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A SEMI-EMPIRICAL PREDICTION METHOD FOR PRESSURES ON THE WINDWARD SURFACE OF CIRCULAR CONES AT INCIDENCE AT HIGH SUPERSONIC AND HYPERSONIC SPEEDS ($M \geq 3$)

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SUMMARY

The so-called impact theory equation $Cp^* = K^* \sin^2\theta$ for a circular cone is written as the sum of three terms associated with axial, combined and transverse flows respectively. By postulating that K^* in each term represents a limiting value as $M_\infty . \theta \to \infty$ of coefficients K_a, K_b, K_c which are functions of variables like $M_\infty . \theta$, a simple, semi-empirical method is devised for predicting pressure distribution on the windward region of a circular cone which agrees well with experimental data up to large incidence angles. Numerically, the method is based on the small incidence theories of Taylor-Maccoll and Stone, and on experimental pressure distributions over cylinders placed normal to the stream.

<u>CONTENTS</u>

<u>ILLUSTRATIONS</u>

1 INTRODUCTION

Theoretical methods for predicting pressures on cones at high supersonic and hypersonic speeds ($M \geq 3$) are strictly limited to moderate incidence angles, and for higher angles semi-empirical methods such as the so-called impact theory and tangent-cone approximations are used. In this note, a method for predicting pressures over the windward region of a circular cone at these speeds and at arbitrary incidence is proposed, which can be regarded as derived from the impact theory equation $Cp^* = K^* \sin^2\theta$. However, unlike earlier semi-impirical methods of this type, the suggested method is consistent with the well-known Taylor-Maccoll and Stone theoretical results, and also with experimental pressure distributions round circular cylinders placed normal to the stream.

No explicit account is taken of boundary-layer displacement effects, real gas effects or separation phenomena. Application of the method is restricted in this note to the special case of a circular cone, and hence the question of possible corrections for centrifugal effects due to body curvature in the longitudinal planes does not arise. Extension of the method to cover other bodies of revolution is obviously feasible.

2 IMPACT THEORY - A BRIEF REVIEW

The general name of impact theory is given to all those equations for the pressure coefficient in hypersonic flow which have the form

$$Cp^* = K^* \sin^2\theta . \quad (1)$$

Here the asterisk is used to denote limiting values for $M_\infty . \theta \to \infty$ and K^* is an impact coefficient. θ is the angle between the free stream and the local surface, measured in a plane normal to the surface, i.e. $\left(\frac{\pi}{2} - \theta\right)$ is the angle between free stream direction and surface normal. θ must be positive for this equation to apply, Cp^* being assumed zero for negative θ.

Formulae of this type may be derived in several ways, one of which employs the original corpuscular theory of Newton and hence has given rise to the name impact theory. Another derivation applies the strong shock approximation to the oblique shock equations. Details of such analyses are given in Refs.1 and 2, but some special examples will be repeated here:-

(a) For plane surfaces (e.g., a wedge) the oblique shock equations give

$$K^* = (1 + \gamma) . \quad (2)$$

for the assumption of small θ and very high Mach number such that $M_\infty . \theta \gg 1$, where θ is the angle between the plane body surface and the free stream (see equation (25) of Ref.1). For $M \to \infty$, equations (1) and (2) in fact predict Cp^* to within 5% of the oblique shock equation results for angles up to about 40°.

(b) For conical flow with an attached shock and again assuming θ small and $M_\infty . \theta >> 1$, Lees[3] obtained the impact coefficient for a cone

$$K^* = \frac{2(\gamma+1)(\gamma+7)}{(\gamma+3)^2} . \qquad (3)$$

Here θ is the semi-angle of the cone, which is at zero incidence. Numerical results obtained by Kopal[4] from the complete Taylor-Maccoll equations for $M_\infty = \infty$ suggest that equation (3) is a close approximation for all cone angles which permit an attached shock.

(c) The above values of K* apply to cases where the angle between the flow and the surface is small enough for the shock to remain attached, so that the flow between the shock and the surface is essentially planar or conical as the case may be. For bluff bodies such as spheres or cylinders placed normal to the stream, K* at the stagnation point, (where θ = 90°), must be equal to the stagnation pressure coefficient behind a normal shock in the limit as $M_\infty \to \infty$. Experimentally it is found that the pressure distribution over the forward-facing surfaces of a bluff body with a detached shock does in fact follow a $\sin^2\theta$ law quite closely, so that the limiting stagnation pressure coefficient may be regarded as an impact coefficient for bluff bodies,

$$K^* = \frac{\gamma+1}{\gamma} \left[\frac{(\gamma+1)^2}{4\gamma}\right]^{\frac{1}{\gamma-1}} . \qquad (4)$$

Now, the well-known Newtonian result K* = 2 is obtained by assuming that all free-stream momentum normal to the surface is lost, the flow moving tangentially to the surface after impact. This implies that the shock is coincident with the body surface, and therefore the density ratio across the shock $\left(= \frac{\gamma+1}{\gamma-1} \text{ for } M_\infty \sin\zeta >> 1, \text{ where } \zeta \text{ is the shock angle}\right)$ must be infinite in order that the flow behind the shock may be accommodated. This will be the case for γ = 1, and with this assumption equations (2), (3) and (4) all give K* = 2.

3 IMPACT THEORY FOR A CONE

Consider a cone with its axis at an angle of incidence α, (see Fig.1). Let any point on this body and the most windward point in the same plane normal to the axes subtend a polar angle ϕ at the axis. Then if ε is the surface slope of the body at the point relative to the axis, (i.e. the cone semi-angle) the angle θ between the free stream direction and the surface at

	x	y	z
Tangent to generator through P	$\cos \varepsilon$	$\sin \varepsilon \sin \phi$	$- \sin \varepsilon \cos \phi$
Tangent to circular section at P	0	$\cos \phi$	$\sin \phi$
Normal to surface at P	$\sin \varepsilon$	$- \cos \varepsilon \sin \phi$	$\cos \varepsilon \cos \phi$
Stream direction	$\cos \alpha$	0	$\sin \alpha$

Hence, the relation between the local inclination of the body surface to the stream, θ, and the variables α, ε and ϕ is:-

$$\sin \theta = \cos \left(\frac{\pi}{2} - \theta\right) = \sin \varepsilon \cos \alpha + \sin \alpha \cos \varepsilon \cos \phi \, . \tag{5}$$

Fig.1 indicates how this result may be derived in another way.

Assuming θ to be positive, i.e. $\cos \phi > - \tan \varepsilon / \tan \alpha$, equations (1) and (5) give

$$Cp^* = K^*(\sin^2\varepsilon \cos^2\alpha + 2 \sin \varepsilon \cos \varepsilon \cos \phi \sin \alpha \cos \alpha + \cos^2\varepsilon \cos^2\phi \sin^2\alpha) \, . \qquad \ldots (6)$$

The three terms in this equation can be identified with three distinct components of Cp^* which will be denoted respectively Cp^*_a, Cp^*_b, Cp^*_c. These will be considered in turn:-

The first component Cp^*_a is given by the following equation

$$Cp^*_a = K^* \sin^2\varepsilon \cos^2\alpha \, . \tag{7}$$

Cp^*_a, is that part of the total pressure coefficient which can be regarded as being generated by the axial flow component of the stream, (velocity $V_\infty \cos \alpha$, where V_∞ is the free stream velocity) and is due entirely to the surface slope of the body ε relative to its axis. At zero incidence Cp^*_a is equal to Cp^*.

The second component is Cp^*_b, given by

$$Cp^*_b = 2K^* \sin \varepsilon \cos \varepsilon \cos \phi \sin \alpha \cos \alpha \, . \tag{8}$$

Cp_b^*, is that part of the total pressure coefficient on a body at incidence which can be regarded as arising from interaction between the axial velocity component of the stream, $V_\infty \cos\alpha$ and the transverse velocity component $V_\infty \sin\alpha$, on a body having a finite surface slope. Thus Cp_b^* depends on all components of the flow. At small angles of incidence Cp_b^* is the major increment in Cp^* due to incidence and in this case is clearly related to the increment derived by linearised theory.

Lastly, Cp_c^* is given by

$$Cp_c^* = K^* \cos^2\varepsilon \cos^2\phi \sin^2\alpha \; . \tag{9}$$

Cp_c^* is that part of the total pressure coefficient which can be considered as being generated entirely by the transverse velocity component of the stream, $V_\infty \sin\alpha$. This component of Cp^* is very little affected by surface slope, and in the limiting case of a cone with $\varepsilon \to 0$, i.e. a cylinder, Cp_c^* is equal to Cp^*.

The remainder of this note is devoted chiefly to showing how, by rewriting these three equations in a more generalised form and by making some intuitive assumptions concerning the interpretation of the supersonic-hypersonic similarity rule, their range of applicability can be usefully extended. However it will be necessary to assume that the three components of Cp^* described above can at all times be treated independently of each other, and this assumption alone must limit the method to high supersonic and hypersonic speeds.

The basis of the proposed method is to replace K^* in equations 7, 8, 9 by variable coefficients which tend to K^* in the limit $M_\infty . \theta \to \infty$. Even in this limiting case, for which the above impact theory equations are strictly valid, although a single value of K^* could be chosen to suit any particular case (depending on the nature of the flow) it would seem logical to associate with the axial and combined flow components Cp_a^* and Cp_b^* an impact coefficient K^* derived from theories which assume flow through an attached shock, (e.g. equation (3)), while the transverse flow component Cp_c^* would be associated with an impact coefficient K^* appropriate to flow round a cylinder, i.e. that for bluff bodies equation (4).

4 SIMILARITY RULES

The hypersonic similarity rule for the pressure coefficient on a body of revolution at a given longitudinal position and a given polar angle ϕ may be written

$$\frac{Cp}{\theta^2} = f\,(M_\infty . \theta_1, M_\infty . \theta_2 \text{ etc.}) \tag{10}$$

where θ_1, θ_2 etc. are small angles defining the flow field. Refs. 1,2,3 give fuller accounts of this rule.

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stream Mach number $M_a = M_\infty . \cos \alpha$, and the cone semi-angle ε. It is proposed to obtain values of K_a from theoretical results, discussed in section 7.

K_b like K_a is essentially related to flow through an attached shock. Since $\overline{Cp}_b$ arises from the interaction between both axial and transverse flow components, the hypotheses will be made that the value of K_b at any point depends only on a similarity parameter involving the stream Mach number M_∞ and the local inclination of the body surface to the stream at that point, θ. Since values of K_b must be derived from theoretical results which assume $\alpha \to 0$, this implies that for a cone at an arbitrary angle of incidence the value of K_b applying at any point where the local surface inclination to the flow is θ is the theoretical value for the same Mach number but at zero incidence and with $\varepsilon = \theta$. Values of K_b are derived, for $\gamma = 1.4$, in section 8.

K_c is associated with the transverse flow, velocity $V_\infty \sin \alpha$, i.e. since the effect of surface slope is small, essentially with the flow round a cylinder normal to the stream with a detached shock. Hence it is proposed to assume that K_c (numerically equal to the maximum pressure coefficient on a cylinder normal to the stream) depends simply on the cross-flow Mach number, $M_\infty . \sin \alpha$. The validity of this assumption is discussed, and values of K_c are derived, in section 9.

(b) In addition to the introduction of variables K_a, K_b, K_c, the original impact theory equation has been modified by the substitution of $\cos^2(1 - \delta)\phi$ for $\cos^2\phi$ in equation (14). For a long cylinder at 90° incidence, equations 11-14 give $Cp/Cp_{max} = \cos^2(1 - \delta)\phi$; the variable δ has been introduced to enable small differences between experimental pressure distributions round cylinders and the impact theory prediction to be taken into account. Data on Cp/Cp_{max} for cylinders and appropriate values of δ are discussed in section 10.

