C.P. No. 746

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Some Simple Calculations Relating to the Generation of an R.F. Plasma

by

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LONDON: HER MAJESTY'S STATIONERY OFFICE

1964

PRICE 4s 6d NET

U.D.C. No. 537.56

C.P. No.746

April, 1963

SOME SIMPLE CALCULATIONS RELATING TO THE GENERATION OF AN R.F. PLASMA

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SUMMARY

A cylindrical plasma is considered to be heated by R.F. currents to a temperature of about 8000°K in argon at a pressure of 50×10^{-6} atmospheres. An estimate is made of the skin depth and of the uniformity of the plasma in the central region both with and without an axial magnetic field. A skew solenoid is suggested to provide a magnetic field inclined to the axis of the plasma.

Replaces R.A.E. Tech. Note No. Aero. 2889 - A.R.C. 24 950.

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1 INTRODUCTION

In two recent Notes[1,2] consideration has been given to the possibility of setting up a laboratory plasma for the purpose of carrying out experiments on ion wave propagation. The method of plasma generation suggested follows those of Knechtli and Wada[3,4], Kino[5] and Rynn and D'Angelo[6]. The experiments of Rynn and D'Angelo were carried out in the Q machine at Princeton, which appears to be on a much bigger scale as regards the plant involved than does the apparatus used by either Knechtli and Wada, or Kino. Wong, D'Angelo and Motley[7] have generated and detected ion acoustic waves which were very heavily damped as in fact the theory of Fried and Gould[8] predicts.

In the present paper a few calculations are given that relate to the possibility of generating a plasma by means of R.F. heating. This is not intended to offer an alternative apparatus for the proposed wave experiments, but it may well be that a plasma of this type would be a useful standby for the development of instrumentation, and it may also be possible at some stage to include such a plasma in a small firing range to investigate the passage of aerodynamic models through an ionised gas. If we consider three possible methods of plasma generation in the laboratory,

(1) by d.c. discharge,

(2) by R.F. discharge,

(3) by direct thermal ionisation,

it may be assumed that the order given represents an order of increasing difficulty of construction. On the other hand the order given probably also represents an order of increasing quiescence, and in addition the presence of a steady current in a d.c. discharge can be a considerable handicap in some contexts. A specific reason for the present paper is the existence of a considerable R.F. power supply within Aerodynamics Department R.A.E.

It is assumed that the plasma is contained within a long circular cylinder surrounded by a helical R.F. coil and further that the R.F. currents in the plasma are confined to an outer skin as shown in Fig.1. In Section 2 the diffusion inwards is considered across an axial magnetic field to get an estimate of the uniformity of the plasma in such conditions: the analysis follows that of Rynn and D'Angelo[6]. In Section 3 the diffusion is considered in the absence of a magnetic field, both for a fully ionised gas and a gas in the presence of a preponderance of neutral atoms. The axial magnetic field impedes the diffusion process, and in order to allow easy diffusion inwards while yet providing a magnetic field, the field due to a skew solenoid is considered in Section 4.

The few calculations given here are not intended in any way to be a thorough investigation of the problem of plasma production by means of an R.F. discharge, but it is hoped that they may give some idea of the possibilities.

2 PLASMA WITH AN AXIAL MAGNETIC FIELD

2.1 Skin depth

The R.F. coil will induce circumferential currents in the plasma which will have their greatest intensity near to the circumference and fall away in an approximately exponential manner if the skin depth is small compared with the cylinder diameter. We assume that the hot gas in the outer skin has reached equilibrium which we take as being the Saha value. Convenient numerical values for Argon which has an ionisation potential of 15·7 volts, are

$$T = 8000^{\circ}K$$

$$p = 50 \times 10^{-6} \text{ atmospheres}$$

$$\left.\begin{aligned} n_n &= 5 \times 10^{13} \text{ cm}^{-3} \\ n_e &= 5 \times 10^{12} \text{ cm}^{-3} \end{aligned}\right\} 10\% \text{ ionisation}$$

where T is the temperature, p the pressure, n_n the neutral number density and n_e the electron number density both per cubic centimetre. The skin depth depends on the conductivity, which may be estimated from Spitzer[9] who gives the specific resistance η to be

$$\eta = 6{\cdot}53 \times 10^3 \frac{\log \Lambda}{T^{3/2}} \text{ ohm cm} . \qquad (1)$$

The value of $\log \Lambda$ is about 8·5 so this gives a value for η of about 0·078 ohm cm. Some justification for using this value in the presence of an axial magnetic field is given below in Section 2.2. The skin depth for a semi-infinite conductor is given by e.g. Sommerfeld[10] in m.k.s.c. units as

$$d_m = \left(\frac{2}{\mu \sigma \omega}\right)^{\frac{1}{2}} \qquad (2)$$

where the suffix m denotes that d is in metres, and in the plasma $\mu = \mu_o = 4\pi \times 10^{-7}$, k.m.c.$^{-2}$, σ is the specific conductivity and ω the circular frequency. For the specific resistance obtained above this expression leads to a skin depth given by $d = 1400/\sqrt{f}$ cm, where f is the frequency in cycles per second.

In order to reduce the skin depth to a small fraction of a centimetre, which is probably ideal for the creation of a large central region free from much current and thus reasonably uniform and quiescent, the exciting frequency needs to be of the order of 100 megacycles. The equipment available in Aerodynamics Department R.A.E. is at present limited to about 0·75 m.c. which corresponds to a skin depth of 1·6 cm. In a plasma with an overall diameter of 10 cm, however, this may still leave a central core of about 5 cm in a reasonably uniform state.

