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An Experimental Investigation into the Determination of Lift from Wake Traverses at Supersonic Speeds

by

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AN EXPERIMENTAL INVESTIGATION INTO THE DETERMINATION OF LIFT FROM WAKE TRAVERSES AT SUPERSONIC SPEEDS

by

J. Wooller

M.C.P. Firmin

SUMMARY

Two theoretical results are derived by an approximate method for relating the lift on an object to the properties of the flow in its wake. An experimental investigation was devised, firstly to compare the flow properties with those indicated by horseshoe-vortex and linearised theories, and secondly, to compare the lift derived from those properties with the lift measured by a conventional strain-gauge balance. Results are presented which indicate a maximum difference of 6% between the measurements for the lift, provided the traverses enclose the complete viscous wake, but improved theory and apparatus are necessary if the lift is to be obtained more precisely by wake traverse methods.

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1 INTRODUCTION

The forces on models in supersonic wind tunnels can be measured in a number of ways. The most widely used method is that of the strain-gauge balance with the model mounted on a sting support. In certain circumstances this method necessitates distorting the shape of the model and therefore leads to an incorrect simulation. In other cases, such as interference problems, a sting support may be inadmissible, or the model may be too small to make the installation of a balance practicable. It is under these conditions that other methods of evaluating forces are required together with a knowledge of their accuracy. This investigation was devised to that end.

This Report compares the results of two methods of evaluating the lift of a wing from traverses in its wake with results obtained from a strain-gauge balance.

The wake traverse methods consist of measuring the local velocity perturbations round a contour in a cross flow plane downstream of the wing with a yawmeter, and from these measurements, either to evaluate the circulation within the contour, or to compute the velocity potential and cross-flow stream function on the contour. In the former method the circulation is assumed to be concentrated at two points in the cross-flow plane and its value is determined by measurements on a contour surrounding one of these points; the lift is then evaluated from the line-vortex method (equation (14)). In the second method, the lift is evaluated from numerical integration of the velocity potential and cross-flow stream function round a contour surrounding the wake. This method avoids making assumptions about the distribution of circulation.

The relations expressing the lift in terms of the induced velocity field (based on the assumption of small perturbations) are well known. Nevertheless it is thought worth while to recapitulate the derivation of these equations, in particular since equation (11) is less well known than its equivalent (8). At the same time it is convenient for an investigation such as this to review the assumptions involved. This is done in Section 2.

In Sections 3, 4 and 5 the test procedure, experimental equipment and experimental errors are discussed and in Section 6 the results are discussed in relation to known theories and the strain-gauge balance measurements.

In addition, in Section 6, the measurements of downwash and sidewash behind the wing are compared with those derived by the linearised theory method of Lagerstrom and Graham[1].

2 WAKE TRAVERSE METHODS

It is well known that from a knowledge of the flow parameters downstream of an object (i.e. wing, body, etc.) in supersonic flow it is theoretically possible to determine the pressure forces acting on it, but, in practice, a detailed enough survey of the flow field is not possible. Hence we will derive equations that may be used in practical applications, paying particular attention to any assumptions that must be made in their derivation. For convenience we will start by quoting Euler's momentum theorem[2], for the force ($\underline{F}$) on an object immersed in a volume of fluid, which may be written in vector notation as

$$\underline{F} = - \iint_S (P - P_\infty)\, d\underline{S} - \iint_S \rho\, U\, [U \,.\, d\underline{S}] \quad . \tag{1}$$

This equation applies to steady inviscid flow and S is the surface of the volume of fluid surrounding the object.

We may consider a sufficiently large volume of fluid surrounding the object so that all the disturbances in the flow due to the object pass through the surface (S_1) as shown in Fig.1, then if S_1 is perpendicular to the x direction and we restrict our attention to the lift we have

$$L = - \iint_{S_1} \rho\, w\, (U_\infty + u)\, dS \quad . \tag{2}$$

At supersonic speeds the flow may not be isentropic but it may be considered so for moderate deflections. It was, in fact, shown by Busemann[3] that the flow may be considered isentropic provided flow deflections are such that third-order terms may be neglected. Now, admitting the error which results from the assumption of an isentropic flow, we have from equation (2)

$$L = - \rho_\infty U_\infty^2 \iint_{S_1} \frac{w}{U_\infty}\, dS + \rho_\infty U_\infty^2 (M_\infty^2 - 1) \iint_{S_1} \frac{w}{U_\infty} \frac{u}{U_\infty}\, dS + \ldots \tag{3}$$

since

$$\rho = \rho_\infty \left[1 - M_\infty^2 \frac{u}{U_\infty} + \ldots \right] \tag{4}$$

The second term in equation (3) may be neglected if we require the lift to second-order accuracy only since $\frac{wu}{U_\infty^2}$ is then anti-symmetric about the plane $z = 0$.

The equation for the lift may therefore be written as

$$L = - \rho_\infty U_\infty^2 \iint_{S_1} \frac{w}{U_\infty} dS \quad . \tag{5}$$

This equation may be transformed into a contour integral by using Stokes theorem

$$\int_C \phi \, d\underline{C} = \iint_S [d\underline{S} \wedge \underline{\nabla}] \, \phi \quad . \tag{6}$$

If ϕ is taken as the velocity potential and S the surface S_1, then to satisfy the conditions of the theorem that ϕ must be defined and continuous on the contour C, and at all points on the surface, it is necessary to distort the contour as shown in Fig.2 and omit the wake. Then on equating vector components in equation (6) we get

$$- \int_{C_2} \phi \, dy = \iint_{S_1} w \, dS \tag{7}$$

since the contributions on the cut cancel and ϕ is constant on C_1.

Hence equation (5) may be written

$$L = \rho_\infty U_\infty \int_{C_2} \phi \, dy \tag{8}$$

which is the same as the well known result for the lift in terms of the change in potential across the wake, but here it has not been assumed that the wake has remained flat.

In practice the integral in equation (8) cannot be evaluated experimentally since the cross-flow velocity perturbations are changing rapidly near the wake, so some further approximation must be made. It has been assumed that in the region of C_2 the second derivative of ϕ in the x direction may be neglected compared with its second derivatives in the cross-flow plane (i.e. slender approximation) and the flow considered as quasi-two-dimensional, then we may write

$$w' = \phi + i \psi \tag{9}$$

where ψ may be considered as a stream function in the cross-flow plane and w' is the cross-flow plane complex potential function. If the "thickness" of the wake (i.e. the region of flow with vorticity) is small we have $\int_{C_2} \psi \, dZ = 0$ since the curve C_2 moves under the influence of the induced velocity field but contributions to the integral on the upper and lower side cancel each other. This assumption on the "thickness" of the vortical region is questionable and may partly explain the difference between balance and wake measurements (Fig.9). Hence

$$L = \rho_\infty U_\infty R\ell \int_{C_2} w' \, dZ \tag{10}$$

where $Z = y + i z$. The contour C_2 may be expanded within the above limitations and we get

$$L = \rho_\infty U_\infty \left\{ \int_{AB+CD} \phi \, dy - \int_{BC+DA} \psi \, dz \right\} \tag{11}$$

which may be used for one possible practical method of determining the lift (hereafter called the potential method).

If it is further assumed that the flow field may be represented by a symmetric distribution of horseshoe vortices then

$$w' = \sum_1^n i \frac{\Gamma_r}{2\pi} [\log (Z - Z_r) - \log (Z + Z_r)] \tag{12}$$

and on solving equation (10) we obtain

$$L = 2\rho_\infty U_\infty \sum_1^n \Gamma_r S_r \; . \tag{13}$$

If the vortex distribution is restricted to a single vortex pair by making the assumption that the vortices are fully rolled up with a span 2S' we get

$$L = 2\rho_\infty U_\infty \Gamma S' \tag{14}$$

and we have another possible practical method of determining the lift (hereafter called the circulation method) provided we can evaluate the circulation, Γ, within a contour surrounding one vortex and 2S' the span between the vortices.

