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Measurement of Air Temperature  
on an Aircraft Travelling at High  
Subsonic and Supersonic Speeds

*by*

*A. A. Woodfield and P. J. Haynes*

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MEASUREMENT OF AIR TEMPERATURE ON AN AIRCRAFT TRAVELLING  
AT HIGH SUBSONIC AND SUPERSONIC SPEEDS

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SUMMARY

Flight tests have been performed on two different designs of impact air thermometer, with two thermometers of one design fitted with different sensing elements, in the altitude range from 30,000 ft to 40,000 ft at Mach numbers between 0.50 and 1.82. The performance of these thermometers is described by two parameters, the recovery factor and the time constant, values of which were obtained. The unusual behaviour of the recovery factor for one of the thermometers is discussed. At subsonic speeds it is shown that the normal straight line method of analysis, plotting indicated temperature versus (Mach number)<sup>2</sup>, can be invalid. At supersonic speeds, the use of an apparent recovery factor, which includes the effects of both the normal shock wave and the thermometer recovery factor, is recommended.

There is a large difference between the flight values and some laboratory values of time constant for one of the thermometers which suggests that the normal laboratory tests are not fully representative.

The flight test technique has been critically examined and suggestions made for any future investigations into the behaviour of air thermometers. The need for an accurate independent method of measuring static temperature during the tests is stressed.





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## 1 INTRODUCTION

With the development of high performance aircraft and airborne navigation systems has come a growing need for more precise measurements of either static or total air temperature.

All direct methods of measuring air temperature indicate a temperature containing some fraction of the local dynamic temperature, because the air is brought to rest at the sensing element. However, the indicated temperature is rarely the same as the free stream total temperature, and the efficiency of the thermometer, known as its recovery factor, is given by the difference between the indicated and the free stream static temperatures divided by that between the free stream total and static temperatures. For a reliable air thermometer the recovery factor should be constant, independent of its position on the aircraft, or of varying flight conditions, and ideally should be either zero or unity, because the thermometer will then indicate either the free stream static or free stream total temperature. A rapid dynamic response is also required to minimize the error caused by the rapid changes in free stream temperatures that can occur in some flight conditions (e.g. climbs and accelerations).

At the present time there are three direct methods of measuring air temperature.

Surface temperature elements:<sup>1-3,5</sup> These measure the temperature of the skin of the aircraft, or a probe, and are therefore very dependent on the state of the local boundary layer. Thermometers of this type are still used but are rapidly being replaced by the other two types. The disadvantages are a large lag, depending on skin thickness and internal air temperatures, a low recovery factor, of the order of 0.8 to 0.9 which depends on the state of the boundary layer, and large, possibly variable, radiation errors.

Vortex thermometers:<sup>4,5</sup> These utilize the reduction in total temperature which occurs at the core of a vortex. The vortex strength is controlled to obtain a core total temperature equal to the free stream static temperature i.e. a recovery factor of zero. Thermometers of this type have been used on subsonic aircraft in the United States<sup>4</sup>. The main problem is maintaining the correct vortex at high subsonic and supersonic speeds, as the presence of shock waves alters the vortex characteristics, and therefore the recovery factor varies.

Impact thermometers:<sup>3,5</sup> These are designed to bring the air to rest at the sensing element by an adiabatic process and so measure the free stream total temperature. This type of thermometer appears to be the most reliable instrument for measuring air temperature, and is the method most commonly used at present. This report presents the results of flight tests on two different designs of impact thermometer.

There are indirect methods for measuring temperature which depend on such temperature sensitive quantities as the velocity of sound, air density, infra-red radiation, etc., but all these methods are too cumbersome for use on aircraft<sup>5</sup>.

Apart from these methods, air temperature can also be obtained from Meteorological office measurements using Radiosonde balloons. However, this method is not very reliable, as air temperatures may vary considerably with time and distance from the measuring station. Meteorological data is normally only measured every twelve hours, and the stations are separated by large distances.

Previous tests<sup>3</sup> (unpublished) at subsonic speeds, made by the College of Aeronautics at R.A.E. Bedford, used four impact and one surface element thermometer installed on a Hunter aircraft; the best results in these tests were obtained from impact thermometers with radiation shields, a ventilated sensing element chamber and thermistors (temperature sensitive semi-conductors) as the sensing element. Both of the thermometer designs used in the present tests were fitted with radiation shields and ventilated, and one of the thermometers used thermistors as the sensing element.

The thermometers were tested at supersonic speeds, using the Fairey Delta 2 aircraft, to extend the work of Ref.3 to higher speeds and demonstrated some of the difficulties in obtaining an accurate measurement of air temperature under these conditions.

It should be noted that the majority of the results were obtained whilst the aircraft was engaged on another project and the time scale was severely restricted by other commitments on the aircraft. As a consequence the results have certain limitations which are considered in the section on Flight Test Technique (Section 5.4).

## 2 INSTRUMENTATION AND PRE-FLIGHT CALIBRATIONS

### 2.1 Thermometers

Four thermometers were used in the tests:-

Two Penny and Giles (A.&A.E.E. type) with nickel wire sensing elements (Serial Nos. 3021-2),

One Penny and Giles (A.&A.E.E. type) with thermistor sensing elements,  
and

One Rosemount Engineering Company Model 102E with a platinum wire sensing element.

All the Penny and Giles thermometers were identical apart from the sensing elements. The standard element is a nickel wire resistance thermometer, but one thermometer was modified by fitting 2 thermistors, as the sensing element. External and sectional views of the Penny and Giles thermometer with the nickel wire elements are given in Fig.1.

The Rosemount 102E thermometer was a sample loaned to the R.A.E. for testing; external and sectional views are given in Fig.2.

Throughout this report the nickel wire Penny and Giles is referred to as the nickel wire thermometer, the thermistor Penny and Giles as the thermistor thermometer, and the Rosemount Model 102E as the platinum thermometer.

Before describing the instruments, the properties of an ideal impact thermometer are briefly considered. The behaviour of impact thermometers is usually described by two parameters, the recovery factor, which defines the fraction of the free stream dynamic temperature rise indicated by the thermometer, and the time constant, which is a measure of the response of the thermometer to changes in temperature.

The ideal impact thermometer should have a recovery factor of unity and zero time constant. To obtain this value of recovery factor the air sample in the instrument must be brought to rest under true adiabatic conditions (i.e. no heat transfer) and without inducing turbulence, because turbulence can produce an uneven distribution of total temperature in the air sample; whereas for a low time constant the sensing element must respond very rapidly to temperature changes and the rate of airflow past the element must be high to give a rapid change of air sample. Thus a recovery factor of unity and zero time constant are conflicting requirements. However, by careful detail design satisfactory results may be achieved.

The design features by which the manufacturers of these thermometers try to achieve these aims can be seen from the sectional views in Figs.1(b) and 2(b), and are as follows:-

(a) Nickel wire and thermistor thermometers

To reduce heat transfer by conduction from the measured air sample, the sensing element chamber is lined with cork. This inner chamber is surrounded by a second, in which the air temperature and velocity are approximately equal to that in the sensing element chamber, so further reducing conduction and radiation losses. Another small annular chamber surrounds the inner two and contains air at static conditions to provide additional insulation. The external casing is of polished metal to reduce radiation effects, and the whole unit is thermally isolated from the aircraft structure by the non-metallic section attaching the thermometer to the strut holding it clear of the aircraft boundary layer.

The velocity of the sample in the innermost chamber is reduced by the throttling effect of the four small exit holes at the rear of the chamber, and the air in the secondary chamber is also throttled through a further four exit holes. The usual sensing element is nickel wire wound non-inductively on a cylindrical former, running nearly the full length of the chamber. Unfortunately, the need for the element to be of robust construction to resist damage by objects passing through the chamber (e.g. rain, ice, insects or sand) leads to the use of thick wire and thus increases the time constant. In an attempt to reduce the time constant to a lower level in one of the instruments used in the present tests two thermistors were fitted in place of the normal element\*.

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\*Tests on Penny and Giles thermometers with 'paper' resistance elements have been carried out at subsonic speeds by the Aircraft and Armament Experimental Establishment, (A. & A.E.E.), Boscombe Down. The results indicate a time constant of approximately 1.2 seconds for these elements (c.f. approximately 10 seconds for the standard nickel wire elements). It was also found that increasing the area of the exit holes further reduced the time constant to an almost negligible value.

It should be noted that since the sensing chamber is a relatively long narrow annulus with rough walls the turbulence will be high.

(b) Platinum thermometer

As in the previous design, the velocity of the sample is reduced by the throttling effect of the exit ports, although the throttling is less than in the nickel wire and thermistor thermometers.

The sensing element chamber is again surrounded by two other chambers (the outer chamber being especially large) but both contain air at conditions similar to those of the measured air sample. The body of the thermometer is again of polished metal, but insulation from the aircraft structure must be arranged at the mounting point. The entrance to the thermometer is clear of the aircraft boundary layer, as in the case of the previous thermometer.

In the platinum thermometer a delicate sensing element can be used, because the axis of the sensing element and secondary chambers is normal to the main flow direction, air being diverted through it by suction at the strut trailing edge exit. This change of direction prevents all but the smallest particles from entering the chamber. In the thermometer tested the element is bare platinum wire, approximately 0.001 inches in diameter, wound with ample spacing on a short cruciform former and placed at the entrance to the chamber. The boundary layer on the inside of the bend is removed by suction through the porous surface before the air enters the chamber, and so the level of turbulence should be small.

2.2 Aircraft instrumentation

Of the four thermometers used one of the nickel wire type was fitted to a De Havilland Comet aircraft on one of the starboard cabin-windows, approximately over the wing quarter chord Fig.3, and the remaining three were mounted on the underside of the nose structure of the Fairey Delta 2 supersonic research aircraft WG 774 (Figs.4 and 5). Unfortunately the circuit used with the nickel wire thermometer on the Fairey Delta 2 developed an intermittent fault\* which could not be cured during the brief period occupied by these tests.

The circuits used with the thermometers are shown in Fig.6<sup>1</sup>. The nickel wire and platinum thermometers use ratiometer circuits (Fig.6(a)), operated by approximately 4 volts. The voltage across the thermometer element was approximately 2 volts, low enough to prevent any significant self-heating under flight conditions. The thermistor thermometer used a galvanometer circuit (Fig.6(b)) supplied with a constant voltage, because the change of resistance with temperature is too large (from 400K $\Omega$  to 2K $\Omega$  for a range of 120 $^{\circ}$ K) for a ratiometer. The voltage was monitored by a second galvanometer. The ratiometers and galvanometers were mounted in continuous trace recorders. The value of the series resistor in the thermistor circuit was adjusted to give maximum sensitivity (mA/ $^{\circ}$ K) over the temperature range required.

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\*Following these tests the nickel wire thermometer was tested and no trace of an intermittent fault was found.

<sup>1</sup>It is essential to note that these non-temperature compensated circuits could only be used because of the small variation of the temperature in the Fairey Delta 2 instrumentation bay (typically  $\pm 5^{\circ}$ K about ground ambient).



The dynamic characteristics of the ratiometers and galvanometers are:

	Undamped natural frequency o/s	Damping ratio
Ratiometers	11	1.0 - 2.0
Galvanometers	80	0.7

The other measurements required were airspeed, altitude and time. The timing for the three recorders in the Fairey Delta 2 was provided from the internal timing system in No. 1 recorder, and this timing was synchronized with the Comet's recorder by simultaneous event marks on the Comet and Fairey Delta 2 records at the moment of passing. The airspeed and altitude of the Fairey Delta 2 were indicated by barometric capsules; those of the Comet were obtained from auto observer records.

### 2.3 Calibrations

Calibration of thermometers when installed on the aircraft is a rather difficult task, because of the problem of applying a known and controlled temperature to the thermometers under conditions where self-heating is negligible. Because of these difficulties it was decided to first obtain a laboratory calibration of resistance against temperature under negligible self-heating conditions; then simulate the thermometers on the aircraft using a sub-standard resistance box, to obtain calibrations of the aircraft circuits in terms of thermometer resistance. From these two calibrations the variation of trace deflection with temperature for zero self-heating, the desired calibration, was derived.

#### 2.3.1 Still air self-heating

Before the thermometers were calibrated in a temperature chamber, the thermometer voltage required to produce less than 0.5°K self-heating error under still air conditions was determined to reduce self-heating effects to an acceptable level.

To obtain this still air self-heating effect, measurements of thermometer resistance were made over a range of power dissipation at constant air temperatures and then, from the nominal rate of change of resistance with temperature for the thermometer, the self-heating temperature error variation can be plotted against the power dissipated. From this the power dissipation equivalent to an error of 0.5°K was obtained. After the thermometers had been calibrated in the temperature chamber the actual rate of change of temperature was compared with the nominal values, but no major differences were found. The self-heating errors for the three thermometers (Fig.7) correspond to the measured rates of change of resistance with temperature. The self-heating effect is greatest for the thermistor thermometer with an error of 0.37°K/mW; with the platinum thermometer it is 0.22°K/mW and the nickel wire thermometer, 0.096°K/mW. This sequence may be explained by the different surface areas of

the elements. At equilibrium conditions the approximate power dissipated by the element  $(I^2R)_{T_R}$  (or rate of flow of heat) is given by (see Appendix 1):-

$$(I^2R)_{T_R} = H(T_R - T_r) \quad (1)$$

where H is the heat transfer rate given by  $H = \bar{H}S$ , S = surface area of the element,  $\bar{H}$  = a heat transfer factor, and  $(T_R - T_r)$  is the self-heating temperature rise. Thus for constant  $T_r$ :

$$\frac{dT_R}{d(I^2R)_{T_R}} = \frac{1}{\bar{H}S} = \phi \quad (2)$$

Therefore the larger the surface area of the element the smaller the effects of self-heating, assuming a constant heat transfer factor. In the present tests the nickel wire thermometer had the largest surface area, the platinum next with a slightly larger area than the thermistor. This agrees with the measured values of heat transfer rate. Although the heat transfer rate is constant for a reasonable range of temperature and is therefore the basic self-heating parameter, the self-heating is normally required in terms of the voltage across the thermometer. Equation (3) then becomes:-

$$T_R - T_r = V^2 \frac{\phi}{R} \quad (3)$$

Hence, for a given voltage, the self-heating error is proportional to  $\phi/R$ . In these terms the self-heating effect is greatest for the platinum thermometer. Comparative values at room temperature (290°K) are:-

platinum thermometer  $4.13^\circ\text{K}/\text{V}^2$

nickel wire thermometer  $0.98^\circ\text{K}/\text{V}^2$

thermistor thermometer  $0.035^\circ\text{K}/\text{V}^2$

### 2.3.2 Resistance versus temperature calibration

The thermometers were calibrated in a thermostatically controlled temperature chamber with a range of 213°K (-60°C) to 333°K (+60°C). The air in the chamber, dried to prevent icing, was circulated by a large fan and, to prevent the formation of pockets of stationary air within the thermometers, air was directed towards the entrances of the thermometers by a small auxiliary fan.

An N.P.L. calibrated mercury-in-glass thermometer was used to measure the temperature in the chamber; and the thermometer resistances were measured and corrected for lead resistance. The results of these calibrations are shown in Fig.8, where the experimentally determined points are compared with the manufacturer's data. The best agreement is given by the platinum thermometer, Fig.8(a). The results for the two nickel wire thermometers, Fig.8(b), do not show such good agreement. Because of possible instability of the thermistors this thermometer was calibrated several times during a period of two years. Fig.8(c) shows the results of these calibrations; there is good agreement between the different calibrations so a single line has been drawn through the results. All previous experience with thermistors suggests the calibration as plotted should be a straight line, however the present experimental results define a slight curve. No manufacturer's data has been plotted in this case, since a resistance tolerance of  $\pm 20\%$  is quoted for these thermistors. The lines through the measured points have been used as the calibration in all cases.

The sensitivity of the three elements expressed as the percentage change in resistance per  $^{\circ}\text{K}$  is:

	Sensitivity % change in R/ $^{\circ}\text{K}$	
	T = 250 $^{\circ}\text{K}$	T = 330 $^{\circ}\text{K}$
Nickel wire	0.52	0.36
Thermistor	-5.25	-3.03
Platinum wire	0.43	0.32

### 2.3.3 Calibration of the aircraft systems

During the calibration of the aircraft system the resistance box was connected into the circuit at the points normally occupied by the thermometer, the connecting leads being kept short and the resistance corrected for them.

The final calibrations showing trace deflection with change in temperature are shown in Fig.9 and a plot of recording sensitivity (inches/ $^{\circ}\text{K}$ ) in Fig.10. The sensitivity of the thermistor thermometer falls off rapidly at both ends of the temperature range. This calibration could be made more linear by placing an additional fixed resistor in parallel with the thermistors.

## 3 FLIGHT TESTS

The flight test programme is considered in two separate sections because of the different techniques used above and below Mach one.

### 3.1 Subsonic

Some flights were made at subsonic speeds by the Fairey Delta 2 in the altitude range from 30,000 ft to 40,000 ft to obtain values of the recovery

factor for the thermometers fitted to the aircraft using the conventional technique. During the tests the aircraft was flown at constant height and a series of speeds for about 20-25 seconds at each speed, to stabilize conditions, and a record taken. The altitude was held constant to reduce changes in static temperature to a minimum, and the speed varied from 200 knots indicated airspeed to that equivalent to a Mach number of unity. To reduce the effects of changes in static temperature at constant altitude with time and distance the stabilized levels were performed in random order of Mach number with each successive level on a reciprocal heading. This produces a random scatter in the results.

### 3.2 Supersonic

Since the Fairey Delta 2 is not able to cruise at both constant speed and height when supersonic, because of the limited thrust control available with reheat on, all measurements have to be made during accelerations or decelerations; thus the simpler steady techniques used in the subsonic tests cannot be used. Continuous flight records were obtained from the Fairey Delta 2 as it accelerated in level flight up to its maximum speed and then decelerated in a turn at a nominally constant height, until the speed was subsonic. It would have been preferable to decelerate in straight and level flight, but, owing to the short endurance of the aircraft, a rapid return to base was essential. The Comet was used to obtain independent measurements of the static air temperature along the same track as the Fairey Delta 2 and at approximately the same time. This time interval was kept as small as possible by arranging for the Fairey Delta 2 to overtake the Comet during the supersonic acceleration. The Comet was flown at a constant Mach number of 0.70 at constant height and obtained a continuous record of air temperature along the track. Checks by an observer in the Comet on two of the flights established that typical variations in altitude and Mach number were  $\pm 30$  ft and  $\pm 0.02$  respectively during a period of five minutes. The Fairey Delta 2 flights took place between 35,000 ft and 40,000 ft and the maximum Mach number achieved was 1.82.

In addition two flights were made with the Comet to measure the recovery factor of its nickel wire thermometer at the operating height and speed, using the technique described in Section 3.1.

## 4 THEORY

### 4.1 The temperature equation

This relates the indicated and the free stream static temperatures, as follows,

$$T_r = T_o \left( 1 + k \frac{\gamma - 1}{2} M^2 \right) \quad (4)$$

where  $T_r$  = indicated temperature (absolute)

$T_o$  = free stream static temperature (absolute)

$k$  = recovery factor

$\gamma$  = ratio of gas specific heats

$M$  = free stream Mach number.

Equation (4) is obtained by combining the definition of recovery factor;

$$k = \frac{T_r - T_o}{T_i - T_o} \quad (5)$$

with the relation between total and static temperatures in adiabatic flow of a perfect gas,

$$T_i = T_o \left( 1 + \frac{\gamma-1}{2} M^2 \right) \quad (6)$$

where  $T_i$  = free stream total temperature (absolute).

The recovery factor is a function of many variables, including air density, static temperature, local flow direction relative to the thermometer, and sensing chamber air velocity. Most thermometers try to reduce the variations in recovery factor with varying flight conditions to a minimum.

The value of the recovery factor may also be influenced by the position of the thermometer on the aircraft and, ideally, thermometers should be mounted with their intakes outside the boundary layer, in an area unaffected by separated flow or discharges from the aircraft (e.g. near fuel or cooling - air vents) and where the local Mach number is equal to the free stream Mach number. Often, this means they must be mounted on the nose of the aircraft.

At supersonic speeds the indicated temperature rise is obtained in two stages, the rise through the shock wave at entry to the thermometer and the rise corresponding to the recovery factor of the thermometer (i.e. the air entering the thermometer is already at a higher static temperature than the free stream value, but at a subsonic Mach number).

Thus, at supersonic speeds, the recovery factor in equation (4) is a combination of the separate recovery factors of the shock wave and the thermometer, and, as the shock wave has a recovery factor of unity, an equation can be derived giving the indicated temperature in terms of the same parameters as equation (4). Thus, for a normal shock wave at entry to the thermometer:

$$T_r = T_o \{ 2\gamma M^2 - (\gamma - 1) \} \left\{ \frac{2 + (\gamma - 1) M^2}{(\gamma + 1)^2 M^2} \right\} \left\{ 1 + k_1 \frac{(\gamma - 1)}{2} \left[ \frac{1 + \frac{\gamma - 1}{2} M^2}{\gamma M^2 - \frac{(\gamma - 1)}{2}} \right] \right\} \quad \dots (7)$$

where  $k_1$  = thermometer recovery factor for the conditions aft of the normal shock wave.

If the thermometer recovery factor is unity, then equation (7) reduces to equation (4) with an overall recovery factor of unity. The relation between the overall recovery factor,  $k$ , and the thermometer recovery factor,  $k_1$ , can be derived from equations (4) and (7) as:

$$\left(\frac{1-k}{1-k_1}\right) = \left(\frac{\partial k}{\partial k_1}\right)_M = \frac{\left\{1 + \frac{2\gamma}{\gamma+1}(M^2-1)\right\} \left\{1 + \frac{\gamma-1}{2}M^2\right\}^2}{\frac{\gamma+1}{2}M^4 \left\{\gamma M^2 - \frac{\gamma-1}{2}\right\}} \quad (8)$$

for  $M \geq 1.0$  .

At subsonic speeds  $k = k_1$

(8a)

The variation of  $\left(\frac{\partial k}{\partial k_1}\right)_M$  with Mach number is shown in Fig.11, and the variation of recovery factor,  $k$ , with Mach number for constant values of thermometer recovery factor,  $k_1$ , is shown in Fig.12. In practice the overall recovery factor is the basic parameter required for measuring air temperature.

In the past it has been common practice to use the thermometer recovery factor,  $k_1$ , as the basic parameter and to neglect variations of this, but in view of the rather complex equation involved at supersonic speeds, it is simpler to use the overall recovery factor,  $k$ , and accept a variation with speed. However it is not advisable to assume a constant value for the overall recovery factor,  $k$ , unless it be unity, as large temperature errors may be introduced as the speed increases. The temperature errors corresponding to an error of 0.05 in both the overall recovery factor,  $k$ , and the thermometer recovery factor,  $k_1$ , at a static temperature of 216°K are shown in Fig.13. The rapid increase in the temperature errors for the error in overall recovery factor,  $k$ , at high Mach numbers is a result of the large change in thermometer recovery factor that the error represents, as shown in Fig.11. In general, if the thermometer recovery factor is reasonably constant and nearly equal to unity, it would be better to assume an overall recovery factor of unity, for the errors would then correspond to those for an error in thermometer recovery factor,  $k_1$ , with a maximum at a Mach number of unity (Fig.13).

It should be noted that the rapid increase in temperature error at high Mach numbers corresponding to a constant overall recovery factor error does not indicate a decrease of measuring accuracy at high speeds because the recovery factor can be measured more precisely at the higher Mach numbers. The temperature error should remain constant.

#### 4.2 The transfer function

This is the equation used to correct for the effects of instrument lag when the stagnation temperature varies with time, as in the supersonic tests. The expression is based on the assumption that:-

$$\frac{dT_R}{dt} \propto (T_r - T_R)$$

or

$$\frac{dT_R}{dt} = \frac{1}{\epsilon} (T_r - T_R) \quad (9)$$

where  $T_R$  = indicated temperature uncorrected for lag

$T_r$  = indicated temperature (steady state)

$t$  = time

$\epsilon$  = is a constant known as the time constant.

This assumption is only valid when there is no self-heating of the element and when the element is cooled or heated by forced convection, i.e. without any heat transfer by conduction or radiation. In practice there will be some self-heating, but this is a function of temperature, not time, and does not affect the dynamic response (Appendix 1). Rewriting equation (9) as a transfer function gives:

$$\frac{T_R}{T_r} = \frac{1}{1 + \epsilon D} \quad (10)$$

Equation (9) can be used directly for correcting continuous flight records, since  $T_R$  is measured as a function of time and  $dT_R/dt$  can be determined by numerical differentiation. However, if a prediction of the dynamic response error is required then equation (9) must be solved to give  $T_R$  as a function of the input temperature,  $T_r$ , and time.

This solution is given in Appendix 1 as:-

$$(T_R - T_{R_0}) = \frac{e^{-t/\epsilon}}{\epsilon} \int_0^t (T_r - T_{R_0}) e^{t_1/\epsilon} dt \quad , \quad (11)$$

where  $T_{R_0} = T_{r_0}$  = the temperature at  $t = 0$ .

Using this solution the effects of various flight conditions on the dynamic response error,  $(T_R - T_r)$ , can be calculated.

In practice the thermometer is connected to a recording system the dynamic response of which must be considered. The recording systems used in the tests of this report have a transfer function of the form:-

$$\frac{\text{Output}}{\text{Input}} = \left( \frac{1}{w_n^2} D^2 + \frac{2F}{w_n} D + 1 \right)^{-1} \quad (12)$$

Appendix 2 considers the response of a thermometer connected to such a recording system. For a ramp input, - a typical flight condition, - the effective time constant for the complete system,  $\epsilon'$ , is given by:-

$$\epsilon' = \epsilon + \frac{2F}{w_n} \quad (13)$$

Usually  $2F/w_n$  is small and the effect of the recording system can be ignored.

