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by

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SUMMARY

A type of rolling manoeuvre suitable for structural design load calculations is defined. The governing equations of motion are given in basic form and in a form suitable for general practical application. Their solution is briefly discussed and it is seen that a strict adherence to the specified conditions leads to difficulties. A simpler approach is given: examples show that results agree acceptably well with those from the strict approach, and also that a qualitative assessment of the closeness of this agreement can in each case be made.

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CONTENTS

1 INTRODUCTION

One of the manoeuvres considered fundamental in the estimation of structural design loads is the aileron-induced manoeuvre, the aircraft responding predominantly in roll but also in other degrees of freedom. The current approach is to divide the load calculation into the determination of the "rigid body" response of the aircraft to aileron application and, subsequently, the corresponding distribution of net external load*. The former part presents greater difficulty from the computational point of view and is the subject of this Report. The equations and analysis are developed with ease of application to digital computers in mind: the availability of automatic computing facilities is essential in tackling the present problem.

The type of manoeuvre to be considered is defined fairly precisely and the equations of motion are developed in a form suitable for general application. This is desirable since, although the "response" method has been widely accepted for some time as the standard technique for this loading problem, the lack of a document describing in some detail both the problem and methods for its solution has led to two major difficulties. The first is that the various groups engaged in such work have adopted a number of variations on the basic scheme and, while each may have its particular merits, such diversity makes communication difficult. Secondly, it is necessary for the aerodynamicist to appreciate what information the structural design team requires and the form in which it should be given: equally, the data given by the aerodynamicist must be immediately understood. Lack of standardisation can cause the waste of time and effort; providing a basis for avoiding this is one of the purposes of this paper.

The other purpose is to present and briefly illustrate an approach, likely to be of practical value to the designer, which produces calculations of the motion of the aircraft for a fraction of the effort involved in a strict adherence to the defined conditions while giving results sufficiently accurate at least for preliminary work. Moreover, the reliability of the approach in any case can be assessed qualitatively from the results produced: this is an important consideration in any approximate method. In general, then, the use of this simplified approach is thought to effect a good compromise

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*The above division of the calculation is strictly valid only if the aircraft can be considered rigid. Some account may be taken of aeroelastic effects by the "method of modification of derivatives"[1]; however, as has been noted[2], care must be taken in the application of this "quasi-static" technique to essentially dynamic problems.

between precision and convenience in most cases, while exceptional cases can be recognised and treated by more rigorous and laborious techniques.

2 DEFINITION OF THE MANOEUVRE

The specification of the type of aileron-induced rolling manoeuvre to be considered for structural design purposes can be approached in a variety of ways since, if it is assumed that the initial conditions are fixed, requirements can be formulated in terms of the aircraft kinematics at the end of the manoeuvre, the aileron input used, and the form of the response itself. The definition of the manoeuvre will usually be achieved by the choice of specified conditions from at least two of these. Only one such choice is considered in detail here, although much of the paper is clearly more generally applicable.

In seeking requirements for modern aircraft we wish to choose a manoeuvre which at any speed and altitude is structurally at least as severe as any feasible rolling manoeuvre at that speed and altitude. Therefore this manoeuvre should accommodate any adverse effects that arise from aerodynamic damping deficiencies, inertia and aerodynamic coupling, or violent pilot action. The following requirements, which allow their numerical specification to be chosen to suit a particular aircraft or class of aircraft, are in accordance with these desires.

R(i) The aircraft is to roll through a given angle of bank.

R(ii) At the final bank angle the rate of roll shall be zero.

R(iii) The aileron input is to be of a specified form.

This last requirement has purposely been stated in a general fashion since, although detailed analysis is given below for only one form of aileron input, the main lines of approach would not be altered should any other form be substituted. Aileron input here refers to the deflection of the control surfaces, not the pilot's control. The above requirements might not be satisfactory should there be an indirect relationship between pilot action and control surface deflection, for example if rate demand control were employed.

3 THE EQUATIONS OF MOTION

The equations of rigid body motion as given herein apply to the principal inertia axes system, Gxyz, shown in Fig.1. The orientation of these axes in space is defined with respect to a system of axes, GXYZ, of fixed orientation by the Eulerian angles θ, ϕ, ψ shown in Fig.2*. GX and GY are horizontal, GZ is vertical; the direction of GX (and hence that of GY) is left undefined since ψ does not enter the present analysis but is included for completeness.

As is usual when considering rapid manoeuvres in which the velocity component u is not of primary interest, we regard this as constant, so eliminating one equation of motion. Further, the total velocity V is taken as constant and it is assumed that one can simplify the kinematic terms involving u by setting u/V equal to unity.

3.1 Basic form of the equations

Under the conditions and assumptions of the preceding section the equations of motion are, in basic form

$$\frac{WV}{g}\left(\frac{d\beta}{dt} + r - p\alpha\right) = Y + W\cos\theta\sin\phi \qquad (1)$$

$$\frac{WV}{g}\left(\frac{d\alpha}{dt} + p\beta - q\right) = Z + W\cos\theta\cos\phi \qquad (2)$$

$$A\frac{dp}{dt} - (B - C)\,qr = L \qquad (3)$$

$$B\frac{dq}{dt} - (C - A)\,rp = M - M_E r \qquad (4)$$

$$C\frac{dr}{dt} - (A - B)\,pq = N + M_E q \qquad (5)$$

where $\alpha = w/V$, $\beta = v/V$**.

M_E is the sum of the angular momenta of the rotating parts of the engines due to rotation about their own axes (here assumed to be parallel to Gx), that of a particular engine being positive for rotation in the sense of positive p. Y,Z are the total aerodynamic forces along Gy and Gz; L,M,N are the total aerodynamic moments about Gx, Gy, and Gz.

*A definition of these angles may be found in Ref.3, the system $G\overline{xyz}$ therein being replaced by the system GXYZ above.

**With these definitions, α and β do not represent easily recognisable angles: however, for small values they are very near to the radian measure of the angles of incidence and sideslip. For convenience, then, they will be termed incidence and sideslip and be regarded as being in radians; their values multiplied by 180/π will be regarded as being in degrees.

3.2 Practical forms of the equations

As the first step in recasting equations (1) to (5) in a form amenable to numerical treatment the forces and moments Y,Z,L,M,N are expressed in terms of velocities and control angles. The expressions which are chosen depend upon the purpose for which the equations are to be used, the relative importance of various quantities in each expression, and, to a considerable extent, upon the amount of theoretical and experimental data which the user hopes will be available to him. Bearing in mind the current state of the art in predicting and measuring aerodynamic forces, and anticipating the initial conditions of Section 3.2.2, the following expressions are chosen:

$$Y = \tfrac{1}{2}\rho V^2 S \left(\frac{\partial C_y^*}{\partial\beta}\beta + \frac{\partial C_y^*}{\partial p} p + \frac{\partial C_y^*}{\partial r} r + \frac{\partial C_y^*}{\partial\xi}\xi\right) \tag{6}$$

$$Z = \tfrac{1}{2}\rho V^2 S \left(\bar{C}_z + \frac{\partial C_z^*}{\partial\alpha}\alpha + \frac{\partial C_z^*}{\partial\eta}\eta\right) \tag{7}$$

$$L = \tfrac{1}{2}\rho V^2 S\, b \left(\frac{\partial C_\ell^*}{\partial\beta}\beta + \frac{\partial C_\ell^*}{\partial p} p + \frac{\partial C_\ell^*}{\partial r} r + \frac{\partial C_\ell^*}{\partial\xi}\xi\right) \tag{8}$$

$$M = \tfrac{1}{2}\rho V^2 S\, c \left(\bar{C}_m + \frac{\partial C_m^*}{\partial\alpha}\alpha + \frac{\partial C_m^*}{\partial\dot\alpha}\dot\alpha + \frac{\partial C_m^*}{\partial q} q + \frac{\partial C_m^*}{\partial\eta}\eta\right) \tag{9}$$

$$N = \tfrac{1}{2}\rho V^2 S\, b \left(\frac{\partial C_n^*}{\partial\beta}\beta + \frac{\partial C_n^*}{\partial p} p + \frac{\partial C_n^*}{\partial r} r + \frac{\partial C_n^*}{\partial\xi}\xi\right) . \tag{10}$$

$$\bar{C}_z = [C_z]_{\alpha,\eta=0} \qquad \bar{C}_m = [C_m]_{\alpha,\dot\alpha,q,\eta=0} .$$

The starred differentials indicate that these are taken to be variable with the quantity with respect to which the differentiation takes place, and with incidence. For example, $\partial C_y^*/\partial\beta$ is assumed to be a function of β and α. In the majority of cases the available data can be adequately fitted by expressing the differentials as linear functions of α alone: this form has been used in the analysis and examples of Section 6. In such a case the differentials are written, for example

$$\frac{\partial C_y^*}{\partial\beta} = \frac{\partial C_y}{\partial\beta}^{(0)} + \alpha\,\frac{\partial C_y}{\partial\beta}^{(1)} .$$

This same notation is retained for the generalisation of the usual aerodynamic derivatives. That is y_v is generalised to y_v^*, for instance, and we write by analogy with the above expression

$$y_v^* = y_v^{(0)} + \alpha\, y_v^{(1)} .$$

When the force and moment expressions (6) to (10) are substituted into equations (1) to (5) the arrangement of these equations in terms of aerodynamic derivatives may be made in a variety of ways. The following set of equations has been found very satisfactory for practical application since a minimum of multiplicative constants is used and real time is preserved.

