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Potential Flow through Cascades
Extensions to an Exact Theory

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Potential Flow through Cascades

Extensions to an Exact Theory

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SUMMARY

The range of application of a conformal transformation, initially given by Merchant and Collar and developed by the author, is explored. Results of practical value are obtained in which the pressure distribution around certain profiles with a rounded trailing edge is computed. The theory is used as a check on the accuracy of approximate methods of solution due to Garrick, Howell, Martensen and Schlichting.

1./

Replaces A.R.C.26 168.

1. Introduction

In an earlier paper[1] the author has given an exposition of a potential flow theory, due to Merchant and Collar[2], for the analytical determination of the outlet angle from a cascade of previously defined aerofoils. The shape of these aerofoils is derived by conformal transformation from the flow past a series of ovals. It was demonstrated that this theory could be extended to give a formula for the pressure distribution over the blade profile without further assumptions or approximations; the analysis was subsequently programmed for a digital computer, giving the outlet angle and pressure distribution for the given profile in cascade to an accuracy of seven decimal places.

These exact calculations were performed for a cascade of cusped Merchant and Collar aerofoils; approximate potential flow methods were then used to calculate the pressure distribution and outlet angle for this cascade and a comparison with respect to the exact solution was thereby obtained. It was found that for the given cascade configuration the Garrick method[3] as developed by Hall[4], gave excellent agreement with the Merchant and Collar analysis; the Schlichting[5,6] method was 0.7° in error for the outlet angle and gave a misplaced suction peak on the aerofoil; and that difficulty was encountered in using the Martensen[7,8] method with such a thin trailing edge.

Notwithstanding the success of the early analysis, it is obvious that blades with cusped trailing edges can not be used in practice. An attempt was therefore made to obtain a wider range of profiles with rounded trailing edges, similar to those used in current industrial practice for compressor blades. It was also desirable that some idea should be formed of the range of application of the Merchant and Collar analysis and in this paper an attempt is made to answer this latter question.

2. Notation

c	chord length
s	blade spacing
t	maximum blade thickness
$l = m + in$	coordinates in plane of ovals
$l^1 = m^1 + in^1$	coordinates of centre of offset oval
$z = x + iy$	coordinates in cascade plane
$Cp = \frac{p - p_1}{\frac{1}{2} u_1^2}$	pressure coefficient
p	local static pressure
p_1	static pressure upstream of cascade
U_1	stream velocity upstream of cascade
α_1	flow angle upstream of cascade
α_2	flow angle downstream of cascade
β	length of semi-major axis of basic oval
β^1	length of semi-major axis of offset oval
$\lambda = l + \sinh^2\beta \coth l$	
$\gamma = \beta + \sinh^2\beta \coth \beta$	
$\sigma =$	stagger angle
ρ	fluid density
θ (fig.6)	angular position of rear stagnation point.

3. Derivation of profiles with a rounded trailing edge

In the previous paper a cascade of aerofoils in the z plane was derived from the conformal transformation

$$z = \lambda \cos\sigma - i\sin\sigma \cosh^{-1}(\operatorname{sech}\gamma \cosh\lambda) \tag{1}$$

applied to a series of ovals in the l plane

$$\cosh 2(m+m^1) = \cos 2(n+n^1) + \frac{\sinh^2\beta^1 \sin 2(n+n^1)}{n+n^1} \tag{2}$$

where

$l = m + in$	coordinates in the plane of the ovals
$z = x + iy$	coordinates in the cascade plane
β	length of basic oval major axis
β^1	length of offset oval major axis
$l^1 = m^1 + in^1$	coordinates of centre of the offset oval
σ	stagger angle of the aerofoils in cascade

$\lambda = l + \sinh^2\beta \coth l$

$\gamma = \beta + \sinh^2\beta \coth \beta$

The ovals of equation (2) were offset with respect to basic ovals

$$\cosh 2m = \cos 2n + \frac{\sinh^2\beta \sin 2n}{n} \tag{3}$$

and were spaced with a period π along the n axis.

The particular case was considered in which m^1 and n^1 were chosen such that the two ovals intersected at one singularity of the transformation and such that the larger (β^1) oval enclosed the other singularity. The transformation of the flow about the ovals into the flow about the aerofoils in the z plane was thus conformal everywhere and a cascade of profils with cusped trailing edges was derived, as in Fig. 1.

This case may be regarded as a limiting case of the shortest distance, between the β^1 oval and one singularity, tending to zero. At the other limit the larger (β^1) oval is concentric with the β oval, thus implying $l^1 = m^1 + in^1 = 0$. Upon transformation to the z plane a cascade of symmetrical uncambered aerofoils which tend towards an elliptic shape is obtained, as in Fig.2. The significance of the relationship between the distance in the l plane from the singularity to the nearest point of the β^1 oval and the radius of curvature of the nose or tail in the z plane thus becomes apparent.

The general case is intermediate between the two extremes mentioned above. As long as both singularities in the l plane are enclosed by the β^1 oval, any position of l^1 will give a transformation which is conformal. It is thus possible, by correct selection of the normal distance in the l plane between the singularity and the basic oval, to obtain any value of radius of curvature at the nose or tail in the z plane, from zero to a high positive value.

By a "trial and error" alternation of the variables β^1, m^1 and n^1 it is possible to approximate to practical compressor cascade profiles, which have rounded leading and trailing edges.

An aid to the convergence of the iterative process is given by a knowledge of the position of maximum thickness in the cusped Merchant and Collar profile of Ref. 1. (At 25% of the chord length) and the knowledge that for the ovals in a concentric configuration the position of the maximum thickness is at 50% chord. With these extremes fixed any desired position of maximum thickness can be obtained by an approximate interpolation on a linear basis for $l^1 = m^1 + in^1$.

The procedure for calculation of the pressure distribution and outlet angle for a profile with a rounded trailing edge follows that of section 3.2 of Ref.1. With this new type of profile, however, there is an additional complication. For profiles with a cusped trailing edge the selection of the position of rear stagnation point, and hence the outlet angle, followed automatically from the Kutta condition, which avoided infinite velocities at the trailing edge by placing the rear stagnation point at the transformation singularity, on the point of the cusp. Unfortunately this condition does not apply to potential flows which have singularity, and which do not tend to give rise to infinite velocities on profiles. The Kutta condition cannot, therefore, be applied to the usual type of compressor cascade profile which has a rounded trailing edge.

The author knows of no alternative condition which can be applied to the potential flow around a cascade of aerofoils with rounded trailing edges. As far as is known, all previous investigators have either:-

a) replaced the aerofoil by an equivalent one with a cusp at the trailing edge, thus facilitating use of the Kutta condition (e.g. the Schlichting method).

b) placed the stagnation point either at the intersection of the profile with the line connecting the trailing edges of each aerofoil in the cascade or at the rear end of the camber line.

c) specified the outlet angle as well as the inlet angle (e.g. the Martensen method).

Hourteux et al [8], had mentioned, in private communications to the author, the large extent of variation in outlet angle when the rear stagnation point had been allocated differing positions on the trailing edge. The author was able to confirm the validity of this effect as is demonstrated in Figs 7 and 8 which give the variation of outlet angle with the position of the stagnation point for a typical compressor cascade profile. Such results as these, predicting a large variation in outlet angle for a comparatively small alteration in the position of the stagnation point, emphasize the fact that the potential flow around a conventional aerofoil in cascade is not completely "determined" by specification of the cascade configuration and the inlet angle but that the position of the rear stagnation point is a further variable.

Figure 6 shows that the effect of the position of the rear stagnation point upon the pressure distribution for a compressor cascade profile is also large. A qualitative explanation of this effect is given in Appendix A.

Since no condition has been discovered which gives a unique solution to the potential flow around a cascade, the quest for such a condition will be postponed until the effect of viscosity on the flow is considered. In this paper the author will confine the investigation to obtaining outlet angles and pressure distributions for clearly stated and arbitrarily selected positions of rear stagnation point. This procedure, whilst introducing a further variable, will provide the maximum amount of information when, with the introduction of the concept of viscous flow, an attempt is made to postulate a flow condition in a manner similar to that suggested by Preston in Ref. 13.

It should be emphasized that the author is not rejecting the Kutta condition, but is recognising the fact that it does not apply in the general case of potential flow past a cascade or isolated aerofoil.

4. The scope and limitations of the theory

An attempt was next made to determine the range of operation of the previously given theory. It is quite obvious at the outset that many limitations prevent the use of the theory as a conventional potential flow "method". The nature of these limitations is discussed in this section.

i) Space-chord ratio

Variation in the space-chord ratio of the cascade is obtained from the analysis by variation of the size of the ovals in the 1 plane. A formula connecting $^s/c$ and β for flat plate cascades is given by Merchant and Collar but this only gives an approximate idea of the range for conventional aerofoils. In practical cases a given value of $^s/c$ can only be obtained by a "trial and error" specification of β. This process can be made to converge quite rapidly to a required space-chord ratio on a digital computer.

For $\beta \ll \pi$ the 1 plane ovals tend to become circles and the space/chord ratio tends to infinity. Values of β around 0.725 give space-chord ratios of approximately unity. As β tends to an infinite value the ovals flatten considerably and the space-chord ratio becomes very small.

