19.5	25.6	-22.6	127.8	125.6	33.0	18.1	91.7	-36.4	4.0	37.5	43.1	23 24 33 34 44 45 55 62 62 65
26*	21.3	1000	77.8	26.4	26.5	4.3	26.8	20.8	-10.2	-50.4	23.2	-19.9	81.0	82.1	34.8	13.0	88.6	-13.2	-0.9	28.9	128.8	04 22 24 34 35 43 44 45 54 54 60 60 64
27	24.3	1000	80.0	29.2	32.1	4.4	31.8	21.2	- 8.4	-38.6	24.7	-22.3	85.0	104.1	17.5	8.6	109.1	-15.1	7.0	7.4	81.6	23 24 25 33 34 43 44 44 45 54 63 66
28	20.2	1000	75.5	26.1	28.1	3.9	27.9	19.0	-12.0	-49.2	20.5	-17.6	83.4	108.6	12.6	5.7	100.8	-11.9	-1.4	25.1	94.8	24 33 34 34 35 43 44 54 55 63 64
29*	31.1	1000	91.0	35.7	39.7	9.1	34.1	32.5	0.4	-21.7	24.2	-24.3	100.0	127.4	32.9	16.7	81.8	-37.1	16.6	39.1	51.4	22 23 34 43

† See Fig.2 for reference axes.

TABLE 3

Measured and superposed strain gauge responses for distributed load (Table 4)

Bridge	A_5	A_7	A_8	A_9	B_2	B_4	C_5	D_1	D_2	D_3	D_6	D_7
Superposed response	183	361	198	362	174	76	308	17	78	-78	284	-115
1st dist.load 2nd dist.load	187 183	350 363	196 197	362 365	173 169	76 78	298 298	14 14	72 75	-78 -85	283 283	-118 -115
Mean	185	356	196	364	171	77	298	14	74	-81	283	-117
% age error on mean	-1·1	1·4	1·0	-0·5	1:8	-1·3	3·4	21·5	5·4	-6·1	0·4	-1·7

TABLE 4

Distributed load

Load lb	1000	500	500	310	290	245	225	190	145	125	100	70
Pad No.	00	61,63	64,66	24	34	35	44	45	04	25	02,03	05

TABLE 5

Regressions for estimating bending moment

Gauge station (a)	Regression coefficient: Individual load (b)	Regression coefficient: Combined load (c)	Gauge station response (d)	Distributed load contribution: Individual load (e)	Distributed load contribution: Combined load (f)
A_5	0.035	-0.062	185	6.5	- 11.5
A_8	0.903	0.721	196	177.0	141.0
A_9	-0.069	-0.083	364	- 25.1	- 30.2
B_2	0.000	0.005	171	0.0	0.8
B_4	0.105	0.324	77	8.1	24.9
D_6	0.002	0.008	283	0.6	2.3
D_7	-0.057	-0.433	-117	6.7	+ 50.7
Predicted B.M. x 10^{-3}				173.7	178.0
Actual B.M. x 10^{-3}				179.0	179.0
Percentage error				-3.0	0.6
Percentage standard error				9.9	7.1

A_8	0.958	0.654	196	188.7	128.2
A_9	-0.072	-0.083	364	- 26.2	- 30.2
B_2	0.001	0.003	171	0.2	0.5
B_4	0.082	0.333	77	6.3	25.6
D_6	0.002	0.006	283	0.6	1.7
D_7	-0.037	-0.459	-117	4.3	53.7
Predicted B.M. x 10^{-3}				173.9	179.5
Actual B.M. x 10^{-3}				179.0	179.0
Percentage error				- 2.9	0
Percentage standard error				9.7	3.3

A_8	0.982	0.620	196	192.5	121.5
A_9	-0.074	-0.080	364	- 26.9	- 29.1
B_4	0.050	0.350	77	3.9	27.0
D_7	-0.033	-0.526	-117	3.9	61.5
Predicted B.M. x 10^{-3}				173.4	180.9
Actual B.M. x 10^{-3}				179.0	179.0
Percentage error				- 3.2	1.1
Percentage standard error				9.4	3.2