6 LIMITATIONS OF METHOD

Clearly an approach of this kind is only valid if both M_∞ and $M_\infty . \theta$ are reasonably large. The lower limit of M_∞ is believed to be about 3 and the method can be applied with greatest confidence where $M_\infty . \theta > 1$. Although values of K_a and K_b can be correlated for $M_\infty . \theta < 1$ the assumption that the transverse flow term Cp_c can be treated as two-dimensional and independent of the rest of the flow is only likely to be applicable if the condition $M_\infty . \theta > 1$ is fulfilled. Fortunately, Cp_c is usually small when $M_\infty . \theta$ is small so that the method may be expected to yield useful results for values of $M_\infty . \theta$ less than unity.

7 DERIVATION OF K_a

Values of the coefficient K_a derived from Kopal's tables[4] for the pressure coefficient on a cone in axial flow (Taylor-Maccoll theory) are plotted for $M \geqslant 3$ in Fig. 2(a) as a function of the supersonic-hypersonic similarity parameter for small angles, i.e.

$$K_a = f\,(\beta_a \cdot \varepsilon) \; . \qquad (15)$$

Here, β_a has the meaning $\beta_a = \sqrt{M_a^2 - 1}$ where M_a is the axial Mach number $M_\infty \cos\alpha$ in the general case. (In fact of course the theoretical points have been derived for zero incidence.) The correlation represented by equation (15) is good for cone semi-angles up to about 20° but less satisfactory for larger angles.

Much better correlation of the Taylor-Maccoll values of K_a at large angles is obtained by means of the relation

$$K_a = f\,(\beta_a \cdot \sin\varepsilon \cdot \cos\varepsilon) \; . \qquad (16)$$

As shown in Fig. 2(b), for $M_a \geqslant 3$ equation (16) gives almost perfect correlation for angles up to at least 40°. For predicting K_a for cones, Fig. 2(b) is therefore used.

Thus the statement of the small angle, hypersonic similarity rule given by equation (10) has been modified empirically, following the arguments presented in section 4, by replacing Cp/θ^2 by $Cp/\sin^2\theta$, and $M_\infty\theta$ by $\beta \sin\theta \cos\theta$. In this way, the range of applicability of the similarity law is greatly increased in the present case. A similar approach has been found[7] to correlate pressures behind plane oblique shocks[†].

8 DERIVATION OF K_b

For the reasons given above, it will be assumed that K_b is a unique function of $\beta \sin\theta \cdot \cos\theta$, where β is derived from stream Mach No. and θ is the local incidence. Theoretical values of K_b must be obtained from small incidence theory. Now Stone[8] has derived pressure perturbation coefficients for a slightly yawed cone in the form

$$(dCp/d\alpha)_{\alpha \to 0} = \cos\phi \cdot f\,(M_\infty, \varepsilon) \; . \qquad (17)$$

From equation (13) it follows that values of K_b can be derived from these results using the relation

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† For this simple case, and also for the Prandtl-Meyer expansion, it may also be noted that the modified law

$$M_\infty^2\, Cp = f(M_\infty^2/\beta \sin\theta)$$

correlates pressures over an even wider range of M_∞.

$$K_b = f(\beta \,.\, \sin\theta \,.\, \cos\theta) = \left[\frac{-(dCp/d\alpha)_{\alpha\to 0}}{2\sin\varepsilon \,.\, \cos\varepsilon \,.\, \cos\phi}\right]_{\varepsilon\to\theta} . \qquad (18)$$

Fig. 3(b) shows a plot of K_b derived from Stone's results (as tabulated by Kopal[9]) and it can be seen that for $M_\infty \geq 3$ the similarity law expressed by equation (18) is well justified up to large angles. Also shown, Fig. 3(a), is a plot of K_b against the small angle parameter $\beta\theta$; as in the case of K_a this does not yield such a good correlation.

It is proposed that K_b be obtained from Fig. 3(b) using stream Mach number for β and the local incidence θ to obtain the similarity parameter $\beta \sin\theta \cos\theta$.

9 DERIVATION OF K_c

The component Cp_c of the pressure coefficient, given by equation (14) is proportional to $\sin^2\alpha$ and can be attributed entirely to the flow in the transverse plane, with a maximum value at $\alpha = 90°$, (at which condition the other components of Cp are zero). The coefficient K_c is related to flow about a bluff body with shock detached, and must therefore tend in the limit $M_\infty . \theta \to \infty$ to the value given by equation (4).

It is proposed to consider the transverse flow as two-dimensional so that the pressures due to it can be derived from the pressure distribution round a long cylinder; (with $\varepsilon = 0$, Cp_a and Cp_b are both zero and $Cp = Cp_c$).

The similarity parameter $\beta \sin\theta \cos\theta$ used for K_a and K_b applies to flows with shock attached and is not appropriate. It is proposed to assume that the relevant 'bluff body' similarity parameter is the cross-flow Mach number, i.e.

$$K_c = f(M_\infty . \sin\alpha) . \qquad (19)$$

Experimental values of K_c from wind-tunnel tests on long cylinders are shown on Fig. 4. The values plotted are in fact $(Cp_{max})_{CYL}/\sin^2\alpha$, (see equation (14)) where $(Cp_{max})_{CYL}$ is of course the pressure coefficient on the most windward generator of a cylinder, $\phi = 0$. The line through these points, which is a very good fit, is the theoretical stagnation pressure coefficient behind a normal shock in a stream of Mach number $M_\infty . \sin\alpha$, with $\gamma = 1.4$. Hence it is valid to write

$$K_c = Cp_s(M_\infty \sin\alpha) . \qquad (20)$$

10 PRESSURE DISTRIBUTION ROUND CYLINDERS - DERIVATION OF δ

In equation (14) the pressure distribution round a cylinder has been virtually represented by

$$(Cp/Cp_{max})_{CYL} = \cos^2 (1-\delta)\phi . \tag{21}$$

Experimental values of $(Cp/Cp_{max})_{CYL}$ from references 10, 11, 12 are plotted against ϕ in Fig. 5 for $M_\infty \sin\alpha \geq 3.5$ and in Fig. 6 for $1.5 < M_\infty \sin\alpha < 3$. We are concerned here only with the distribution for $\phi \leq 90^\circ$.

If $M_\infty \sin\alpha > 3.5$ (Fig. 5) all the results lie near the full-line curve which corresponds to, (see inset graph on Fig. 5),

$$\delta = 0.1\,\phi \quad \text{for} \quad M_\infty \sin\alpha \geq 3.5 \tag{22}$$

where ϕ is in radians. Fig. 6 shows experimental values of $(Cp/Cp_{max})_{CYL}$ for $1.5 < M_\infty \sin\alpha < 3$ and in this case the points suggest a variation of δ with $M_\infty \sin\alpha$ from approximately zero at $M_\infty \sin\alpha \simeq 1.5$ towards equation (22) at $M_\infty \sin\alpha \simeq 3.5$.

If $M_\infty \sin\alpha < 1$ large variations of $(Cp/Cp_{max})_{CYL}$ for a given ϕ occur with variations of $M_\infty \sin\alpha$ and M_∞. In any case, if $M_\infty \sin\alpha < 1$ the principle of adding the transverse flow pressure distribution to the other components cannot be justified; however since the method is limited to cases where $M_\infty \geq 3$, quite large errors in Cp_c can be tolerated if $M_\infty \sin\alpha < 1$, since this component of Cp must then be small.

For prediction purposes, the assumption will be made

$$\delta = 0 \quad \text{for} \quad M_\infty \sin\alpha \leq 1.5 \tag{23}$$

interpolation between equations (22) and (23) being necessary for $1.5 < M_\infty \sin\alpha < 3.5$.

11 COMPARISONS WITH EXPERIMENT

On Figs. 7(a) to 7(j), comparisons are made between the predicted pressure coefficient on cones by the method of this note and experimental data from Refs. 13,14,15 and 16. Each figure shows, for particular values of cone semi-angle and Mach No., pressure coefficient Cp vs. meridian angle ϕ for a range of angles of incidence, α. The predicted pressure coefficients are shown by full lines for $M_\infty \sin\theta > 1$, and continued as chain dotted lines for $1.0 > M_\infty \sin\theta > 0.3$. Also shown on each figure by a broken line is the prediction using the simple Newtonian impact coefficient of 2.

On the whole, for $M_\infty \sin\theta > 1$, agreement between the predicted pressure coefficients using the method of this note and experimental values is very good, the only serious - and somewhat puzzling - discrepancy being the case shown on Fig. 7(g) for $\phi > 90^\circ$.

For moderate or large cone semi-angles and small angles of incidence the method is much superior to simple impact theory, especially at the lower Mach numbers - see for example Figs. 7(a) and 7(b), $\alpha = 5^o$. This is because in these cases that part of the pressure perturbation generated by incidence is small compared with that generated by the cone volume itself, and since the latter corresponds to a coefficient K_a rather greater than 2, (Fig. 2(a)) Newtonian impact theory underestimates the pressure. At the other extreme, exemplified by Fig. 7(c), where the cone angle is small but incidence large, a large part of the pressure perturbation is generated by the transverse flow which corresponds to a coefficient K_c less than 2, (Fig. 4). Hence in this case, Newtonian theory overestimates the pressure coefficient. Between these two extremes, for example Fig. 7(a), $\varepsilon = 15^o$, $\alpha = 25^o$, the differences between the prediction of the method described herein and of simple impact theory may be small, due to a fortuitous combination of axial and transverse flow pressure components.

Where the surface normal Mach number $M_\infty . \sin\theta$ is less than one, the prediction would be expected to be less reliable than at higher normal Mach numbers, and the experimental data confirm this. In some cases, for example Figs. 7(a) and 7(h), the prediction is quite good for $M_\infty . \sin\theta$ as low as 0.3, but in others, for example Fig. 7(j), it is poor. In general, successful prediction for $M_\infty . \sin\theta < 1$ must be regarded as fortuitous, but it is worth noting that there is no sudden divergence between prediction and experiment as $M_\infty . \sin\theta$ decreases through unity; instead the likely error apparently increases steadily as $1 - M_\infty . \sin\theta$ increases.

12 CONCLUSIONS

A simple semi-empirical method has been devised for predicting pressure on the windward surface of a cone at Mach numbers in excess of 3, and arbitrary angle of incidence.

Agreement between predicted pressures and experimental data is good so long as the local Mach number component normal to the surface exceeds unity. The method is more accurate than the simple impact theory equation $Cp = 2\sin^2\theta$ especially for small angles of incidence or small cone angles.

