C.P. No. 823

C.P. No. 823

MINISTRY OF AVIATION

AERONAUTICAL RESEARCH COUNCIL

CURRENT PAPERS

A Parametric Study of Take-Off and Landing Distances for High-Lift Aircraft

By

W.A. Mair and B.J. Edwards
Engineering Department, Cambridge University

LONDON: HER MAJESTY'S STATIONERY OFFICE

1965

Price 6s. 6d net

C.P. No. 823

A Parametric Study of Take-Off and Landing Distances for High-Lift Aircraft

- By -
W. A. Mair and B. J. Edwards*
Engineering Department, Cambridge University

October, 1963

SUMMARY

Ignoring the possibility of engine failure and making some other idealised assumptions, particularly about margins of lift coefficient, calculations have been made to study the effects on take-off and landing distances of varying the main aircraft parameters such as maximum lift coefficient, wing loading, aspect ratio and thrust/weight ratio.

The results show that unless the thrust/weight ratio is large the take-off distance will usually be at least as great as the landing distance, especially if reversed thrust is used for landing. For this reason attention is concentrated on the take-off results. It is shown that if high lift coefficients can be obtained large reductions of take-off distance are possible. The maximum lift coefficiert that can usefully be employed is shown to be a function of aspect ratio and thrust/weight ratio, almost independent of wing loading.

1. Introduction

Various methods of increasing the maximum lift coefficient of a wing are now well known. These include the use of slots and flaps at the leading and trailing edges and control of the boundary layer by suction or blowing. The devices may be used in various combinations and unless the thrust/weight ratio is exceptionally low the increase of maximum lift coefficient may be expected to improve the airfield performance for a given wing loading. Alternatively, a high-lift device may be used to enable the wing loading to be increased while maintaining the same airfield performance.

All high-lift devices involve some additional weight and complication as a price to be paid for improved airfield performance, although in some cases the penalty may be small. The decision whether to use a high-lift device must be made by the designer in the light of information about the additional weight and the expected reduction of take-off and landing distances. The reliability

of/

---

*Now at British Aircraft Corporation Ltd., Guided Weapons Division, Bristol.
Replaces A.R.C.25 173

of the device and the possible consequences of failure of a suction or blowing plant must also be considered.

The field length required for take-off is greatly affected by the possibility of engine failure and by operating requirements designed to maintain safety in the event of such failure. In the calculations to be described the possibility of engine failure has been ignored and no attempt has been made to match the assumed piloting procedure to any particular set of operating requirements. Thus the calculations do not give realistic values of the actual field length required, but they do show the effect of various parameters on the take-off and landing distances that could be achieved in ideal conditions, with all engines working and no consideration of possible failure.

Since the use of a high-lift device to obtain low flight speeds necessarily increases the trailing vortex drag it is to be expected that the advantages to be gained will be strongly dependent on the aspect ratio, wing loading and thrust/weight ratio of the aircraft. The calculations were therefore planned to cover a wide range of these variables in order to assess their effect on take-off and landing distances. The distances have been calculated in terms of the maximum lift coefficient C_{L_S}, without any detailed consideration of the method used for obtaining high lift or the weight penalty involved. In any given case where a realistic assessment is required it will be necessary to consider not only the effects of possible engine failure and operating requirements but also the effect of the additional weight of any proposed high-lift device.

It has been assumed throughout that the engine thrust acts in the direction of flight and that the specified lift coefficients can be achieved at any value of the thrust. Thus the results are applicable to aircraft using BLC systems in which the suction or blowing can be adjusted independently of the propulsive thrust. Aircraft using jet flaps or deflected thrust or slipstream have not been considered, although it is recognised that these are powerful methods of achieving STOL and they may well be used in conjunction with BLC. Thrust deflection is particularly valuable at high values of the thrust/weight ratio T_o/W, and because this has not been considered the range of T_o/W has been restricted to values below about 0·6.

With a well designed high-lift system using boundary-layer control, unseparated flow can be maintained so that the profile drag coefficient is not much greater at high values of C_L than in cruising flight. This is in contrast to some trailing-edge flap systems without BLC, in which high lift coefficients can only be obtained at the expense of high profile drag. In the calculations to be described the values assumed for the profile drag coefficient are low, so that the results are applicable to BLC systems designed to obtain low profile drag at high incidence and not to BLC systems or other high-lift devices giving a large profile drag.

The calculations were made with an electronic digital computer (EDSAC 2). In making assumptions about the take-off and landing procedures some simplifications were introduced. These tended to be optimistic, so that the calculated distances represent the minimum obtainable in ideal conditions; in most cases the actual distances achieved would be greater than the calculated ones because the pilot would have to follow a more realistic procedure. Nevertheless the results should

enable/

enable valid comparisons to be made between aircraft having differing values of the main parameters, because the same simplifying assumptions are made in all cases.

Although calculations have been made for both take-off and landing, the emphasis in this paper is on the former because the results show that in most cases, using the family of assumptions defined later, the landing distance is less than the take-off distance.

The airfield is assumed to be horizontal and the calculated take-off distance is the minimum distance required for the aircraft to accelerate from rest and reach a height of 50 ft, while the landing distance is the minimum distance required for the aircraft to descend from a height of 50 ft and come to rest. All the calculations were made for zero wind.

Theoretical studies on the same lines as the present one have been made by a number of other authors, e.g. Refs. 1, 2 and 3. In Ref. 1 the emphasis is on aircraft using deflected thrust, but some calculations of limiting conditions with undeflected thrust, neglecting C_{D_o} and C_{D_W}, are in fair agreement with the present results. Ref. 2 is restricted to a particular class of aircraft with a fixed payload of 1 500 lb and the assumptions made are appropriate only to this class. The weights of aircraft considered are in the range 4000 to 10 000 lb and for these weights some of the assumptions made in the present work are not expected to be valid. Nevertheless, the results given here agree fairly well with those of Ref. 2.

Ref. 3 covers a wider field and the present investigation may be regarded as an extension of that work. The use of an electronic digital computer in the present work has made it possible to consider a large number of different cases and to adjust the take-off and landing procedures to give minimum distances. It is believed that the results obtained give a useful indication of the classes of aeroplane that will show the greatest benefit from the use of high-lift devices. They also give an indication of the greatest values of the maximum lift coefficient that are likely to be useful for various classes of aircraft in ideal conditions, but it must be remembered that various operational margins and safety requirements have not been considered and these have an important effect on the highest useful value of the maximum lift coefficient.

Notation

A		aspect ratio of wing
a	ft	height of wing above ground
a_v	ft/sec^2	vertical acceleration during transition or flare
B		see V_B
bC_{L_S}		constant lift coefficient in steady climb
c	(sec/ft)^2	constant in thrust equation (1)
c'	(sec/ft)^2	constant in reversed thrust equation (5)
C_D		total drag coefficient of aircraft
C_{D_o}		drag coefficient of aircraft at zero lift, with undercarriage retracted

C_{D_W}		drag coefficient of undercarriage, based on wing area
C_L		lift ocefficient of aircraft
C_{L_F}		lift coefficient during flare (assumed constant for the purpose of analysis)
C_{L_M}		lift coefficient in level flight at speed V_M
C_{L_S}		maximum lift coefficient of aircraft (with appropriate engine power)
C_{L_U}		value of C_{L_S} giving take-off distance 15% greater than minimum possible
D_i	lb	induced drag in level flight at light coefficient C_{L_U}
E		see V_E
F	lb	total retarding force acting on aircraft during take-off ground run
g	ft/sec²	acceleration due to gravity
h_L	ft	height at which landing flare begins
h_T	ft	height at which transition to steady climb is complete
K		induced drag factor in absence of ground effect (assumed to be 1·25)
K'		induced drag factor, allowing for ground effect (assumed to be 0·875)
L_L	ft	total landing distance required from a height of 50 ft to a standstill
L_1	ft	horizontal distance required for steady approach from height of 50 ft to start of landing flare
L_2	ft	horizontal distance required for landing flare
L_3	ft	horizonatal distance required for landing float, if any
L_4	ft	length of ground run required in landing, to bring aircraft to rest
ℓ_T	ft	total take-off distance required from standstill to a height of 50 ft
ℓ_1	ft	length of take-off ground run
ℓ_2	ft	distance, if any, flown close to the ground while accelerating to speed at which climb is started
ℓ_3	ft	horizontal distance required for transition from level flight to steady climb at constant angle
ℓ_4	ft	horizontal distance required, if any, in addition to ℓ_3, for climb at constant angle to a height of 50 ft

S	sq ft	gross wing area
s	ft	semi-span of wing
T	lb	total thrust of all engines
T_B	lb	total braking thrust of propellers
T_o	lb	total static thrust of all engines
V	ft/sec	speed of aircraft
V_a	ft/sec	speed at which aircraft leaves ground, during take-off
V_b	ft/sec	speed in steady climb and during transition to steady climb
$V_B = BV_s$	ft/sec	speed during steady approach
$V_E = EV_s$	ft/sec	speed at end of flare
V_M	ft/sec	mean speed during flare. See equation (6)
V_S	ft/sec	stalling speed (with appropriate engine power)
V_T	ft/sec	speed at touch-down
V_{16}	ft/sec	stalling speed of aircraft in landing condition with BLC shut off
W	lb	total weight of aircraft
$w = \frac{W}{S}$	lb/ft^2	wing loading
x	ft	horizontal distance
α_T		wing incidence during landing ground-run, measured from no-lift condition with flaps up
γ		angle of steady climb
δ		inclination of steady approach path to horizontal
μ		coefficient of rolling resistance
μ_B		mean effective braking coefficient of friction
ρ	slug/ft^3	air density (standard sea-level conditions assumed throughout).

2./

2. Assumed Characteristics of Aircraft

Both jet and propeller aircraft have been considered, with parameters varying over the following ranges.

Wing loading: 15 to 250 lb/ft^2

Aspect ratio: 5 to 18.

Ratio of static thrust to weight: 0·26 to 0·47 for jets,
0·32 to 0·58 for propellers.

Calculations were made for aircraft weights from 10 000 to 200 000 lb, but some of the assumptions made are not considered to be valid for weights below 30 000 lb. The variation of weight had some effect on the take-off calculations, because the drag coefficient was assumed to depend to some extent on the weight. In calculating the landing distances the effect of drag was relatively small, so that simpler assumptions could be made with no dependence on weight.