2.2 Electrical conductivity

The assumption of a simple conductivity law in Section 2.1 requires justification when a powerful axial magnetic field is present. The induced E.M.F. acts circumferentially in such a way as to try to drive current across the magnetic field lines and this results in a number of complications. After making several simplifying assumptions, Rose and Clark[14] quote a conductivity matrix for a plasma in the presence of a magnetic field using rectilinear coordinates in which $\underline{B}$ takes the form $(0, 0, B_z)$. They treat two models, the first of which is a fully ionised gas subject to no restraint other than that of the uniform magnetic field. This yields a conductivity matrix given by

$$\sigma_1 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1/\eta \end{bmatrix} \qquad (3)$$

This implies that an electric field drives no current across the magnetic field, which can be seen to be true in the absence of restraints and if the collision frequency is small compared with the cyclotron frequency. In these circumstances both electrons and ions follow the paths of free particles in the presence of perpendicular electric and magnetic fields and both types of particle drift in the same direction as each other and at right angles to both the applied fields. The two drift velocities are identical and of magnitude E/B since (see e.g. Ref.9) in a system moving with velocity $\underline{E} \wedge \underline{B}/B^2$ there is no electric field and both particles in the moving coordinates simply spiral around the magnetic field lines; and thus no net current flows across the magnetic field. Since the relative frequencies are important in connection with this reasoning it is of interest to note the values that apply in the conditions mentioned above in Section 2.1 and for a typical magnetic field such as the 350 gauss used in Section 2.4 below. The frequencies in increasing order are then given roughly by the following table.

ν_{in}	=	collision frequency, ions with neutrals	$= 5 \times 10^4$ sec^{-1}
ω_{bi}	=	ion cyclotron frequency	$= 8 \times 10^4$ sec^{-1}
ν_{ii}	=	collision frequency, ions with ions	$= 8{\cdot}3 \times 10^5$ sec^{-1}
ν_{ie}	=	collision frequency, ions with electrons	$= 1{\cdot}2 \times 10^6$ sec^{-1}
ω	=	applied frequency	= order 10^6 sec^{-1}
ν_{en}	=	collision frequency, electrons with neutrals	$= 10^7$ sec^{-1}
ν_{ee}	=	collision frequency, electrons with electrons	$= 2{\cdot}2 \times 10^8$ sec^{-1}
ν_{ei}	=	collision frequency, electrons with ions	$= 3{\cdot}2 \times 10^8$ sec^{-1}
ω_{be}	=	electron cyclotron frequency	$= 6 \times 10^9$ sec^{-1}

In this Table the cyclotron frequencies are given in radians per second since this is relevant for comparisons with collision frequencies; the applied frequency ω may be up to $0{\cdot}75 \times 10^6$ cycles per second for the available equipment, i.e. $4{\cdot}7 \times 10^6$ radians per second, and the value of 10^6 quoted in the table is intended to indicate this order. The charged particle collision frequencies include an allowance for long range collisions and are based on the deflection times given by Spitzer[9].

We can return now to equation (3) and note that for the electrons the cyclotron frequency is greater than any collision frequency so that the electrons would start to drift radially on the appearance of the induced electric field and in accordance with (3). The magnitude of the electric field has been left as an unknown in the present Note but a value of, say, 35 volts/cm would result in a radial drift velocity E/B of 10^7 cm/sec so that in a half cycle of the applied frequency the electrons would have drifted the order of the tube radius. A drift velocity of such magnitude is, of course, out of the question since a large pressure gradient would build up in the electron gas to oppose it. With regard to the ions the drift velocity would be interrupted by collisions since $\omega_{bi} < \nu_{ii}, \nu_{ie}$ and this suggests that the ions would lag behind the electrons in any radial drift thus setting up a space charge field that would also restrain the electrons.

The conductivity matrix (3) must thus be rejected as regards the behaviour in a constrained plasma and we turn to the second model given by Rose and Clark in which fixed scattering centres are present. In a weakly ionised gas, for example, where collisions with neutrals predominate, the electrons and ions lose momentum to the neutrals (which are supposed to be so numerous as not to be greatly affected by the momentum transfer) instead of to each other; thus the electron and ion equations are uncoupled and the conductivity matrix for either species takes the form[14]

$$[\sigma_2]_{i,e} = \frac{ne^2}{m}\begin{bmatrix} \dfrac{\nu}{\nu^2+\omega_b^2} & -\dfrac{\omega_b}{\nu^2+\omega_b^2} & 0 \\ \dfrac{\omega_b}{\nu^2+\omega_b^2} & \dfrac{\nu}{\nu^2+\omega_b^2} & 0 \\ 0 & 0 & 1/\nu \end{bmatrix} \qquad (4)$$

where m and ν each take the appropriate suffix. Here the conductivity across the magnetic field is much reduced (and in fact the ions begin to carry an appreciable amount of the current) and the off-diagonal elements which represent the Hall currents are large by comparison with the direct elements in the (x,y) plane. It is of interest to quote the electrical resistance matrix that corresponds with the conductivity matrix (4) and which may at least be expected to apply during the early stages of the R.F. heating process. If the ion current can be neglected we can write

$$\underline{I} = [\sigma_2]\,\underline{E} \qquad (5)$$

where $\underline{I}$ and $\underline{E}$ are the current and electric field vectors respectively. Then

$$\underline{E} = [\sigma_2]^{-1}\,\underline{I} \equiv [\eta_2]\,\underline{I} \qquad (6)$$

and $[\eta_2]$ is given by[14]

$$[\eta_2] = \frac{m}{n\,e^2}\begin{bmatrix} \nu & \omega_b & 0 \\ -\omega_b & \nu & 0 \\ 0 & 0 & \nu \end{bmatrix} \qquad (7)$$

so that the direct resistance does not vary with the direction of excitation (Spitzer[9] finds that the resistance is increased across the magnetic field by a factor of just over 3). The heating effect, P, of a current vector $\underline{I}$ will be given by

$$P = I'\,[\eta_2]\,I \qquad (8)$$

where I' is the transpose of I, and it can be seen that P is independent of Hall currents.

