C.P. No. 770

C.P. No. 770

MINISTRY OF AVIATION

AERONAUTICAL RESEARCH COUNCIL

CURRENT PAPERS

High Pressure Real Gas Drivers and Tailoring in Shock Tunnels

By

L. Davies

LONDON. HER MAJESTY'S STATIONERY OFFICE

1965

FOUR SHILLINGS NET

C.P. No.770

High Pressure Real Gas Drivers and Tailoring in Shock Tunnels

- By -

L. Davies

December, 1963

List of Contents

Nomenclature

c	velocity of sound
m	molecular weight
M_s	primary shock Mach number
p	pressure
P_{ij} =	p_i/p_j
s	entropy
t	time
T	temperature

u/

Replaces N.P.L. Aero Report No.1088 - A.R.C.25 486.
Published with the permission of the Director, National Physical Laboratory.

u velocity

x distance parameter

$Z = PV/RT$ compressibility factor

$\gamma = c_p/c_v$ ratio of specific heats

σ Riemann characteristic

1. Introduction

The diaphragm pressure ratio required to produce a particular shock Mach number in N_2 (say) depends not only on the driver gas used but also on the absolute pressure of the driver gas[1]. The deviation from ideal P_{41} values for real hydrogen is substantial. This non-ideal behaviour is due to the variation in the specific-heat ratio (γ) with temperature as the hydrogen expands and cools during the process of shock initiation. When the absolute value of the driver pressure increases, however, the compressibility of the gas must be considered, and the agreement with ideal shock tube performance that is obtained (calculated by Huber[1]) is shown in Fig.1(a and b).

Huber states that "Helium is close to a truly ideal gas both as to low compressibility and constancy of specific heat .....". Whilst the latter part of this statement is not disputed, the first is certainly not true[3,4]. In the case of helium, the lack of significant variation of γ with temperature[5] in the range 300°K - 90°K (at 300°K, $\gamma = 1{\cdot}63$, at 90°K, $\gamma = 1{\cdot}66$) means that the dominant factor will be the compressibility of helium $Z = 1{\cdot}11$ at 3 000 p.s.i. and room temperature, where $Z = PV/RT$. (A plot of compressibility factor at room temperature vs. pressure for helium is shown in Fig.2).

The Mach number at which tailoring occurs has been observed to vary from facility to facility and it is shown below that this variation in tailoring Mach number can be predicted from a study of the effects of non-ideal driver pressures on the expanded driver gas flow.

2. Derivation of Real-Gas Diaphragm-Pressure Ratio vs. Shock Mach Number Data

The one-dimensional isentropic flow equations may be written as follows:-

$$\frac{\partial s}{\partial t} + u\frac{\partial s}{\partial x} = 0 \qquad \ldots (1)$$

$$\frac{\partial \rho}{\partial t} + u\frac{\partial \rho}{\partial x} = -\rho\frac{\partial u}{\partial x} \qquad \ldots (2)$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} = -\frac{c^2}{\rho}\frac{\partial \rho}{\partial x} . \qquad \ldots (3)$$

Equations (2) and (3) may be solved by the method of Riemann giving the following relation

$$\left(\frac{\partial}{\partial x} + (u \pm c)\frac{\partial}{\partial x}\right)(u \pm \sigma) = \left(\left(\frac{\partial\sigma}{\partial\rho}\right)_s - \frac{c}{\rho}\right)\left(\frac{c\partial\rho}{\partial x} \mp \rho\frac{\partial u}{\partial x}\right). \qquad \ldots (4)$$

The importance of this method[6] is the definition of the quantity $\sigma(\rho,s)$. This is known as the Riemann characteristic and is defined by

$$\sigma(\rho,s) = \int_{\rho_o}^{\rho} \frac{c}{\rho}\, d\rho\, . \qquad \ldots (5)$$

The required P_{41} vs. M_s data was computed using Huber's method. The quantity conserved in the driver gas is $(\sigma + u)$. This is evaluated along an isentrope using data presented by Akin[7]. The corresponding pressure and fluid velocity are then matched across the contact surface, and the shock Mach number and diaphragm pressure ratio derived. The result of such a computation for helium (at p_4 = 2 500 and 6 000 lb/in^2) driver and with nitrogen as the low-pressure gas is shown in Fig.3a. Experimental points are included for comparison. It is seen that the real gas curves are again significantly higher than the ideal curve. This is at variance with Huber's theory but agrees with Hufton's[9]. Hufton shows (Fig.1b) that with increasing p_4 values for hydrogen (i.e., compressibility increasing) then the required value of P_{41} becomes increasingly greater than ideal. This second case agrees with the present calculations.

As the nitrogen is compressed and heated during the formation of the shock wave the increase in sound speed across each compression wave will not only be a result of increasing temperature but also a result of changing γ and compressibility in the gas. Relaxation effects have also to be considered when considering actual shock velocity. In this present work, however, the shock Mach number which would be measured before attenuation effects become considerable will be considered to be that given by the evaluation of the Riemann invariant.

3. The Tailored Interface Technique

The tailored interface technique refers to the conditions under which the reflected primary shock wave passes through the contact surface without giving rise to additional disturbances.

After the reflected shock wave has passed through the interface the pressure and velocity must be constant across the interface, as in the case behind the primary shock wave.

For a reflected shock wave to pass through the contact surface without refraction the relationship[8]

γ_4/

$$\frac{\dfrac{\gamma_4 + 1}{\gamma_4 - 1} + P_{25}}{\dfrac{\gamma_2 + 1}{\gamma_2 - 1} + P_{25}} = \frac{m_2(\gamma_2 - 1)\, T_3}{m_4(\gamma_4 - 1)\, T_2} \qquad \ldots (6)$$

must be satisfied and this is so at a unique shock Mach number for a given driver-driven gas combination. The left-hand side of the above relationship is a very slowly varying function of Mach number. The right-hand side is very sensitive to changes in the value of T_3 (γ_4 remains sensibly constant, the molecular weights, m_2, m_4, are constants, and γ_2 and T_2 refer to real gas values at a chosen primary shock Mach number). T_3 is very sensitive to the values of p_4 used to drive a given shock wave. It is clear then that in the case of helium, the tailoring Mach number is a function of diaphragm pressure ratio or

$$(M_s)_{\text{tailoring}} \;\alpha\; f\left(\frac{(P_{41})_{\text{ideal}}}{(P_{41})_{\text{actual}}}\right).$$

If P_{41} is less than ideal then T_3 is greater than ideal and the tailoring Mach number is greater, and vice-versa for P_{41} greater than ideal. In the case of hydrogen, however, the effect of change in γ will have to be considered. With increase in temperature at constant pressure γ for hydrogen decreases and this has the same effect as T_3 increases. The variation in tailoring Mach number is then dictated by both factors. Even if $(P_{41})_{\text{actual}} = (P_{41})_{\text{ideal}}$ at the higher pressures (5 000 - 10 000 lb/in²) for helium T_3 will still increase due to real-gas considerations as is shown in Table 1.