2.1 Thrust

To simplify the take-off calculations the variation of thrust with forward speed has been represented approximately by the empirical equation

$$T = T_o (1 - cV^2), \qquad \dots (1)$$

where T_o is the static thrust. It was found that the values

$$c = 0{\cdot}25 \times 10^{-5} \ (\text{sec/ft})^2 \quad \text{for jets}$$

and

$$c = 1{\cdot}0 \times 10^{-5} \ (\text{sec/ft})^2 \quad \text{for propellers}$$

gave reasonable agreement with data for a number of current jet and propeller aircraft over the appropriate speed range.

2.2 Drag during take-off

The coefficient of ground rolling resistance μ was varied from 0·02 (concrete runway) to 0·10 (long grass). Most of the results given are for $\mu = 0{\cdot}02$.

The total drag coefficient of the aircraft (based on wing area) has been represented by

$$C_D = C_{D_o} + C_{D_W} + \frac{KC_L^2}{\pi A}, \qquad \dots (2)$$

where/

where C_{D_0} is the drag coefficient at zero lift with the undercarriage retracted and C_{D_W} is the drag coefficient of the undercarriage. When the aircraft is close to the ground the induced drag factor K is replaced by K' to allow for ground effect.

The value of C_{D_0} during take-off was estimated as the sum of two terms, the first due to the wing and the second due to the fuselage and tail. The wing contribution was taken to be 0·0065 for jet aircraft and 0·0080 for propeller aircraft, the latter being greater because of slipstream effects. The contribution of the fuselage and tail to C_{D_0} can be expressed in terms of the wetted area of these parts. Plotting this wetted area against $W^{\frac{1}{2}}$ for a number of current aircraft gave the approximate relationship.

Wetted area of fuselage and tail surfaces = $12{\cdot}2\ W^{\frac{1}{2}}$ sq ft, where W is the weight in lb.

Taking drag coefficients, based on wetted area, of 0·00285 for jet aircraft and 0·0040 for propeller aircraft (multi-engined, with slipstream), the contributions of the fuselage and tail to C_{D_0} are found to be

$$\frac{0{\cdot}035\ W^{\frac{1}{2}}}{S} \quad \text{for jet aircraft}$$

and

$$\frac{0{\cdot}049\ W^{\frac{1}{2}}}{S} \quad \text{for propeller aircraft,}$$

where S is the wing area in sq ft. Thus the total values of C_{D_0} during the take-off are taken to be

$$\left.\begin{array}{ll} \text{Jet:} & 0{\cdot}0065 + \dfrac{0{\cdot}035\ W^{\frac{1}{2}}}{S} \\ \text{Propeller:} & 0{\cdot}0080 + \dfrac{0{\cdot}049\ W^{\frac{1}{2}}}{S} . \end{array}\right\} \quad \ldots (3)$$

There is very little published information on the drag of modern undercarriages and in the absence of more complete information the curves given by Perkins and Hage[4] have been used. These can be represented approximately by the equation

C_{D_W}/

$$C_{D_W} = \frac{0{\cdot}143\, W^{\frac{1}{2}} - 10}{S} . \qquad \ldots (4)$$

The induced drag factor K has been assumed to be 1·25 in the absence of ground effect. The reduction of K due to ground effect depends on the ratio a/2s, where a is the height of the wing above the ground and 2s is the wing span. With conventional undercarriages a/2s is usually between about 0·1 and 0·2 when the aircraft is on the ground. High-lift aircraft will perhaps tend to have rather high undercarriages and a value of a/2s of about 0·2 may be typical. Theoretical and experimental results reproduced by Hoerner[5] show that for this value of a/2s the ground effect reduces the induced drag factor by about 30%. Thus K' is assumed to be 0·875 for the aircraft on the ground.

Two further assumptions affecting the drag during take-off are:

(a) The undercarriage is assumed to be retracted immediately after the aircraft leaves the ground and the time required for retraction is neglected.

(b) The induced drag factor is assumed to change discontinuously from K' = 0·875 when the aircraft is on the ground to K = 1·25 as soon as the aircraft begins to climb.

Since some time is required for retraction of the undercarriage, assumption (a) leads to an underestimate of the drag immediately after the aircraft leaves the ground. This error may be roughly compensated by assumption (b) which leads to an overestimate of drag.

2.3 Drag during landing

It is assumed that the engine thrust needed to follow the required approach path is less than the maximum available thrust. It is also assumed that during the flare, with the engines throttled back as necessary, the drag of the aircraft is sufficient to produce the assumed loss of speed. Then in the usual case with no float the ground run is the only part of the total landing distance that is affected by the drag of the aircraft. Since the effect of drag on the ground run is not large, an accurate assessment of drag is not necessary.

It is assumed that the incidence throughout the ground run remains constant and equal to α_T, the value corresponding to level flight at the touch-down speed V_T. This speed is either V_E or V_{16}, whichever is the smaller. (See §3.2). The incidence α_T is measured from the zero-lift condition with flaps up and can easily be estimated for given values of V_T and aspect ratio.

It is assumed that the BLC system is shut off at the end of the flare, so that the wing will usually be stalled during the ground run. The total wing drag is then estimated roughly, in the absence of ground effect, by assuming that the resultant force acts in a direction normal to the

zero-lift/

zero-lift line. The normal force coefficient for the stalled wing is taken to be $\left(1 - \frac{0 \cdot 9}{A}\right)$, where A is the aspect ratio, so that the drag coefficient in the absence of ground effect is $\left(1 - \frac{0 \cdot 9}{A}\right) \sin \alpha_T$. To allow for ground effect this value is reduced by $\frac{0 \cdot 375\, C_L^2}{\pi A}$ as for the take-off calculations.

Since α_T is usually not large, the lift coefficient is nearly equal to $\left(1 - \frac{0 \cdot 9}{A}\right)$.

To simplify the calculations α_T has been arbitrarily assumed to be equal to 16° in the few cases where it is actually less than this value. For these cases the assumption that the wing is stalled during the ground run may be incorrect and the actual wing drag may be less than the value used in the calculations.

The expressions used in the take-off calculations for the fuselage and tail and undercarriage drag coefficients depended on both the weight W and the wing loading w. For the calculation of ground run during landing there was less need for accuracy in estimating drag and simpler expressions were used in order to eliminate W. These were:

Fuselage and tail contribution to $C_D = 0 \cdot 017$

Undercarriage drag coefficient $= C_{D_W} = 0 \cdot 0005\, w$.

Since slipstream effects are small during the landing ground run there is no need to distinguish between jet and propeller aircraft in estimating the drag.

The total aircraft drag coefficient C_D during the ground run is obtained by adding the wing, fuselage and tail, and undercarriage drag coefficients.

In the few cases where there is a "float" between the end of the landing flare and the start of the ground run the total drag coefficient is taken to be $C_{D_o} + \frac{K' C_L^2}{\pi A}$, where $C_{D_o} = 0 \cdot 07$. This value of C_{D_o} is an arbitrary one, but tests with the computer showed that reasonable variations in the chosen value had negligible effects on the calculated distances.

2.4 Braking and reversed thrust

Two values of the braking coefficient of friction μ_B were used in the calculations, 0·17 representing a runway covered with water and 0·35 representing a dry concrete runway for which the limiting factor is the brake capacity rather than the runway friction.

Calculations were made for propeller aircraft with reversed thrust, assuming that the maximum reversed thrust at zero speed is only 0·64 of T_o, the maximum static thrust when the propellers are operating normally. The maximum braking thrust at a speed V ft/sec is then assumed to be

$$T_B = 0{\cdot}64\, T_o\, (1 + c'\, V^2)\;. \qquad \ldots (5)$$

Comparison with measurements on typical modern propellers has shown that reasonable agreement is obtained if c' is taken to be $4{\cdot}3 \times 10^{-5}$ (sec/ft)^2.

In calculating the ground run using reversed thrust the effect of the wake from the braking propellers on the lift and drag of the aircraft was taken into account. In doing this it was necessary to consider separately the two stages of the ground run:

(a) for which the propeller wake is a simple expanding slipstream

and (b) for which there is either a turbulent wake behind the propeller or a vortex ring.

For stage (a) the velocity behind the propeller can be calculated easily in terms of the thrust. For stage (b) the air velocity relative to the aircraft was assumed to be zero behind the propeller.

3. Piloting Technique and Other Assumptions

3.1 Take-off

It is assumed that during the ground run the attitude of the aircraft is maintained at the value giving maximum acceleration. In practice, with a nose-wheel undercarriage, the attitude during the greater part of the ground run will be determined by the undercarriage design and the assumed ideal condition will usually not be satisfied exactly.

The transition from horizontal motion on or near the ground to a steady climb at an angle γ is assumed to be made in a parabolic flight path at a constant speed V_b and a constant lift coefficient $0{\cdot}9\, C_{L_S}$. In a few cases the available thrust will not be sufficient to enable the transition arc to be flown at a lift coefficient as high as $0{\cdot}9\, C_{L_S}$. This means that, for each thrust/weight ratio, there is a range of high values of C_{L_S} for which the results are not strictly valid. This limitation is not important, however, because the range of C_{L_S} for which the results are not valid is

just/

just that which is shown by the results to be above the useful range. Even when sufficient thrust is available there may be cases in which the effects of induced drag mean that a slightly shorter take-off could be achieved by keeping the lift coefficient below $0{\cdot}9\ C_{L_S}$ during the transition, but these cases are also likely to be unimportant.

It was found in a few cases, with large ground-rolling resistance and high aspect ratio, that the shortest total take-off distance ℓ_T is obtained by leaving the ground at a speed V_a which is less than the optimum speed V_b for the transition. The aircraft then accelerates from speed V_a to V_b while flying horizontally just above the ground. (In this state the induced drag factor is taken to be $K' = 0{\cdot}875$.) The induced drag coefficient varies during the acceleration, but for simplicity in calculating the distance travelled the arithmetic mean of the two extreme values has been used.

Both V_a and V_b are limited by the condition that the lift coefficient must not exceed 90% of the maximum value C_{L_S}. Because the flight path is curved during the transition the speed V_b has to be greater than the speed for a lift coefficient of $0{\cdot}9\ C_{L_S}$ in level flight. This explains why it is possible, and in some cases desirable, to lift the aircraft off the ground at a speed V_a that is less than V_b, even though the lift coefficient at speed V_a must not exceed $0{\cdot}9\ C_{L_S}$, the value during the transition.

In considering the limitations on lift coefficient the value of C_{L_S} should always be the appropriate power-on maximum lift coefficient. Within the limitations already stated the computer was programmed to choose the values of V_a and V_b giving the shortest total distance for take-off over a 50 ft obstacle. For take-off from normal concrete runways $(\mu = 0{\cdot}02)$ it is always found that $V_a = V_b$ so that the transition starts immediately the aircraft leaves the ground.