Hence we have two possible methods of determining the lift on a wing by wake traverse methods, the first of which is applicable at stations downstream of the wing where the flow may be assumed quasi-two-dimensional and the other where the wake may be assumed fully rolled up into a pair of discrete vortices.

3 TEST PROCEDURE

Having devised two possible methods for determining the lift which involve the perturbation velocities up to second order, we now formulate a method to determine these perturbation velocities.

3.1 The measurement of circulation

The circulation within the contour A'BCD' of Fig.2 may be defined as

$$\Gamma = \int_{A'BCD'} \underline{q} \cdot \underline{dC} \tag{15}$$

from which we get

$$\frac{\Gamma}{U_\infty} = \int_{A'B+CD'} \frac{v}{U_\infty}\, dy + \int_{BC+D'A'} \frac{w}{U_\infty}\, dz \tag{16}$$

The velocity perturbations v and w may be determined from the measurements made with a conical yawmeter since, for inviscid flow, we have from Appendix A

$$\frac{v}{U_\infty} = \frac{M}{M_\infty}\left[\frac{\left(1+\frac{\gamma-1}{2}M^2\right)}{\left(1+\frac{\gamma-1}{2}M_\infty^2\right)\left(1+\tan^2\alpha+\tan^2\beta\right)}\right]^{\frac{1}{2}} \tan\beta \tag{17}$$

$$\frac{w}{U_\infty} = \frac{M}{M_\infty}\left[\frac{\left(1+\frac{\gamma-1}{2}M^2\right)}{\left(1+\frac{\gamma-1}{2}M_\infty^2\right)\left(1+\tan^2\alpha+\tan^2\beta\right)}\right]^{\frac{1}{2}} \tan\alpha \tag{18}$$

where α, β and M are measured. Thus Γ/U_∞ may be determined from equation (16) by numerical integration using equations (17) and (18) for the velocity perturbations.

In practice the use of a conical yawmeter for the measurement of flow parameters in a region where velocity gradients are large may be inaccurate due to the finite distance between the pressure points on the cone. Care must therefore be taken to minimise the influence of pressure gradients on the measurements by taking contours far enough away from the centres of vorticity and choosing contours with complementary pressure gradients.

As mentioned in the previous section in order to obtain the lift by the circulation method some assumption has to be made about the distribution of vorticity. If the contour is well behind the wing then it may be sufficient to assume that the wake is fully rolled up and then the vortex centre may be determined by locating the point of minimum pressure in the cross-flow plane. Where vorticity is measured away from the main centre of vorticity it has been assumed that it is concentrated at the centre of the contour within which it was measured and then equation (13) was used to determine the lift.

3.2 The measurement of velocity potential and cross-flow stream function

The velocity potential and stream function as defined in paragraph 2 may also be determined from the measurements made with the conical yawmeter. Since

$$\frac{v}{U_\infty} = \frac{1}{U_\infty}\frac{\partial\psi}{\partial y} = \frac{1}{U_\infty}\frac{\partial\psi}{\partial z} \tag{19}$$

and

$$\frac{w}{U_\infty} = \frac{1}{U_\infty}\frac{\partial\phi}{\partial z} = -\frac{1}{U_\infty}\frac{\partial\psi}{\partial y} \tag{20}$$

the change of potential or stream function between any two points (y, z) and (y_1, z_1) in the (y, z) plane can be determined by numerical methods from the following integrals.

$$\phi = \phi_1 + \int_{y_1}^{y} \frac{v}{U_\infty}\,dy + \int_{z_1}^{z} \frac{w}{U_\infty}\,dz \tag{21}$$

$$\psi = \psi_1 - \int_{y_1}^{y} \frac{w}{U_\infty}\,dy + \int_{z_1}^{z} \frac{v}{U_\infty}\,dz \quad . \tag{22}$$

Here the velocity perturbations are obtained in the same way as before for the circulation method. Hence the velocity potential and stream function may be determined at all points round the contour ABCD of Fig.2. It now follows that the lift of the wing may be determined by a further numerical integration using equation (11).

4 EXPERIMENTAL EQUIPMENT

To check the wake traverse methods it was necessary to mount a wing in a supersonic flow such that the loads acting on it could be determined from both the strain gauge balance technique and the wake traverse methods without the usual uncertainties about the influence of the supporting system. A wing of rectangular planform and a modified double wedge section was mounted on a reflection plate by a spanwise cantilever as shown in Fig.3. The support was enclosed in a wind-shield of the same section as the wing. The wind-shield was necessary to obtain a region on the wing where the flow is two-dimensional, so that the flow field due to an isolated tip could be studied. The dimensions of the cantilever and the wind-shield were chosen so that no shear existed between the fixed wind-shield and the wing when the latter was loaded at the estimated centre of pressure. The gap between the wind-shield and the wing was sealed (see Section 6.3.1). The balance consisted of four Baldwin AB19 strain gauges mounted on the spanwise cantilever so as to form a difference bridge to measure normal force.

The wing was set at a fixed incidence of 8 degrees so that reasonably large velocity perturbations existed without the leading edge shockwave being detached or the tip flow influencing the load on the wind-shield. The panel aspect ratio (A), using the exposed span as the wing semi-span, was 3.0, giving a value of $A\sqrt{M_\infty^2 - 1}$ of 4.5, The wind-shield was of sufficient span to eliminate any direct influence of the boundary layer on the reflection plate but if the boundary layer on the plate was distorted by the flow over the wind-shield it is possible that the flow in the wake of the wing could be influenced.

In a certain region behind the wing the flow was free from wall interference other than that mentioned above and was suitable for wake traversing. An isometric view of the model together with the general arrangement of the contours round which wake traverses were made is given in Fig.4. It will be noted that the contours were restricted to two chord lengths behind the wing due to wall interference. The five hole conical yawmeter of 0.1 inch diameter used to measure Mach number, total pressure and the inclination of the local flow to the main stream was the same as that described by Andrews and Sawyer[4].

All the measurements were made in the R.A.E. No.18 (9" × 9") supersonic wind tunnel which is a continuous-flow closed-circuit tunnel with a dry air interchange system. The tests were made at $M_\infty = 1.81$, the highest Mach number at which the tunnel would run with the present traversing gear, and under atmospheric stagnation conditions corresponding to a chord Reynolds number of 0.7×10^6. The absolute humidity was kept below the critical value of 0.0002 given by Raney and Beastall[5].

5 EXPERIMENTAL ERRORS

The yawmeter used for the wake traverse measurements was firstly calibrated in the empty tunnel, and an estimate of the possible errors indicated that for conditions of nearly uniform flow but otherwise similar to those to be encountered in the test the measurements should be within the following limits

M	±0.005
α, β	±0.15°
$\frac{v}{U_\infty}$, $\frac{w}{U_\infty}$	±0.003

As mentioned in Section 3.1 at points close to a line vortex the Mach number and pressure change rapidly, and therefore it is possible that measurements made with a yawmeter are in error due to the non-uniformity of the flow over the head. In the present experiment care was taken to avoid making measurements too close to the wake of the wing, and hence any errors involved were small enough to be neglected in the determination of the lift since all the contours were arranged to have complementary pressure gradients.

An investigation of the flow in the wind tunnel with the reflection plate mounted from the side wall, but without the wing or wind-shield, indicated that the Mach number in the region where measurements were made was 1.805 ±0.010. The flow inclinations showed negligible vorticity but indicated a mean sidewash (β) of 0.10° and a mean upwash (α) of 0.35°. Corrections to all measurements were made for these flow inclinations.

Since the strain-gauge balance measured the force normal to the chordal plane only, a correction was necessary due to the chordwise force in order to evaluate the lift. Further details of this correction will be given in the discussion. Taking into account a possible error in this correction of ±25%, and inaccuracies in the measurement of the normal force, it is unlikely that the error in the lift exceeded ±1%.