Table 1

$M_s = 3{\cdot}4$

P_4 (lb/in².)	T_3 (ideal)	$(T_3)_{\text{real}}$ (for ideal P_{41})
900	142°K	142·5°K
1 500	"	142·5
2 000	"	143
2 500	"	146
4 000	"	147
5 000	"	148
6 000	"	155

A plot of equation (6) for different Mach numbers for ideal and real driving conditions is shown in Fig.4. A Mollier diagram for helium from data given by Akin is reproduced in Fig.5.

A consequence of an increase in tailoring Mach number in helium for lower than ideal driver pressures is an increase in range of tailoring Mach numbers that can be obtained using helium-nitrogen mixtures, and hence an increase in range of stagnation enthalpies. The reverse is true for higher than ideal P_{41} values.

4. Experimental Evidence

Supporting evidence for these ideas has been obtained from the N.P.L. 6 in. shock tunnel and 2 in. shock tunnel. In the 6 in. tunnel helium at 2 500 lb/in^2 is used to drive shock waves in nitrogen and it is found that a value of P_{41} some 25% less than ideal is required to give a prescribed shock Mach number. Under these conditions it is found that the tailoring Mach number (obtained by noting the ratio P_{84} to P_{54} over a range of Mach numbers, see Fig.6a) is approximately 4·0. This is in complete agreement with the result obtained from Fig.4a. In the N.P.L. 2 in. shock tunnel when running at 1 800 $lb/in.^2$ driver P_{41} is approximately 23% greater than ideal and the tailoring Mach number is 3·1, and at 1 000 $lb/in.^2$ driver P_{41} is approximately 25% greater than ideal, giving a tailoring Mach number, again, of 3·1 (Fig.6b).

5. Determination of Tailoring Mach Number

The Mach number at which tailoring is said to occur has been determined from examination of the reflected shock pressure-time profiles measured near the nozzle of the shock tunnel. The tailoring Mach number is that at which the pressure remains constant and equal to p_5 until the arrival of the head of the expansion wave from the high pressure section. In the case of helium driver gas there is an initial hump in the pressure-time profile and in the case of hydrogen a dip which is due to boundary-layer interaction. Both these have to be ignored (and tolerated in tunnel operation) and the mean pressure level determined as in Fig.7. The pressure level p_5 has been chosen as shown from a study[11] (at U.C.W. Aberystwyth) of the first 100 microseconds of reflected shock pressure time profiles using a pressure bar gauge[13,14], where a steady pressure level, p_5, was found to prevail before perturbations occurred.

6. Conclusions

When helium is used as the driver gas in a shock tunnel the effects of compressibility at high pressures on P_{41} and T_3 are large due to the absence of any great variation in specific heat ratio. P_{41} becomes greater than ideal and hence T_3 becomes less. This results in a decrease in tailoring Mach number. In the case of hydrogen as driver gas, variation in γ will play an important part in determining the change in tailoring Mach number with increasing absolute driver pressures.

Shock attenuation has not been considered in the above calculations. This will certainly have an effect on the tailoring Mach number but this, it is considered, will not be significant unless very strong attenuation is present.

Inefficiency due to bad diaphragm opening will also affect the energy relations, but this will be noted from experimental P_{41} vs. M_s plots.

Even though the tailoring Mach number has been shown to be lower than ideal this does not mean that the best pressure-time profile for shock-tunnel work will be at this lower Mach number. A higher Mach number may be chosen if the overall pressure time profile is more suitable since it is known[10,15] that acceptable tailoring conditions exist over a small range of Mach numbers around the ideal value.

References

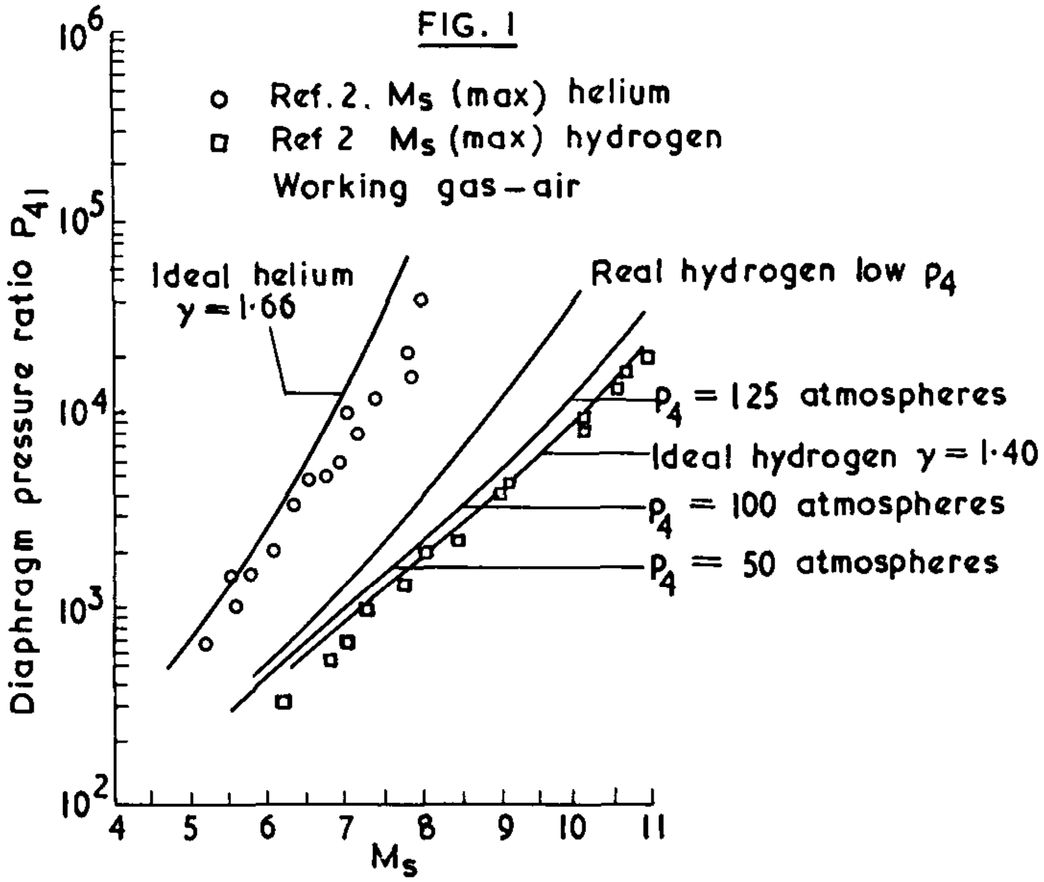
No.	Author(s)	Title, etc.
1	P. W. Huber	Note on hydrogen as a real gas driver for shock tubes. J. Aero. Sci. Vol.25, No.4, 1958.
2	J. J. Jones	Experimental investigation of attenuation of strong shock waves in a shock tube with hydrogen and helium as driver gases. NACA TN 4072, July, 1957.
3	W. D. Erickson	An extension of estimated hypersonic flow parameters for helium as a real gas. NACA TN D-1632, April, 1963.
4	L. Davies, L. Pennelegion, P. Gough and K. Dolman	The effects of high pressure on the flow in the reflected shock tunnel. A.R.C. C.P. No.730. September, 1963.
5	W. H. Keeson	Helium, Elsevier, Amsterdam, Holland, 1942.
6	J. O. Hirschfelder, C. F. Curtiss and R. B. Bird	Molecular theory of gases and liquids. John Wiley and pars. New York, 1954.
7	S. W. Akin	The thermodynamic properties of helium. Trans. A.S.M.E., Vol.72, 1950.
8	D. H. Henshaw	Refraction of a shock at a gaseous interface. UTIA Report No.11, October, 1950.
9	P. A. Hufton	Hypersonic facilities in the aerodynamic department, Royal Aircraft Establishment. AGARD Report No.146, 1957.
10	C. E. Wittliff, M. R. Wilson and A. Hertzberg	The tailored-interface hypersonic shock tunnel. J. Aero/Space Sci. Vol.26, 4, 1959.