Gauge station (a)	Regression coefficient: Individual load (b)	Regression coefficient: Combined load (c)	Gauge station response (d)	Distributed load contribution: Individual load (e)	Distributed load contribution: Combined load (f)
A_5	1.152	1.016	185	213.1	188.0
A_7	-0.142	-0.177	356	- 50.6	- 63.0
B_2	0.017	0.048	171	2.9	8.2
C_5	-0.013	0.008	298	- 3.9	2.4
D_6	0.001	-0.009	283	0.3	- 2.5
D_7	-0.193	-0.415	-117	22.6	48.6
Predicted B.M. x 10^{-3}				184.4	181.7
Actual B.M. x 10^{-3}				179.0	179.0
Percentage error				3.0	1.5
Percentage standard error				9.9	3.1

A_8	0.961	0.645	196	188.5	126.5
A_9	-0.072	-0.082	364	- 26.2	- 29.9
B_4	0.082	0.359	77	6.3	27.7
D_6	0.002	0.007	283	0.6	2.0
D_7	-0.037	-0.461	-117	4.3	53.9
Predicted B.M. x 10^{-3}				173.5	180.2
Actual B.M. x 10^{-3}				179.0	179.0
Percentage error				- 3.2	0.7
Percentage standard error				9.5	2.9

A_5	0.468	0.308	185	86.6	57.0
A_9	-0.006	-0.045	364	- 2.2	- 16.4
B_4	0.608	0.749	77	46.8	57.7
D_7	-0.398	-0.744	-117	46.6	87.0
Predicted B.M. x 10^{-3}				177.8	185.3
Actual B.M. x 10^{-3}				179.0	179.0
Percentage error				- 0.7	3.5
Percentage standard error				20.4	4.2

Gauge station (a)	Regression coefficient: Individual load (b)	Regression coefficient: Combined load (c)	Gauge station response (d)	Distributed load contribution: Individual load (e)	Distributed load contribution: Combined load (f)
A_5	0.442	0.326	185	81.8	60.3
A_9	-0.006	-0.049	364	- 2.2	-17.8
B_2	-0.010	-0.016	171	- 1.7	- 2.7
B_4	0.705	0.764	77	54.3	58.8
D_6	0.011	-0.003	283	3.1	- 0.9
D_7	-0.371	-0.767	-117	43.4	89.7
Predicted B.M. x 10^{-3}				178.7	187.4
Actual B.M. x 10^{-3}				179.0	179.0
Percentage error				- 0.2	4.7
Percentage standard error				18.1	4.3

A_5	0.011	0.040	185	2.0	7.4
A_8	0.998	0.836	196	195.6	163.9
A_9	-0.077	-0.120	364	- 28.0	- 43.7
D_7	-0.029	-0.430	-117	3.4	50.3
Predicted B.M. x 10^{-3}				173.0	177.9
Actual B.M. x 10^{-3}				179.0	179.0
Percentage error				- 3.5	- 0.6
Percentage standard error				9.5	3.5

A_5	-0.011	-0.0072	185	- 2.0	- 13.3
A_8	1.034	1.113	196	202.7	218.1
A_9	-0.077	-0.093	364	- 28.0	- 33.9
Predicted B.M. x 10^{-3}				172.7	170.9
Actual B.M. x 10^{-3}				179.0	179.0
Percentage error				- 3.6	- 4.5
Percentage standard error				9.4	5.2

TABLE 16

Regressions for estimating shear

Gauge station (a)	Regression coefficient: Individual load (b)	Regression coefficient: Combined load (c)	Gauge station response (d)	Distributed load contribution: Individual load (e)	Distributed load contribution: Combined load (f)
A_7	1.996	1.753	356	710.6	624.1
A_9	2.260	1.862	364	822.6	677.8
B_2	1.623	1.939	171	277.5	331.6
C_5	2.684	3.703	298	799.8	1103.5
D_1	2.521	3.178	14	35.3	44.5
D_6	2.147	2.348	283	607.6	664.5
M	4.036	3.166	179	722.4	566.7
Predicted shear				3975.8	4012.7
Actual shear				4170.0	4170
Percentage error				-4.9	-3.8
Percentage standard error				9.4	1.4