6 DISCUSSION OF RESULTS

6.1 Cross-flow velocities behind the wing

Figs.5 and 6 show the downwash and sidewash measured at two stations ($x/\bar{c}$ = 1.75 and 3.0) aft of the leading edge of the wing. For all the theoretical predictions the origin of the coordinate system has been fixed with respect to the wake. The z = 0 plane is taken to pass through the point of minimum pressure by a translation of the axes system in the z direction. The origin is therefore different for each downstream location according to the movement of the point of minimum pressure which is also taken to indicate the movement of the tip vortex for the appropriate test incidence of eight degrees (Fig.7).

At the station furthest downstream the measured flow inclinations (Fig.6) agree well with those predicted by linearised theory using the method of Lagerstrom and Graham. At the station closest to the trailing edge (Fig.5) there is some evidence of the inadequacy of linearised theory, especially in the region of the tip Mach cones, where the Mach cones obtained by linearised theory are modified by the local Mach number changes over the wing, and are different on the upper and lower surfaces.

According to the linearised theory assumptions the flow leaves the trailing edge as a flat vortex sheet in the $z = 0$ plane and all the vorticity is contained in that part of the wake downstream of the trailing edge between the tip and the point where the Mach cone cuts the trailing edge. Moreover the vorticity and its distribution remain constant as it passes downstream. Since the experimental velocity distributions are in general agreement with the linearised theory no major change in the distribution of vorticity is evident from these results, but the velocity distribution for the line vortex method of Mirels and Haefeli[6], which corresponds to the distribution for a fully rolled up wake at all stations, is not sufficiently different from the linearised result to allow a firm conclusion to be drawn about the redistribution of vorticity. The results from the line vortex method of Mirels and Haefeli are given in Figs.5 and 6. In general the experimental results are more in accordance with those given by linearised theory than by the lifting line theory. Near the Mach cone the line vortex method fails since it gives infinite values for the velocity perturbations. The flow field given by Mirels and Haefeli degenerates into a simple infinite line vortex in the Trefftz plane, and the flow inclinations due to such a vortex have been included in Figs.5 and 6 to show the marginal difference between the station $x/c = 3.0$ and the Trefftz plane.

6.2 Measurements of circulation

As mentioned before, according to linearised theory all the trailing vorticity is confined to a certain region in the wake and remains constant as it passes downstream. Therefore any contour in a plane perpendicular to the mainstream surrounding this region of the wake contains all the vorticity generated by the wing and the value of the circulation is independent of the streamwise location of the contour.

In order to check the application of the method of Lagerstrom and Graham for the cross-flow velocity perturbations, the value of the circulation was calculated for a number of contours of various sizes and at different streamwise locations. The larger contours enclosed all the vorticity, according to linearised theory, and gave the maximum value of $\frac{\Gamma}{U_\infty C} = \frac{2i}{\sqrt{M_\infty^2 - 1}}$, while the contours which did not enclose all the vorticity gave a lower result as shown in Fig.8.

Fig.8 also shows how the circulation measured around a contour of fixed size varies for different streamwise locations. There is no evidence from these results to suggest that the wake rolls up in such a way that the circulation becomes more concentrated as it moves downstream. In fact if any trend exists it is for the circulation within the smaller contour to decrease with streamwise location. Linearised theory would suggest that about 6% of the vorticity remains outside the small contour, while all the circulation should remain within the large contour as it moves downstream.

The oil-flow technique was used on the surface of the wing and it indicated that the flow separated from the second shoulder on the upper surface. This is consistent with all the experimental results being well below the linearised theory values.

In addition to the linearised theory, a simple wave theory and two shock expansion theory results have been given. In obtaining them it has been assumed that the circulation within a contour surrounding the two dimensional part of the wing has all been turned into trailing vorticity in the region of the wing tip. The simple wave theory results and the upper value for the shock expansion theory are for fully attached flow, while for the lower value the flow was assumed to separate from the second shoulder, with the free streamline in the free stream direction. This result, together with the one for fully attached flow, gives a possible limit to the change in circulation to be expected due to a separation from the upper surface. The measured circulations are all consistent with a loss in circulation due to flow separation.

In deriving the results of Section 2 it was assumed that the flow was inviscid and small velocity perturbations were considered such that terms higher than the second order in the velocity perturbations could be neglected. This could well lead to inaccuracies due to the finite strength of the shockwaves, since then the flow is rotational and it is not clear what part of the vorticity generated at the wing is converted into trailing vorticity.

If all the vorticity generated at the wing is converted into trailing vorticity then an idea of the order of possible inaccuracies may be obtained by considering the applicability of the Kutta-Joukowski theorem at supersonic speeds. At subsonic speeds and also for small perturbations at supersonic speeds it is well known that the theorem holds. Further, on applying simple wave theory i.e. isentropic flow, it is found that the Kutta-Joukowski theorem still holds but when shock-expansion theory, i.e. inviscid flow but not necessarily isentropic or small perturbations, is applied $L \neq \rho_\infty U_\infty \Gamma$. This must be entirely due to the gain in entropy through the shockwaves. For the particular wing tested the leading edge shockwave was fairly strong and applying the shock-expansion method showed that $\rho_\infty U_\infty \Gamma$ was greater than the lift per unit span (L) by about 9%. Hence a large uncertainty exists about the influence of entropy changes on the wake traverse methods. It should also be noted that if vorticity is created as a result of entropy changes the analysis in Section 2 does not apply and this could explain, in part, the low values for the measured circulation.

6.3 Results for the lift

6.3.1 Strain-gauge balance result

On the assumption that it was necessary to prevent flow through the gap between the wing and the wind-shield from the lower to the upper surface of the wing, the gap was sealed with adhesive tape for the wake traverse measurements*. Contrary to previous experience, however, the tape introduced hysteresis and a constraint on the balance. In determining the lift by the strain-gauge balance, therefore, the gap of 0.008 inch was sealed with rubber solution for which the hysteresis and constraint proved negligible. Also in determining the lift coefficient a small correction for the chordwise force was made, since the balance measured only the force normal to the chordal plane. For the correction, shock-expansion theory was used, ignoring the tip effect, to estimate the pressure force; and an estimate of the skin friction, assuming a turbulent boundary layer, was also included. The total correction due to the chordwise force amounted to approximately 2% of the normal force. If the skin friction had been estimated assuming a laminar boundary layer the correction would have been 1.8% instead of 2%. The resultant lift coefficient (C_{L_B}) was 0.311.

6.3.2 Wake traverse results

In Fig.9 results derived for the lift by both wake traverse methods are given as a fraction of the lift measured by the strain-gauge balance. The results derived from the measured circulations were computed as stated in Section 3.1 and are in fair agreement with those derived from the potential method. The results from the small contours are, of course, low, but the results from the large contours are not as high as might be expected in view of the discussion in Section 6.2. At the station $x/\bar{c} = 3.0$ the results from the lift coefficient were different for each wake traverse method but were identical for each of the large contours of height $0.5\bar{c}$ and $0.6\bar{c}$. Extensions of the contours outboard as shown in Fig.4 produced results for both wake traverse methods within 1% of the result for the large contours A'B'CD and E'F'GH. At the forward station $x/\bar{c} = 1.75$ the result for the large contour A'B'CD using the circulation method is about 3% greater than that for the potential method, but is less than 1% greater than the result for the small contour ABCD using the circulation method. As before, extending the contour outboard made little difference to the results; +1% to 2% for the circulation method and ±1% for the potential method.

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* Subsequent measurements without any sealing showed that the loss of lift through a gap of 0.008 inch was negligible, suggesting that, in fact, sealing was unnecessary.

7 CONCLUSIONS

Two theoretical results are derived relating the lift of a wing to the properties of the flow in its wake. An experimental investigation was devised, firstly, to compare the flow properties so obtained with those indicated by theory, and secondly, to compare the lifts derived from those properties with the lift measured by a conventional strain-gauge balance. Results of this analysis lead to the following conclusions.

(1) The cross-flow velocities behind a wing are reasonably well predicted by linearised theory and the agreement improves with distance downstream.

(2) The difference between the circulation derived from the cross-flow velocities and that given by linearised theory can be accounted for by flow separation on the upper surface.

