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A Simple Method for Real Gas Flow Calculations

By

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A Simple Method for Real Gas Flow Calculations

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SUMMARY

A method is found for calculating correction factors which enables perfect gas flow tables to be used with any isentropic real gas flow which may also include a normal shock wave. Two correction factors, which are independent of Mach number, are used to relate the reservoir to the freestream conditions. A further factor, which is independent of Mach number when the latter is greater than three, corrects the ratio of the freestream to total pressure behind a normal shock. Using these three factors all the flow parameters may be easily computed.

The method is used to calculate the correction factors for nitrogen in the temperature range 600°K to 2000°K at pressures up to 1000 atmospheres.

Notation

C_p	specific heat at constant pressure
H	enthalpy
m	mass flow rate
P	pressure
R	gas constant
S	entropy
T	temperature
u	velocity
Z	bulk compressibility factor
γ	ratio of specific heats
ρ	density

Subscripts/

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Subscripts

o	thermodynamic reference state
1	conditions ahead of shock ≡ freestream conditions
2	conditions behind a normal shock
EQ	equivalent perfect gas
s	signifies conservation across a normal shock wave
t_1	total conditions ahead of shock ≡ reservoir conditions
t_2	total conditions behind a normal shock

Introduction

When operating a wind tunnel it is convenient to find the Mach number and other flow variables from tables, rather than to calculate them from the perfect gas or real gas formulae.

At moderate temperature and low pressures, the perfect gas formulae are adequate. Tables such as "Compressible Airflow: Tables"[1] and NACA Report 1135[2] tabulate the properties of the reservoir conditions, etc., divided by the freestream conditions, as a function of flow Mach number. The condition that the flow is adequately described by the perfect gas formulae is that throughout the flow the enthalpy is proportional to the temperature, i.e., $H = H(T)$.

At higher temperatures when vibrational or electronic excitation are significant, this proportionality fails but the enthalpy remains a function of temperature, i.e., $H = H(T)$. Under these conditions, corrections can be applied to the perfect gas tables which allow for this departure from ideal. Each of the tabulated values will have a correction factor which will in general be a function of Mach number and reservoir temperature. These correction factors may be found in Ref.2 and also in Bouniol[3].

However, modern shock tunnels and hotshot tunnels operate at sufficiently high pressures for the bulk compressibility factor Z to have an appreciable effect on the thermodynamic properties of the gas, and, in particular, on the enthalpy, i.e., $H = H(P,T)$. Each correction factor now depends on three variables: the flow Mach number, and the reservoir temperature and pressure. Their tabulation or graphical representation, therefore, becomes rather unwieldy.

In addition, if the Mach number is an unknown, an iterative procedure is necessary, although in practice the correction factors tend to a constant value at high Mach numbers. Two sets of graphs of correction factors valid for Mach numbers greater than 10 have been published by Erickson and Creekmore[4] for air, and more recently by Clark and Johnson[5] for nitrogen.

It/

It would, however, be convenient if a method of applying the correction factors at lower Mach numbers could be found which did not depend on Mach number. That such a correction factor should exist can be seen from the following argument.

A divergent nozzle expanding a real gas to a supersonic velocity can be divided into two regions: an upstream region where the gas must be regarded as real and a downstream region where at each point $H = C_p T$. Clearly, a further expansion to a higher Mach number from anywhere in this downstream region can be calculated using the perfect gas formulae and therefore the correction factor for the nozzle as a whole is only affected by the upstream section of the nozzle.

Providing therefore that the gas in the freestream condition obeys the relation $H = C_p T$, it should be possible to express the correction factor in such a way that it is independent of Mach number.

Equivalent Perfect Gas

The form of this correction factor can be found by an extension of the above argument.

If we apply the perfect gas formulae or tables to the freestream flow conditions, which we shall assume for the moment to be known, then we shall find the reservoir conditions which would exist if the gas remained perfect upstream. We shall call the gas in this fictitious reservoir the 'equivalent perfect gas'. It is clear that if we can find the thermodynamic properties of the equivalent perfect gas as a function of the real gas reservoir properties, we can then immediately find all the freestream conditions from the perfect gas tables.