SYMBOLS

Cp	local pressure coefficient, $2(p-p_\infty)/\gamma\, p_\infty\, M_\infty^2$
Cp_a, Cp_b, Cp_c	components of Cp, see section 5
Cp_s	stagnation pressure coefficient
Cp*	pressure coefficient in the limit $M_\infty \cdot \theta \to \infty$
K_a, K_b, K_c	variable coefficients in equations 12, 13, 14, Figs. 2, 3, 4
K*	impact coefficient in the equation $C_p^* = K^* \sin^2\theta$
M_∞	free stream Mach number
M_a	$M_\infty \cdot \cos\alpha$
M_c	$M_\infty \cdot \sin\alpha$ in Fig. 4
p	local static pressure
p_∞	free stream static pressure
r	local cone radius (Fig. 1)
V_∞	free stream velocity
x,y,z	Cartesian coordinates for a cone, (Fig. 1)
α	cone incidence, measured from axis
β	$\sqrt{M_\infty^2 - 1}$
β_a	$\sqrt{M_a^2 - 1}$
γ	ratio of specific heats
δ	empirical variable discussed in section 10
ε	cone semi-angle
ζ	angle between shock wave and free stream direction
θ	angle between free stream and local surface, measured in a plane normal to the surface
ϕ	polar angle subtended at the cone axis between any point on the surface and the most windward point in the same plane normal to the axis

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15	Peckham, D.H.	Experiments at hypersonic speeds on circular cones at incidence. ARC CP No.702, January 1963
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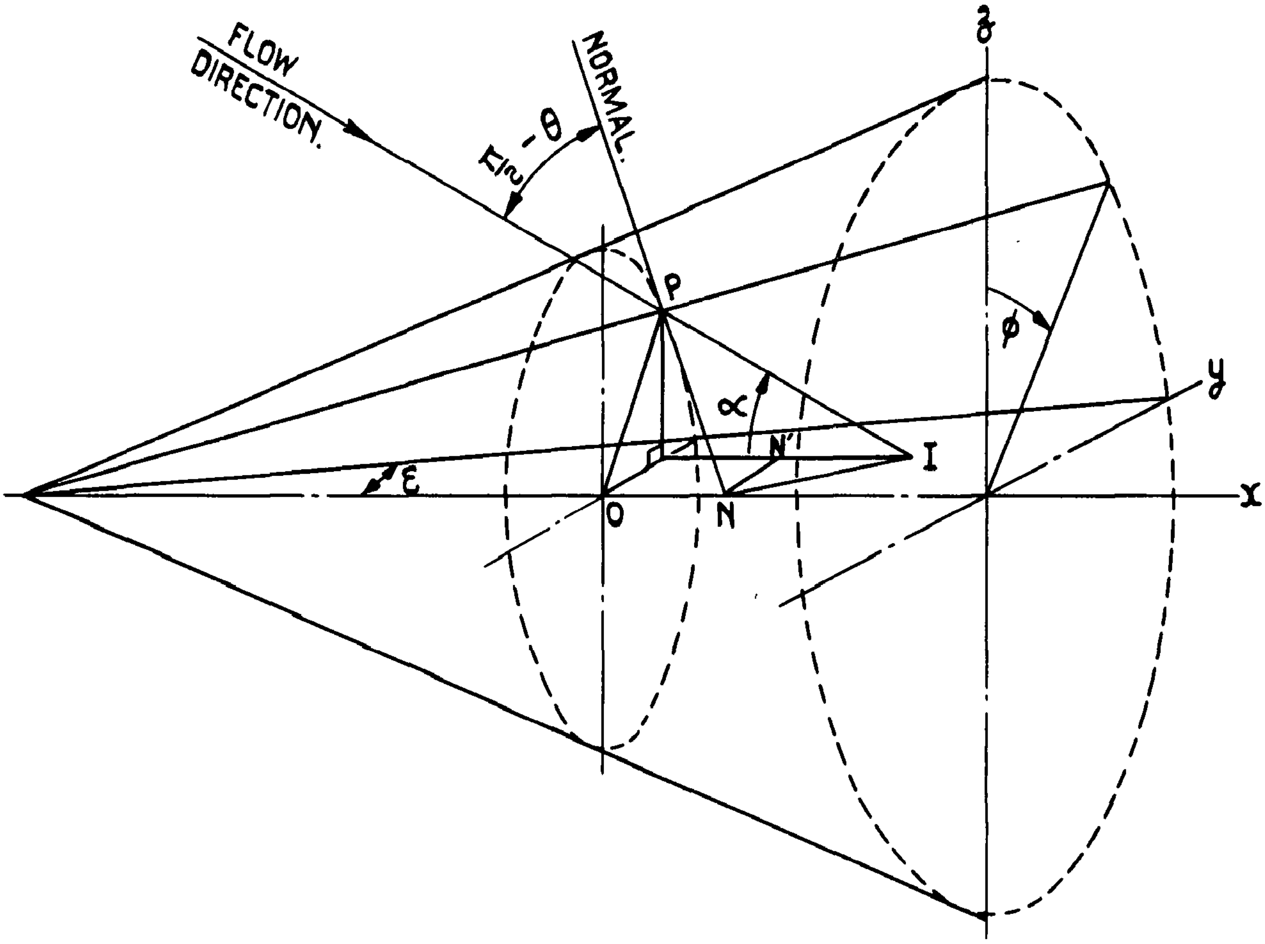
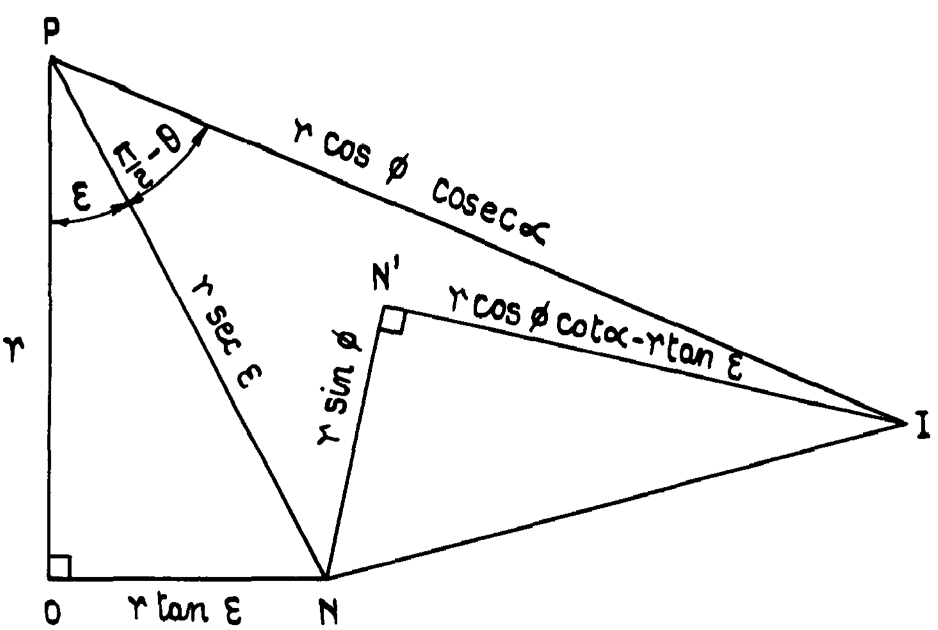




$$\cos\left(\frac{\pi}{2}-\theta\right) = \sin\theta = \frac{PN^2 + PI^2 - NI^2}{2\,PN.\,PI}.$$