Gauge station (a)	Regression coefficient: Individual load (b)	Regression coefficient: Combined load (c)	Gauge station response (d)	Distributed load contribution: Individual load (e)	Distributed load contribution: Combined load (f)
A_7	2.931	2.809	356	1043.4	1000.0
B_2	1.466	1.727	171	250.7	295.3
C_5	3.386	4.349	298	1009.0	1296.0
D_6	2.030	2.058	283	574.5	582.4
M	5.906	4.299	179	1057.2	769.5
Predicted shear				3934.8	3943.2
Actual shear				4170	4170
Percentage error				-6.0	-5.4
Percentage standard error				10.7	2.5

Gauge station (a)	Regression coefficient: Individual load (b)	Regression coefficient: Combined load (c)	Gauge station response (d)	Distributed load contribution: Individual load (e)	Distributed load contribution: Combined load (f)
A_7	1.969	1.757	356	701.0	625.5
A_8	3.601	2.668	196	705.8	522.9
A_9	2.031	1.674	364	739.3	609.3
B_2	1.672	1.988	171	285.9	340.0
C_5	2.766	3.836	298	824.3	1143.1
D_1	3.581	4.328	14	50.1	60.6
D_6	2.234	2.424	283	623.3	686
Predicted shear				3938.7	3987.4
Actual shear				4170	4170
Percentage error				-5.9	-4.4
Percentage standard error				10.2	1.4

Gauge station (a)	Regression coefficient: Individual load (b)	Regression coefficient: Combined load (c)	Gauge station response (d)	Distributed load contribution: Individual load (e)	Distributed load contribution: Combined load (f)
A_7	3.858	3.283	356	1373.4	1168.7
C_5	2.953	3.514	298	880.1	1047.2
D_6	1.877	2.338	283	531.3	661.5
M	6.651	7.097	179	1190.5	1270.3
Predicted shear				3975.3	4147.7
Actual shear				4170.0	4170.0
Percentage error				-4.9	-0.5
Percentage standard error				18.4	4.1

Gauge station (a)	Regression coefficient: Individual load (b)	Regression coefficient: Combined load (c)	Gauge station response (d)	Distributed load contribution: Individual load (e)	Distributed load contribution: Combined load (f)
A_7	5.859	5.288	356	2058.8	1882.4
C_5	5.294	5.792	298	1577.5	1725.4
Predicted shear				366.38	3608.3
Actual shear				4170.0	4170.0
Percentage error				-12.2	-15.6
Percentage standard error				49.0	9.9

Gauge station (a)	Regression coefficient: Individual load (b)	Regression coefficient: Combined load (c)	Gauge station response (d)	Distributed load contribution: Individual load (e)	Distributed load contribution: Combined load (f)
A_7	1.980	1.493	356	705.0	531.5
A_9	2.149	1.821	364	782.3	662.8
B_2	1.663	2.064	171	284.4	352.9
C_5	2.517	3.333	298	750.1	993.2
D_6	1.987	2.274	283	562.4	643.5
M	5.119	4.640	179	916.4	830.6
Predicted shear				4001	4014.5
Actual shear				4170	4170
Percentage error				-4.2	-3.8
Percentage standard error				9.9	1.7

Gauge station (a)	Regression coefficient: Individual load (b)	Regression coefficient: Combined load (c)	Gauge station response (d)	Distributed load contribution: Individual load (e)	Distributed load contribution: Combined load (f)
A_7	3.821	3.477	356	1360.3	1237.1
C_5	4.294	5.581	298	1279.7	1663.1
M	6.77	5.798	179	1202.4	1037.8
Predicted shear				3842.4	3938.6
Actual shear				4170.0	4170.0
Percentage error				-7.9	-5.5
Percentage standard error				25.2	7.1