In general any temperature changes in flight are continuous and can be represented by a polynomial of the form:

$$T_r - T_{r_0} = a_1 t + a_2 t^2 + a_3 t^3 \dots \dots \dots a_n t^n \quad (14)$$

and the response of a thermometer to this function is given in Appendix 1. This shows that, if  $\epsilon$ , and all the differential coefficients, other than the first, are small, the solution is given by:-

$$(T_R - T_{R_0}) \simeq (T_r - T_{r_0}) - \epsilon \frac{dT_r}{dt} \quad (15)$$

or, as  $T_{R_0} = T_{r_0}$

$$(T_R - T_r) \simeq - \epsilon \frac{dT_r}{dt} \quad (16)$$

From equation (9) an expression for  $dT_r/dt$  can be obtained in terms of basic flight parameters, such as Mach number, altitude, and static temperature.



$$\frac{dT_r}{dt} = \left[ \lambda \frac{dh}{dt} + \left( \frac{dT_o}{dt} \right)_{\text{const.h}} \right] \left( 1 + k \frac{\gamma-1}{2} M^2 \right) + \frac{\gamma-1}{2} T_o M \left[ M \frac{dk}{dM} + 2k M^2 T_o \frac{d\gamma}{dT_i} + 2k \right] \frac{dM}{dt} \quad (17)$$

If  $\frac{dT_r}{dt}$  is required in terms of the thermometer recovery factor then, at supersonic speeds, differentiating equation (8) w.r.t.M.

$$\frac{dk}{dM} = \left( \frac{\partial k}{\partial k_1} \right)_M \frac{dk_1}{dM} - (1 - k_1) \frac{d}{dM} \left( \frac{\partial k}{\partial k_1} \right)_M \quad (18)$$

The first term in equation (17) contains the derivatives giving the rate of change of static temperature, and the second all the derivatives associated with thermometer performance and air conditions.

The relative importance of the various parts of the second term in equation (17) can be shown by considering a typical thermometer with a constant recovery factor of 0.95 travelling at Mach numbers of 1.0 and 2.0 with a static air temperature of 220°K.

M	$\frac{d}{dM} \left( \frac{\partial k}{\partial k_1} \right)_M$	k	$\frac{d\gamma}{dT_i} \text{ } ^\circ\text{K}^{-1}$	M $\frac{dk}{dM}$	$2k M^2 T_o \frac{d\gamma}{dT_i}$	2k
1.0	3.33	0.950	$5 \times 10^{-5}$	-0.165	0.021	1.90
2.0	0.15	0.993	$5 \times 10^{-5}$	-0.015	0.087	1.99

The largest part, under these conditions, is the 2k term. A reasonable approximation to  $dT_r/dt$  can be obtained if k is assumed to be unity and  $d\gamma/dT_i$  is neglected. Neglecting the change of static temperature with time at constant altitude equation (17) can be rewritten:-

$$\frac{dT_r}{dt} \approx \lambda \frac{dh}{dt} \left( 1 + \frac{\gamma-1}{2} M^2 \right) + (\gamma-1) T_o M \frac{dM}{dt} \quad (19)$$

This is the equation that is generally used for predicting rates of change of indicated temperature, and the dynamic response error of a thermometer.

## 5 ANALYSIS AND DISCUSSION OF RESULTS

### 5.1 Measuring accuracy

The estimated accuracies of the basic measurements, based on calibration and reading accuracies are as follows:

Dynamic pressure, $q_c$	$\pm 1 \text{ lb/ft}^2$
Static pressure, $p_a$	$\pm 1 \text{ lb/ft}^2$
Corrected altitude, $h$	$\pm 50 \text{ ft (changes)}, \pm 200 \text{ ft (absolute)}$
Corrected Mach number, $M$	$\pm 0.004 \text{ (changes)}, \pm 0.01 \text{ (absolute)}$
Indicated temperature, $T_R$	$\pm 0.5^\circ\text{K}$
Comet static temperature, $T_{O_{N_i}}$	$\pm 1.0^\circ\text{K}$
Temperature lapse rate, $\lambda$	$\pm 0.2^\circ\text{K/1000 ft.}$

The dynamic pressure,  $q_c$ , is that behind a normal shock at supersonic speeds, and the free stream value at subsonic speeds. The errors in altitude and Mach number are somewhat uncertain in the Mach number range 0.98 to 1.04, due to the large variation of the static pressure error at transonic speeds, and there may also be larger errors when the rate of descent is high (i.e. greater than 150 ft/sec), as no correction has been applied for lag in the static pressure system. For the thermistor thermometer, the error in indicated temperature of  $\pm 0.5^\circ\text{K}$  only applies over the temperature range 245-320 $^\circ\text{K}$ . Outside this range the error for this thermometer rises rapidly to  $\pm 1.5^\circ\text{K}$  at 220 $^\circ\text{K}$  and 360 $^\circ\text{K}$ .

### 5.2 Time constants

The effective time constant of a thermometer,  $\epsilon'$ , can be obtained from flight tests by considering the difference in indicated temperature between two points with different rates of change of indicated temperature with time. These points were usually chosen as nearly equal Mach numbers on the acceleration and deceleration to give the largest difference between rates of change of temperature and the smallest change in temperature due to changes in flight conditions. Using the symbol  $\Delta$  to indicate differences between the first and second points, the time constant is obtained from equations (4) and (9) as:

$$\epsilon' = \frac{\Delta T_o \left( 1 + k \frac{\gamma - 1}{2} M^2 \right) - \Delta T_R}{\Delta \left( \frac{dT_R}{dt} \right)} \quad (20)$$

where the Mach number at both points is the same.

In general, the value of  $k$  will not be known exactly, although it will be close to unity, but if the  $\Delta T_o$  term is small compared with  $\Delta T_R$  the approximation  $k = 1.0$  can be used. This assumption has been used to analyse the tests of this report, using only those pairs of points where  $\Delta T_o < 1^\circ\text{K}$  and where  $\frac{d}{dt} \left( \frac{dT_R}{dt} \right)$  was small. From an original selection of 33 pairs only 12 remained after applying these restrictions. The value of  $\Delta T_o$  was calculated from the change of altitude between the two points using the temperature lapse rate determined from meteorological measurements. No data were available to correct for changes of static temperature at constant altitude with time and distance. The results of the tests are given in Table 1.

Since, to a first approximation

$$\frac{dT_R}{dt} = \frac{dT_r}{dt} \quad (\text{see Appendix 1})$$

the same 12 points have been analysed using equation (19) which gives:-

$$\Delta \left( \frac{dT_R}{dt} \right) = - \Delta \left( \frac{dh}{dt} \right) \lambda \left( 1 + \frac{\gamma-1}{2} M^2 \right) + \Delta \left( \frac{dM}{dt} \right) (\gamma-1) T_o M \quad (21)$$

The results for this indirect analysis are given in Table 2.

The possible error in  $\epsilon'$ , using the accuracies of 5.1 is of the order  $\pm 50\%$ , so, although there is a slight apparent variation with Mach number, this probably has little significance. Therefore, the mean value of  $\epsilon'$  for each thermometer has been determined and then corrected to the true thermometer time constant,  $\epsilon$ , using equation (13) i.e.

$$\epsilon' = \epsilon + \frac{2F}{w_n} \quad (13)$$

The results are:\*

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\*No flight results have been obtained from the nickel thermometer on the Fairey Delta 2, because of an intermittent fault in the associated circuit.

Thermometer		Thermistor	Platinum
Direct	Mean $\epsilon'$ , sec	1.16	1.50
Method	Standard deviation, sec	0.42	0.45
Indirect	Mean $\epsilon'$ , sec	1.17	1.48
Method	Standard deviation, sec	0.49	0.38
$2F/w_n$ , sec		0.0028	0.035
Thermometer time constant, $\epsilon$ , sec		1.2	1.5

The effective time constants of the recording systems,  $2F/w_n$ , are negligible compared with the thermometer time constants,  $\epsilon$ .

The close agreement between the mean values of  $\epsilon'$  determined by the direct and indirect methods may be fortuitous because there are quite large differences between corresponding values of  $dT_R/dt$  and  $dT_r/dt$ . However, the standard deviations also remain fairly constant for both methods so that it would appear that both methods give similar accuracy. The main cause of the large scatter indicated by the large standard deviation is the small difference in  $T_R$  measured because of the low rates of change of temperature attainable and the small values of the time constants. However, for small values of time constant, errors are less significant, because the difference between the indicated and true temperature is also small. The absolute error in temperature is related to the percentage error in the effective time constant,  $\epsilon'$  by:-

$$\delta(T_R - T_r) = - \frac{dT_R}{dt} \epsilon' \frac{\delta\epsilon'}{\epsilon'} \quad (25)$$

Thus for a given allowable error in  $(T_R - T_r)$  and a given value of  $dT_R/dt$ , the corresponding percentage error in  $\epsilon'$  is inversely proportional to  $\epsilon'$ .

The major contribution to the temperature changes comes from the change in Mach number, and the effects of rates of climb are generally small, but may occasionally become more noticeable when the Mach number is high, since this increases the  $\left(1 + \frac{\gamma-1}{2} M^2\right)$  term, and the flight path angle for high rates of climb is small (e.g. when flying at  $M = 1.5$ , when the speed of sound is 1000 ft/sec, then  $\left(1 + \frac{\gamma-1}{2} M^2\right) = 1.45$ , and the flight path angle for a climb of 250 ft/sec (15,000 ft/min) is only  $9.6^\circ$ .)

In these tests it was difficult to find points on the acceleration and deceleration where the Mach number and height were both the same, and, as the majority of the flights were below the tropopause, any change in height was a major contributor to the difference in indicated temperature. Maintaining constant height with the Fairey Delta 2 is particularly difficult as the pilot has no vertical speed indicator, and was made more difficult by his having to pass the Comet within a small height band.

The value of  $\epsilon = 1.2$  sec obtained for the thermistor thermometer is a considerable improvement over the 10 sec for the almost identical nickel wire thermometer quoted in some unpublished flight tests by A. & A.E.E., but is still rather large for use on a high performance aircraft.

The platinum thermometer  $\epsilon$  of 1.5 sec is considerably larger than the value of 0.013 sec obtained in laboratory tests. In Appendix 1 the value of  $\epsilon$  for the element alone is shown to be  $m/H$ . The value of  $m$  for this thermometer is 0.5 mW sec/ $^{\circ}$ K, and  $H$  is related to the self-heating error by equation (1)

$$\text{viz } T_R - T_r = \frac{(I^2 R)}{H} T_R \quad (3)$$

where  $(T_R - T_r)$  is the steady state self-heating error.

Using the above analysis, the values of self-heating and time constant obtained in the laboratory are consistent with a value of  $m = 0.5$  mW sec/ $^{\circ}$ K. However, if the measured value of the time constant is used, the calculated self-heating error of 3 $^{\circ}$ K/mW, is far greater than the still air value of 0.22 $^{\circ}$ K/mW, and this is unlikely.

There are two possible explanations for the difference between the time constant as measured in flight and in the laboratory:

- (a) The accuracy of the flight test results is insufficient for measuring small time constants.
- (b) The laboratory tests for measuring time constants were not fully representative of flight conditions.

Despite the large possible error of  $\pm 50\%$ , and the standard deviation of 0.45 sec, the first explanation is unlikely, because the laboratory value is still outside the range of probable values, and the probability of the laboratory value being correct for flight is small.

It appears more likely that the laboratory tests were not fully representative and, as the method used is typical of those generally employed to measure the time constants of air thermometers, a critical appraisal of the technique is required. This is included in Appendix 3, and it is concluded that the laboratory method may be unrepresentative for two reasons:

The method studies the response to a step input, and this is not representative of the continuous changes experienced in flight; it introduces the complication of transient aerodynamic and heat transfer conditions.

The step input is not applied to the complete thermometer environment, but only to the air entering the thermometer, and occasionally to the sensing element alone by varying the electrical power dissipation. Therefore, the input is not a complete step function.

Under these conditions it is possible that the effects of the time constant of the inner radiation shield may not be adequately represented. This is also considered in Appendix 3.

It would appear that the only fully representative test method is to measure the time constant in flight, unless a suitable variable stagnation temperature wind tunnel can be constructed. Further laboratory and flight tests would be required to substantiate this statement, possibly by studying the effects of varying the time constant of the inner radiation shield on the difference between flight and laboratory results.

### 5.3 Recovery factors

#### 5.3.1 Analysis and results of tests at subsonic speeds

The measurements of indicated temperature obtained from the thermometers on the Fairey Delta 2 during 8 flights at subsonic speeds are plotted against (Mach number)<sup>2</sup> in Fig.15. These tests at subsonic speeds were performed as a series of stabilized level runs as described in Section 3.1, and the indicated temperatures have been corrected for altitude changes using the temperature lapse rate from Meteorological office measurements and assuming a recovery factor of unity.

The usual method of analysing the results of tests of this type assumes that the recovery factor,  $k$ , and static temperature,  $T_0$ , are constant during each flight at constant altitude. If these assumptions are valid, then, from equation (1), if the indicated temperature is plotted against (Mach number)<sup>2</sup>, as in Fig.15, the points should all be on a straight line passing through  $T_0$

at  $M^2 = 0$  and with a slope equal to  $\frac{\gamma-1}{2} k T_0$ . The measurements of Fig.15 have been analysed on this basis using the method of least squares and the results obtained for recovery factors,  $k$ , static temperatures,  $T_0$ , and the standard deviations of indicated temperature, S.D., are summarized as follows:-

Flt.No.	Altitude (ft)	Thermometer						
		Thermistor			Platinum			Met
		k	T <sub>0</sub> °K	S.D. °K	k	T <sub>0</sub> °K	S.D. °K	T <sub>0</sub> °K
34	30,000	0.823	230.1	0.84	1.001	222.6	0.34	221.3
35	30,000	0.853	228.8	0.84	0.967	222.8	0.84	220.3
25	35,000	0.893	223.4	0.64	1.014	218.8	0.28	215.3
29	35,000	0.749	225.1	0.75	0.896	218.3	1.05	216.3
33	35,000	0.824	222.9	0.86	1.071	213.6	0.62	211.3
22	39,000	0.749	221.0	0.63	1.030	210.6	0.13	206.1
27	40,000	0.708	221.5	0.61	0.993	210.7	0.75	207.3
39	45,000	1.085	221.7	0.92	1.191	217.7	1.07	218.3

The results for Flt.39 are not consistent with the remainder, probably because 2 out of the 6 points are considerably lower than the other 4 points. The results for this flight are not used in the subsequent analysis.

The large difference between the values of T<sub>0</sub> obtained by extrapolating the results for the two thermometers on the same flight is completely unexpected and unlikely to be a true condition. The difference, which varies between 6° and 10.8° with a mean of 7.9°, is larger than the experimental error of approximately ±1.0° and indicates that the assumption of constant k is not valid for one, or both, of the thermometers.

If k varies it can only be calculated if the static temperature, T<sub>0</sub>, is known, and the only available independent measurement of temperature was the Meteorological office data which cannot be expected to have an accuracy better than ±3°K because of the interpolation in time and distance required to obtain the local temperature. The static temperature deduced from the platinum thermometer gives the best agreement with the Meteorological office data, but is consistently about 3°K higher. This same order of difference was also noted in the results from the nickel thermometer on the Comet and is similar to results obtained by Meteorological Research Flight, R.A.E., Farnborough using yet another type of aircraft mounted thermometer. It has been suggested that the radiosonde results from which the Meteorological office data are obtained are being over-corrected for radiation errors and thus giving temperatures lower than ambient.

Because the platinum thermometer gives the best agreement with the Meteorological office data and its recovery factor is compatible with the supersonic results, the subsonic results have been analysed using the platinum thermometer T<sub>0</sub>, (i.e., assuming constant K for the platinum thermometer) Fig.16(a). The effects of assuming either the thermistor thermometer T<sub>0</sub> or an intermediate value are shown in Fig.16(b). However, it is shown in Section 5.3.3 that these alternatives are incompatible with the supersonic results.

The large difference between the predicted values of static temperature for the two thermometers emphasises the importance of reliable independent measurements of static temperature, rather than relying on the 'straight line' method of analysis when testing any new design of thermometer. It should be noted that, with this method, the standard deviations for both thermometers are similar and compare well with the expected experimental accuracy, therefore the results for either thermometer tested alone would be quite acceptable, although one, or both, must be predicting an incorrect static temperature with an error of at least 4°K.

In the absence of any independent measurement of static temperature the nickel wire thermometer fitted to the Comet aircraft was calibrated using the same technique, however, as the  $T_0$  derived from this thermometer was only used to provide an independent value to analyse measurements at supersonic speeds, the effects of an error in measuring the recovery factor of the Comet thermometer are not very large. e.g. an error of 5% in the recovery factor for the Comet thermometer gives errors in apparent recovery factor for the Fairey Delta 2 thermometers of 2.7% and 1.5% at Mach numbers of 1.0 and 1.5 respectively. These errors are well within the experimental accuracy governed by other limitations.

The results of the calibration of the nickel wire thermometer fitted to the Comet are shown in Fig.17 and indicate a recovery factor of unity.

### 5.3.2 Analysis and results of tests at supersonic speeds

Seven flights were performed at supersonic speeds using the technique described in Section 3.2, and the variation of the quantities measured during a typical flight (Flt. No.38) is shown in Fig.18. As the speed was continuously changing during all the flights, the indicated temperatures must be corrected for the effects of the dynamic response of the measuring system. This correction has been evaluated for Flts.37 and 38 and the results for Flt.38 are shown in Fig.19(a), and, apart from the deceleration (i.e. after 220 sec), the average correction is less than  $\frac{1}{2}$ °K, which can be neglected.

As the conditions during the deceleration are varying far more rapidly than during the acceleration and the corrections are difficult to apply, the analysis has been restricted to the acceleration phase only for flights other than 37 and 38. Flts.37 and 38 were the only two flights where the tests took place above the tropopause and the general rates of change of temperature varied more steadily than on the other five flights, probably due to the reduced effect of altitude changes on the static temperature. Therefore, the dynamic response correction has been applied to these two flights and both the acceleration and deceleration phases analysed.

The variation of static temperature along the track of the Fairey Delta 2 during the test runs is derived from the total temperature measured by the nickel wire thermometer fitted to the Comet using the measured recovery factor of unity. Unfortunately the Comet speed and altitude could only be recorded at the moment when the Fairey Delta 2 passed, although the temperature was recorded continuously. In view of this the derived static temperature at the moment of passing has been used as the basic measurement and the variation along the

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\*Independent tests at A&AEE confirm that the recovery factor should be within 5% of unity, and thus the error in  $T_0$  should not exceed 1°K.



Fairey Delta 2 track has been obtained using the altitude difference from the point of passing and the forecast temperature lapse rate. The continuous temperature record obtained from the Comet has been analysed using the Mach number at the moment of passing, and, as the typical variations in altitude and Mach number of  $\pm 30$  ft and  $\pm 0.02$  only result in errors of approximately  $\pm 1^\circ\text{K}$  in the static temperature, the results have been used to give an indication of any large scale variations of static temperature with time and distance at constant altitude. Apart from the beginning of some flights the variations of static temperature are less than  $\pm 1^\circ\text{K}$ , therefore, no large variations in static temperature are apparent. The variations at the beginning of some flights are probably caused by starting the recording before the speed and altitude of the Comet had been stabilized. The Comet static temperature results for two flights (Flts.28 and 38) are shown in Fig.20. The results for Flt.28 are typical of the initial variation obtain during the stabilizing period.

As independent values of static temperature are available for these flights at supersonic speeds the recovery factors,  $k$ , can be calculated directly. The results for the platinum and thermistor thermometers are shown in Figs.21 and 22 respectively.

### 5.3.3 Discussion of recovery factor results

It is estimated from the accuracies stated in 5.1 that the accuracy of the recovery factor results is within the following limits:

Mach number	0.7	1.0	1.5
Recovery factor accuracy	$\pm 0.07$	$\pm 0.05$	$\pm 0.03$

The effects of errors of this magnitude can be estimated from Fig.23, which shows the error in  $k$  or  $k_1$  corresponding to a  $1^\circ\text{K}$  error in  $T_0$ . A systematic error of less than a third of the above magnitude, and in the negative direction, may exist for those flights at supersonic speeds where no dynamic response correction is applied.

As the results from the tests at supersonic speeds are the only ones analysed using independent values of static temperature, these will be considered first. A general comparison of the results for the two thermometers on the same flight (Figs.21, 22) shows that many of the disturbances are common to both thermometers - e.g. the hump at  $M^2 = 3.05$  on Flt.28. These disturbances are almost certainly caused by errors in the static temperature used in the analysis, or by high rates of change of temperature for which no correction is applied. The hump in Flt.28 is equivalent to 2 or  $3^\circ\text{K}$  error in static temperature. If some allowance is made for these common deviations, the general trend in the variation of recovery factor,  $k$ , with (Mach number)<sup>2</sup>, Figs.21, 22 shows that the platinum thermometer recovery factor is virtually constant at a mean value of unity, whereas that of the thermistor thermometer increases slightly, but significantly, from unity at  $M^2 = 1.0$  to 1.05 ( $k_1 = 1.26$ ) at  $M^2 = 3.0$ . The error in stagnation temperature, if it was assumed that the thermistor thermometer recovery factor is unity, would be rather large, approximately  $6^\circ\text{K}$  at  $M^2 = 3.0$ , as may be deduced from Figs.13 and 19(o).

The rise in the thermistor thermometer recovery factor to values significantly greater than unity is difficult to understand. There are two possible causes of such results:

Calibration errors.

Aerodynamic peculiarities.

The first suggestion is unlikely to be correct, because the calibration (Fig.8(b)) is well established, and also the required error in the calibration to give a recovery factor of unity, or less, would increase the non-linearity of the calibration; whereas previous experience with thermistors suggests that the calibration should be a straight line. The overall scatter in the graphs of the difference in indicated temperature between the thermistor and platinum thermometers versus the platinum thermometer indicated temperature (Figs.24(a), 25) is nearly 3 times the experimental error and this is far more than would be expected if a calibration error was responsible. Therefore, an aerodynamic cause appears more likely.

The recovery factor can be greater than unity if the conditions of uniform adiabatic flow are not satisfied; caused, for example, by the presence of heat addition, or uneven distribution of total temperature due to turbulence or vortex-type flow. As the thermometer is shielded and insulated from radiation and conduction effects, the main source of energy is the electrical power dissipated in the sensing element. This, however, is less than 0.75 mW per thermistor and, despite the small surface area, is unlikely to produce any significant temperature rise, as the still air self-heating error for this power dissipation is only 0.5°K, Fig.7(b).

It, therefore, appears most likely that the rise in recovery factor is caused by the presence of non-uniform flow either in the sensing element chamber or external to the thermometer. This suggestion appears very relevant, because the small size of the thermistors - approximately 0.015 inches - makes them very sensitive to variations of total temperature in the air sample. The characteristics of such a flow are probably controlled mainly by Mach number, Reynolds number and temperature.

Because the platinum thermometer recovery factor is approximately unity and the time constants of both thermometers are similar, any graph of the difference in indicated temperature between the two thermometers can be regarded as showing the difference between the thermistor thermometer indicated temperature and the stagnation temperature. Graphs of this difference are plotted against the platinum thermometer indicated temperature ( $\approx$  stagnation temperature) and (Mach number)<sup>2</sup> in Figs.24, 25, 26. It is not possible to decide which of the two parameters, temperature or Mach number, is most significant, because total temperature is a function of both static temperature and Mach number, and the range of static temperature covered was small.

The results of the subsonic tests, Fig.24, assume a subsonic platinum thermometer recovery factor of unity, and this is compatible with the supersonic results. Other assumptions for analysing the subsonic results, (Fig.16(b)), give a discontinuity between the subsonic and supersonic

recovery factors for both thermometers. And, as there is an overlap of subsonic and supersonic results without any evidence of such a discontinuity, the use of the other assumptions about  $T_0$  cannot be justified.

The increase of the difference in temperature at both ends of the Mach number range suggests that the results might be condensed by considering the effects of the Mach number at the entrance to the thermometer (i.e. the Mach number downstream of a normal shock for the supersonic case). The variation of the Mach number downstream of a normal shock with free stream Mach number is given by Fig.27, and the variation of the temperature difference with entry Mach number is shown in Fig.28. The general increase in temperature difference occurs at about the same entry Mach number but the scatter increases as the Mach number decreases from unity. This would be so if there was also a dependence on Reynolds number, so a check has been made by plotting all the points at  $M = 0.7 \pm 0.01$  against entry Reynolds number (Fig.29). There appears to be a tendency to vary with Reynolds number, but the scatter is still large. It would seem that the difference in indicated temperature between the thermistor and platinum thermometers may well be due to aerodynamic peculiarities depending on temperature, Mach number and Reynolds number. Any aerodynamic effects may be encouraged or caused by the wake from the whip aerial mounted approximately 15 inches in front of the thermistor thermometer (Fig.5). Unfortunately this thermometer could not be tested in any other position, because its mounting was not interchangeable with the similar nickel wire thermometer.

#### 5.4 Discussion of flight test technique

Because the flight tests had to be completed by a given date (see para 1) there was insufficient time available to develop the technique. However, several suggestions can be made about the requirements for any future flight programme to investigate the behaviour of impact thermometers at high subsonic and supersonic speeds.

The first essential is an accurate knowledge of the static temperature and lapse rate in the air space used for the tests. The most reliable way of achieving this is to fly an aircraft, fitted with a thermometer of known characteristics, at constant speed and altitude through the air space immediately before the test run. This aircraft and thermometer should be thoroughly calibrated, if possible using static temperature measurements such as measurements from a static balloon - low altitude, or meteorological radiosonde balloon measurements - high altitude. The speed and altitude must be kept constant to minimize the changes in indicated temperature, and hence the dynamic response error. Also, the speed should be low to reduce the errors in static temperature caused by any errors in the assumed recovery factor. This aircraft can be used to find the lapse rate by recording a constant Mach number climb through the airspace. To avoid large variations of static temperature, flights should not be made in regions with unsteady meteorological conditions (e.g. fronts, clouds, jetstreams, regions of clear air turbulence, etc.). The above conditions are restrictive but the accuracy of the static temperature measurements is the main factor dominating the final accuracy of the analysis.