(3) An uncertainty exists in the lift coefficient derived from the measured flow inclinations due to neglecting entropy gains through the shock-waves in deriving the theoretical results for the wake traverse methods, but even so, all the measurements using the largest contours were within 6% of the strain-gauge balance measurement.

(4) The wake-traverse method has been shown to be an acceptable method of measuring the lift on a wing at supersonic speeds within the accuracy of the present equipment.

Appendix A

DERIVATION OF THE CROSS-FLOW VELOCITIES FROM THE CROSS-FLOW INCLINATIONS

The velocities in the cross-flow plane are given by

$$\frac{v}{U_\infty} = \frac{(U_\infty + u)}{U_\infty} \tan\beta = \left(1 + \frac{u}{U_\infty}\right) \tan\beta \tag{23}$$

and

$$\frac{w}{U_\infty} = \frac{(U_\infty + u)}{U_\infty} \tan\alpha = \left(1 + \frac{u}{U_\infty}\right) \tan\alpha \quad . \tag{24}$$

Now from the energy equation

$$a^2 + \tfrac{1}{2}(\gamma - 1) q^2 = a_\infty^2 + \tfrac{1}{2}(\gamma - 1) U_\infty^2 \tag{25}$$

we have

$$\left(\frac{a}{a_\infty}\right)^2 = \left[\frac{1 + \frac{1}{2}(\gamma - 1) M_\infty^2}{1 + \frac{1}{2}(\gamma - 1) M^2}\right] \tag{26}$$

and so

$$\left(\frac{M}{M_\infty}\right)^2 = \left[\frac{(U_\infty + u)^2 + v^2 + w^2}{U_\infty^2}\right] \left[\frac{1 + \frac{1}{2}(\gamma - 1) M_\infty^2}{1 + \frac{1}{2}(\gamma - 1) M^2}\right] \tag{27}$$

hence,

$$\left(1 + \frac{u}{U_\infty}\right)^2 + \left(\frac{v}{U_\infty}\right)^2 + \left(\frac{w}{U_\infty}\right)^2 = \left[\frac{M^2}{M_\infty^2} \quad \frac{1 + \frac{1}{2}(\gamma - 1) M^2}{1 + \frac{1}{2}(\gamma - 1) M_\infty^2}\right] . \tag{28}$$

On substituting for $\frac{v}{U_\infty}$ and $\frac{w}{U_\infty}$ from equations (23) and (24) we have,

$$\left(1 + \frac{u}{U_\infty}\right)^2 [1 + \tan^2\beta + \tan^2\alpha] = \left[\frac{M^2}{M_\infty^2} \quad \frac{1 + \frac{1}{2}(\gamma - 1) M^2}{1 + \frac{1}{2}(\gamma - 1) M_\infty^2}\right] \tag{29}$$

We may now substitute for $\left(1 + \frac{u}{U_\infty}\right)$ in equations (23) and (24) and derive the cross-flow velocities in terms of M_∞ , which is known, and M, α and β, which are measured.

Hence

$$\frac{v}{U_\infty} = \frac{M}{M_\infty}\left[\frac{1 + \frac{1}{2}(\gamma - 1)\ M^2}{(1 + \frac{1}{2}(\gamma - 1)\ M_\infty^2)\ (1 + \tan^2 \beta + \tan^2 \alpha)}\right]^{\frac{1}{2}} \tan \beta \qquad (30)$$

and

$$\frac{w}{U_\infty} = \frac{M}{M_\infty}\left[\frac{1 + \frac{1}{2}(\gamma - 1)\ M^2}{(1 + \frac{1}{2}(\gamma - 1)\ M_\infty^2)\ (1 + \tan^2 \beta + \tan^2 \alpha)}\right]^{\frac{1}{2}} \tan \alpha \qquad (31)$$

SYMBOLS

a	sonic velocity
C, C_1, C_2	contours of integration
$\bar{c}$	wing chord
C_L	lift coefficient of wing
$d\underline{S}$	element of area $\underline{S}$ having the direction of the outward normal
$\underline{F}$	vectorial force
i	incidence of wing to free stream
L	lift of wing
L_B	lift of wing as measured on balance
M	Mach number
Ox, y, z	orthogonal right hand system of axes with origin at wing root leading edge; Ox in free stream direction, Oy to starboard
p	static pressure
q	total velocity in cross flow plane; $\sqrt{v^2 + w^2}$
s	wing-panel span
S'	semi-span of horseshoe vortex
S_r	position in y direction of rth vortex
S_1	area of surface of integration perpendicular to free stream
U	total velocity
u, v, w	perturbation velocities in x,y,z directions
w'	cross-flow plane complex potential function
y'	distance inboard from tip
α	angle of flow in xz plane; $\tan^{-1} \frac{w}{U_\infty + u}$
β	angle of flow in xy plane; $\tan^{-1} \frac{v}{U_\infty + u}$
γ	ratio of the specific heats
Γ_r	circulation due to rth vortex
ρ	density
ϕ	perturbation velocity potential
ψ	perturbation stream function

Suffix

∞	refers to free stream conditions

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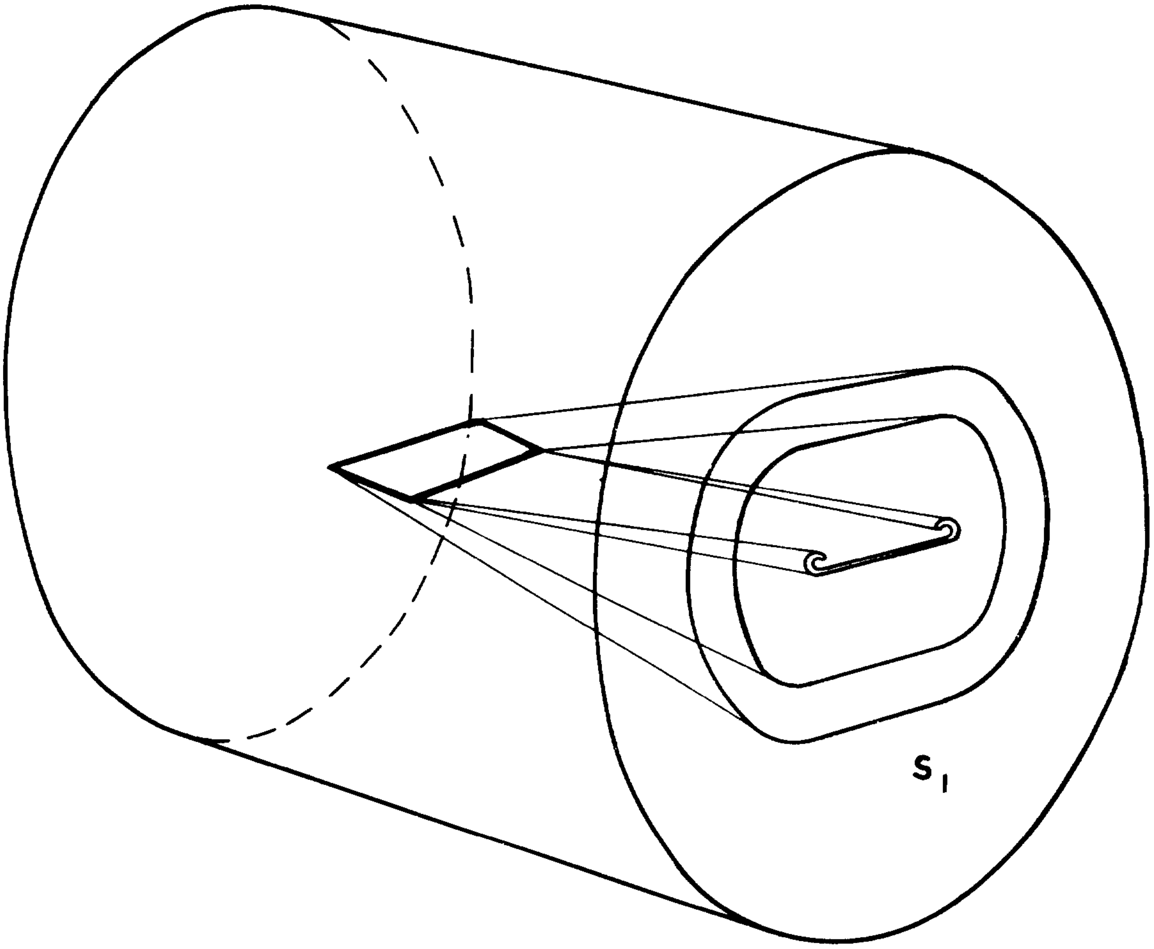