TABLE 7

Regressions for estimating torque

Gauge station	Regression coefficient: Individual load	Regression coefficient: Combined load	Gauge station response	Distributed load contribution: Individual load	Distributed load contribution: Combined load
A_7	0.182	0.259	356	64.8	92.2
A_9	0.145	0.064	364	52.8	23.3
B_2	0.288	0.279	171	49.2	47.7
C_5	0.109	0.238	298	32.5	70.9
D_1	0.097	0.371	14	1.4	5.2
D_6	0.070	0.075	283	19.8	21.2
M	1.023	0.777	179	183.1	139.1
Predicted Torque x 10^{-3}				403.6	399.6
Actual Torque x 10^{-3}				409.6	409.6
Percentage error				-1.5	-2.4
Percentage standard error					2.1

Gauge station	Regression coefficient: Individual load	Regression coefficient: Combined load	Gauge station response	Distributed load contribution: Individual load	Distributed load contribution: Combined load
A_7	0.185	0.249	356	65.9	88.6
A_9	0.155	0.054	364	56.4	19.6
B_2	0.274	0.269	171	46.9	46.0
C_5	0.115	0.211	298	34.3	62.9
D_2	0.050	0.234	74	3.7	17.3
D_6	0.080	0.076	283	22.6	21.5
M	0.963	0.795	179	172.4	142.3
Predicted Torque x 10^{-3}				402.2	398.2
Actual Torque x 10^{-3}				409.6	409.6
Percentage error				-1.8	-2.7
Percentage standard error					2.0

Gauge station	Regression coefficient: Individual load	Regression coefficient: Combined load	Gauge station response	Distributed load contribution: Individual load	Distributed load contribution: Combined load
A_7	0.252	0.306	356	89.7	108.9
B_2	0.271	0.266	171	46.3	45.5
C_5	0.154	0.274	298	45.9	81.7
D_1	-0.155	0.376	14	-2.2	5.3
D_6	0.069	0.067	283	19.5	19.0
M	1.111	0.765	179	198.9	136.9
Predicted Torque x 10^{-3}				398.1	397.3
Actual Torque x 10^{-3}				409.6	409.6
Percentage error				-2.9	-3.0
Percentage standard error					2.2

A_7	0.423	0.318	356	150.6	113.2
B_2	0.172	0.239	171	29.4	40.9
B_4	1.327	1.683	77	102.2	129.6
C_5	0.236	0.299	298	70.3	89.1
D_6	0.092	0.087	283	26.0	24.6
Predicted Torque x 10^{-3}				378.5	397.4
Actual Torque x 10^{-3}				409.6	409.6
Percentage error				-8.2	-3.0
Percentage standard error					3.4

A_7	0.256	0.268	356	91.1	95.4
B_2	0.267	0.283	171	45.7	48.4
C_5	0.165	0.230	298	49.2	68.5
D_6	0.076	0.059	283	21.5	16.7
M	1.056	0.944	179	189	169.0
Predicted Torque x 10^{-3}				396.5	398.0
Actual Torque x 10^{-3}				409.6	409.6
Percentage error				-3.3	-2.6
Percentage standard error					2.5

TABLE 8

Influence of size of sample

Shear

Gauges	Coefficients based on 29 pt data (b)	Coefficients based on 10 pt data (c)
A_7	1.753	3.848
A_9	1.862	0.821
B_2	1.939	1.696
C_5	3.703	3.660
D_1	3.178	6.336
D_6	2.348	2.153
M	3.166	1.411
% standard error	1.4	0.8

Bending moment

Gauges	Coefficients based on 29 pt data (b)	Coefficients based on 10 pt data (c)
A_8	0.645	0.726
A_9	-0.082	-0.100
B_4	0.359	0.200
D_6	0.007	-0.009
D_7	-0.461	-0.543
% standard error	2.9	3.1

Torque

Gauges	Coefficients based on 29 pt data (b)	Coefficients based on 10 pt data (c)	Coefficients based on 10 pt data
A_7	0.249	0.875	0.354
A_9	0.054	-0.371	
B_2	0.269	0.115	0.229
C_5	0.211	0.404	0.214
D_2	0.234	1.404	0.305
D_6	0.076	0.057	0.061
M	0.795	-0.396	0.718
% standard error	2.1	5.9	1.5

