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Static Tests of Ground Effect on Planforms Fitted with a Centrally-Located Round Lifting Jet

by

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STATIC TESTS OF GROUND EFFECT ON PLANFORMS FITTED WITH A CENTRALLY-LOCATED ROUND LIFTING JET

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L. A. Wyatt, Ph.D.

SUMMARY

Static measurements of the thrust losses due to ground effect on a range of plane circular, rectangular and delta wings of various aspect ratios with a single round lifting jet located centrally on the planform are described. A satisfactory correlation of the results is presented.

Replaces R.A.E. Tech. Note Aero 2826 - A.R.C. 24099.

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1 INTRODUCTION

Under static conditions, the effect of ground proximity on the resultant upward thrust on planforms fitted with lifting jet systems can range from wholly favourable to wholly unfavourable. If the planform is bounded by a peripheral jet, substantial thrust augmentation results from the formation beneath the wing of a closed high-pressure cushion, whose strength is a function of ground clearance. The opposite extreme arises if the planform surrounds a central jet (or closely-spaced group of jets): here, again depending on the ground clearance, an adverse ground effect is developed beneath the wing due to the generation of low pressures by air being accelerated from rest and entrained by the jet efflux. Intermediate cases can obviously arise, for example, if a number of discrete jets is so disposed on a planform that regions of favourable and adverse interference exist between and outside the jets respectively.

The present series of tests is relevant to configurations with centrally-located jets which experience an unfavourable ground effect under static conditions. It extends the range of previously published results[1] which were restricted to two planforms, one circular and one of delta shape; both planforms had a single centrally-located round jet, and the ratio of jet area to wing area remained unchanged. Additional results are available from N.A.C.A. tests[2] on rectangular and square wings with a central jet, and from N.P.L. tests[3] on a hexagonal (or circular) wing with a central fan. Unpublished work on a rectangular wing with a central fan has been completed by Boulton Paul Aircraft Ltd.

Measurements have been made of the thrust loss due to ground effect on a circular wing, rectangular wings with aspect ratios between 0.25 and 4.0, and delta wings with aspect ratios between 0.5 and 4.0. A single round jet was located at or near the centre of area of each wing, the ratio of jet area to wing area being kept constant. Additional circular planforms were used to investigate the effect of varying the ratio of jet area to wing area.

Using a reference dimension derived from the planform geometry to make the ground clearance non-dimensional, it has been found possible to correlate the variation of thrust loss with ground clearance for the complete range of planforms tested.

2 EXPERIMENTAL DETAILS

2.1 Planform geometry

The range of planforms used in these experiments is illustrated in Fig.1, and all the relevant data are listed in Table 1. The main series of wings comprised a circular planform (of diameter D), rectangular planforms with aspect ratios of 1, 2 (or $\frac{1}{2}$) and 4 (or $\frac{1}{4}$), and, finally, delta planforms with aspect ratios of $\frac{1}{2}$, 1, 2 and 4: each wing had an area of 9 sq. ft. Using a round jet of fixed size, the ratio of jet diameter d to equivalent circular wing diameter D ($= 2\sqrt{S/\pi}$) was maintained constant and equal to 0.108. The jet was located at the centre of the circular and rectangular planforms, and at 0.707 of the root chord of the delta planforms, so that equal amounts of wing area lay fore and aft of the jet.

To assess the influence of the ratio of jet area to wing area on the ground effect experienced by a given planform, additional circular planforms of reduced diameter were provided having d/D = 0.146, 0.175, 0.219 and 0.292. The ratio of jet to wing area S_J/S ($= d^2/D^2$) was varied from 0.01 to 0.10 approximately, thus covering the range of practical interest for high-velocity lifting fan and jet applications.

2.2 Experimental method

The experimental rig has been fully described and illustrated previously[1] and only the more important details are repeated here.

The wings were made from 1 in. thick plywood and had a central clearance hole cut to accommodate a round jet mounted with its exit flush with the wing lower surface. The rig was set up in the inverted position with the jet nozzle supported from the floor and exhausting upwards. Each wing was hung in a horizontal plane and was supported independently of the jet nozzle on three overhead mechanical balances which measured the vertical components of the loads induced on the wing. A movable ground board was located above the wing. The gap between the jet nozzle and the wing was maintained by suitable horizontal bracing around the wing perimeter: Section 3.5 discusses errors due to the presence of the gap between the wing and the jet nozzle, and due to the width of the nozzle wall.

2.3 Range of tests

For each planform, the ground effect was determined as a function of the clearance H between the ground and the wing lower surface, the two being maintained parallel. The three measured vertical forces were summed to give the total thrust loss (or ground suction) G, and, for the delta planforms, the readings were combined to give the resultant pitching moment about an axis through the jet centre. The ground heights varied from 0.15 to 0.85 of the diameter of the largest circular planform.

To enable an estimate to be made of possible Reynolds Number effects on the thrust losses, readings were taken at each ground clearance with four distinct jet thrusts. Throughout this note, the reference jet thrust T is the gross jet momentum flux determined by the "reaction" method described in Appendix 2 of Ref.1. The following table lists corresponding values of the jet thrust T, the wing thrust loading T/S, an approximate mean jet efflux velocity V_J and an approximate jet Reynolds Number $V_J d/\nu$.

Jet thrust T lb.	Thrust loading T/S lb./sq.ft.	Jet velocity V_J ft./sec.	Jet Reynolds Number $V_J d/\nu$
33	3.67	400	1.0×10^6
66	7.33	560	1.4×10^6
132	14.67	760	2.1×10^6
252	28.00	980	3.1×10^6

A 3 to 1 range of Reynolds Number is thus covered by the tests.

2.4 Presentation of results

The initial representation of results is identical with that of Ref.1 i.e. the thrust loss G is expressed as a fraction of the gross jet thrust T and plotted as a function of the ratio of ground clearance H to equivalent circular wing diameter D $(= 2\sqrt{S/\pi})$. G_∞ signifies the free air thrust loss i.e. the value of G as $H \to \infty$. $(T - G_\infty)$ therefore represents the

The pitching moments for the delta planforms are referred to the product of the gross thrust T and the geometric mean chord $\bar{c}$. The pitching moment axis passes through the centre of the jet in all cases.

3 DISCUSSION OF RESULTS

3.1 Effect of Reynolds Number

At a fixed ground clearance, the results obtained with different wing thrust loadings confirmed previous evidence[1] on the effect of Reynolds Number, namely that the thrust losses tend to decrease as the Reynolds Number of the test increases. Nevertheless, it was obvious that Reynolds Number was a parameter of minor importance in that the average spread of the four values of G/T obtained at each ground clearance was only about 5% of the mean value, despite the 3 to 1 range of jet Reynolds Number. The spread of values became more random and was almost doubled at low thrust loadings and large ground clearances, but this merely reflects the decreased accuracy of measurement under these conditions.

The results for the four thrust loadings have therefore been combined to give a mean thrust loss G/T appropriate to each ground height, and it is this mean value over a range of jet Reynolds Number of 1 to 3×10^6 which is quoted in succeeding sections.

3.2 Ground effect on circular planforms with varying ratio of jet area to wing area

The thrust losses measured on the circular planforms with differing values of d/D are presented in Fig.2. The curves of thrust loss against non-dimensional ground clearance have the usual shape, and show an anticipated fall-off in the thrust losses as the ratio of jet area to wing area is increased. An increase in d/D from 0.1 to 0.3 results in a halving of the ground suction G/T at fixed H/D.

For practical purposes, the results may be reduced to a universal curve by plotting $(G - G_\infty)/T$ as a function of $H/(D-d)$, see Fig.3. The difference of the wing and jet diameters appears to be the relevant parameter to use in making the ground clearance non-dimensional. G_∞/T is the asymptotic value of the ground effect as $H \to \infty$ i.e. the free air thrust loss, and is a function of the diameter ratio d/D. G_∞/T decreases as the proportion of jet area is increased, varying from 0.020 to 0.0075 as d/D increases from 0.1 to 0.3.

The mean line inserted on Fig.3 is sufficient to predict $(G - G_\infty)/T$ to within ±0.01. Small systematic differences do exist between the curves for fixed d/D, but each set of points will satisfy an equation of the form

$$\frac{G - G_\infty}{T} = A\left(\frac{H}{D-d}\right)^{-B}$$

where the constants A, B and G_∞/T are functions of d/D. Values of these constants are listed in the following table for each d/D value used, together with values of A and B appropriate to the mean line.

d/D	A	B	G_∞/T
0.108	0.0158	2.02	0.020
0.146	0.0128	2.22	0.017
0.175	0.0091	2.50	0.015
0.219	0.0088	2.55	0.012
0.292	0.0107	2.11	0.0076
Mean line	0.0113	2.30	-

It is possible to represent the variation of G_∞/T with d/D by an exponential expression, the limiting values for G_∞/T being an asymptotic value as $d/D \to o$ and zero as $d/D \to 1$. The experimental values for G_∞/T suggest that, as the ratio of wing area to jet area increases, the free-air thrust loss tends to an asymptotic value of order 0.03. Lack of knowledge of the limiting values as d/D tends to 0 and 1 prevents the fitting of empirical expressions to the values of the constants A and B.

3.3 Ground effect on range of planforms with constant ratio of jet area to wing area

The thrust loss results for the planform range are plotted in Fig.4 and show the variations arising from changes in planform shape and aspect ratio, d/D being constant at 0.108. Differences between the wings only become marked at smaller values of H/D. Typically, at H/D = 0.15, the values of G/T range from 0.4 to 0.7, whereas at H/D = 0.75 the spread of values is only 0.04 to 0.05.

The thrust losses for rectangular wings of aspect ratio 1 and 2(or $\frac{1}{2}$) are little different from those of the circular wing, and the aspect ratio has to be as extreme as 4(or $\frac{1}{4}$) before a substantial fall-off in G/T occurs even at small H/D. Similarly the aspect ratio of the delta planform has to be reduced below 1 before the thrust losses become appreciably smaller than the circular wing values. Attempts to correlate the results from the various planforms concentrated on making the ground clearance H non-dimensional by a suitable parameter, defined so that it reduced to the diameter D in the case of a circular wing. First attempts made use of parameters dependent on the planform geometry only e.g. 4S/P, where $S = \pi (D^2 - d^2)/4$ is the net wing area outside the jet and $P = \pi (D + d)$ is the total wing perimeter. However, such parameters proved to be unsuitable, usually having a variation with planform larger than that required to correlate the results: moreover, they had the deficiency of being independent of the jet position and hence of not distinguishing between regions of the wing at different distances from the jet.