In the equilibrium phase covered by the frequency table given above, however, it is less clear that the conductivity matrix (4) applies. It can be seen at once from the respective collision frequencies that the neutral particles are too few in number to provide the predominant collisions, but one might argue that the pressure gradients and the electric field set up when the type of motion considered under equation (3) tries to develop would themselves act to destroy the gathering momentum of the electrons and ions, and thus take the place of the fixed neutral scattering particles that were assumed in the derivation of (4). But we must still conclude that equation (4) cannot apply for very long because of the large radial Hall currents that would be set up. These would lead to the development of a space charge and be counteracted by the corresponding radial electric field. In fact it seems that if bodily drift and Hall currents are both prevented by the geometry, as they appear to be in our case, the conductivity that finally results will be similar to that assumed in Section 2.1, and the skin depth of 1·6 cm found in that section may well be of the right order.

It is possible, however, that for very much higher applied frequencies, of the order of 100 m.c., the geometrical restraints may not have time to take effect and much of the conductivity in the direction of the circumferential electric field could then be lost. In such circumstances it might be more efficient to generate the plasma by axially induced currents, in which case the R.F. coil would be replaced by longitudinal R.F. conductors, carrying suitably phased alternating currents, surrounding the plasma cylinder in the manner shown in Fig.1(b).

2.3 Plasma uniformity

For simplicity the cylindrical plasma is assumed to consist of two regions, an outer skin in which the heating occurs and an inner core maintained in a steady state by inward diffusion and in which the only loss is due to volume recombination of electrons and ions. The axial magnetic field is supposed to be large so that the magnetic pressure is large compared with the gas pressure and the diffusion process is supposed to be dominated by the Coulomb cross section, i.e. the gas is effectively fully ionised. The analysis of Rynn and D'Angelo[6] is applicable (in their case the diffusion is outwards) and we repeat here the relevant equations. It is assumed that only singly charged ions exist so that the basic continuity and momentum equations (which can also be obtained from Spitzer[9]) are

$$\mathrm{div}(n_i \underline{v}_i) + \alpha\, n_i\, n_e = 0 \qquad (9)$$

$$\mathrm{div}(n_e \underline{v}_e) + \alpha\, n_i\, n_e = 0 \qquad (10)$$

$$n_i\, m_i(\underline{v}_i \cdot \mathrm{grad})\, \underline{v}_i + k\, T_i\, \mathrm{grad}\, n_i - n_i\, e(\underline{E} + \underline{v}_i \times \underline{B}) = -\eta e^2\, n_e(n_i\, \underline{v}_i - n_e\, \underline{v}_e) \qquad \ldots (11)$$

$$n_e\, m_e(\underline{v}_e \cdot \mathrm{grad})\, \underline{v}_e + k\, T_e\, \mathrm{grad}\, n_e + n_e\, e(\underline{E} + \underline{v}_e \times \underline{B}) = \eta e^2\, n_e(n_i\, \underline{v}_i - n_e\, \underline{v}_e) \qquad \ldots (12)$$

Here α is the recombination coefficient

$\underline{v}_i$ is the ion velocity vector

$\underline{v}_e$ is the electron velocity vector

e is the electronic charge

m_i is the mass of an ion

m_e is the mass of an electron

k is Boltzmann's constant

$\underline{E}$ is the electric vector

$\underline{B}$ is the magnetic vector

The terms on the right hand side of equations (11) and (12) represent the momentum exchange between electron and ions, and η is the electrical resistance of the plasma in the absence of a magnetic field.

In the limiting case of a long circular cylinder, the physical parameters become functions of the single independent variable r. We also make a number of simplifying assumptions, of which the first is that departures from electrical neutrality are negligible, so that

$$n_i = n_e = n \text{ , say,} \tag{13}$$

and hence equation (9) becomes

$$\frac{1}{r} \frac{d}{dr} (r n v_r) + \alpha n^2 = 0 \tag{14}$$

where v_r is the radial component of velocity and because of assumption (13) the values for the ions and electrons are identical. There is no other component of v present in equation (14) since the z and θ components must be constants by reason of symmetry. The momentum equations are now linearised by neglecting terms of order v^2; the electron and ion temperatures are assumed to be equal and constant and the electric field is assumed to be zero*, so that after subtracting equations (11) and (12) we have

$$2 n e \underline{v} \times \underline{B} = 2 \eta e^2 n^2 (\underline{v}_i - \underline{v}_e) \quad . \tag{15}$$

This gives an equation for the radial velocity v_r in terms of the circumferential velocities $v_{i\theta}$ and $v_{e\theta}$.

$$- v_r B = \eta e n (v_{i\theta} - v_{e\theta}) \quad . \tag{16}$$

A further equation, also in the circumferential velocity components, is obtained by the addition of equations (11) and (12) to give

$$2 kT \frac{dn}{dr} = e n B (v_{i\theta} - v_{e\theta}) \quad . \tag{17}$$

From equations (16) and (17) we have

$$v_r = - \frac{2 \eta kT}{B^2} \frac{dn}{dr} \quad . \tag{18}$$

Maxwell's equation (curl $\underline{B} = 4\pi j$) can be used[6] to show that variations in B^2 can be neglected in the conditions of a strong magnetic field, so that substitution for v_r from equation (18) in equation (14) leads to

$$\frac{d^2(n^2)}{dr^2} + \frac{1}{r} \frac{d(n^2)}{dr} - \frac{2\alpha}{A} n^2 = 0 \tag{19}$$

where $A = 2 \eta kT/B_o^2$.