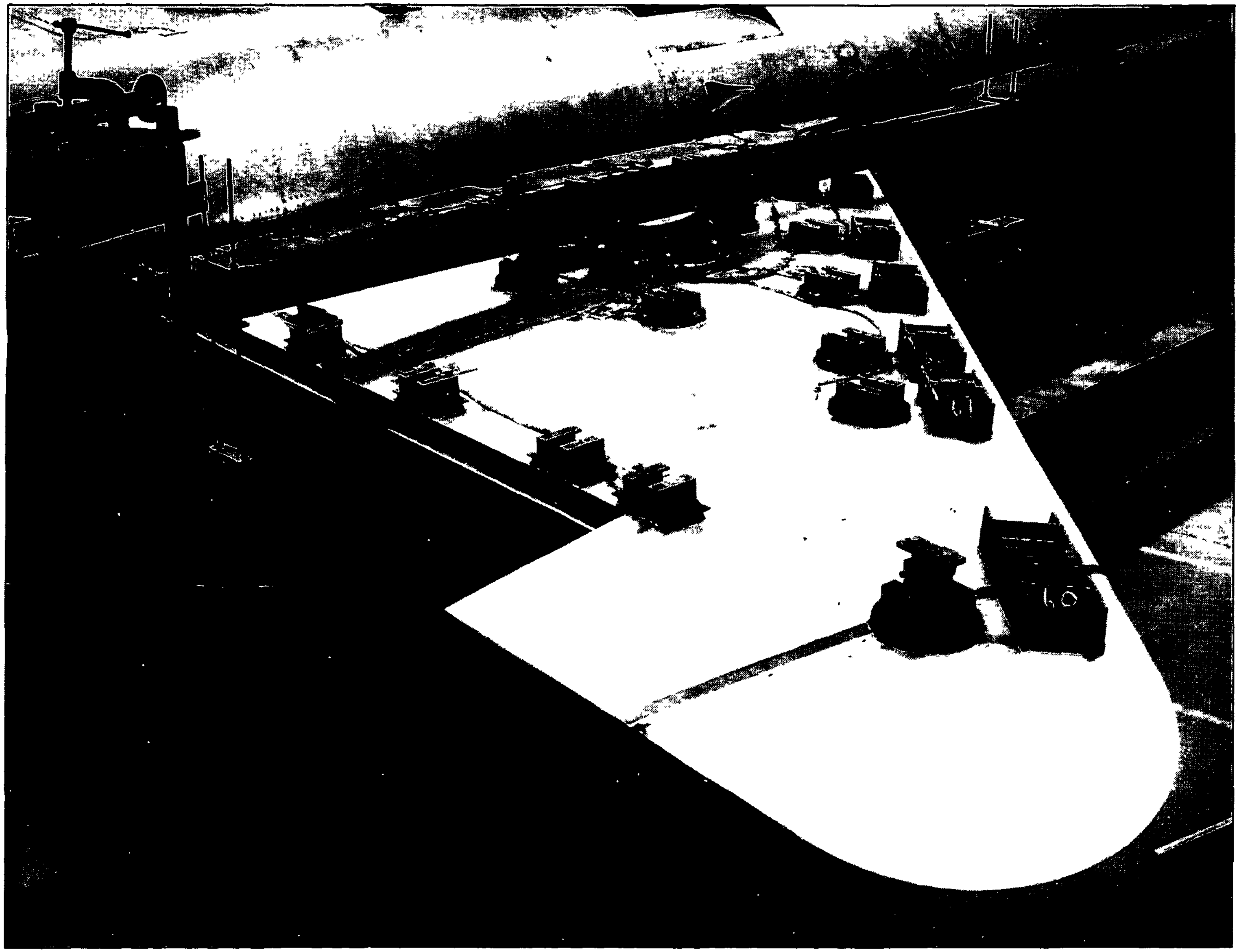

FIG. 1. GENERAL VIEW OF LIGHTNING FIN