No.	Author(s)	Title, etc.
11	L. Davies	Ph.D. Thesis, University of Wales, 1962.
12	L. Pennelegion and P. Gough	The change in shock-tunnel tailoring Mach number due to driver gas mixtures of helium and nitrogen. A.R.C. R.& M. 3398. October, 1963.
13	D. H. Edwards	J. Sci. Instr., 35, 346, (1958).
14	D. H. Edwards, L. Davies and R. Lawrence	(To be published)
15	D. W. Holder and D. L. Schultz	On the flow in a reflected-shock tunnel. A.R.C. R.&M.No.3265. August, 1960.

DR

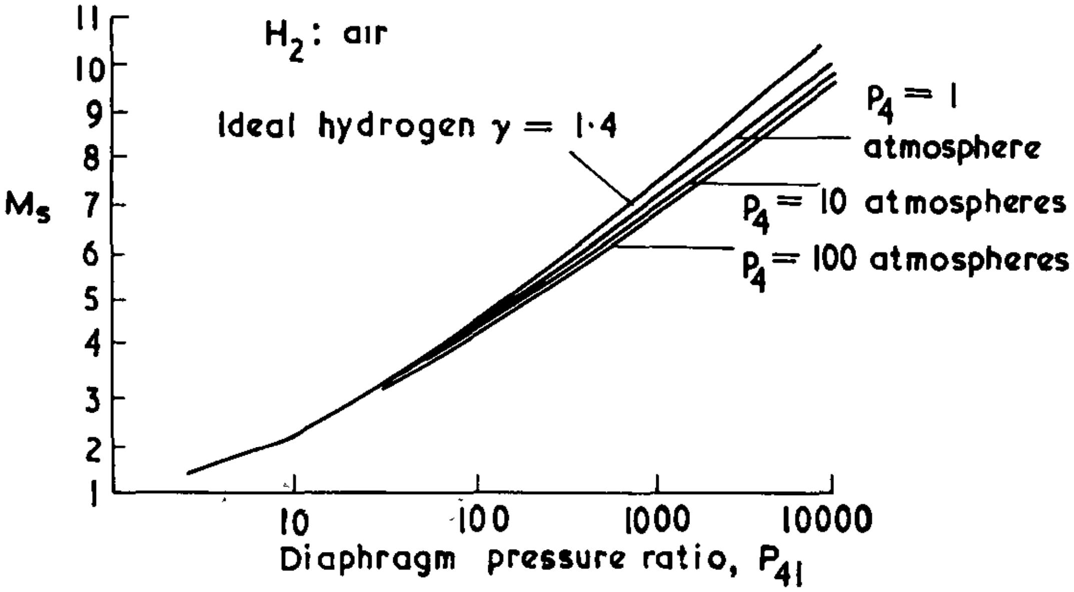
D 19192/1/Wt.60 K4 10/64 R & CL

FIG. 1



Diaphragm pressure ratio vs. shock Mach number (Ref. 1)

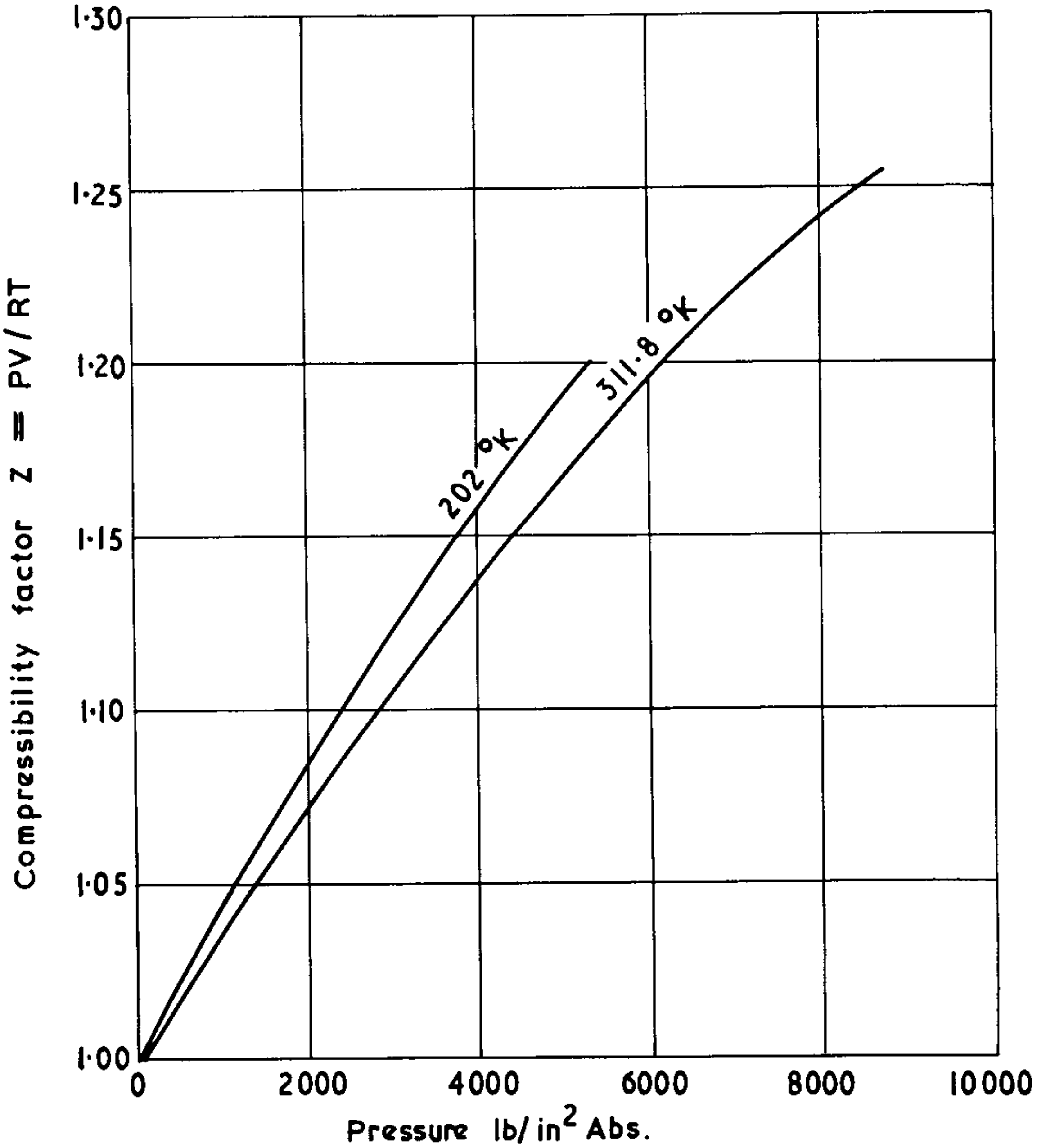
(a)