$$\hat{t}\,\frac{d\beta}{dt} = \hat{t}\,(p\alpha - r) + y_v^*\,\beta + \left(\frac{b}{2V}\right) y_p^*\,p + \left(\frac{b}{2V}\right) y_r^*\,r + y_\xi^*\,\xi + F\cos\theta\sin\phi \qquad (11)$$

$$\hat{t}\,\frac{d\alpha}{dt} = \hat{t}\,(q - p\beta) + \bar{z} + z_w^*\,\alpha + z_\eta^*\,\eta + F\cos\theta\cos\phi \qquad (12)$$

$$\gamma_A\,\frac{dp}{dt} = \varepsilon_A\,qr + \ell_v^*\,\beta + \left(\frac{b}{2V}\right) \ell_p^*\,p + \left(\frac{b}{2V}\right) \ell_r^*\,r + \ell_\xi^*\,\xi \qquad (13)$$

$$\gamma_B\,\frac{dq}{dt} = \varepsilon_B\,rp + \bar{m} + m_w^*\,\alpha + \left(\frac{\ell}{V}\right) m_{\dot{w}}^*\,\dot{\alpha} + \left(\frac{\ell}{V}\right) m_q^*\,q + m_\eta^*\,\eta - E_1\,r \qquad (14)$$

$$\gamma_C\,\frac{dr}{dt} = \varepsilon_C\,pq + n_v^*\,\beta + \left(\frac{b}{2V}\right) n_p^*\,p + \left(\frac{b}{2V}\right) n_r^*\,r + n_\xi^*\,\xi + E_2(q - q_o) \qquad (15)\dagger$$

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† When equation (5) is recast to give equation (15), the right hand side contains the term $E_2\,q$. However, retention of this term is contrary to the initial conditions of 3.2.2 since in symmetric flight there would be an unbalanced yawing moment giving non-zero acceleration in yaw. We assume that this is balanced by a small rudder deflection which contributes negligible side force and rolling moment, and hence replace $E_2\,q$ by $E_2(q - q_o)$ as above.

where

$$\bar{z} = \tfrac{1}{2}\,\bar{C}_z \qquad \bar{m} = \frac{c}{2\ell}\,\bar{C}_m$$

$$F = \frac{W}{\rho S V^2} \qquad \hat{t} = \frac{VF}{g} = \frac{W}{g \rho S V}$$

$$E_1 = \frac{F M_E}{W\ell} \qquad E_2 = \frac{F M_E}{W(b/2)}$$

$$\gamma_A = \frac{FA}{W(b/2)} \qquad \gamma_B = \frac{FB}{W\ell} \qquad \gamma_C = \frac{FC}{W(b/2)}$$

$$g_A = \frac{F(B - C)}{W(b/2)} \qquad g_B = \frac{F(C - A)}{W\ell} \qquad g_C = \frac{F(A - B)}{W(b/2)} \; .$$

Some further notation may conveniently be given here, although it does not occur in the analysis of this paper. This is

$$f_A = \frac{B - C}{A} \qquad f_B = \frac{C - A}{B} \qquad f_C = \frac{A - B}{C} \; .$$

It will be noted that $g_A = \gamma_A f_A$ etc.

Equations (11) to (15), together with the kinematic relations

$$\frac{d\theta}{dt} = q \cos \phi - r \sin \phi \tag{16}$$

$$\frac{d\phi}{dt} = p + q \tan \theta \sin \phi + r \tan \theta \cos \phi \tag{17}$$

are those which are used to compute the response of the aircraft.

3.2.1 Notes on the aerodynamic derivatives

Two points concerning the aerodynamic terms in the above equations should be noted.

(1) To simplify the kinematic terms, α is defined as w/V and β as v/V. The aerodynamic terms must be given with regard to these same definitions, and not alternative forms such as $\arctan(w/V)$. Also, principal inertia axes are used rather than stability axes. Wind tunnel results should be analysed with these points in mind as subsequent transformation of derivatives is time-consuming and inaccurate.

(2) The incremental values of response quantities can be quite large and hence it is more important to be able to represent the aerodynamic forces with good accuracy for fairly large departures from the initial motion than with closer accuracy over a restricted range. Expressions for the derivatives must be chosen accordingly. In some cases a set of derivatives, all of which are adequate over the whole range of initial normal accelerations, n_o, cannot be found; it is then necessary to use different expressions when n_o lies in particular ranges, the computed response indicating whether the range of validity of a particular expression has been exceeded, in which case the computation is repeated with an alternative one.

3.2.2 The initial conditions

The above practical equations of motion, (11) to (15), have been developed while anticipating the initial conditions, i.e. the aircraft's motion at the start of the manoeuvre, described in this section.

The aircraft is in a symmetric pull-up or push-over at a specified normal acceleration, n_o, the flight path being approximately horizontal and $\dot{\alpha}$ and $\dot{q}$ being instantaneously zero. This condition is not in general steady due to the changing normal component of the weight; however the resulting accelerations are small over a fairly long time. (The term "quasi-steady flight in a vertical circle" is sometimes applied to this condition.) More precisely, the plane of symmetry xGz (Fig.1) is taken to be vertical and the aircraft's initial motion is in this plane and unaccelerated with respect to the axes Gxyz. Then from equations (12) and (14) are obtained

$$-\left(\bar{z} + z_w^* \alpha_o + z_\eta^* \eta_o\right) = \hat{t}\, q_o + F \cos \theta_o \tag{18}$$

and

$$\bar{m} + m_w^* \alpha_o + \left(\frac{\ell}{V}\right) m_q^* q_o + m_\eta^* \eta_o = 0 \ . \tag{19}$$

Now the normal acceleration, n_o, is given and

$$n_o = \cos \theta_o + q_o V/g \tag{20a}$$

or

$$n_o F = \hat{t}\, q_o + F \cos \theta_o \ . \tag{20b}$$

Equation (18) then becomes

$$-\left(\bar{z} + z_w^* \alpha_o + z_\eta^* \eta_o\right) = n_o F \ . \tag{21}$$

For simplicity it is usual to take $\theta_0 = 0$ and hence q_0 is obtained from equation (20a), and α_0 and η_0 from equations (19) and (21).

It has been usual to obtain from the equations of motion only incremental values of response quantities, α_0 and q_0 having been found by preliminary calculations. However, with the above representation of the aerodynamics, which is meant to cover large variations in incidence, it is possible to retain the same aerodynamic data for varying values of n_0. Then equations (19) and (21) may be solved within the main response calculation program: this is of particular value if the aerodynamic derivatives depend markedly on α, necessitating an iterative method of solution.

4 THE AILERON INPUT FUNCTION

The form of the aileron input function demanded by R(iii) is assumed in the present report to be as shown in Fig.3(a), that is a "double trapezoidal" function. This is uniquely defined by seven parameters: the rates of aileron application $1/\kappa_1$, $1/\kappa_2$, $1/\kappa_3$; the angles ξ_1, ξ_2; and the times t_1 and t_2. Of these $1/\kappa_1$, $1/\kappa_2$, $1/\kappa_3$ are in all cases taken to be specified and, with the exception mentioned in section 6.2, it is further assumed that ξ_1 and ξ_2 are also given. Then requirements R(i) and R(ii) are sufficient to determine t_1 and t_2.

In the majority of cases each of the five given quantities will take its numerically greatest value (appropriate to the flight conditions) so that the manoeuvre is a limiting one, the possible exceptions being in cases where the manoeuvre loads imply that because of jack stalling, for instance, these values cannot be attained. We assume that modifications introduced to cover these cases will not alter the general form of the requirements and so do not pursue this point any further. It may be mentioned that the idealisation to infinite rates of aileron application ($\kappa = 0$) is not considered desirable since this leads to unrealistic responses, particularly in roll, as well as causing programming difficulties.

5 CALCULATION OF THE RESPONSE TO AILERON INPUT

The set of differential equations (11) to (17) may be solved on a digital computer for known input $\xi(t)$ by any suitable marching process: there is usually a standard process readily available to the programmer and so this report will not discuss the various alternatives. Experience with a particular process will soon enable questions such as the best choice of marching step to be decided.

The greatest difficulty in practice is that, within R(iii), the exact function $\xi(t)$ which will satisfy R(i) and R(ii) cannot be computed before the full response calculation is performed. One can only find an approximation, compute the response, and then adjust $\xi(t)$ in the light of the results obtained. If R(i) were the only one not satisfied this would cause little trouble as one could then merely perform the response calculations for differing bank angles and interpolate to find the required information: however, it is often the case that the initial estimate of $\xi(t)$ leads to R(ii) being badly violated, thus necessitating adjustment of $\xi(t)$. Such a state of affairs is very serious from the designer's standpoint since much time and effort is needed in this adjustment process. In fact if the aircraft's flight regime is to be adequately investigated the employment of such a technique for every flight condition considered would be quite unpractical. The designer's need is first to decide on a fairly small number of manoeuvres among which are those giving critical design loads and then, possibly, to investigate these more fully. To satisfy this a technique is sought which can be applied with a minimum of effort, still assuming the use of a digital computer, and which yields results accurate enough to allow reliable assessment of a manoeuvre's severity.

6 A SIMPLIFIED APPROACH TO CALCULATING THE RESPONSE

6.1 Outline of the method

For practical purposes it is sufficiently accurate to apply R(i) to $\int p\,dt$ rather than to bank angle and thus both R(i) and R(ii) are requirements applying to the time history in roll, p(t), alone (see equation (17)). Hence by choice of p(t) one can ensure that R(i) and R(ii) are simultaneously satisfied. If, then, this function p(t) is substituted into equations (11) to (17) the remainder of the response quantities may be computed, again by a marching process.

Within this basic idea there are many possible practical approaches, the differences between them stemming from the ways in which p(t) is determined. If p(t) be chosen with little or no regard for the actual dynamic properties of the aircraft in a particular flight condition then the results of the response calculation cannot be expected to be reliable. This lack of realism is often most clearly shown in the prediction of rather curious aileron movements.