A successful correlation of the results was finally achieved in terms of an "angular mean diameter" $\bar{D}$, whose definition is illustrated in Fig.8. A system of polar co-ordinates (r,θ) is set up with the origin at the centre of the jet, and $\bar{D}$ is defined by

$$\bar{D} = \frac{1}{\pi}\int_0^{2\pi} r \, . \, d\theta .$$

The above formula may be written as

$$\bar{D} = \frac{2}{\pi}\int_0^{2\pi} \frac{dS}{r}$$

where dS is the polar element of area ($= \frac{1}{2} r^2 \, d\theta$). Thus it may be seen that $\bar{D}$ is proportional to an area integral in which each polar element is weighted inversely by the local radius i.e. the closer an element of area is to the origin the more it contributes to $\bar{D}$. The definition satisfies the requirement that $\bar{D}$ should equal the diameter D in the case of a circular planform with the origin at the centre. Values of $\bar{D}$ are most easily derived graphically. In Appendices 1 - 3, $\bar{D}$ is computed explicitly for circular, rectangular and delta planforms of various aspect ratios with the jet origin along the centre-line. Explanatory sketches and computed values of $\bar{D}$ for these shapes are presented in Figs. 9, 10, 11.

Fig.5 illustrates how well the results from the range of planforms correlate if the parameter $(\bar{D} - d)$ is used to make the ground clearance non-dimensional. The following table gives values of $(\bar{D} - d)$ for the various planforms with the jet positions used, relative to the value $(D - d)$ for the circle.

Planform	Aspect ratio	$(\bar{D} - d)/(D - d)$
Rectangle	1	0.99
	2,0.5	0.95
	4,0.25	0.85
Delta	0.5	0.83
	1	0.92
	2	0.97
	4	0.95

Since the circular planform is included in the correlation, it follows that the mean line equation found for the range of circular planforms (Fig.3) also applies for the complete range of planforms, provided $\bar{D}$ is substituted for D and a suitable value used for G_∞/T. A value of G_∞/T of 0.03 suits the results of Fig.5 for the range of planforms. The value of d/D, i.e. 0.108, is rather low for high-velocity lifting jet applications; and estimates for a practical layout would need to make use of a more appropriate value for G_∞/T - the values of Fig.3 for the circular planforms would probably suffice as a first estimate.

It should be stressed that the correlation has been demonstrated for centrally-located jets only. Although the method of correlation encourages one to think that it will be satisfactory for more extreme configurations, it should not be applied to layouts with non-central jet positions until suitable confirmatory tests have been made.

3.4 Pitching moments on delta planforms

The pitching moments for the delta planforms are given in Fig.6, together with the longitudinal position of the centre of the suction force due to ground effect: the reference length for the pitching moments is the geometric mean chord $\bar{c} = c_o/2$.

The pitching moments are generally negligible for ground clearances greater than 0.4 D. In the practical range of ground heights, say $0.2 < H/D < 0.4$, it appears that ±2% of the jet thrust applied at an arm $\bar{c}$ would be adequate for pitch control of any of the wings concerned with the jet positioned at $x/c_o = 0.707$. The position of the centre of the ground suction force does not show any consistent trend with either ground clearance or aspect ratio: unfortunately (at small ground clearances) the experimental rig was unsuitable for surface flow visualization which would probably have helped interpretation of the measured trends.

It is notable that the sense of the pitching moments for the A = 4 wing opposes that for the remainder. This difference correlates qualitatively with the contributions to the integral expression for $\bar{D}$ arising from regions of the wing respectively fore and aft of the jet: the lowest section of Fig.6 demonstrates that A = 2 represents the change over point between the greater contribution arising for θ between 0 and π/2 and for θ between π/2 and π.

3.5 Remarks on experimental technique

The thrust losses due to ground effect measured by the technique adopted in these experiments will be somewhat too small due to (a) the gap present between the jet nozzle and the independently-mounted wings, and (b) the appreciable wall thickness of the jet nozzle. A calculation based on wing pressure distributions on an integral wing-jet model (see Fig.19b of Ref.2) suggested that the results should not be in error by more than 3% of the jet thrust.

This estimate has been confirmed by testing a circular wing with a central jet, on which the thrust of the jet and the induced loads on the wing could be measured jointly or independently. At a large ground clearance, the presence of the gap and the nozzle wall did not significantly affect the measured thrust loss. At a ground clearance of $H/(D-d) = 0.24$, where G/T is about 0.33, the deficiency in the measured thrust loss due to the gap and the nozzle wall was 0.02 T i.e. about 6% of the actual ground effect. Since the deficiency might be expected to be approximately proportional to the true thrust loss, it is suggested that the quoted values of $(G - G_\infty)/T$ are about 6% below those which would have been measured on an integral wing-jet model i.e. the formula of Fig.3, when written in general terms, should be adjusted to read $(G - G_\infty)/T = 0.012\,[H/(\bar{D} - d)]^{-2.30}$.

3.6 Correlation of NACA results on ground effect on square planforms

The method of correlation described in the previous sections has been applied to a selection of the data on ground effect extracted from the NACA tests reported in Ref.2. The data selected refers to square planforms with d/D in the range of 0.08 to 0.42. Use has been made of results obtained at a jet pressure ratio of 1.45, which lies within the range covered by the R.A.E. tests; a further restriction has been made to ground clearances not less than one jet diameter, thus avoiding any uncertainty

Fig.7 confirms that the method of correlation is successful - the increasing scatter as $H/(\bar{D}-d)$ tends towards 1.0 is due to difficulty experienced in reading off accurate values from the graphs of Ref.2. Data taken at higher pressure ratios of 2.12 and 2.70 has also been found to correlate satisfactorily on the same basis.

For comparison, Fig.7 includes the mean curve obtained from the R.A.E. results of Fig.3 - the ordinates of this curve have been increased by 6% in accordance with the considerations of section 3.5 and by a further 2% to allow for the fact that the reference thrust for the NACA results is the installed thrust i.e. $T - G_\infty$. It may be seen that an appreciable difference exists between the two curves, the NACA results giving thrust losses about 50% larger than those indicated by the R.A.E. tests. The mean curve drawn through the NACA results satisfies a power law of the form representing the R.A.E. results: the power index is -2.02 and the constant A is 0.025.

The cause of the discrepancy between the results from the two apparently almost identical experiments is not known. The magnitude of the correction applied to the R.A.E. results in section 3.5 makes it clear that differing experimental techniques cannot account for the discrepancy. It has been verified that the air jets used in the experiments both had uniform velocity distributions. Lack of knowledge of the free-air thrust loss G_∞/T of the NACA tests would not imply a correction which could account for more than a fraction of the discrepancy. The jet Reynolds Number of the NACA tests is in the region of 0.5×10^6, compared with 1.0 to 3.0×10^6 for the R.A.E. experiments, but the evidence presented in section 3.1 of this note makes it unlikely that this difference in Reynolds Number is sufficient to cause the disagreement.

4 CONCLUDING REMARKS

This note has shown how it is possible to correlate measurements of the thrust losses due to ground effect experienced by a wide range of planforms of varying aspect ratio with a single centrally-located lifting jet. The variation of the thrust loss G with ground clearance H may be expressed as

$$(G - G_\infty)/T = 0.012\,[H/(\bar{D} - d)]^{-2.30}$$

where G_∞ is the free-air thrust loss, T is the gross jet thrust, d is the jet diameter, and $\bar{D}$ is a suitably defined "angular mean diameter" (see Section 3.3).

Although the method of correlation has worked equally well in collapsing a selection of NACA data on square wings, the comparison of results from similar R.A.E. and NACA tests has not proved completely satisfactory because the results in non-dimensional form were significantly different.

It should be emphasized that these results only apply to the thrust losses which would be expected on simple configurations with a central jet or closely-spaced group of jets. Attempted applications to configurations with widely spaced jets are likely to be unrewarding, unless account is taken of the varying favourable ground effect developed on the wing surface between the jets.

LIST OF SYMBOLS

A	aspect ratio
b	span
c	chord
$\bar{c}$	geometric mean chord
c_o	root chord of delta planform
d	jet diameter
D	$= 2\sqrt{S/\pi}$, diameter of equivalent circular planform
$\bar{D}$	angular mean diameter of planform (see Fig. 8)
H	ground clearance
P	perimeter of planform
R	radius of circular planform
S	area of planform
S_J	area of jet
x	distance of jet centre from L.E. of planform
x_s	distance of centre of suction from L.E. apex of delta planform
X	$= x/c$, x/c_o, non-dimensional jet position
r, θ	radial and angular polar co-ordinates
ν	kinematic viscosity
V_J	mean jet efflux velocity
T	gross jet momentum flux
G	thrust loss due to ground effect
G_∞	free-air thrust loss ($H = \infty$)
M	pitching moment about jet centre
A,B	constants

LIST OF REFERENCES

No.	Author	Title etc.
1	Wyatt, L.A.	Tests on the loss of vertical jet thrust due to ground effect on two simple VTOL planforms, with particular reference to the Short S.C.1 aircraft. A.R.C. R & M 3313. May 1958
2	Spreeman, K.P., Sherman, I.R.	Effects of ground proximity on the thrust of a simple downward-directed jet beneath a flat surface. N.A.C.A. Tech. Note 4407. September, 1958
3	Gregory, N. Walker, W.S.	Measurements of lift and ground interference on a lifting-fan at zero forward speed. A.R.C. R & M 3263. March 1958

APPENDIX 1

EVALUATION OF $\bar{D}/D$ FOR CIRCULAR PLANFORM

The evaluation of $\bar{D}/D$ for a circular planform of diameter D and radius R is illustrated by Fig.9. The jet origin lies on the centre-line at a distance x = XD from the leading edge. Due to symmetry, we may write

$$\bar{D} = \frac{1}{\pi}\int_0^{2\pi} r\,d\theta = \frac{2}{\pi}\int_0^{\pi} r\,d\theta \,.$$

Simple geometry yields

$$R^2 = (R - x)^2 + r^2 + 2\,(R - x)\,r\cos\theta$$

whence

$$r = -(R - x)\cos\theta + [R^2 - (R - x)^2 \sin^2\theta]^{\frac{1}{2}} \,,$$

and

$$\frac{\pi}{2}\bar{D} = -(R - x)\int_0^{\pi}\cos\theta\,d\theta + R\int_0^{\pi}\left[1 - \left(1 - \frac{x}{R}\right)^2 \sin^2\theta\right]^{\frac{1}{2}} d\theta$$

$$= 0 + 2R\int_0^{\pi/2} [1 - (1 - 2X)^2 \sin^2\theta]^{\frac{1}{2}}\,d\theta$$

whence

$$\bar{D}/D = \frac{2}{\pi}\,E\left(\frac{\pi}{2}\,,\ 1 - 2X\right)$$

where $E\left(\frac{\pi}{2},\ 1 - 2X\right)$ is the complete elliptic integral of the second kind.