Thus for all practicable compressor cascades, variation of $^s/c$ is achieved quite simply by a corresponding variation in β

ii) Camber

There seems to be no means in the analysis for variation in shape of the camber line, which, as far as can be determined, must be almost a circular arc. This is very convenient for aerofoils of the "N.G.T.E." 'C' series of profile shapes which are mainly intended for use with a circular arc camber line. Attempts have been made to match profiles with conic camber lines but no success has been achieved.

Determination of the camber angle of the z plane profiles can be obtained by measurement, or alternatively could be obtained analytically by differentiation of the equations for the l plane ovals prior to transformation. Inspection of Fig. 15. (in conjunction with Appendix B), will indicate the range of camber obtainable, which varies with the stagger and thickness of the profile. The hypothetical considerations of Fig. 15. indicate that a reasonable range of camber is available for most profile configurations, ranging from negative values to high positive values, (in the conventional sense for compressor cascades), indeed, c4 type profiles have been obtained from the analysis for up to 70° camber and in certain cases a camber angle of over 100° has been obtained. It will be seen, therefore, that a large range of camber angles may be studied although the interdependence in the analysis of camber, position of maximum thickness; maximum thickness and stagger must be considered.

iii) Position of maximum thickness

The position of maximum thickness is to a certain extent at the users' disposal although, like the camber, it is dependent on the other variables, especially the radii of leading and trailing edges. It appears that the position of maximum thickness may be varied from around 23% of the chord to 50% or more, a range which should cover any likely requirements.

iv) Value of maximum thickness

The value of $^t/_c$, is influenced by variation of the ratio $^{\beta^1}/_\beta$. For a value $^{\beta^1}/_\beta = 1$ a cascade of flat plates is generated in the z plane; as $^{\beta^1}/_\beta$ increases, so does $^t/_c$, and any practicable blade thickness may be easily obtained.

v) Leading and trailing edge radii

As outlined in Appendix A a substantial variation in these radii from zero to a high finite value is obtainable. Once more the variables must be studied in conjunction with others.

vi) Stagger

All of the previous variables, with the exception of $^{s}/c$ and $^{t}/c$ are dependent on the stagger angle , through the positioning of the 1 plane singularities, however there is no special limitation to the value of stagger itself, which should be specified by the user.

vii) Inlet Angle

There is no limit on the inlet angle. Pressure distributions and outlet angles have been obtained for very large ranges of incidence.

viii) Profile shape

Other than by adjustment of the previously-mentioned parameters there is no control of profile shape.

It will therefore be seen that the main limitations on the Merchant and Collar analysis are:-

a) the shape of the camber line, which must approximate to a circular arc.

b) the interdependence of the position of maximum thickness leading and trailing edge radii, and camber angle, giving difficulty in varying one without the others.

If it were not for these limitatations the analysis would be capable of taking its place as a potential flow "method" and would have the advantage over all known methods of complete accuracy. Because of these limitations, however, its application is confined to the role of an exact standard for checking the accuracy of more general approximate methods, or alternatively, of deriving profiles for which the aerodynamic parameters can easily be computed.

5. The use of a 10C4/30C50 profile for comparisons

In section 3 a method was given for establishing a Merchant and Collar blade profile with a rounded trailing edge. In this section the analysis is used to obtain an approximation to a typical compressor cascade profile. The profile chosen was a 10C4/30C50 section set at a stagger of 36° with a space-chord ratio of unity.

It was found that following the procedure of section 3 and appendix ? only three iterations were necessary to give the agreement with the standard C4 form shown in Fig. 4. The y axis has been magnified in order to reveal any profile discrepancies and the maximum difference between the 10C4/30C50 section and the Merchant and Collar approach to it is 0.003 of the chord length. Since the difference is most marked at the trailing edge, a magnified view of the trailing edge has also been included (Fig. 7): in this region, although the ordinates are still accurate to within 0.3% of the chord, the radius of curvature is only 40% of the given C4 trailing edge radius.

Having established a profile which was similar to blade profilesused in industrial applications, the author was able to proceed towards a comparison of the results of several well-known potential flow methods with the analytical result. In order to eleminate one variable for the purpose of the comparison, the position of the rear stagnation point was selected at the end of the camber line, the coordinates of this position being (1,0) in the z(x,y) plane. For the used inlet angle of 51° this resulted in an outlet angle given by $\tan \alpha_2 = 0.5359714$ according to the analysis. The selection of this stagnation point position was entirely arbitrary.

Approximate methods available were:- The Garrick[5] method, computer programs for which were evolved and run by Hall[4].* The Martensen[7] method, computer programs for which were developed by Price and Heurteux[8]. The Howell[9] and Schlichting[5] methods; programs for which were developed by Pollard and Wordsworth[6].

The procedure for the Martensen method was that the users were supplied with the cascade configuration and profile for the analytical approximation to the C4 (curve B in Fig.4.), inlet angle and position of rear stagnation point and were asked to compute the pressure distribution - the results for which are shown in Fig. 10. compared with the analytical result.

The computer programs for the Garrick, Howell and Schlichting methods were not available to the author at the time of the comparison; however, these computations had been carried out previously for a 10C4/30C50 profile with the same stagger of 36°, the same inlet angle of 51° and a slightly different space-chord ratio of unity (as compared with 0.9901985). As a result of the assumptions implicit in the Schlichting method, the trailing edge becomes cusped and the Kutta condition is applied. As mentioned previously, the discrepancy between the 10C4/30C50 profile and the ordinates of the 'Merchant and Collar' type approach to this was nowhere greater than 0.3% of the chord. The previously computed results from the Garrick, Howell and Schlichting methods are thus compared directly with the results of the Martensen method in Fig. 10.

From an inspection of Fig. 10. it will be seen that the Garrick method gives good agreement, apart from one point near the suction peak. The Martensen method is seen to give complete agreement apart from a very slight error at the suction peak. The Howell method gives reasonable agreement, the maximum error in C_p being of the order of 10%. The Schlichting method in general gives a similar order of accuracy but misplaces the suction peak, in an identical fashion to Fig 4 of Ref. 1., and has a maximum error in C_p of 15%.

6. Generation and use of a highly cambered profile for comparisons.

The need for an analytical solution to a highly cambered blade arose from doubts as to the applicability of the Schlichting method at high cambers. In the Schlichting theory the profile is generated by a source distribution. Ideally the sources would be distributed along the camber line, but due to the complexity of the mathematics involved the sources are usually distributed along the chord line, (this difficulty is partially overcome in Ref. 10) As a result of this simplification, errors were thought to be present in the prediction of the performance of highly cambered profiles. The following comparison was an attempt to assess the magnitude of these errors.

Reference should be made to Figs. 11 and 16 which demonstrate that it was possible to obtain a highly cambered profile from the analysis. In this instance the stagger was zero and the space-chord ratio was 0.9003643 - a fairly typical impulse cascade. It will be noted that for a cascade of zero stagger both singularities of the analysis are on the real axis in the 1 plane. For a positively cambered aerofoil with the position of maximum thickness forward of 50% chord the point 1 should be within the area ABC. For the profile to have a large camber, n^1 should have a high negative value; for the trailing edge to be rounded m^1 should be within the arc CB and the position of the centre of the β^1 oval between the n axis and the arc CB will determine how far back the position of maximum thickness of the z plane profile is to be. In this way a suitable position was selected at E in the schematic diagram.

Upon application of the conformal transformation to the configuration of ovals thus described a cascade of aerofoils with approximately 70° camber was generated, as shown in Fig. 11.

The procedure for the comparison was similar to that of section 5. The rear stagnation point was arbitrarily selected at the point (1,0) in the z plane. The profile coordinates were supplied to the users who were asked to run their computer programs for $\alpha_1 = +35°$ and $\alpha_1 = -35°$.

Methods available for the comparison were:- The Martensen method, as developed by Price and Heurteux[8]. The Schlichting method, as developed by Lewis[11], and independently the Schlichting method as developed by Chauvin and Breugelmans[12].

Once more, in the Martensen method the trailing edge was treated in its correct rounded form; the position of the rear stagnation point was selected by specifying the correct value of outlet angle in the computer data. In the Schlichting method the trailing edge became cusped and the Kutta condition was applied. The results, as shown in Figs. 12 and 13, indicate that for such a cascade the Martensen method is the most accurate, although a considerable scatter is present.

The Schlichting method, as developed independently by Lewis and Chauvin, gives surprisingly good agreement with the analysis under what one would expect to be the least favourable conditions for such a theory.

7. Discussion and Conclusions

The analysis has been extended to give cascade aerofoils of practical significance whilst retaining the strict accuracy of the original Merchant and Collar theory. A profile similar to the standard 10C4/30C50 profile was produced as an example of this generalisation and a comparative survey of several approximate potential flow methods was carried out. Results indicated that for this practicable compressor cascade profile the Martensen method gave results of high accuracy; the results for the Garrick, Schlichting and Howell methods suggested that for this type of profile and cascade configuration a reasonable accuracy was attainable and that these methods could be used for 'engineering' applications with confidence.