FIG.1 SURFACE OF INTEGRATION AROUND MODEL

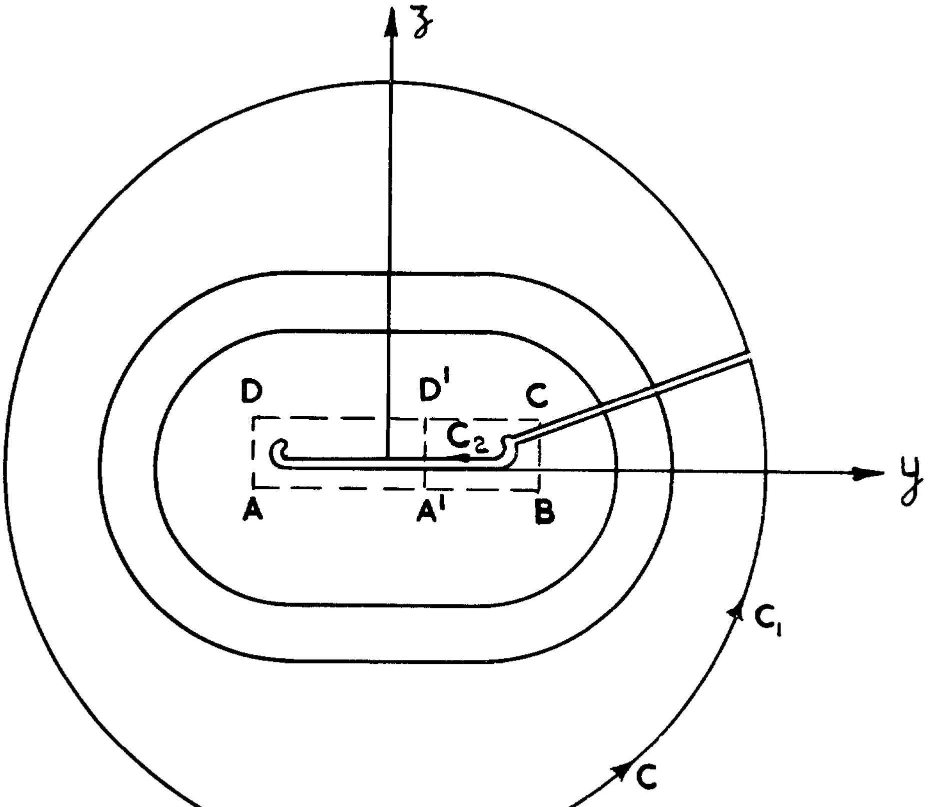
z
y
D
D'
C
C2
A
A'
B
C1
C

FIG.2 CROSS FLOW SURFACE S_1

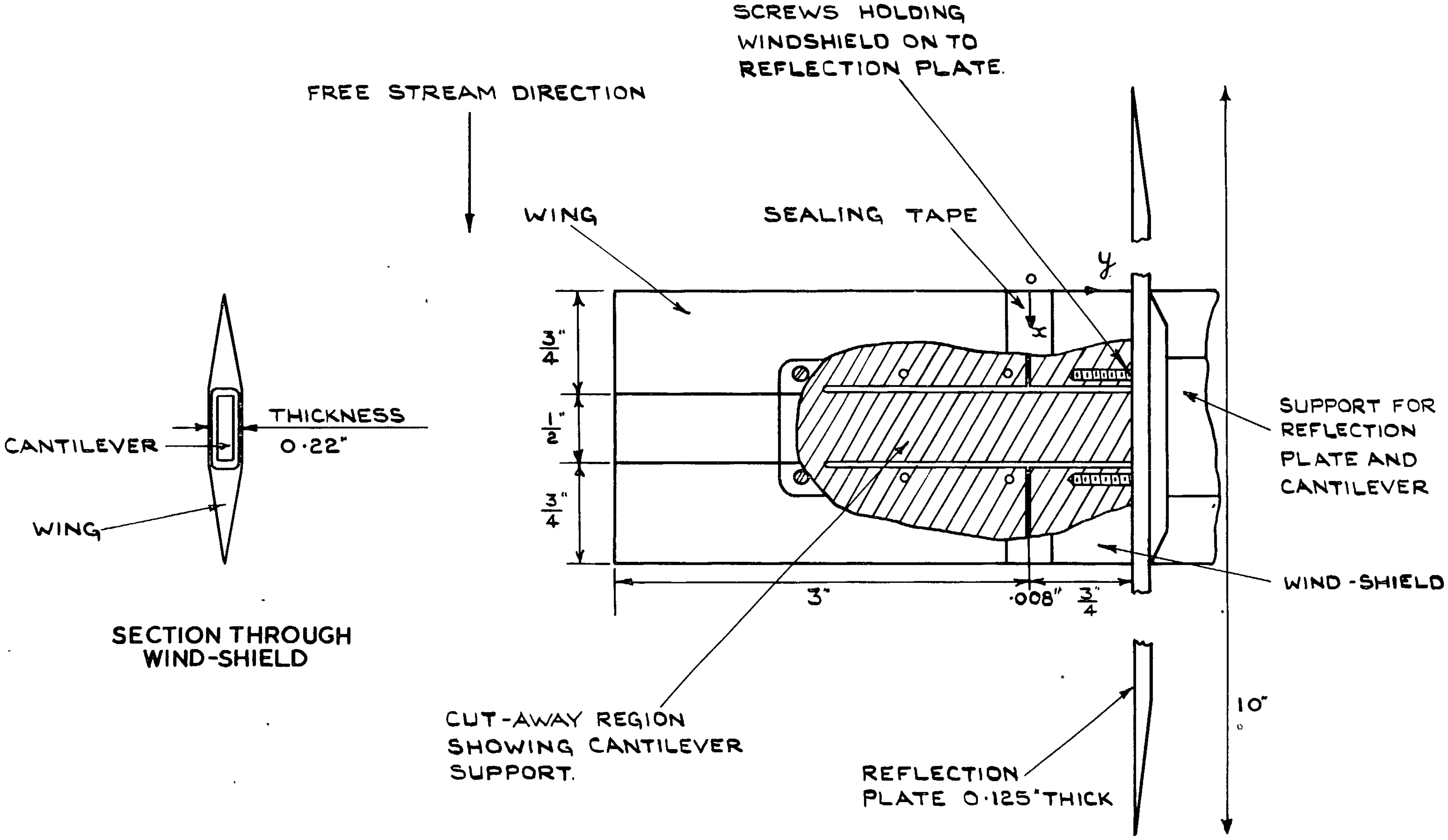
SCREWS HOLDING WINDSHIELD ON TO REFLECTION PLATE.
FREE STREAM DIRECTION
WING
SEALING TAPE
y
x
o
3"/4
1"/2
3"/4
SUPPORT FOR REFLECTION PLATE AND CANTILEVER
WIND-SHIELD
3"
·008"
3"/4
10"
THICKNESS 0·22"
CANTILEVER
WING
SECTION THROUGH WIND-SHIELD
CUT-AWAY REGION SHOWING CANTILEVER SUPPORT.
REFLECTION PLATE 0·125" THICK