Fig.9 shows the values of $\bar{D}/D$ derived from the above formula: $\bar{D}/D$ has a maximum of unity with the origin at the centre of the planform, and falls off to about 65% of the maximum value as $X \rightarrow 0$ or 1.

APPENDIX 2

EVALUATION OF $\bar{D}/D$ FOR RECTANGULAR PLANFORMS

The evaluation of $\bar{D}/D$ for a rectangular planform of aspect ratio A is illustrated by Fig.10. The jet origin lies on the centre-line at a distance $x = Xc$ from the leading edge of the planform. Due to symmetry, we again write

$$\bar{D} = \frac{2}{\pi}\int_0^{\pi} r \, d\theta \, .$$

The integration may be divided into three sections, in each of which the expression for r as a function of θ takes a different form. We find that

$$r = x/\cos\theta \qquad 0 \leqslant \theta \leqslant \theta_A$$

$$r = b/2\sin\theta \qquad \theta_A \leqslant \theta \leqslant \theta_B$$

$$r = -(c - x)/\cos\theta \qquad \theta_B \leqslant \theta \leqslant \pi \, .$$

Hence

$$\frac{\pi}{2}\bar{D} = x\int_0^{\theta_A} \frac{d\theta}{\cos\theta} + \frac{1}{2}b\int_{\theta_A}^{\theta_B} \frac{d\theta}{\sin\theta} - (c - x)\int_{\theta_B}^{\pi} \frac{d\theta}{\cos\theta}$$

$$\therefore \frac{\pi}{2}\frac{\bar{D}}{c} = \sqrt{\pi A}\,\frac{\bar{D}}{D}$$

$$= \frac{X}{2}\left[\log\frac{1+\sin\theta}{1-\sin\theta}\right]_0^{\theta_A} - \frac{A}{4}\left[\log\frac{1+\cos\theta}{1-\cos\theta}\right]_{\theta_A}^{\theta_B}$$

$$- \frac{1-X}{2}\left[\log\frac{1+\sin\theta}{1-\sin\theta}\right]_{\theta_B}^{\pi} \, .$$

Limiting values of functions of θ_A and θ_B are as follows

$$\sin\theta_A = A/(A^2 + 4X^2)^{\frac{1}{2}} \qquad \sin\theta_B = A/[A^2 + 4(1-X)^2]^{\frac{1}{2}}$$

$$\cos\theta_A = 2X/(A^2 + 4X^2)^{\frac{1}{2}} \qquad \cos\theta_B = -2(1-X)/[A^2 + 4(1-X)^2]^{\frac{1}{2}} .$$

By substitution, a final expression for $\bar{D}/D$ is derived as

$$2\sqrt{\pi A}\,\frac{\bar{D}}{D} = X\log\left(\frac{[A^2 + 4X^2]^{\frac{1}{2}} + A}{[A^2 + 4X^2]^{\frac{1}{2}} - A}\right)$$

$$+ \frac{A}{2}\log\left(\frac{[A^2 + 4(1-X)^2]^{\frac{1}{2}} + 2(1-X)}{[A^2 + 4(1-X)^2]^{\frac{1}{2}} - 2(1-X)}\right)$$

$$+ \frac{A}{2}\log\left(\frac{[A^2 + 4X^2]^{\frac{1}{2}} + 2X}{[A^2 + 4X^2]^{\frac{1}{2}} - 2X}\right)$$

$$+ (1-X)\log\left(\frac{[A^2 + 4(1-X)^2]^{\frac{1}{2}} + A}{[A^2 + 4(1-X)^2]^{\frac{1}{2}} - A}\right) .$$

For a fixed aspect ratio, the expression for $\bar{D}/D$ is symmetrical in X and $(1-X)$. Values of $\bar{D}/D$ have been computed for aspect ratios between 0.25 and 4 for values of X between 0 and 1, and these are presented in Fig.10. $\bar{D}/D$ is a maximum for each aspect ratio when the origin is at the centre of the planform ($X = 0.5$). As $X \to 0$ or 1, $\bar{D}/D$ falls smoothly to a value which is about 70% of the maximum. The highest values of $\bar{D}/D$ are those for the square planform, reduced values occurring for both higher and lower aspect ratios.

APPENDIX 3

EVALUATION OF $\bar{D}/D$ FOR DELTA PLANFORMS

The evaluation of $\bar{D}/D$ for a delta planform of aspect ratio A is illustrated in Fig.11. The jet origin lies on the centre-line at a distance $x = Xc_o$ from the leading edge apex. Due to symmetry, we have

$$\bar{D} = \frac{2}{\pi}\int_0^{\pi} r \, d\theta \, .$$

The integration may be divided into two sections, in which the expressions for r are as follows

$$r = xb/(2\, c_o \sin\theta + b\cos\theta) \qquad 0 \leqslant \theta \leqslant \theta_A$$

$$r = -(c_o - x)/\cos\theta \qquad \theta_A \leqslant \theta \leqslant \pi \, .$$

For $0 < \theta \leqslant \theta_A$, the first expression may be re-written as

$$r = \frac{2\,Xb}{(16 + A^2)^{\frac{1}{2}} \sin(\theta + \gamma)} \qquad \text{where} \quad \tan\gamma = A/4 \, .$$

Hence

$$\frac{\pi}{2}\bar{D} = \frac{2\,Xb}{(16 + A^2)^{\frac{1}{2}}}\int_0^{\theta_A} \frac{d\theta}{\sin(\theta + \gamma)} - (c_o - x)\int_{\theta_A}^{\pi} \frac{d\theta}{\cos\theta}$$

$$\frac{\pi}{2}\frac{\bar{D}}{\frac{1}{2}c_o} = \sqrt{\pi A}\,\bar{D}/D$$

$$= \frac{2\,AX}{(16 + A^2)^{\frac{1}{2}}}\int_0^{\theta_A} \frac{d\theta}{\sin(\theta + \gamma)} - 2\,(1 - X)\int_{\theta_A}^{\pi} \frac{d\theta}{\cos\theta}$$

$$= \frac{AX}{(16 + A^2)^{\frac{1}{2}}}\left[-\log\frac{1 + \cos(\theta + \gamma)}{1 - \cos(\theta + \gamma)}\right]_0^{\theta_A} - (1 - X)\left[\log\frac{1 + \sin\theta}{1 - \sin\theta}\right]_{\theta_A}^{\pi} \, .$$

Limiting values of functions of θ_A and γ are as follows

$$\sin\theta_A = A/[A^2 + 16(1-X)^2]^{\frac{1}{2}} \qquad \sin\gamma = A/(16 + A^2)^{\frac{1}{2}}$$

$$\cos\theta_A = -4(1-X)/[A^2 + 16(1-X)^2]^{\frac{1}{2}} \qquad \cos\gamma = 4/(16 + A^2)^{\frac{1}{2}}$$

$$\cos(\theta_A + \gamma) = -[16(1-X) + A^2]/[16(1-X)^2 + A^2]^{\frac{1}{2}}(16 + A^2)^{\frac{1}{2}}$$

By substitution, a final expression for $\bar{D}/D$ is derived as

$$\sqrt{\pi A}\,\frac{\bar{D}}{D} = \frac{AX}{(16+A^2)^{\frac{1}{2}}}\log\left(\frac{[16(1-X)^2 + A^2]^{\frac{1}{2}}(16+A^2)^{\frac{1}{2}} + [16(1-X)+A^2]}{[16(1-X)^2 + A^2]^{\frac{1}{2}}(16+A^2)^{\frac{1}{2}} - [16(1-X)+A^2]}\right)$$

$$+ \frac{AX}{(16+A^2)^{\frac{1}{2}}}\log\left(\frac{[16 + A^2]^{\frac{1}{2}} + 4}{[16 + A^2]^{\frac{1}{2}} - 4}\right)$$

$$+ (1-X)\log\left(\frac{[16(1-X)^2 + A^2]^{\frac{1}{2}} + A}{[16(1-X)^2 + A^2]^{\frac{1}{2}} - A}\right) .$$

Values of $\bar{D}/D$ have been computed for aspect ratios between 0.25 and 4.0 for values of X between 0 and 1, and these are presented in Fig.11. $\bar{D}/D$ is seen to reach its maximum when X lies between 0.6 and 0.8. As $X \to 1$, $\bar{D}/D$ decreases to about 70% of the maximum value. Forward movement of the origin towards X = 0 leads to a large decrease in $\bar{D}/D$, the limiting values being between 20% and 50% of the maximum value depending on the aspect ratio.

TABLE 1

GEOMETRY OF PLANFORMS

A **Jet**

Diameter d = 0.365 ft.

Area S_J = 0.105 sq ft.

Nozzle wall thickness = 1.0 in.