*The assumption of equal ion and electron temperatures is justified in the numerical examples that follow, and the unimportance of the V^2 term is demonstrated in Section 3.1.

The solution of equation (19) is

$$n^2 = a_1 I_o\left(r \cdot \sqrt{\frac{2\alpha}{A}}\right) \qquad (20)$$

where a_1 is an arbitrary constant and I_o a modified Bessel function of the first kind. If we use the suffix 1 to denote conditions at the circumference of the plasma where it meets the outer skin, and the suffix o to denote conditions on the centreline, then

$$\left(\frac{n_1}{n_o}\right)^2 = I_o\left(r_1 \sqrt{\frac{2\alpha}{A}}\right) . \qquad (21)$$

2.4 Numerical values

The relevant value of α to use in equation (21) is uncertain. For pure radiative (two-body) recombination a value has been measured for Argon at 3100°K of 2×10^{-10} cm^3/sec (see Ref.11) and this may be expected to reduce with the square root of the temperature[12] to about $1 \cdot 2 \times 10^{-10}$ cm^3/sec. The practical value, however, may well be anything up to an order greater than this as a result of three-body recombination, and the value used here is 3×10^{-10} cm^3/sec which is the same as that used by Rynn and D'Angelo[6] for caesium at 2000°K. The magnetic field is no doubt a parameter that the experimenter would wish to vary: we take 350 gauss as a typical value that gives about the same ratio of magnetic to gas pressure (roughly a hundred to one) in the laboratory as obtains in the ionosphere. Then we find

$$\sqrt{\frac{2\alpha}{A}} = 0 \cdot 66 \text{ cm}^{-1} . \qquad (22)$$

If we specify that n_1 should not exceed n_o by more than 20%, equations (21) and (22) lead to a limiting radius r_1 of about 2 cm which in view of all the uncertainties is much too small from the point of view of relying on a reasonable diameter of fairly uniform plasma. A graph of the distribution given by equation (21) is shown in Fig.2. It is not to be expected that a highly uniform distribution would be obtained in the presence of a strong axial magnetic field since this would imply rapid diffusion inwards which the magnetic field opposes.

We have assumed that the ion and electron temperatures are equal at the inner surface of the skin. As a check on this we can estimate the order of the radial distance travelled by an ion before it is heated up to the temperature of the electrons by collisions. Spitzer[9] gives an expression for the equipartition time, t_{eq}, for a mixture of two kinds of particles each with its own initial Maxwellian distribution and its own temperature. The temperature difference $T_e - T_i$ diminishes exponentially and after a time t_{eq} it has reduced by a factor e. For electrons and singly charged ions, Spitzer's expression becomes

$$t_{eq} = \frac{5{\cdot}87\, A_e\, A_i}{n_e \log \Lambda} \left(\frac{T_e}{A_e} + \frac{T_i}{A_i}\right)^{3/2} \text{ sec} \qquad (23)$$

where A_e and A_i are the atomic weights of the electron and ion respectively. The numerical value of t_{eq} is found to be $1{\cdot}7 \times 10^{-4}$ sec. We can estimate the radial drift velocity of the ions at the edge of the plasma from equation (18) where, by equation (20), (dn/dr) is given by

$$\left(\frac{dn}{dr}\right)_1 = \frac{n_1^2}{I_o\left(r_1\sqrt{\frac{2\alpha}{A}}\right)} \times \sqrt{\frac{2\alpha}{A}}\; \frac{I_o'\left(r_1\sqrt{\frac{2\alpha}{A}}\right)}{2\,n_1} = \frac{n_1}{2}\sqrt{\frac{2\alpha}{A}}\; \frac{I_1\left(r_1\sqrt{\frac{2\alpha}{A}}\right)}{I_o\left(r_1\sqrt{\frac{2\alpha}{A}}\right)} \,. \qquad \ldots (24)$$

With the same numerical values as before this gives $\left(\frac{dn}{dr}\right)_1 = 0{\cdot}86 \times 10^{12}$ cm^{-4}. The radial velocity is then

$$v_r = -A\left(\frac{dn}{dr}\right)_1 = -1{\cdot}2 \times 10^3 \text{ cm sec}^{-1} \,, \qquad (25)$$

leading to a radial equipartition distance of 0·2 cm. In the circumstances the assumption of equal electron and ion temperatures at the inner skin surface appears reasonable.

3 PLASMA IN THE ABSENCE OF A MAGNETIC FIELD

3.1 Fully ionised plasma

Consideration of the equations of a fully ionised plasma is no longer a good approximation for the condition of 10% ionisation assumed in Section 2, because in the absence of an axial magnetic field the main resistance to diffusion inwards will come from collisions with the neutral atoms. It is, however, of interest to consider the fully ionised plasma first since this gives a measure of the importance of the non-linear velocity terms in equations (11) and (12). The continuity relation is still given by equation (14), but the right hand side of equation (17) vanishes so that the pressure gradient must be balanced by the second order velocity terms in equations (11) and (12), whence with $T_i = T_e$ and $m = m_i + m_e$, we have

$$n\, m v_r \frac{dv_r}{dr} + 2\, kT \frac{dn}{dr} = 0 \,. \qquad (26)$$

Equation (26) is not strictly correct since in deriving equation (11) in the form given from the Boltzmann equation[9] a continuity equation was used that did not include the recombination term given in equation (14). The correct form can be obtained by differentiating Bernoulli's equation, $p + \frac{1}{2}\rho V^2$ = constant where $p = 2\,nkT$ since there are two fluids each with a partial pressure of $n\,kT$. This leads to an additional term $\frac{1}{2} m v^2\, dn/dr$ which is however negligibly small since it can be verified from the solution given below that $\frac{1}{2} m v^2$ is less than 0·1% of 2 kT.