FIG. 3 DETAILS OF THE MODEL

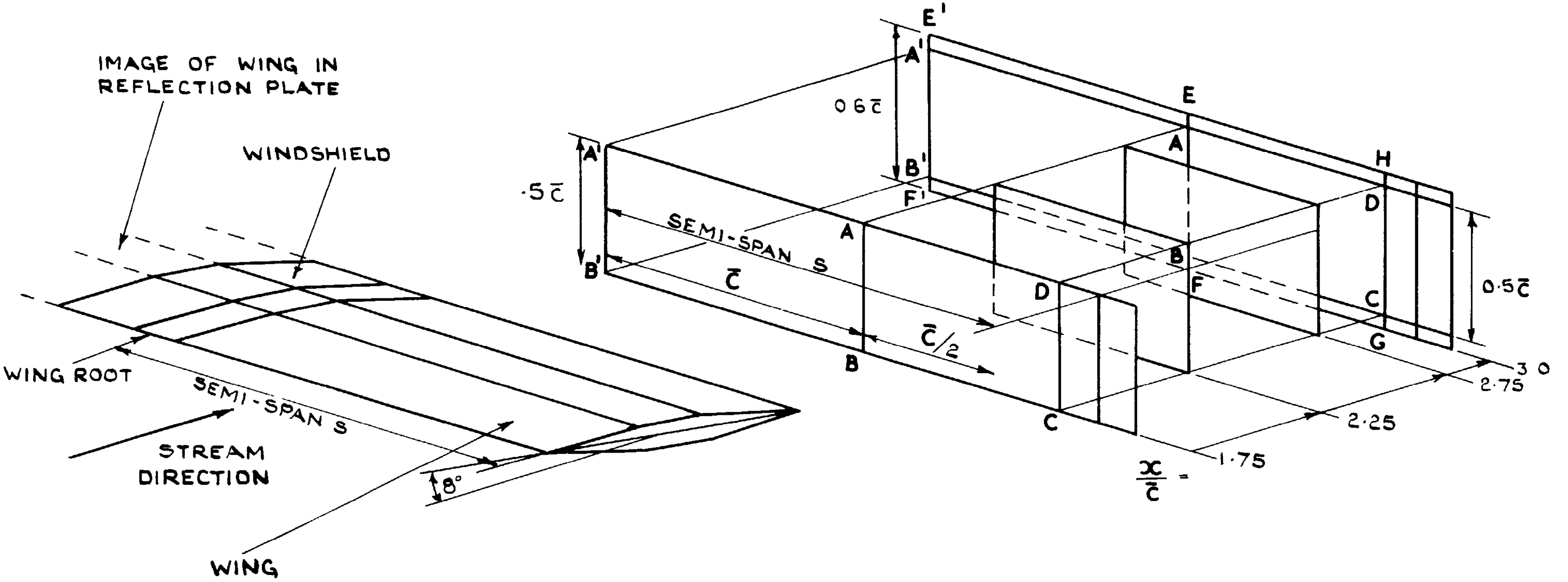
IMAGE OF WING IN REFLECTION PLATE
WINDSHIELD
WING ROOT
SEMI-SPAN S
STREAM DIRECTION
8°
WING
E'
A'
0.6$\bar{c}$
B'
F'
E
A
H
D
B
F
C
G
A'
·5$\bar{c}$
B'
SEMI-SPAN S
$\bar{c}$
A
B
$\bar{c}/2$
D
C
0.5$\bar{c}$
3.0
2.75
2.25
1.75
$\frac{x}{\bar{c}}$ =

WING CHORD = $\bar{c}$

x MEASURED FROM L E

NOTE :- EACH CONTOUR WAS CENTRALISED IN z DIRECTION
WITH RESPECT TO MEASURED LOCATION OF VORTEX (FIG 7)
WHEREAS IN y DIRECTION IT WAS FIXED WITH
A' B' DOWNSTREAM OF WING ROOT

FIG. 4 DIAGRAMMATIC VIEW OF MODEL SHOWING POSITIONS OF CONTOURS

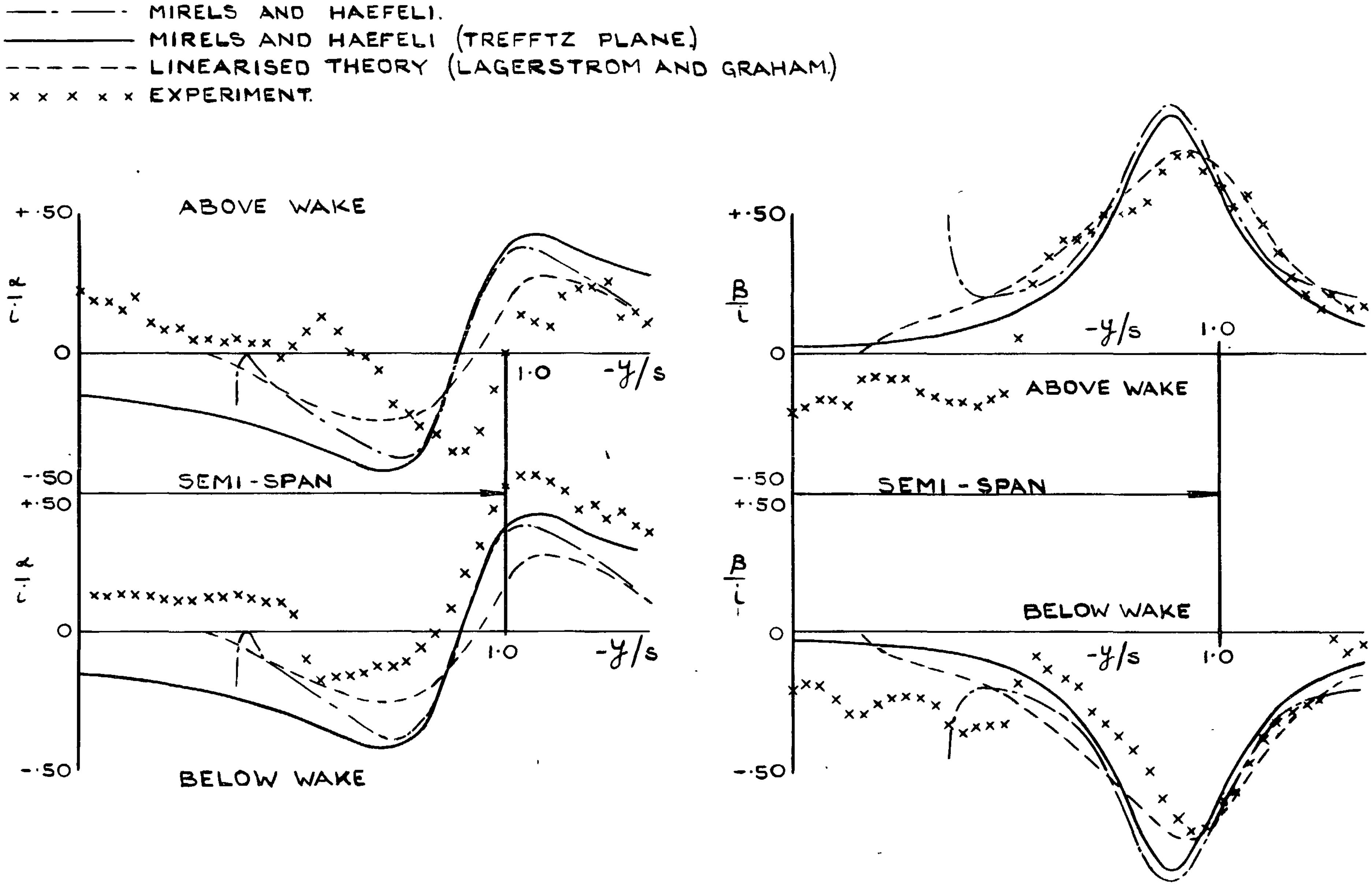

FIG. 5 CROSS-FLOW ANGLES AT $x/\bar{c} = 1{\cdot}75$, CONTOUR HEIGHT $0{\cdot}5\,\bar{c}$