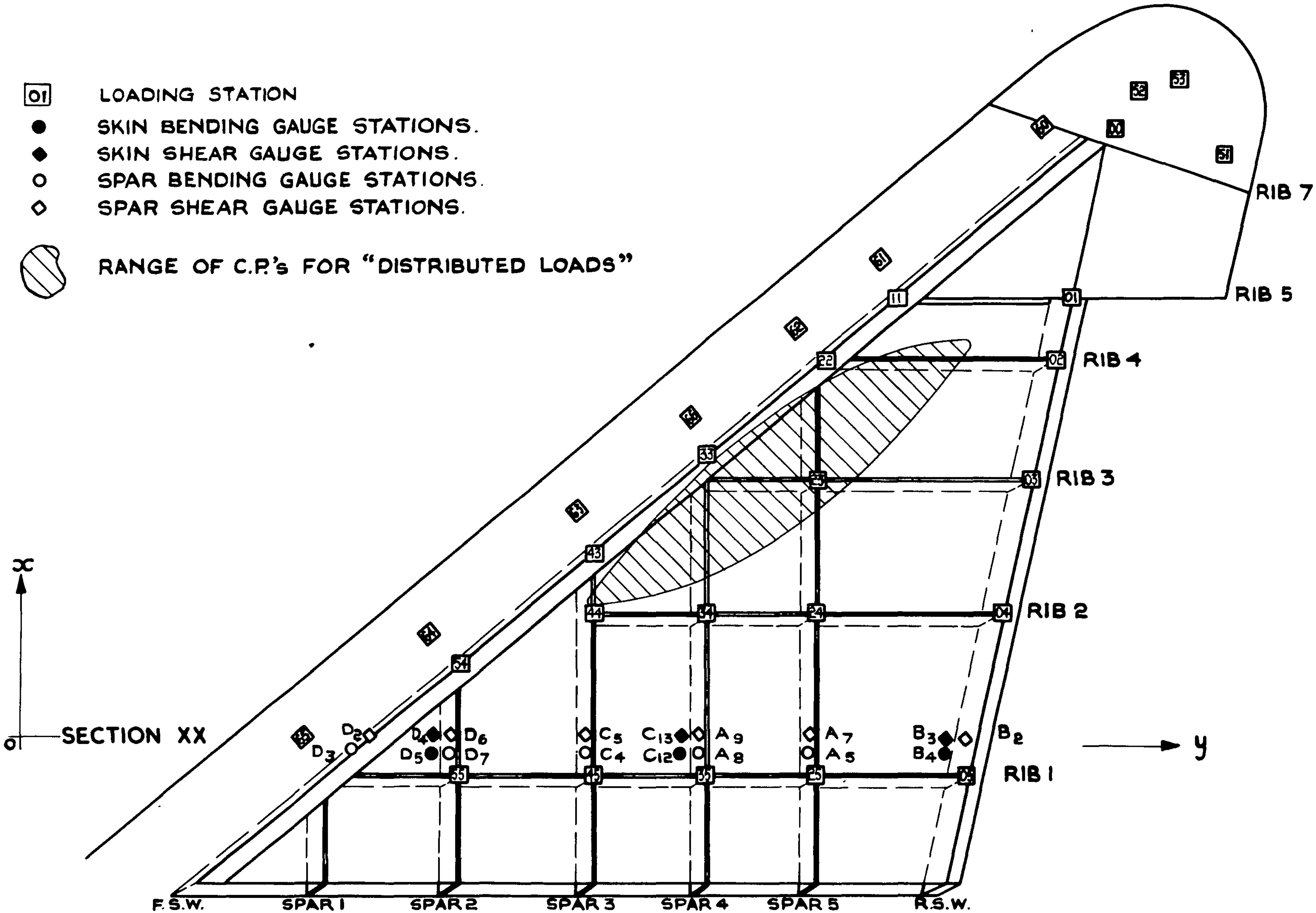

FIG. 2. STRAIN GAUGE STATIONS AND LOADING ARRANGEMENT.