Equations (14) and (26) may be simplified to give

$$\left.\begin{aligned} \frac{dv_r}{dr} &= 2\,kTW \\ \frac{dn}{dr} &= -\,nm\,v_r\,W \\ W &= \frac{v_r + \alpha n r}{r m v_r^2 - 2\,kTr}\,. \end{aligned}\right\} \qquad (27)$$

where

The boundary condition is that when $r = 0$, $v_r = 0$ and $n = n_o$, and starting from this condition it is an easy matter to integrate equation (27) on a digital computer as far as any desired radius. Such a calculation has been performed for the same conditions as before and ignoring the neutral particles with the result that for a plasma of 10 cm radius the ratio n_o/n_1 exceeds unity by rather less than 0·001. In obtaining this result it is assumed that n_1 and n_o are nearly equal, because the ions are supplied from the outer skin and it is strictly n_1 that is known rather than n_o; if n_1 differed appreciably from n_o the boundary condition at $r = 0$ would not be completely specified. For comparison with equation (25), the radial velocities calculated from equations (27) are $-7{\cdot}1 \times 10^3$ cm sec^{-1} at 10 cm radius and $-3{\cdot}8 \times 10^3$ cm sec^{-1} at 5 cm radius.

3.2 Diffusion through the neutral gas

In this section we consider quasi static diffusion of the ions through the neutral gas. Neglect of the momentum term $n m v_r\, dv_r/dr$ is justified in view of the results of the previous section. Strictly the gas mixture consists of three components, ions, electrons and neutrals, but the electrons on their own can diffuse very rapidly in the absence of a magnetic field so it is sufficient to consider diffusion of the ions with the electron gas diffusing at the same rate and having a partial pressure equal to that of the ion gas. We assume for simplicity that both argon ions and atoms are hard spheres with a diameter of 3·1 Angstroms. The equation of diffusion is then (see Ref.13)

$$v = \frac{N^2}{n_i \rho} m_i D_{12} \text{ grad} \left(\frac{N - 2 n_i}{N}\right) \tag{28}$$

with

$$D_{12} = \frac{0{\cdot}00186 \, T^{3/2}}{p(m_i/2)^{\frac{1}{2}} r_o^2} \tag{29}$$

where N is the total number of particles per unit volume

ρ is the density

p is the pressure in atmospheres

r_o is the particle diameter in angstroms

In equations (28) and (29) masses of the order of m_e are neglected by comparison with m_i, so that since N is a constant on our assumption of equal temperatures, and since $n_i = n_e$ it follows that $\rho = m_i(N - n_i)$. The velocity v can be eliminated between the continuity equation (14) and equation (28) to give a second order non linear equation for n_i,

$$\left.\begin{aligned} &\frac{d^2 n_i}{dr^2} + \frac{1}{r} \frac{d n_i}{dr} - \beta n_i^2 = 0 \\ \text{where} \qquad & \\ &\beta = \frac{\alpha(N - n_i)}{2 N D_{12}} . \end{aligned}\right\} \tag{30}$$

To a good approximation we can use the same boundary condition as before, that $n_i = n_o$ at $r = 0$, where $n_o = 0{\cdot}1$ N, and in addition $dn_i/dr = 0$ at $r = 0$. The equations (30) have been integrated numerically and the results are given in Table 1. It can be seen that whereas for the fully ionised gas the number density n_i increased towards the centre, it now reduces. This is because in the dynamical equation (26) the radial deceleration of the particles was only possible if a pressure gradient (2 kT dn/dr) developed, whereas in the quasi static equation (28) there is no pressure gradient and the neutral particles are pressed inwards to make up for any pressure loss due to recombination.

TABLE 1

Radius (cm)	n_i/n_o	Radial velocity (cm/sec)
1	1·0003	- 750
2	1·0011	-1510
3	1·0025	-2260
4	1·0044	-3020
5	1·0069	-3770
6	1·0099	-4530
7	1·0135	-5280
8	1·0177	-6040
9	1·0225	-6790
10	1·0278	-7550

To check on the assumption of equal ion and electron temperatures, we note that for a plasma of 5 cm radius the radial velocity is -3770 cm/sec which with an equipartition time of $1·7 \times 10^{-4}$ seconds leads to an equipartition distance of 0·64 cm. This is still considerably less than the estimated skin depth of 1·6 cm so that the electron and ion temperatures should be nearly equal. The fact that the margin is small in this case offers some possibility of control should it be desired to increase the electron temperature above that of the ions; this could be done in principle by reducing the gas pressure.

4 MAGNETIC FIELD DUE TO A SKEW SOLENOID

It is apparent from Sections 2 and 3 that the charged particles diffuse rapidly enough to give nearly uniform conditions over a considerable diameter, unless they are impeded by a magnetic field. A circular solenoid wrapped around a circular cylinder produces an axial magnetic field which has two disadvantages:

(i) it gives maximum impedance to inward diffusion,

(ii) if models are to be fired lengthways down the tube the direction of firing will be along the magnetic field, whereas in general the desired direction will be at some angle to the magnetic field.