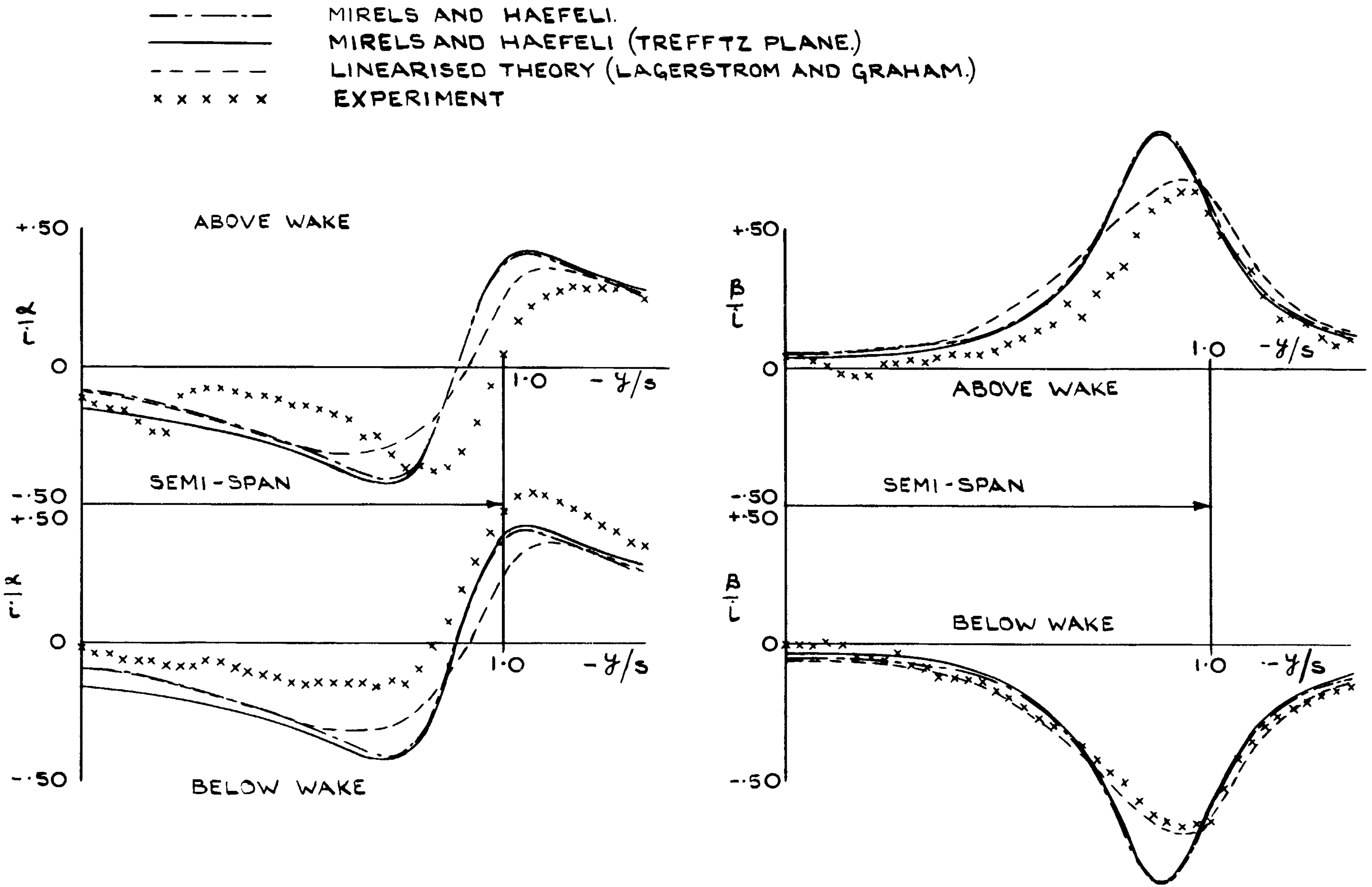

FIG. 6 CROSS-FLOW ANGLES AT $x/\bar{c} = 3{\cdot}0$, CONTOUR HEIGHT $0{\cdot}5\,\bar{c}$