The relationship between the equivalent perfect gas and the real gas can be found by considering the basic equations describing the flow. For a real gas these are:

$$S_1 = S_{t_1} \quad \text{... (1a)}$$

$$H_1 = H_{t_1} - \tfrac{1}{2}u_1^2 \quad \text{... (1b)}$$

i.e., the entropy and total enthalpy are conserved.

These equations must also hold for a perfect gas. In fact, the elementary perfect gas equations in terms of Mach number and γ are derived from them.

Hence we have:-

$$S_1 = S_{EQ} \quad \text{... (2a)}$$

$$H_1 = H_{EQ} - \tfrac{1}{2}u_1^2 \quad \text{... (2b)}$$

and/

and it therefore follows that:-

$$S_{EQ} = S_{t1} \qquad \ldots (3a)$$

and

$$H_{EQ} = H_{t1} . \qquad \ldots (3b)$$

It is thus apparent that the equivalent perfect gas is that perfect gas which has the same entropy and enthalpy as the real gas in the reservoir.

The temperature and pressure of the equivalent perfect gas immediately follow, since for a perfect gas:-

$$H_{EQ} = C_P T_{EQ} \qquad \ldots (4a)$$

and

$$S_{EQ} = C_P \ell n \; T_{EQ}/T_o - R\ell n \; P_{EQ}/P_o + S_o . \qquad \ldots (4b)$$

We have therefore found a one to one relationship between a real gas and its equivalent perfect gas, since each state of the real gas may be defined by its entropy and enthalpy which in turn determines the pressure and temperature of the equivalent perfect gas.

The correction factor required to convert the real gas pressure to the equivalent perfect gas pressure is obviously P_{EQ}/P_{t1} and similarly for temperature it is T_{EQ}/T_{t1} . These may be plotted or tabulated against suitable real gas thermodynamic functions such as P_{t1} and T_{t1} or, in the case of a shock tunnel, the equivalent perfect gas properties themselves may be plotted against initial channel pressure and shock velocity.

In Figs.1 and 2 the correction factors are given for nitrogen as a function of the real reservoir pressure and temperature over the range of temperatures from 600°K to 2000°K at pressures up to 1000 atmospheres. The real gas tables from which the data was taken were those by the present authors[6].

Total Conditions behind a Normal Shock

In high Mach number tunnels where the static pressure is low, it is more convenient to determine the ratio of reservoir pressure to pitot pressure in order to calculate the Mach number. At the point downstream where the pressure is measured, the gas has crossed a normal shock and has been brought isentropically to rest.

Across the normal shock we have the relations:

$$\rho_1 u_1 = \rho_2 u_2 = m_s \qquad \ldots (5a)$$

$$P_1 + \rho_1 u_1^2 = P_2 + \rho_2 u_2^2 = P_s \qquad \ldots (5b)$$

$$H_1 + \tfrac{1}{2} u_1^2 + H_2 + \tfrac{1}{2} u_2^2 = H_s \qquad \ldots (5c)$$

and/

and for the adiabatic compression:-

$$S_{t2} = S_2 \qquad \ldots (6a)$$

$$H_{t2} = H_2 + \tfrac{1}{2}u_2^2 \; . \qquad \ldots (6b)$$

The correction factors to the ideal case can be found by solving these equations numerically over the range of interest.

An iterative procedure is not required if the conditions behind the shock are used as independent variables and if use is made of the fact that the freestream gas is perfect.

Thus we may put:-

$$H_1 = C_p T_1 \qquad \ldots (7a)$$

and

$$P_1 = \rho_1 R T_1 \; . \qquad \ldots (7b)$$

We obtain by substitution of these equations into equations (5):-

$$\rho = \frac{C_p}{2RH_s}\left\{ P_s \mp \left[P_s^2 - \frac{4RH_s m_s^2}{C_p}\left(1 - \frac{R}{2C_p}\right)\right]^{\frac{1}{2}} \right\} . \qquad \ldots (8)$$

The shock equations (5) show that this solution is equally applicable to the two sides of the shock, and in fact the negative sign refers to the upstream conditions while the positive sign gives the density downstream of the shock which would occur if the gas there were perfect. By substituting these two values of ρ into equations (5), we may determine all the freestream conditions and also those of a perfect gas just downstream of the shock in terms of the known quantities m_s, P_s, and H_s.