As a result of the need for a comparison with highly cambered blade profiles such a profile was obtained from the analysis and a further comparative survey was made. The Martensen method produced accurate results, although a slight scatter was present. The results of the Schlichting method suggested that the performance of this method at high camber need not be as poor as had hitherto been suspected and that results of engineering accuracy could be obtained even at high camber.

The range of application of the theory has been discussed and the three limitations to this range were found to be:-

i) the profile must have a circular arc camber line.

ii) Interdependence of camber and profile form makes selection of a particular profile difficult.

iii) lack of control of blade form, other than through these interdependent variables.

Apart from these limitations a quick method of deriving the profile form was given and the attainment of a wide range of cascades was found to be quite possible.

No unique solution was found within potential flow theory to the problem of the location of the rear stagnation point; The position was arbitrarily specified in all computations.

8. Acknowledgments

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Appendix A

Potential flow near the rear stagnation point

In order to obtain a qualitative understanding of the pressure distribution in Fig. 6, it is necessary to show the conditions in an exaggerated fashion in Fig. 14.

Fig. 14(a) shows the stagnation point located at the extreme rear of the aerofoil. Since the flow does not obtain very high velocities anywhere, the suction surface pressure rises smoothly to the stagnation point, at exactly $\frac{x}{c} = 1$. . Since the path of a particle travelling along the pressure surface towards the stagnation point would follow similar curvatures, the velocity attained is similar and the pressure distribution approaches the stagnation point in a similar way.

In Fig. 14(b) the stagnation point is well up on the suction surface. Consequently, a fluid particle travelling along this surface is soon decelerated to approach the stagnation condition before the end of the chord. On the other hand, the pressure surface route entails a path of high curvature and an associated high velocity (which would be infinite in the limiting case of a cusped trailing edge). After negotiation of this region of high curvature, a rapid deceleration would bring the particle to the stagnation point. The upper and lower surface pressure distributions intersect somewhat upstream of the stagnation point position.

Fig. 14(c) shows the stagnation point on the pressure surface. Such configurations have possible parallels in 'jet flap work'. In this case it is the pressure surface which gives low velocities and an early stagnation point. The suction surface velocity reaches high values in the region of high curvature. As in the case of the pressure surface velocity of Fig 14(b), deceleration to the stagnation velocity is then rapid.

This exaggerated flow model should explain the pressure distribution of Fig.6 and provide a qualitative picture of the potential flow conditions at the trailing edge for use when the effect of viscosity is considered in a further paper.

Appendix B

The significance of oval positions in the l plane

Fig 13 shows the ovals in the l plane which were used to obtain an approximation to the 10C4/30C50 profile. In order to give a space-chord ratio of nearly unity, β is given a value of 0.725. The singularities in the transformation for a stagger of 36° are calculated as in Ref.1. Appendix B and are indicated.

It then becomes necessary to postulate a value of β^1 which would give the required value of $^t/c$. This is arrived at by a 'trial and error' process and was around 10% higher than β for a value of $^t/c = 0.1$. With the selected values of β and β^1 and the position of the two singularities fixed by the stagger, it is desirable to attempt to predict the range of possible values of camber, position of maximum thickness and leading and trailing edge radii. To do this, the author used the following procedure. Firstly an arc of radius β^1 is struck off from each singularity, as shown in the figure. For the transformation to be conformal the centre of the β^1 oval, $l^1 = m^1 + in^1$, may be chosen anywhere within the area common to these arcs, i.e. the area E,F,G,C.

In order that the correct leading and trailing edge radii may be obtained, points P,Q, are selected at small distances away from the singularity point, upon the normal from the β oval at the singularities. The distances of points P and Q from their respective singularities are again selected by experience, being approximately proportional to the required radius of curvature. From P.Q, arcs of radius β^1 are struck, as shown dotted in Fig. 13. The area D, A, H, B is common to these arcs and the centre of the offset oval may be placed anywhere within this area and satisfy all conditions imposed. Thus in the example of Ref.1., which had a cusp at the trailing edge, l^1 was chosen to lie on the arc G, C, E between C and E. This ensured that the β^1 oval passed through the singularity P, giving a cusp at the trailing edge, and also gave ample radius of curvature at the leading edge.

The only further variables needing specification are the camber and the position of maximum thickness. Incorporation of these variables is now a simple process. If 1^1 is selected near to D, a highly cambered profile will result; if near to B, the camber will be low and the position of maximum thickness will be well forward.

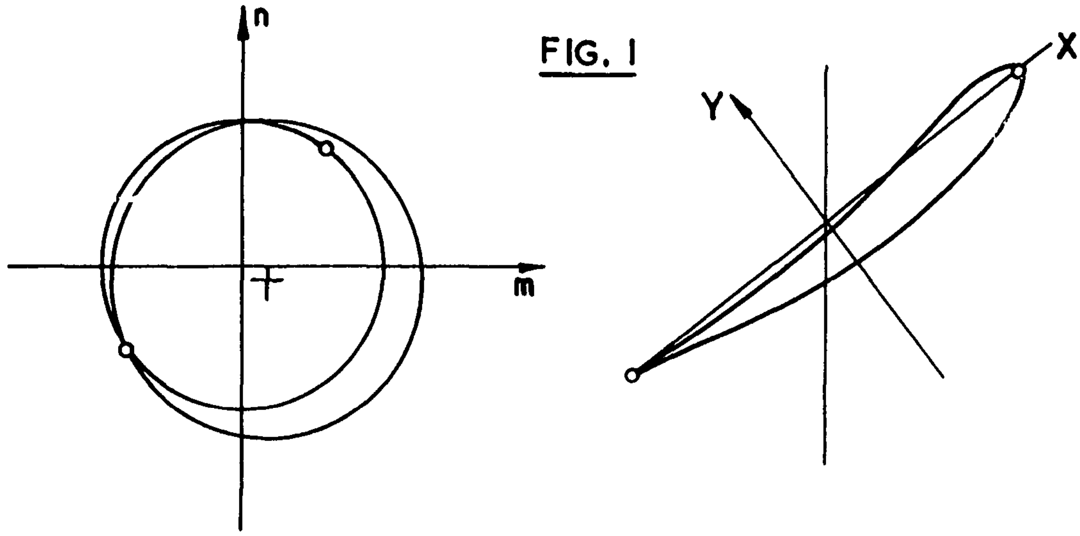
This schematic diagram approach has proved useful in obtaining the previously mentioned approximation to a C4 profile and many other profiles with a large range of camber and stagger. As a further illustration, the schematic diagram of the 1 plane configuration for a highly cambered profile is shown in Fig.16.

D 37266/1/Wt.60 K4 5/65 R & CL

FIGS. 1 – 3

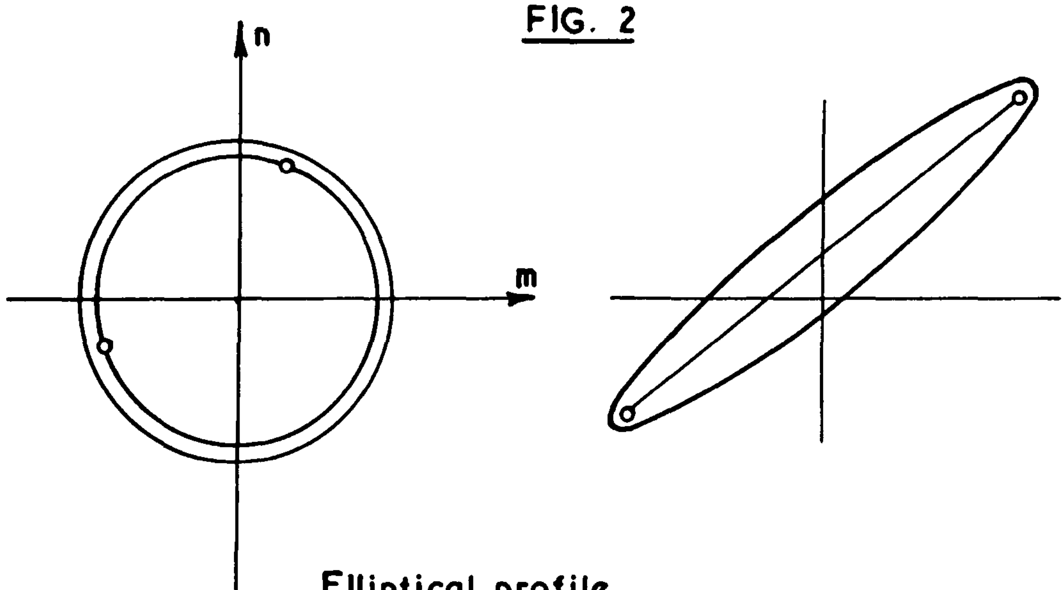
Series of ovals transforms to cascade of aerofoils.