The aircraft with the test thermometer should be flown through the same air space as the datum aircraft, along the same track, at the same altitude, and should fly past the datum aircraft at close range.

For determining time constants the maximum possible acceleration and deceleration should be used, and the height kept constant to reduce the change in static temperature along the flight path.

For recovery factor measurements the speed and height should both be constant to minimize the dynamic response correction.

Other test requirements are accurate pressure error corrections for the pitot-static systems of both the test and datum aircraft, and careful calibration of the complete temperature measuring systems on these aircraft. The effects of sensing element self-heating should also be measured in flight.

The flight tests described in this report fall short, in some respects, of the ideal procedure. The most important limitations are:-

- (a) The static temperature was only measured at one point.
- (b) The lapse rate was deduced from meteorological data.
- (c) The recovery factors at supersonic speeds were determined from measurements under non-stabilized conditions.
- (d) Self-heating effects were not measured in flight.

## 6 CONCLUSIONS

The report covers laboratory and flight tests on three impact air thermometers and presents associated theoretical work. The results lead to the following conclusions.

It is desirable to have a time constant less than 0.2 sec if the temperature error is not to exceed 1.0°K under most flight conditions.

The platinum thermometer time constant of 1.5 sec is much larger than the value of 0.013 sec obtained in laboratory tests, and it is concluded that present laboratory tests, particularly the step input method, may be unrepresentative of flight conditions, and only tests in flight or a variable stagnation temperature wind tunnel are adequate. The accuracy of the recovery factor results obtained is considered sufficient to enable the static temperature to be obtained to within  $\pm 1.5^\circ\text{K}$ . In the section on theory the use of the overall recovery factor is advocated for supersonic speeds, rather than the thermometer recovery factor, as using the overall value removes the necessity to consider the effects of the normal shock wave formed at the entrance to the thermometer. The use of the straight line method of analysis, using plots of indicated temperature versus (Mach number)<sup>2</sup>, to obtain recovery factors at subsonic speeds is not recommended for tests on any new design of thermometer, because the basic assumption involved, that the recovery factor does not vary with Mach number, is not necessarily valid.

The unusual behaviour of the thermistor thermometer, with recovery factors greater than unity, is probably caused by an aerodynamic disturbance producing non-uniform flow, and possibly vortices, in the sensing element chamber. The small size of the thermistors makes them very sensitive to such effects. The

difference in indicated temperature between the thermistor and platinum thermometers (platinum thermometer temperature  $\approx$  stagnation temperature), which is a measure of the flow non-uniformity, appears to be mainly dependent on the thermometer entry Mach number, with some variation with indicated temperature and Reynolds number. The aerodynamic disturbance may well be caused or enhanced by the whip aerial 15 inches forward of the thermistor thermometer.

It is concluded that the most vital measurement required for all flight tests to investigate the behaviour of air thermometers is an independent value of the static air temperature. More than half of the  $\pm 1.5^\circ\text{K}$  uncertainty in obtaining static temperature from the measured recovery factors is due to the errors in the independent measurements of static temperature. The use of an adequately calibrated thermometer mounted on a datum-aircraft, and the use of the fly-past, or a formation technique is recommended for all future tests on air thermometers.

## 7 ACKNOWLEDGEMENTS

The authors wish to acknowledge the advice and comments obtained from staff of the Aeroplane and Armament Experimental Establishment during the writing of this report.

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## SYMBOLS

$a_{1,2,\dots,n}$	coefficients in the temperature polynomial $\Delta T_r = a_1 t + a_2 t^2 + a_3 t^3 \dots a_n t^n$
D	differential operator, $d/dt$
F	damping ratio of the recording system
H	surface heat transfer rate of the sensing element watts/ $^\circ\text{K}$
$\bar{H}$	surface heat transfer factor of the sensing element watts/cm <sup>2</sup> $^\circ\text{K}$
h	pressure altitude, ft
I	current through the sensing element, amps
j	gradient of the ramp function, $\Delta T_r = jt$
k	recovery factor, $k = \frac{T_t - T_o}{T_i - T_o}$
$k_1$	thermometer recovery factor, related to k by equations (8) and (8a)
M	Mach number

SYMBOLS (Contd.)

$\hat{M}$	Mach number downstream of a normal shock
$m$	thermal capacity of the sensing element, watts-sec/ $^{\circ}$ K
$1/m_{1,2}$	time constants of the recording system, sec
$N$	a constant in the step function equation
$p$	the Laplace operator
$R$	thermometer resistance, ohms
$S$	surface area of the sensing element, $\text{cm}^2$
$T_i$	free stream stagnation temperature, $^{\circ}$ K
$T_o$	static temperature, $^{\circ}$ K
$T_R$	indicated temperature, $^{\circ}$ K
$T_r$	temperature of air surrounding the element, $^{\circ}$ K
$T_r'$	stagnation temperature at the centre of the measuring chamber, $^{\circ}$ K (Appendix 3)
$t$	time
$t_1$	$0 \leq t_1 \leq t$ (Appendix 1)
$U$	constant (Appendix 3)
$V$	voltage across the sensing element, volts
$\bar{V}$	sensing element volume, $\text{cm}^3$
$w_n$	recording system undamped natural frequency, radians/sec
$\gamma$	ratio of the specific heats of air, $\gamma = 1.40$
$\varepsilon$	thermometer time constant, sec
$\varepsilon'$	effective time constant of the complete temperature measuring system, sec
$\lambda$	temperature lapse rate, $^{\circ}$ K/1000 ft
$\sigma$	specific heat per unit volume watts-sec/ $^{\circ}$ K $\text{cm}^3$
$\phi$	still air self-heating constant, $^{\circ}$ K/mW

SYMBOLS (Contd.)

Suffices

Th	thermistor thermometer
Pl	platinum thermometer
Ni	nickel wire thermometer
Met	meteorological office data
I	inner radiation shield (Appendix 3)

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## APPENDIX 1

### THE DYNAMIC RESPONSE OF SYSTEMS WITH THE TRANSFER FUNCTION

#### $(1 + \epsilon D)^{-1}$ (FIRST ORDER SYSTEMS) WITH PARTICULAR REFERENCE TO IMPACT THERMOMETERS

A first order system is one where the rate of change of output signal with time is proportional to the difference between input and output signals. The heat exchange equation for a thermometer sensing element, when all the heat transfer at its surface takes place under forced convection, with no conduction or radiation, is:

$$H(T_r - T_R) + (I^2R)_{T_R} = m \frac{dT_R}{dt} \quad (26)$$

where  $H$  = heat transfer rate in watts/degree

$(I^2R)_{T_R}$  = electrical power dissipated at temperature  $T_R$  in watts

$m$  = thermal capacity of the element in watts-sec/degree.

The electrical power dissipated introduces a self-heating error, which under stabilized conditions is given by:

$$T_R - T_r = \frac{(I^2R)_{T_R}}{H} \quad (3)$$

Usually the electrical power dissipated is a function of  $T_R$  only and it can be assumed that the power changes instantaneously with any change in  $T_R$ . Hence  $(I^2R)_{T_R}$  is not time dependent and the element temperature at any instant may be obtained by finding the value of  $T_R$  without any power dissipation and adding the effect of self-heating to this value. Thus in determining the dynamic response of the system the effects of self-heating may be neglected and the heat exchange equation becomes:

$$H(T_r - T_R) = m \frac{dT_R}{dt} \quad (27)$$

or, written as a transfer function

$$\frac{T_R}{T_r} = \frac{1}{(1 + \epsilon D)} \quad (2)$$

where  $\epsilon = m/H$

and  $D = d/dt$ , the differential operator.

The units of  $\epsilon$  are time, and it is known as the time constant of the system.

Using the Laplace transform technique, the response of the system to any input function can be determined as follows:

Defining the Laplace transform of  $x(t)$  as

$$\bar{x}(p) = \int_0^{\infty} x(t) e^{-pt} dt$$

then the transform of the transfer function is

$$\frac{\bar{T}_R(p)}{\bar{T}_r(p)} = \frac{1}{1 + \epsilon p} \quad (28)$$

provided that  $T_R = T_r = 0$  at  $t = 0$ . Thus in all the following, the difference between the present and initial temperature is considered. Since the Laplace transform of a unit impulse is unity, the transform of the response to a unit impulse is:

$$\bar{T}_R(p) = \frac{1}{\epsilon} \frac{1}{\left(p + \frac{1}{\epsilon}\right)} \quad (29)$$

Solving equation (29) gives the response to a unit impulse

$$\Delta T_R = \frac{1}{\epsilon} e^{-t/\epsilon} \quad (30)$$

Equation (29) can now be substituted in Duhamel's integral and the response to any input function obtained.

Duhamel's integral states that

$$\Delta T_R = \int_0^t \Delta T_r A(t-t_1) dt_1 \quad (31)$$

where  $t$  is any given time and  $0 \leq t_1 \leq t$

and  $A(t)$  is the response to a unit impulse.

Hence from equations (30) and (31), the response to any input function,  $\Delta T_r$ , is

$$\Delta T_R = \frac{e^{-t/\epsilon}}{\epsilon} \int_0^t \Delta T_r e^{t_1/\epsilon} dt_1 \quad (32)$$

where  $\Delta T_R = T_R - T_{r_0}$  and  $\Delta T_r = T_r - T_{r_0}$ , and at  $t = 0$ ,  $T_R = T_r = T_{r_0}$ .

This may now be used to find the response of the system to three typical input functions:

1 Step input This function is frequently used for laboratory tests to determine the time constant and gives the familiar exponential response associated with first order systems.

2 Ramp input This a simple approximation to the flight conditions and illustrates the effects of the initial exponential response and the final steady lag condition.

3 Polynomial input All flight conditions are continuously changing and can be represented by a polynomial. The response indicates the effects of the basic assumptions that the time constant is small and all differential coefficients, other than the first, are negligible.

1 Step input

A step input can be represented by

$$\Delta T_r = \Delta T(1 - e^{-N t/\epsilon}) \quad (33)$$

as  $N \rightarrow \infty$ .

Substituting for  $\Delta T_r$  in equation (32), and integrating:

$$\Delta T_R = \Delta T \left[ (1 - e^{-t/\epsilon}) - \frac{e^{-t/\epsilon}}{N-1} \left( 1 - e^{-\frac{(N-1)t}{\epsilon}} \right) \right] \quad (34)$$

Thus for  $N \rightarrow \infty$

$$\Delta T_R \rightarrow \Delta T (1 - e^{-t/\epsilon}) \quad .$$

Therefore the response to a step input is:

$$\Delta T_R = \Delta T (1 - e^{-t/\epsilon}) \quad . \quad (35)$$

Equation (35) forms the basis of the usual methods for determining the value of  $\epsilon$ , since when  $t = \epsilon$

$$\Delta T_R = \Delta T \left( 1 - \frac{1}{e} \right) = 0.632 \Delta T \quad (36)$$

and also

$$\left( \frac{d\Delta T_R}{dt} \right)_{t=0} = \frac{\Delta T}{\epsilon} \quad . \quad (37)$$

In practice the time for  $\Delta T_R$  to reach  $0.632 \Delta T$  is used, as the initial slope is more sensitive to imperfections in the step input.

## 2 Ramp input

$$\Delta T_r = jt \quad (38)$$

where  $j$  is a constant.

Substituting in equation (32) and integrating:

$$\Delta T_R = \Delta T_r - j\epsilon(1 - e^{-t/\epsilon}) \quad . \quad (39)$$

It is desirable to have  $(1 - e^{-t/\epsilon}) > 0.99$ , or  $t/\epsilon > 4.6$ .

Then, for large  $t/\epsilon$

$$\Delta T_R = \Delta T_r - \epsilon j . \quad (40)$$

### 3 Polynomial input

$$\Delta T_r = a_1 t + a_2 t^2 + a_3 t^3 \dots a_n t^n . \quad (41)$$

Most continuous inputs may be represented by a polynomial of this form. Substituting in equation (32) and integrating:

$$\begin{aligned} \Delta T_R = e^{-t/\epsilon} \left\{ 1! a_1 \epsilon - 2! a_2 \epsilon^2 + 3! a_3 \epsilon^3 \dots (-1)^{n-1} n! a_n \epsilon^n \right\} \\ + \Delta T_r \{ 1 - \epsilon D + \epsilon^2 D^2 - \epsilon^3 D^3 \dots (-1)^n \epsilon^n D^n \} . \end{aligned} \quad (42)$$

Again considering large values of  $t/\epsilon$ , we have:

$$\Delta T_R = \Delta T_r \{ 1 - \epsilon D + \epsilon^2 D^2 - \epsilon^3 D^3 \dots (-1)^n \epsilon^n D^n \} .$$

Thus, if  $\epsilon$  is small, and all differentials higher than  $D$  are small, the equation becomes:

$$\Delta T_R = \Delta T_r - \epsilon \frac{dT_r}{dt} . \quad (43)$$

Therefore, if the above conditions are satisfied, from equation (2)

$$\frac{dT_r}{dt} = \frac{dT_R}{dt} .$$



APPENDIX 2

THE EFFECTS OF A RECORDING SYSTEM WITH THE TRANSFER FUNCTION

$\left(1 + \frac{2F}{w_n} D + \frac{1}{w_n^2} D^2\right)^{-1}$  ON THE RESPONSE OF A FIRST ORDER SYSTEM

The overall transfer function for a first order system in series with a second order system is given by the product of their individual transfer functions:

$$\frac{T_R}{T_r} = \frac{1}{(1 + \epsilon D)} \frac{1}{\left(1 + \frac{2F}{w_n} D + \frac{1}{w_n^2} D^2\right)} \quad (44)$$

where  $F$  is the damping ratio of the recording system

$w_n$  is the undamped natural frequency of the recording system.

Using the Laplace transform method of Appendix 1 the response to any input function is given by

$$\Delta T_R = \frac{(w_n \epsilon)^2}{\left(1 - 2 \epsilon w_n F + \epsilon^2 w_n^2\right)} \left[ \frac{e^{-t/\epsilon}}{\epsilon} \int_0^t \Delta T_r e^{t_1/\epsilon} dt_1 - \frac{1}{2 \epsilon^2 w_n \sqrt{F^2 - 1}} \right. \\ \left. \left\{ \left(1 - \epsilon w_n F + \epsilon w_n \sqrt{F^2 - 1}\right) e^{-m_1 t} \int_0^t \Delta T_r e^{m_1 t_1} dt_1 - \right. \right. \\ \left. \left. \left(1 - \epsilon w_n F - \epsilon w_n \sqrt{F^2 - 1}\right) e^{-m_2 t} \int_0^t \Delta T_r e^{m_2 t_1} dt_1 \right\} \right] \quad \dots (45)$$

where  $m_{1,2} = F w_n \pm w_n \sqrt{F^2 - 1}$  .

Appendix 2

Considering only the response to a polynomial input, and making the same assumptions as in Appendix 1 (i.e.  $t \rightarrow \infty$ ,  $\epsilon$ ,  $\frac{1}{m_1}$ ,  $\frac{1}{m_2}$  all small, and differential coefficients other than the first are negligible) the solution is

$$\Delta T_R = \Delta T_r - \left( \epsilon + \frac{2F}{w_n} \right) \frac{dT_r}{dt} \quad (46)$$

and the effective time constant  $\epsilon'$  is given by:

$$\epsilon' = \epsilon + \frac{2F}{w_n} \quad (47)$$

As the damping of recording systems is usually about 0.7 and rarely greater than 2.0, the major factor contributing to the increase in time constant is the undamped natural frequency as would be expected.



APPENDIX 3

THERMOMETER TIME CONSTANTS AND THEIR MEASUREMENT

In Appendix 1 the time constant of a thermometer element was given by:

$$\epsilon = \frac{m}{H} = \frac{m}{\bar{H}S} \quad (48)$$

This equation also applies to any other portion of the thermometer, such as the radiation shields and the former on which the sensing element is wound. If the inner radiation shield is considered using suffix I, then

$$\epsilon_I = \frac{m_I}{\bar{H}_I S_I} \quad (49)$$

and during any change in free stream stagnation temperature the temperature of the radiation shield will lag behind the true temperature. This will set up a temperature gradient in the element chamber, and in general the element will be in a region where the stagnation temperature is not the same as the free stream stagnation temperature. The ambient element temperature,  $T_r$ , can then be given as:

$$T_r = T_r' - U (T_r' - T_I) \quad (50)$$

where 'U' is a constant and  $T_r'$  is the stagnation temperature at the centre of the measuring chamber. The value of 'U' increases to unity as the element approaches the radiation shield.

Using the methods of Appendix 1 the total response of the thermometer to a ramp input can be found

viz. 
$$T_R = T_r' - j(\epsilon + U \epsilon_I) \quad (51)$$

and for a step input

$$T_r = T_r' \left( 1 - U e^{-t/\epsilon_I} \right) \left( 1 - e^{-t/\epsilon} \right) \quad (52)$$

From the above equations it can be seen that the time constant of the radiation shield could have quite an appreciable effect on the overall effective time constant.

If the specific heat per unit volume of the element and shield are  $\sigma$  and  $\sigma_I$  respectively, and their volumes  $\bar{V}$  and  $\bar{V}_I$ ,

then

$$\frac{\epsilon_I}{\epsilon} = \frac{\sigma_I}{\sigma} \frac{\bar{H}}{\bar{H}_I} \frac{\bar{V}_I}{\bar{V}} \frac{S}{S_I} \quad (53)$$

If the element is an open wire of diameter 'd' and the shield is a thin cylinder of thickness,  $\Delta d$ , then equation (53) becomes

$$\frac{\epsilon_I}{\epsilon} = \frac{\sigma_I}{\sigma} \frac{\bar{H}}{\bar{H}_I} \frac{2\Delta d}{d} \quad (54)$$

In general  $\frac{\sigma_I}{\sigma} \simeq \frac{\bar{H}}{\bar{H}_I} \simeq 1.0$  and  $\Delta d$  will be larger than  $d$  for structural

stiffness, unless  $d$  is abnormally large.

Therefore  $\epsilon_I$  will normally be much larger than the element time constant  $\epsilon$  and the overall effect depends very much on the value of 'U'.

If the effect of the element former is considered then similar equations apply and the value of  $U$  will be unity where the wire is touching the former. Thus the overall value of  $U$  will be proportional to the fraction of the total sensing element length touching the former. The time constant of the former is not so easy to obtain as it is usually made of a material with high electrical resistivity and thus low heat conductivity. This low conductivity produces steep temperature gradients and under these conditions the time constant will probably be less than that given by the ratio of the thermal mass to the heat transfer rate. In general the presence of the former will increase the time constant of the thermometer.

From consideration of the above effects it can be seen that it is important to maintain the same relative temperatures throughout the thermometer during any time constant measurements as those present in flight, as any local heating or cooling may have a considerable effect on the time constant.

A common laboratory method for measuring time constants is to mount the thermometer in a wind tunnel with a flow of hot air from a pipe near the thermometer entrance. Conditions are stabilized and then a step function is applied by suddenly stopping the hot air supply. The time constant is determined from the response to this input. This method is not fully representative of operating conditions in flight, since the step function is not applied to the complete thermometer, and furthermore any step input cannot be representative as the thermometers are only subject to continuous changes in normal use. The above method is, however, slightly better than the use of element

self-heating to provide the step input, as this completely ignores the effects of the radiation shield and element former. In both these laboratory tests the results obtained will tend to give lower values of time constant than would be obtained in flight. It would seem that the only truly representative methods are, either flight measurements, or measurements in some form of variable stagnation temperature wind-tunnel.

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TABLE 1

Measurement of time constants (direct method)

Flt. No.	M	Δt sec	Δh ft	λ °K/ 1000 ft	ΔT <sub>o</sub> °K	ΔT <sub>R</sub>		ΔT <sub>o</sub> $\left(1 + \frac{\gamma-1}{2} M^2\right) - \Delta T_R$		-Δ $\left(\frac{dT_R}{dt}\right)$ °K/sec	Thermistor thermometer ε' Th sec	Platinum thermometer ε' Pl sec
						Th	Pl	Th	Pl			
38	1.046	256	-750	-0.5	-0.4	0.9	2.2	1.2	2.7	1.15	1.04	2.34
37	1.069	144	-690	"	-0.4	1.6	3.6	2.1	4.1	1.70	1.23	2.41
37	1.149	120	+300	"	0.1	1.6	2.6	1.5	2.5	1.65	0.91	1.51
38	1.154	226	-450	"	-0.2	1.3	2.3	1.6	2.6	1.70	0.94	1.53
38	1.268	194	-350	"	-0.2	2.8	2.0	3.1	2.3	2.05	1.51	1.12
37	1.293	86	-840	"	-0.4	4.4	5.3	4.9	5.8	3.60	1.36	1.61
31	1.396	20	-200	2.0	0.4	1.5	3.2	0.9	2.6	2.05	0.44	1.27
38	1.408	154	-640	-0.5	-0.3	2.2	3.1	2.6	3.5	2.55	1.02	1.37
28	1.411	216	-400	2.0	0.8	3.5	3.3	2.4	2.2	2.15	1.12	1.02
37	1.484	40	+ 50	-0.5	0	1.2	1.9	1.2	1.9	1.80	0.67	1.05
37	1.532	22	+380	"	0.2	2.4	1.7	2.1	1.4	1.35	1.56	1.04
38	1.558	100	-890	"	-0.4	4.3	3.4	4.9	4.0	2.35	2.08	1.70

N.B. No results available for the nickel thermometer, because of an intermittent circuit fault

TABLE 2

Measurement of time constants (indirect method)

Flt. No.	M	$\Delta T_o \left(1 + \frac{\gamma-1}{2} M^2\right) - \Delta T_R$ °K		$-\Delta \left(\frac{dT_R}{dt}\right)$ °K/sec	Thermistor thermometer ε' Th sec	Platinum thermometer ε' Pl sec
		Th	Pl			
38	1.046	1.2	2.7	1.53	0.78	1.76
37	1.069	2.1	4.1	2.04	1.03	2.01
37	1.149	1.5	2.5	1.21	1.24	2.07
38	1.154	1.6	2.6	1.81	0.88	1.44
38	1.268	3.1	2.3	2.05	1.51	1.12
37	1.293	4.9	5.8	3.64	1.31	1.59
31	1.396	0.9	2.6	1.71	0.53	1.52
38	1.408	2.6	3.5	2.55	1.02	1.37
28	1.411	2.4	2.2	2.04	1.18	1.08
37	1.484	1.2	1.9	2.35	0.51	0.81
37	1.532	2.1	1.4	1.25	1.68	1.12
38	1.558	4.9	4.0	2.11	2.32	1.89

TABLE 3

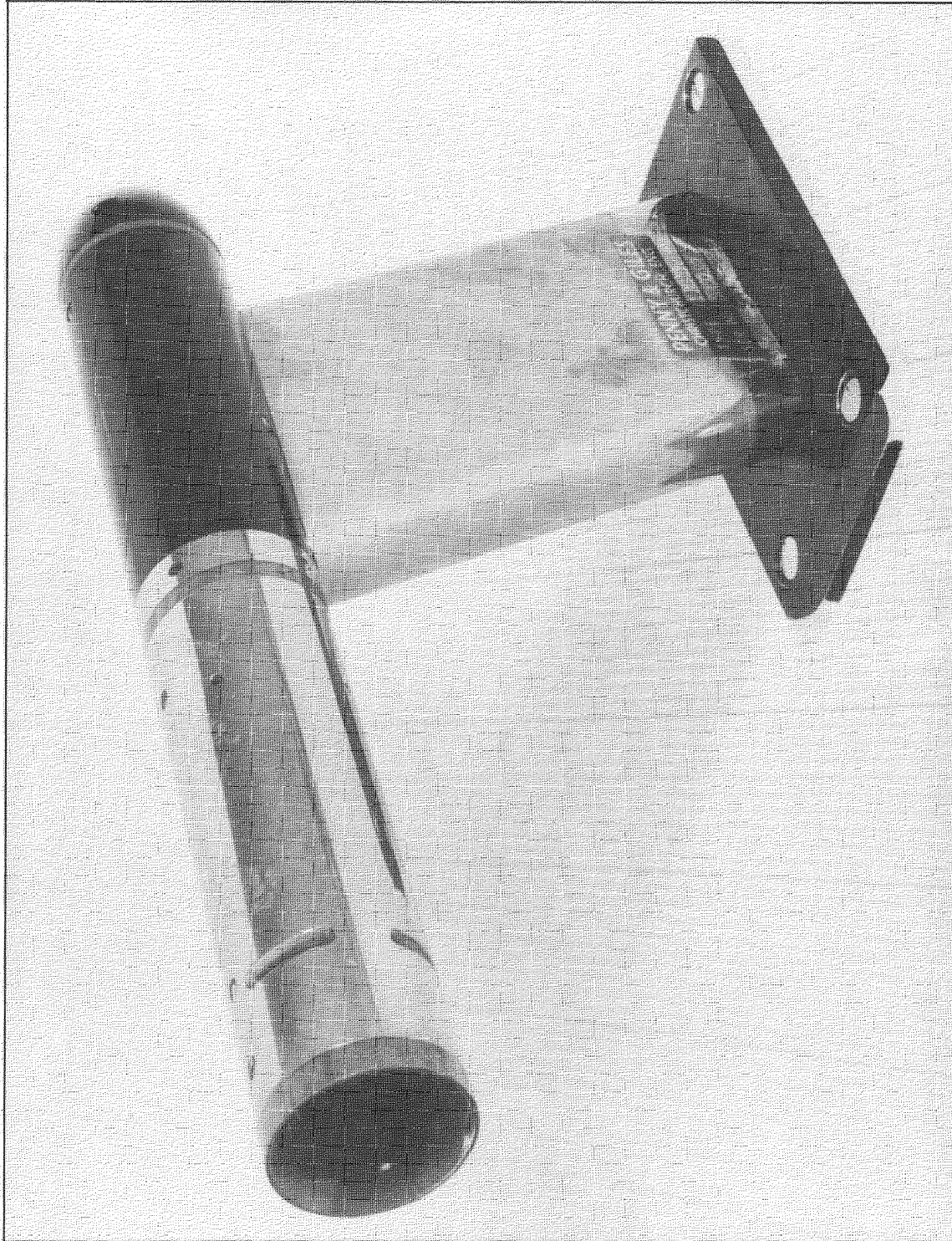
Summary of results

Thermometer		Laboratory						Flight					
		Self-heating		Resistance sensitivity		Recording sensitivity		Time constants		Recovery factors			
Type	Sensing element	Constant $\phi$ °K/mW	Factor (at 290°K) $\phi/R$ °K/V <sup>2</sup>	$\frac{1}{R} \frac{dR}{dT} \% \text{ } ^\circ\text{K}^{-1}$		inches/°K		$\epsilon$ sec	Standard deviation sec	Supersonic, k		Subsonic, k (2)	
				250°K	330°K	250°K	330°K			M = 1.0	M = 1.7	M = 0.7	M = 1.0
Penny and Giles A. & A.E.E. type	Nickel wire	0.10	0.98	0.52	0.36	0.022 <sup>(1)</sup>	-	-(3)	-	-	-	1.00 <sup>(1)</sup>	-
	Thermistor	0.37	0.04	-5.25	-3.03	0.012	0.009	1.2	0.42	1.00	1.05	1.15	1.00
Rosemount Type 102E	Platinum	0.22	4.13	0.43	0.32	0.019	3.015	1.5	0.45	1.00	1.00	1.00	1.00

NOTES: 1 Nickel wire thermometer on Comet.