One way of avoiding these disadvantages is to use a solenoid made up of elliptical coils so oriented that they fit closely round a tube of circular cross-section as in Fig.3. In this section we consider what field is produced by a long coil of this type.

Cylindrical coordinates (r,θ,z) are used and the planes of the ellipses are taken to make an angle ψ with planes normal to the z axis. The axis $\theta = 0$ is also the x axis. We represent the coil by a thin current sheet I of which the θ component I_θ is constant. The z component is given by

$$I_z = I_\theta \tan\psi \cos\theta \quad . \tag{31}$$

Since I and I_θ are current intensities, e.g. in amperes per metre if m.k.s.c. units are used, the field strength $H(I_\theta)$ is given by the usual value for a long solenoid

$$H(I_\theta) = I_\theta \tag{32}$$

where $H(I_\theta)$ is the field due to the component of current I_θ and is directed axially along the tube; it is a constant at all points within the solenoid. For the skew solenoid we have to superimpose the field due to the z component of current given by equation (31) and this field will clearly be normal to the z axis. Furthermore the current distribution (31) is symmetric about the x axis, so that we can say at once that the x component of the field is zero on the z axis.

Inside the coil where no current flows curl $\underline{H}$ and div $\underline{H}$ are both zero, so the magnetic field can be expressed in terms of a scalar potential Ω given by

$$H = -\operatorname{grad}\Omega \quad . \tag{33}$$

For a long straight wire carrying a current i the potential Ω is given by

$$\Omega = -\frac{i\varepsilon}{2\pi} \tag{34}$$

at a point whose coordinates are (r', ε) relative to the wire. We wish to find the potential and hence the field at a point (r,ϕ) within the coil and to do this we use rectangular coordinates such that the point (r,ϕ) is also described by (ξ,η) where $\xi = x/r_1$ and $\eta = y/r_1$ as in Fig.4. Then $\varepsilon = \pi + \beta$ and

$$\Omega = -\frac{I_\theta \tan\psi \cdot r_1}{2\pi}\int_0^{2\pi} d\theta \,.\, (\pi+\beta)\cos\theta$$

where from Fig.4, β is given by (35)

$$\beta = \tan^{-1}\left(\frac{\sin\theta-\eta}{\cos\theta-\xi}\right) \quad .$$

It follows that

$$\left.\begin{aligned} H_x &= -\frac{I_\theta \tan\psi}{2\pi}\int_0^{2\pi} d\theta\,\frac{(\eta-\sin\theta)\cos\theta}{(\sin\theta-\eta)^2+(\cos\theta-\xi)^2} \\ &\text{and} \\ H_y &= -\frac{I_\theta \tan\psi}{2\pi}\int_0^{2\pi} d\theta\,\frac{(\cos\theta-\xi)\cos\theta}{(\sin\theta-\eta)^2+(\cos\theta-\xi)^2}\ . \end{aligned}\right\} \qquad (36)$$

By using the relations $\cos\phi = \frac{\xi}{\sqrt{\xi^2+\eta^2}}$ and $\sin\phi = \frac{\eta}{\sqrt{\xi^2+\eta^2}}$ the denominator in the integrand of (36) can be written in terms of $\cos(\theta-\phi)$ and since the integration is over a complete period of 2π we can replace $\theta-\phi$ by θ throughout the integrands. The second half of the range is then covered by replacing θ by $2\pi-\theta$ with the results that

$$\left.\begin{aligned} H_x &= -\frac{I_\theta \tan\psi}{2\pi}\int_0^{\pi}\frac{d\theta}{D}\,(\sin 2\phi + 2\eta\cos\phi\cos\theta - 2\sin 2\phi\cos^2\theta) \\ H_y &= -\frac{I_\theta \tan\psi}{2\pi}\int_0^{\pi}\frac{d\theta}{D}\,(1-\cos 2\phi - 2\xi\cos\phi\cos\theta + 2\cos 2\phi\cos^2\theta) \end{aligned}\right\} \qquad (37)$$

where the numerators of the integrands are quadratics in $\cos\theta$ and the denominator D is linear, and given by

$$D = 1 + \xi^2 + \eta^2 - 2\sqrt{\xi^2+\eta^2}\cos\theta\ . \qquad (38)$$

We now substitute back in equations (37) for ϕ in terms of ξ and η, divide through by the denominator and carry out the integrations to find

$$\left.\begin{aligned} H_x &= 0 \\ H_y &= -\tfrac{1}{2} I_\theta \tan\psi\ . \end{aligned}\right\} \qquad (39)$$

The field inside this skew solenoid is thus constant and the magnitude is such that if the individual coils are wound in planes at 45° to the axis of the solenoid, the cross component of the field will be in the plane of the major axes of the ellipses and will have a magnitude of half the axial field.

This analysis shows that it is possible to design a simple coil that gives a constant magnetic field inclined to the axis of the coil and thus overcome both the objections (i) and (ii).

5 CONCLUSIONS

The calculations made in this paper suggest that a plasma can be generated by circumferentially induced R.F. currents with a cylindrical core that is largely free from current and of nearly uniform density provided:

(1) the R.F. has sufficiently high frequency

and (2) a strong axial magnetic field is not present.

For a plasma of practical size, one megacycle is about the lower limit of exciting frequency and existing equipment within Aerodynamics Department might be just acceptable. If a strong magnetic field is provided it would seem desirable to use a skew solenoid of the type discussed in Section 4 so as to give a constant angled magnetic field that should not unduly impede diffusion. In some circumstances it may be desirable to induce the R.F. currents in an axial direction, but for the frequencies considered this does not appear to be necessary and conductivity across the field should be satisfactory.