FIG. 1



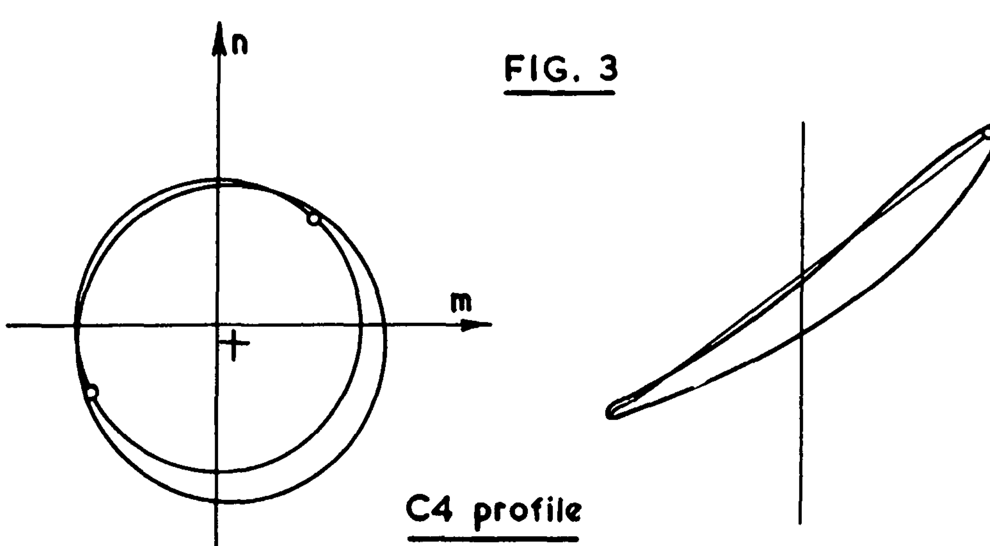
Profile with cusped trailing edge

FIG. 2



Elliptical profile

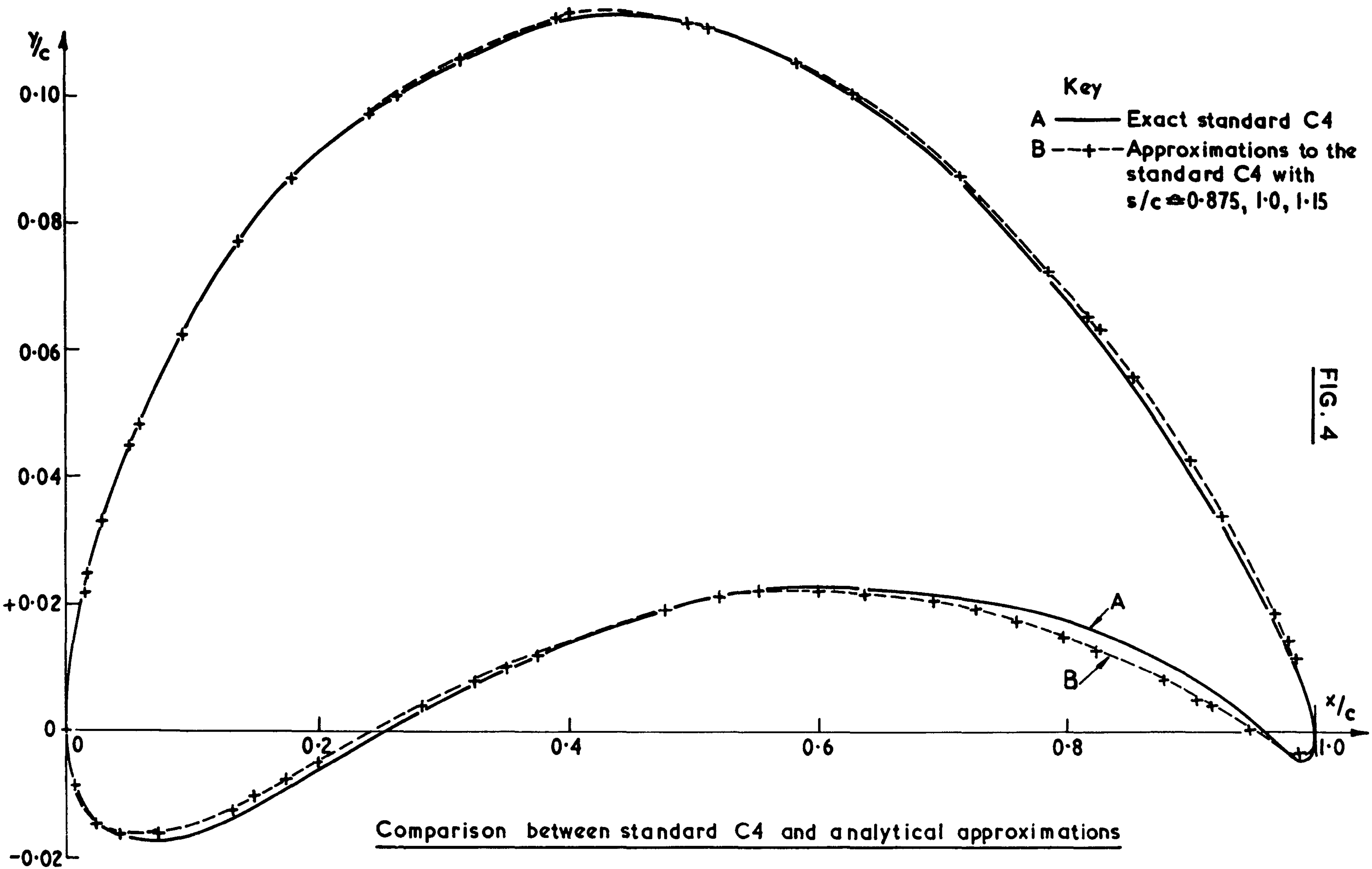
FIG. 3



C4 profile

Profile shapes obtainable (not to scale)

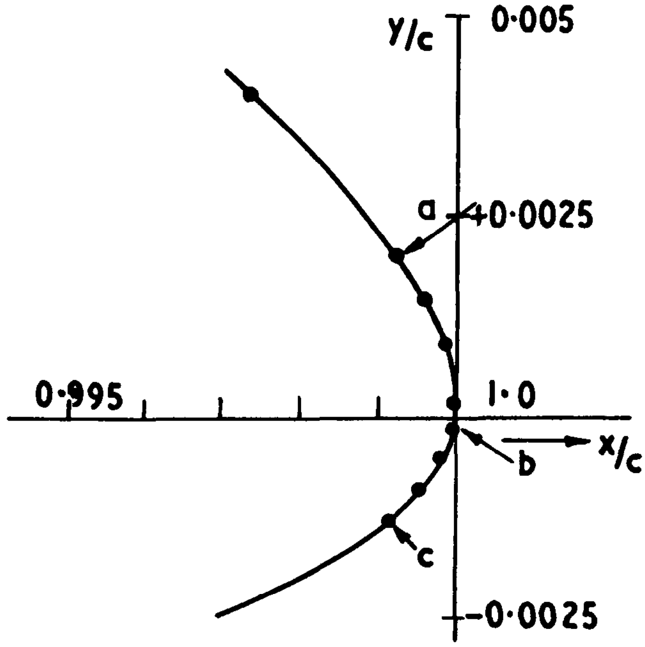
o marks a singularity of the transformation