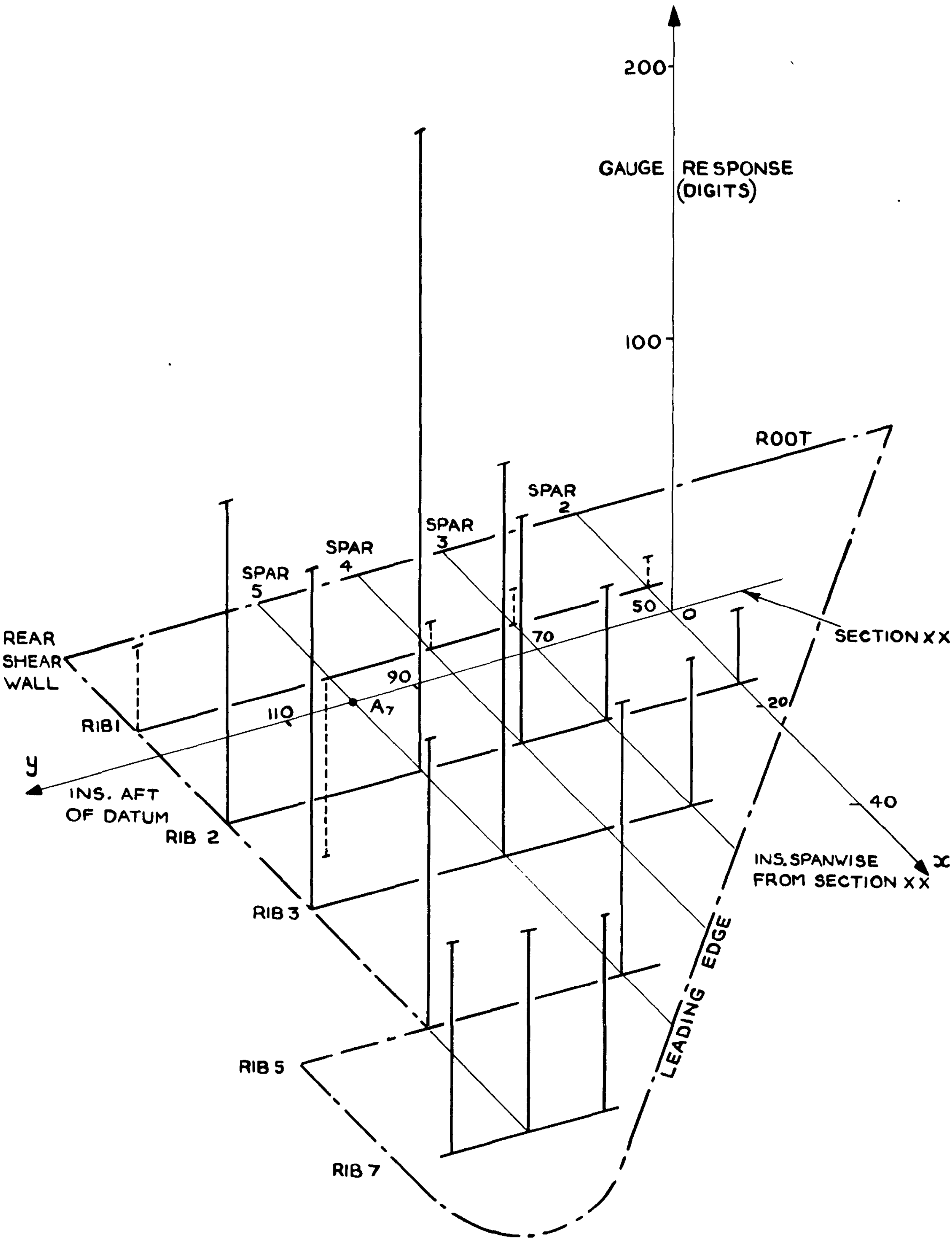

FIG. 3. RESPONSE OF GAUGE A_7 WITH POSITION OF LOAD. (1000 LB).