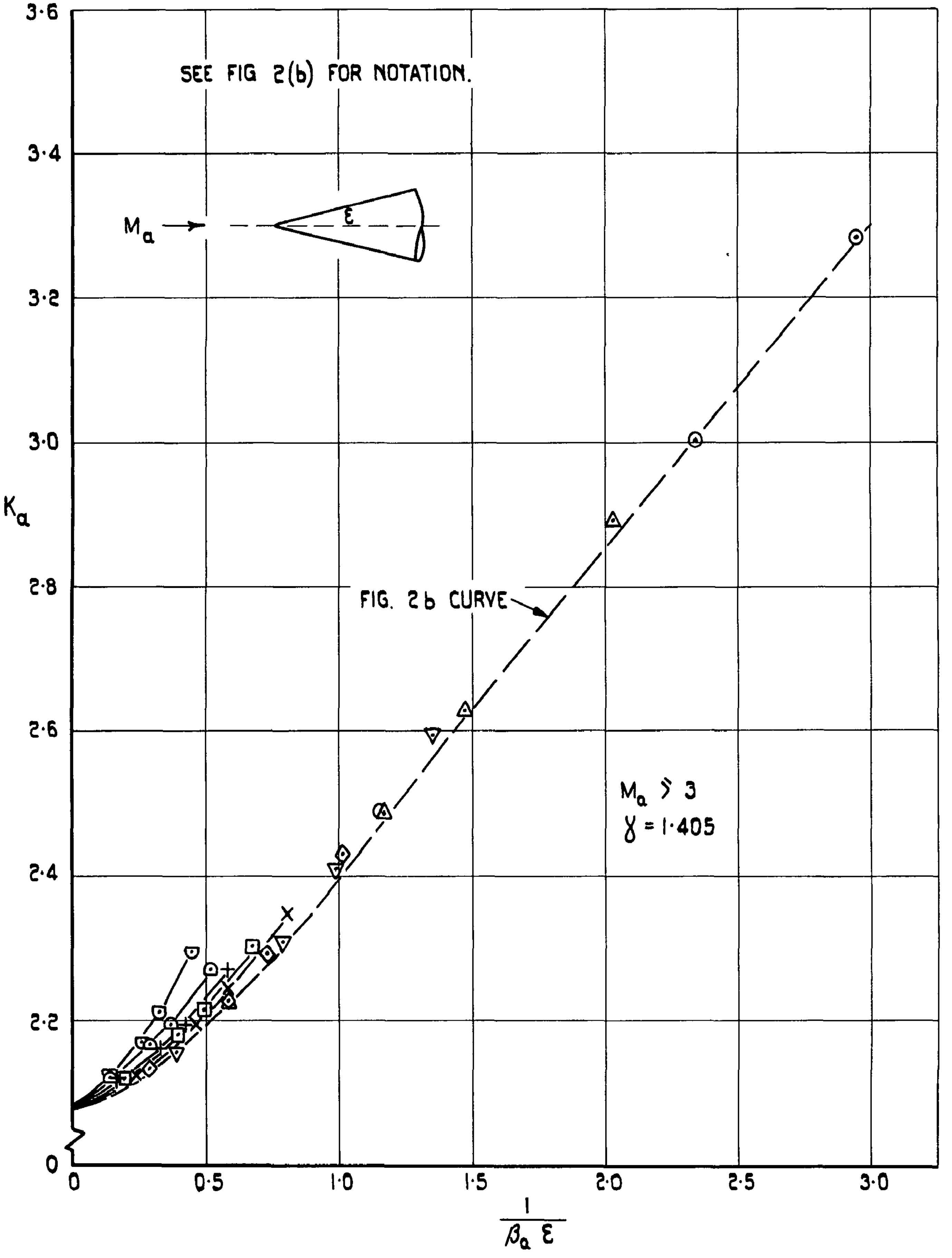


FIG. 2(a). COEFFICIENT K_a FOR A CONE IN AXIAL FLOW -CORRELATION BY SUPERSONIC–HYPERSONIC SIMILARITY RULE

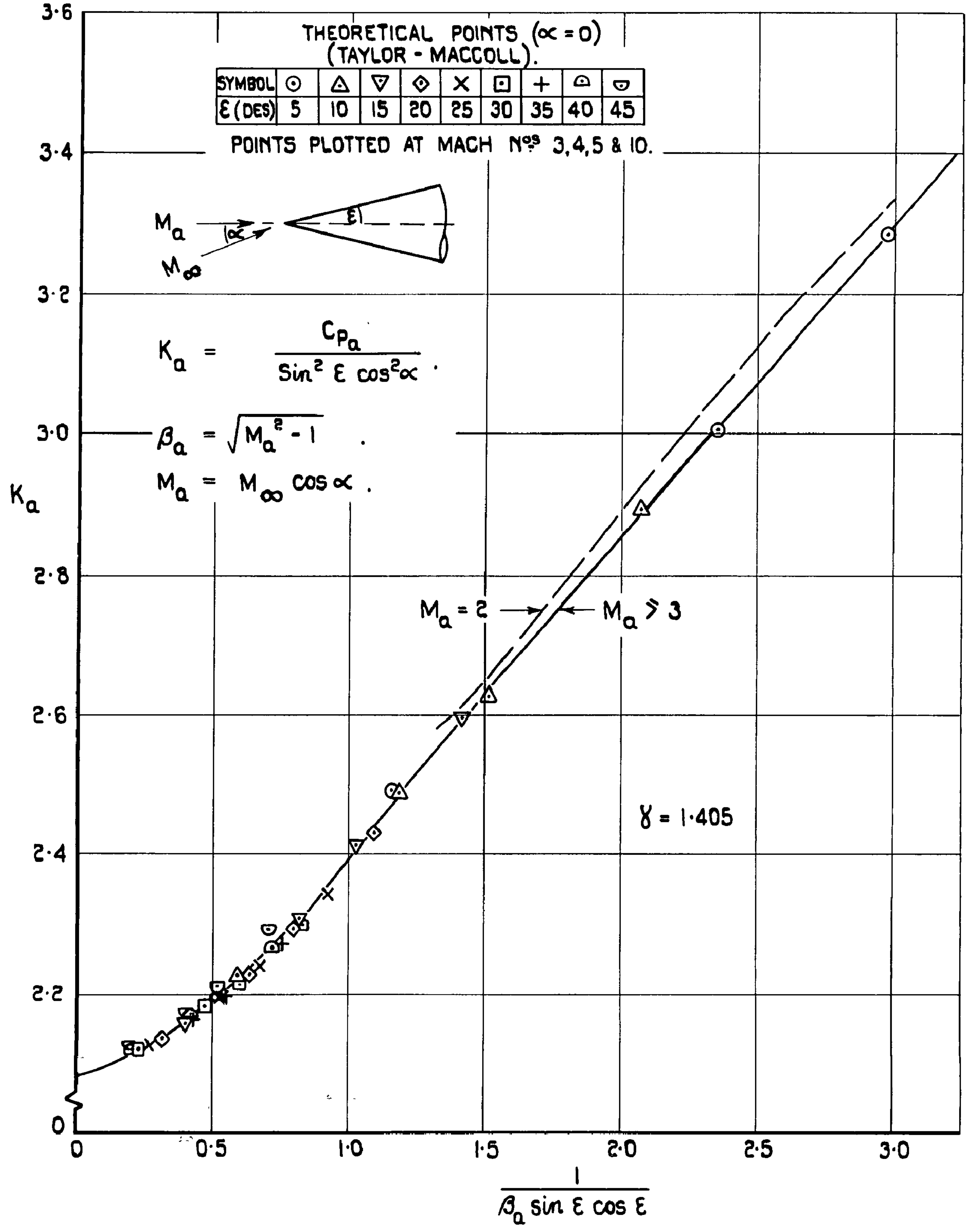


FIG. 2.(b) COEFFICIENT K_a FOR A CONE IN AXIAL FLOW. -CORRELATION BY LARGE ANGLE EXTENSION OF SUPERSONIC-HYPERSONIC SIMILARITY RULE.

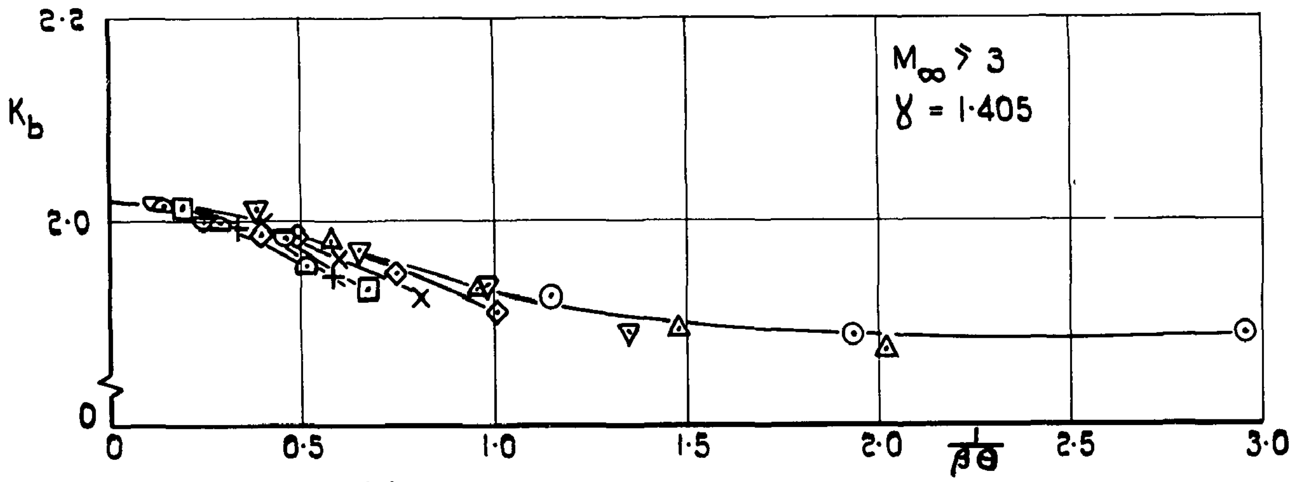
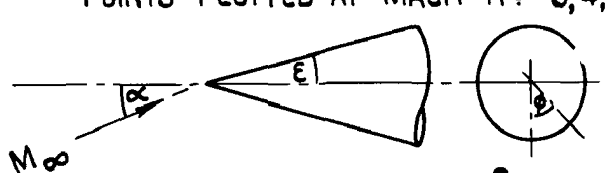


FIG. 3. (a) COEFFICIENT K_b FOR A CONE -CORRELATION BY SUPERSONIC–HYPERSONIC SIMILARITY RULE FOR SMALL ANGLES.

THEORETICAL POINTS ($\alpha \to 0$).
(STONE).

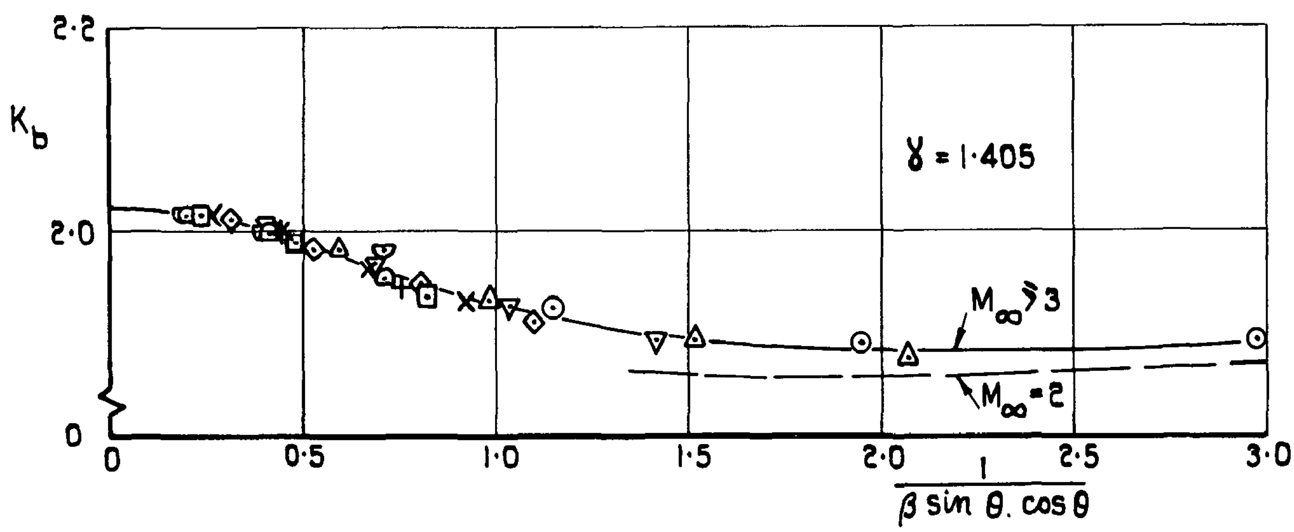
SYMBOL	⊙	△	▽	◇	×	⊡	+	◓	◒
ε (DEG.)	5	10	15	20	25	30	35	40	45

POINTS PLOTTED AT MACH N^os 3, 4, 5, 6 & 10.



$$K_b = \frac{C_{p_b}}{2 \sin \varepsilon \cos \varepsilon \cos \phi \sin \alpha \cos \alpha}$$

IN SMALL INCIDENCE THEORY $K_b = \dfrac{(dC_p/d\alpha)_{\alpha \to 0}}{2 \sin \varepsilon . \cos \varepsilon \cos \phi}$ AND $\theta \to \varepsilon$.



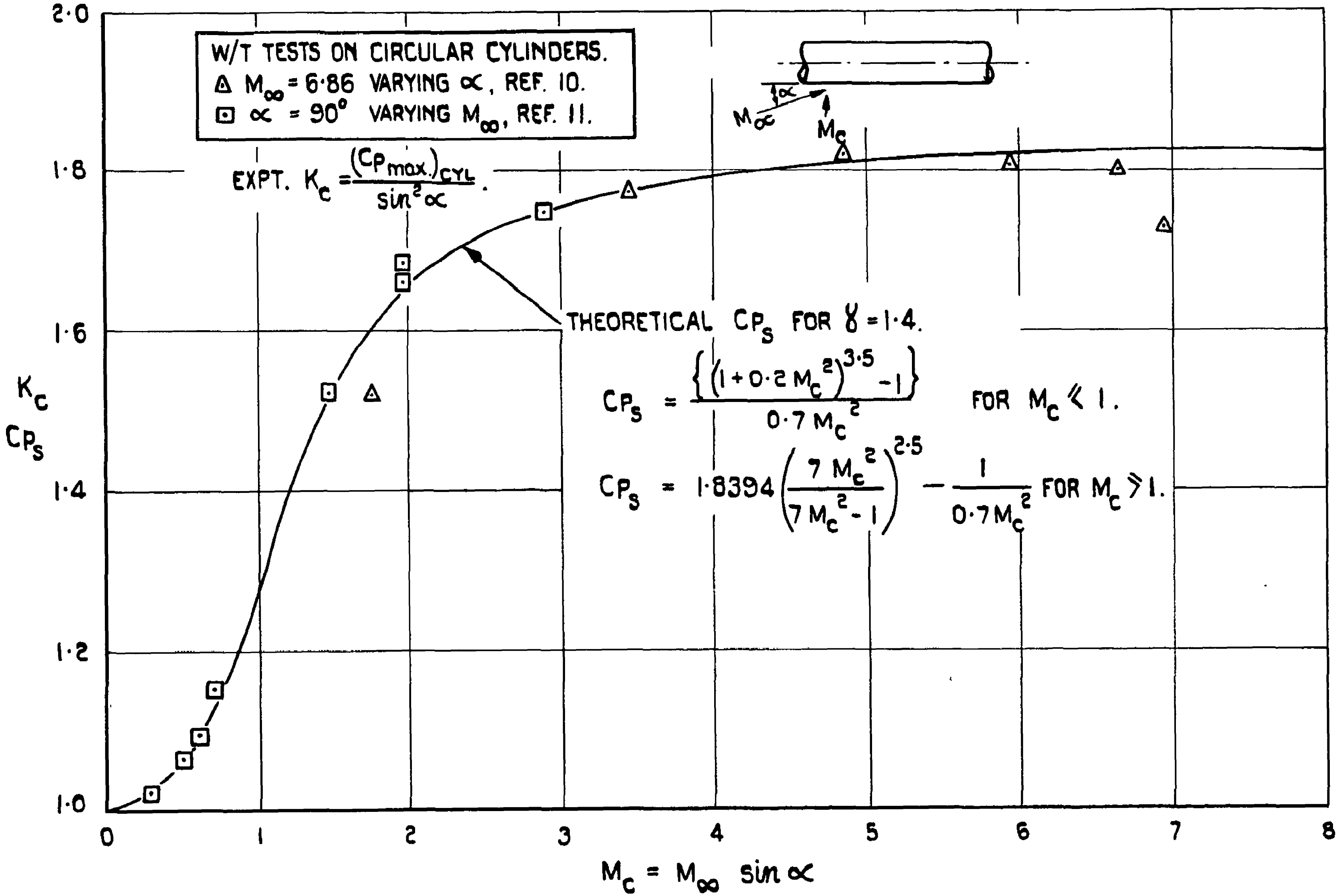


FIG. 4. COEFFICIENT K_c FOR PRESSURES DUE TO CROSS FLOW.