Comparison between standard C4 and analytical approximations

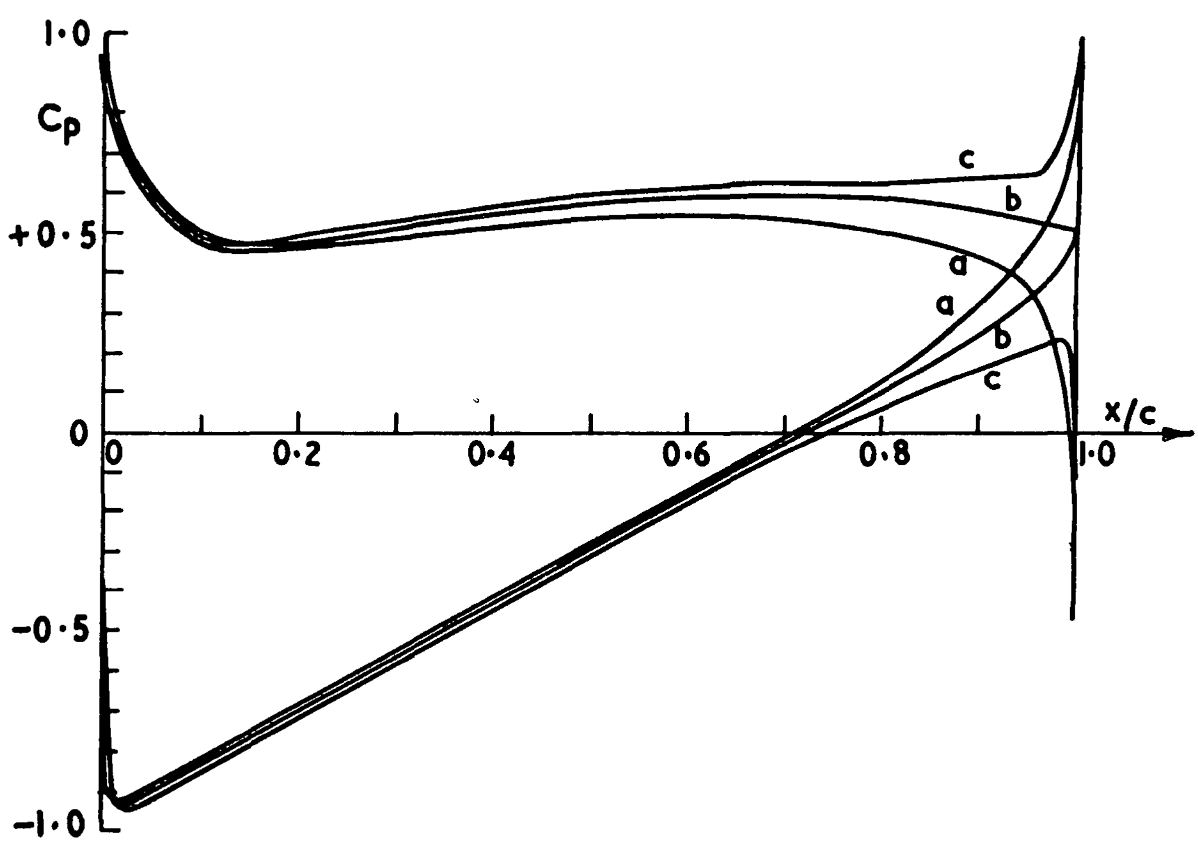
FIG. 4

FIG. 5



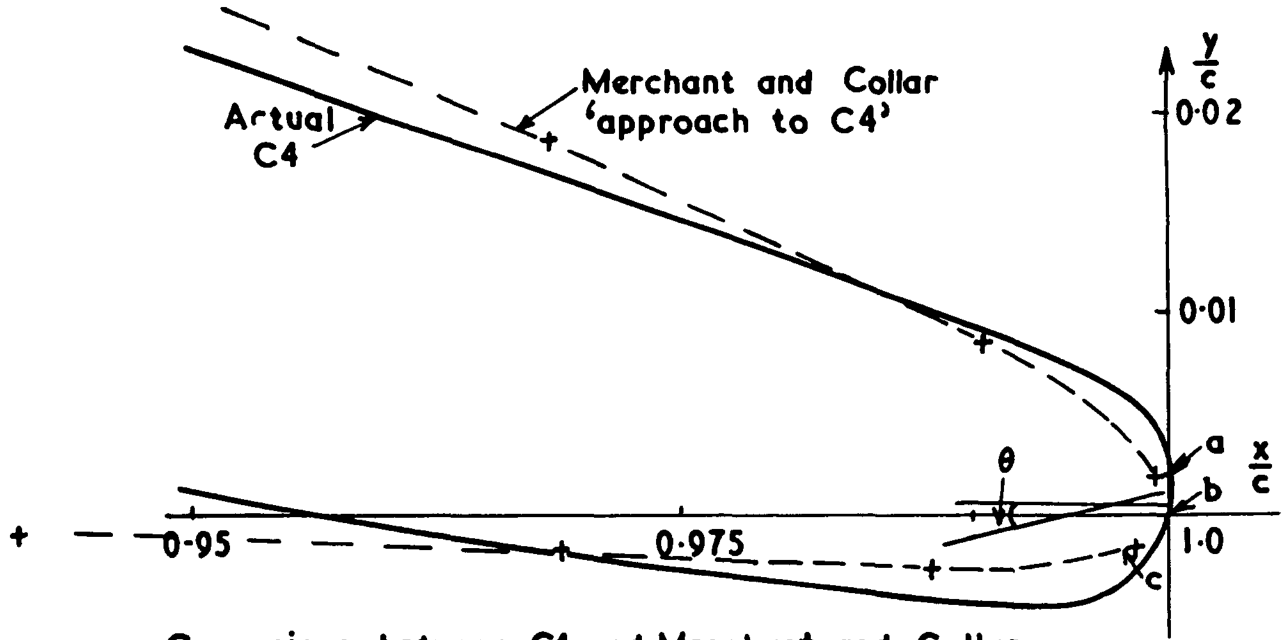
Trailing edge detail $s/c = 1{\cdot}15$

FIG. 6



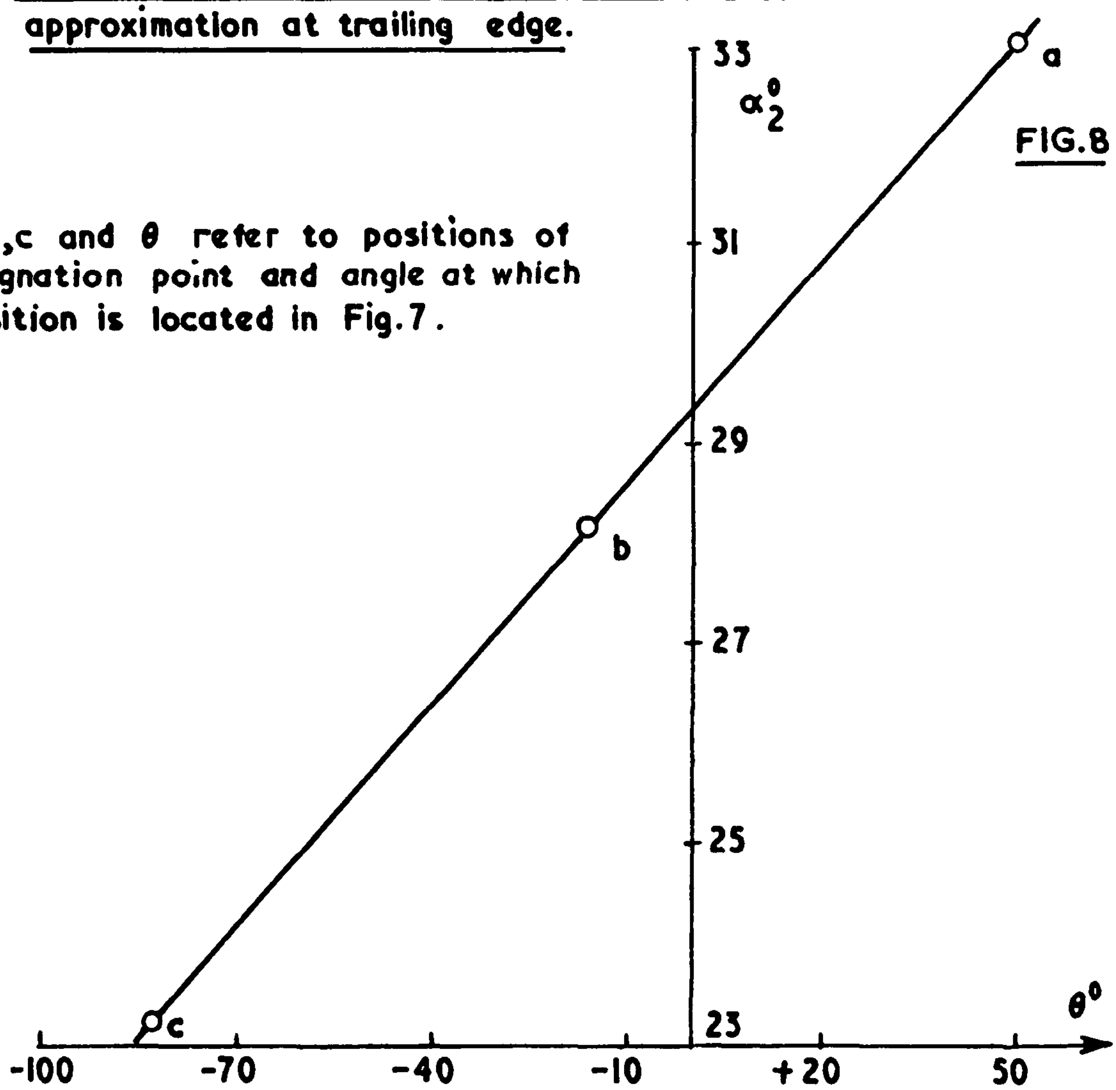
Effect of variation of rear stagnation point on pressure distribution for Merchant and Collar type 'approximation to 10C4/30C50' with $s/c = 1{\cdot}15$, $\alpha_1 = 52° 50'$

FIG. 7



Comparison between C4 and Merchant and Collar approximation at trailing edge.