ACKNOWLEDGEMENT

The author wishes to acknowledge his indebtedness to Messrs. D.M.Gilbey and A.H.Mitchell for many helpful discussions.

LIST OF PRINCIPAL SYMBOLS

A a constant defined under equation (19)

$\underline{B}$ magnetic induction vector = $\mu\underline{H}$

D_{12} diffusion coefficient defined by equation (29)

$\underline{E}$ electric vector

I current density in amperes per metre

N total number density of particles per cubic centimetre

T absolute temperature

LIST OF PRINCIPAL SYMBOLS (Contd)

d	skin depth
e	charge on an ion
k	Boltzmann's constant
m	particle mass
n	number density per cubic centimetre
r	distance from centreline
v	particle velocity
x,y,z	rectilinear coordinates
α	recombination coefficient
β,θ,ϕ	angles defined by Fig.4
η	specific electrical resistance*
μ	permeability
ν	collision frequency
σ	specific electrical conductivity
ψ	angle of inclination of coils to z axis (Fig.3)
ω	frequency

Suffixes

e	electron
i	ion
n	neutral
r	radius
o	centreline value
1	value at inner skin

*ξ,η are also used in Section 4 as non dimensional coordinates x/r_1 and y/r_1 respectively

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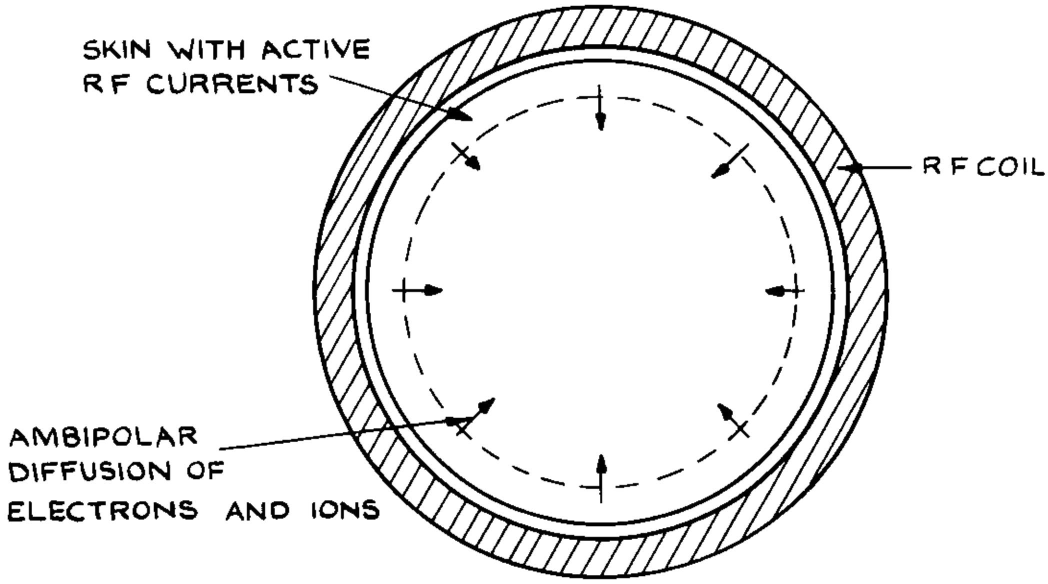

FIG 1. CROSS SECTION OF R.F. PLASMA FIG. 1.a WITH R F COIL

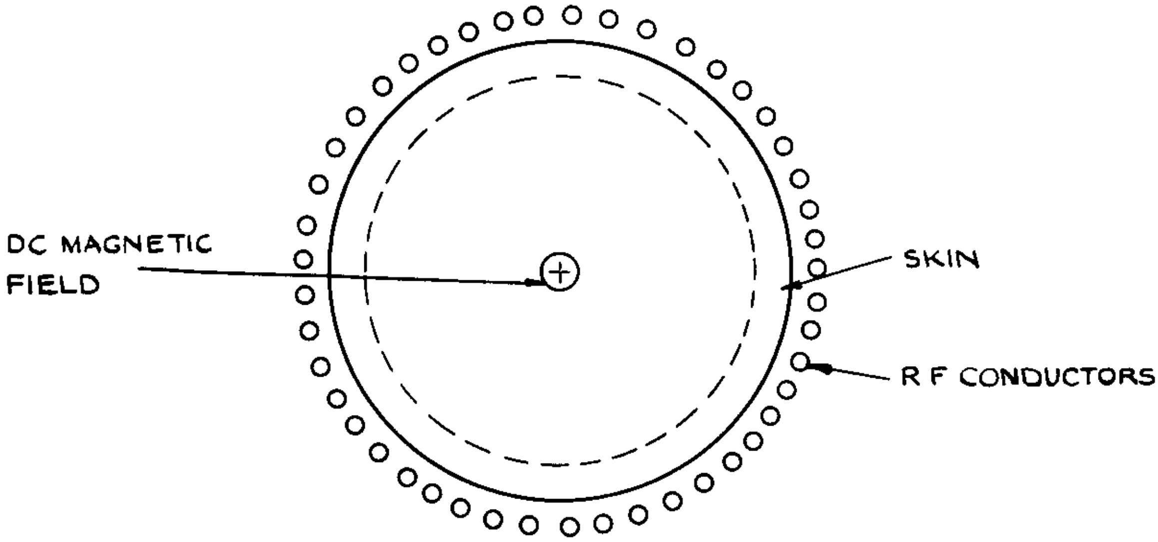

FIG. 1.b. WITH LONGITUDINAL R.F. CURRENTS.