The steady climb after the transition is assumed to be made at the same speed V_b as the transition.

3.2 Landing

In calculating the landing distance the aircraft is assumed to approach the airfield at a constant speed V_B in a straight path inclined at an angle δ to the horizontal. Values of δ up to 35° were considered, but the results show that there is usually little to be gained by increasing δ above about 8°.

At a height h_L the flare is started. In this phase of the landing the speed decreases from V_B to V_E and the vertical component of velocity decreases from $V_B \sin\delta$ to zero. The flight path in the flare is assumed to

be/

be a parabolic arc with constant lift. To simplify the analysis this constant lift is expressed in terms of a constant lift coefficient C_{L_F} and a constant speed V_M given by

$$V_M^2 = \tfrac{1}{2}(V_B^2 + V_E^2) \ . \qquad \dots (6)$$

It is clear that to achieve a short landing distance the lift coefficient should be as high as possible during the flare. A margin of safety from stalling is required and it seems reasonable to relate this safety margin in the flare to the speed margin in the steady approach. Although the assumption is optimistic the lift coefficient in the flare has been taken to be

$$C_{L_F} = (1{\cdot}24 - \tfrac{1}{4}B)\ C_{L_S} \ , \qquad \dots (7)$$

where V_S is the appropriate power-on stalling speed and $V_B = BV_S$ is the speed in the steady approach. Thus for a normal landing by a civil aircraft B might be $1{\cdot}3$, giving $C_{L_F} / C_{L_S} = 0{\cdot}915$, whereas for some military purposes a value of B as low as $1{\cdot}1$ might be acceptable, giving $C_{L_F} / C_{L_S} = 0{\cdot}965$.

It is assumed that at the instant the flare is completed the engine thrust is reduced to zero and the BLC system is shut off. In most cases the loss of BLC will cause the wing to stall and the ground run then commences immediately. There may be some cases, however, in which the wing does not stall when the BLC is shut off at the end of the flare and a "float" may be desirable, with the aircraft flying close above the ground while losing speed. For consideration of these cases some further assumptions are required.

It is assumed that when the BLC system has been shut off at the end of the flare the wing has a stalling incidence 16° above that giving zero lift with flaps up. Denoting the speed for level flight at this incidence by V_{16}, it is assumed that if the speed V_E at the end of the flare is greater than V_{16} there is a float in which the speed falls to V_{16}. This assumption is perhaps a little unrealistic, because in practice it is not necessary for the wing to be stalled at touch-down, particularly if the aircraft has a nose-wheel undercarriage. Nevertheless the assumption is probably better than the opposite extreme, complete neglect of the possibility of a float, and in any case for most of the aircraft considered V_E was found to be less than V_{16}, so that no float was included. It is only for aircraft having low maximum lift coefficients or using large values of B that V_E is found to be greater than V_{16}.

It is assumed that the brakes are applied and reach the skidding or brake-capacity limit immediately after touch-down. Reversed thrust is considered only for propeller aircraft and where this is available it is

assumed/

assumed that after touch-down there is a delay period of 2 seconds before the pilot takes any action to reverse the thrust; during this period the engine thrust is assumed to be zero. A further period of 2 seconds is assumed after this, during which the pilot operates the pitch-reversing mechanism and increases the engine speed to the maximum permissible. For simplicity in the calculations a mean reversed thrust of half the maximum is assumed in this further 2 second period.

4. Calculation of Take-Off Distance

The total distance required to accelerate from rest and climb to a height of 50 ft is

$$\ell_T = \ell_1 + \ell_2 + \ell_3 + \ell_4 \cdot$$

The computer programme was arranged to give the minimum value of ℓ_T for each selected combination of aircraft and runway, the other variables being adjusted automatically to their optimum values. As already mentioned, ℓ_2 is zero for minimum ℓ_T except for a few special cases.

4.1 The ground run ℓ_1

The total retarding force acting on the aircraft, when travelling at a speed V along level ground in zero wind, is

$$F = \tfrac{1}{2}\rho V^2 S\left(C_{D_o} + C_{D_W} + \frac{K' C_L^2}{\pi A}\right) + \mu(W - \tfrac{1}{2}\rho V^2 S C_L) \ . \qquad \ldots (8)$$

For a given speed V, F is a minimum when

$$C_L = \frac{\pi A \mu}{2K'} \ . \qquad \ldots (9)$$

During the ground run the attitude of the aircraft is assumed to be maintained at the value required by equation (9). The length of the ground run ℓ_1 is then obtained by integration of the equation of motion

$$\frac{W}{g} V \frac{dV}{dx} = T - F \ , \qquad \ldots (10)$$

from zero speed to the speed V_a at which the aircraft leaves the ground.

4.2 Level flight close to the ground, ℓ_2

The calculation of this distance is straightforward when V_a, V_b and the thrust and drag characteristics are given.

4.3 Distance ℓ_3 for transition to steady climb.

In the steady climb following the transition the speed is V_b, the lift coefficient bC_{L_S} and the angle of climb γ. For a given value of C_{L_S}, b and V_b are simply related and γ is easily found from the thrust and drag. During the transition the lift coefficient is $0{\cdot}9\ C_{L_S}$ and hence for small γ the vertical acceleration is

$$a_v = g\left(\frac{0{\cdot}9}{b} - 1\right). \qquad \ldots (11)$$

Thus the height required for completion of the transition is

$$h_T = \frac{V_b^2 \sin^2\gamma}{2a_v}. \qquad \ldots (12)$$

It is necessary now to distinguish two cases, (a) $h_T > 50$ ft and (b) $h_T < 50$ ft. In the former case the assumed 50 ft obstacle is cleared before transition is complete and the required horizontal distance is

$$\ell_3 = 10\, V_b a_v^{-\frac{1}{2}} = \left[\frac{200\, w}{\rho g C_{L_S}(0{\cdot}9 - b)}\right]^{\frac{1}{2}}. \qquad \ldots (13)$$

In the case where equation (12) gives $h_T < 50$ ft the required length ℓ_3 is the total horizontal distance travelled during transition. This is

$$\ell_3 = V_b\left(\frac{2h_T}{a_v}\right)^{\frac{1}{2}} = \frac{bV_b^2 \sin\gamma}{g(0{\cdot}9 - b)}. \qquad \ldots (14)$$

4.4 Horizontal distance ℓ_4 in steady climb

For $h_T > 50$ ft, $\ell_4 = 0$.

For $h_T < 50$ ft, $\ell_4 = (50 - h)\cot\gamma$ ft.

5./

5. Calculation of Landing Distance

The total distance required for the aircraft to descend from a height of 50 ft and come to rest is

$$L_L = L_1 + L_2 + L_3 + L_4 .$$

As for the take-off calculations, the computer programme was arranged to give minimum L_L. For most of the cases considered the distance L_3 was zero.

5.1 The steady approach

It is only necessary to consider this phase of the landing if the height h_L at which the flare starts is less than 50 ft. In this case

$$L_1 = (50 - h_L) \cot \delta \text{ ft.} \qquad \dots (15)$$

5.2 The flare

The speeds at the beginning and end of the flare are $V_B = BV_S$ and $V_E = EV_S$ respectively. The lift coefficient in the flare is C_{L_F}, given by equation (7), whereas the lift coefficient in level flight at the mean flare speed V_M is

$$C_{L_M} = \frac{2C_{L_S}}{E^2 + B^2} . \qquad \dots (16)$$

Thus if $\cos \delta \approx 1$ the upward vertical acceleration during the flare is

$$a_v = g \left(\frac{C_{L_F}}{C_{L_M}} - 1 \right) . \qquad \dots (17)$$

The height at which the flare begins is

$$h_L = \frac{(BV_S \sin \delta)^2}{2a_v} \qquad \dots (18)$$

and with a parabolic flight path the horizontal distance required is

L_2/

$$L_2 = \frac{2h_L}{\tan\delta} . \qquad \ldots (19)$$

If $h_L > 50$ ft the length L_1 will be zero and only the part of the flare below a height of 50 ft needs to be considered. For this part

$$L_2 = \frac{10\sqrt{2h_L}}{\tan\delta} \text{ ft} . \qquad \ldots (20)$$

5.3 The float

In cases where there is a float the required distance L_3 is easily calculated from the initial and final speeds and the drag.

5.4 The ground run

The distance L_4 required for the ground run is calculated by integration of the equation of motion, using the drag and reversed thrust data already given.

6. Results

Consideration of the results showed that some of the variables had only small effects and that the main trends could be presented fairly concisely by choosing appropriate methods of plotting. Thus only a selection of typical results is given here. It was found that in most cases, except for high values of T_0/W, the calculated distances for take-off were greater than for landing, especially when reversed thrust was used for landing. For this reason the results are presented with a greater emphasis on take-off than on landing.

Since the take-off results for jet and propeller aircraft showed similar trends, attention has been concentrated on propeller aircraft in presenting the results. STOL aircraft incorporating boundary-layer control systems derive considerable benefit from propeller slipstream, and for relatively low cruising speeds propellers are perhaps more likely to be used than jet engines. When no reversed thrust is used the calculated landing distances apply either to jet or propeller aircraft. The use of reversed thrust was only considered for propeller aircraft.

6.1 Take-off

It was expected that two of the most important variables affecting the take-off distance would be the thrust/weight ratio and the stalling speed. The take-off distance was therefore plotted, at first, against stalling speed V_S, but later it was found that the curves were more nearly straight if w/C_{L_S} (equal to $\frac{1}{2}\rho V_S^2$) was used instead of V_S .

Fig.1/

Fig.1 shows the take-off distance plotted in this way, for propeller aircraft with $A = 8$. When w/C_{L_S} is fairly large the take-off distance is a function of T_o/W and w/C_{L_S} as expected, and variation of wing loading while maintaining constant w/C_{L_S} (i.e., constant stalling speed) has no effect. At smaller values of w/C_{L_S} there is an important effect of varying wing loading, even at constant w/C_{L_S}, especially for small T_o/W, and the take-off distance approaches a limiting minimum value which is independent of C_{L_S}. This limiting condition, dependent on wing loading, is due to the effect of induced drag, which is directly proportional to wing loading for given weight, speed and aspect ratio. When the limiting minimum take-off distance is reached the highest lift coefficient used in an optimised take-off is restricted by induced drag and not by stalling. Thus further increase of C_{L_S} has no effect because higher lift coefficients are not used.