We assume that the "direct" terms in the rolling equation are sufficient to describe the manner in which the aircraft is manoeuvred to a given bank angle, the aileron input being of the required form. Consideration of these terms alone allows the response in roll which satisfies R(i) and R(ii) to be computed. This derived function p(t) is then used as an input to the set of equations (11) to (17): equation (13) is now not a differential equation but is used simply to obtain ξ.

In determining p(t) one also finds, at least implicitly, the function $\xi(t)$ which would lead to the satisfaction of R(i) (with ϕ replaced by $\int p\,dt$) and R(ii) if the response in roll were indeed governed only by the "direct" terms: in this ideal case the substitution of $\xi(t)$ into equations (11) to (17) would produce results identical with those of substituting p(t) into the same equations. The approach suggested here of considering p(t) as the input function results in the effects of additional terms in the rolling equation being reflected in $\xi(t)$ differing from the form of R(iii). However, it will be seen in some later examples that this lack of conformity is often not gross and that the remainder of the response quantities agree fairly well with those found by the more laborious technique of adjusting $\xi(t)$ so that R(i) and R(ii) are met while preserving precisely the form demanded by R(iii).

The method is similar to that of Pinsker[4] but extends his analysis to more general responses in roll, and also places emphasis on $\xi(t)$ remaining close to the required form.

6.2 Analysis of the method

It is convenient to introduce an alternative notation to describe the aileron input function, as illustrated in Fig.3(b). This notation and that of Fig.3(a) will be used concurrently as economy and convenience demand.

Retaining only the "direct" terms in equation (13) we obtain

$$\gamma_A \frac{dp}{dt} = \left(\frac{b}{2V}\right) \bar{\ell}_p\, p + \bar{\ell}_\xi\, \xi \quad . \tag{22}$$

$\bar{\ell}_p$ and $\bar{\ell}_\xi$ are some constant values of ℓ_p^* and ℓ_ξ^*, the choice of which is somewhat arbitrary: they will often be chosen to agree with ℓ_p^* and ℓ_ξ^* at or near α_0, but if they depend markedly on α, and the incremental values of α are large, other values may be more suitable.

Equation (22) is rewritten in the form

$$\left\{\frac{d}{dt} - \varepsilon\right\} p = \frac{\bar{\ell}_\xi}{\gamma_A}\, \xi \tag{23}$$

where $\varepsilon = \dfrac{b}{2V}\,\dfrac{\bar{\ell}_p}{\gamma_A}$.

The general solution of this is

$$p = A \exp(\varepsilon t) + p^* \tag{24}$$

in which $p^* = \frac{\bar{\ell}_\xi}{\gamma_A}\left\{\frac{d}{dt} - \varepsilon\right\}^{-1} \xi$.

†With $\xi(t)$ as defined in Section 4 we have ξ taking one of two forms, depending on the value of t:

$$\xi = \bar{\xi}$$

or

$$\xi = \bar{\xi} + \frac{1}{\kappa}(t - \bar{t}) \quad ,$$

with the corresponding expressions for p^*

$$p^* = -\frac{\bar{\ell}_\xi}{\varepsilon\,\gamma_A}\,\xi \tag{25a}$$

$$p^* = -\frac{\bar{\ell}_\xi}{\varepsilon\,\gamma_A}\left(\xi + \frac{1}{\varepsilon\kappa}\right) . \tag{25b}$$

Now let $A = A_n$ for $T_n \leqslant t \leqslant T_{n+1}$; then

$$p = A_0 \exp(\varepsilon t) - \varepsilon\delta t/\kappa_1 - \delta/\kappa_1 \ , \quad 0 \leqslant t \leqslant T_1 \tag{26.0}$$

$$p = A_1 \exp(\varepsilon t) - \varepsilon\delta\xi_1 \ , \quad T_1 \leqslant t \leqslant T_2 \tag{26.1}$$

$$p = A_2 \exp(\varepsilon t) - \varepsilon\delta\xi_1 - \varepsilon\delta(t - T_2)/\kappa_2 - \delta/\kappa_2 \ , \quad T_2 \leqslant t \leqslant T_3 \tag{26.2}$$

$$p = A_3 \exp(\varepsilon t) - \varepsilon\delta\xi_2 \ , \quad T_3 \leqslant t \leqslant T_4 \tag{26.3}$$

$$p = A_4 \exp(\varepsilon t) - \varepsilon\delta\xi_2 - \varepsilon\delta(t - T_4)/\kappa_3 - \delta/\kappa_3 \ , \quad T_4 \leqslant t \leqslant T_5 \tag{26.4}$$

where $\delta = \frac{\bar{\ell}_\xi}{\varepsilon^2\,\gamma_A}$.

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†The subsequent analysis assumes that none of κ_1, κ_2, κ_3 is zero. As has been stated above, the use of infinite rates of aileron application is not desirable; should they be used the analysis may easily be modified by eliminating the redundant time intervals and proceeding to the limiting forms of certain expressions.

Since $p(0) = 0$, and $p(t)$ is continuous

$$A_o = \frac{\delta}{\kappa_1} \tag{27.0}$$

$$A_1 = A_o - \frac{\delta}{\kappa_1} \exp(-\varepsilon T_1) \tag{27.1}$$

$$A_2 = A_1 + \frac{\delta}{\kappa_2} \exp(-\varepsilon T_2) \tag{27.2}$$

$$A_3 = A_2 - \frac{\delta}{\kappa_2} \exp(-\varepsilon T_3) \tag{27.3}$$

$$A_4 = A_3 + \frac{\delta}{\kappa_3} \exp(-\varepsilon T_4) \quad . \tag{27.4}$$

As ξ_1, ξ_2, κ_1, κ_2, κ_3 are assumed to be given, $\xi(t)$ is uniquely defined by the quantities t_1 and t_2: the corresponding function $p(t)$ then follows from equations (26) and (27). t_1 and t_2 are determined as follows.

First, condition R(i) is applied to $p(t)$, in the form

$$\int_0^{T_5} p(t)\, dt = \bar{\phi} \quad .$$

Integrating equation (23) and applying the conditions $p(0) = p(T_5) = 0$ we have

$$-\varepsilon \bar{\phi} = \frac{\bar{\ell}_\xi}{\gamma_A} \int_0^{T_5} \xi \, dt \tag{28}$$

which in this case becomes

$$\bar{\phi} = -\varepsilon\delta \{t_1 \xi_1 + t_2 \xi_2 + \tfrac{1}{2}(\kappa_1 \xi_1^2 - \kappa_2 \xi_1^2 + \kappa_2 \xi_2^2 - \kappa_3 \xi_2^2)\} \quad . \qquad \ldots (29)$$

Applying now R(ii), i.e. $p(T_5) = 0$, to equations (26), (27) we obtain

$$\frac{1}{\kappa_1} [1 - \exp\{-\varepsilon T_1\}] + \frac{1}{\kappa_2} \exp(-\varepsilon T_2) [1 - \exp\{-\varepsilon (T_3 - T_2)\}] + \frac{1}{\kappa_3} \exp(-\varepsilon T_4) [1 - \exp\{-\varepsilon (T_5 - T_4)\}] = 0 \qquad \ldots (30)$$

or

$$\frac{1}{\kappa_1}\,(1-a_1) + \frac{a_1}{\kappa_2}\,(1-a_2)\,\exp(-\varepsilon t_1) + \frac{a_1 a_2}{\kappa_3}\,(1-a_3)\,\exp(-\varepsilon t_1)\,\exp(-\varepsilon t_2) = 0 \qquad \ldots (31)$$

where

$$a_1 = \exp\{-\varepsilon\,\kappa_1\,\xi_1\}$$

$$a_2 = \exp\{-\varepsilon\,\kappa_2(\xi_2 - \xi_1)\}$$

$$a_3 = \exp\{\varepsilon\,\kappa_3\,\xi_2\} .$$

Equations (29) and (31) now enable t_1 and t_2 to be found uniquely. In certain cases the resulting value of t_2 becomes negative: the approach is then to set t_2 to be zero and regard ξ_2 as variable. The solution of these equations is discussed in Appendix A.

The function $p(t)$ is thus determined and used as an input to equations (11) to (17) which can be solved by a marching procedure. Equation (13) is replaced by

$$\xi = \frac{1}{\ell^*_\xi}\left\{\gamma_A\,\frac{dp}{dt} - g_A\,qr - \ell^*_v\,\beta - \left(\frac{b}{2V}\right)\ell^*_p\,p - \left(\frac{b}{2V}\right)\ell^*_r\,r\right\} \qquad (32)$$

which is used to compute that aileron function $\xi(t)$ which is in fact necessary to perform the manoeuvre, when $p(t)$ takes the previously calculated form.

6.3 Examples

To attempt to give a set of examples covering all practical cases would be prohibitive and the use of the above simplified method in any particular case must be judged on the merit of the results obtained in that case. One must therefore be able to decide, having only the results of this method, whether such results give a reliable measure of the manoeuvre's severity. One can do this for the examples below, and most cases are expected to allow this facility.

The examples chosen and shown in Figs.4(a), (b), and (c) are 180° rolling manoeuvres. They all apply to the same aircraft, a high speed delta wing research aircraft. However, variations in the initial conditions of speed, height, and normal acceleration (indicated in the figures) lead to differences in the aircraft's dynamic properties and so to different responses. In particular, the differing characteristics of the responses in roll lead one to consider that these examples should test the applicability of the above method to a wide range of manoeuvres. (For more detailed information the interested reader may refer to Table 1, which lists the data for these examples.)

In each figure rate of roll, aileron angle, incidence, and sideslip are plotted*. These quantities are not, of course, sufficient to determine all the significant manoeuvre loads but they are indicative of the degree of success of the present method. To reduce further the amount of discussion, while still enabling a critical assessment to be made, the comments below concentrate on the two quantities incidence and sideslip. The full lines are the results obtained by the use of the simplified method while the dashed lines, termed "exact", were produced by successively altering the aileron input function prescribed by R(iii) until R(i) and R(ii) were closely met.