B **Circular planforms**

Jet at centre of each planform

d/D	Diameter D ft.	Area S sq.ft.
0.108	3.383	9.000
0.146	2.500	4.909
0.175	2.083	3.409
0.219	1.667	2.182
0.292	1.250	1.227

C **Rectangular planforms**

Jet at centre of each planform

$S = 9.00$ sq.ft., $D = 2\sqrt{S/\pi} = 3.383$ ft., $d/D = 0.108$

Aspect ratio, A	Span b ft.	Chord c ft.	Angular mean diameter, $\bar{D}$ ft.
1	3.000	3.000	3.349
2(½)	4.242 (2.121)	2.121 (4.242)	3.214
4(¼)	6.000 (1.500)	1.500 (6.000)	2.876

D **Delta planforms**

Jet at 0.707 of root chord from L.E. of each planform

$S = 9.00$ sq.ft., $D = 2\sqrt{S/\pi} = 3.383$ ft., $d/D = 0.108$

Aspect ratio, A	Span b ft.	Mean chord $\bar{c} = c_o/2$ ft.	Angular mean diameter, $\bar{D}$ ft.
½	2.121	4.242	2.808
1	3.000	3.000	3.112
2	4.242	2.121	3.282
4	6.000	1.500	3.214

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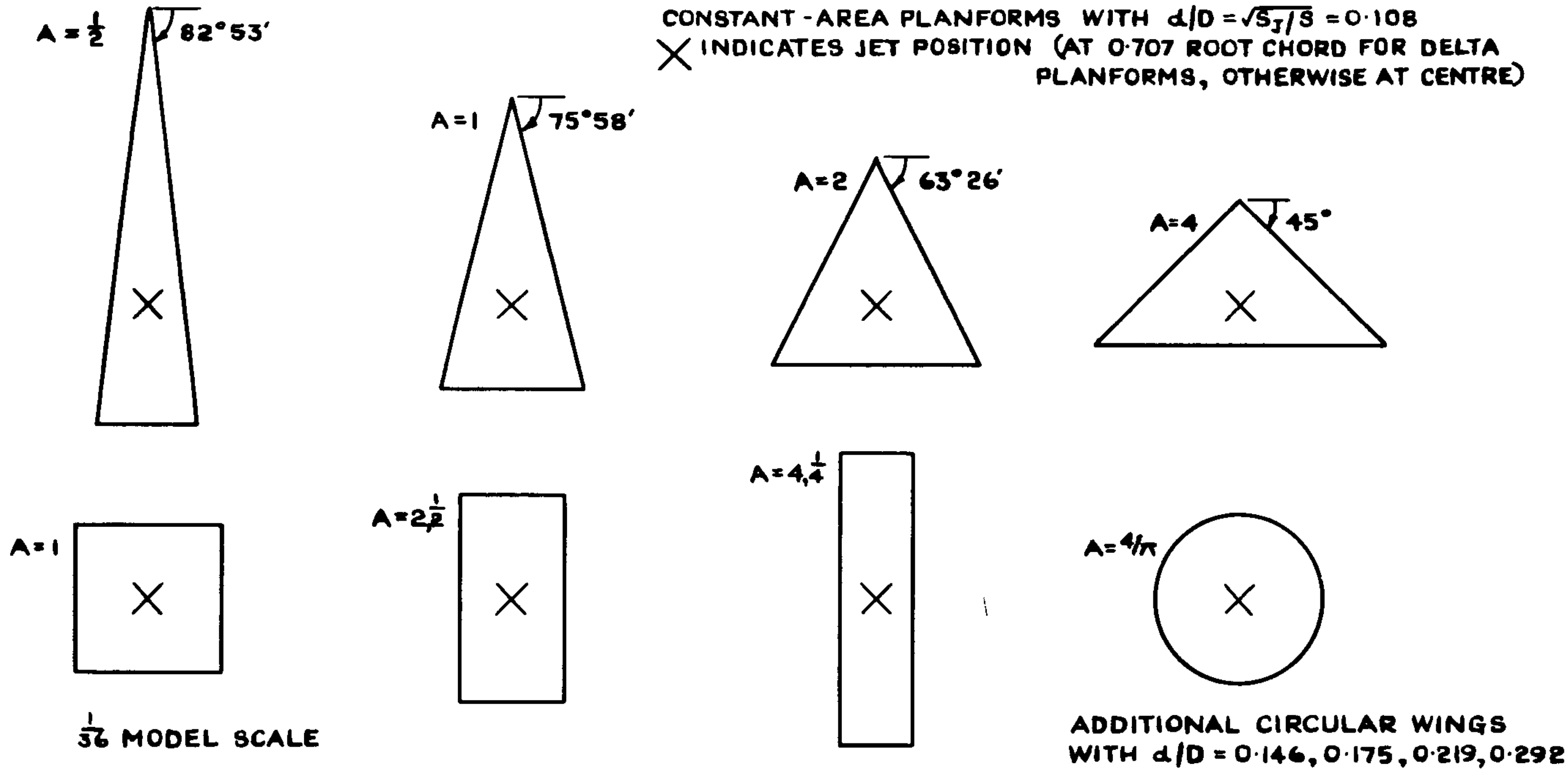