The total downstream conditions of the perfect gas can be found from the perfect gas equations or tables while the real gas total conditions are calculated from real gas tables using entropy and total enthalpy conservation.

Hence we may compare the ratio of the freestream to total conditions behind a normal shock, for a real gas and an ideal gas, with the same freestream conditions.

It was found first of all, that the values of these ratios were almost independent of pressure, varying only by 0·2% for total pressures of 40 atmospheres behind the normal shock. It was also found empirically that the correction factors did not vary with Mach number by more than 0·2% when the Mach number was greater than three.

The reason for the correction factor being insensitive to Mach number is that for a perfect gas the Mach number behind a normal shock varies only

slowly/

slowly with freestream Mach number, tending to the limit $[(\gamma - 1)/2\gamma]^{\frac{1}{2}}$ in the hypersonic approximation. The same is approximately true for a real gas.

Thus the temperature behind the normal shock is almost constant and in fact differs from the stagnation temperature by only about 30°K (for nitrogen).

Thus the correction factor for the pressure ratio can be found virtually as a function of the downstream total temperature only. The latter is not a known quantity but since the enthalpy remains constant through the whole of the flow we may convert this factor to be a function of the equivalent perfect gas temperature of the reservoir.

In Fig.3 are plotted the correction factors for nitrogen which have been calculated using the real gas tables of Hilsenrath[7].

As stated above, it was found that this graph holds for Mach numbers greater than three, but at very high temperatures when dissociation or ionisation occurs, the entropy and enthalpy depend also on pressure. In this case a set of graphs could be constructed which are a function of temperature and pressure. Similarly at Mach numbers below three a set of graphs could be calculated which give the correction factor as a function of temperature and Mach number.

Correction factors for other flow properties behind the normal shock, such as velocity, density or stagnation density, etc., can be found in a similar manner if required, but they will have a greater dependence on Mach number.

Use of the Graphs

It is assumed that the reservoir conditions are known so that the equivalent perfect gas temperature and pressure may be found immediately by multiplying the correction factors read off the graphs by the real temperature and pressure.

Similarly the correction factor for the pitot pressure may be found and the correction made.

Using these corrected values all the other freestream quantities such as density, temperature, Reynold's number, etc., may be found from perfect gas tables.

The tables may not be used to find the remaining total conditions behind the normal shock, but these are easily found. The pressure is known, or may be found if the Mach number is known by using the perfect gas tables and the correction in reverse, and also the enthalpy can be determined from the relation $H_{t_2} = C_P T_{EQ}$ where C_P is the perfect gas value. The remaining total conditions can now be found from real gas tables.

Validity of the Method

It is assumed throughout that the flow is isentropic except across a shock wave, so the effects of viscosity, heat transfer, relaxation, etc., have been ignored.

It/

It is further assumed that the freestream conditions may be regarded as being perfect. This is true for nitrogen and air provided that the temperature is less than about 300°K and the pressure very much less than the vapour pressure at the freestream temperature. If these conditions hold then the perfect gas tables give all the freestream conditions correctly.

The correction factor for total conditions behind a normal shock is in general dependent on Mach number, pressure, and temperature. However, it has been shown that in most cases this simplifies to a dependence on temperature only without introducing appreciable errors, but the range of validity is extended by having two variables.

Accuracy

The method is limited by the degree to which $H = C_p T$ holds in the freestream conditions. This is usually a very good approximation, but if it is not true, then the values found for the correction factors will be in error and care must be exercised.

The correction factor for the total pressure behind a normal shock in nitrogen has an error of 0·2% at a Mach number of three, falling to 0·1% at a Mach number of five. Similarly the error from ignoring the variation with pressure is 0·2% when P_{ta} is 40 atmospheres, and is roughly proportional to pressure.

Apart from these considerations and the limitations set out in the previous section, the accuracy depends only on the accuracy of the tables from which the factors were calculated and the accuracy to which the graphs can be read.