In Fig.4(a) it is seen that $\xi(t)$ differs somewhat from the "exact" time history but remains similar in character over most of the manoeuvre: incremental incidence and sideslip reflect this in that despite some differences in their histories the maximum value of the former agrees with the "exact" value within about 7% and that of the latter differs from the "exact" value by a barely perceptible amount. By contrast, the results shown in Fig.4(b) are far from satisfactory: maxima of incremental incidence and sideslip are overestimated by the simplified method by nearly 100%. A strong indication of this lack of agreement could be obtained by a glance at $\xi(t)$ in this case. The deviation from the correct form is considerable over almost all the manoeuvre so casting doubt on the reliability of the simplified method. With Fig.4(c) we return to more acceptable results, as evidenced by the generally good agreement of $\xi(t)$ with the "exact" form. In this case agreement between maxima of incremental incidence is obtained to within 2·5% and the maxima of sideslip agree to within 8%.

6.4 A modification of the simplified method

The reason for unsatisfactory results in cases such as the second example above must be that the response in degrees of freedom other than rolling has considerable influence on the response in roll. It is then

*In order to roll through a positive bank angle, ξ_1 must be negative. Therefore $(-\xi)$ is plotted to give complete correspondence with the aileron function of Fig.3. Similarly in the Appendix when it is convenient to choose a particular sign for κ_3, and hence for the other quantities defining the aileron function, this is taken to be negative so that $\bar{\phi}$ may be positive.

natural to consider whether account can be taken of this and more reliable results obtained. A method of achieving this is now described: its basis must be regarded as semi-empirical but, as will be seen, considerable advantages can be gained from its application.

It is assumed that the effect of other degrees of freedom is to change the damping in roll such that this now has the (constant) effective value $\bar{\bar{\ell}}_p$. Then the roll response is governed by the equation, analogous to equation (22)

$$\gamma_A \frac{dp}{dt} = \left(\frac{b}{2V}\right) \bar{\bar{\ell}}_p \, p + \bar{\ell}_\xi \, \xi \quad . \tag{33}$$

Suppose that results have been obtained using as input the time history in roll given by equation (22) and that at time $t = T_2$ the value of ξ obtained from equation (32) is ξ'. Then at $t = T_2$

$$\gamma_A \left[\frac{dp}{dt}\right]_{T_2} = \left(\frac{b}{2V}\right) \bar{\ell}_p \, p(T_2) + \bar{\ell}_\xi \, \xi_1 \tag{34}$$

$$\gamma_A \left[\frac{dp}{dt}\right]_{T_2} = \left(\frac{b}{2V}\right) \bar{\bar{\ell}}_p \, p(T_2) + \bar{\ell}_\xi \, \xi' \quad . \tag{35}$$

Hence

$$\bar{\bar{\ell}}_p = \bar{\ell}_p - \left(\frac{2V}{b}\right) \frac{\bar{\ell}_\xi(\xi' - \xi_1)}{p(T_2)} \quad . \tag{36}$$

If $\dot{p}$ is small at this point equation (36) tends to the simple relation

$$\bar{\bar{\ell}}_p = \bar{\ell}_p \, \xi'/\xi_1 \quad . \tag{37}$$

With $\bar{\bar{\ell}}_p$ calculated from equation (36) or (37) the simplified method may be applied again with $\bar{\bar{\ell}}_p$ replacing $\bar{\ell}_p$. Results obtained from this "modified" method are shown in Figs.4(b) and (c), denoted by chain-dotted lines. (In the cases of α in Fig.4(b) and p in Fig.4(c) the "modified" results are so near the "exact" ones that plotting them is impracticable.)

In the example of Fig.4(b) the above modification has been successful in bringing the form of $\xi(t)$ into almost complete agreement with R(iii) and in consequence the maxima of incremental incidence and sideslip are within 13% and 6% of the "exact" values. In Fig.4(c) the already good agreement between the "simplified" and "exact" values of incidence and sideslip is somewhat improved by the modification, the form of the incidence history showing particular improvement.

The choice of the point $t = T_2$ for identifying equations (22) and (33) has been made in an arbitrary fashion but it seems intuitively that this choice is a suitable one. In the majority of cases the aileron time histories are similar to those of the second and third examples above in that t_1 is large compared with T_1, and in such cases the success of the simplified approach depends mainly on keeping the value of ξ near to ξ_1 for most of the time interval t_1: it can be expected that agreement will be excellent at T_1 and so attempting to achieve agreement at T_2 recommends itself.

7 CONCLUDING DISCUSSION

The early sections of the paper described the type of aileron induced rolling manoeuvre thought to be the most suitable for structural design purposes. The equations of motion of the aircraft were then developed in a form suitable for practical application to calculating the aircraft response, retaining sufficient generality for their universal use in this problem while tailoring them to its particular requirements.

It was seen that the production of rigorous solutions of these equations under the specified conditions caused serious practical difficulties, the effort involved in overcoming these being so large that any attempt by a designer to produce such solutions for all flight conditions he would like, or be required, to cover would be unpractical. The designer's need was suggested to be a simpler method which could be used to sort out the most severe manoeuvres, such manoeuvres being the subjects of subsequent more rigorous calculations. Section 6 presented such a method, which it is thought will give, in the great majority of cases, reliable indications of the severity of particular manoeuvres. In addition the degree of reliability can be assessed, qualitatively, from the results obtained.

The recommended approach to dealing with a large number of rolling manoeuvres for structural design purposes is, then

(i) Apply to each manoeuvre the method of Section 6, in its basic form.

(ii) Apply the modified method of section 6.4 to those cases giving results thought to be unreliable.

(iii) If any cases remain in doubt these should be treated rigorously, as described in section 5.

(iv) Having decided from the results of (i), (ii) and (iii) which cases produce the most severe loadings on various aircraft components, these cases can be treated in a rigorous manner to obtain the final "design" loads.

Appendix A

THE SOLUTION OF EQUATIONS (29) AND (31)

The purpose of this Appendix is to discuss methods for the solution of the pair of simultaneous equations (29) and (31), i.e.

$$\bar{\phi} = -\varepsilon\delta\,\{t_1\,\xi_1 + t_2\,\xi_2 + \tfrac{1}{2}(\kappa_1\,\xi_1^2 - \kappa_2\,\xi_1^2 + \kappa_2\,\xi_2^2 - \kappa_3\,\xi_2^2)\} \qquad (29)$$

and

$$0 = (1-a_1)/\kappa_1 + a_1(1-a_2)\exp(-\varepsilon t_1)/\kappa_2 + a_1 a_2(1-a_3)\exp(-\varepsilon t_1)\exp(-\varepsilon t_2)/\kappa_3 \;. \qquad \ldots (31)$$

It is assumed that there is at least one solution (t_1, t_2) such that $t_1 \geqslant 0$ and that we are interested only in such solutions. From equation (29) it can be seen that for most practical cases, where the absolute values of κ_1, κ_2 and κ_3 are fairly similar and also ξ_2 is close to $-\xi_1$, $t_2 < t_1$. Hence a positive value for t_1 may correspond to a negative value for t_2: in this event an alternative approach must be used. Before discussing numerical methods for obtaining a solution, then, it is necessary to establish its nature.

It is shown that there can be only one solution with $t_1 \geqslant 0$; the criterion for deciding on the sign of the corresponding t_2 then follows immediately. Firstly we note that if ξ_1 and ξ_2 are assumed fixed and of opposite signs then, from equation (29), t_2 is a linear and increasing function of t_1. Hence if the right hand side of equation (31) be denoted by F and this be regarded as a function of t_1

$$F(t_1) = \frac{1}{\kappa_1}(1-a_1) + \frac{a_1}{\kappa_2}(1-a_2)\exp(-\varepsilon t_1) + \frac{\mu a_1 a_2}{\kappa_3}(1-a_3)\exp\{-\varepsilon(1+\lambda)\,t_1\} \qquad \ldots (38)$$

where $\lambda = -\xi_1/\xi_2 > 0$ and $\mu > 0$.

Since $\varepsilon\,\kappa_3\,\xi_2 > 0$, $a_3 > 1$ and so

$$F \to -\infty\,\mathrm{sgn}(\kappa_3) \quad \text{as} \quad t_1 \to \infty \;.$$

With no loss in generality we assume that $\kappa_3 < 0$, i.e. $\mathrm{sgn}(\kappa_3) = -1$. Then we can write (38) in the form

$$F = a - b\,\tau + c\,\tau^{1+\lambda}$$

where $\tau = \exp(-\varepsilon\, t_1)$ and $a,b,c > 0$.

An expression of this form has either no real positive zeros or has exactly two. Here we have assumed that there is such a zero and, moreover, this corresponds to a positive value of t_1. That is there exists $\bar{\tau} \geq 1$ for which

$$a - b\,\bar{\tau} + c\,\bar{\tau}^{1+\lambda} = 0 \quad . \tag{39}$$

We now show that the other root of $F = 0$ corresponds to a value of τ less than unity, i.e. to a negative value of t_1. It is sufficient to show that

$$\left[\frac{dF}{d\tau}\right]_{\bar{\tau}} > 0 \quad \text{for any} \quad \bar{\tau} \geq 1$$

or

$$-b + c(1+\lambda)\,\bar{\tau}^{\lambda} > 0 \quad .$$

Now

$$-b + c(1+\lambda)\,\bar{\tau}^{\lambda} = \lambda\, b - (1+\lambda)\, a/\bar{\tau} \quad \text{from (39)}$$

and since the second term on the right hand side is negative, if the inequality be proved for $\bar{\tau} = 1$ it will hold a fortiori for $\bar{\tau} > 1$. Therefore we require

$$\lambda b - (1+\lambda)\, a > 0$$

or, substituting for a,b,λ and after some elementary algebra

$$\frac{\exp\{-\varepsilon\,\kappa_2(\xi_2 - \xi_1)\} - 1}{-\varepsilon\,\kappa_2(\xi_2 - \xi_1)} - \frac{\exp\{\varepsilon\,\kappa_1\,\xi_1\} - 1}{\varepsilon\,\kappa_1\,\xi_1} > 0 \quad . \tag{40}$$

But

$$\exp(x) - 1 > x \quad \text{for all} \quad x \neq 0$$

or

$$\frac{\exp(x) - 1}{x} > 1 \quad \text{for} \quad x > 0 \; ; \qquad \frac{\exp(x) - 1}{x} < 1 \quad \text{for} \quad x < 0 \quad .$$

Applying the above two inequalities to the first and second terms respectively in (40) the desired result is obtained.