Fig.2 is similar to Fig.1 except that it refers to jet aircraft. The curves are of the same form, but the value of T_o/W required to achieve a given take-off distance is less than that for a propeller aircraft with the same w/C_{L_S}, because the constant c in the thrust equation (1) is less for jets than for propellers and also the drag as given by equation (3) is less for the jet case.

Figs.1 and 2 both refer to an aspect ratio of 8, and the effect of varying aspect ratio is shown in Fig.3, where results are plotted for aspect ratios of 5 and 12. As expected, the effect of aspect ratio is small in the region where induced drag is unimportant, i.e., where the take-off distance depends on w/C_{L_S} rather than wing loading. At the smaller values of w/C_{L_S}, where induced drag is important and the take-off distance is approaching its limiting minimum value, there is a large effect of aspect ratio. Since, for a given speed, the ratio of induced drag to weight is proportional to w/A, it is to be expected that the lower ends of the dotted curves in Fig.3 will depend mainly on w/A. Inspection of Fig.3 shows that this is at least approximately true; for example, the curves for $(A = 5, \ w = 40)$ and $(A = 12, \ w = 100)$ are nearly coincident. The point is shown more exactly in Fig.4, where for several values of T_o/W the minimum possible take-off distance is plotted against w/A. The points for $T_o/W = 0{\cdot}3$ and $0{\cdot}4$ cover a range of aspect ratios and show clearly that the limiting take-off distance is a function of T_o/W and w/A. The curves in Fig.4 for $T_o/W = 0{\cdot}5$ and $0{\cdot}6$ were obtained by cross-plotting and the individual points are not shown, but in these cases also there was no evidence of any inconsistency in this method of plotting. Thus it may be concluded that all the results shown in Figs.1 and 2 may be applied with reasonable accuracy to any aspect ratio, provided the dotted parts of the curves are considered to be dependent on w/A and not simply on wing loading. Thus, for example, the curves shown in Figs.1 and 2 for $w = 80$ (and $A = 8$) would be correct for $w = 50$ and $A = 5$, or for $w = 100$ and $A = 10$.

In considering future research on boundary-layer control systems it is of interest to estimate the maximum value of C_{L_S} that is likely to be

useful./

useful. In practice the useful maximum value is not that which gives the minimum distance as shown in Fig.4, because a smaller C_{L_S} will give a smaller weight of BLC system and only slightly greater take-off distance. For the present purpose the "maximum useful lift coefficient" C_{L_U} has been arbitrarily defined as the value of C_{L_S} giving a take-off distance 15% greater than the minimum possible (with unlimited C_L available).

Values of C_{L_U} were found from the results and plotted against aspect ratio for several constant values of T_o/W. This showed that, for given T_o/W, C_{L_U} was almost directly proportional to aspect ratio and nearly independent of wing loading. The results were therefore re-plotted as C_{L_U}/A against T_o/W, as shown in Fig.5. The curve shown applies with good accuracy over a wide range of wing loading, except at the highest values of T_o/W. For $T_o/W = 0{\cdot}58$ the values of C_{L_U}/A were respectively 1·00, 1·11 and 1·16 at wing loadings of 40, 70 and 100 lb/ft.^{2}

It is easy to see why it is to be expected that C_{L_U}/A should be roughly proportional to T_o/W. For,

$$\frac{C_{L_U}}{A} = \frac{\pi}{K}\frac{D_i}{W} = \frac{\pi}{K}\left(\frac{T_o}{W}\right)\left(\frac{D_i}{T_o}\right),$$

where K is the induced drag factor and D_i is the induced drag in level flight at the lift coefficient C_{L_U}. Thus C_{L_U}/A will be nearly proportional to T_o/W if D_i/T_o is roughly constant for the conditions giving $C_L = C_{L_U}$. The curve given in Fig.5 may be represented roughly by the equation

$$\frac{C_{L_U}}{A} = 1{\cdot}8\ T_o/W$$

and with $K = 1{\cdot}25$ this gives $D_i = 0{\cdot}72\ T_o$. Thus for propeller aircraft the maximum useful lift coefficient, as defined here, is determined by the condition that at this lift coefficient the induced drag is about 70% of the static thrust. Results for jet aircraft would be similar, but the corresponding percentage of the static thrust would be slightly greater.

The results given in Fig.5 are re-plotted in a different form in Fig.6. This shows that lift coefficients as high as 5 or 6 may be usefully employed without going to exceptionally high values of A or T_o/W.

Consideration of the proportion of total take-off distance occupied by the ground run showed that this proportion decreased slightly with increasing C_{L_S}. The decrease was rather greater for propeller aircraft than for jets.

All/

All the results given here refer to an aircraft of 40 000 lb weight. As already explained, the assumptions made about drag during the take-off led to some effect of varying weight on the calculated take-off distance, but the effect was very small for weights between 30 000 and 200 000 lb. The assumptions were not intended to apply at weights below 30 000 lb and are likely to become progressively less accurate as the weight is decreased below this value. Some calculations of take-off distance were made, however, for weights down to 10 000 lb.

Fig.7 shows the effect of ground-rolling resistance on the calculated take-off distance of a typical propeller aircraft. As would be expected, the effect of increasing μ is greatest when T_o/W and C_{L_S} are both small. Increasing wing loading has the same effect as a proportionate reduction of C_{L_S}. It should be emphasised, however, that the effect of increasing μ has been minimised in these calculations by the assumption that the optimum lift coefficient is maintained during the ground run (equation (9)). In many cases this is likely to be impracticable, and increasing μ will then have a greater effect on the take-off distance.

It is of interest to compare the results of these take-off calculations with those given by G. W. Johnston[3]. Johnston assumes that the speed during the transition and steady climb is always equal to $1{\cdot}2\ V_S$, whereas in the calculations described here the speed V_b is adjusted to give minimum total take-off distance. Thus when C_{L_S} is large Johnston finds the take-off distance actually increasing with C_{L_S}. This means that the take-off is being limited by induced drag, and not by stalling, and the best transition speed to use is well above the stalling speed.

The "optimum" C_{L_S} given by Johnston is the value of C_{L_S} required to achieve the minimum take-off distances shown in Fig.4. As already noted, the maximum useful lift coefficient in practice will be rather less than this because of the weight of the BLC system.

Apart from the points mentioned above, the results given by Johnston are substantially in agreement with those given here.

6.2 Landing

The distance required for landing is not important if it is less than the take-off distance. The calculations have shown that this condition is always satisfied with a dry concrete runway ($\mu_B = 0{\cdot}35$) for the range of T_o/W considered, even without using reversed thrust, provided the wing loading is at least 20% less at landing than at take-off. For short-range aircraft the reduction of wing loading between take-off and landing may sometimes be less than this, so that for the higher values of T_o/W the landing distance without reversed thrust may be a little greater than the take-off distance. When reversed thrust is used on a dry concrete runway, the landing distance is always less than the take-off distance, even at the same wing loading.

With/

With a wet runway ($\mu_B = 0{\cdot}17$) the landing distance is more critical, and in presenting the results of the landing calculations attention has been concentrated on this condition. Figs.8 and 9 show landing and take-off distances at two values of T_o/W, for propeller aircraft with $A = 8$ on an icy runway. Fig.8 shows that when $T_o/W = 0{\cdot}32$, a normal value for conventional propeller aircraft, the landing distance is never much greater than the take-off distance for the same value of w/C_{L_S}, even for this low value of μ_B and without thrust reversal. When allowance is made for a reasonable reduction of wing loading between take-off and landing, the landing distance without thrust reversal is hardly ever greater than the take-off distance. When reversed thrust is used the distance required for landing is always considerably less than for take-off.

In Fig.9 the landing and take-off distances are compared for $T_o/W = 0{\cdot}58$, with μ_B still equal to $0{\cdot}17$. In this case, provided reversed thrust is used, the landing distance at a given value of w/C_{L_S} never exceeds the take-off distance by more than 200 ft, even at this low value of μ_B.

For aircraft having still higher values of T_o/W the landing distance would be greater than the take-off distance even with $\mu_B = 0{\cdot}35$, but with such aircraft there is a strong case for using some form of thrust deflection and this has not been considered in the present study. It may therefore be concluded that for values of T_o/W up to about $0{\cdot}5$ the take-off is likely to be more critical than the landing.

For a given value of w/C_{L_S} the effect on the landing distance of varying wing loading or aspect ratio is small. The take-off distances shown in Fig.8 refer to propeller aircraft, but the landing distances without reversed thrust apply equally to jet aircraft.

The landing distances given in Figs.8 and 9 were calculated for an approach path inclined at 8° to the horizontal. This is considerably steeper than the approach normally used by civil aircraft but is believed to be quite practicable.

Fig.10 shows the results of calculations made for one case to investigate the effect on the landing distance of varying the approach angle. The ratio of landing distance at an approach angle δ to the distance for an angle of 8° is plotted against the angle δ. It can be seen that increasing δ above 8° gives relatively little improvement, but reduction of δ to small angles gives a considerable increase of landing distance. Fig.10 refers to a wet runway ($\mu_B = 0{\cdot}17$) and for greater values of μ_B the ground run would be shorter and hence for a given reduction of δ the percentage increase of total landing distance would be even greater. The effect of δ on landing distance is more pronounced at higher values of C_{L_S}, because the horizontal distance required for descent from 50 ft to ground level is then a greater

proportion/

proportion of the total landing distance. The dotted curve of $\frac{\cot \delta}{\cot 8°}$ in Fig.10 represents the limiting case of $C_{L_S} \rightarrow \infty$ for which $L_1 = L_L$.

The landing distances shown in Figs.8, 9 and 10 were all calculated for speeds during the steady approach and flare represented by $B = 1 \cdot 3$ and $E = 1 \cdot 2$. These speed ratios may be regarded as typical values for conventional civil aircraft. Some further calculations were made to investigate the effect of reducing E, with and without a reduction of B. The results showed that in typical cases a reduction of E from 1·2 to 1·1, while keeping $B = 1 \cdot 3$, reduced the total landing distance by about 20% without thrust reversal and by about 10% with thrust reversal. With $B = E = 1 \cdot 1$ the total landing distance was slightly greater than with $B = 1 \cdot 3$ and $E = 1 \cdot 1$. Thus to land in the shortest possible distance the speed loss during the flare $(B - E)V_S$ should be as large as possible. The amount of speed loss that can be achieved in practice during the flare will depend on the drag characteristics of the aircraft and will not be in the control of the pilot unless it is considered practicable to apply reversed thrust during the flare.