FIG.1. RANGE OF PLANFORMS FOR STATIC MODEL TESTS OF GROUND EFFECT ON DIRECT JET LIFT

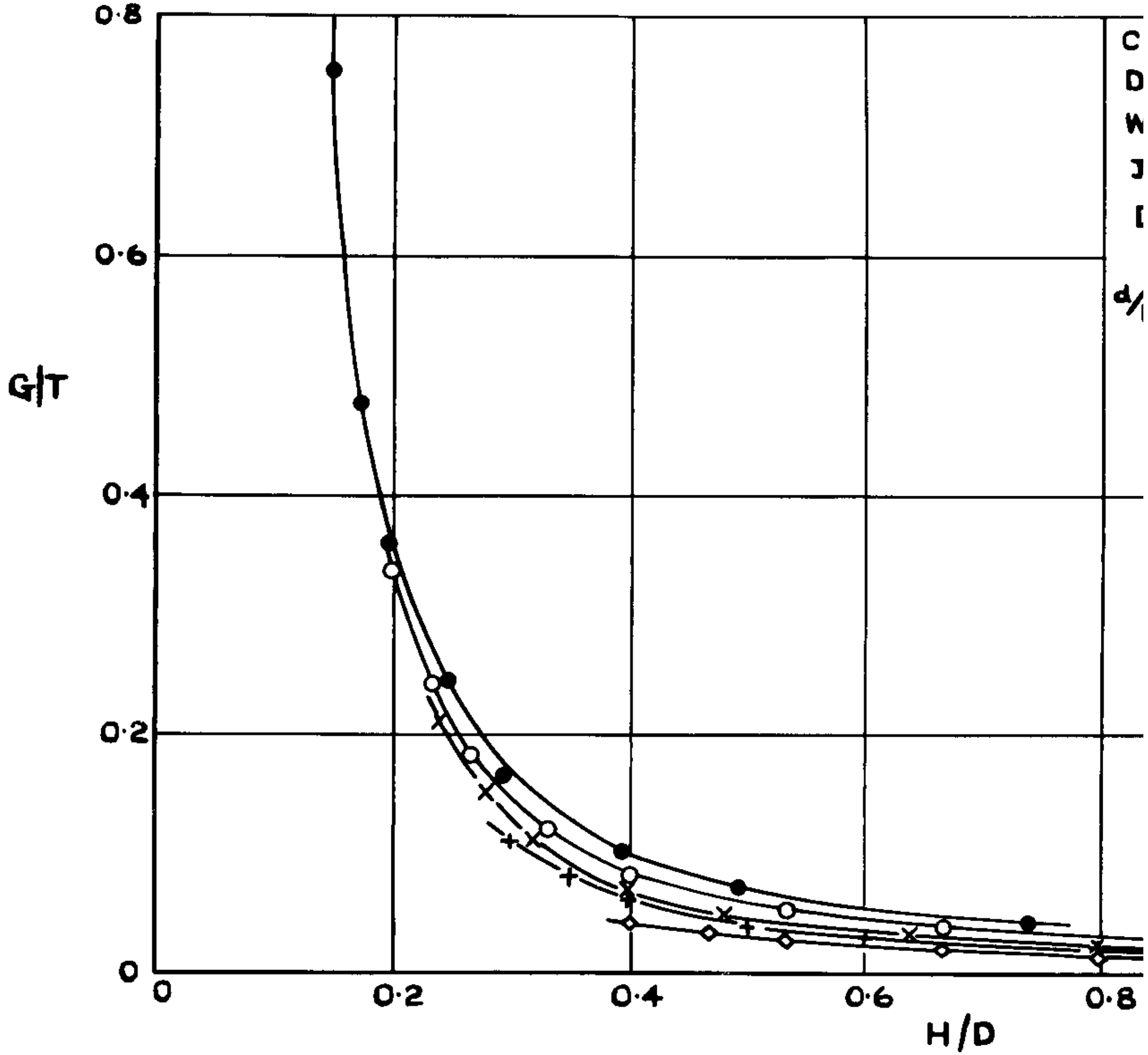


FIG.2. EFFECT OF d/D RATIO
CIRCULAR PLANF

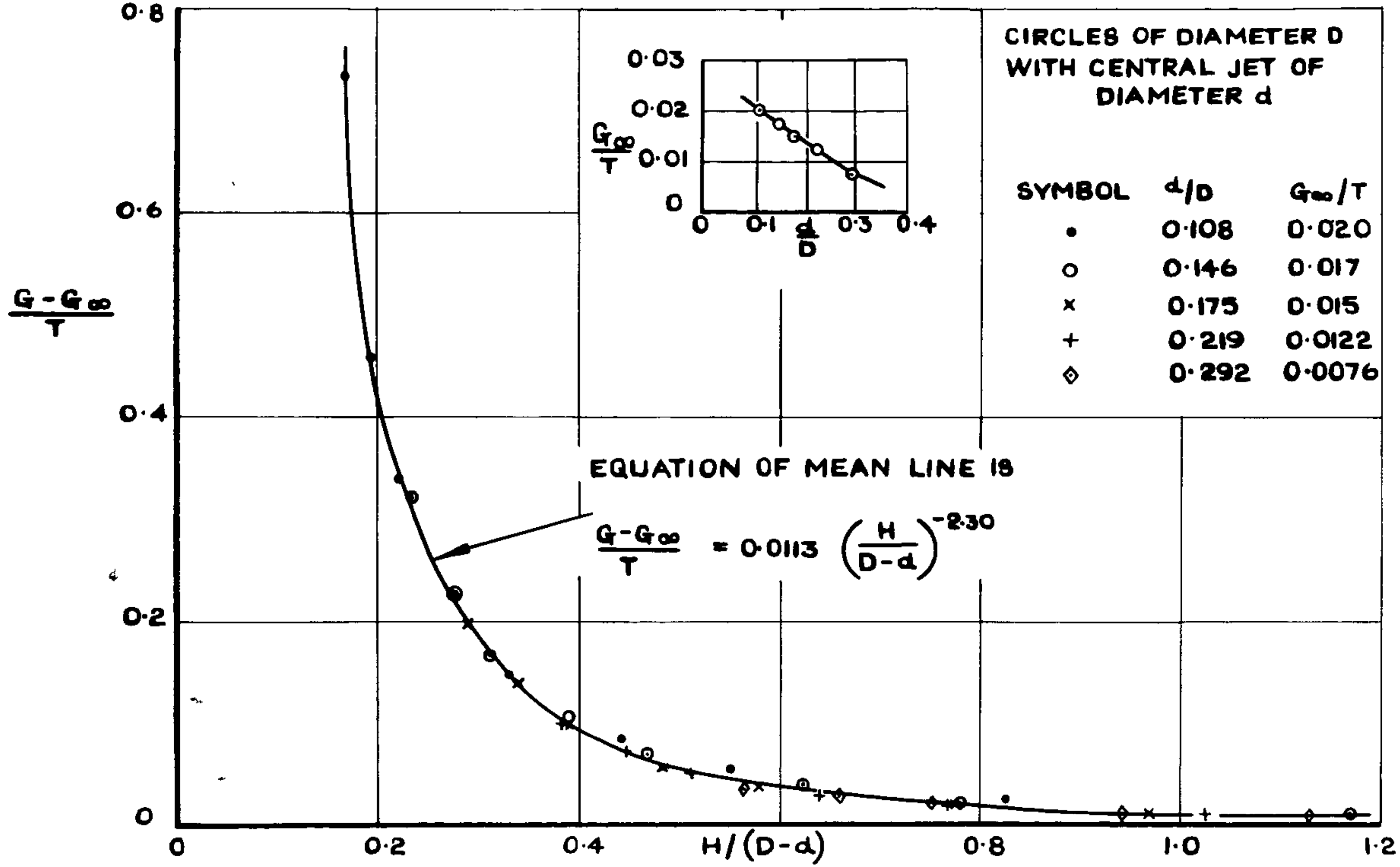


FIG. 3. CORRELATION OF GROUND EFFECT ON CIRCULAR PLANFORMS WITH O·1 < d/D < O·3.

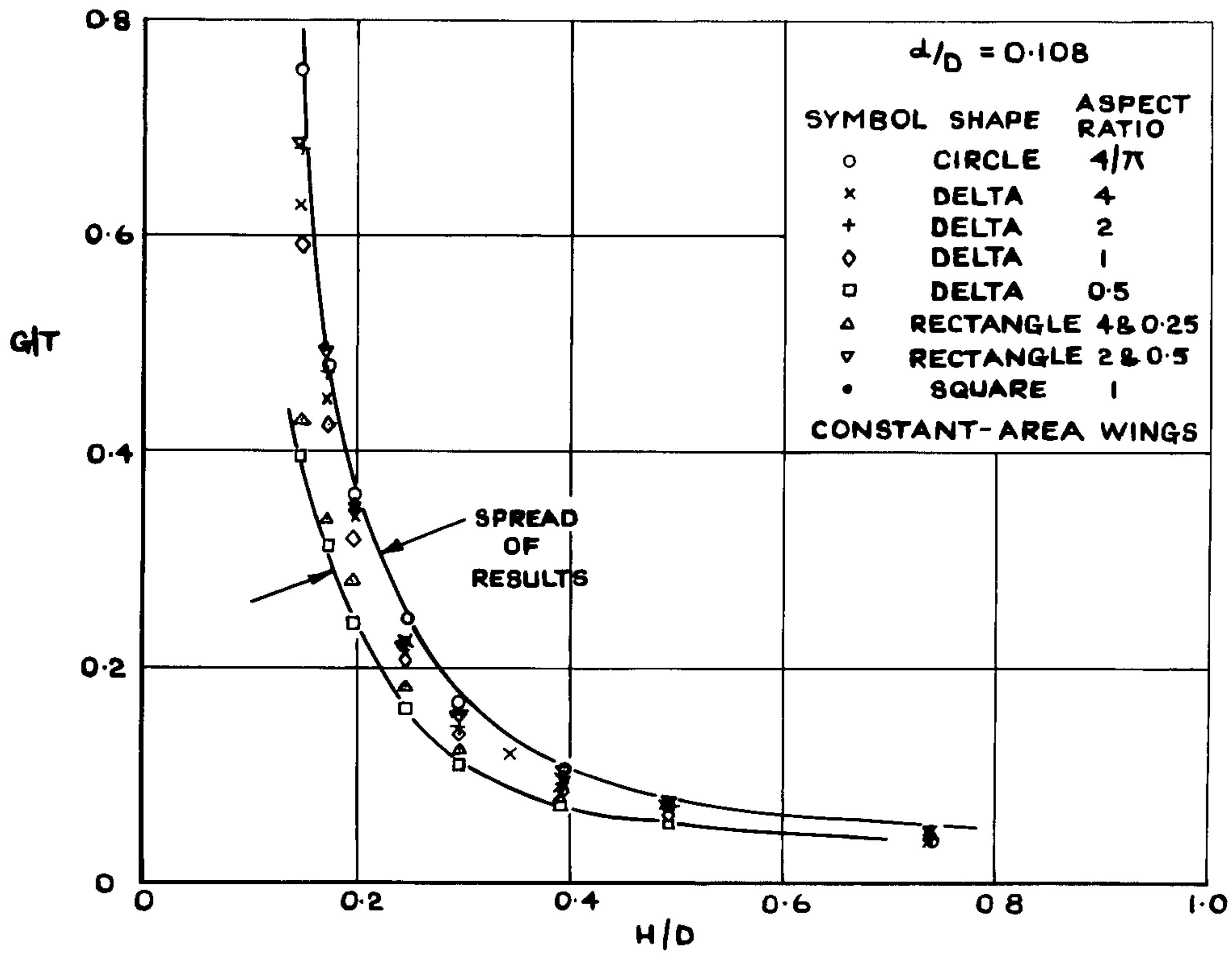


FIG. 4. EFFECT OF PLANFORM SHAPE AND ASPECT RATIO ON GROUND EFFECT.