The criterion for the sign of t_2 may now be immediately deduced. For $t_2 = 0$

$$-t_1 \xi_1 = \frac{\bar{\Phi}}{\varepsilon\delta} + \tfrac{1}{2}(\kappa_1 \xi_1^2 - \kappa_2 \xi_1^2 + \kappa_2 \xi_2^2 - \kappa_3 \xi_2^2)$$

and substitution of this value of t_1, say t^*, into $F(t_1)$ will produce a negative or positive result according to whether t^* is less than or greater than the positive root for t_1. Since t_2 is an increasing function of t_1 we therefore obtain

$$t_2 < 0 \quad \text{if and only if} \quad F(t^*) > 0 \ .$$

When $F(t^*) > 0$, then, we set $t_2 = 0$ and regard ξ_2 as variable; also, for the convenience of eliminating square roots, we regard F as a function of ξ_2.

In general equation (31) can be solved only by an iterative method: the practical application of two such methods will now be discussed.

One method which is practicable is the Newton-Raphson method[5]: for this the first derivative of the function with respect to the independent variable is required. The expressions for this in the two possible cases are given below.

If $F = F(t_1)$

$$\frac{dF}{dt_1} = -\varepsilon \exp(-\varepsilon t_1) \left\{ \frac{a_1}{\kappa_2}(1-a_2) + \frac{a_1 a_2}{\kappa_3}(1-a_3)\left(1 - \frac{\xi_1}{\xi_2}\right) \exp(-\varepsilon t_2) \right\}$$

or if $F = F(\xi_2)$

$$\frac{dF}{d\xi_2} = \varepsilon \exp(-\varepsilon t_1) \left\{ a_1 a_2 + \frac{a_1 a_2}{\kappa_3}[-\kappa_2 + (\kappa_2 - \kappa_3)a_3] + (\kappa_2 - \kappa_3)\frac{\xi_2}{\xi_1}\left[\frac{a_1}{\kappa_2}(1-a_2) + \frac{a_1 a_2}{\kappa_3}(1-a_3)\right] \right\} .$$

An immediate question with such a method is as to the choice of a starting point for the iteration which will ensure convergence to the required root. Since it is necessary to compute $F(t^*)$ to decide whether t_1 or ξ_2 should be considered the independent variable, it may be hoped that t^* is a suitable point and it has indeed been shown that in either case this choice does ensure convergence, at least for the most common situation, namely $\kappa_1 = -\kappa_2 = \kappa_3$. (The proof is omitted since it is a little tedious and devoid of interest.)

An alternative method is to apply a search procedure: the following description is for t_1 as the variable, that for the alternative situation being exactly similar. As in this case the required value of t_1 is greater than t^* the first stage is to march forward in t_1, with step h say, until a value of t_1 is found such that $F(t_1) > 0$. Then the root lies in the interval $(t_1 - h, t_1)$: we step back $h/2$ and compute F at this point. The sign of F will then indicate whether the step back has passed over the root or has not reached it: we next step $h/4$ in the appropriate direction. By this method an interval of length $h2^{-n}$, where n is a specified positive integer, can be found in which the root must lie. (This is mathematically sound but the process has a practical limit: see discussion below.)

Of these two methods the former is rather the better; the more complicated nature being offset by more rapid convergence (and hence fewer exponentials to compute), as well as by the certainty of each iteration producing an answer closer to the root than the previous estimate, a property not of course possessed by the search procedure. In practice both methods are severely limited by the accuracies to which the constituent terms in F (and its derivative) can be calculated, and the specification of too large a number of iterations or too small an absolute error should be avoided as uneconomical and self-deceptive. Experience with the facilities which are available to him will soon indicate to the individual the accuracy for which he is entitled to ask. From this point of view also, the Newton-Raphson method with its inherent properties of self-correction is the better one to use.

Table 1

DATA FOR THE EXAMPLES; FIGS.4(a), (b), AND (c)

	a		b		c	
y^*_v	−0·182	0	−0·167	0	−0·171	0
y^*_p	0	0	0	0	0	0
y^*_r	0	0	0	0	0	0
y^*_ξ	0	0	0	0	0	0
$\bar{z}$	0·051		0·018		−0·012	
z^*_w	−1·472	0	−1·215	0	−0·909	0
z^*_η	−0·346	0	−0·156	0	−0·156	0
ℓ^*_v	−0·032	−0·62	−0·034	−0·80	−0·042	−0·33
ℓ^*_p	−0·237	0·012	−0·144	−1·10	−0·127	0·93
ℓ^*_r	0·0205	0	0·0945	0	0·034	0
ℓ^*_ξ	−0·110	0	−0·0665	0	−0·0673	0
$\bar{m}$	0·00713		0·00774		−0·0026	
m^*_w	−0·042	0	−0·128	0	−0·087	0
$m^*_{\dot{w}}$	−0·0503	0	0·034	0	0·034	0
m^*_q	−0·129	0	−0·297	0	−0·297	0
m^*_η	−0·118	0	−0·087	0	−0·087	0
n^*_v	0·083	−0·195	0·0745	0·046	0·081	0·054
n^*_p	−0·04	0·066	−0·0072	−0·022	−0·0017	−0·14
n^*_r	−0·329	0	−0·193	0	−0·185	0
n^*_ξ	−0·0106	0·014	−0·0172	0·10	−0·005	0·17
A	7602		7602		7602	
B	53815		53815		53815	
C	60319		60319		60319	
b	25·0		25·0		25·0	
ℓ	20·8		20·8		20·8	
W	17500		17500		17500	
F	0·09		0·028		0·028	
V	422		1550		1550	
n_o	2·0		2·0		−0·5	
$\bar{\ell}_p$	−0·235		−0·2105		−0·153	
$\bar{\bar{\ell}}_p$			−0·309		−0·099	
$\bar{\ell}_\xi$	−0·110		−0·0665		−0·0673	
$\bar{\phi}$(deg)	180		180		180	
κ_1	−0·0125		−0·0125		−0·0125	
κ_2	0·0125		0·0125		0·0125	
κ_3	−0·0125		−0·0125		−0·0125	
ξ_1	−21		−5		−5	
ξ_2	21		5		5	

Table 1 (Contd)

Notes: 1. The aerodynamic derivatives are presented in the form

$$y_V^{(0)} , \; y_V^{(1)}$$

2. The "exact" values were produced by a computer program which gives only incremental values of response quantities; therefore the values of $\bar{z}$ and $\bar{m}$ corresponding to $\eta = 0$ were determined and are shown above. Since z^*_η and m^*_η are independent of α in these examples there is no effect on the response.

3. The quantities κ_1, κ_2, κ_3, ξ_1, ξ_2 above are those which are used in equations (29) and (31) to determine p(t) in the simplified approach. They are in terms of degrees and seconds.

SYMBOLS

(i) General notation

A moment of inertia about Gx slugs ft^2

B moment of inertia about Gy slugs ft^2

C moment of inertia about Gz slugs ft^2

$$C_\ell = \frac{L}{\frac{1}{2}\rho V^2 Sb}$$

$$C_m = \frac{M}{\frac{1}{2}\rho V^2 Sc}$$

$$C_n = \frac{N}{\frac{1}{2}\rho V^2 Sb}$$

$$C_y = \frac{Y}{\frac{1}{2}\rho V^2 S}$$

$$C_z = \frac{Z}{\frac{1}{2}\rho V^2 S}$$

$$E_1 = \frac{F M_E}{W\ell}$$

$$E_2 = \frac{F M_E}{W(b/2)}$$

$$F = \frac{W}{\rho V^2 S}$$

M_E sum of angular momenta of rotating parts of engines slug ft^2/sec

S area of reference ft^2

$T_1, T_2, \ldots, T_5$ times in definition of aileron function (see Fig.3(b)) sec

V total linear velocity ft/sec

W aircraft weight lb

L aerodynamic moment about Gx lb ft

M aerodynamic moment about Gy lb ft

N aerodynamic moment about Gz lb ft

Y aerodynamic force along Gy lb

Z aerodynamic force along Gz lb

SYMBOLS (Contd)

a_1, a_2, a_3 coefficients in equation (31)

b aircraft span ft

c chord of reference ft

$f_A = \frac{B - C}{A}$

$f_B = \frac{C - A}{B}$

$f_C = \frac{A - B}{C}$

g gravitational acceleration ft/sec^2

$g_A = \frac{F(B - C)}{W(b/2)}$

$g_B = \frac{F(C - A)}{W\ell}$

$g_C = \frac{F(A - B)}{W(b/2)}$

ℓ length of reference ft

$\bar{\ell}_p$, $\bar{\bar{\ell}}_p$, $\bar{\ell}_\xi$ } Aerodynamic derivatives defining the aircraft response in roll in the simplified approach

n aircraft normal acceleration g-units

p rolling velocity radians/sec

q pitching velocity radians/sec

r yawing velocity radians/sec

t_1, t_2 times in definition of aileron function (see Fig.3(a)) sec

$\hat{t} = \frac{VF}{g} = \frac{W}{g \rho SV}$

u velocity component along Gx ft/sec

v velocity component along Gy ft/sec

w velocity component along Gz ft/sec

$\alpha = w/V$

$\beta = v/V$

$\gamma_A = \frac{FA}{W(b/2)}$

$\gamma_B = \frac{FB}{W\ell}$

SYMBOLS (Contd)