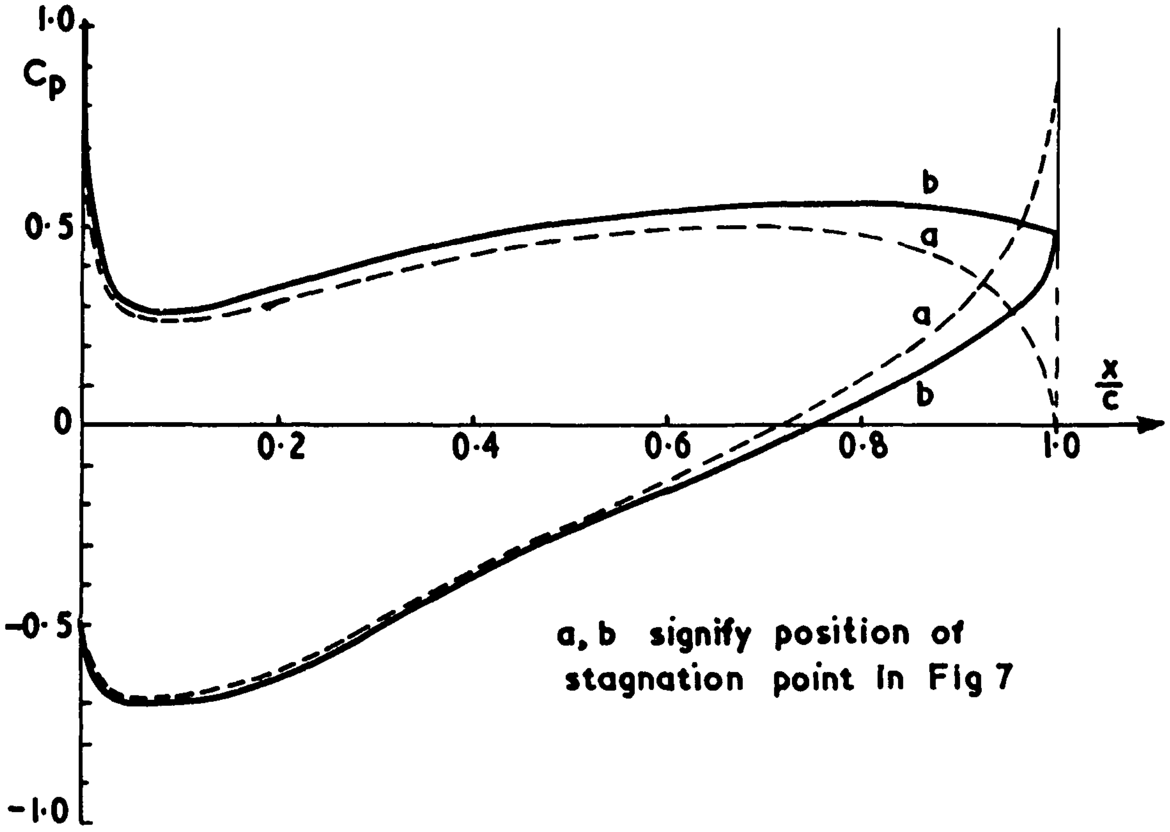
FIG. 8



a, b, c and θ refer to positions of stagnation point and angle at which position is located in Fig. 7.

Outlet angle as a function of position of rear stagnation point.

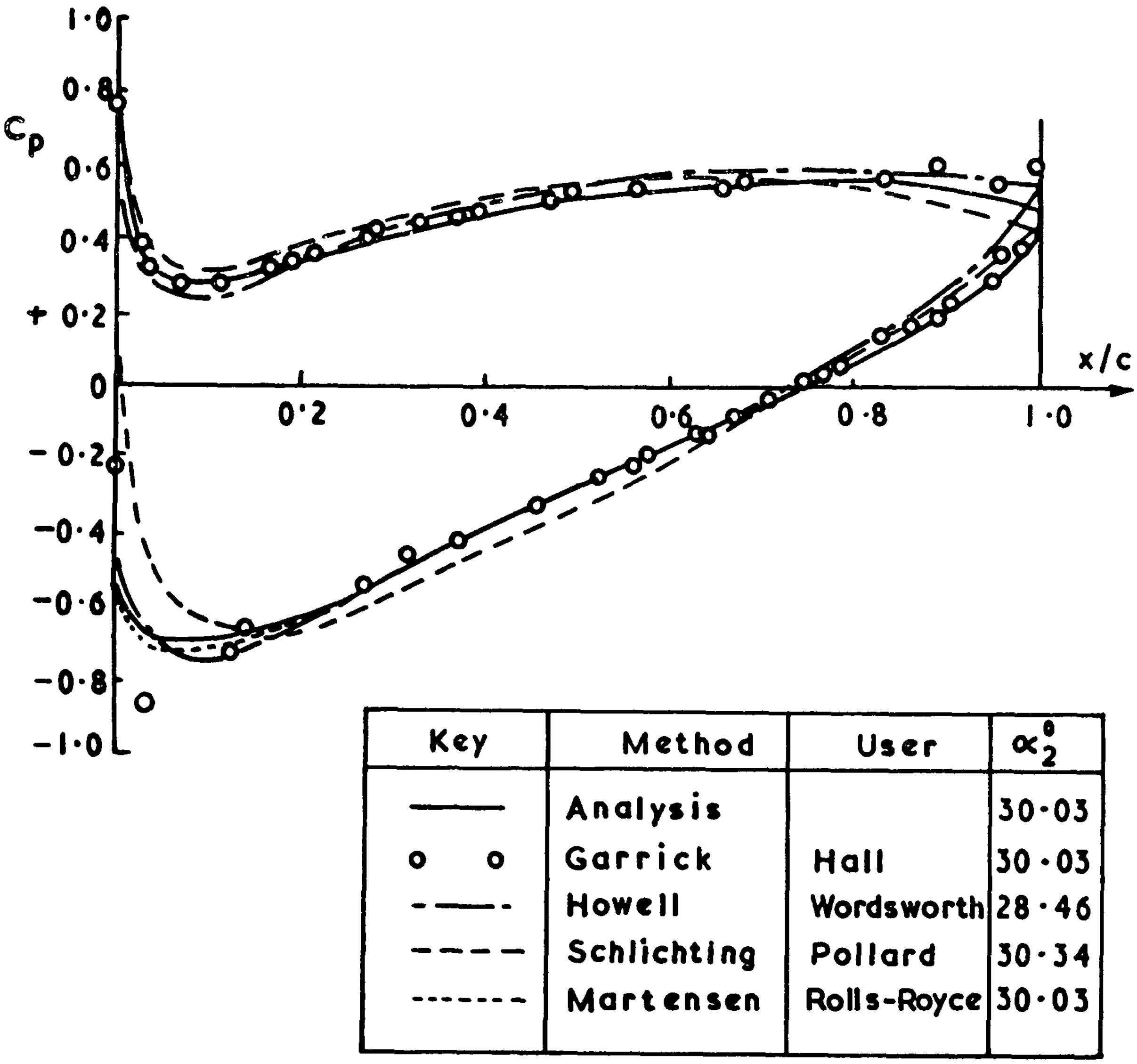
FIG. 9



Effect of variation of rear stagnation point on pressure distribution. Merchant and Collar 10C4/30C50 approximation

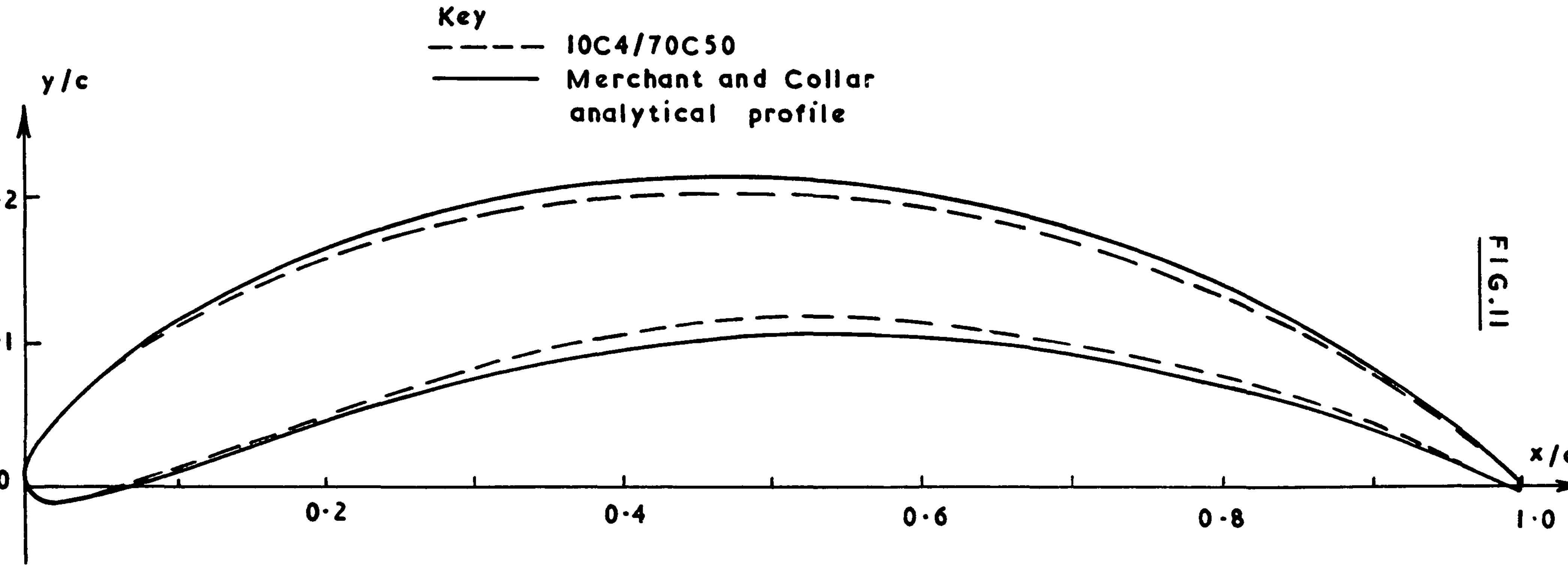
$s/c \simeq 1{\cdot}0$, $\sigma = 36^{\circ}$, $\alpha = 51^{\circ}$