2 Results derived using static temperature predicted by platinum thermometer measurements.

3 Unpublished A. & A.E.E. data gives a time constant of approximately 10 sec for the nickel wire thermometer.



**FIG.1a. PENNY AND GILES (A. & A.E.E. TYPE) THERMOMETER  
GENERAL VIEW**



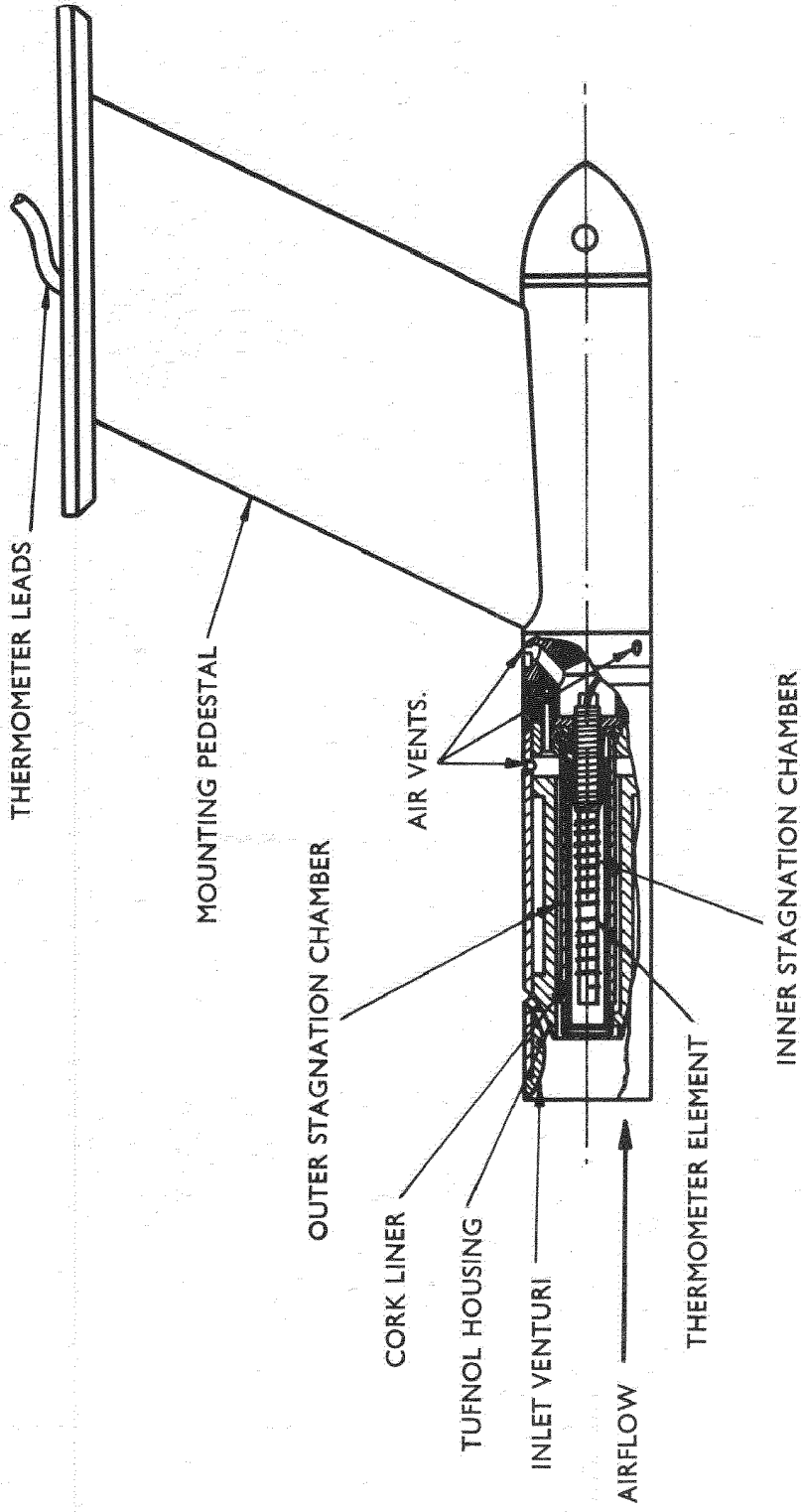
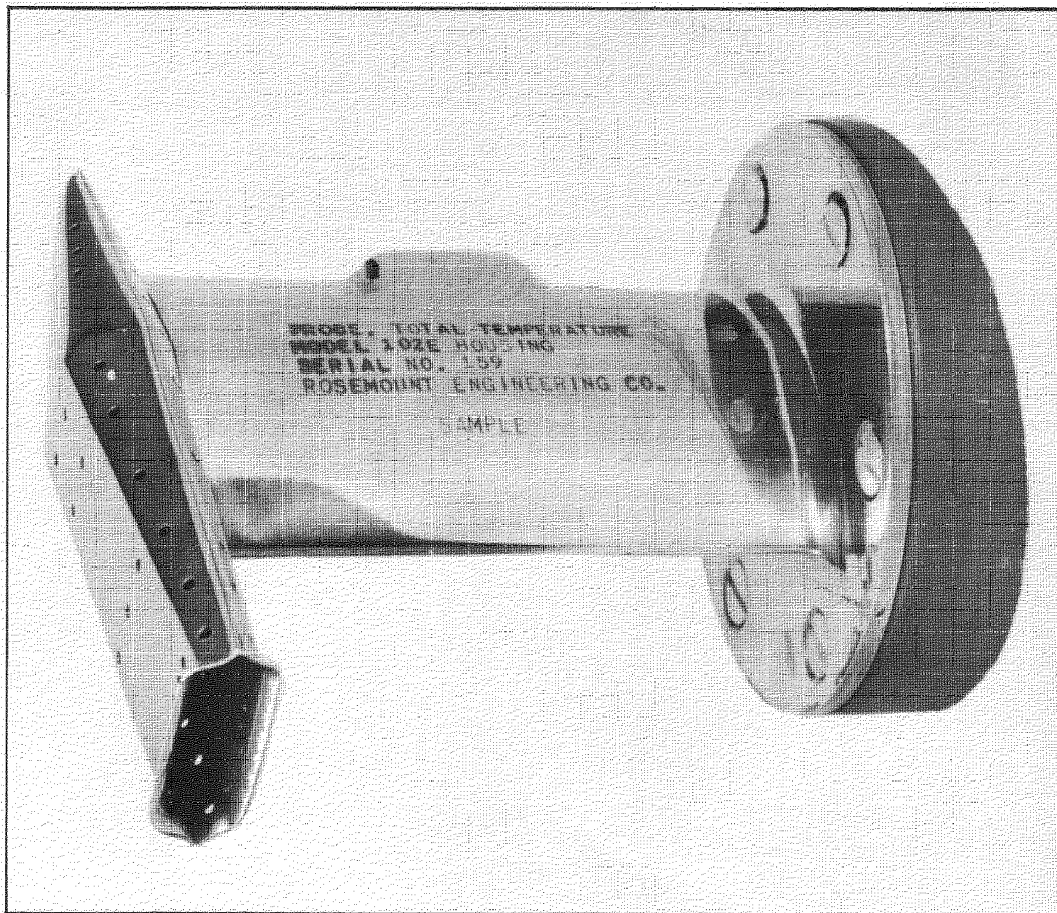
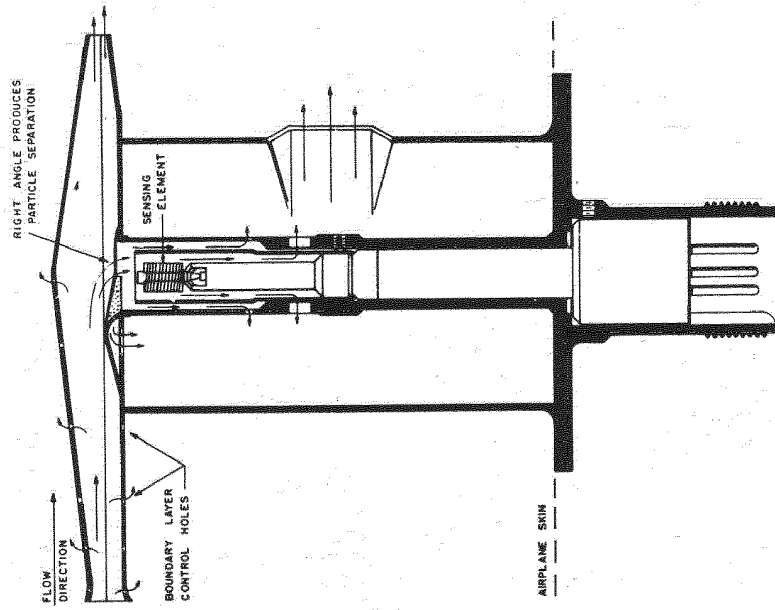


FIG. 1b. PENNY AND GILES (A. & A.E.E. TYPE)  
 THERMOMETER: INTERNAL CONFIGURATION



a. GENERAL VIEW



(REPRODUCED FROM ROSEMOUNT ENGINEERING Co. BULLETIN 7597, FIG.1)

b. INTERNAL CONFIGURATION

FIG.2. ROSEMOUNT ENGINEERING COMPANY TYPE 102E THERMOMETER

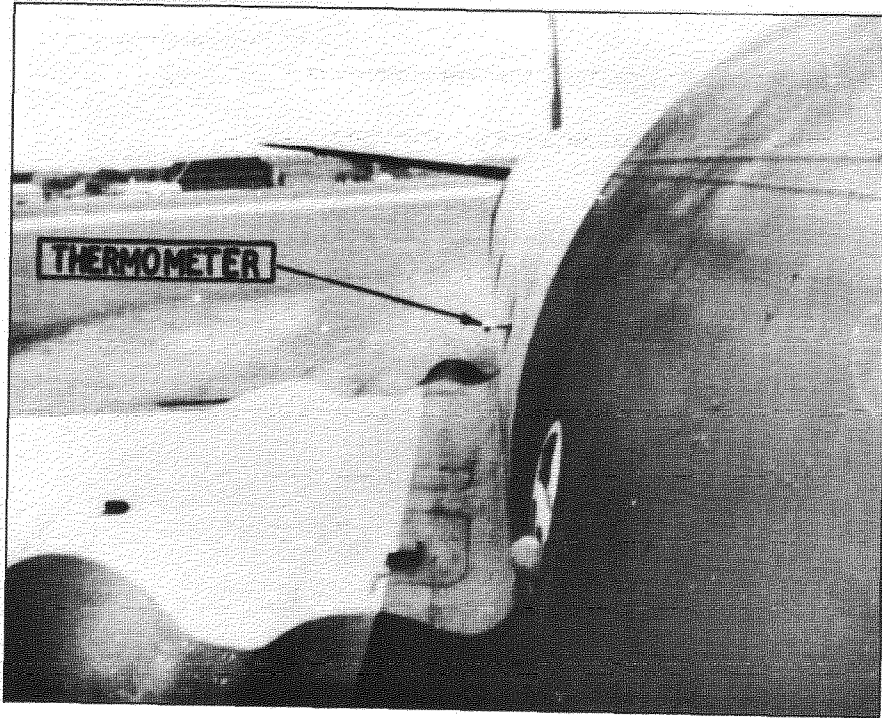


FIG.3. POSITION OF THE NICKEL WIRE THERMOMETER ON THE COMET

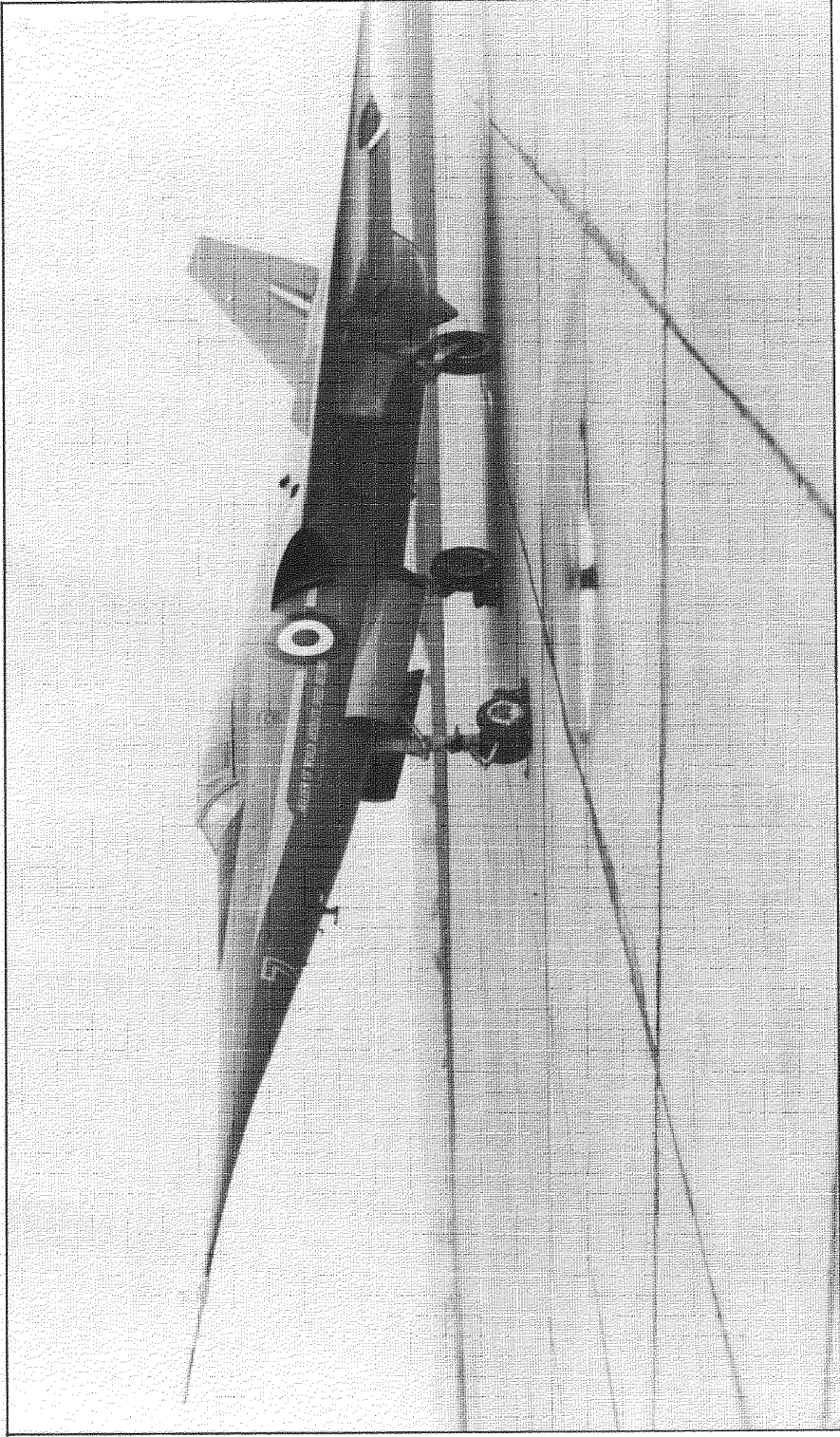
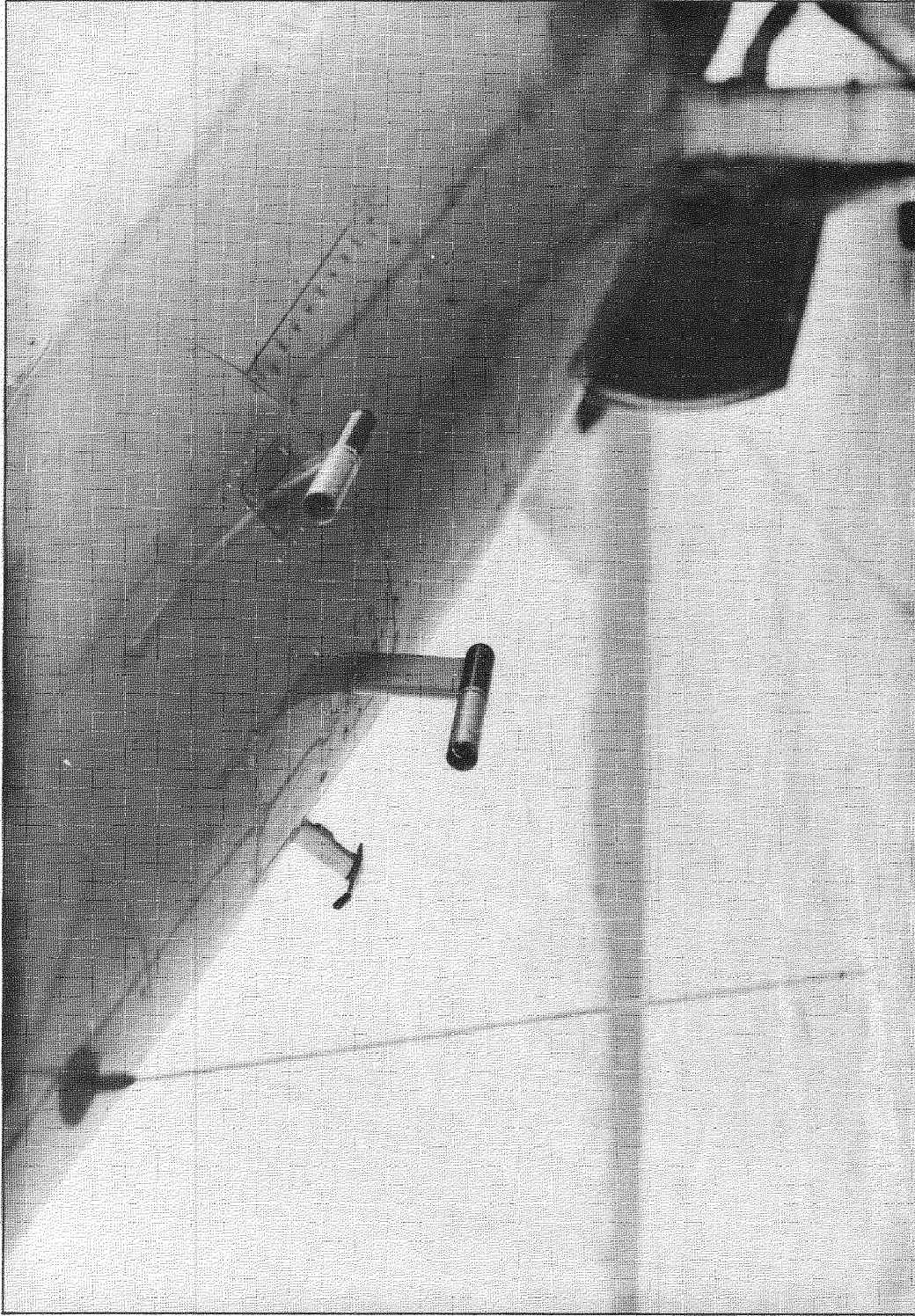


FIG.4. FAIREY DELTA 2 - WG774

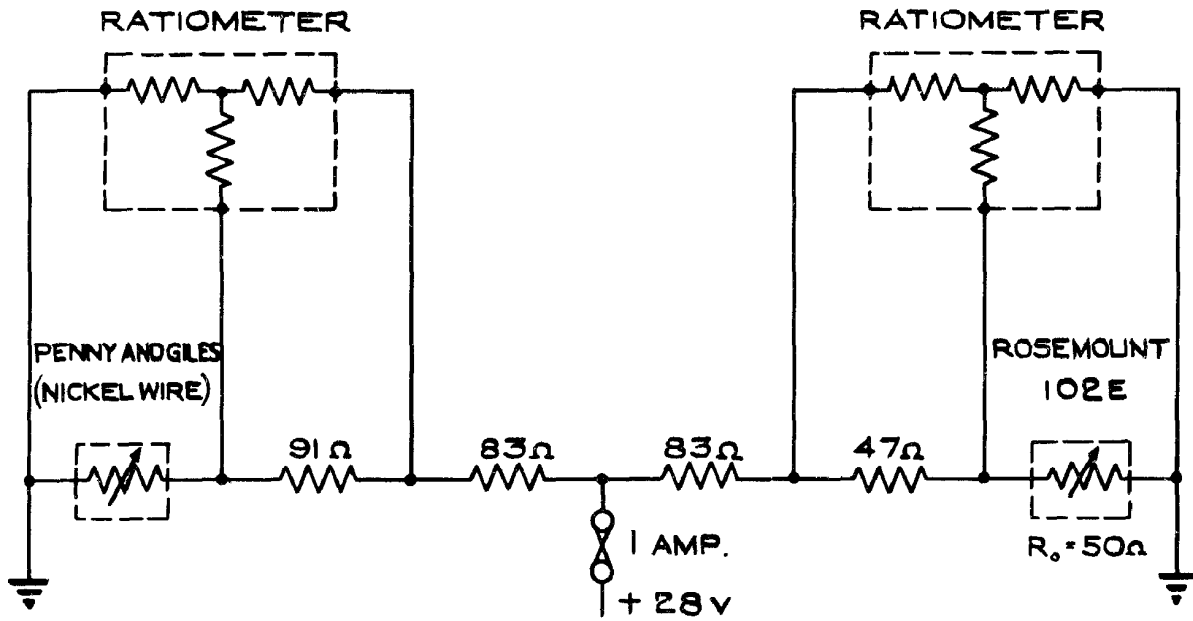


(NOTE THE WHIP AERIAL)

FROM LEFT TO RIGHT:-

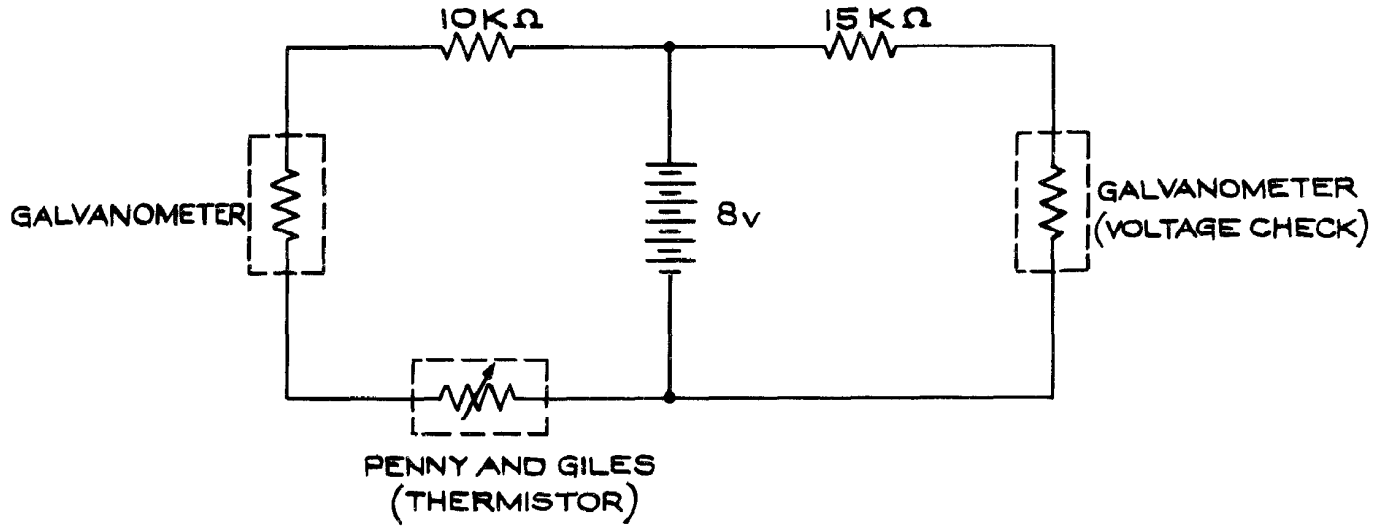
- a. ROSEMOUNT 102E (PLATINUM)
- b. PENNY AND GILES (THERMISTOR)
- c. PENNY AND GILES (NICKEL WIRE)

FIG.5. MOUNTING OF THERMOMETERS ON FAIREY DELTA 2 - WG774



N.B a) INSTRUMENTATION COMMON EARTH POINT USED.  
NOT LOCAL EARTH POINT.  
 b) CIRCUIT FOR PENNY AND GILES PROBE ON THE COMET AS THE LEFT HALF ABOVE.

(a). ROSEMOUNT AND PENNY AND GILES PROBE CIRCUITS.  
 (WIRE TYPES.)