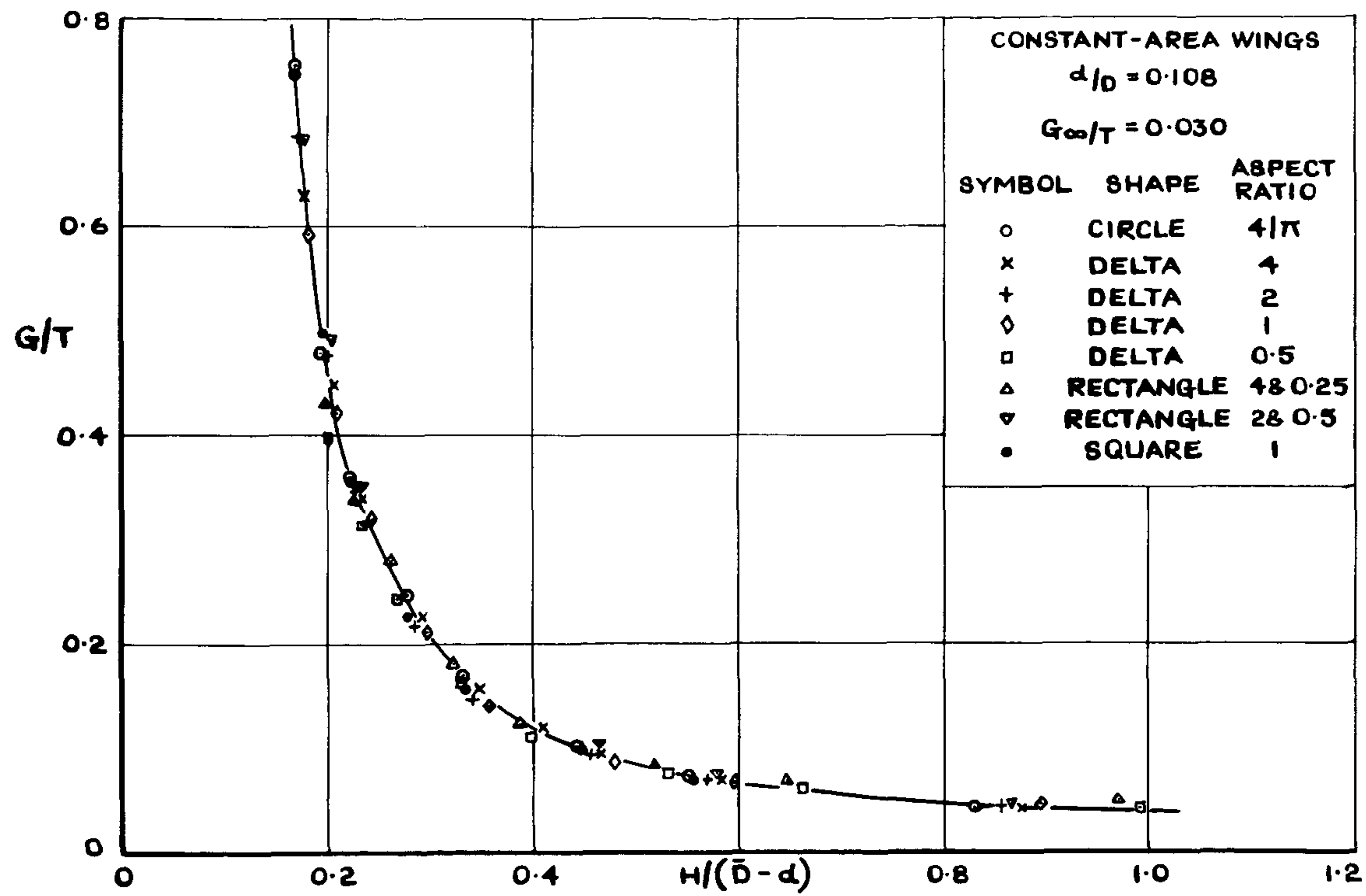
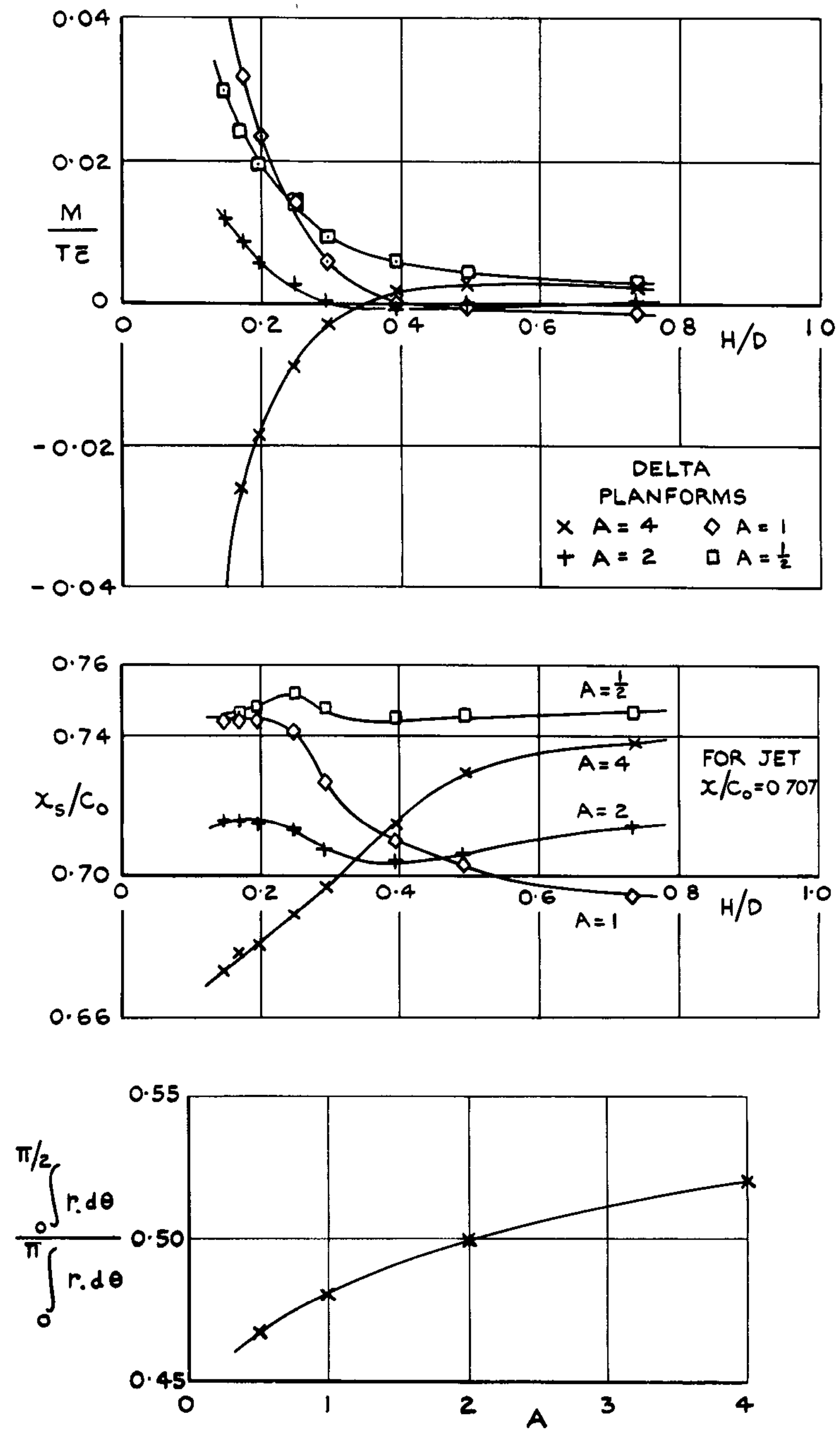


FIG. 5. CORRELATION OF GROUND EFFECT ON PLANFORMS OF VARYING SHAPE AND ASPECT RATIO, d/D = 0·108.


0·04
0·02
0
-0·02
-0·04
M / TĒ
0
0·2
0·4
0·6
0·8
1·0
H/D
DELTA PLANFORMS
× A = 4
◇ A = 1
+ A = 2
□ A = ½
0·76
0·74
0·70
0·66
x_s/c_0
A = ½
A = 4
A = 2
A = 1
FOR JET
x/c_0 = 0·707
0·55
0·50
0·45
$\int_0^{\pi/2} r.d\theta \Big/ \int_0^{\pi} r.d\theta$
0
1
2
3
4
A

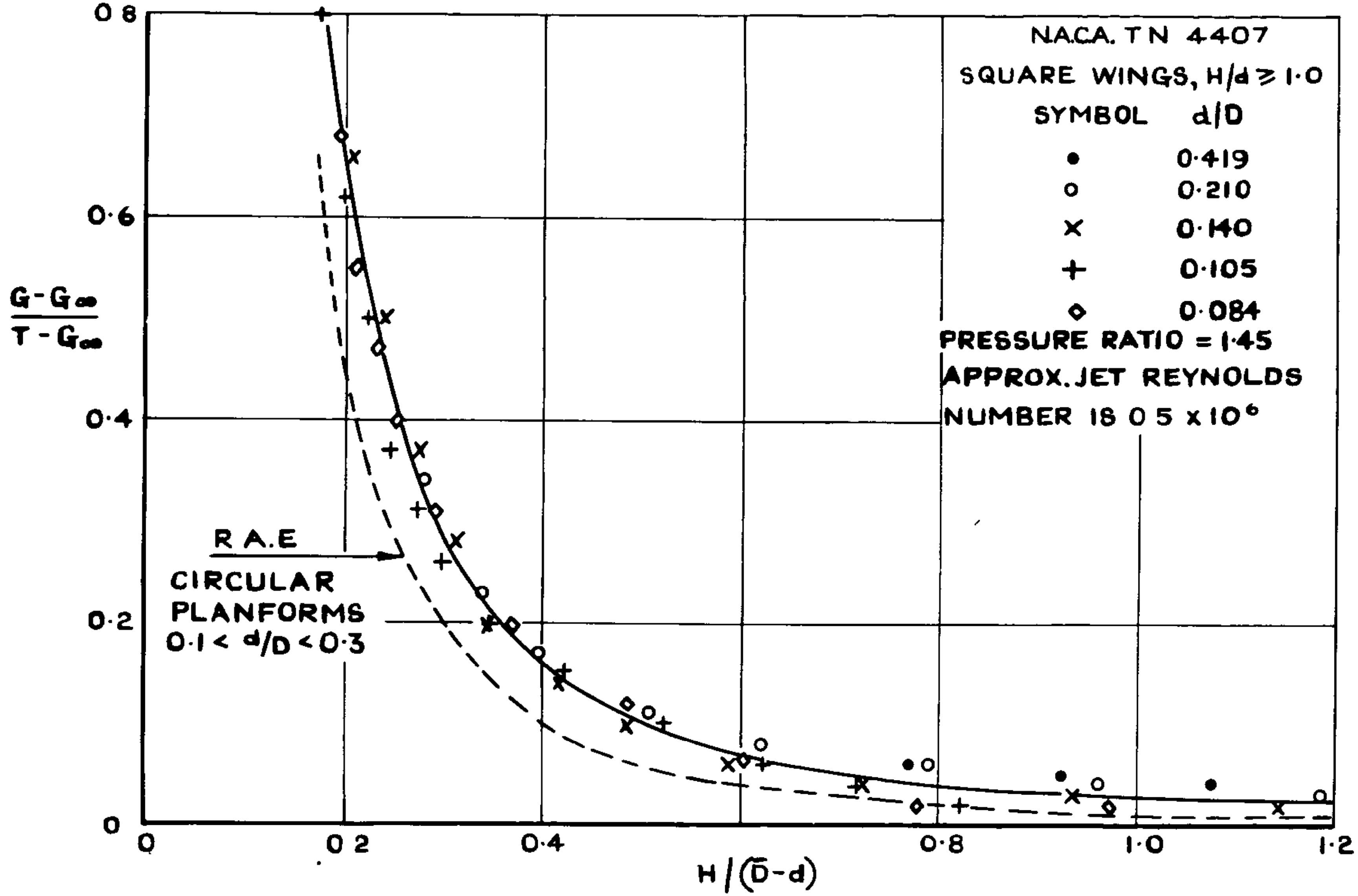
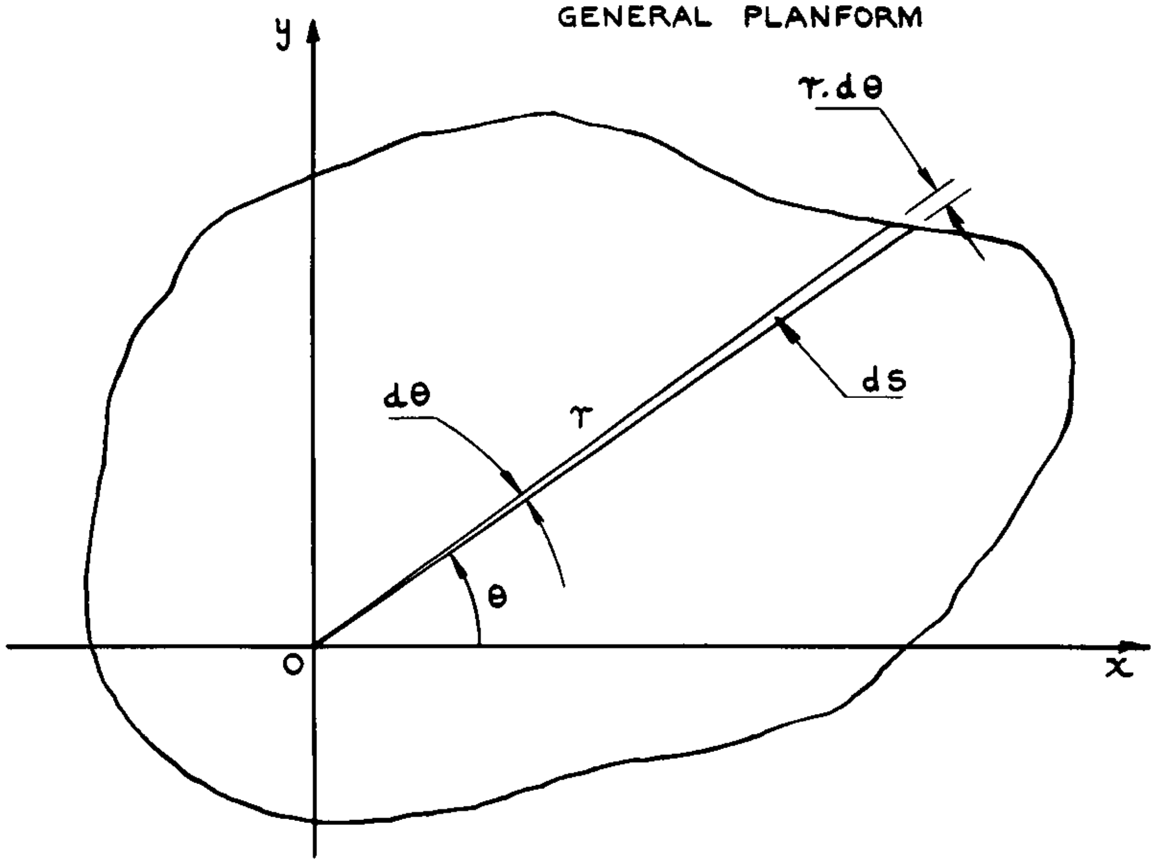