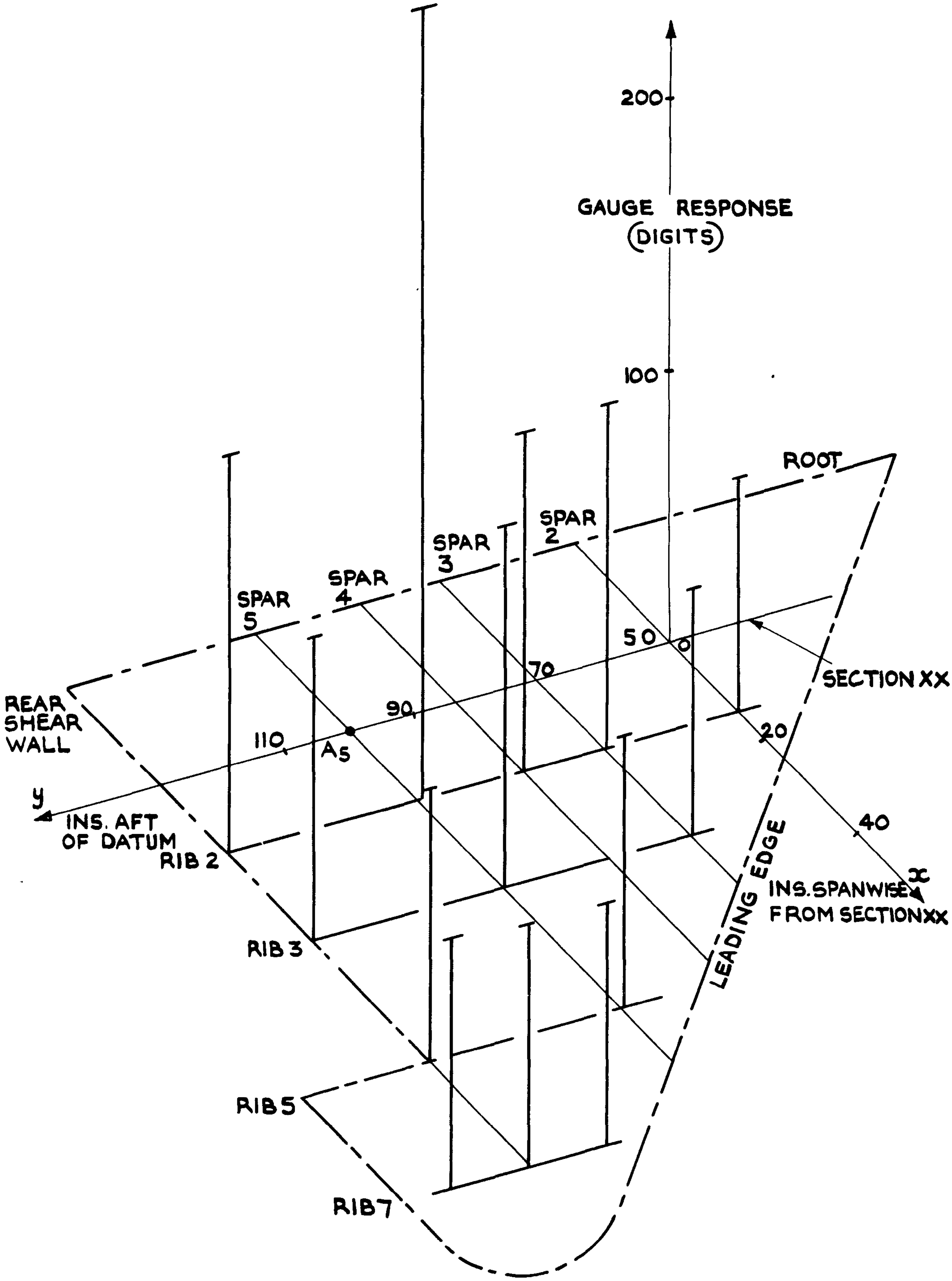

FIG. 4. RESPONSE GAUGE A_5 WITH POSITION OF LOAD.
(LOAD SCALED TO PRODUCE BM = 10^5 LB. IN. AT SECTION XX)

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