Diaphragm pressure ratio vs shock Mach number (Ref. 9)

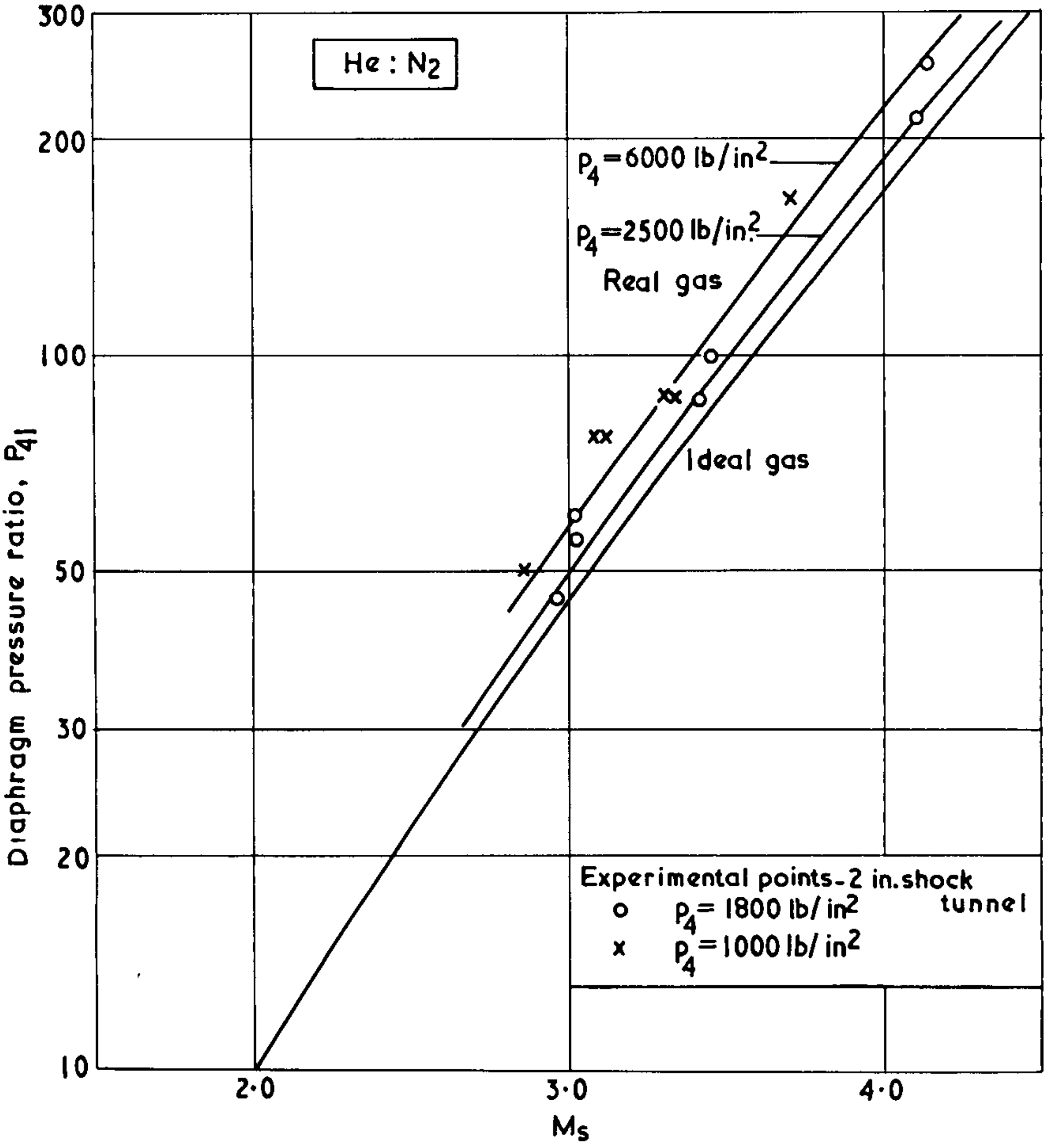
(b)

FIG. 2



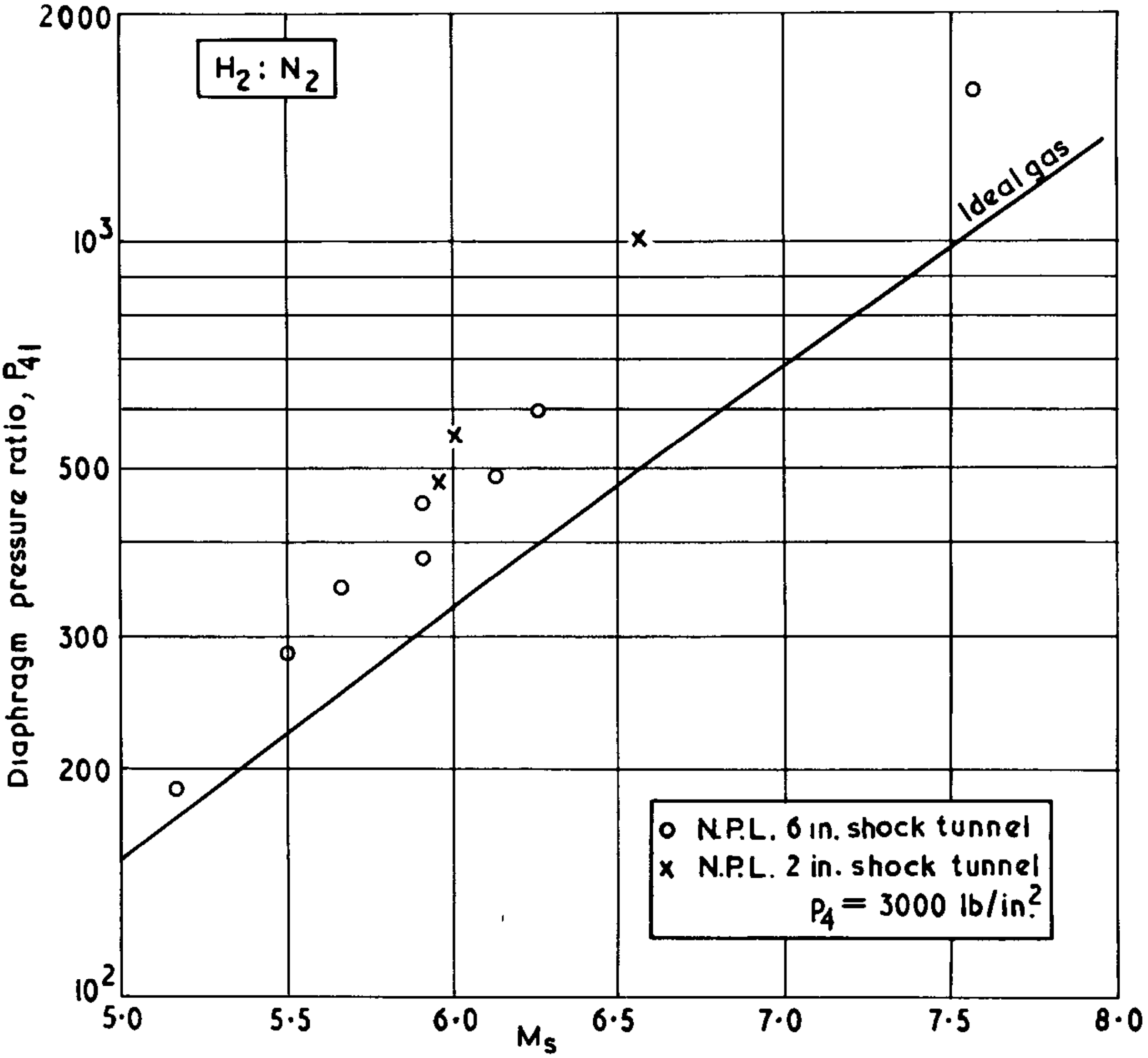
Compressibility factor vs pressure for helium (Ref. 3)