The calculations of landing distance given by G. W. Johnston[3] are based on an assumed technique in which there is no flare (and also no float) and the approach speed ratio B is only 1·1. The vertical component of velocity at touchdown (and during the glide) is assumed to be 10 ft/sec. Thus the assumed approach angle (which is maintained all the way to the ground) is given by

$$\tan \delta = \frac{9 \cdot 1}{V_S},$$

where V_S is in ft/sec. This angle is less than the value of 8° assumed here provided V_S is greater than about 38 knots $\left(\frac{W}{C_{L_S}} > 5 \cdot 0 \text{ lb/ft}^2\right)$.

The landing distances given by Johnston are based on an assumed constant deceleration during the ground run, so that no useful comparison with the present results can be made.

7. Conclusions

The main conclusions may be summarised as follows:-

(1) For values of T_0/W less than about 0·5 the take-off distance will usually be greater than the landing distance, especially if reversed thrust is used on landing.

(2) The use of BLC to increase the maximum lift coefficient (say, from 2 to 5) gives a large reduction of take-off distance, especially if T_0/W is large. (See Figs.1 and 2.)

(3)/

(3) The maximum useful lift coefficient for take-off is given approximately by Fig.6.

(4) With undeflected thrust in the range of T_o/W considered here a take-off distance as short as 500 ft can only be achieved with a very small value of $\frac{w}{C_{L_S}}$ (about 7 lb/ft^2 for a propeller aircraft even with T_o/W as high as 0·6). When such short take-off distances are required T_o/W should be large and thrust deflection is obviously useful.

(5) Take-off distances of the order of the order of 1 000 ft can be achieved much more easily. Thus, for a propeller aircraft with $T_o/W = 0{\cdot}45$, this distance can be achieved with $\frac{w}{C_{L_S}} < 12{\cdot}5$ lb/ft^2 and w/A less than about 9. With $C_{L_S} = 5$, these conditions are satisfied by making $w = 60$ lb/ft^2 and $A = 7$.

8. Acknowledgements

The authors gratefully acknowledge the advice and assistance given by Mrs. G. M. Birch in programming the computer. They are also grateful to Dr. M. R. Head and Mr. D. G. Clark for many helpful suggestions.

References/

References

No.	Author(s)	Title, etc.
1	P. L. Sutcliffe, V. K. Merrick and A. R. Howell	Aerodynamic and propulsion considerations of minimum-field aircraft. 8th Anglo-American Aeronautical Conference, London, 1961.
2	E. A. Horton, L. K. Loftin, S. F. Racisz and J. H. Quinn	Analysis of the effects of boundary-layer control on the take-off and power-off landing characteristics of a liaison type of airplane. N.A.C.A. Report 1057 (1951).
3	G. W. Johnston	Factors affecting the field length of S.T.O.L. aircraft. A.G.A.R.D. Report 81 (1956).
4	C. D. Perkins and R. E. Hage	Airplane performance, stability and control. John Wiley, New York, 1949, page 96.
5	S. F. Hoerner	Fluid dynamic drag. Published by the author, 1958, pages 7-14.

DR

D 037336/1/Wt.61 K4 7/65 XL & CL

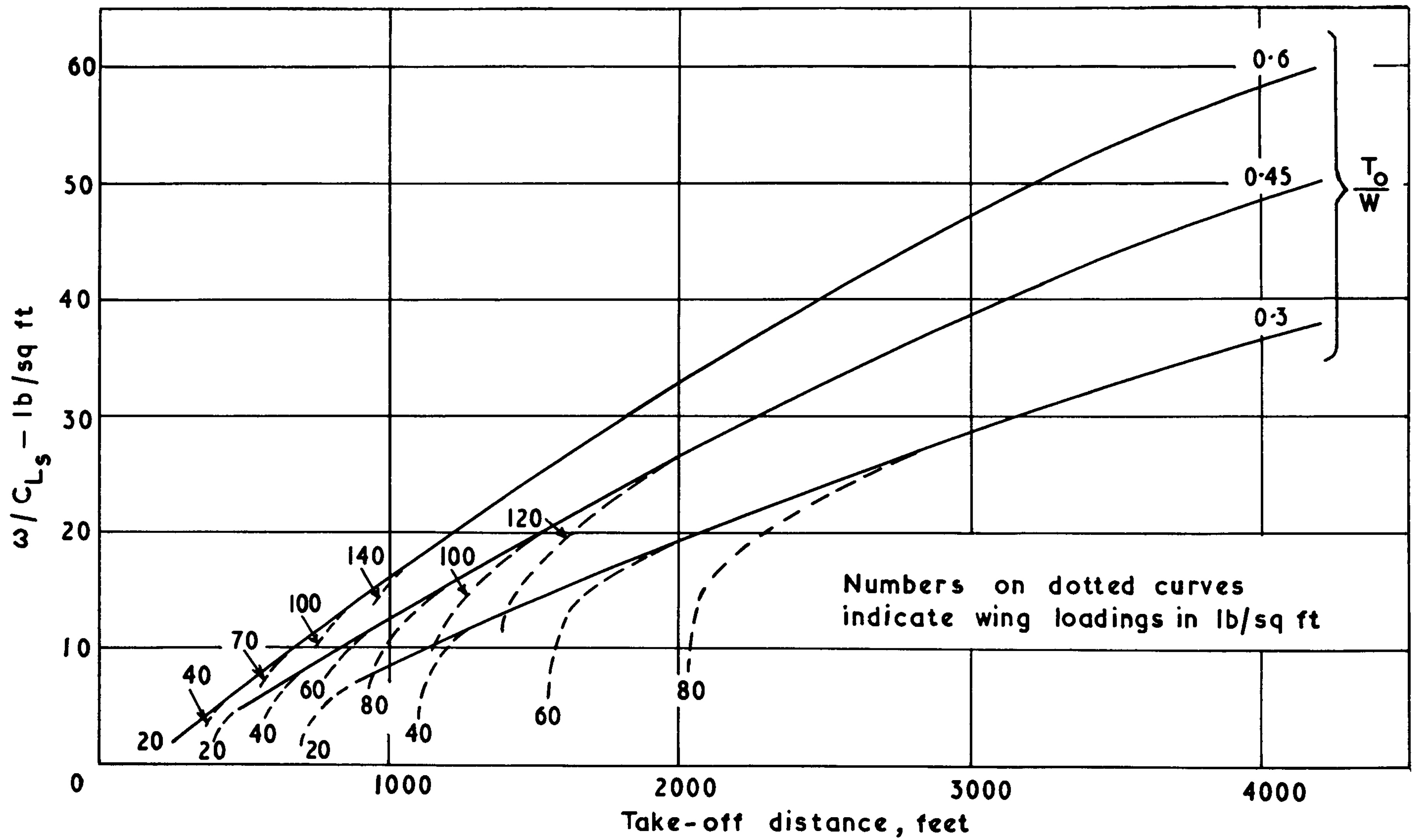


FIG. 1

Take-off distances for propeller aircraft over 50 ft obstacle, $A = 8$, $\mu = 0{\cdot}02$, $W = 40\ 000$ lb