Conclusions

A general method of wind tunnel flow calculations has been described which allows perfect gas tables to be used on a gas having a perfectly general equation of state. As an example correction factors for the freestream and total conditions behind a normal shock in nitrogen are calculated.

The method is most suited to Mach numbers greater than three and temperatures less than those at which dissociation or ionisation occur, but even if this is not the case the method may be generalised without loss of accuracy.

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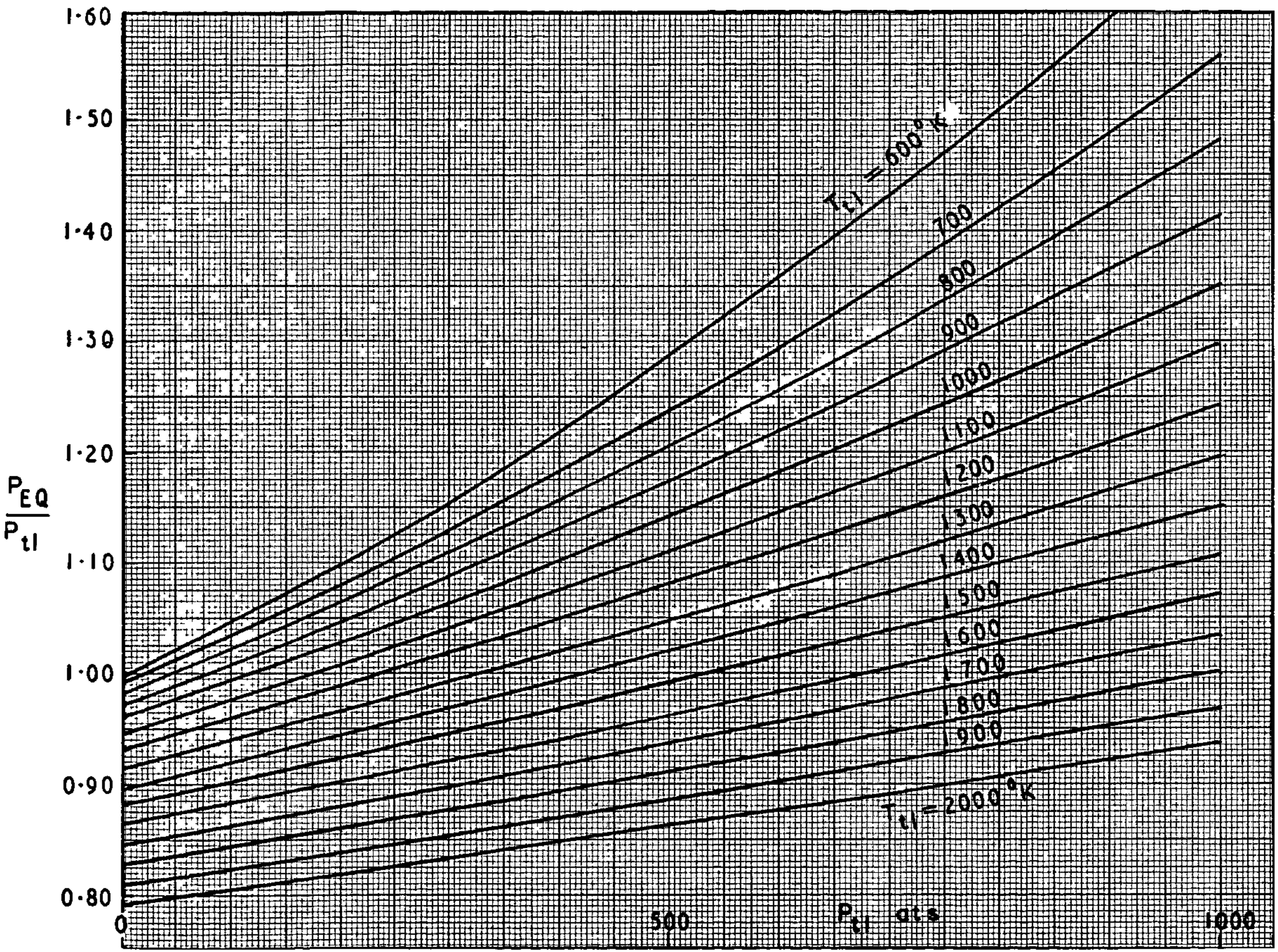