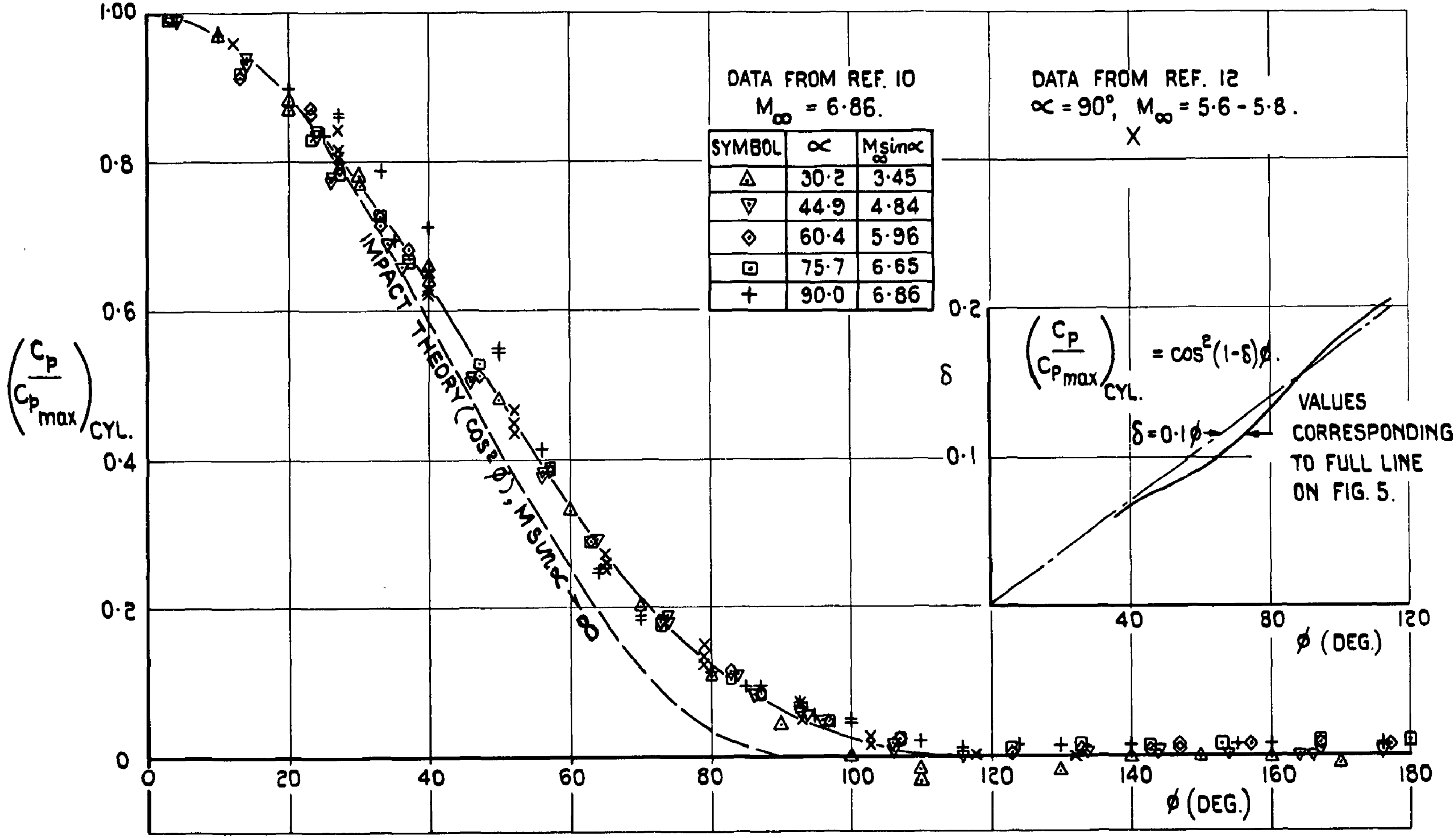


FIG. 5. EXPERIMENTAL PRESSURE DISTRIBUTION ROUND A CIRCULAR CYLINDER AT INCIDENCE ($M_\infty \sin\alpha \geqslant 3{\cdot}5$)

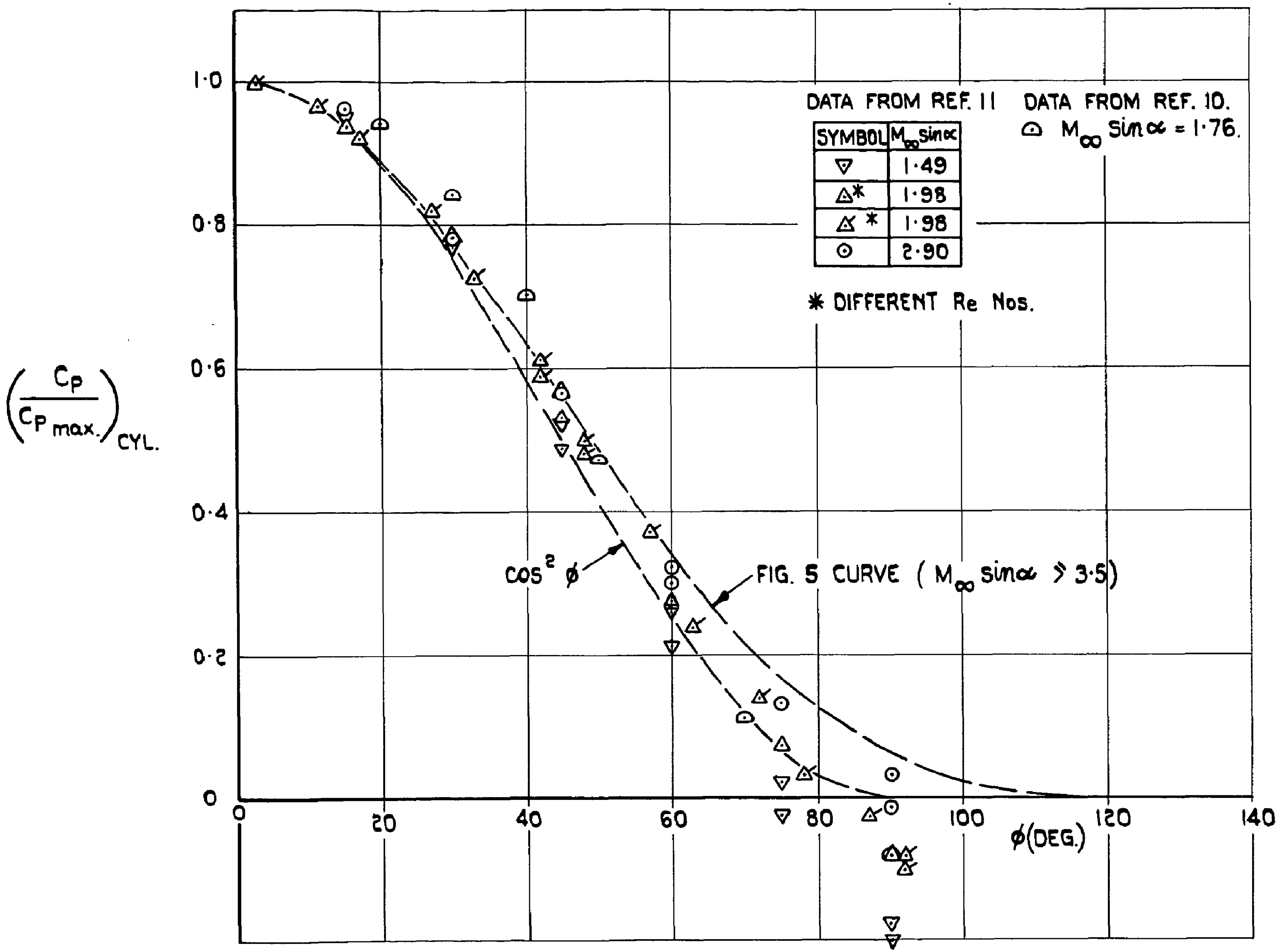
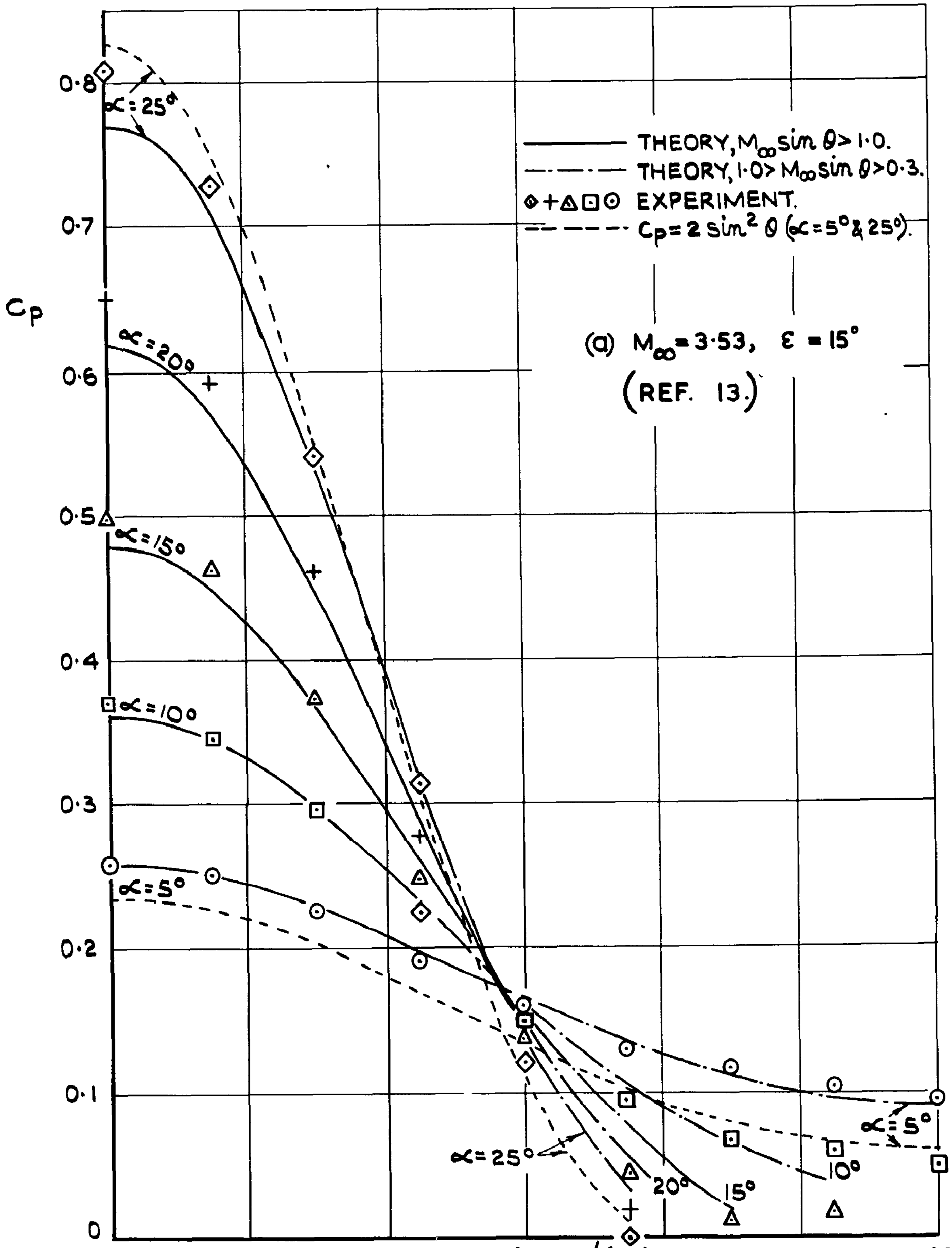
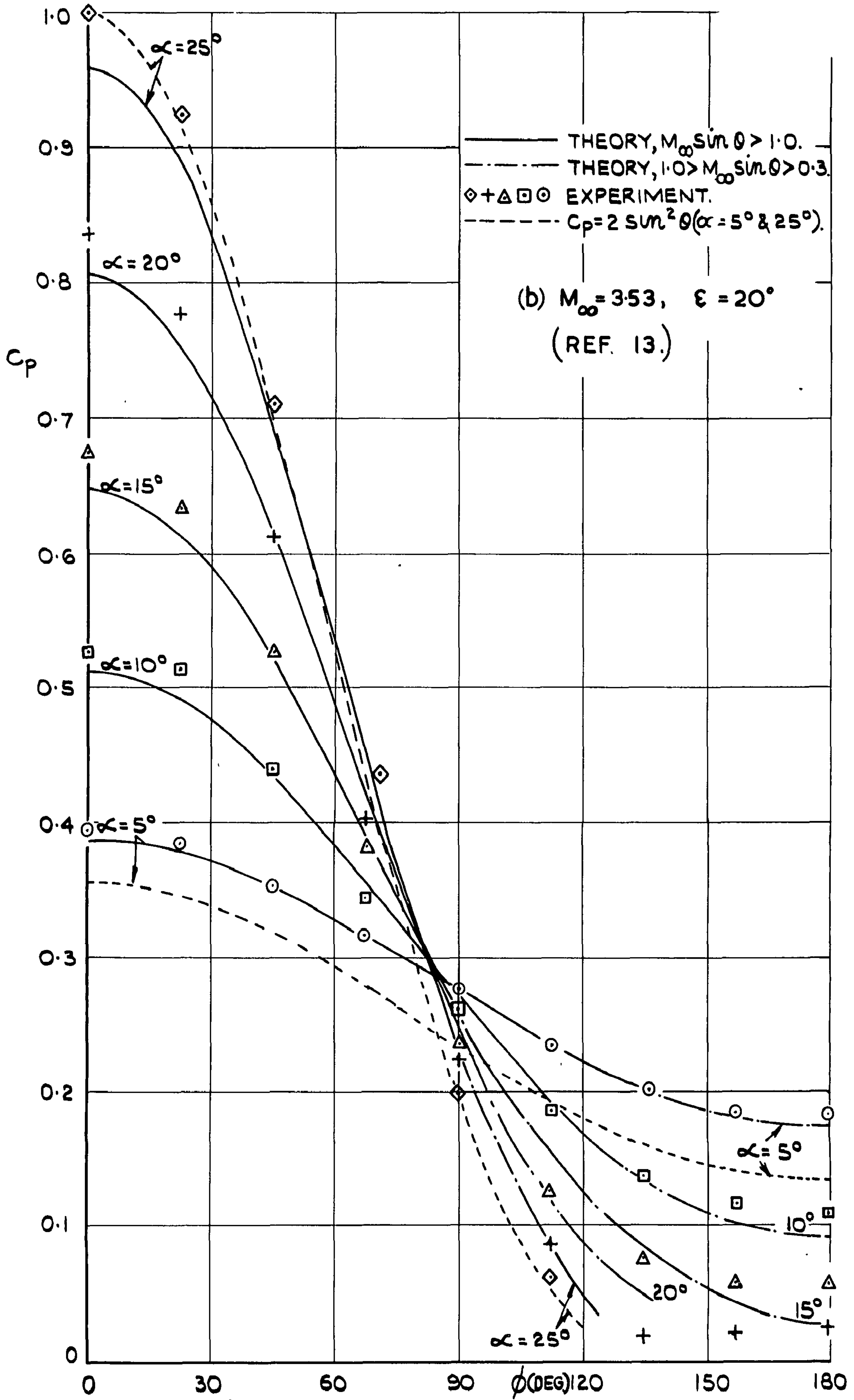