FIG. 3 a



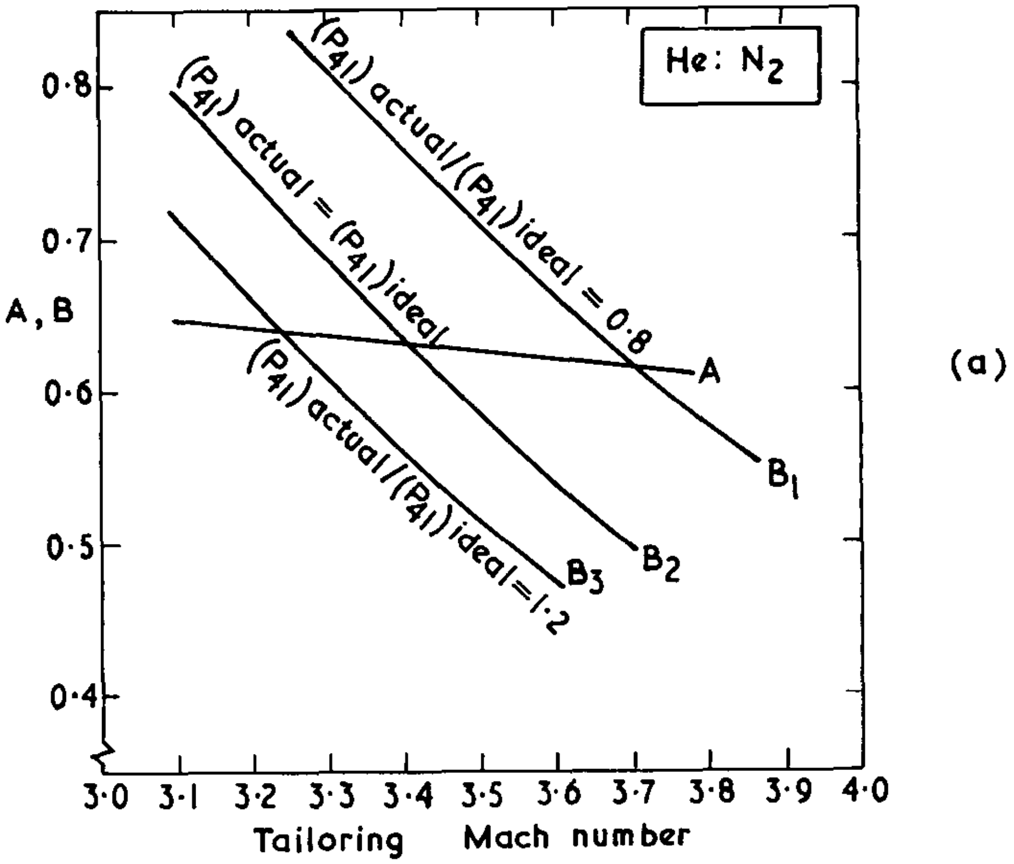
Diaphragm pressure ratio vs Mach number

FIG. 3 b



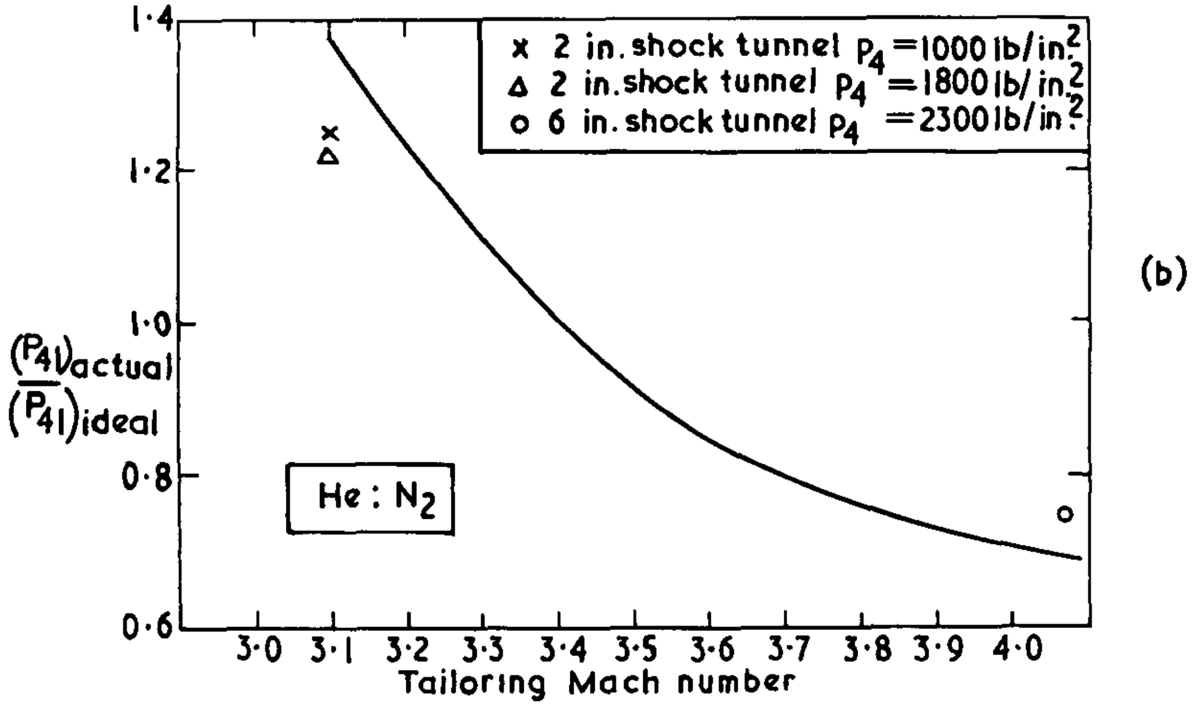
Diaphragm pressure ratio vs Mach number

FIG. 4



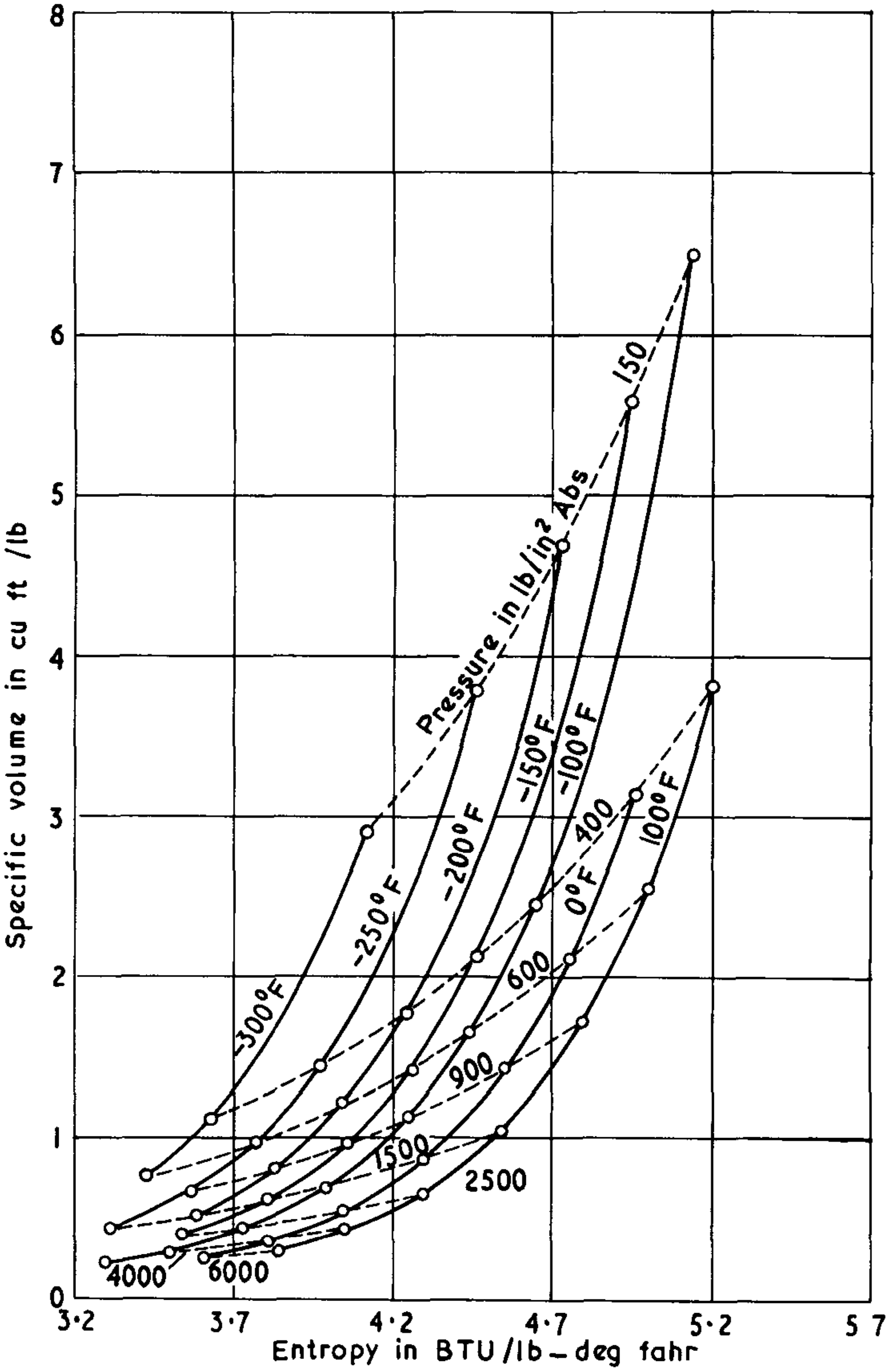
$$\text{where } A = \frac{\left(\frac{\gamma_4+1}{\gamma_4-1}\right) + P_{25}}{\left(\frac{\gamma_2+1}{\gamma_2-1}\right) + P_{25}} \quad , \qquad B = \frac{m_2}{m_4}\left(\frac{\gamma_2-1}{\gamma_4-1}\right)\frac{T_3}{T_2}$$

Graph of equation 6 vs Mach number



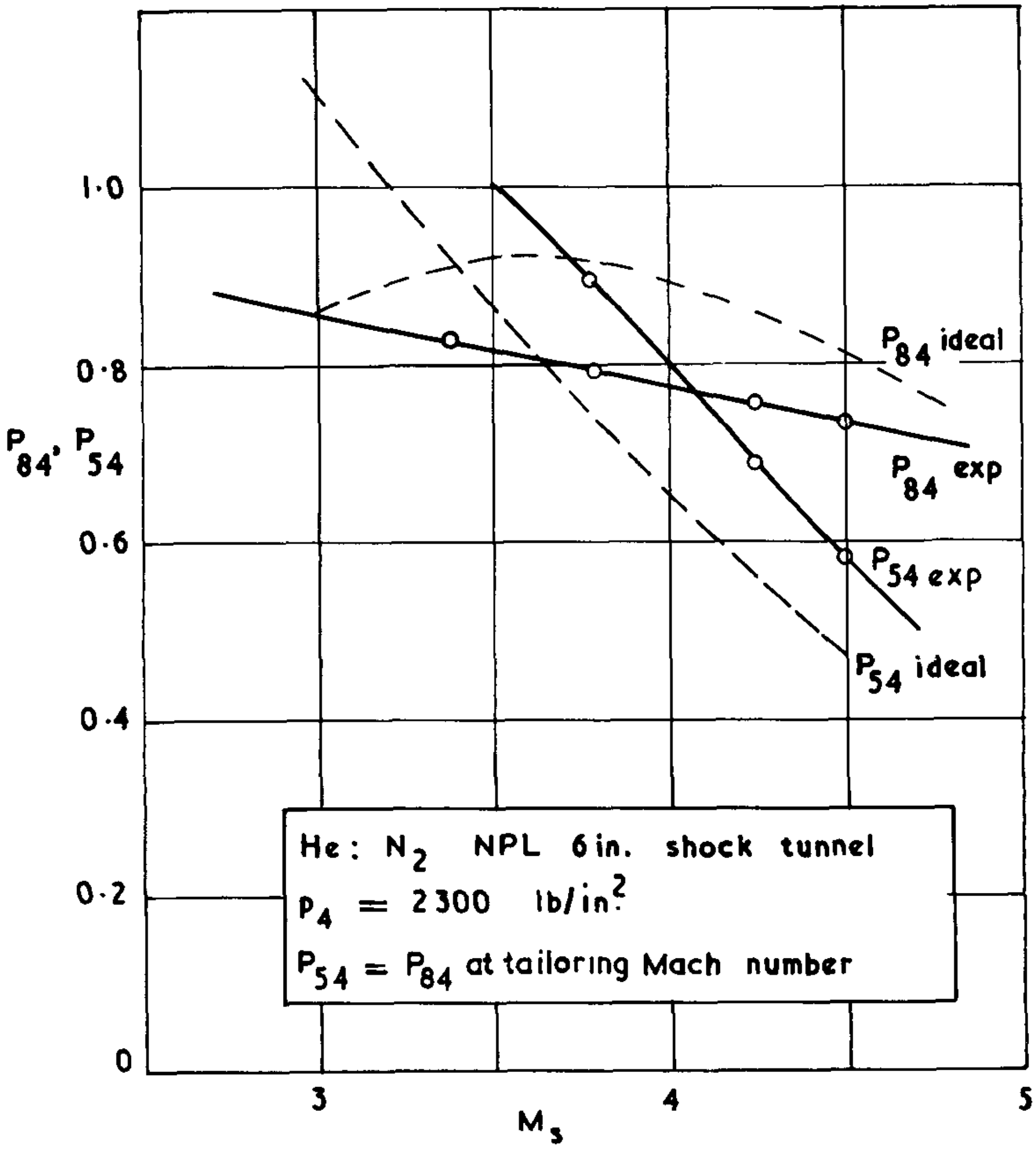
Graph of tailoring Mach number vs $(P_{41})_{actual} / (P_{41})_{ideal}$