FIG. 10



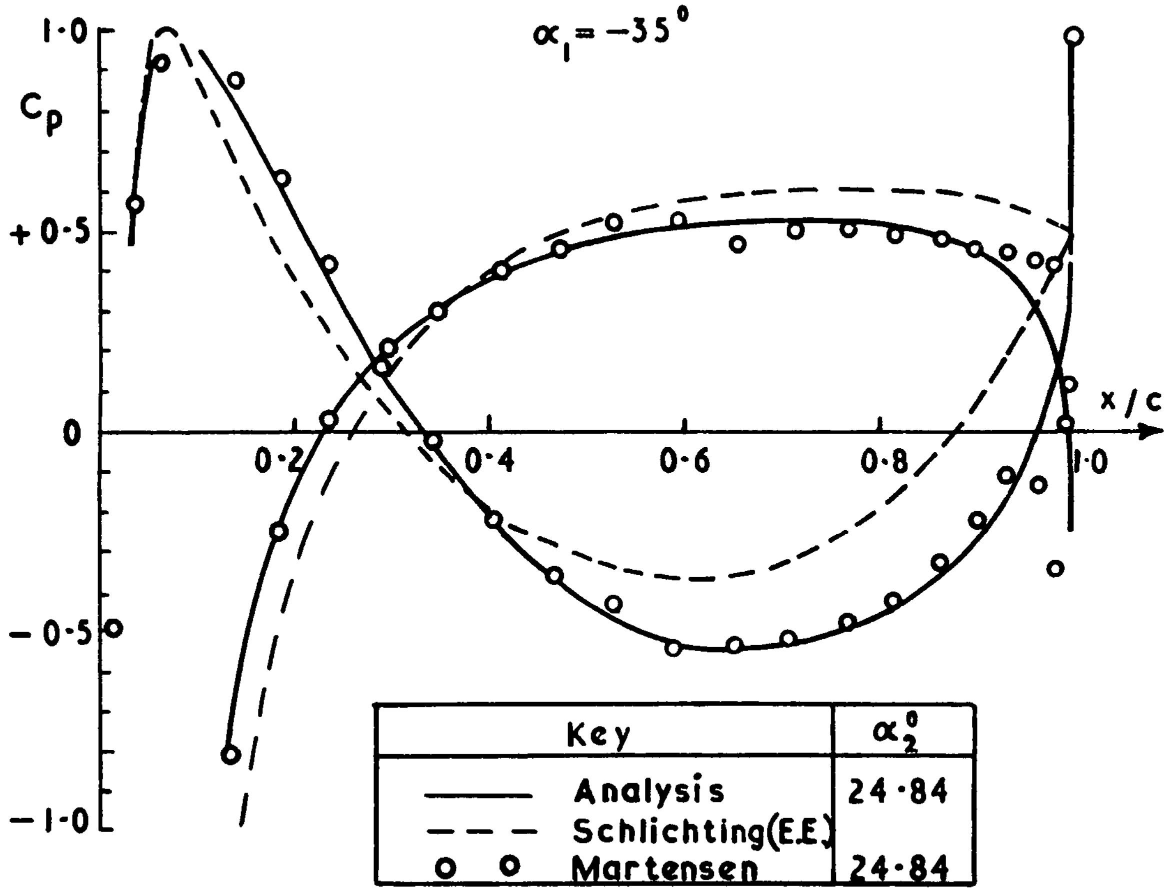
Pressure distributions for Merchant and Collar 'C4' type profile

FIG.11



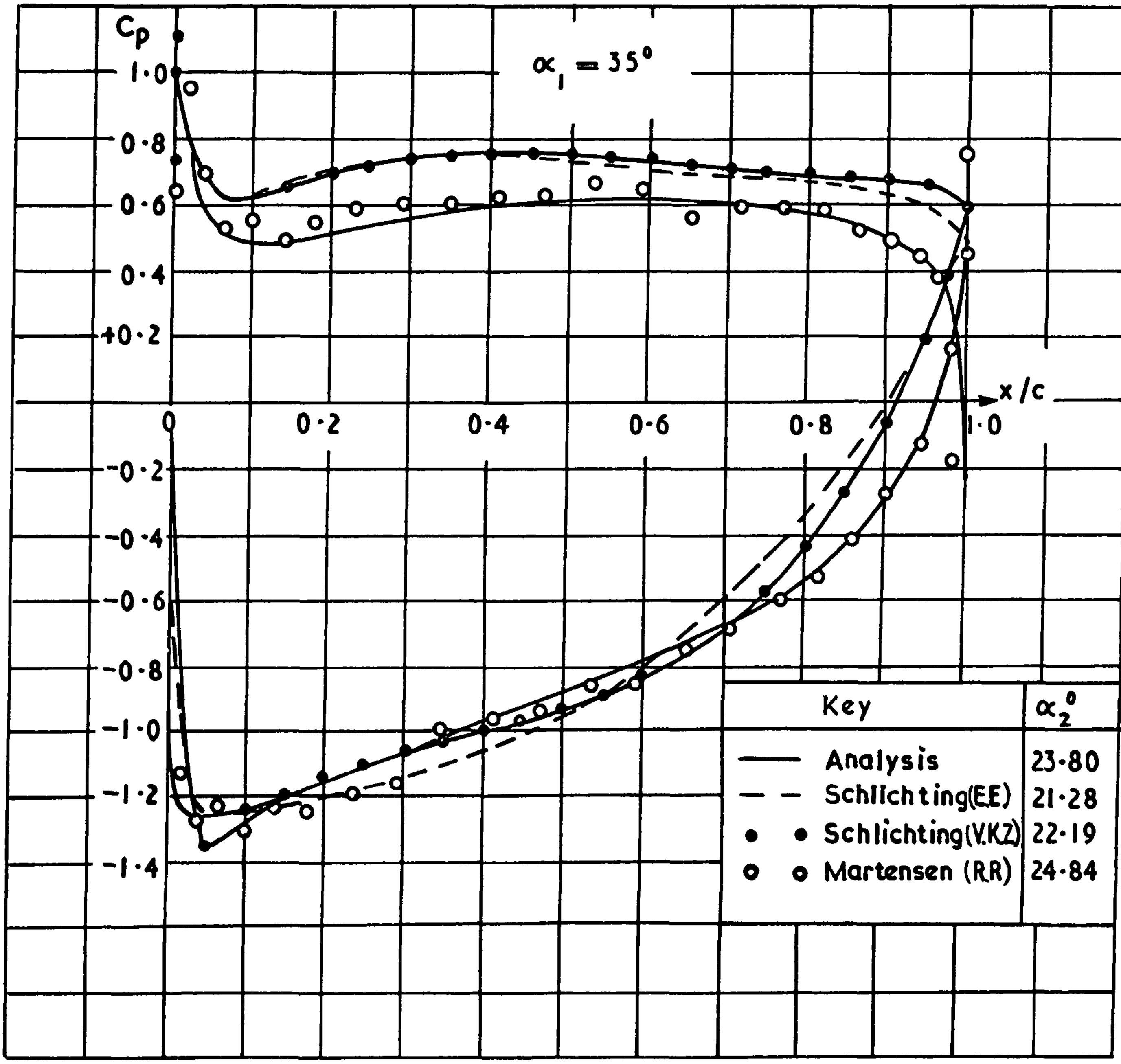
Highly cambered profile   Stagger = zero $s/c = 0 \cdot 90036434$