FIG.6. EXPERIMENTAL PRESSURE DISTRIBUTION ROUND A CIRCULAR CYLINDER NORMAL TO THE STREAM ($1{\cdot}5 < M_\infty \sin\alpha < 3$)



THEORY, $M_\infty \sin\theta > 1\cdot0$.
THEORY, $1\cdot0 > M_\infty \sin\theta > 0\cdot3$.
EXPERIMENT.
$C_p = 2\sin^2\theta$ ($\alpha = 5°$ & $25°$).
(a) $M_\infty = 3\cdot53$, $\varepsilon = 15°$
(REF. 13.)
C_p
0·8
0·7
0·6
0·5
0·4
0·3
0·2
0·1
0
$\alpha = 25°$
$\alpha = 20°$
$\alpha = 15°$
$\alpha = 10°$
$\alpha = 5°$
$\alpha = 25°$
20°
15°
10°
$\alpha = 5°$



THEORY, $M_\infty \sin\theta > 1.0$.
THEORY, $1.0 > M_\infty \sin\theta > 0.3$.
EXPERIMENT.
$C_p = 2\sin^2\theta$ ($\alpha = 5°$ & $25°$).
(b) $M_\infty = 3.53$, $\varepsilon = 20°$
(REF. 13.)
C_p
ϕ (DEG)
$\alpha = 25°$
$\alpha = 20°$
$\alpha = 15°$
$\alpha = 10°$
$\alpha = 5°$
10°
20°
15°
1.0
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
0
30
60
90
120
150
180

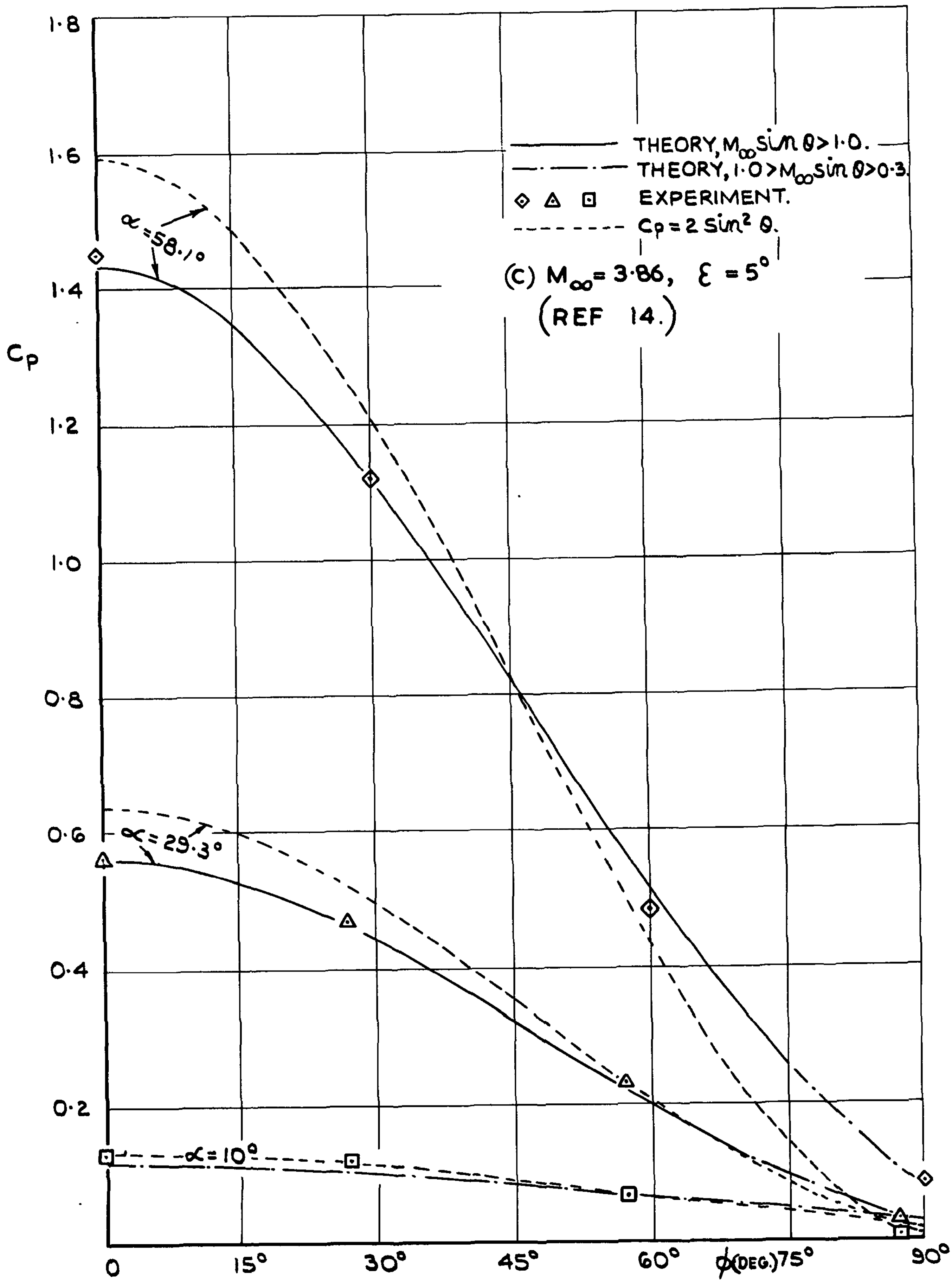


FIG.7 (CONTD.) COMPARISON BETWEEN MEASURED AND PREDICTED PRESSURES ON CONES.