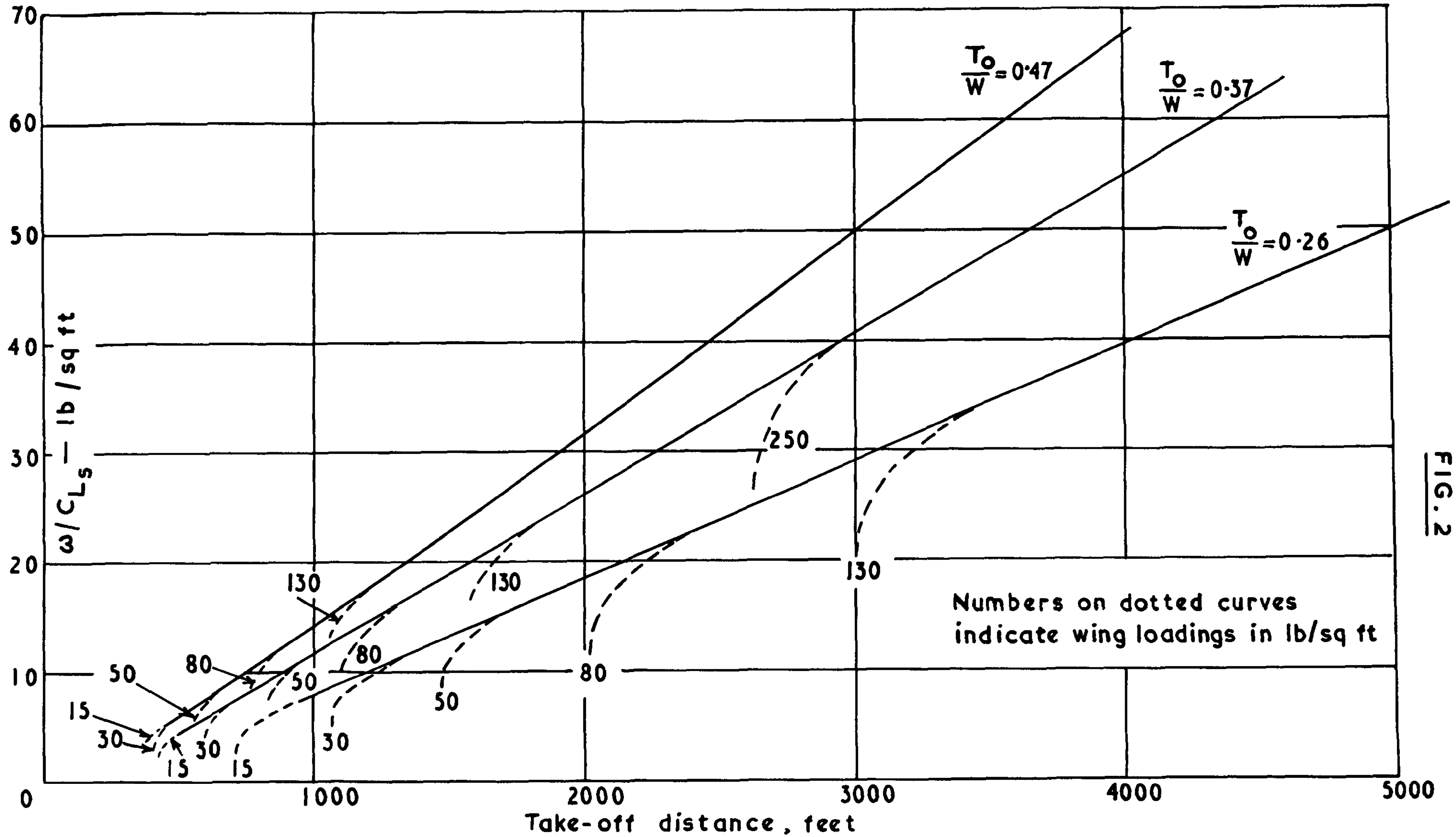


FIG. 2

Take-off distances for jet aircraft over 50 ft obstacle,

$A = 8$, $\mu = 0{\cdot}02$, $W = 40\,000$ lb

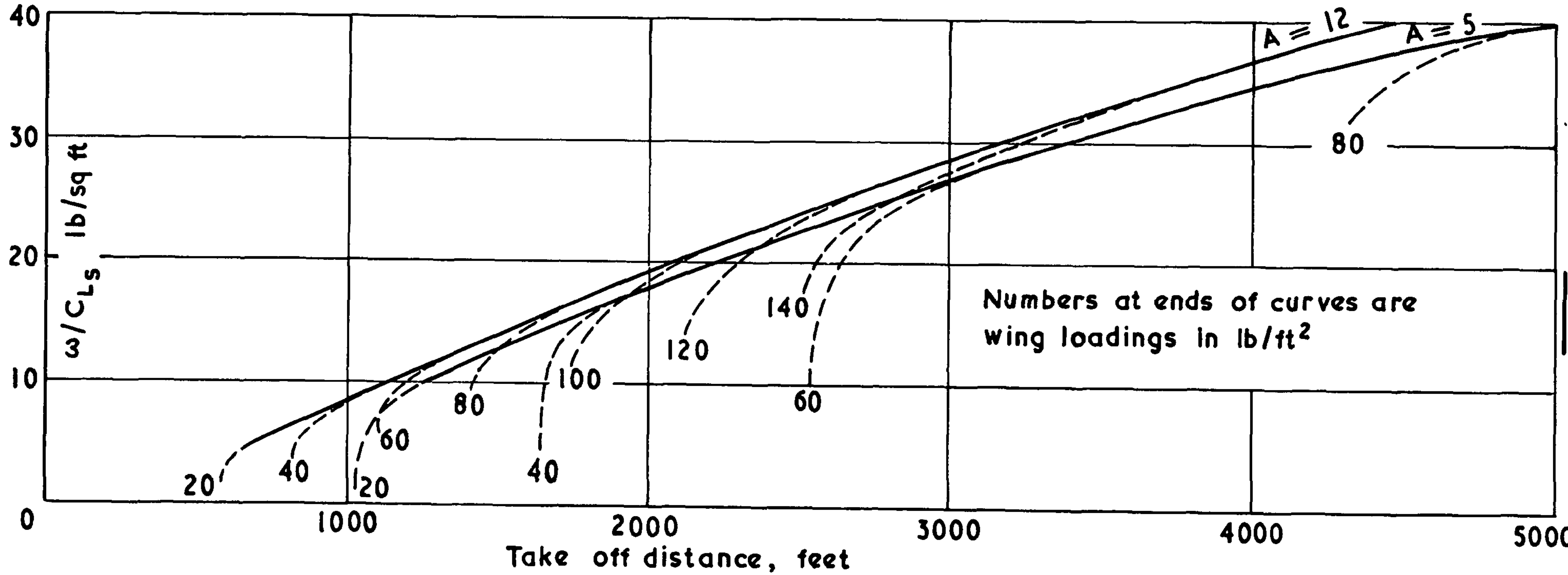
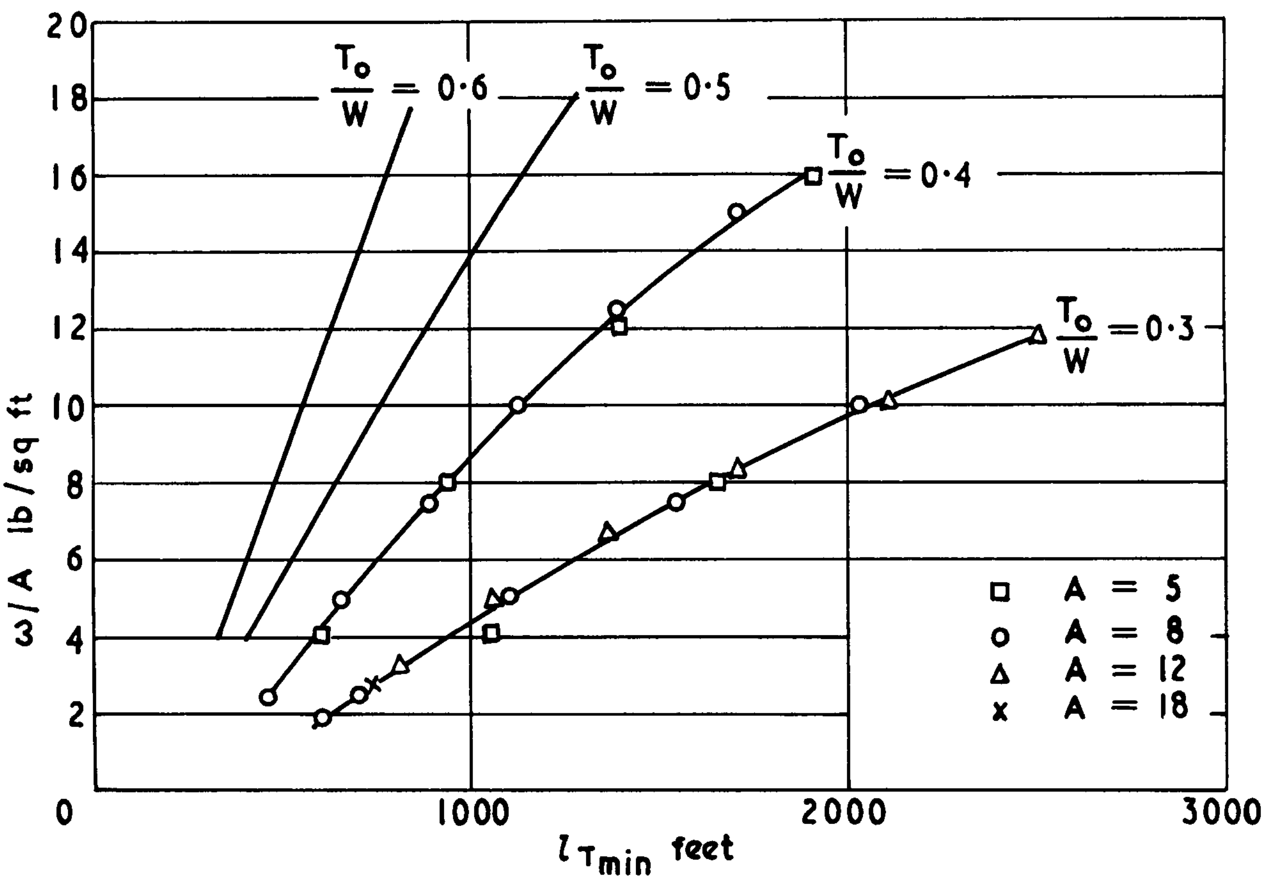


FIG. 3

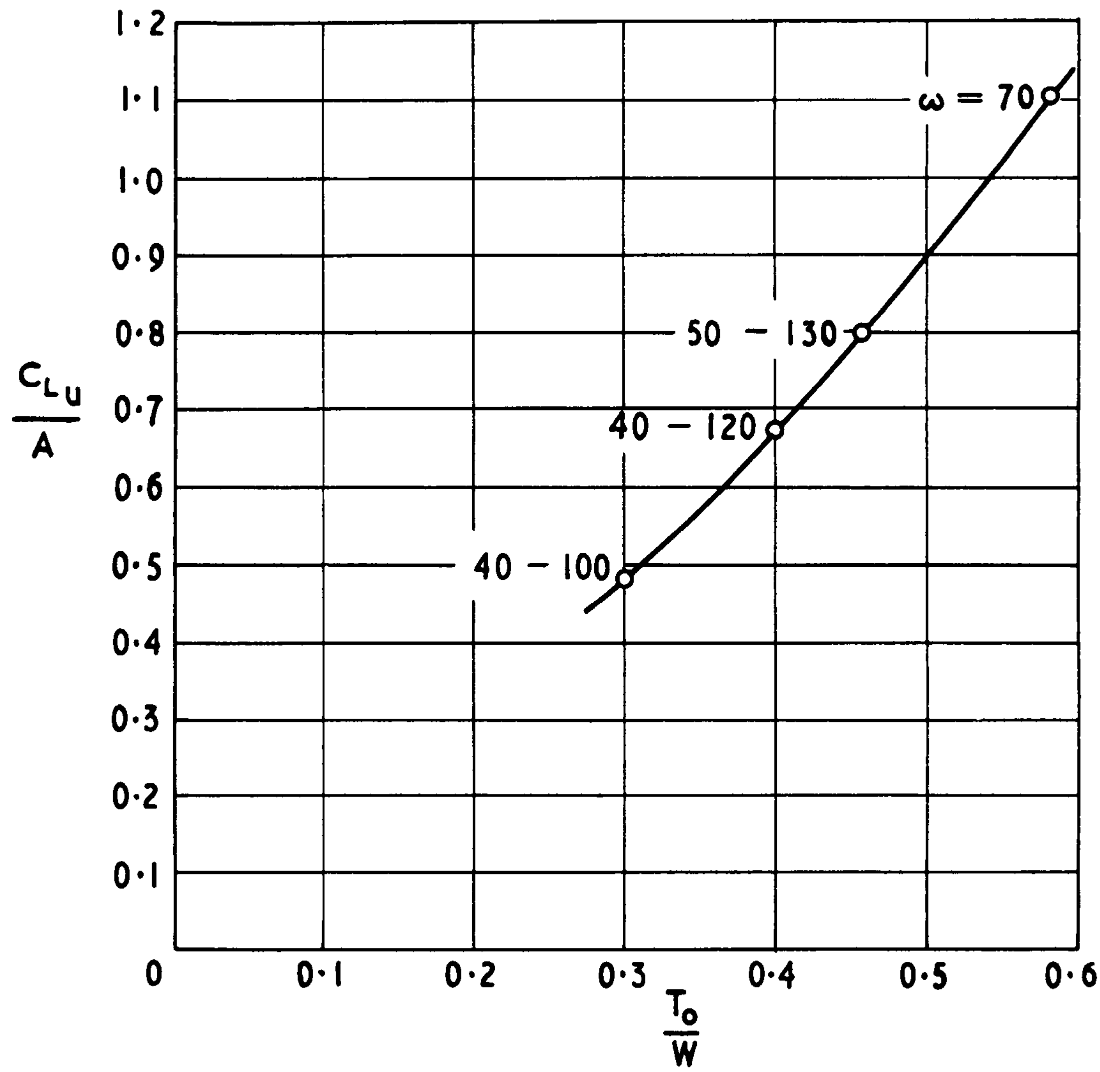
Take-off distances for propeller aircraft over 50 ft obstacle, $T_o / W = 0{\cdot}3$, $\mu = 0{\cdot}02$, $W = 40\,000$ lb