FIG. 7. CORRELATION OF N.A.C.A. RESULTS ON GROUND EFFECT ON SQUARE PLANFORMS, $0{\cdot}1 < d/D < 0{\cdot}4$.



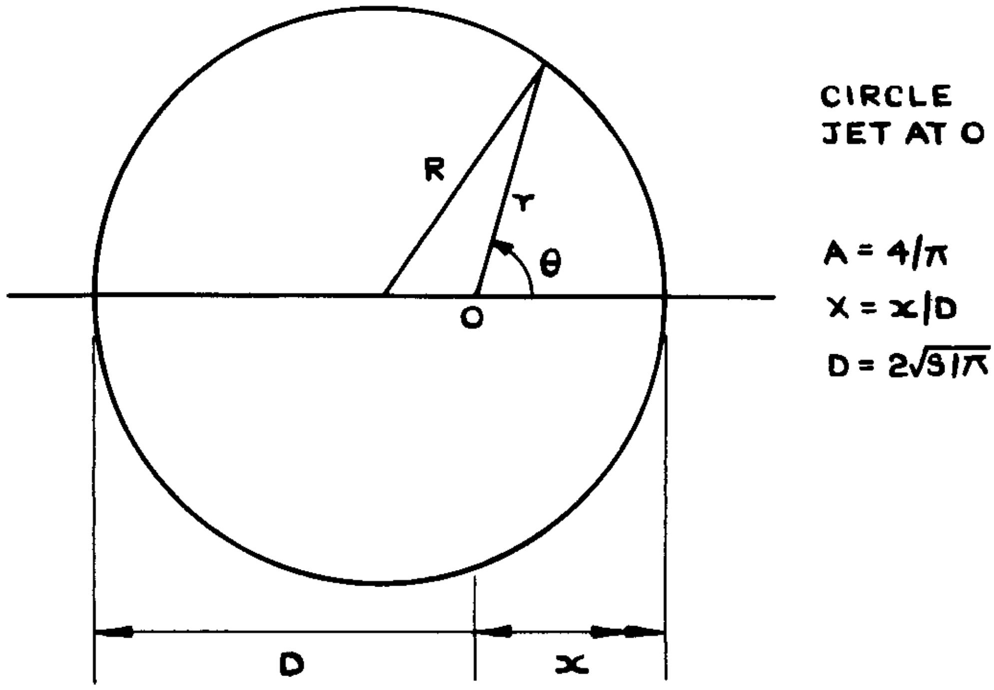
ORIGIN OF POLAR CO-ORDINATE SYSTEM CHOSEN TO COINCIDE WITH CENTRE OF JET (OR GROUP OF CLOSELY-SPACED JETS)

DEFINE

$$\frac{1}{2}\bar{D} = \frac{1}{2\pi}\int_0^{2\pi} r \, d\theta$$

$$= \frac{1}{2\pi}\int_0^{2\pi} \frac{ds}{r/2}$$

FIG. 8. DEFINITION OF ANGULAR MEAN DIAMETER $\bar{D}$ OF GENERAL PLANFORM.