FIG. 12



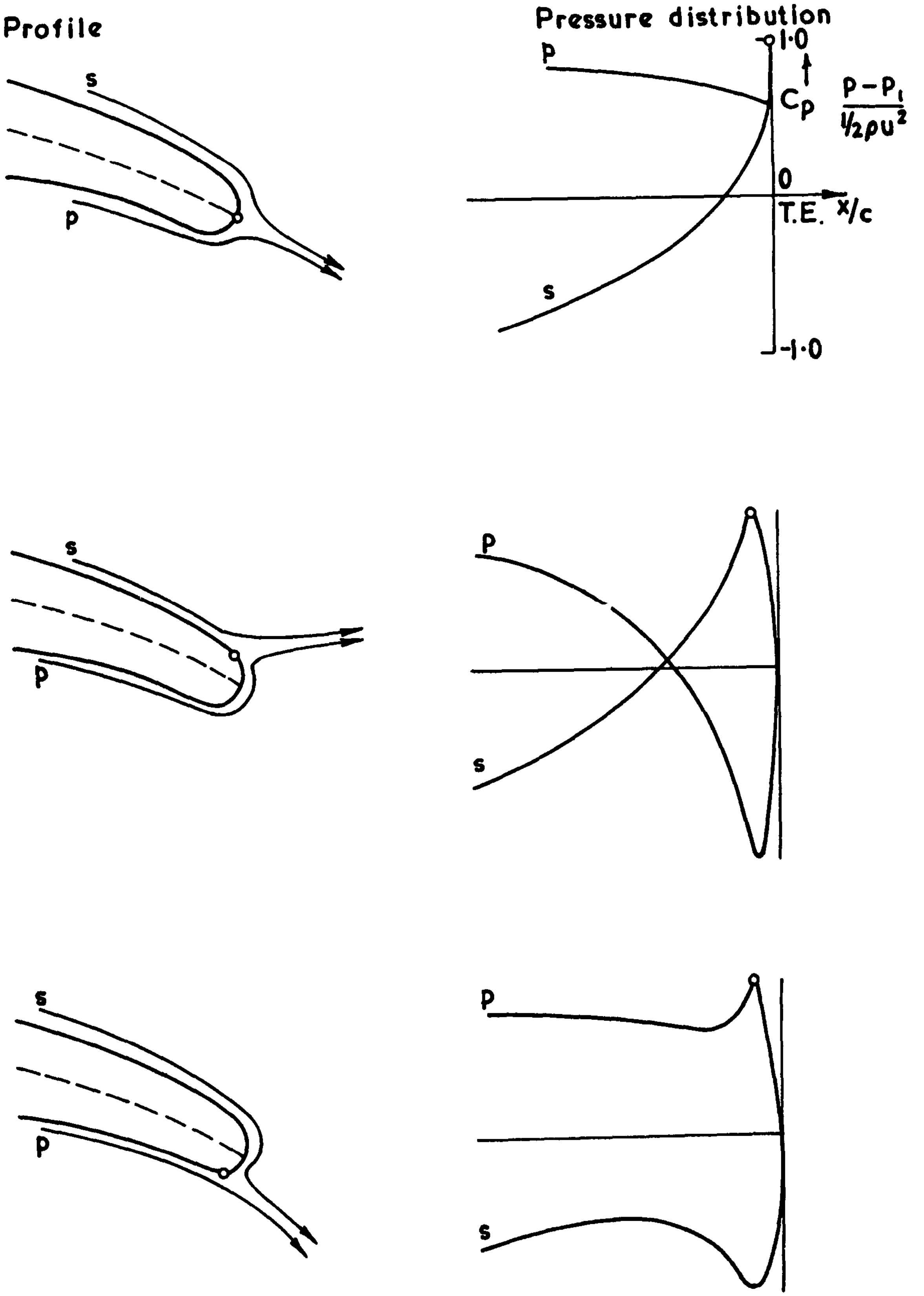
Pressure distributions for 70° camber aerofoil in cascade

FIG. 13



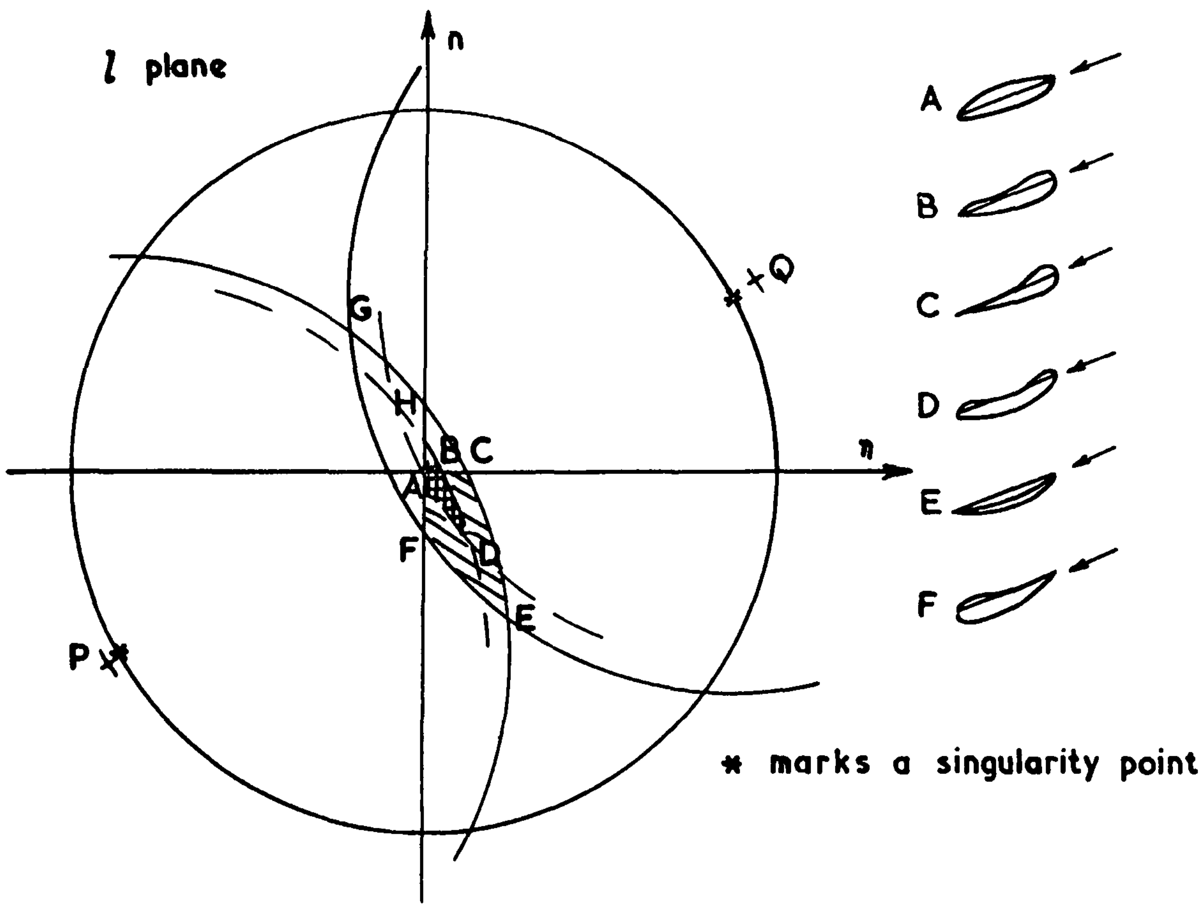
Pressure distributions for 70° camber aerofoil in cascade

FIG. 14



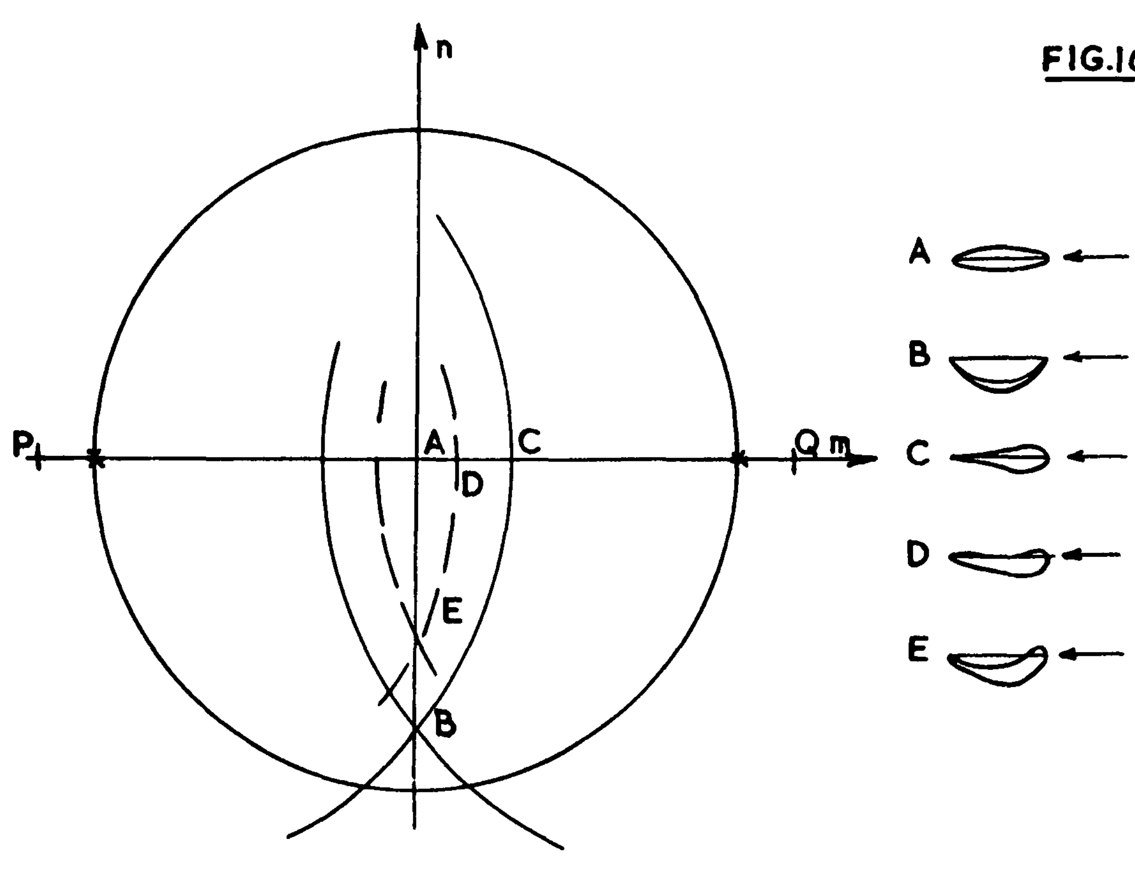
Exaggerated view of flow conditions at trailing edge

FIG.15



Schematic diagram – moderate stagger.

FIG.16



Schematic diagram – zero stagger.

A.R.C. C.P. No. 808
July, 1964
J. P. Gostelow

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EXTENSIONS TO AN EXACT THEORY

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