FIG. 5



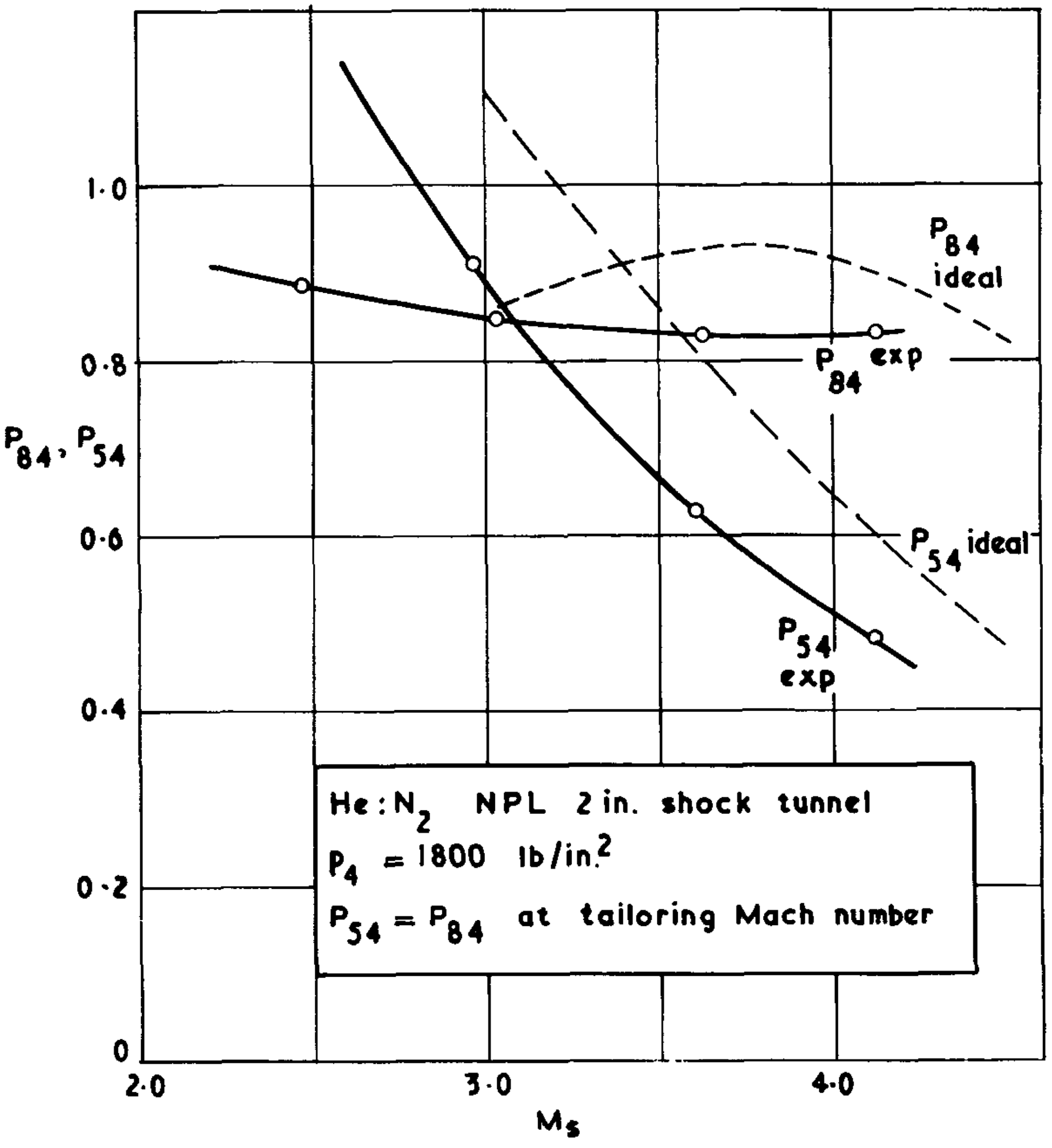
Mollier diagram for helium

FIG. 6 a



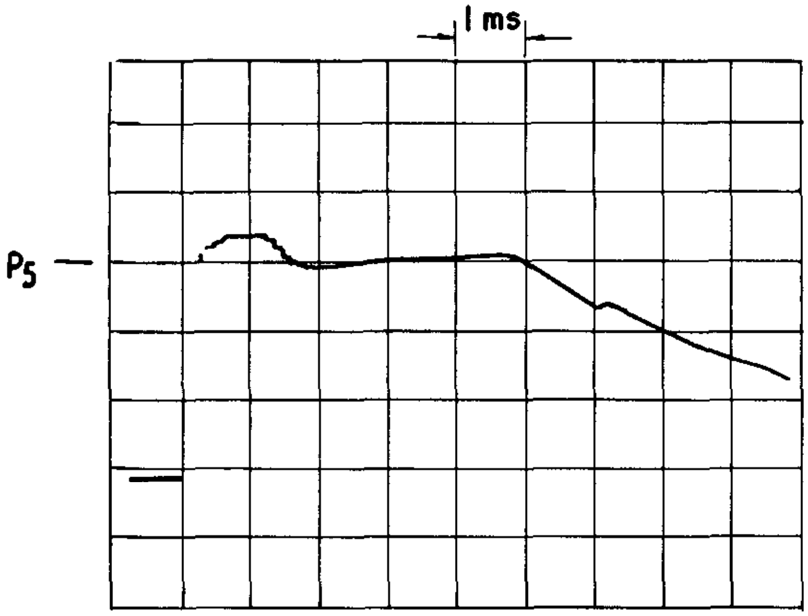
Experimental and ideal tailoring Mach numbers