$\gamma_C = \frac{FC}{W(b/2)}$

$\delta = \frac{\bar{\ell}_\xi}{\varepsilon^2 \gamma_A}$

$\varepsilon = \frac{b}{2V} \frac{\bar{\ell}_p}{\gamma_A}$

$\kappa_1, \kappa_2, \kappa_3$ reciprocals of rates of aileron application sec/radian

ξ aileron angle radians

η elevator angle radians

ρ air density slugs/ft^3

θ angle of pitch radians

ϕ angle of bank radians

$\bar{\phi}$ angle of bank through which the aircraft is required to manoeuvre radians

ψ angle of yaw radians

(ii) Definitions of aerodynamic derivatives in terms of aerodynamic coefficients

See also section 3.2

$$y_v^* = \frac{1}{2} \frac{\partial C_y^*}{\partial \beta}$$

$$y_p^* = \frac{\partial C_y^*}{\partial \left(\frac{pb}{2V}\right)}$$

$$y_r^* = \frac{\partial C_y^*}{\partial \left(\frac{rb}{2V}\right)}$$

$$y_\xi^* = \frac{1}{2} \frac{\partial C_y^*}{\partial \xi}$$

$$\bar{z} = \frac{1}{2} \bar{C}_z$$

$$z_w^* = \frac{1}{2} \frac{\partial C_z^*}{\partial \alpha}$$

$$z_\eta^* = \frac{1}{2} \frac{\partial C_z^*}{\partial \eta}$$

SYMBOLS (Contd)

$$\ell_v^* = \frac{\partial C_\ell^*}{\partial \beta}$$

$$\ell_p^* = \frac{\partial C_\ell^*}{\partial\left(\frac{pb}{2V}\right)}$$

$$\ell_r^* = \frac{\partial C_\ell^*}{\partial\left(\frac{rb}{2V}\right)}$$

$$\ell_\xi^* = \frac{\partial C_\ell^*}{\partial \xi}$$

$$\bar{m} = \frac{c}{2\ell}\bar{C}_m$$

$$m_w^* = \frac{c}{2\ell}\frac{\partial C_m^*}{\partial \alpha}$$

$$m_{\dot{w}}^* = \frac{c}{2\ell}\frac{\partial C_m^*}{\partial\left(\frac{\dot{\alpha}\ell}{V}\right)}$$

$$m_q^* = \frac{c}{2\ell}\frac{\partial C_m^*}{\partial\left(\frac{q\ell}{V}\right)}$$

$$m_\eta^* = \frac{c}{2\ell}\frac{\partial C_m^*}{\partial \eta}$$

$$n_v^* = \frac{\partial C_n^*}{\partial \beta}$$

$$n_p^* = \frac{\partial C_n^*}{\partial\left(\frac{pb}{2V}\right)}$$

$$n_r^* = \frac{\partial C_n^*}{\partial\left(\frac{rb}{2V}\right)}$$

$$n_\xi^* = \frac{\partial C_n^*}{\partial \xi}$$

REFERENCES

No.	Author	Title, etc.
1	W. J. Duncan	Control and stability of aircraft. Cambridge University Press, p 303, 1952
2	A. S. Taylor	The present status of aircraft stability problems in the aeroelastic domain. Jnl. Roy. Aero. Soc., 63, No.580, 230-237, 1959
3	Royal Aeronautical Society	Data sheets - Aerodynamics Vol.3, No. Aircraft 00.00.02, Section 1.2, 1947
4	W. J. G. Pinsker	Charts of peak amplitudes in incidence and sideslip in rolling manoeuvres due to inertia cross-coupling. A.R.C. R & M 3293, April, 1958
5	A. M. Ostrowski	Solution of equations and systems of equations. London, Academic Press, 1960

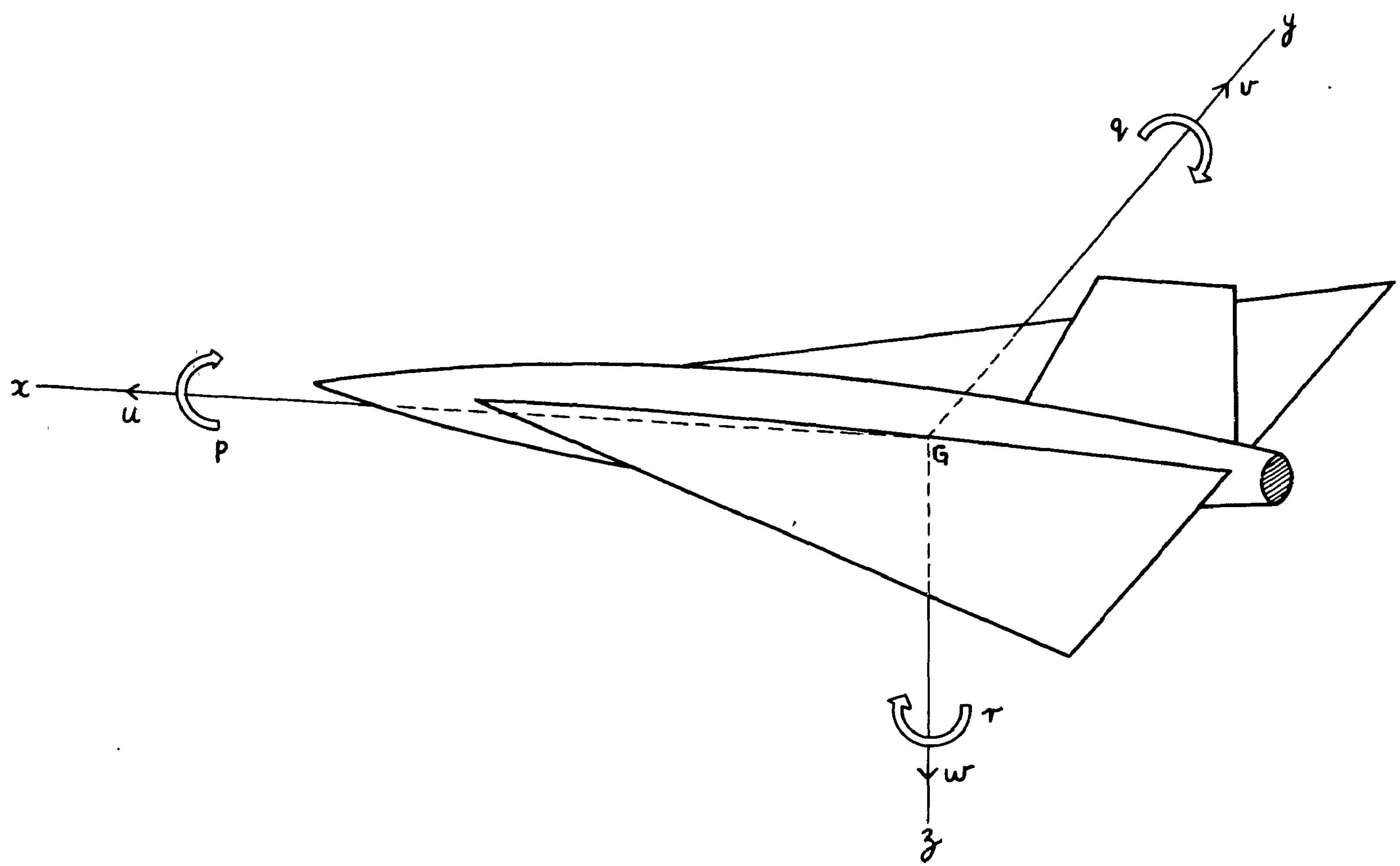

FIG. 1 THE SYSTEM OF PRINCIPAL INERTIA AXES.