FIG. 4



Minimum take-off distances over 50 ft obstacle for propeller aircraft, $\mu = 0{\cdot}02$, W = 40 000 lb

FIG. 5

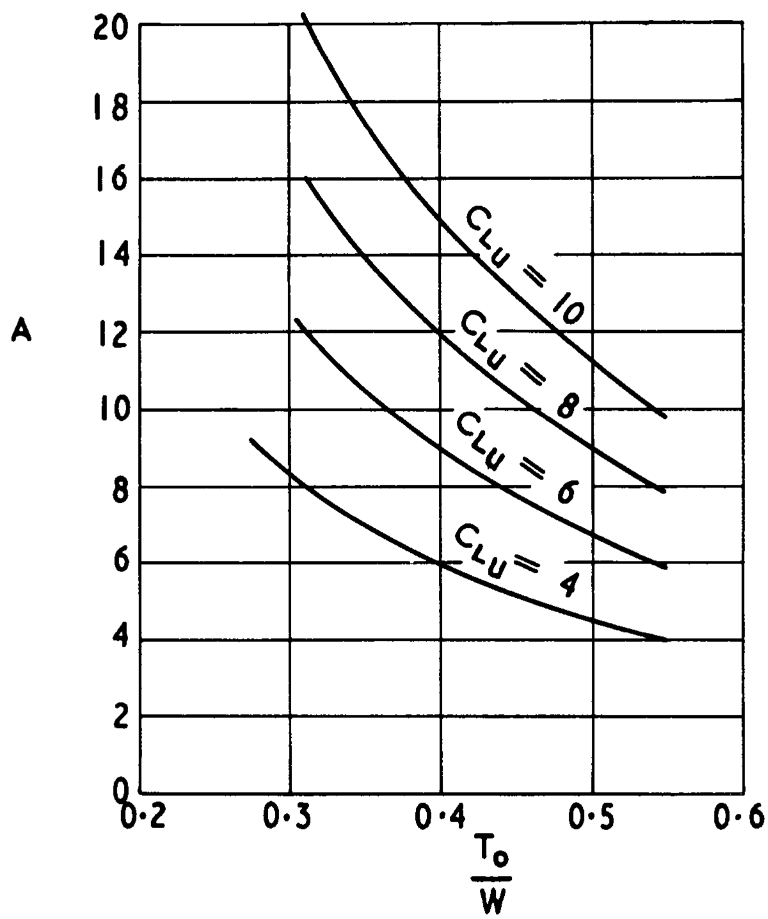


Maximum useful lift coefficient for propeller aircraft,

$\mu = 0{\cdot}02$ $W = 40000$ lb

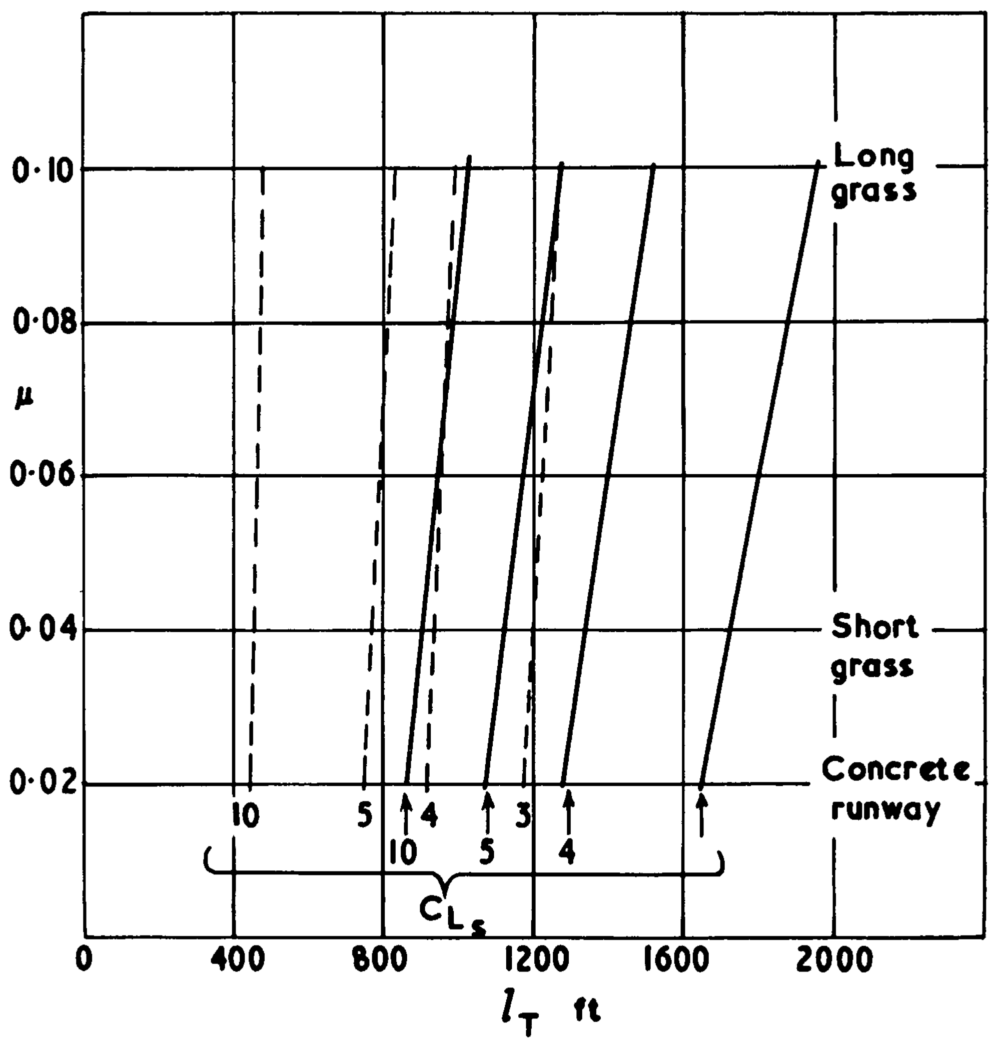
Numbers beside points show range of wing loading in lb/sq ft for which C_{L_u}/A is close to the value shown

FIG. 6



Maximum useful lift coefficient for propeller aircraft as function of aspect ratio and thrust / weight ratio

FIG. 7



———— $\frac{T_0}{W} = 0{\cdot}32$ - - - - - - - - $\frac{T_0}{W} = 0{\cdot}58$

Effect of ground-rolling resistance on take-off distance of propeller aircraft, A = 12, ω = 50 lb/sq ft.