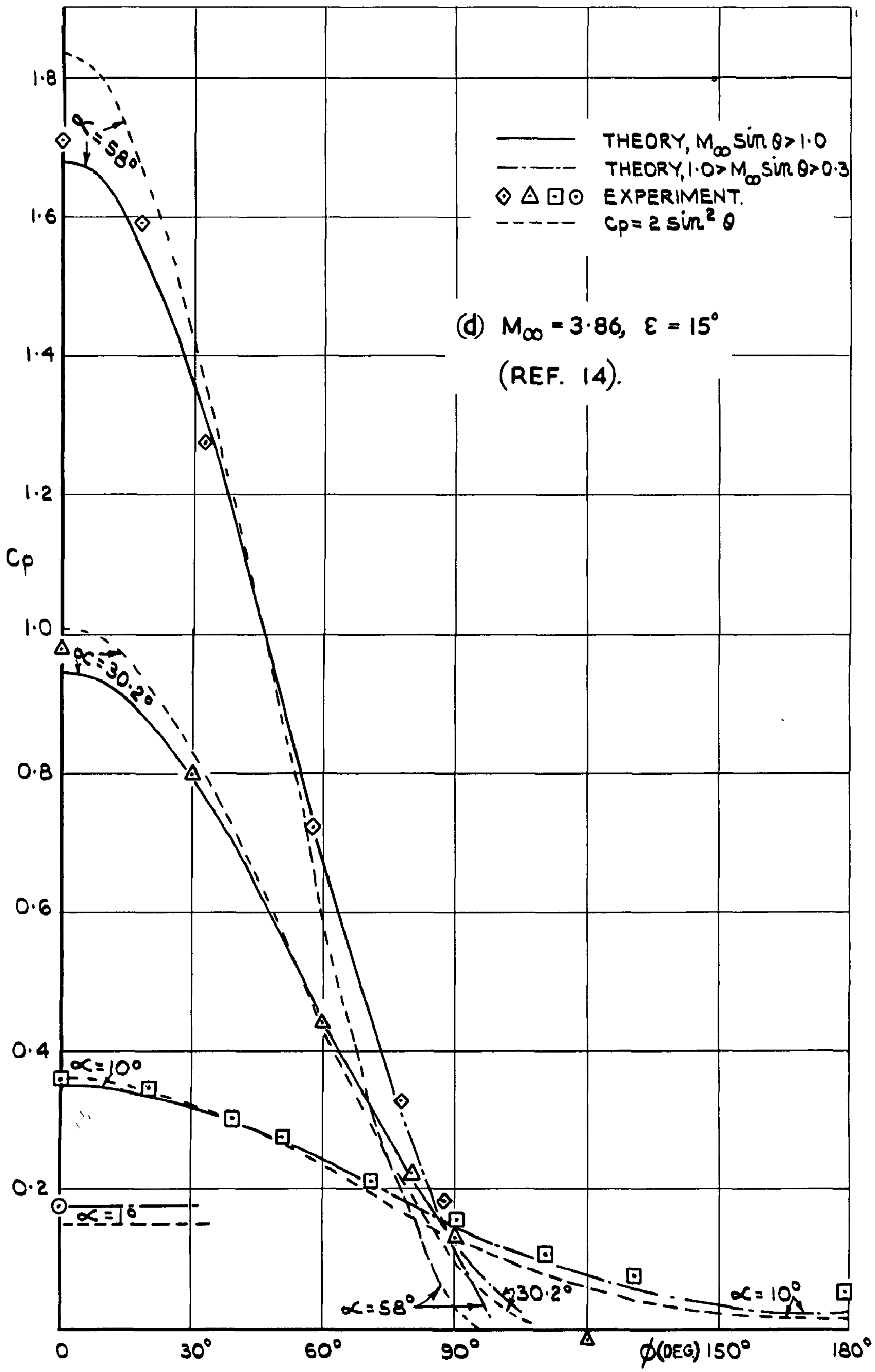


FIG.7. (CONTD.) COMPARISON BETWEEN MEASURED AND PREDICTED PRESSURES ON CONES.

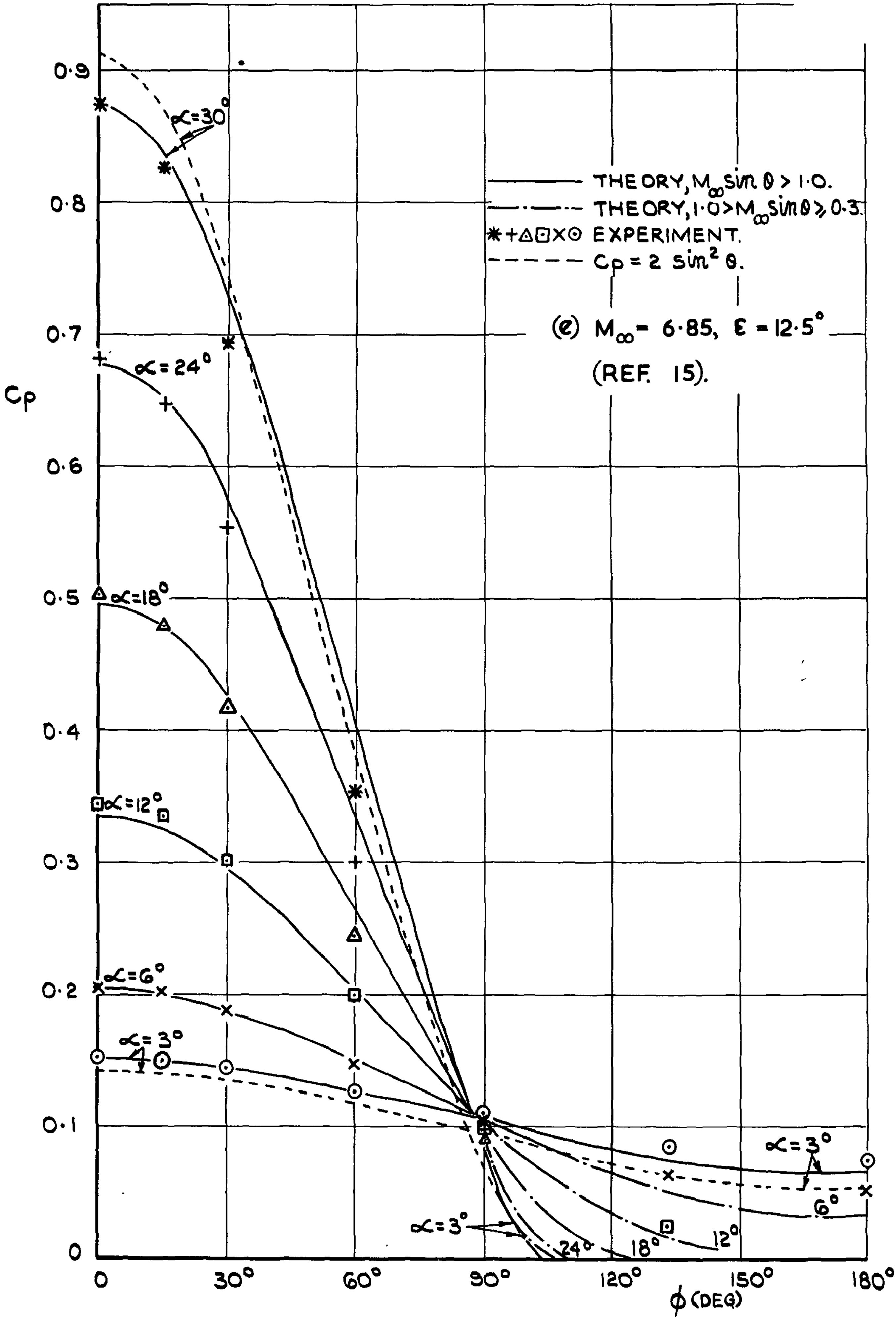


FIG.7. (CONTD.) COMPARISON BETWEEN MEASURED AND PREDICTED PRESSURES ON CONES.

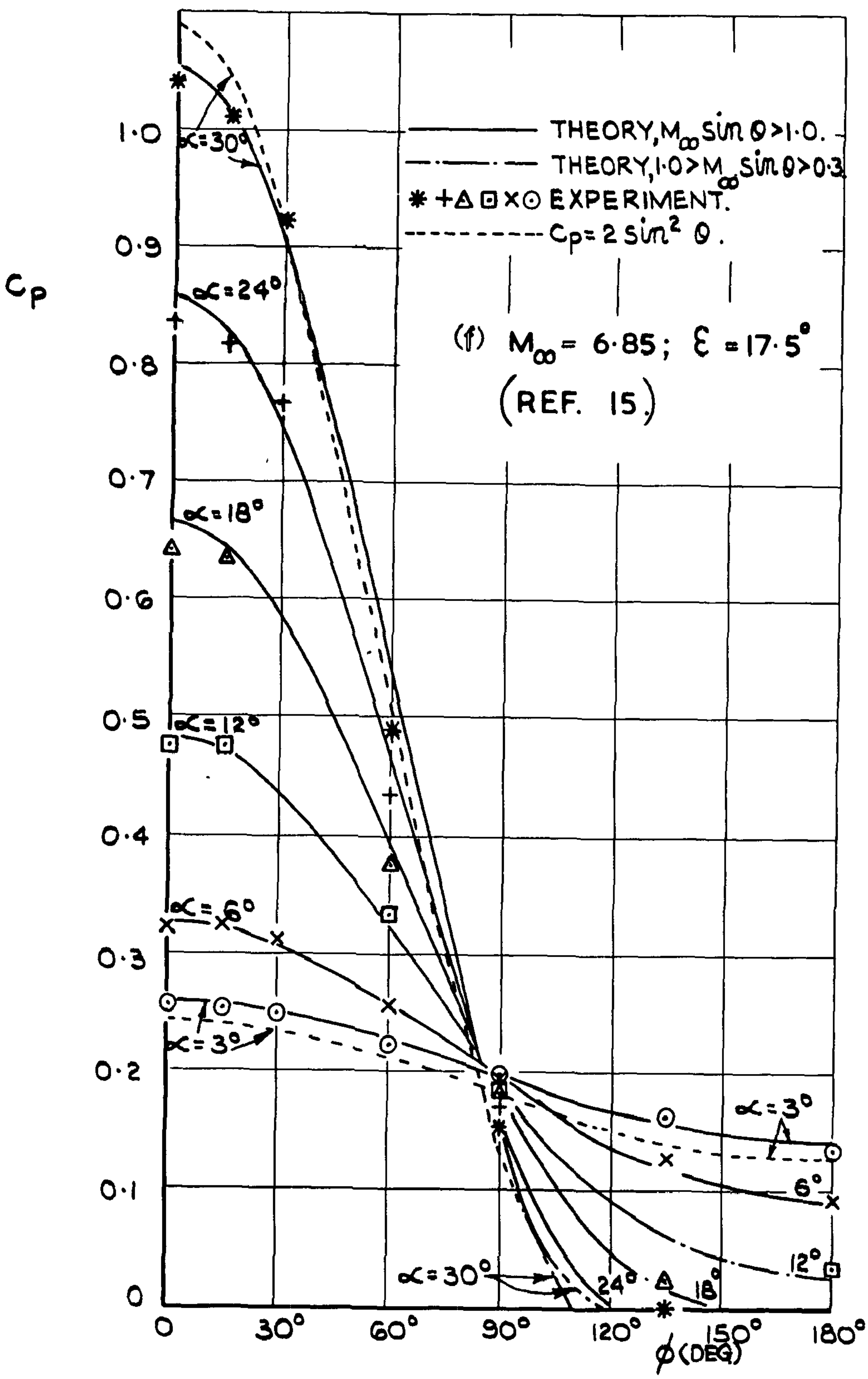


FIG. 7 (CONTD.) COMPARISON BETWEEN MEASURED AND PREDICTED PRESSURES ON CONES.