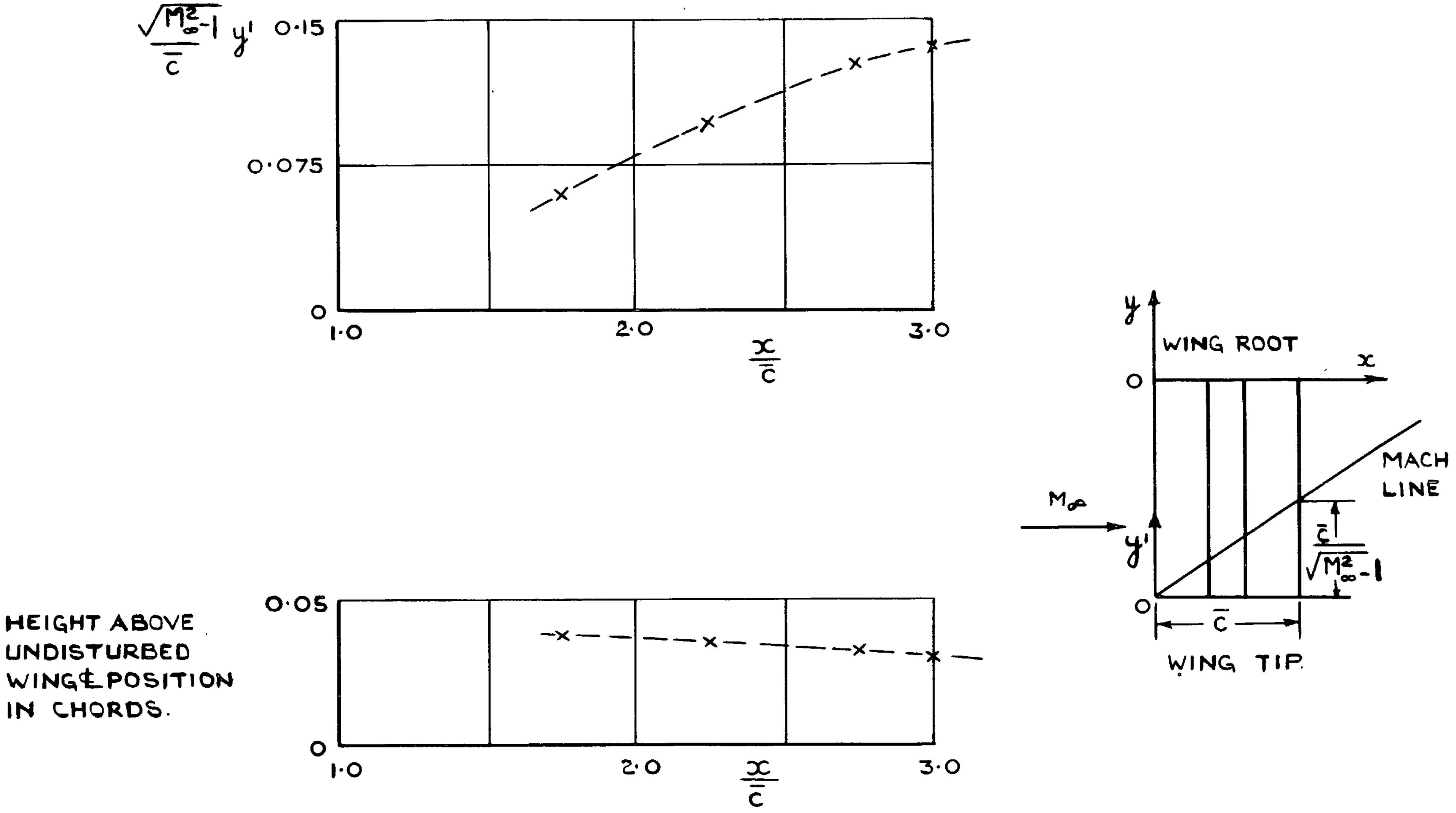

FIG. 7. MOVEMENT OF TIP VORTEX

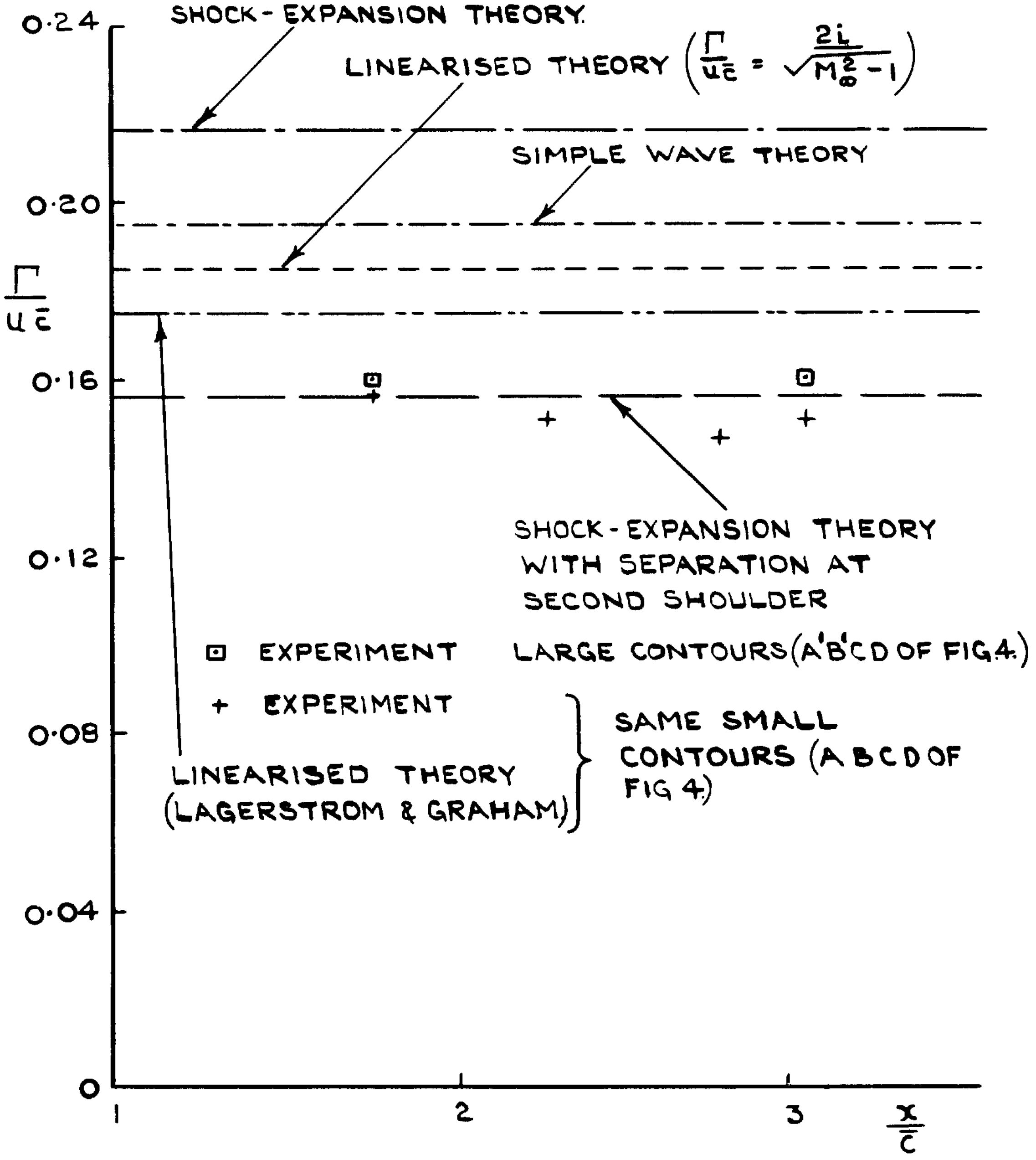

FIG. 8 VARIATION OF CIRCULATION WITH DISTANCE DOWNSTREAM OF LEADING EDGE

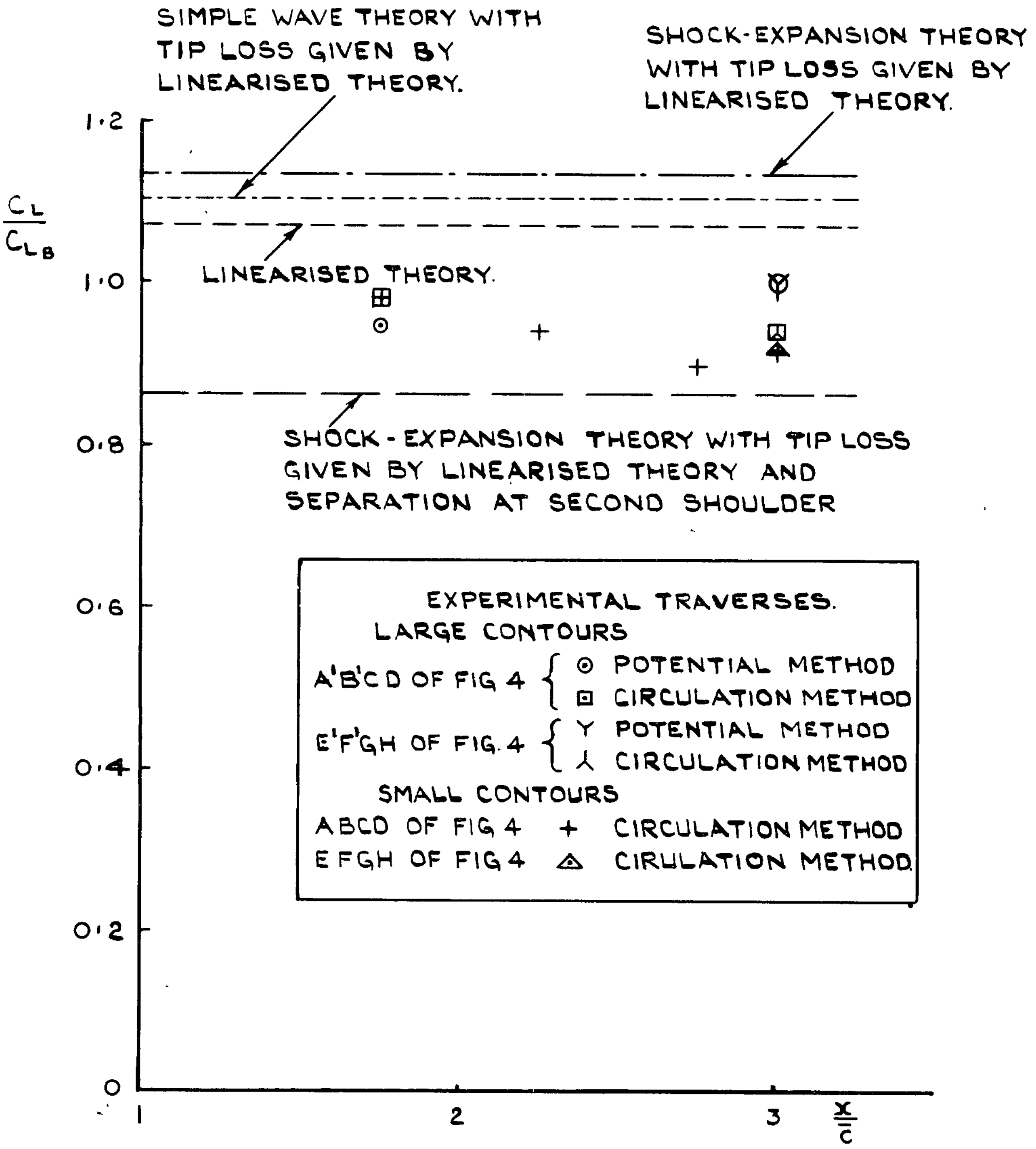

FIG.9 LIFT COMPARED WITH THAT MEASURED BY THE BALANCE

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Wooller, J. 533.6.048.3:
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533.6.011.5

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Two theoretical results are derived by an approximate method for relating the lift on an object to the properties of the flow in its wake. An experimental investigation was devised, firstly to compare the flow properties with those indicated by horseshoe-vortex and linearised theories, and secondly, to compare the lift derived from those properties with the lift measured by a conventional strain-gauge balance. Results are presented which indicate a maximum difference of 6% between the measurements for the lift, provided the traverses enclose the complete viscous wake, but improved theory and apparatus are necessary if the lift is to be obtained more precisely by wake traverse methods.

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