(b). PENNY AND GILES THERMISTOR CIRCUIT.  
 (WITH VOLTAGE MONITORING CIRCUIT.)

FIG.6. THERMOMETERS - WIRING DIAGRAM (FAIREY DELTA 2.)



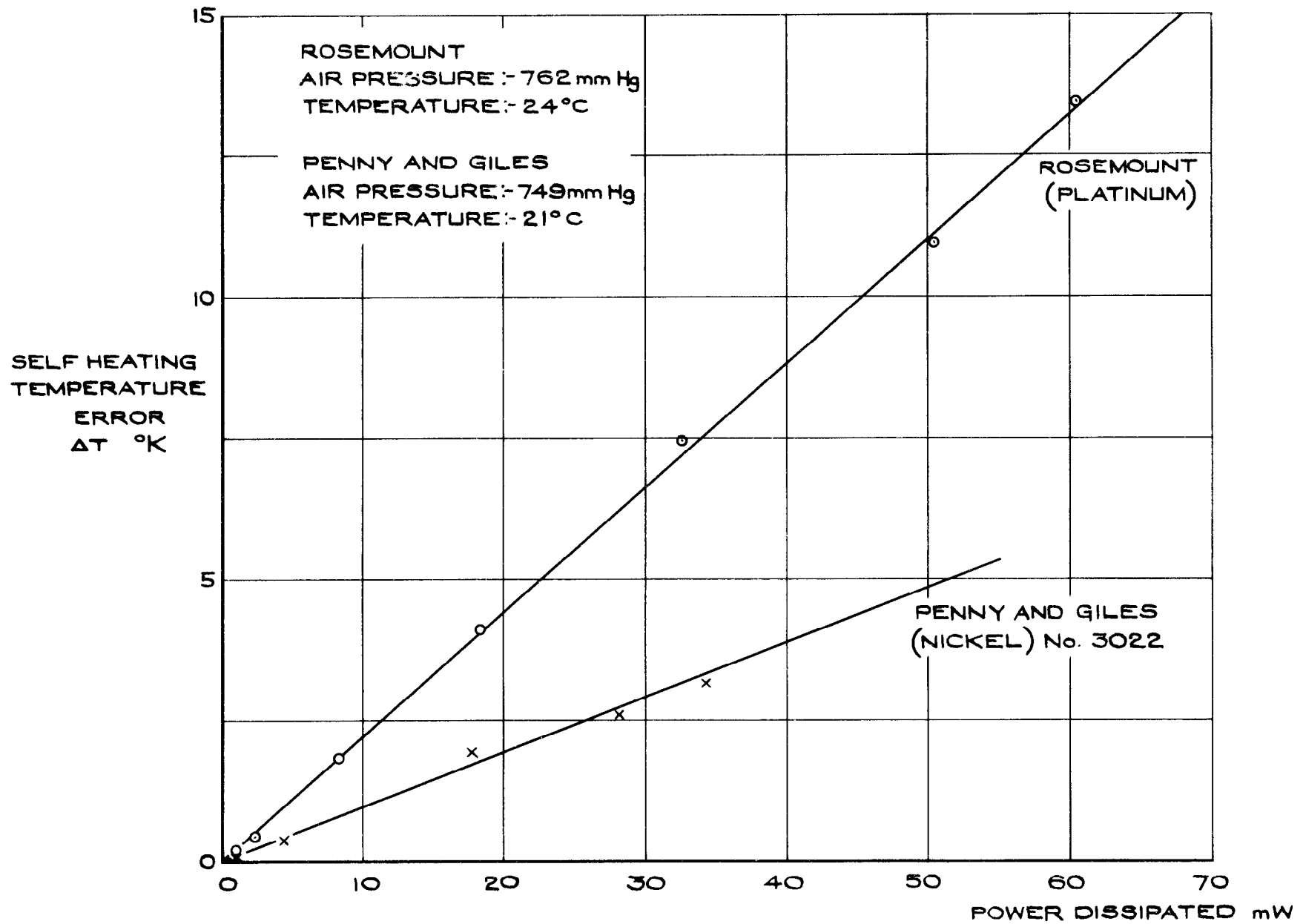


FIG. 7. (d) STILL AIR SELF HEATING :-  
ROSEMOUNT (PLATINUM) AND PENNY AND GILES (NICKEL) THERMOMETERS.

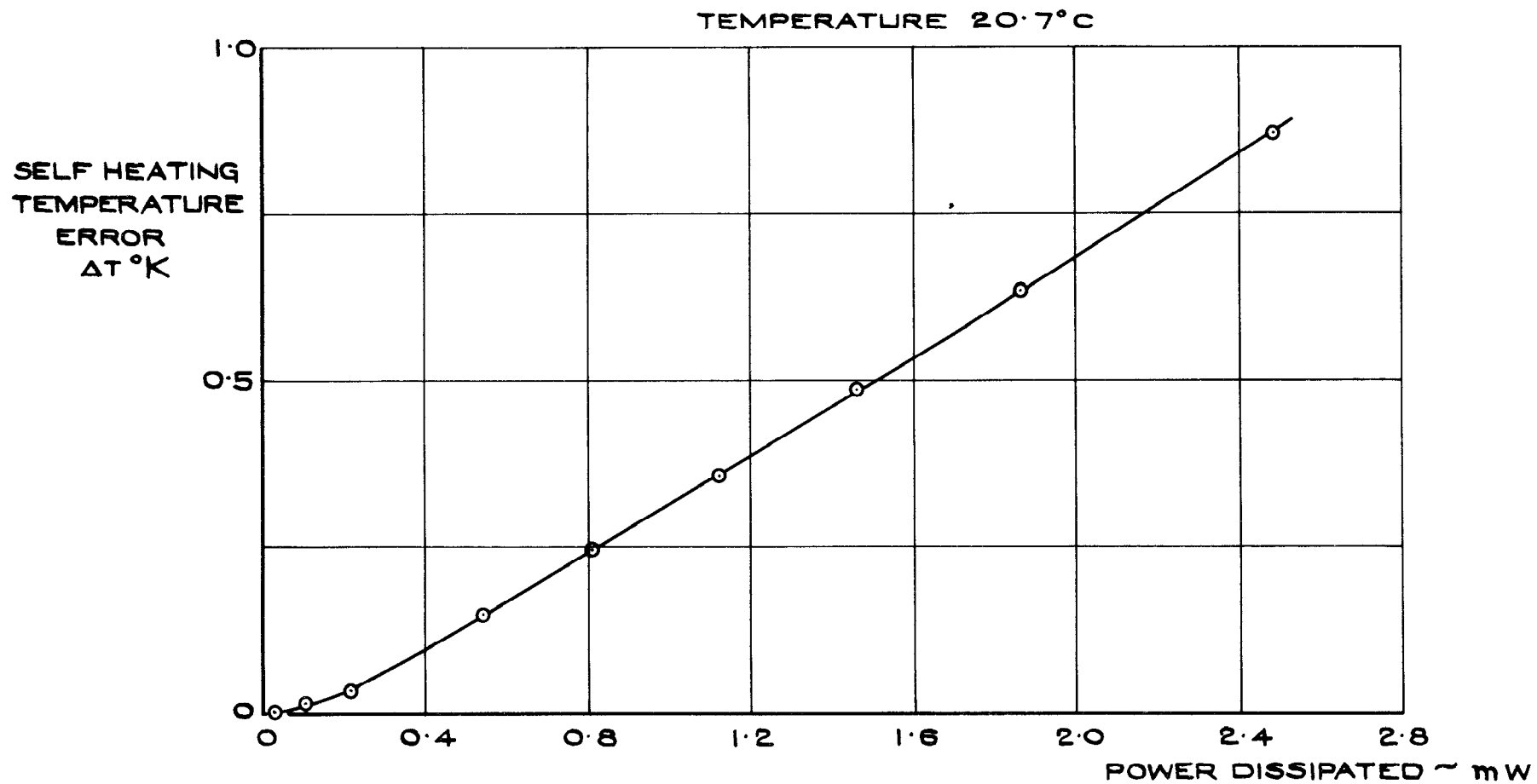


FIG. 7.(b). STILL AIR SELF HEATING:-  
PENNY AND GILES (THERMISTOR) THERMOMETER.



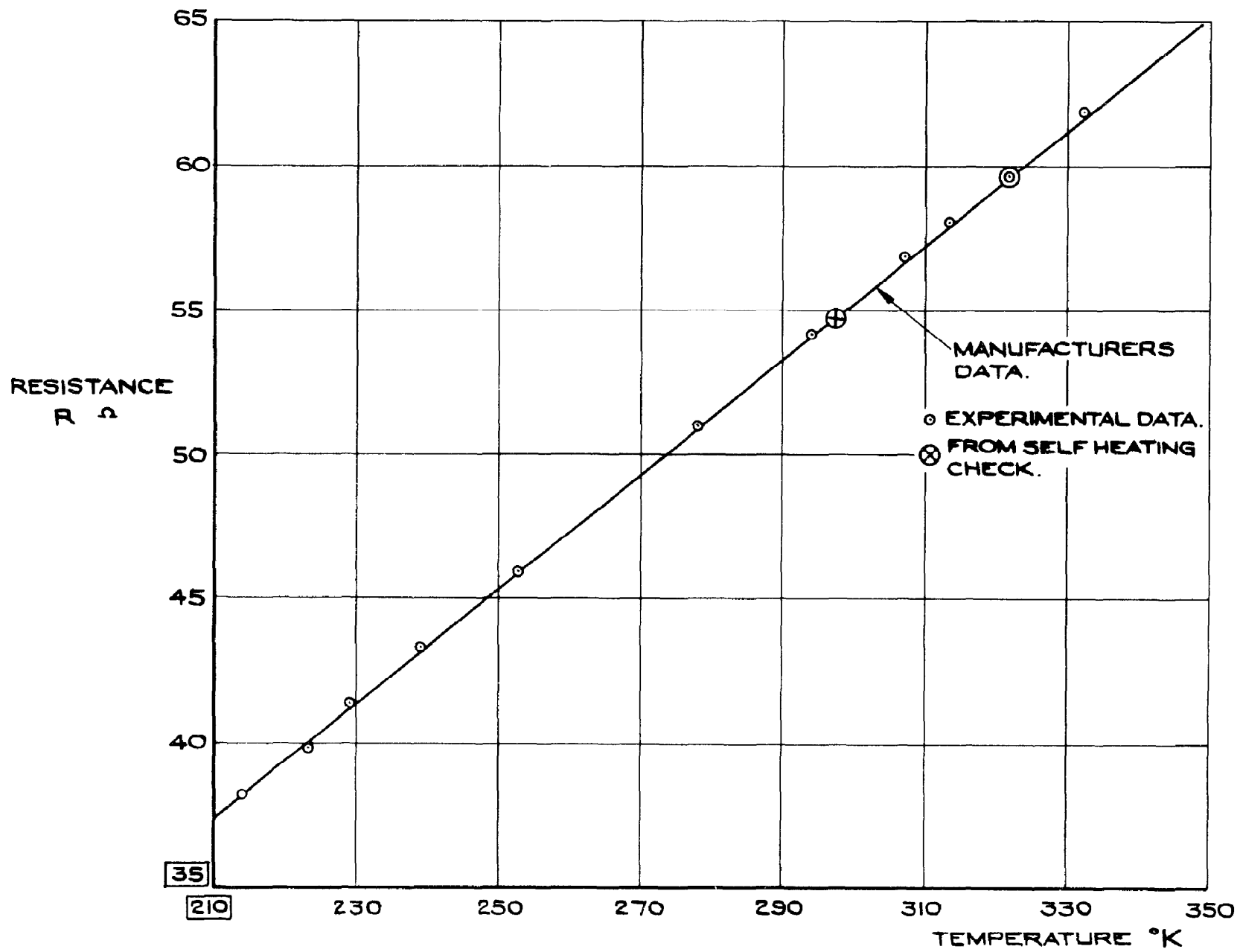


FIG. 8.(a). RESISTANCE VERSUS TEMPERATURE CALIBRATIONS:-  
ROSEMOUNT (PLATINUM) THERMOMETER.

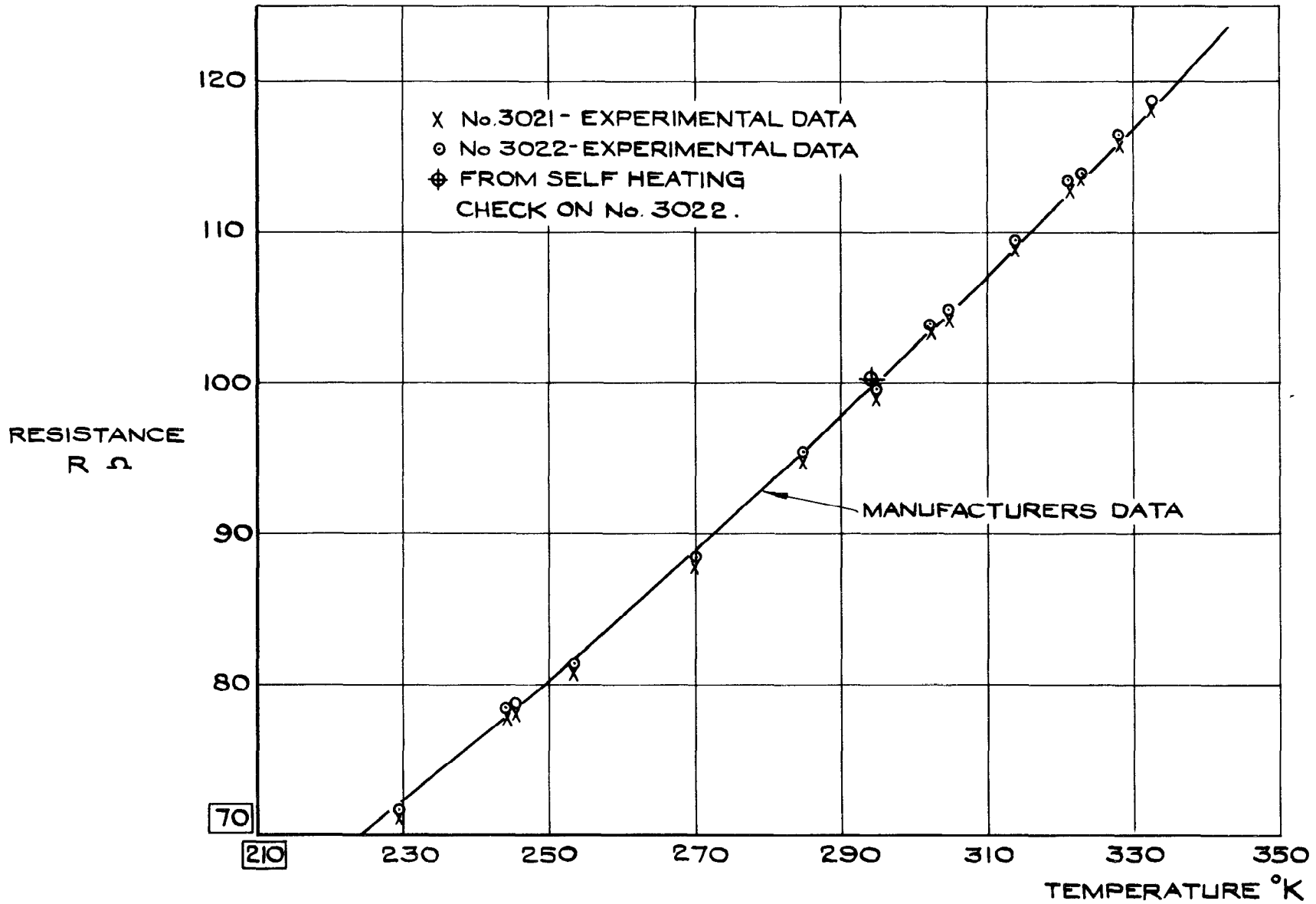


FIG. 8.(b) RESISTANCE VERSUS TEMPERATURE CALIBRATIONS:-  
PENNY AND GILES (NICKEL) THERMOMETER.

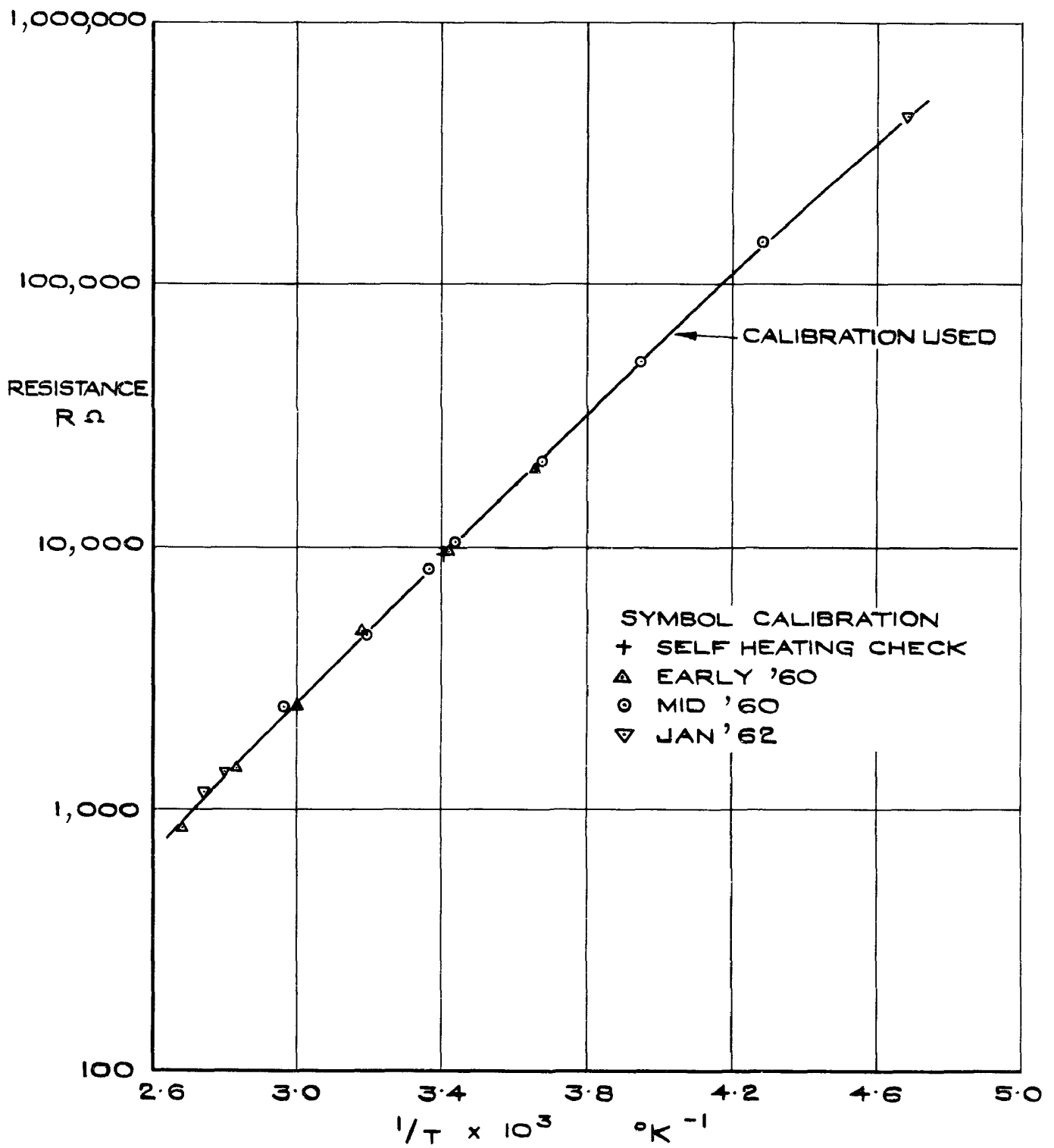


FIG.8(c) RESISTANCE VERSUS TEMPERATURE CALIBRATIONS:-  
PENNY AND GILES THERMISTOR THERMOMETER.

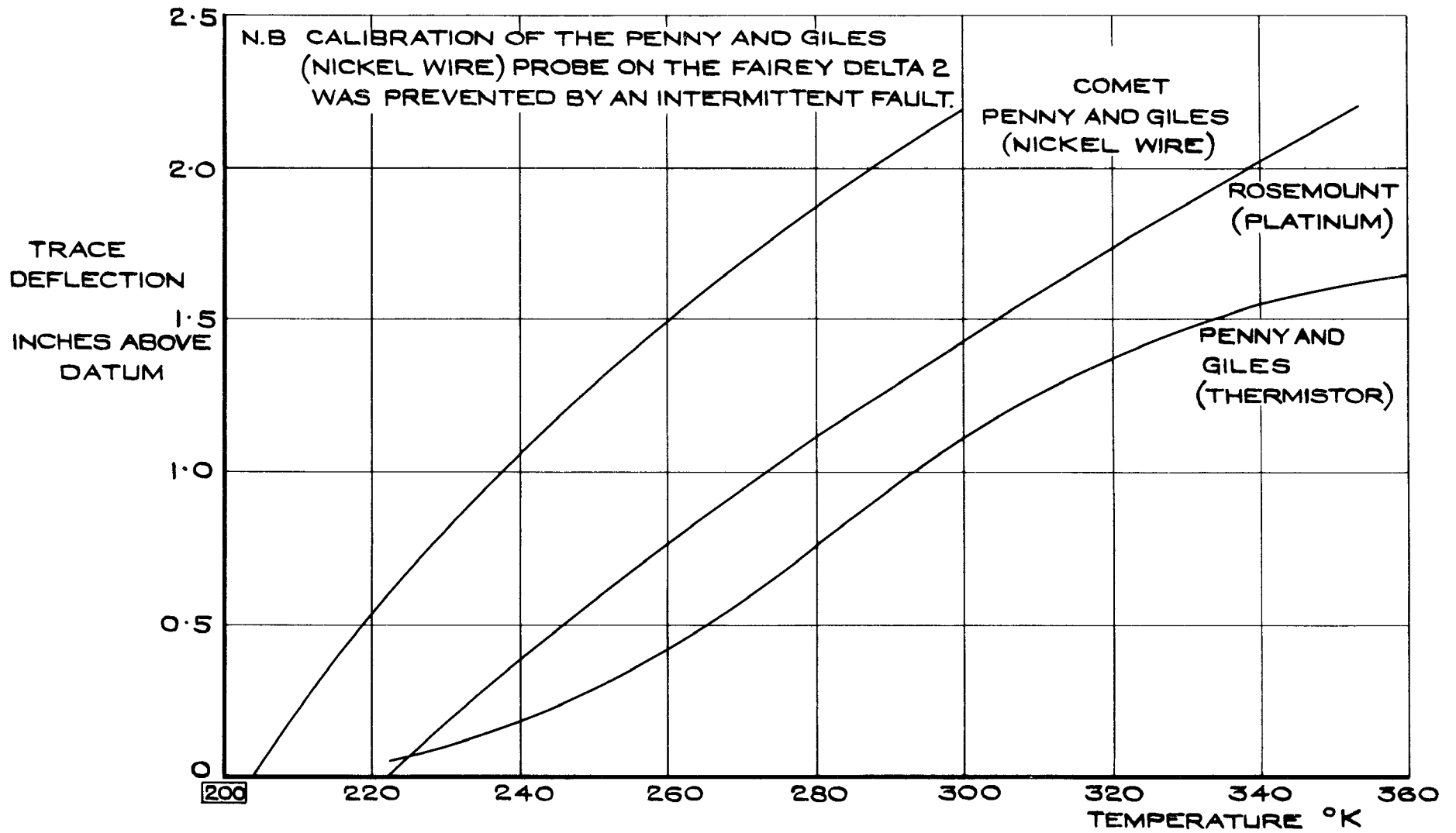


FIG.9. FINAL CALIBRATION:-TRACE DEFLECTION VERSUS TEMPERATURE.

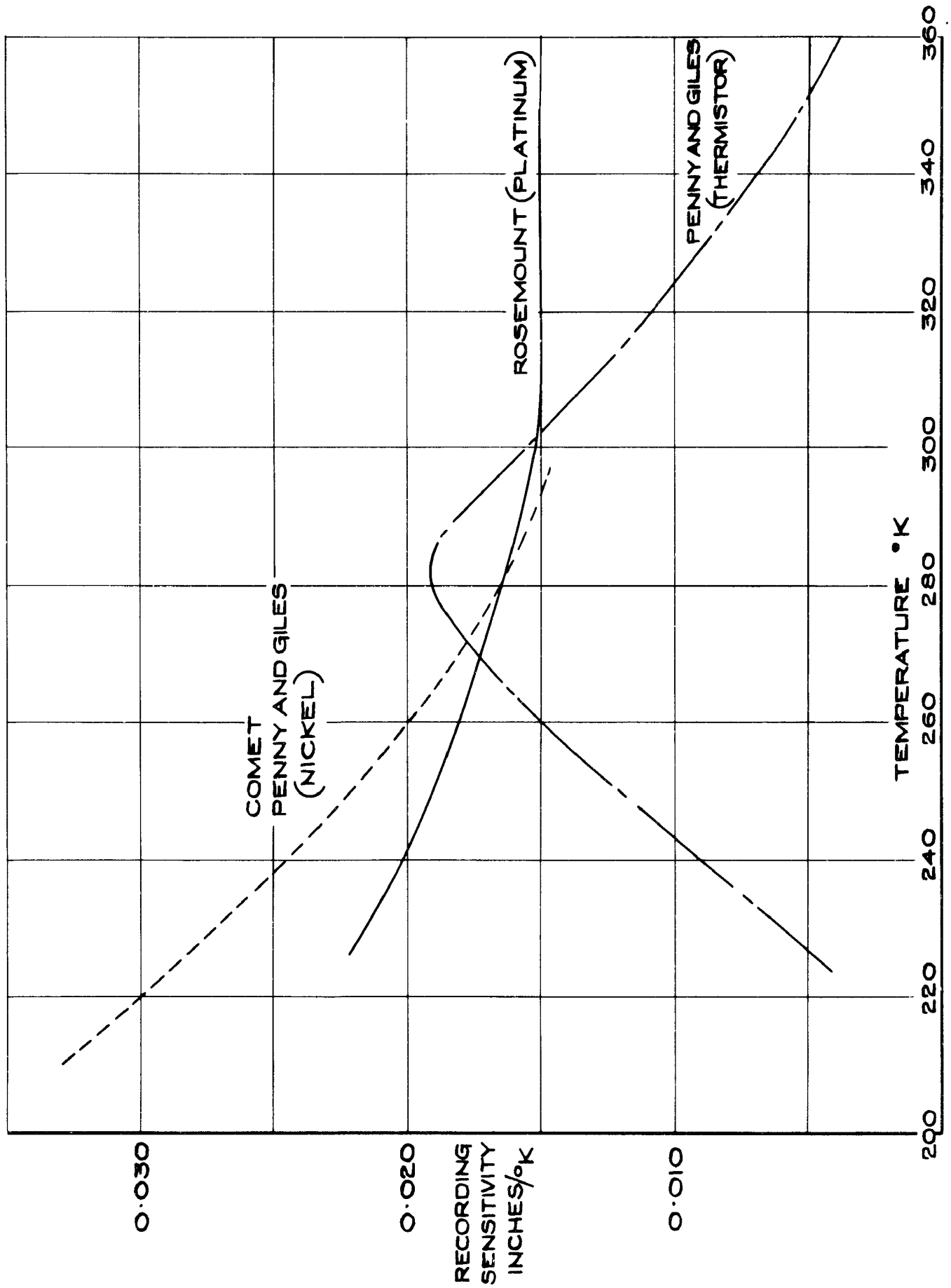


FIG. 10. RECORDING SENSITIVITY.