FIG.1 Pressure correction factor for nitrogen

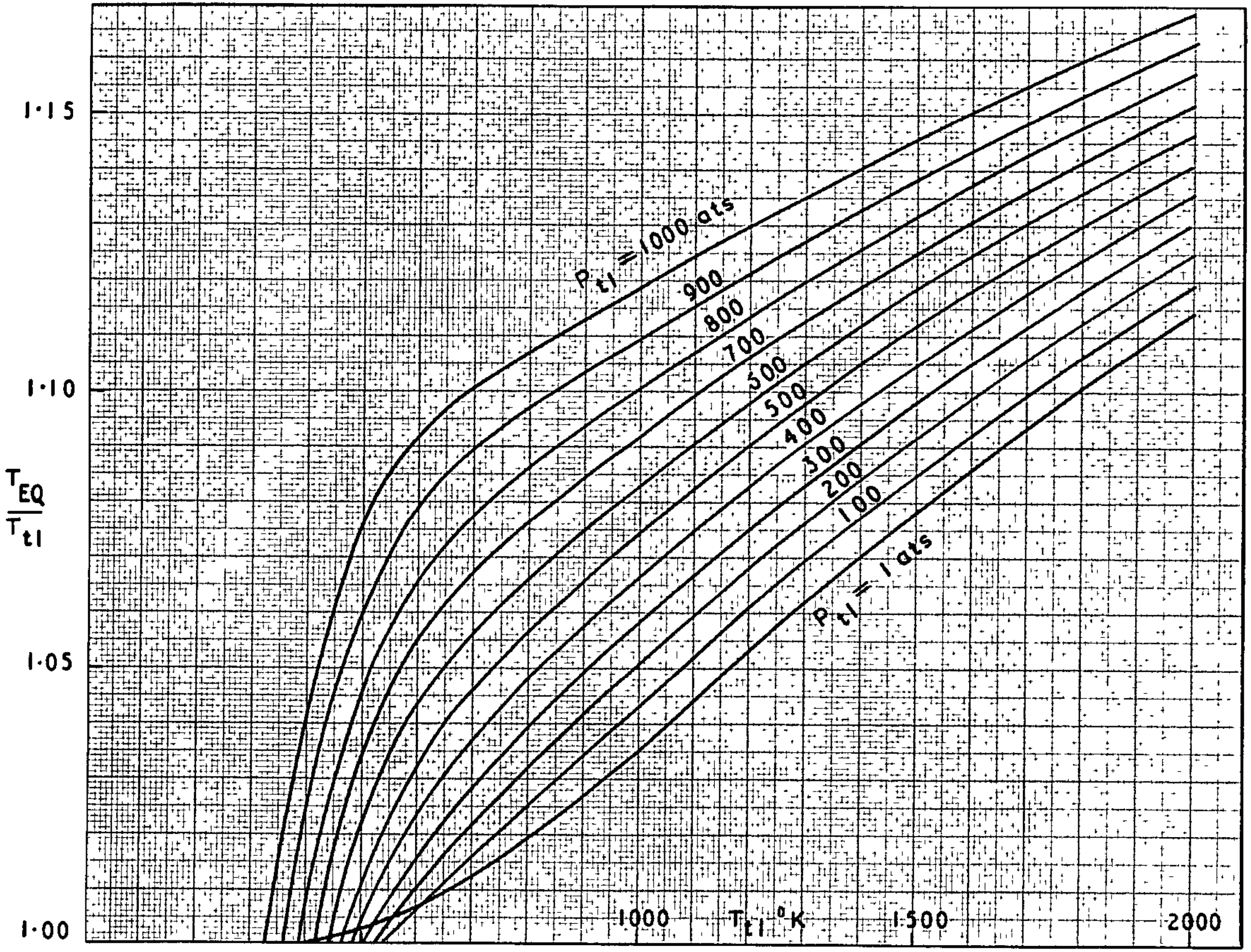

FIG. 2 Temperature correction factor for nitrogen