FIG. 6b



Experimental and theoretical tailoring Mach numbers

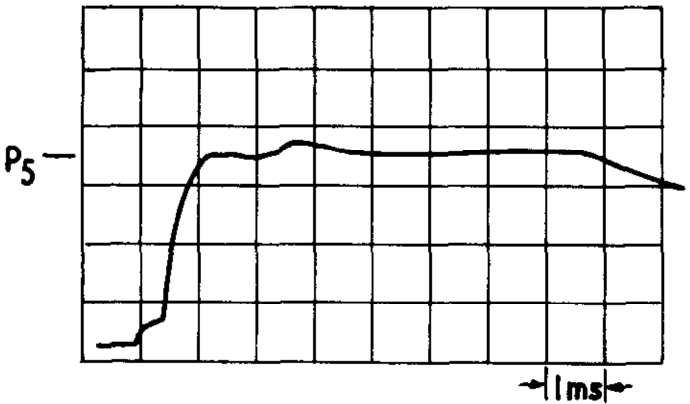
FIG. 7



p_4 = 1000 lb/in^2 p_1 = 13 lb/in^2

M_s = 3·09

(a) Pressure – time profile of reflected shock pressure for He: N_2. N.P.L. 2 in. shock tunnel

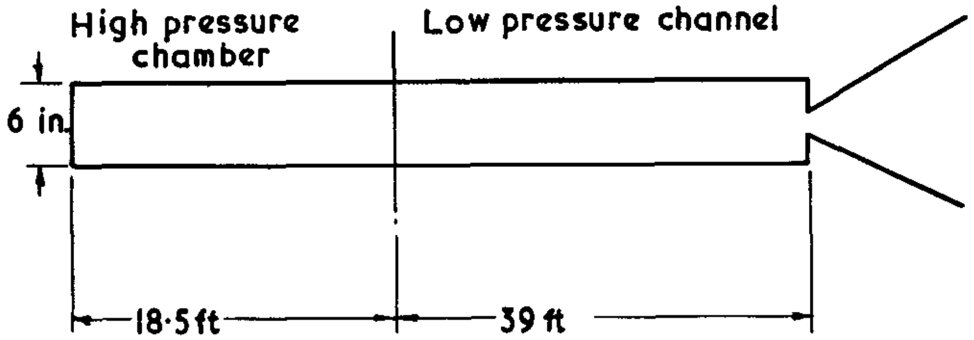


p_4 = 2300 lb/in^2 p_1 = 15 lb/in^2

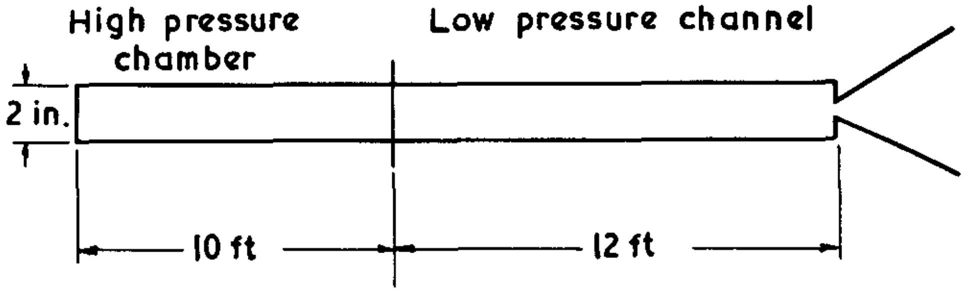
M_s = 4·1

(b) Pressure time profile of reflected shock pressure for He: N_2. N.P.L. 6 in. shock tunnel

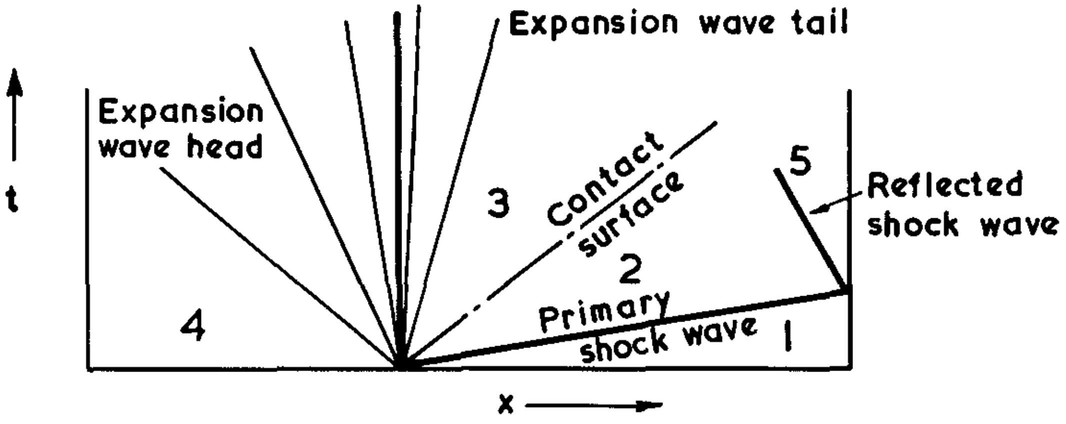
FIG. 8



N.P.L. 6 in. shock tunnel dimensions



N.P.L. 2 in. shock tunnel dimensions



Flow diagram

A.R.C. C.P. No.770
December, 1963
Davies, L.

HIGH PRESSURE REAL GAS DRIVERS AND TAILORING IN SHOCK TUNNELS

The lack of significant variation in specific heat ratio in helium with temperature ($300^{\circ}K$ - $90^{\circ}K$) results in a marked change in tailoring Mach number with change in diaphragm pressure ratio requirements at increasing driving pressures ($\leqslant 10\,000$ lb/in.^{2}).

A.R.C. C.P. No.770
December, 1963
Davies, L.

HIGH PRESSURE REAL GAS DRIVERS AND TAILORING IN SHOCK TUNNELS

The lack of significant variation in specific heat ratio in helium with temperature ($300^{\circ}K$ - $90^{\circ}K$) results in a marked change in tailoring Mach number with change in diaphragm pressure ratio requirements at increasing driving pressures ($\leqslant 10\,000$ lb/in.^{2}).

A.R.C. C.P. No.770
December, 1963
Davies, L.

HIGH PRESSURE REAL GAS DRIVERS AND TAILORING IN SHOCK TUNNELS

The lack of significant variation in specific heat ratio in helium with temperature ($300^{\circ}K$ - $90^{\circ}K$) results in a marked change in tailoring Mach number with change in diaphragm pressure ratio requirements at increasing driving pressures ($\leqslant 10\,000$ lb/in.^{2}).

C.P. No. 770

Printed and published by
HER MAJESTY'S STATIONERY OFFICE
To be purchased from
York House, Kingsway, London W C 2
423 Oxford Street, London W 1
13A Castle Street, Edinburgh 2
109 St Mary Street, Cardiff
39 King Street, Manchester 2
50 Fairfax Street, Bristol 1
35 Smallbrook, Ringway, Birmingham 5
80 Chichester Street, Belfast 1
or through any bookseller

Printed in England

C.P. No. 770

S.O. Code No. 23-9015-70