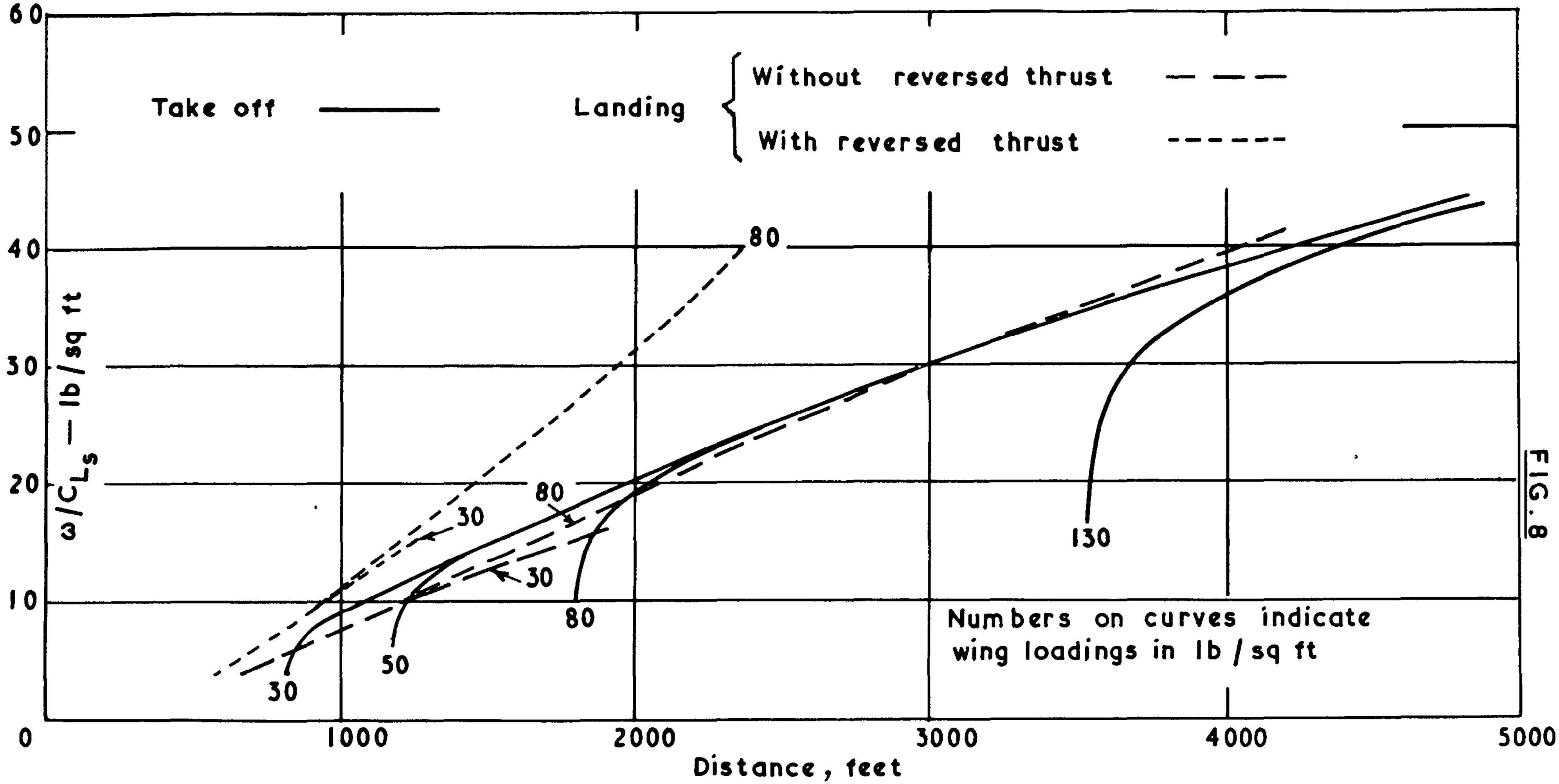
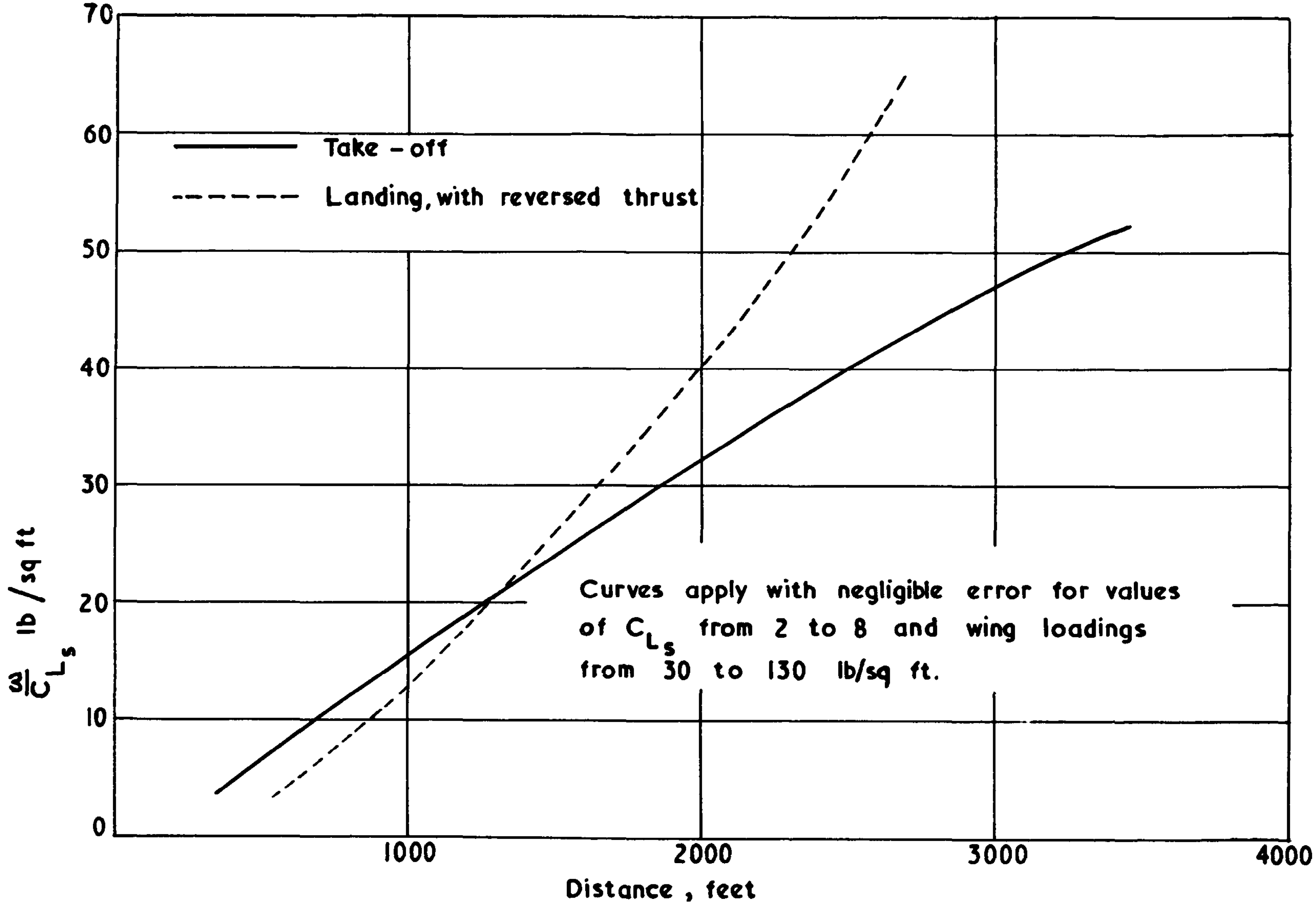


FIG. 8

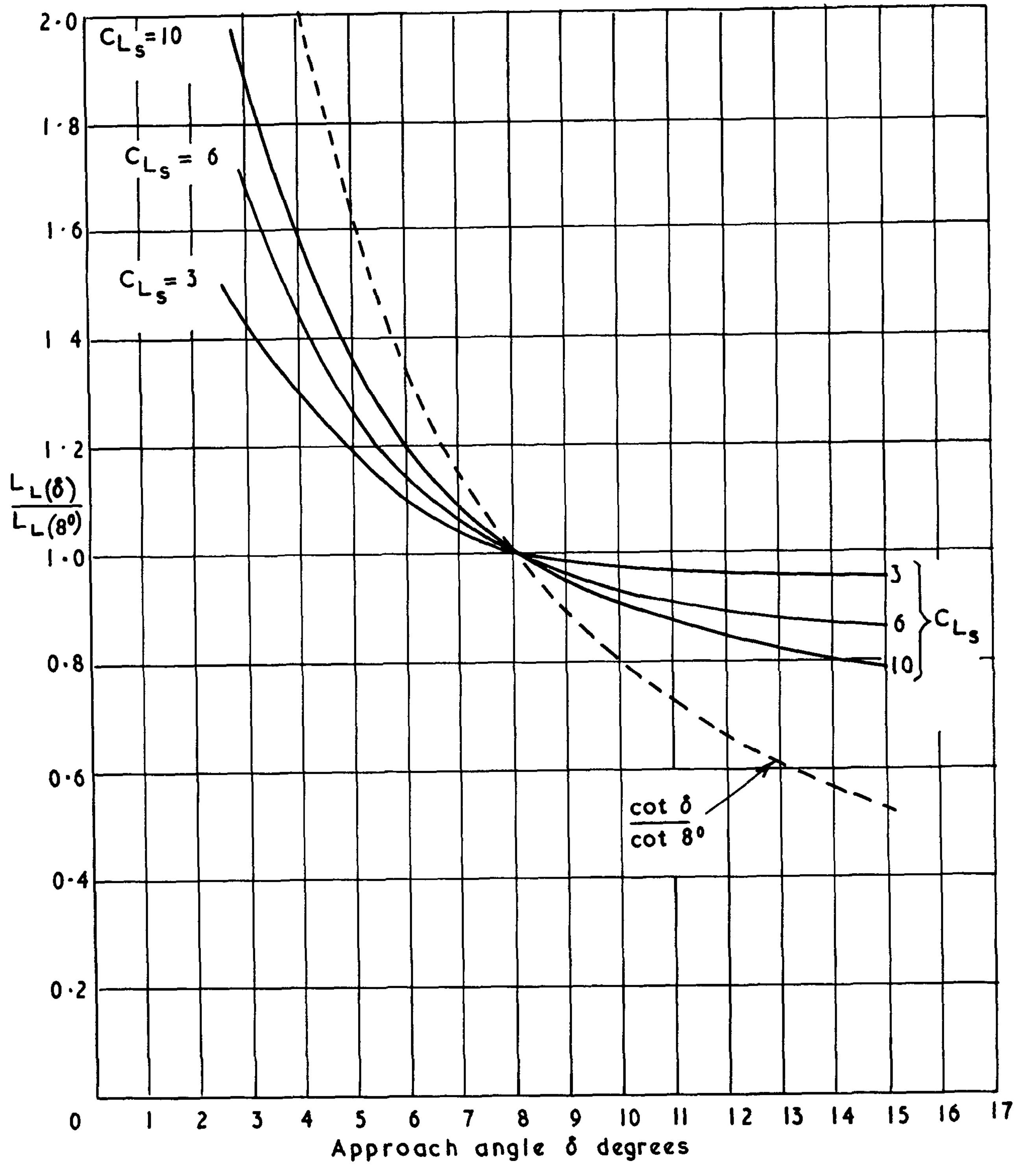
Landing and take-off distances for propeller aircraft over 50 ft obstacle, $A=8$, $T_o/W=0\cdot32$, $\mu_B=0\cdot17$, $\delta=8^{\circ}$

FIG.9



Landing and take-off distance for propeller aircraft over 50 ft obstacle. $A = 8$, $\frac{T_o}{W} = 0{\cdot}58$, $\mu_B = 0{\cdot}17$, $\delta = 8^0$

FIG. 10



Effect of approach angle on landing distance, A=8, T_0/W=0·32, μ_B=0·17, ω =50 lb/sq ft, (Propeller with reversed thrust)

A.R.C. C.P. No.823
October, 1963
Mair, W. A. and Edwards, B. J. Cambridge University

A PARAMETRIC STUDY OF TAKE-OFF AND LANDING DISTANCES FOR HIGH-LIFT AIRCRAFT

Take-off and landing distances have been calculated for a wide range of high-lift aircraft without deflected thrust.

It is shown that if high lift coefficients can be obtained large reductions of take-off distance are possible. The maximum useful lift coefficient is shown to be a function of aspect ratio and thrust/weight ratio, almost independent of wing loading.

A.R.C. C.P. No.823
October, 1963
Mair, W. A. and Edwards, B. J. Cambridge University

A PARAMETRIC STUDY OF TAKE-OFF AND LANDING DISTANCES FOR HIGH-LIFT AIRCRAFT

Take-off and landing distances have been calculated for a wide range of high-lift aircraft without deflected thrust.

It is shown that if high lift coefficients can be obtained large reductions of take-off distance are possible. The maximum useful lift coefficient is shown to be a function of aspect ratio and thrust/weight ratio, almost independent of wing loading.

A.R.C. C.P. No.823
October, 1963
Mair, W. A. and Edwards, B. J. Cambridge University

A PARAMETRIC STUDY OF TAKE-OFF AND LANDING DISTANCES FOR HIGH-LIFT AIRCRAFT

Take-off and landing distances have been calculated for a wide range of high-lift aircraft without deflected thrust.

It is shown that if high lift coefficients can be obtained large reductions of take-off distance are possible. The maximum useful lift coefficient is shown to be a function of aspect ratio and thrust/weight ratio, almost independent of wing loading.

DETACHABLE ABSTRACT CARDS

C.P. No. 823

Printed and published by
HER MAJESTY'S STATIONERY OFFICE
To be purchased from
York House, Kingsway, London W C 2
423 Oxford Street, London W 1
13A Castle Street, Edinburgh 2
109 St Mary Street, Cardiff
39 King Street, Manchester 2
50 Fairfax Street, Bristol 1
35 Smallbrook, Ringway, Birmingham 5
80 Chichester Street, Belfast 1
or through any bookseller

Printed in England

C.P. No. 823

S.O. Code No. 23-9016-23