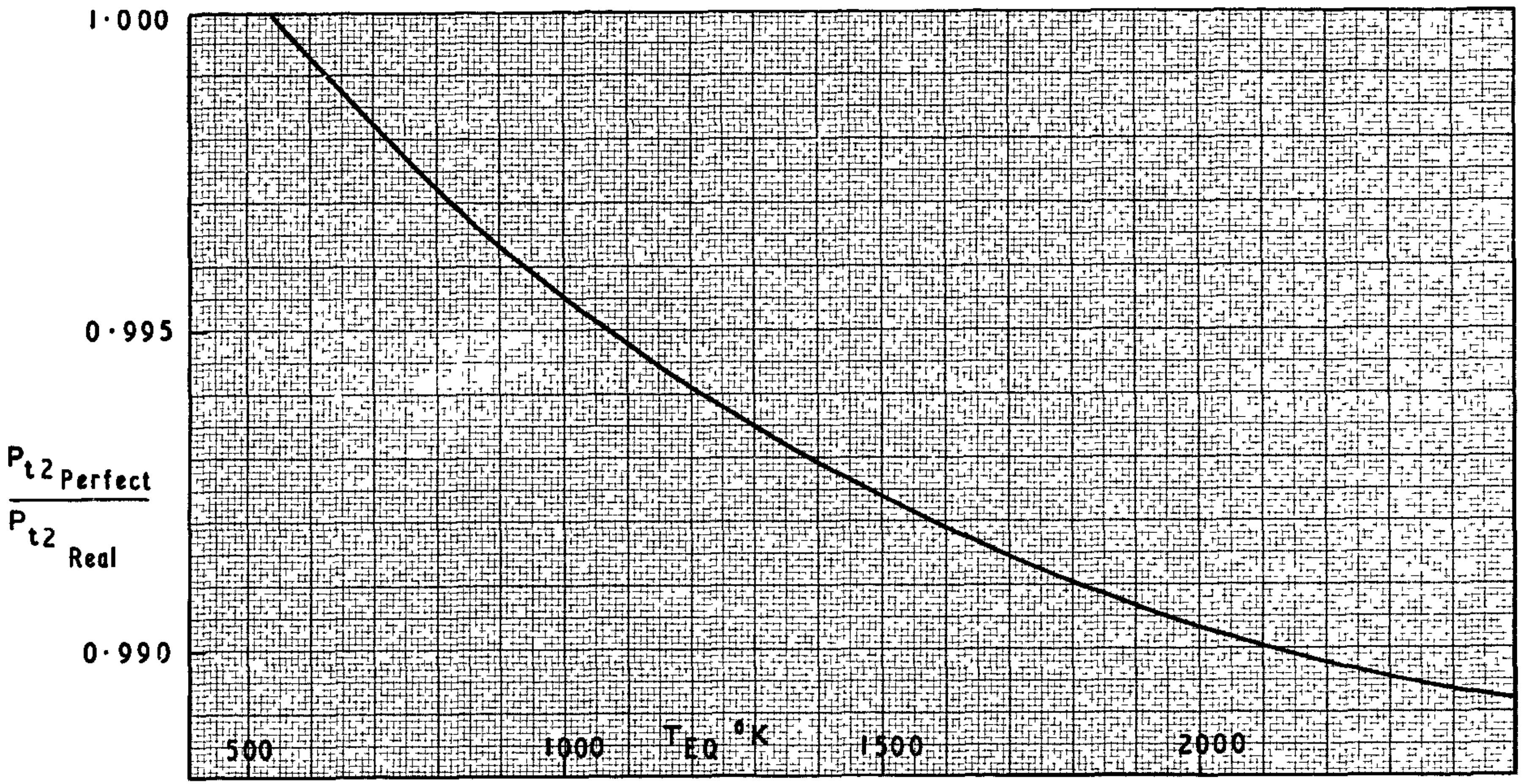

FIG. 3 Correction factor for total pressure behind a normal shock in nitrogen

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