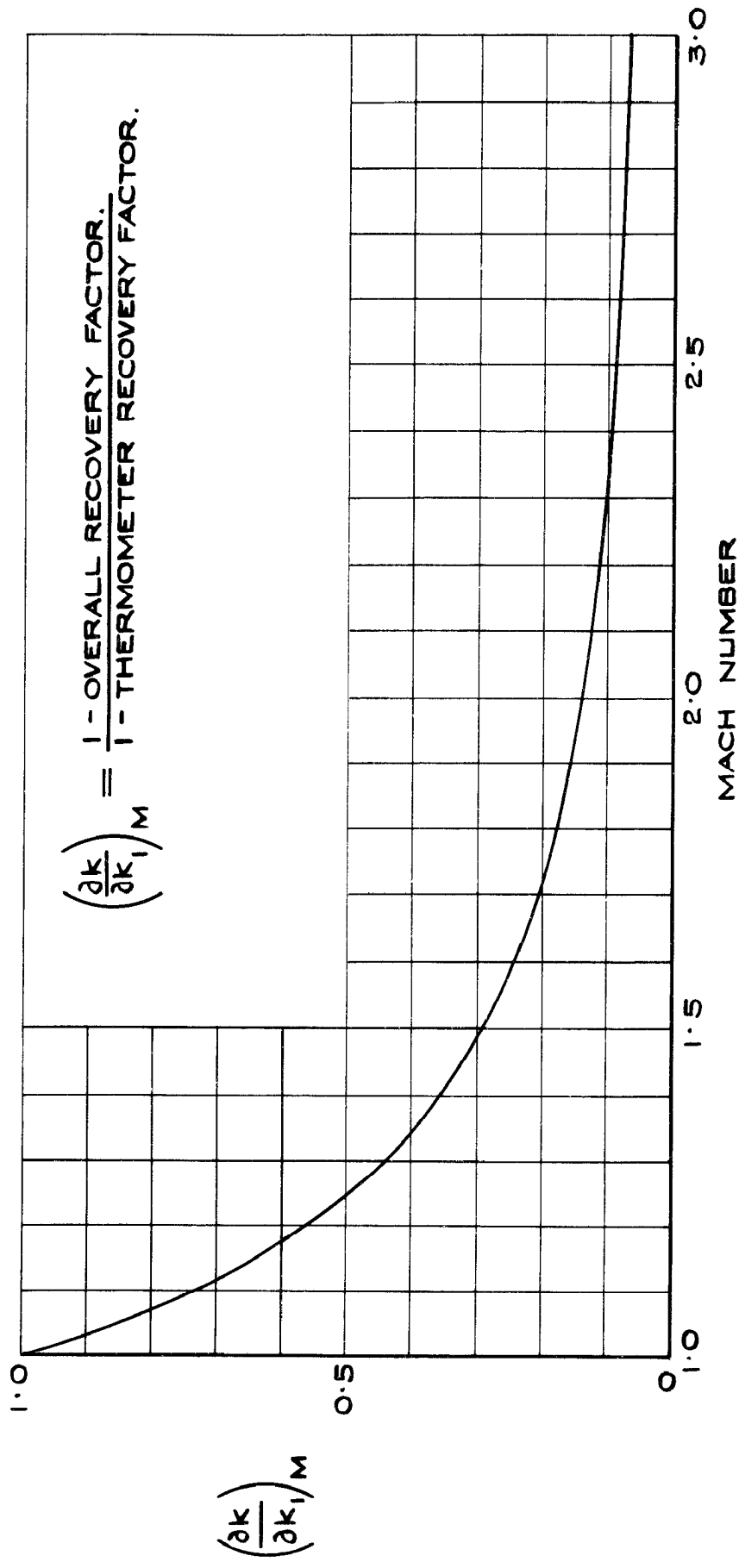


FIG. II. VARIATION OF  $\left(\frac{\partial k}{\partial k_1}\right)_M$  WITH MACH NUMBER ( $\gamma = 1.40$ )

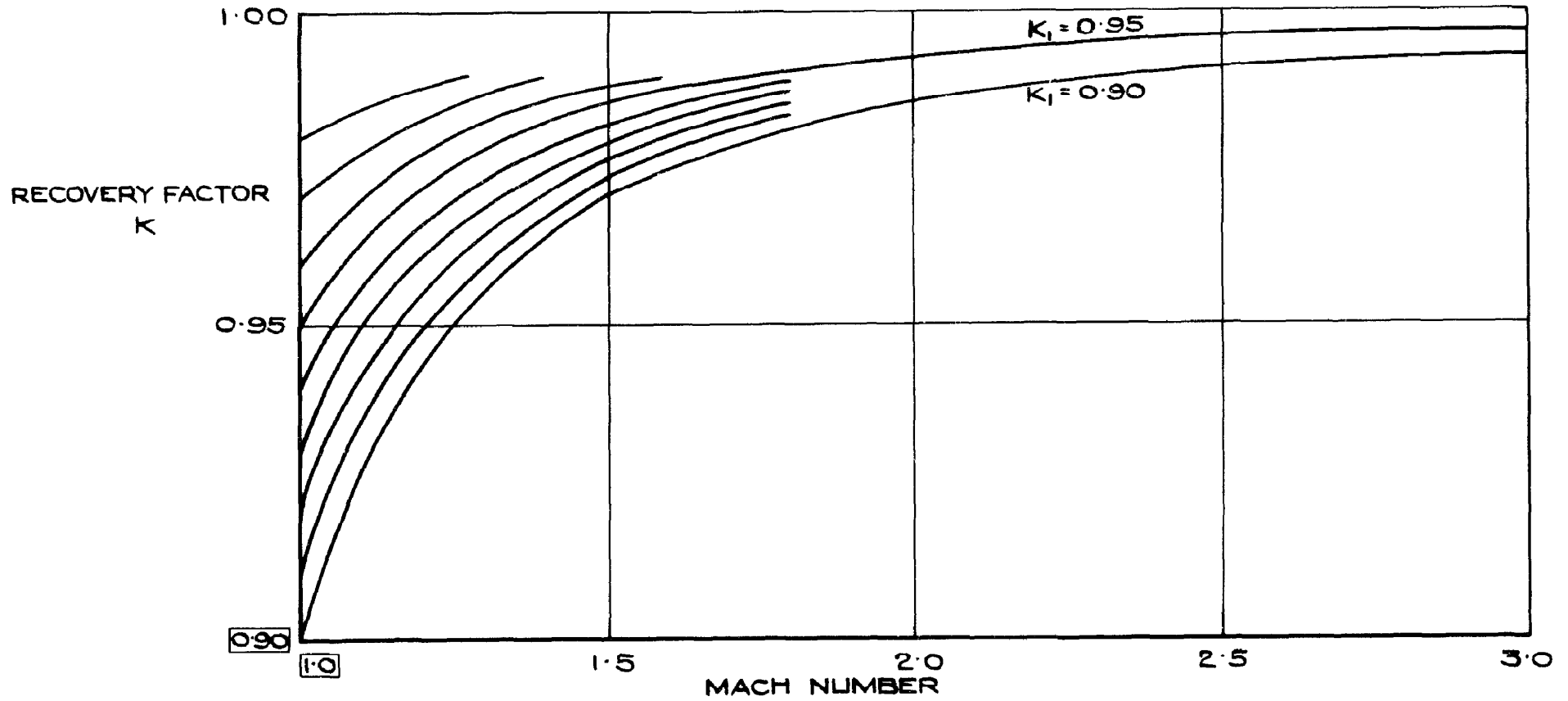


FIG.12. VARIATION OF RECOVERY FACTOR,  $k$ , WITH MACH NUMBER AT CONSTANT THERMOMETER RECOVERY FACTOR,  $K_1$  ( $\gamma = 1.40$ ).

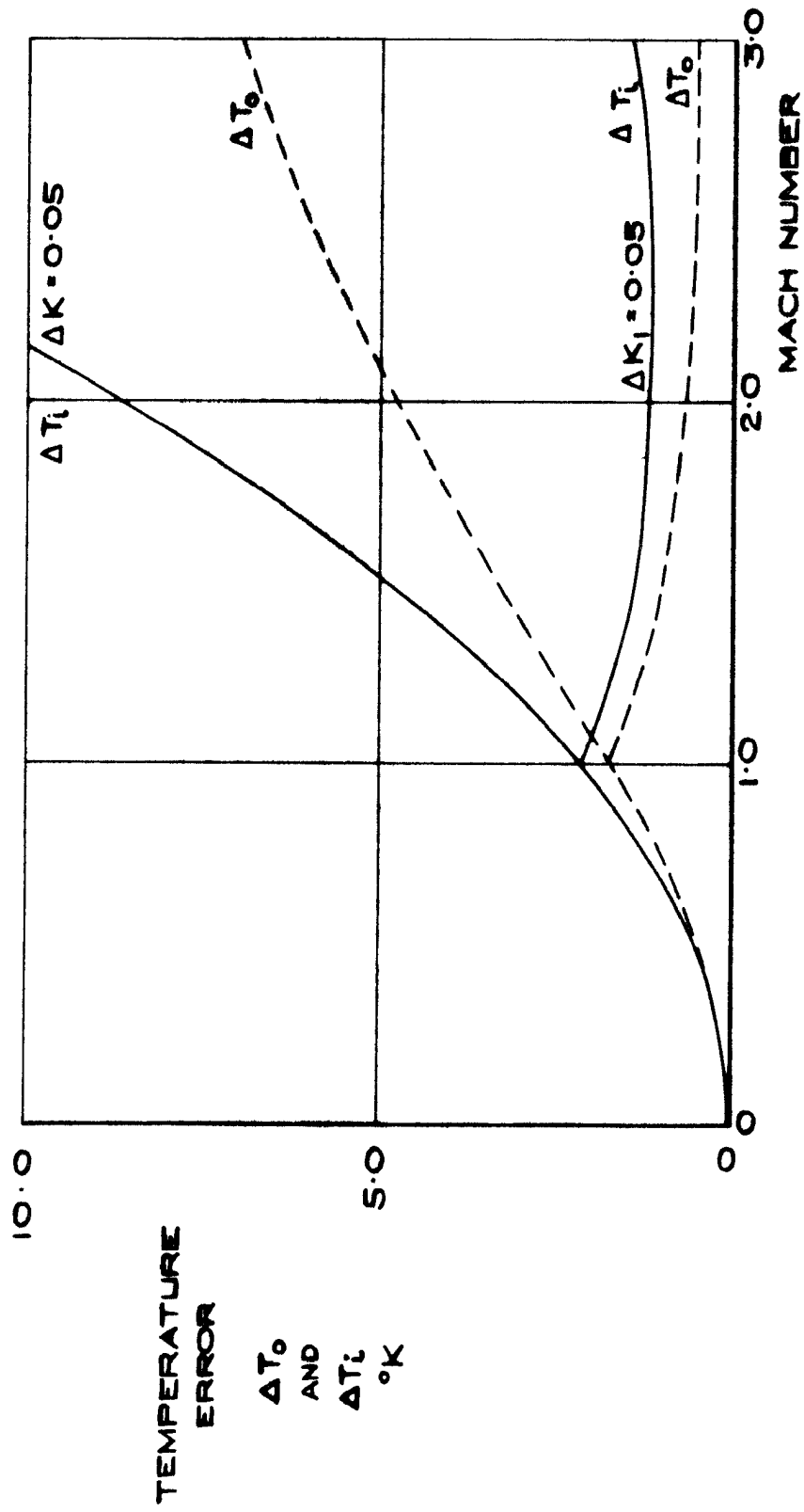


FIG.13. TEMPERATURE ERROR DUE TO ASSUMING  $K_1$  OR  $K=1.0$  WHEN  $K_1$  OR  $K=0.95$  ( $T_0=216$  °K)



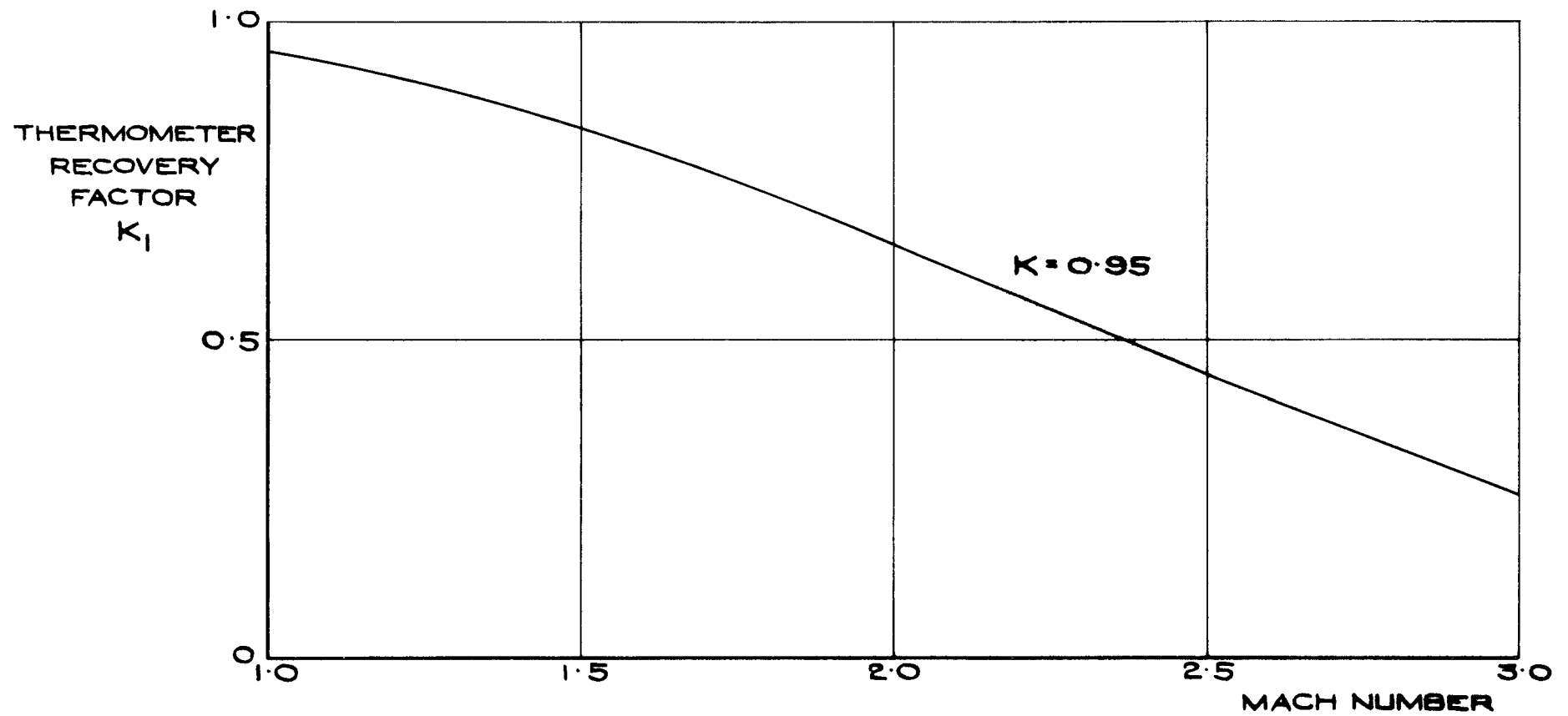


FIG.14. VARIATION OF  $K_1$  WITH MACH NUMBER FOR  $K=0.95$

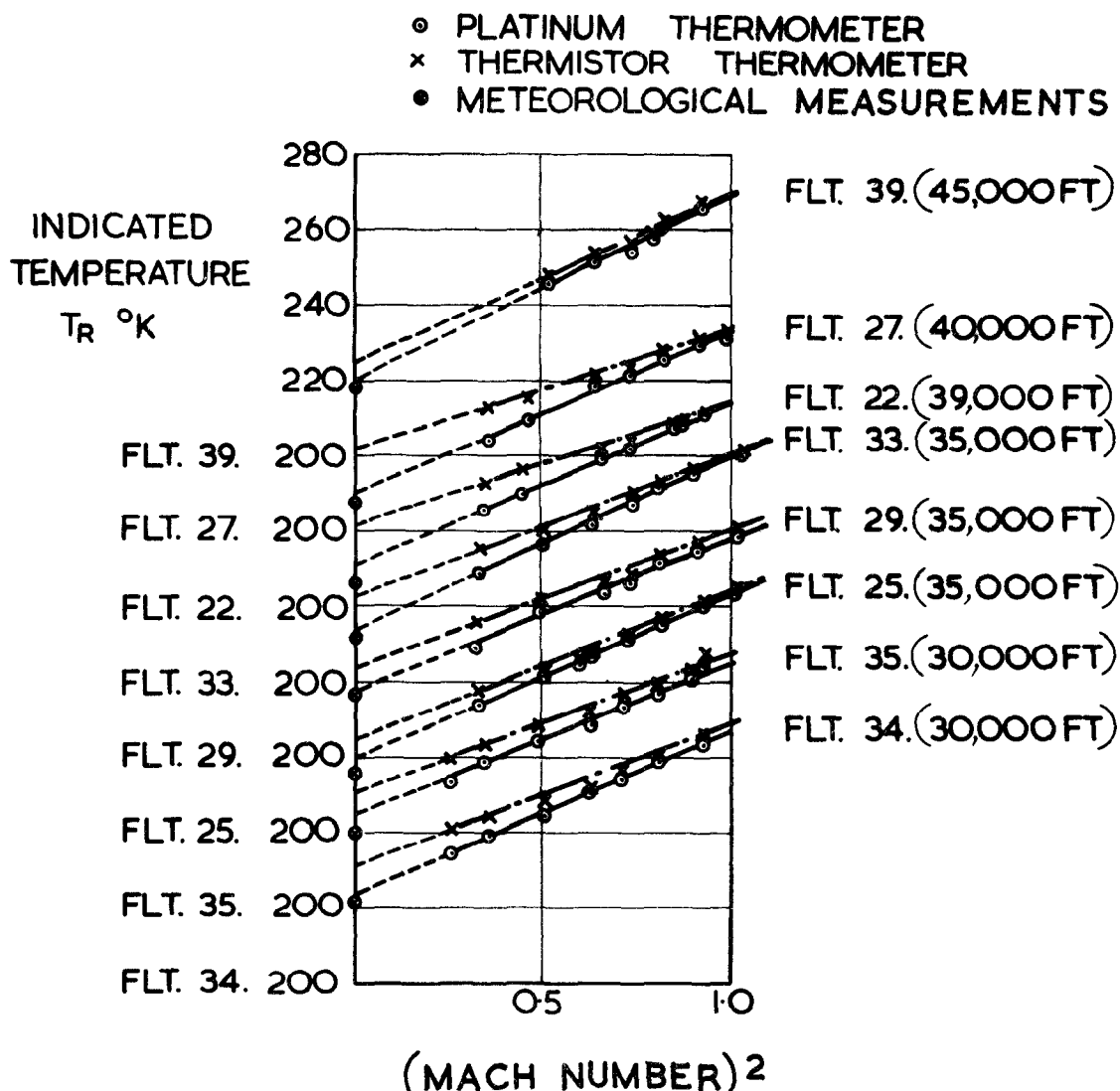
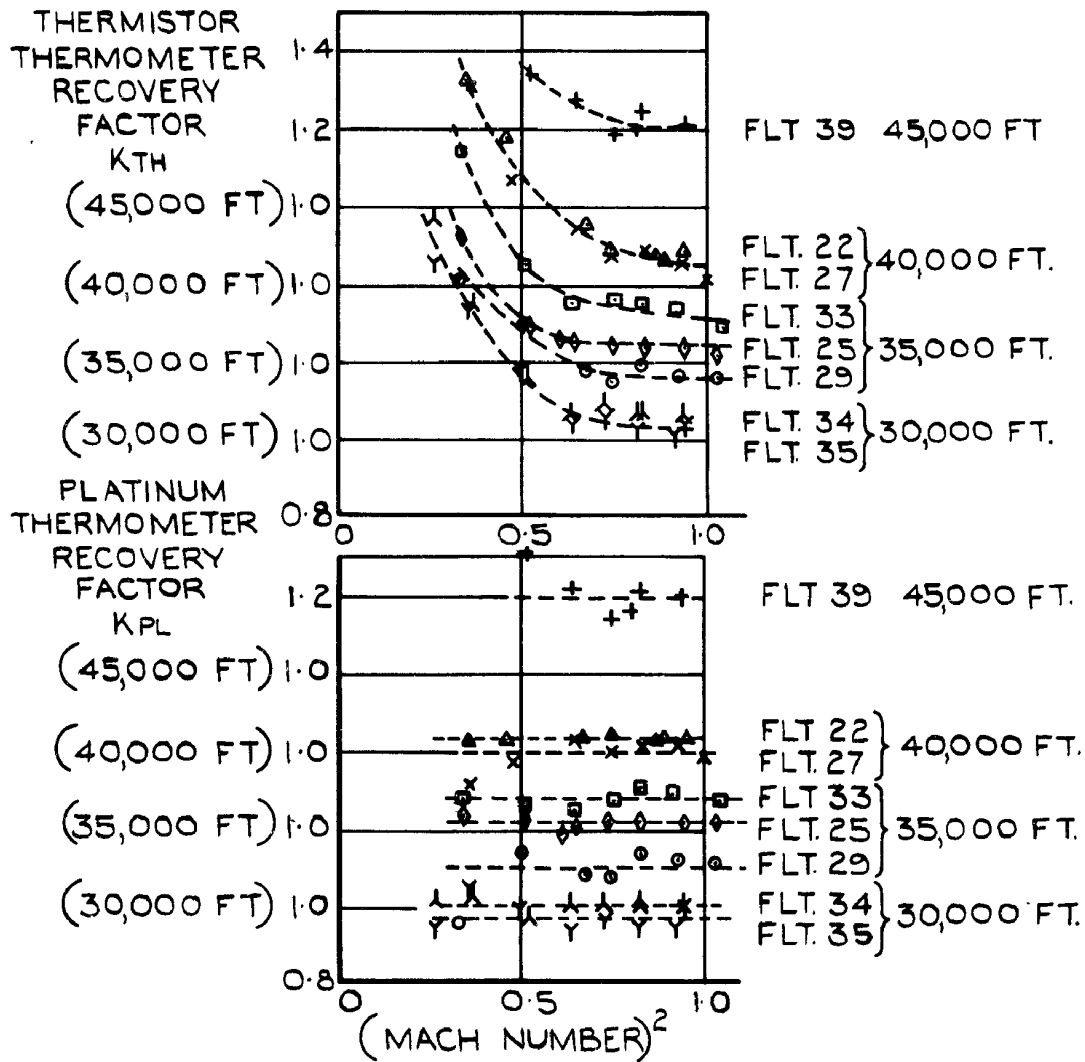
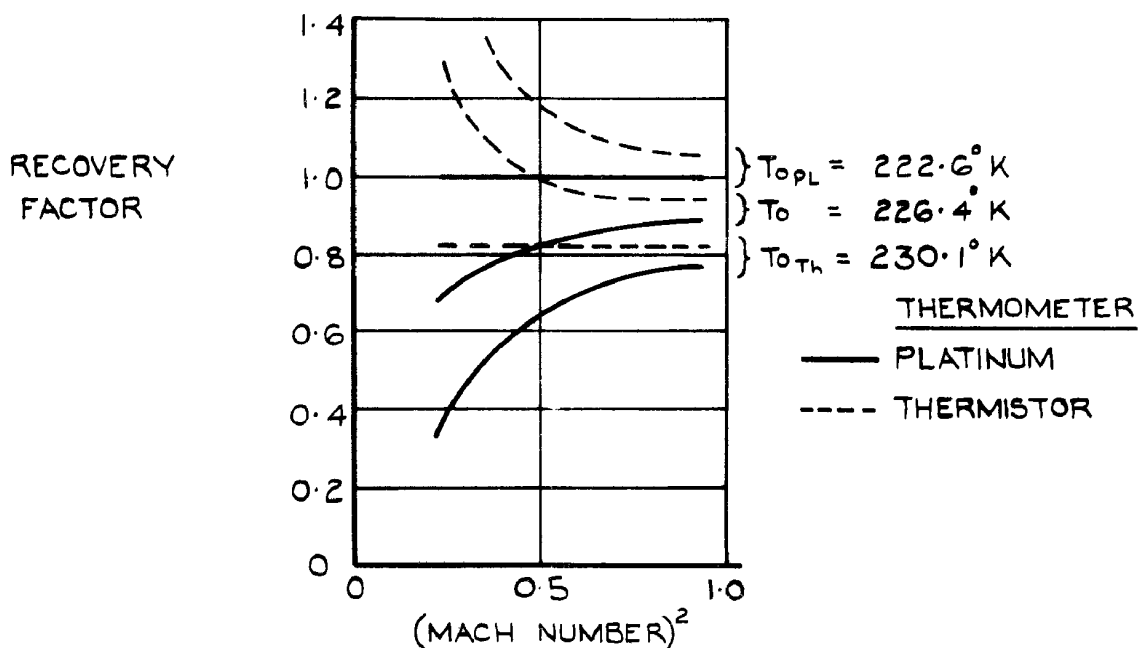


FIG.15. VARIATION OF INDICATED TEMPERATURE  
 WITH (MACH NUMBER)<sup>2</sup> AT SUBSONIC  
 SPEEDS.