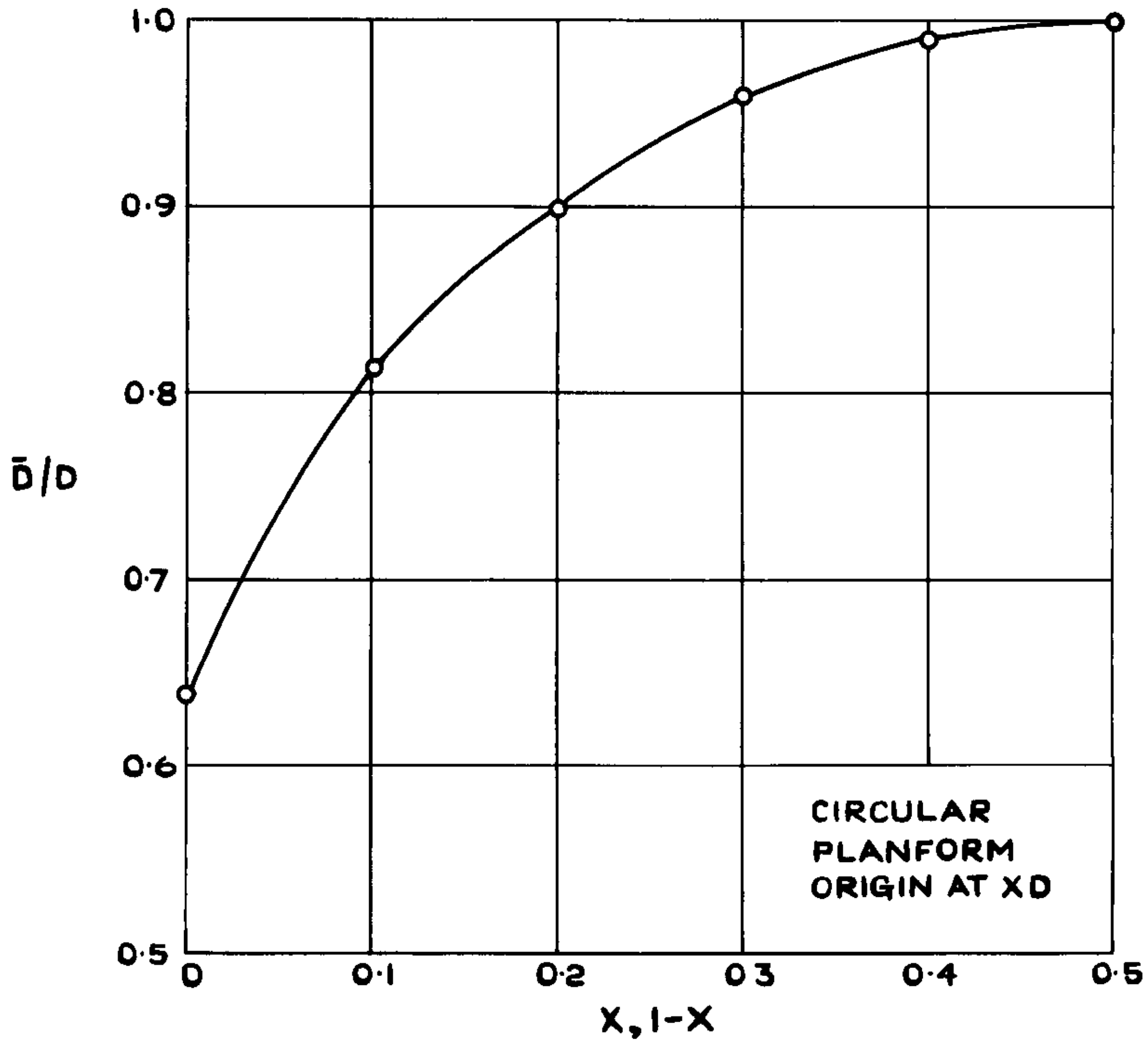
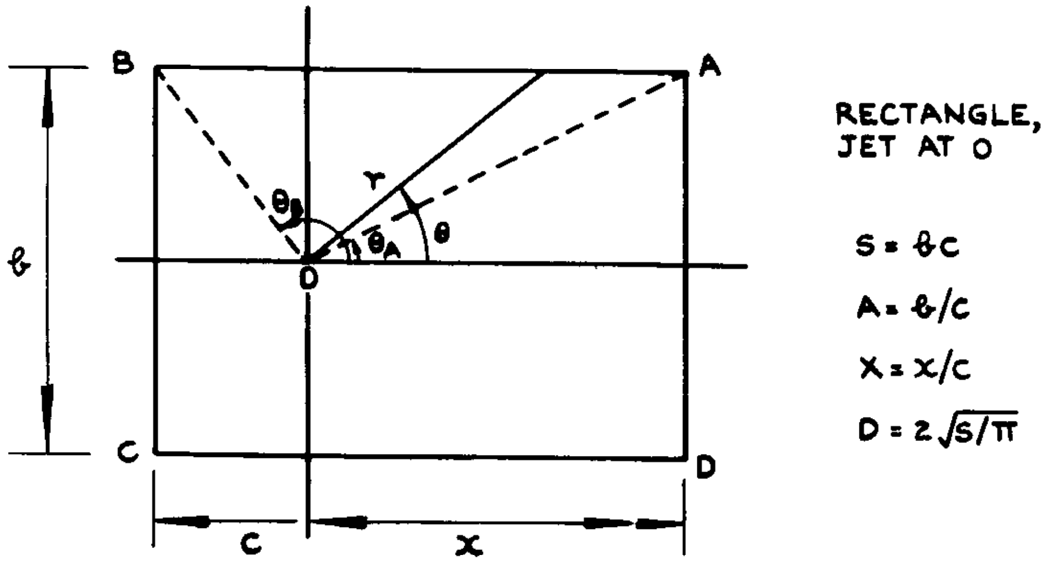
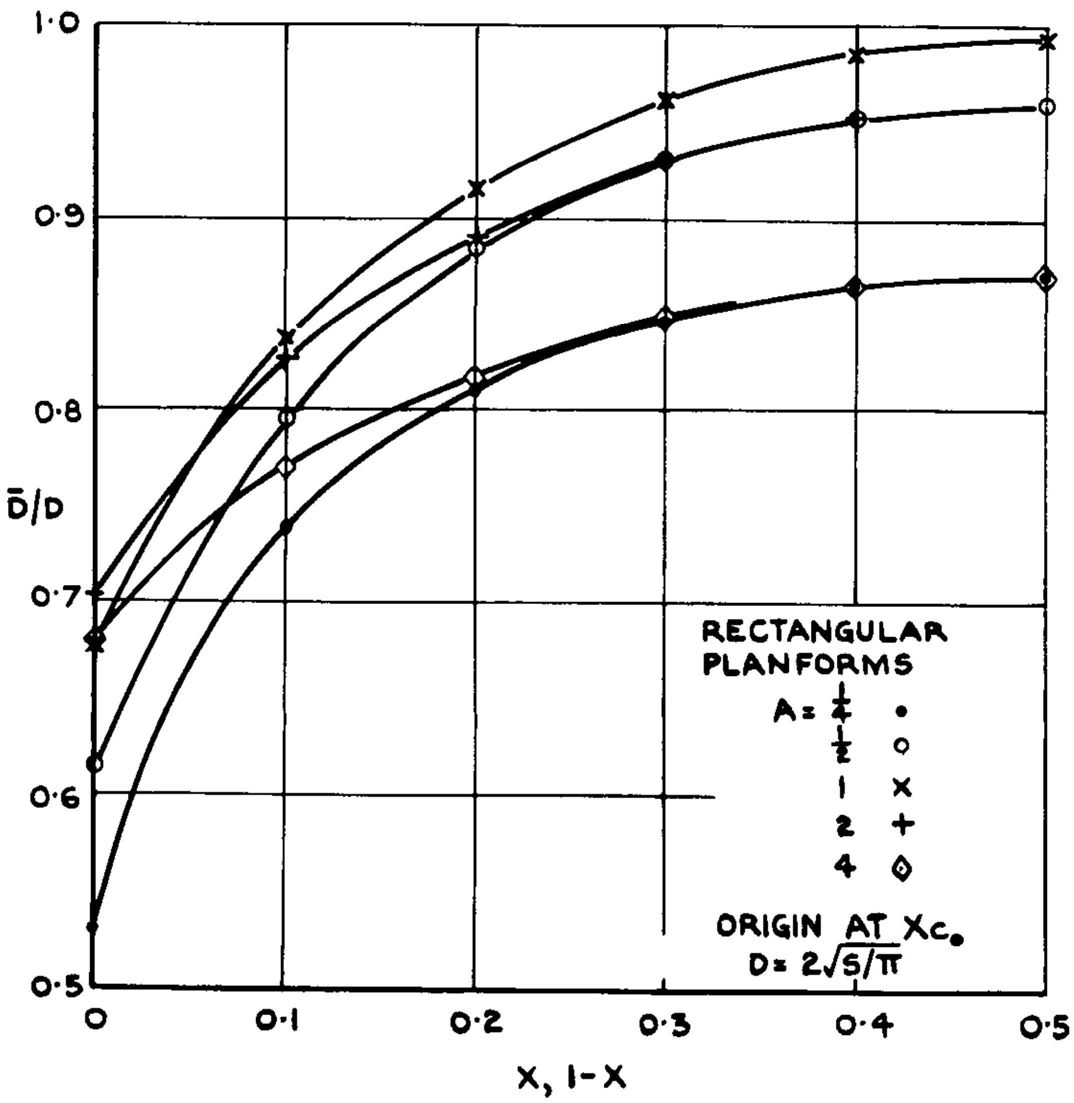


FIG.9. VALUES OF $\bar{D}$ FOR CIRCULAR PLANFORM WITH ORIGIN ON CENTRE-LINE



B
A
C
D
O
r
θ
θ_B
θ_A
b
c
x
RECTANGLE,
JET AT O
S = bc
A = b/c
X = x/c
D = 2√(S/π)



1·0
0·9
0·8
0·7
0·6
0·5
D̄/D
0
0·1
0·2
0·3
0·4
0·5
X, 1-X
RECTANGULAR
PLANFORMS
A = 1/4 •
1/2 ○
1 ×
2 +
4 ◇
ORIGIN AT Xc₀
D = 2√(S/π)

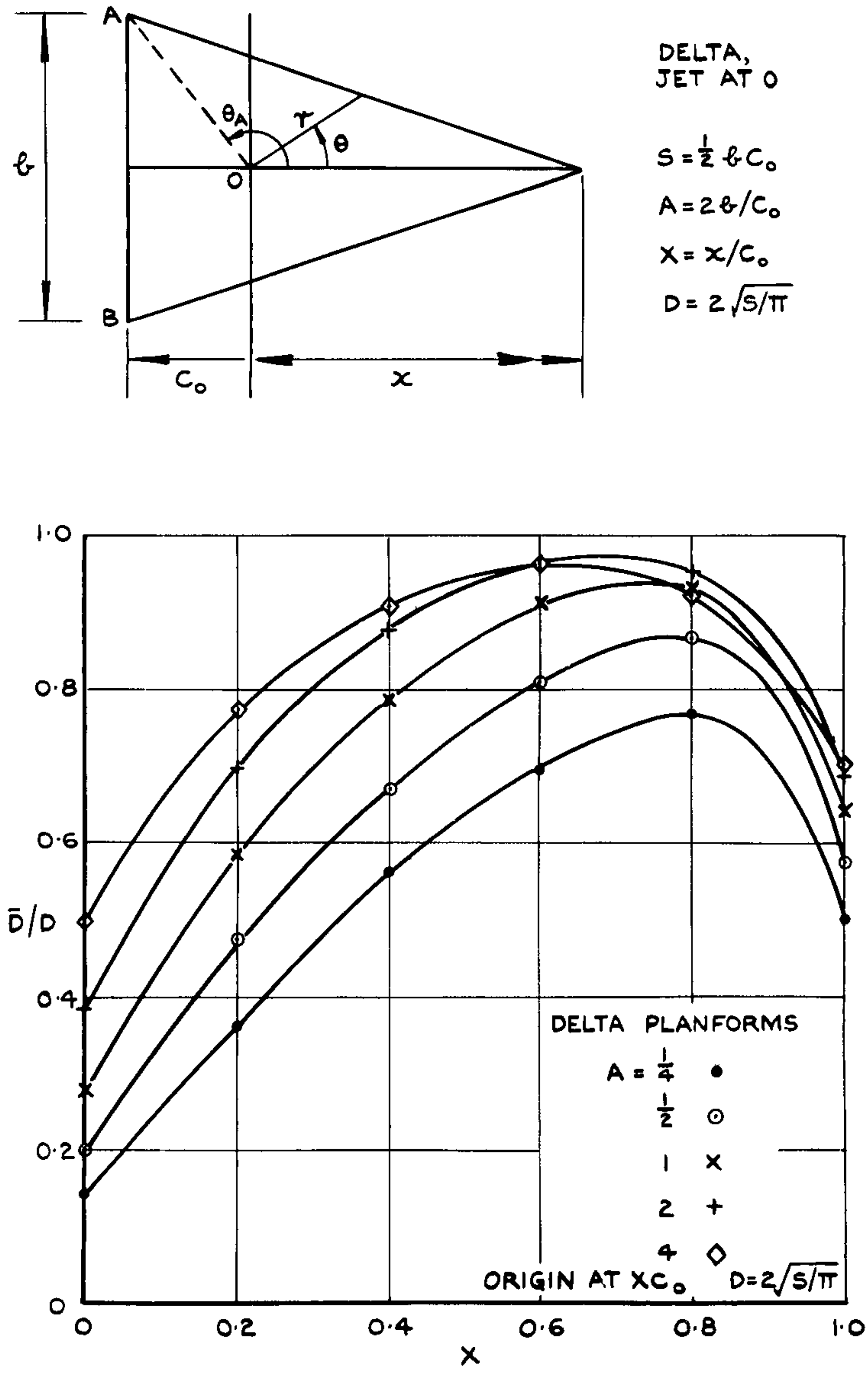


FIG. 11. VALUES OF $\bar{D}$ FOR DELTA PLANFORMS OF VARYING ASPECT RATIO WITH ORIGIN ON CENTRE-LINE.

A.R.C. C.P. No. 749

533.693.3:
533.693.5:
533.693.6:
533.694.6:
533.682

STATIC TESTS OF GROUND EFFECT ON PLANFORMS FITTED WITH A CENTRALLY-LOCATED ROUND LIFTING JET. Wyatt, L.A. June 1962.

Static measurements of the thrust losses due to ground effect on a range of plane circular, rectangular and delta wings of various aspect ratios with a single round lifting jet located centrally on the planform are described. A satisfactory correlation of the results is presented.

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