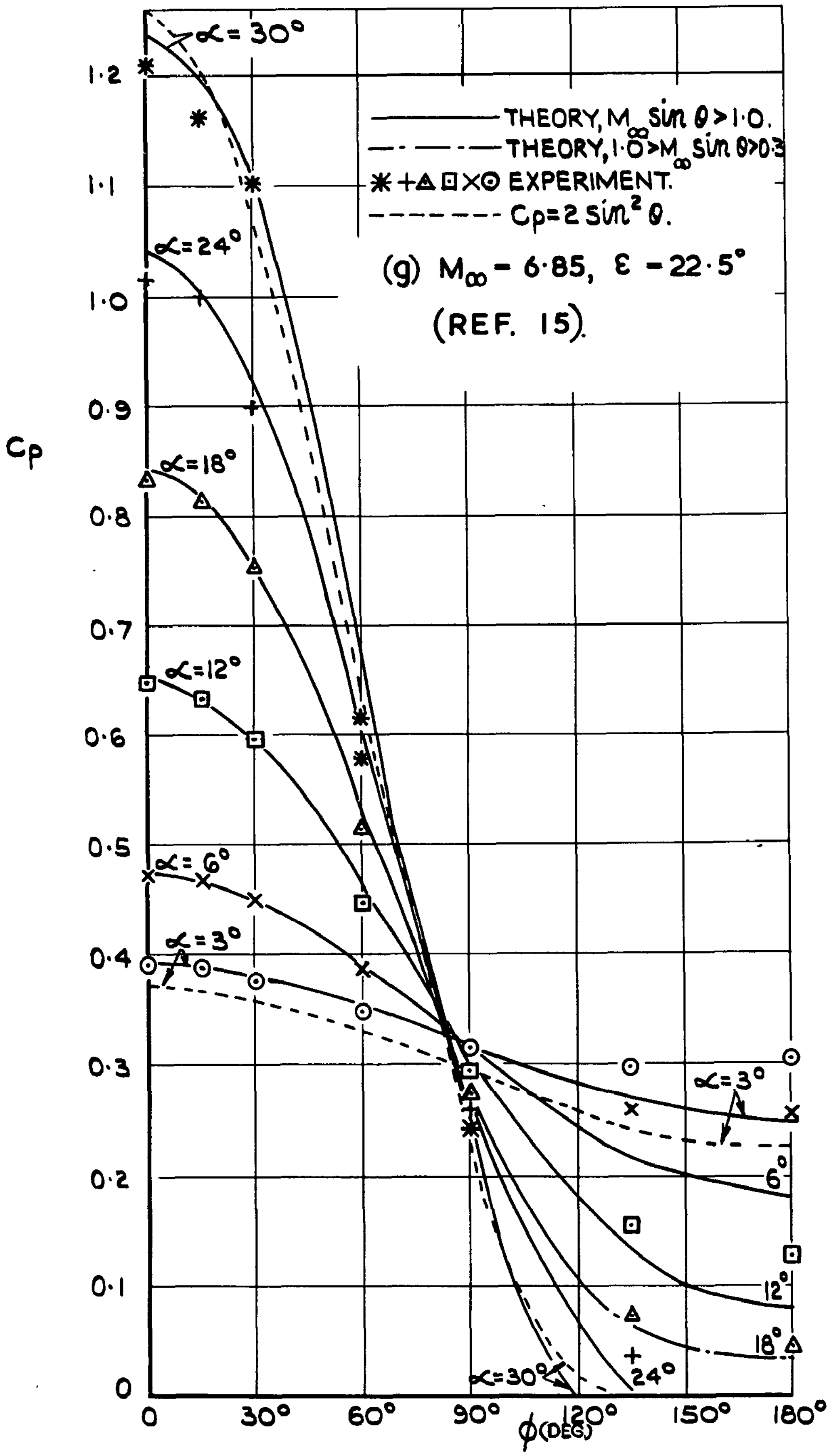


FIG.7. (CONTD.) COMPARISON BETWEEN MEASURED AND PREDICTED PRESSURES ON CONES.

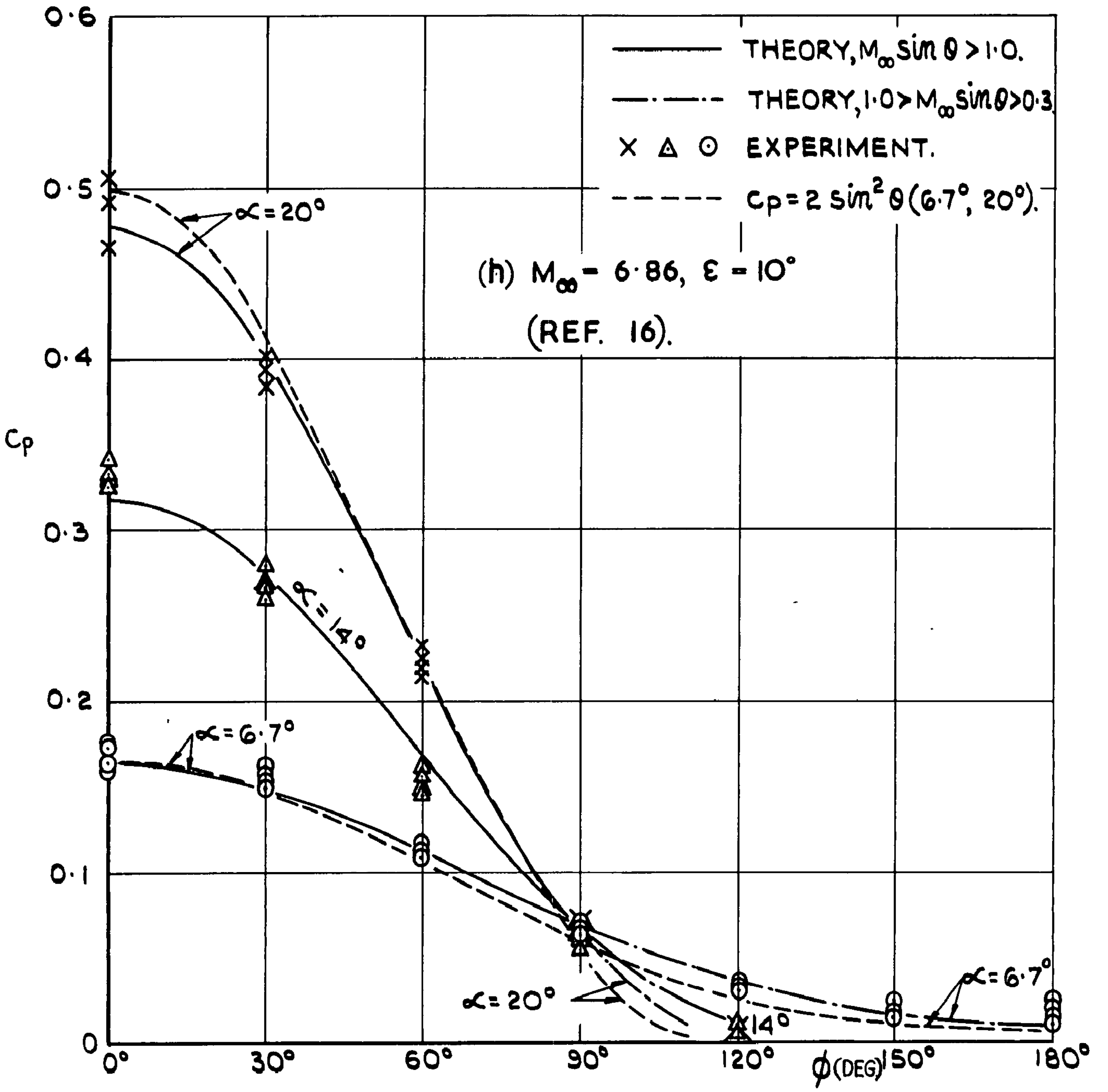
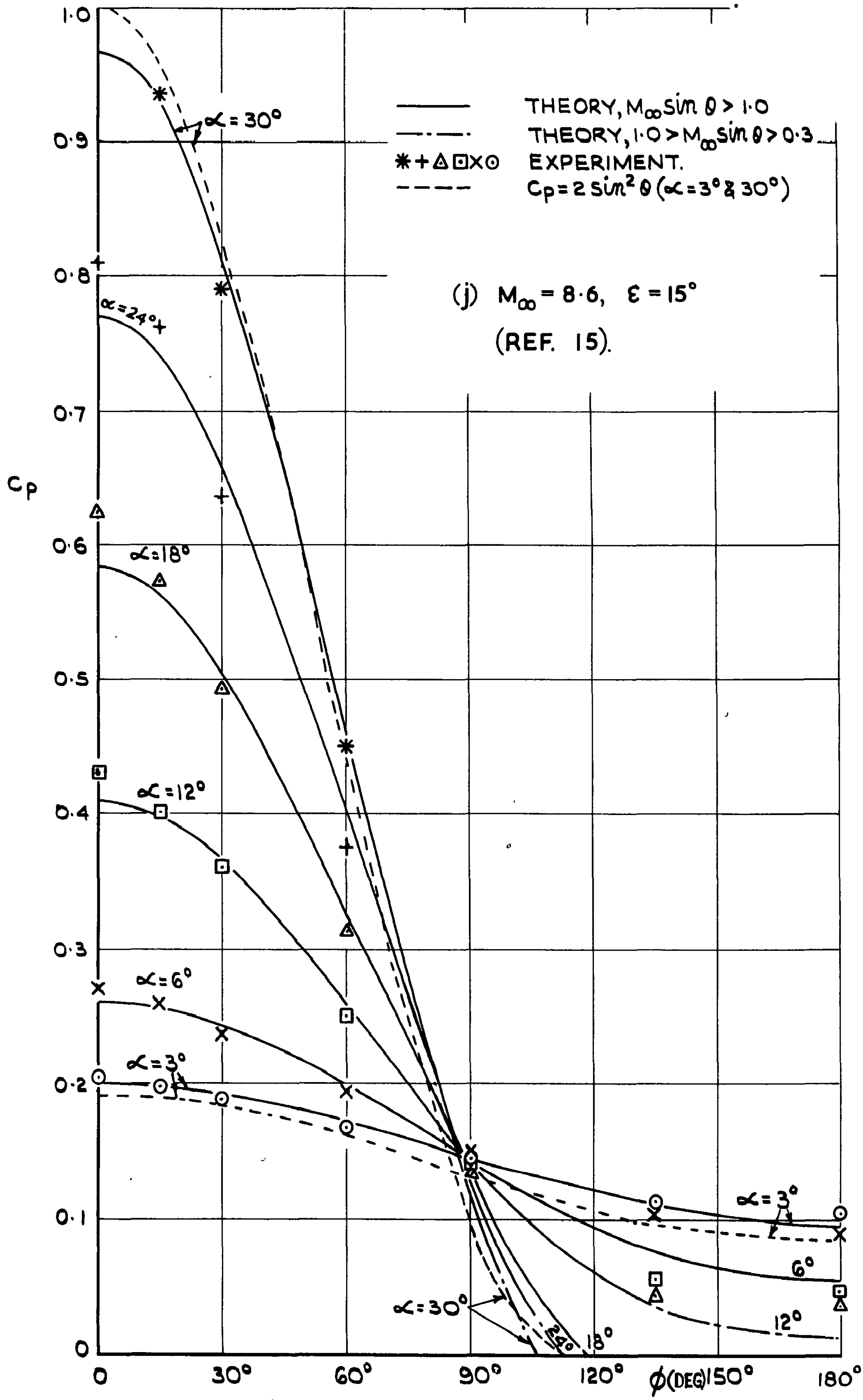


FIG.7. (CONTD) COMPARISON BETWEEN MEASURED



THEORY, M∞ sin θ > 1·0
THEORY, 1·0 > M∞ sin θ > 0·3
✱ + △ ⊡ × ⊙ EXPERIMENT.
Cp = 2 sin² θ (α = 3° & 30°)
(j) M∞ = 8·6, ε = 15°
(REF. 15).
Cp
φ (DEG)
α = 30°
α = 24°
α = 18°
α = 12°
α = 6°
α = 3°
6°
12°
18°
24°
1·0
0·9
0·8
0·7
0·6
0·5
0·4
0·3
0·2
0·1
0
30°
60°
90°
120°
150°
180°

A.R.C. C.P. No.792

533.696.4 :
533.6.048.2 :
533.6.011.5 :
533.6.011.55

A SEMI-EMPIRICAL PREDICTION METHOD FOR PRESSURES ON THE WINDWARD SURFACE OF CIRCULAR CONES AT INCIDENCE AT HIGH SUPERSONIC AND HYPERSONIC SPEEDS ($M \geq 3$)
Collingbourne, J.R., Crabtree, L.F., Bartlett, W.J. June 1964

The so-called impact theory equation $C_p^* = K^* \sin^2\theta$ for a circular cone is written as the sum of three terms associated with axial, combined and transverse flows respectively. By postulating that K^* in each term represents a limiting value as $M_\infty.\theta \to \infty$ of coefficients K_a, K_b, K_c which are functions of variables like $M_\infty.\theta$, a simple, semi-empirical method is devised for predicting pressure distribution on the windward region of a circular cone which agrees well with experimental data up to large incidence angles. Numerically, the method is based on the small incidence theories of Taylor-

p.t.o.

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p.t.o.

Maccoll and Stone, and on experimental pressure distributions over cylinders placed normal to the stream.

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