(a) USING  $T_{oPL}$



(b) USING VARIOUS  $T_o$  VALUES FLT. 34.

FIG. 16. VARIATION OF RECOVERY FACTORS IN THE SUBSONIC TESTS

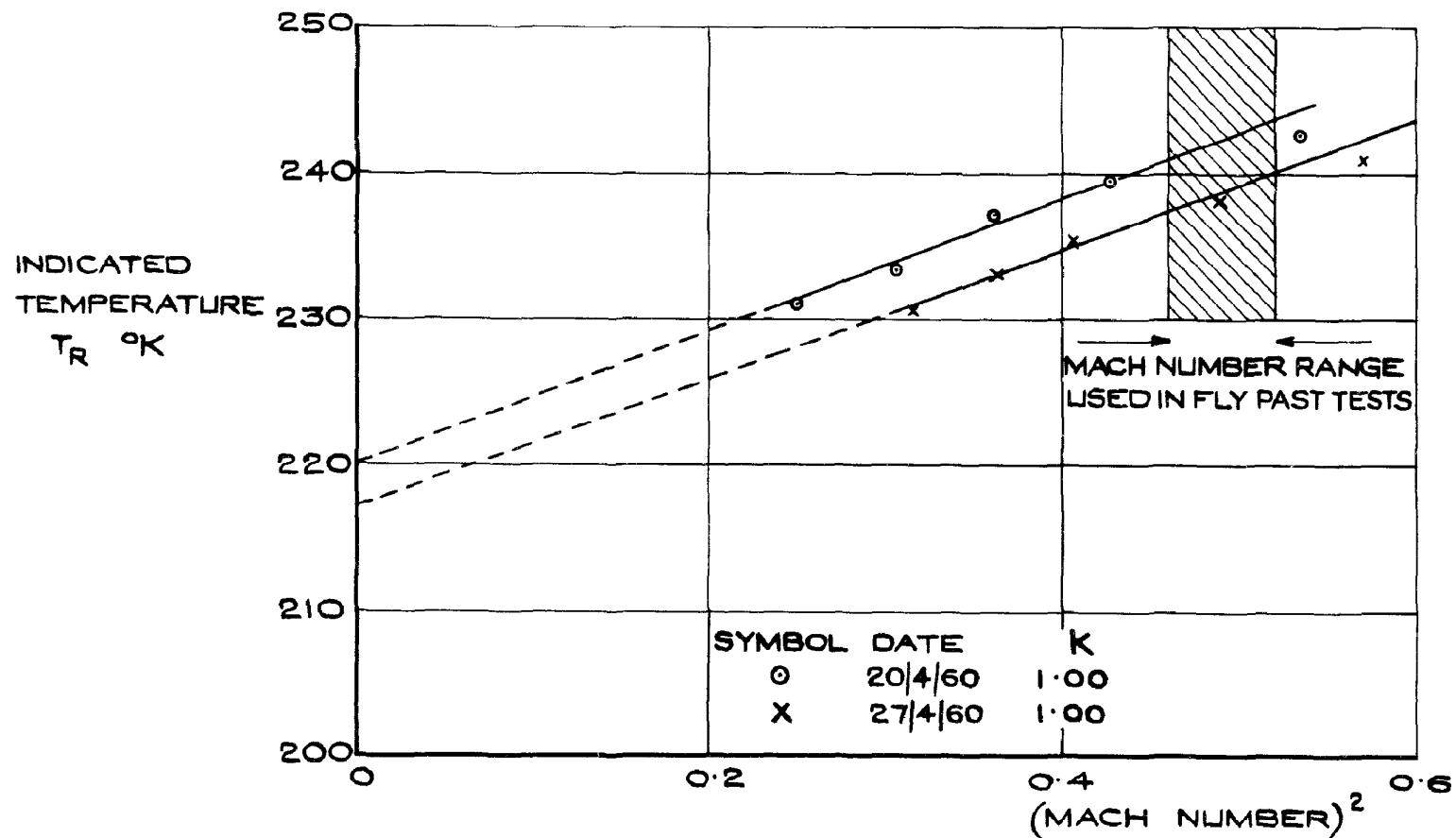


FIG. 17. RECOVERY FACTOR CALIBRATIONS OF THE NICKEL WIRE THERMOMETER ON THE COMET (35,000 FT.)

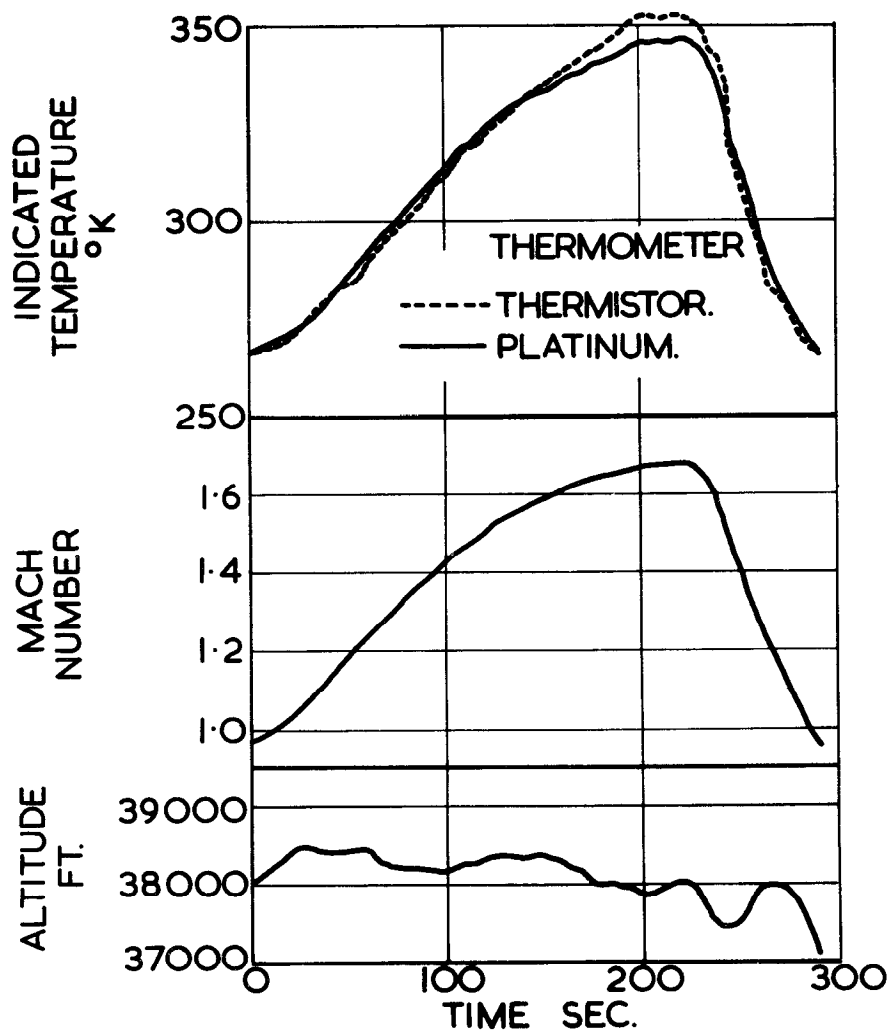


FIG. 18. VARIATION OF THE BASIC QUANTITIES DURING A TYPICAL SUPERSONIC FLIGHT (FLT. 38.)

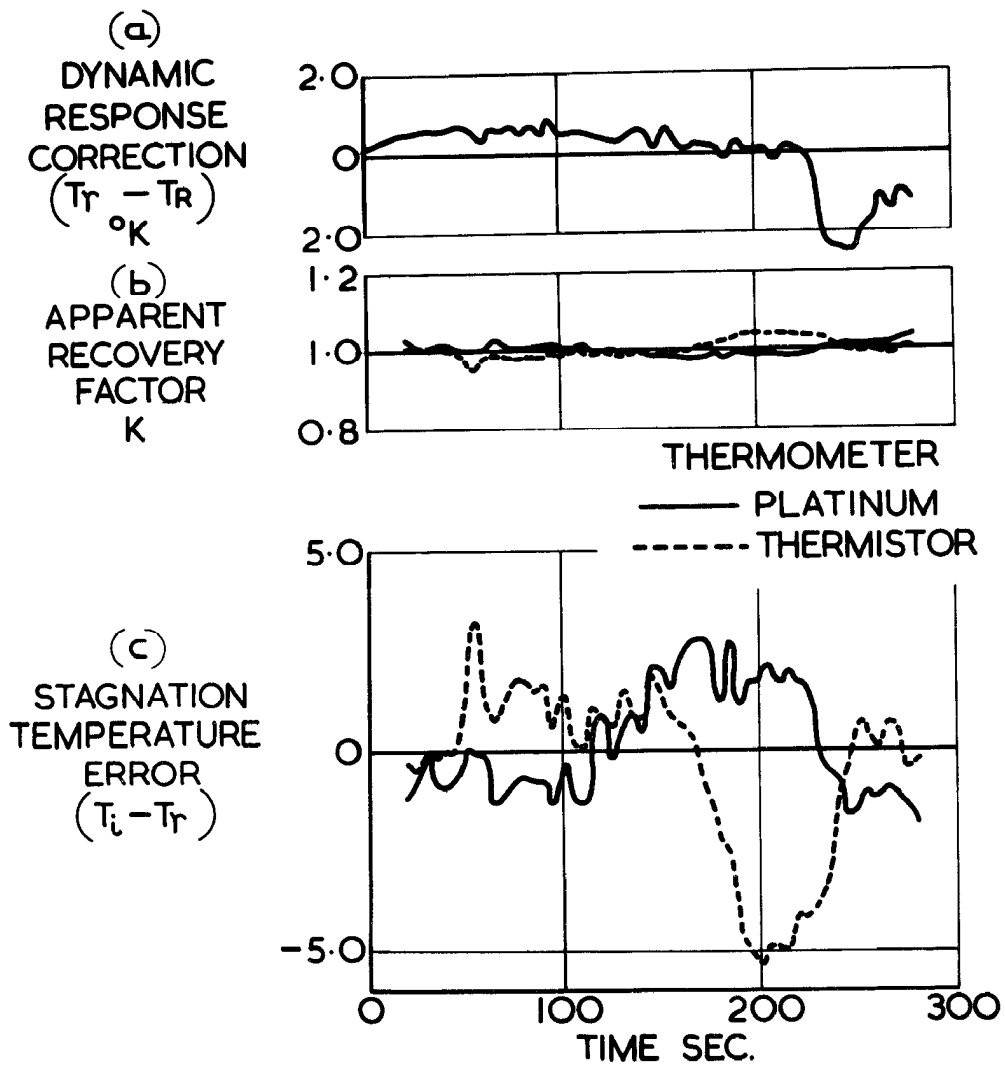


FIG. 19. FLT. 38. VARIATION WITH TIME OF  
 (a) DYNAMIC RESPONSE CORRECTION  $(T_r - T_R)$   
 (b) APPARENT RECOVERY FACTOR, K  
 (c) STAGNATION TEMPERATURE ERROR  $(T_i - T_r)$

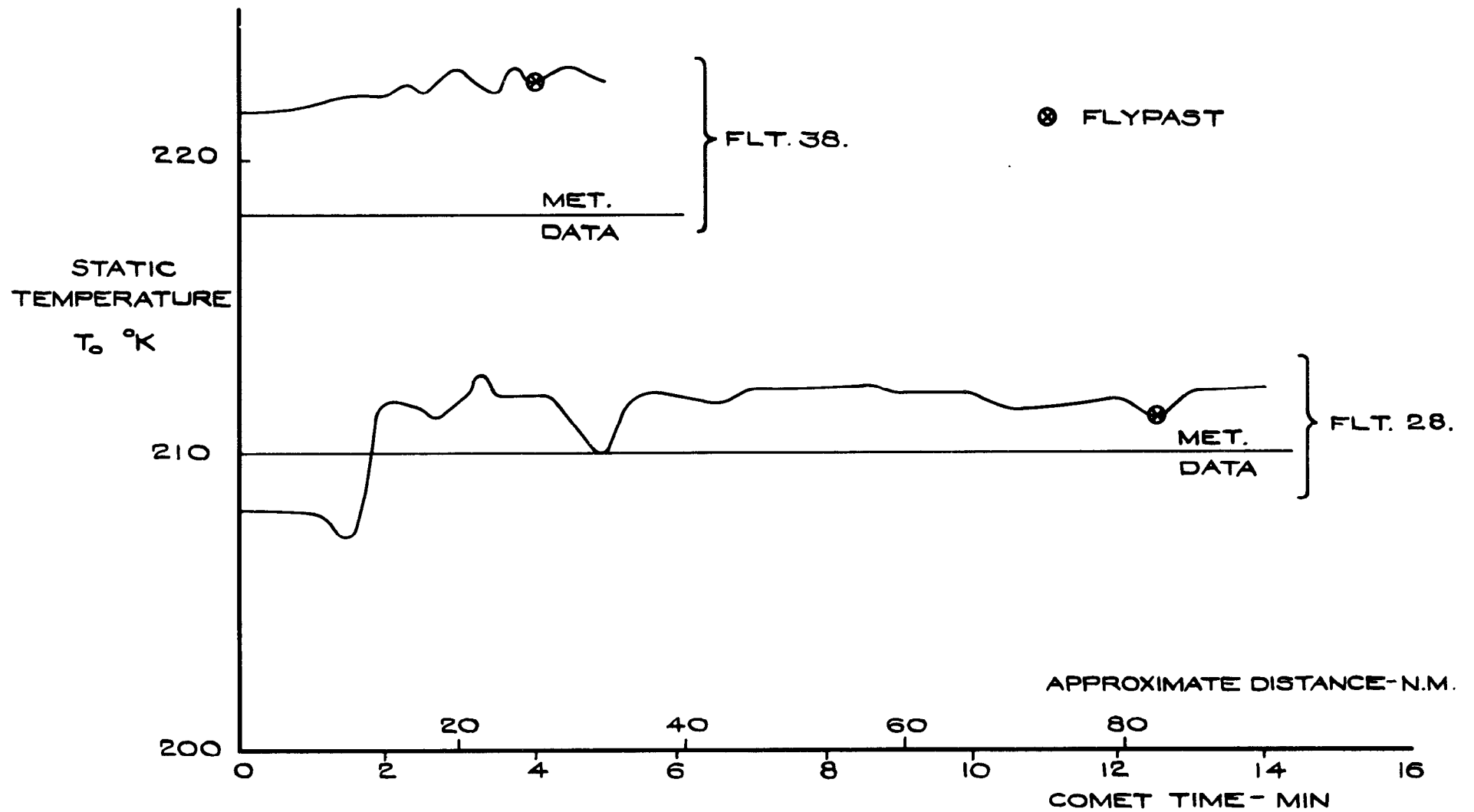


FIG.20. VARIATION OF COMET STATIC TEMPERATURE WITH TIME DURING FLTS, 28 AND 38, ASSUMING COMET SPEED AND HEIGHT CONSTANT.

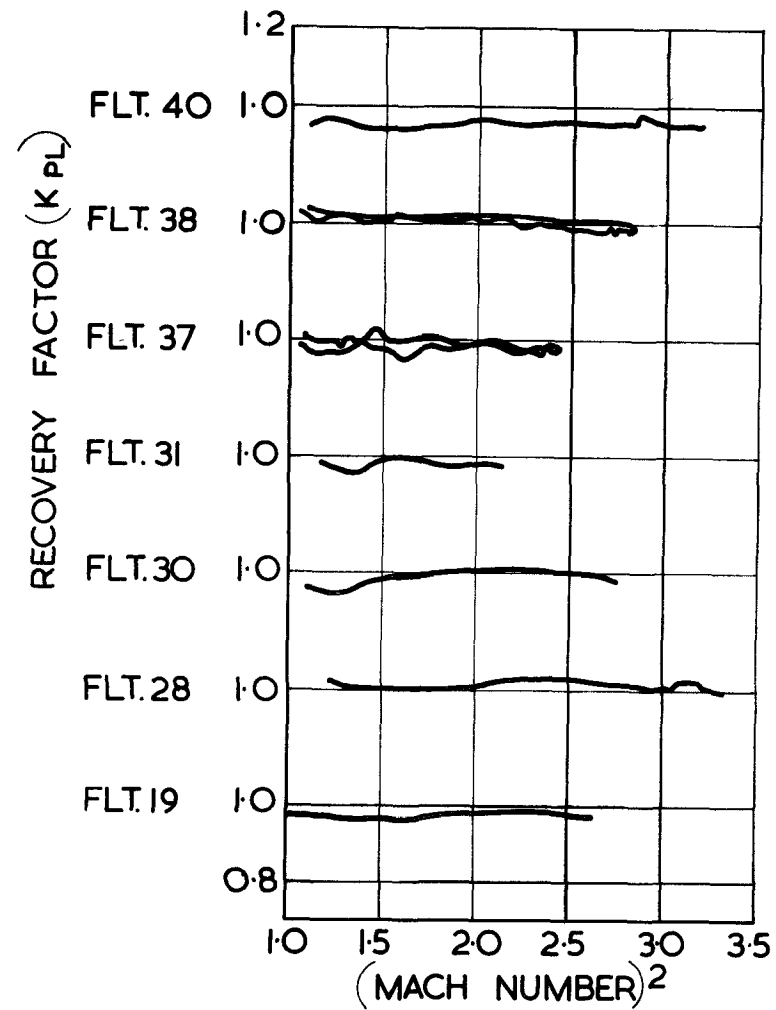


FIG. 21. VARIATION OF THE PLATINUM THERMOMETER RECOVERY FACTOR,  $K_{PL}$ , WITH  $(MACH\ NUMBER)^2$  AT SUPERSONIC SPEEDS.



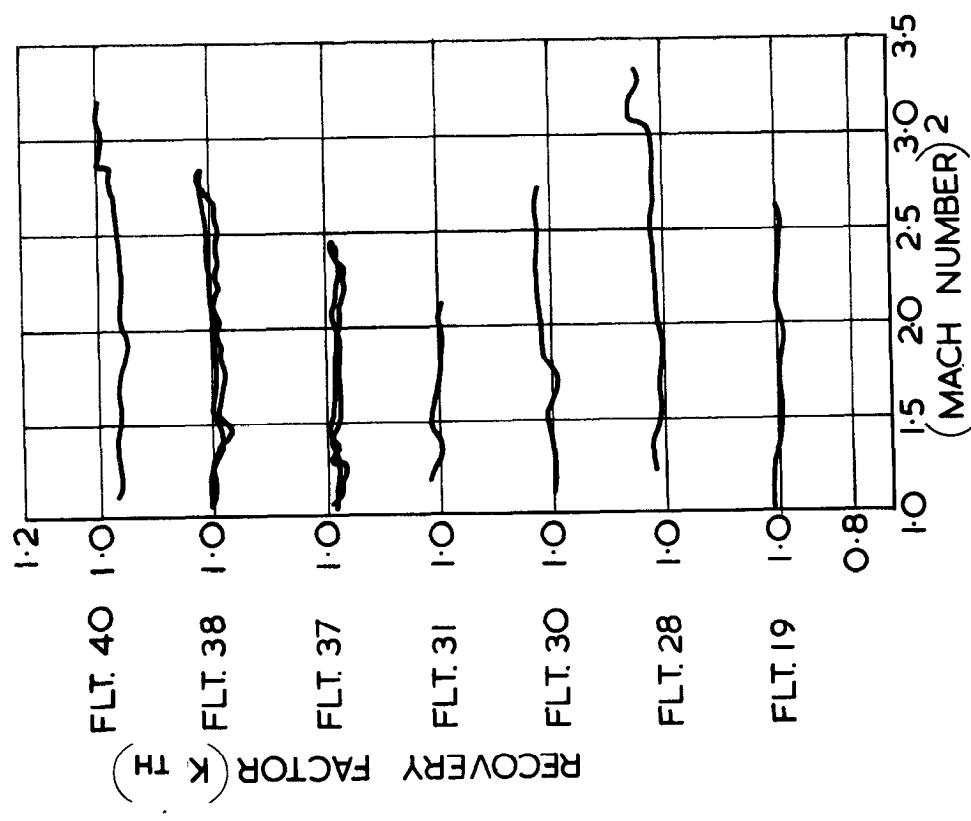


FIG.22 VARIATION OF THE THERMISTOR THERMOMETER RECOVERY FACTOR,  $K_{TH}$ , WITH  $(MACH\ NUMBER)^2$  AT SUPERSONIC SPEEDS

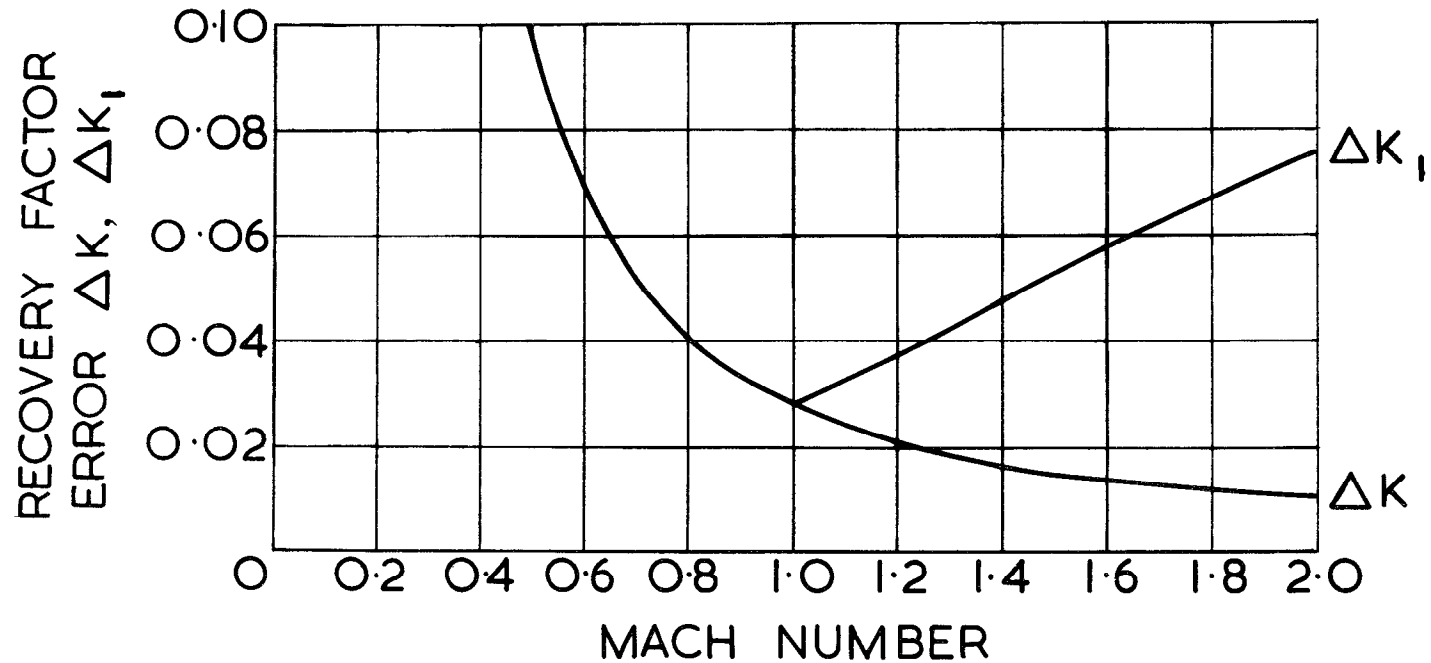


FIG. 23. ERROR IN RECOVERY FACTOR CORRESPONDING TO A 1°K ERROR IN STATIC TEMPERATURE;  $T_0$ , WHEN  $T_0 = 216^\circ\text{K}$