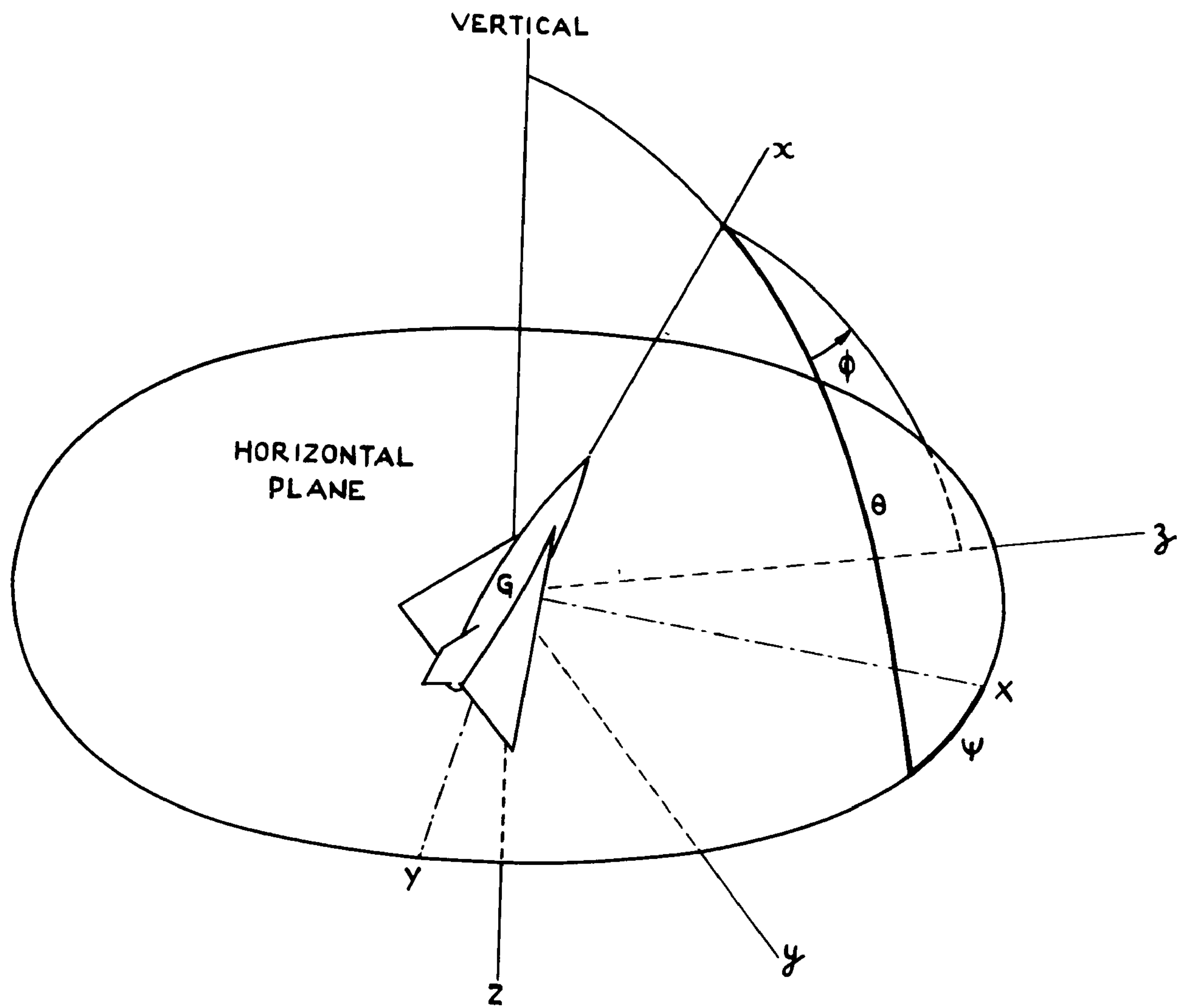

FIG. 2 ORIENTATION OF THE AXES IN SPACE

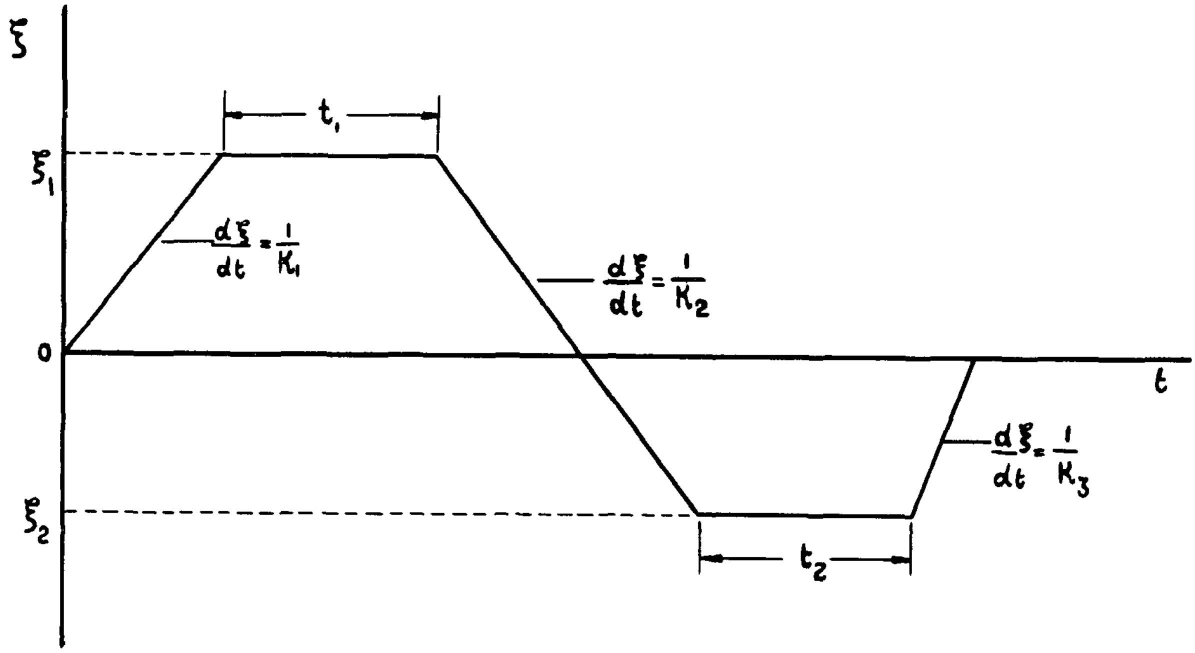

FIG. 3 (a) THE AILERON INPUT FUNCTION

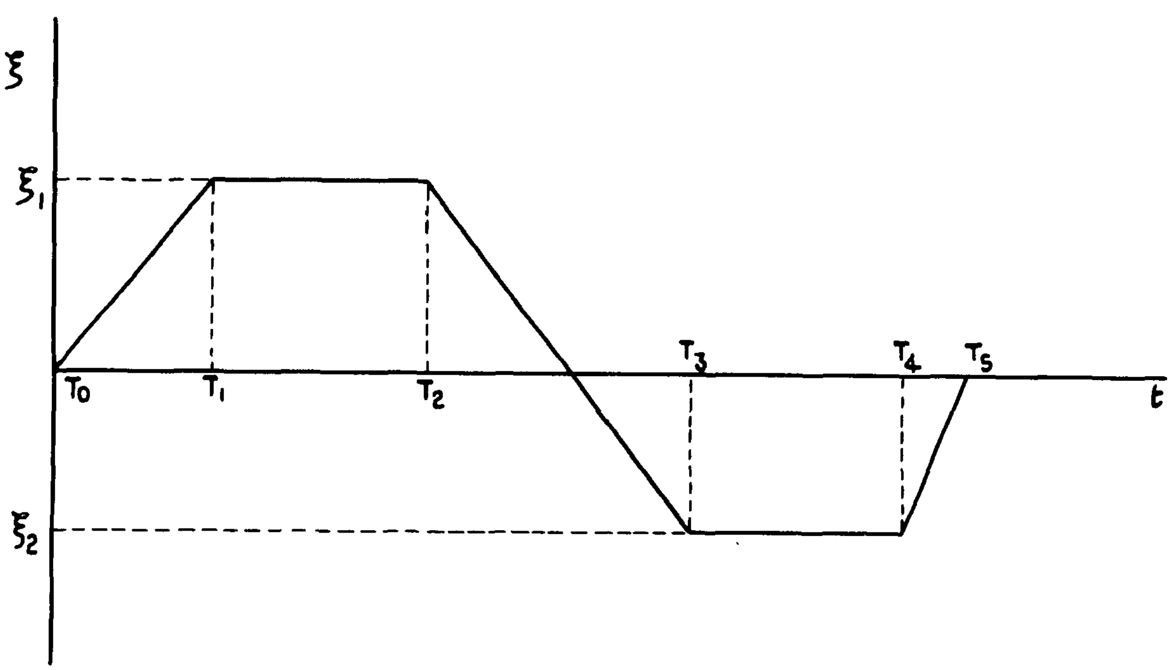

FIG. 3 (b) THE AILERON INPUT FUNCTION – ALTERNATIVE NOTATION

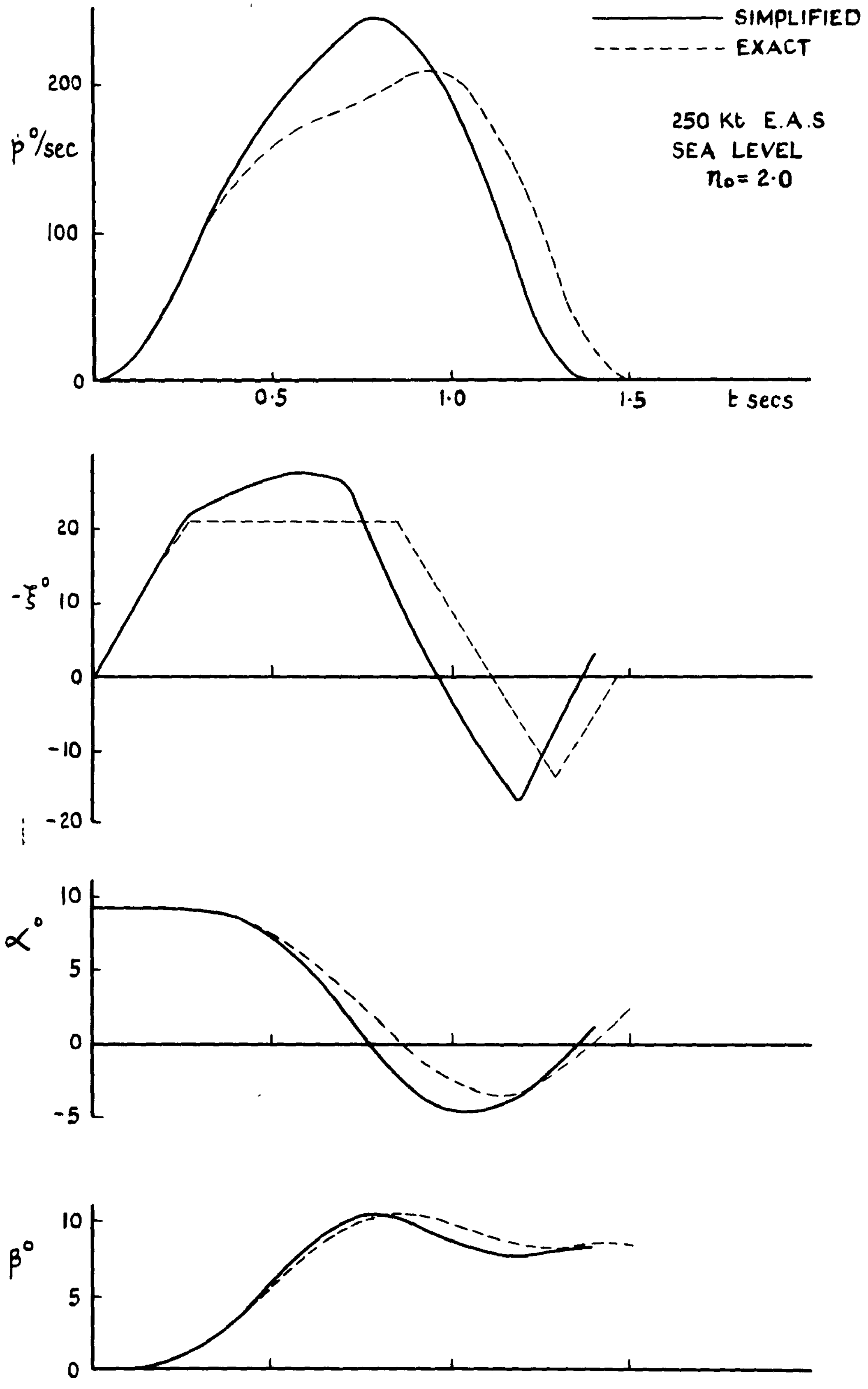

FIG.4 (a) COMPARISON OF SIMPLIFIED AND EXACT METHODS