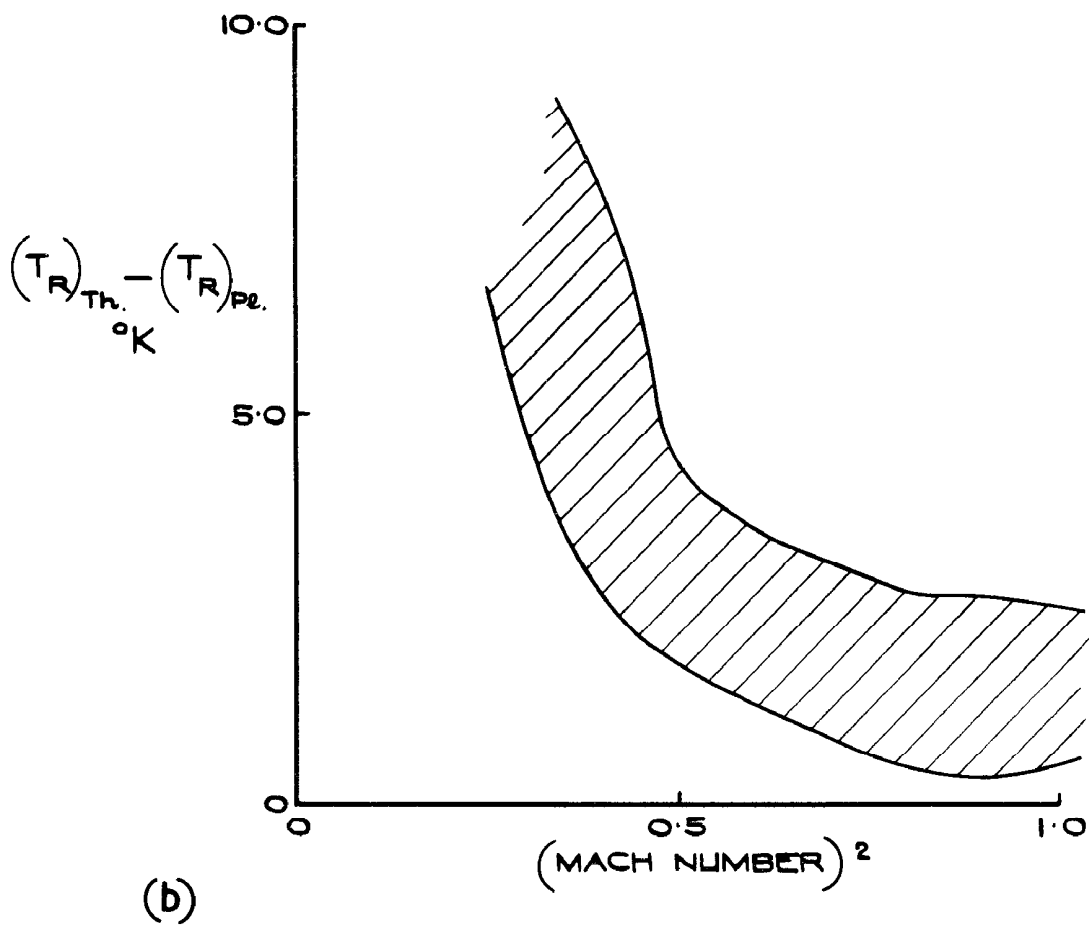
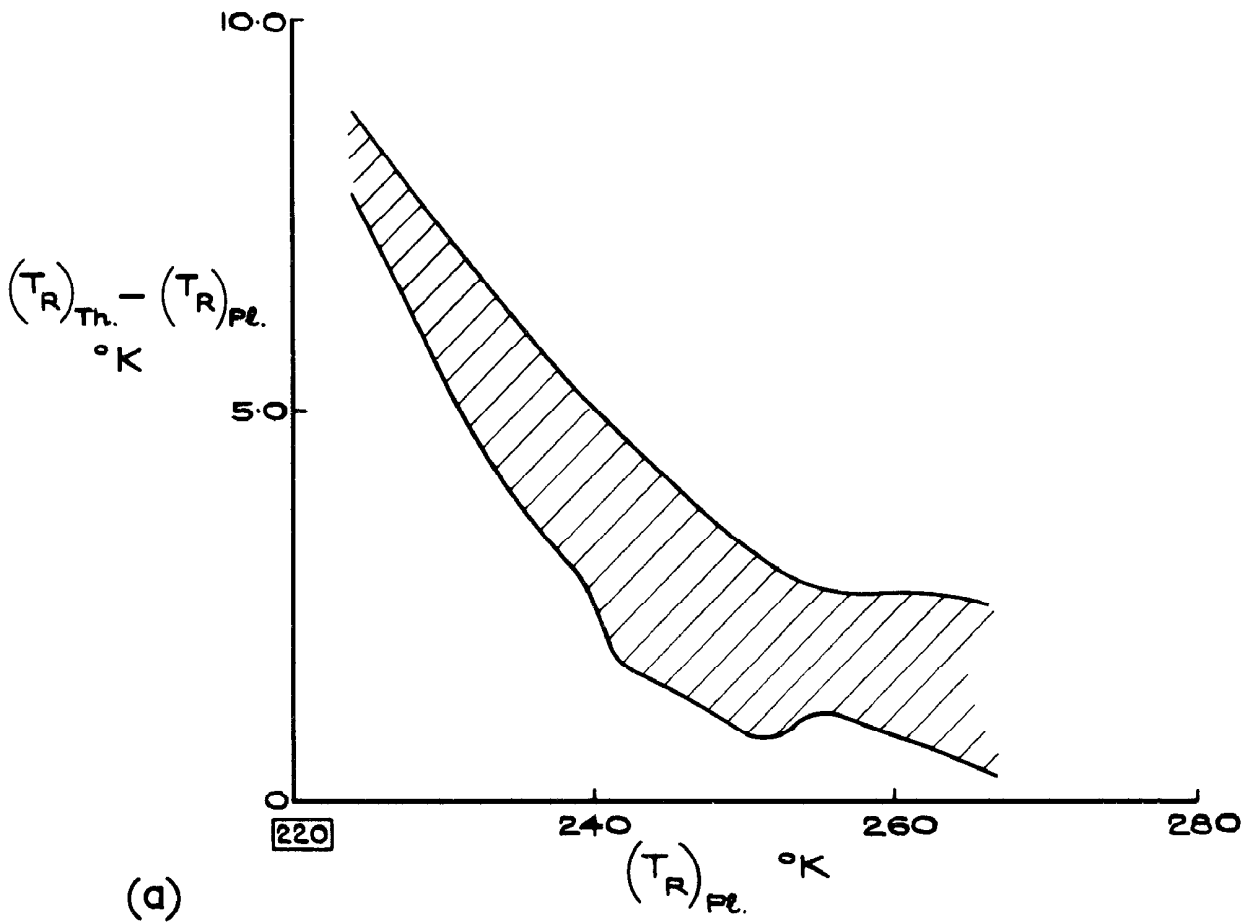


FIG.24. VARIATION OF THE DIFFERENCE IN INDICATED TEMPERATURE AT SUBSONIC SPEEDS.

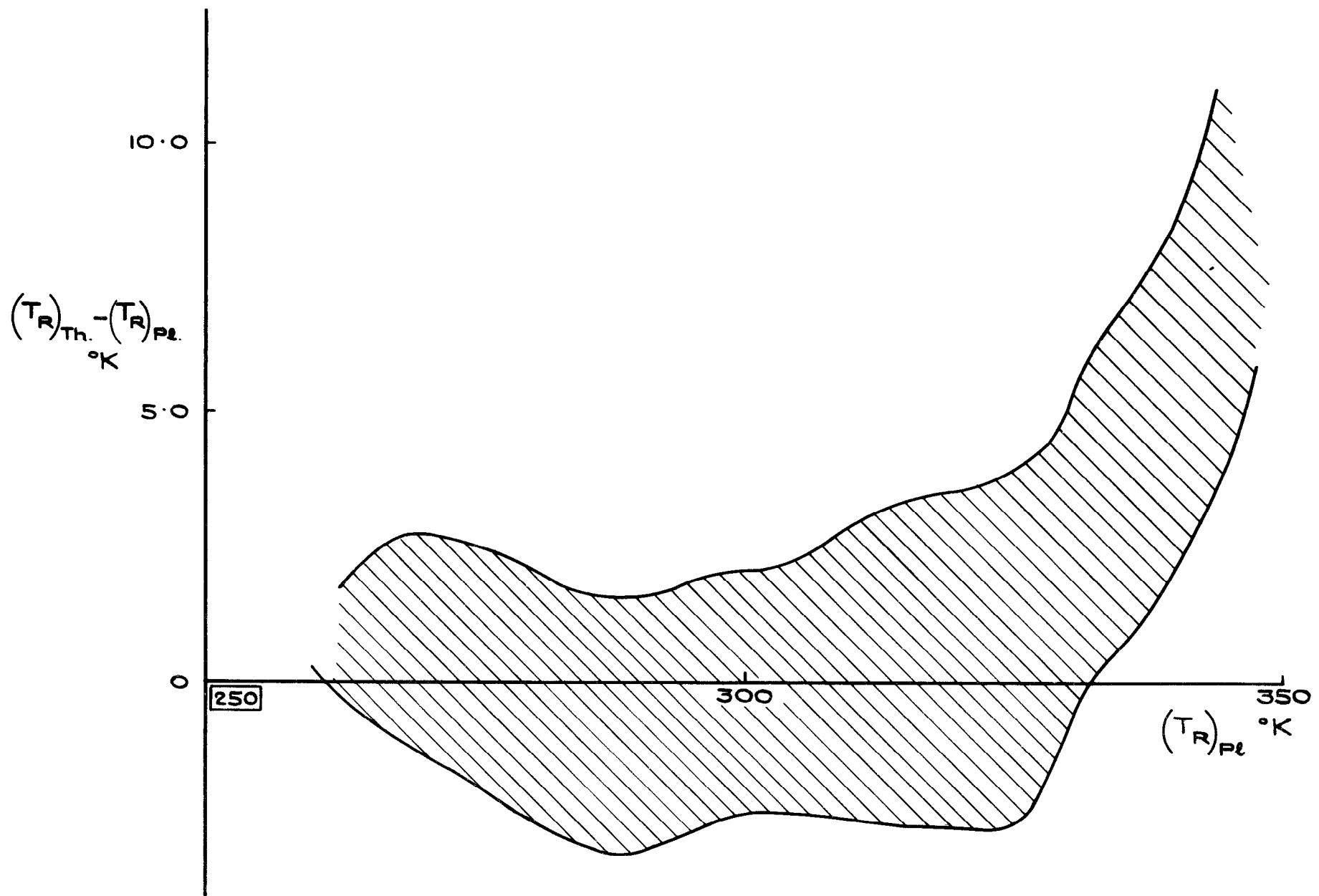


FIG. 25. VARIATION OF THE DIFFERENCE IN INDICATED TEMPERATURE WITH THE PLATINUM THERMOMETER INDICATED TEMPERATURE DURING THE SUPERSONIC FLIGHTS.

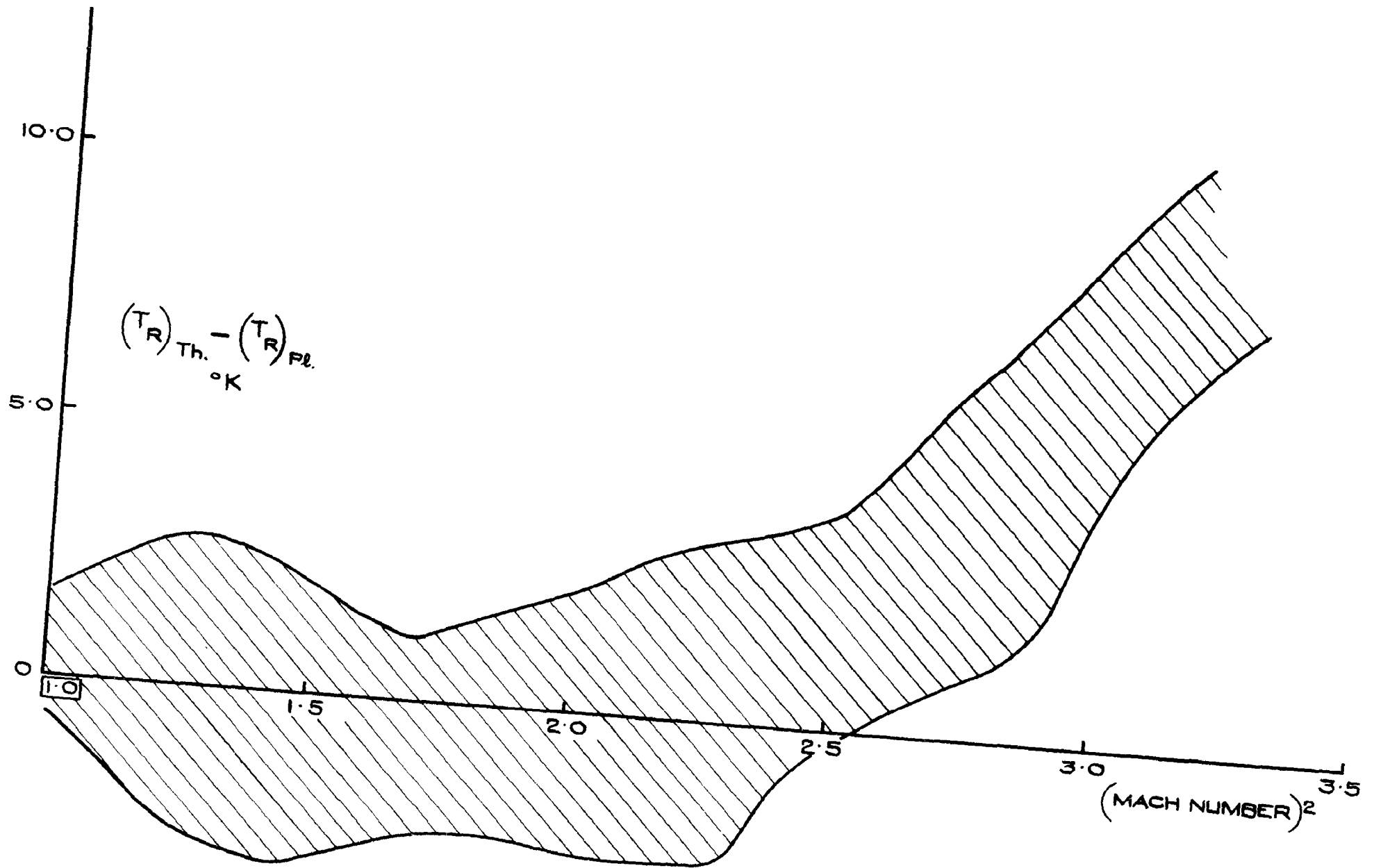


FIG.26. VARIATION OF THE DIFFERENCE IN INDICATED TEMPERATURE WITH  $(MACH\ NUMBER)^2$  DURING THE SUPERSONIC FLIGHTS.

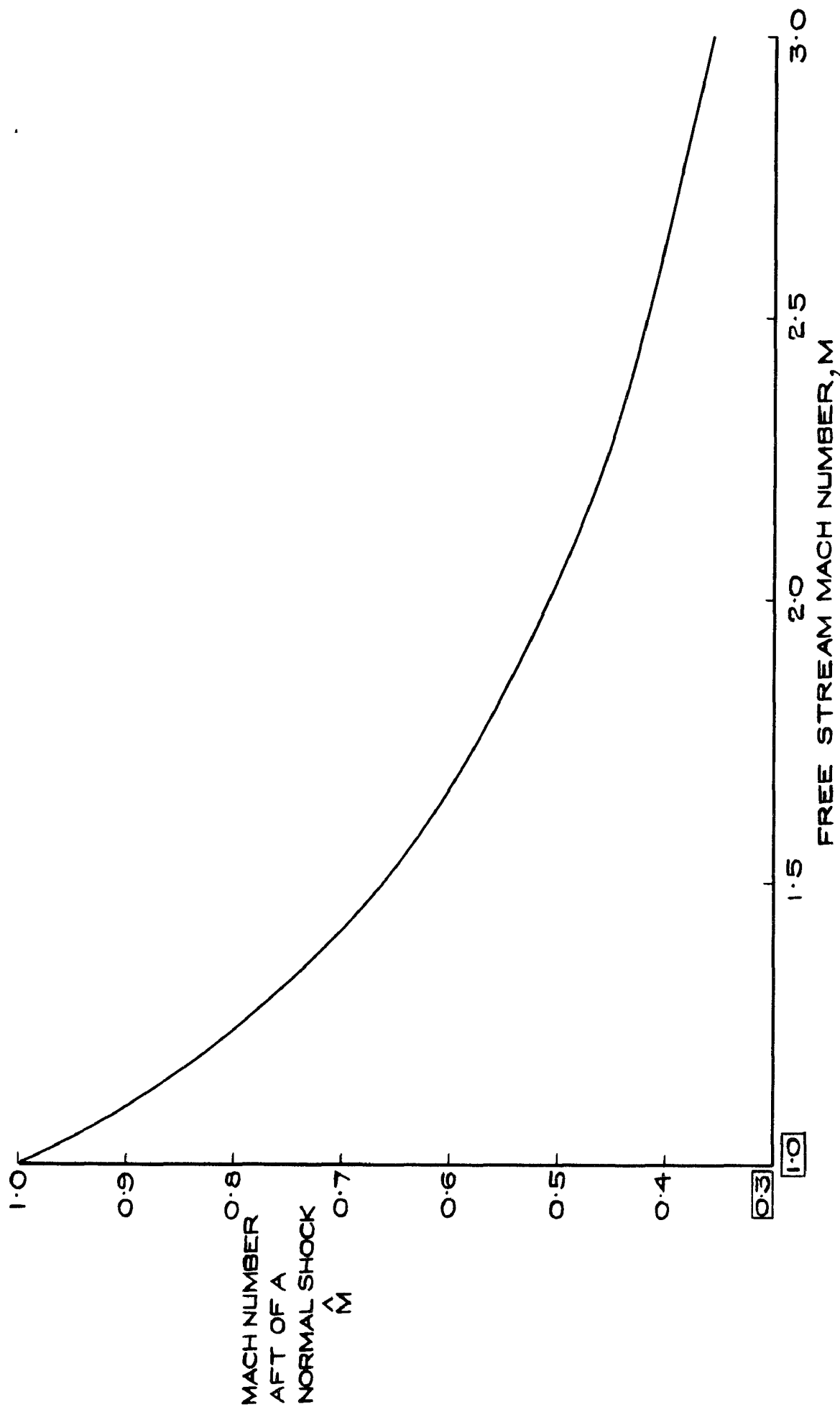


FIG.27. MACH NUMBER AFT OF A NORMAL SHOCK

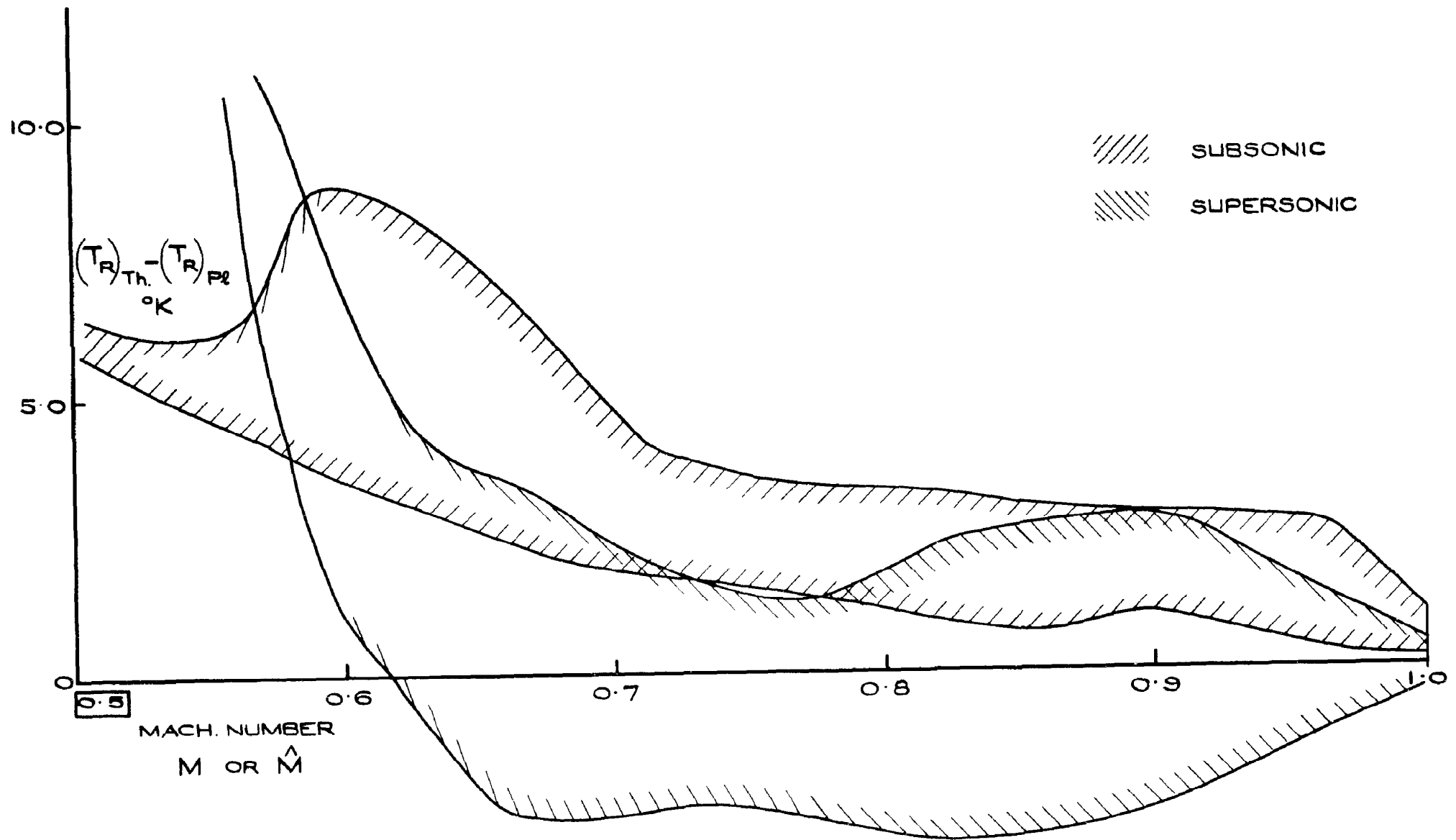


FIG.28. VARIATION OF THE DIFFERENCE IN INDICATED TEMPERATURE WITH THERMOMETER ENTRY MACH NUMBER.

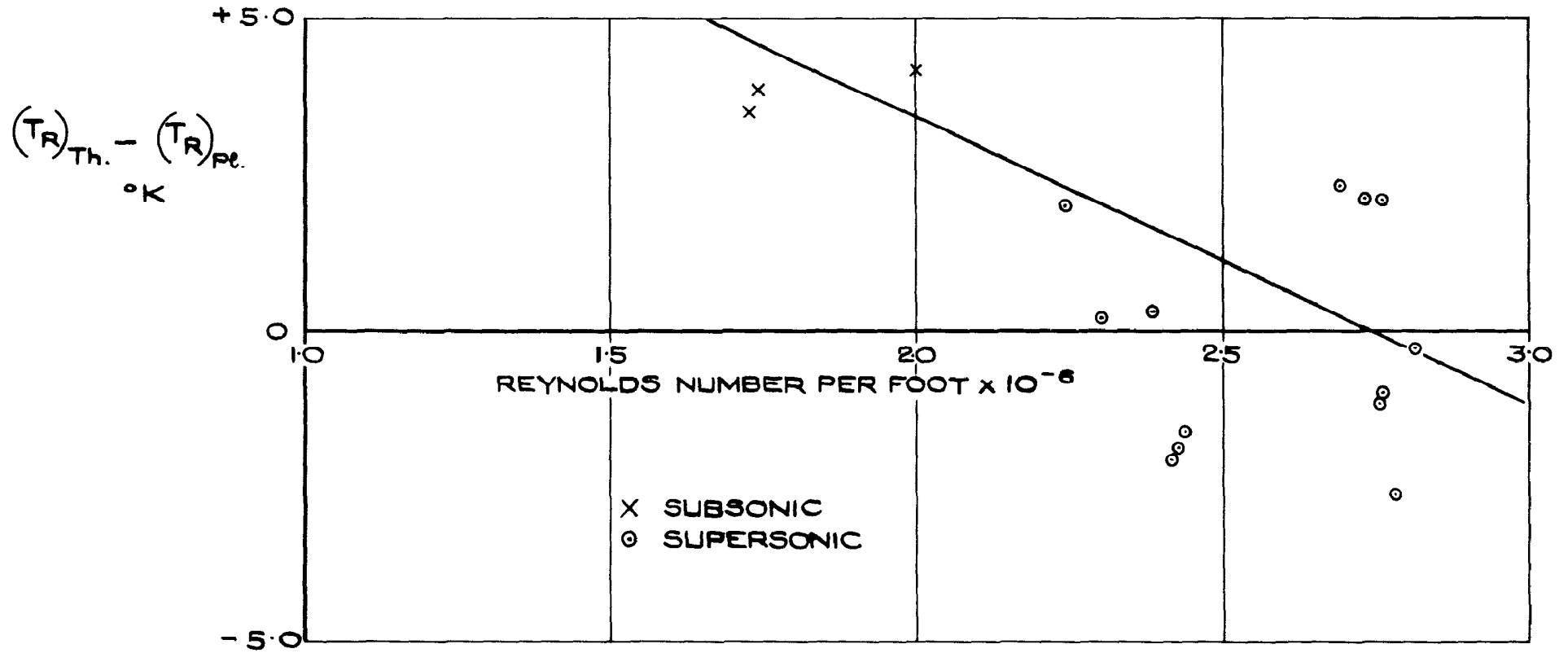


FIG. 29. VARIATION OF THE DIFFERENCE IN INDICATED TEMPERATURE WITH REYNOLDS NUMBER FOR AN ENTRY MACH NUMBER OF  $0.70 \pm 0.01$





A.R.C. C.P. No. 809

533.652.1 :  
533.6.011.341.5 :  
536.51 :  
533.6.011.6

MEASUREMENT OF AIR TEMPERATURE ON AN AIRCRAFT  
TRAVELLING AT HIGH SUBSONIC AND SUPERSONIC SPEEDS.  
Woodfield, A.A., and Haynes, P.J. September, 1963.

Flight tests have been performed on two different designs of impact air thermometer, with two thermometers of one design fitted with different sensing elements, in the altitude range from 30,000 ft to 40,000 ft at Mach numbers between 0.50 and 1.82. The performance of these thermometers is described by two parameters, the recovery factor and the time constant, values of which were obtained. The unusual behaviour of the recovery factor for one of the thermometers is discussed. At subsonic speeds it is shown that the normal straight line method of analysis, plotting indicated temperature versus (Mach number), can be invalid. At supersonic speeds,

(Over)

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(Over)

the use of an apparent recovery factor, which includes the effects of both the normal shock wave and the thermometer recovery factor, is recommended.

There is a large difference between the flight values and some laboratory values of time constant for one of the thermometers which suggests that the normal laboratory tests are not fully representative.

The flight test technique has been critically examined and suggestions made for any future investigations into the behaviour of air thermometers. The need for an accurate independent method of measuring static temperature during the tests is stressed.

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