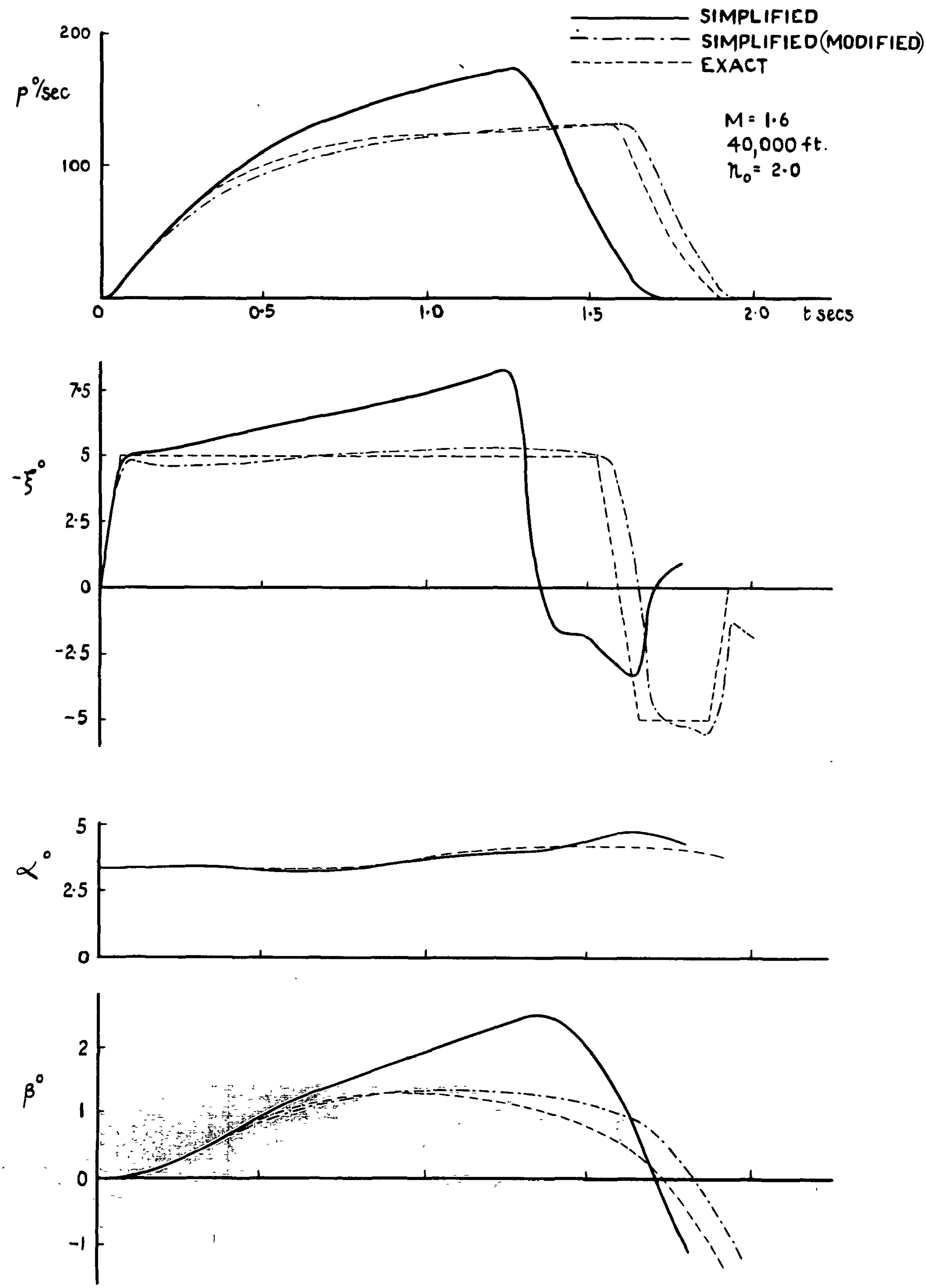

FIG. 4 (b) COMPARISON OF SIMPLIFIED AND EXACT METHODS (CONTINUED)

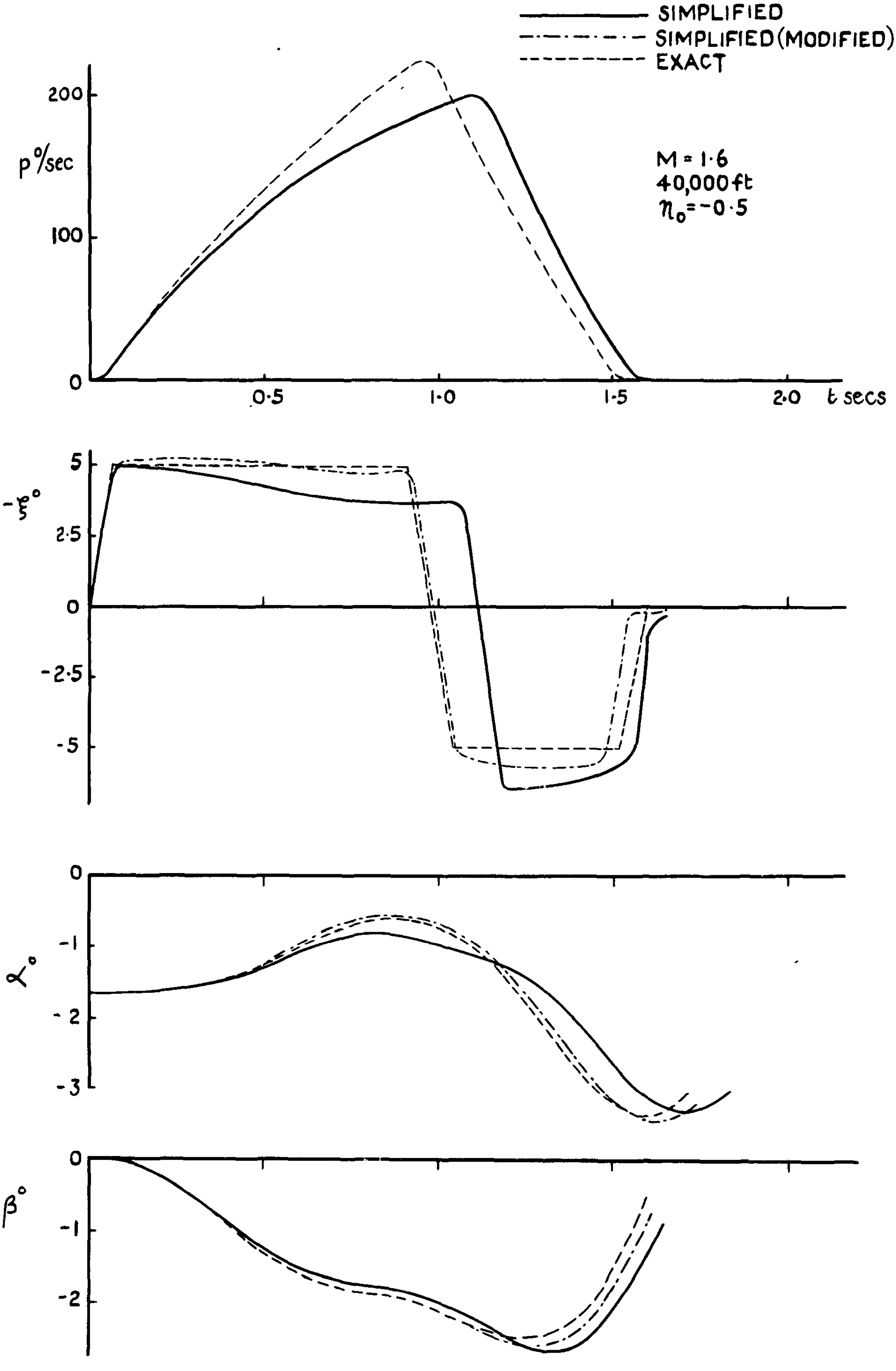

FIG. 4 (c) COMPARISON OF SIMPLIFIED AND EXACT METHODS (CONCLUDED)

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THE CALCULATION OF AIRCRAFT MOTION IN DESIGN ROLLING MANOEUVRES.

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