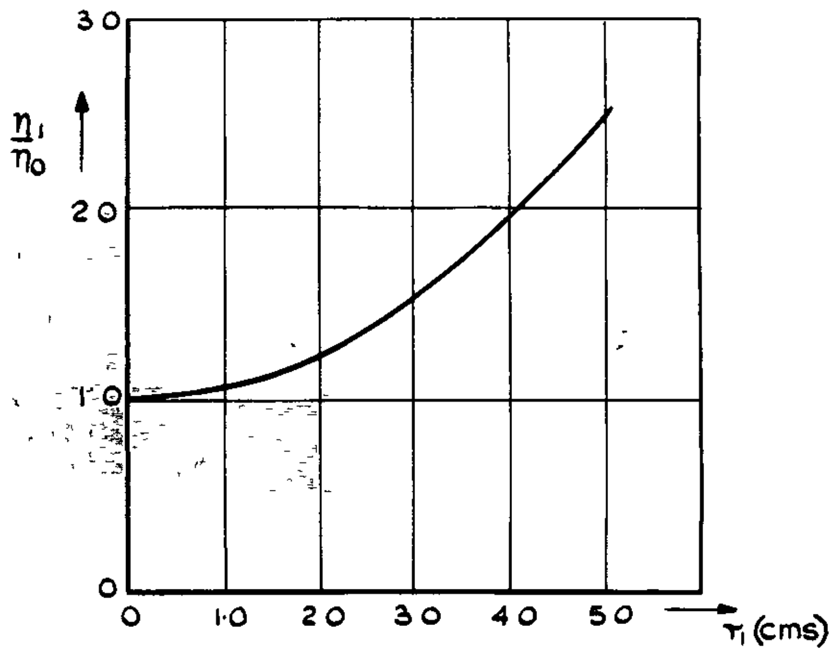

FIG. 2. DENSITY DISTRIBUTION IN PRESENCE OF AXIAL MAGNETIC FIELD.

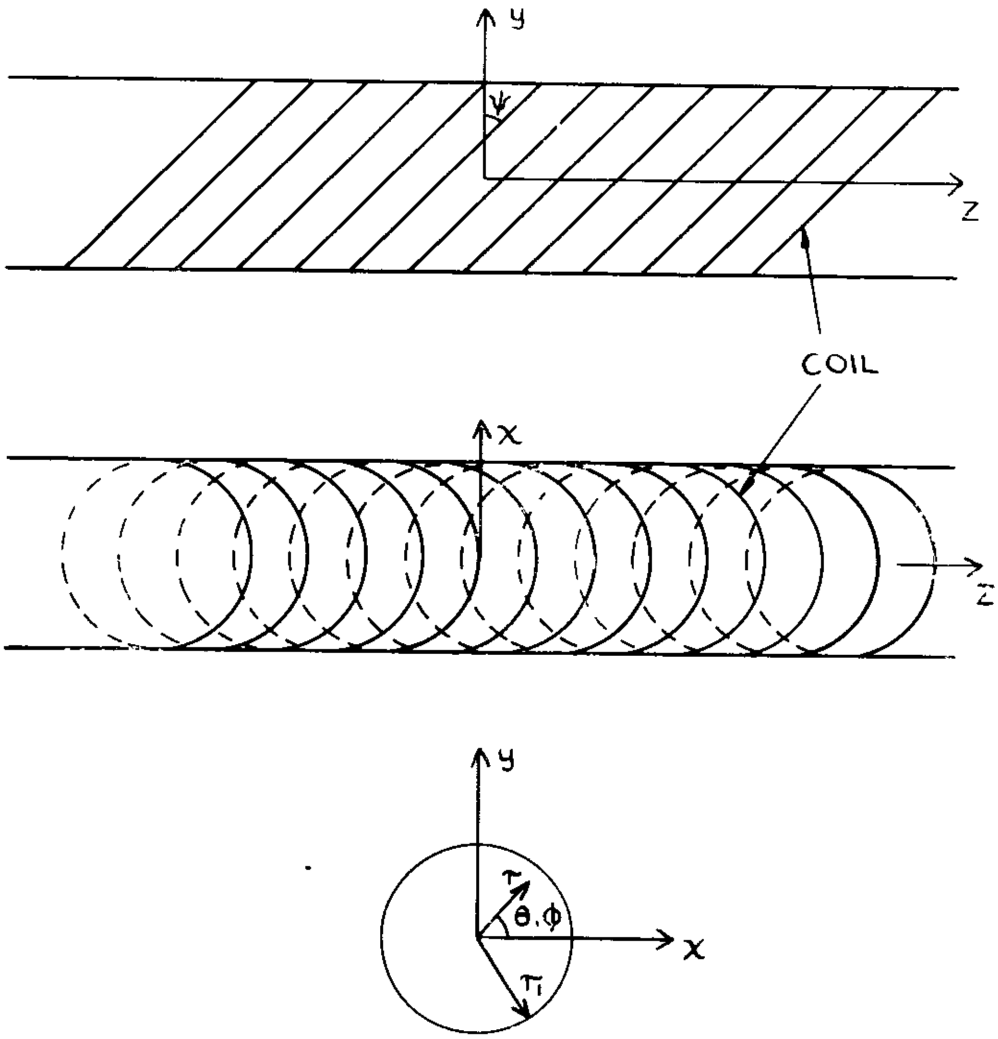

FIG. 3. SKEW SOLENOID

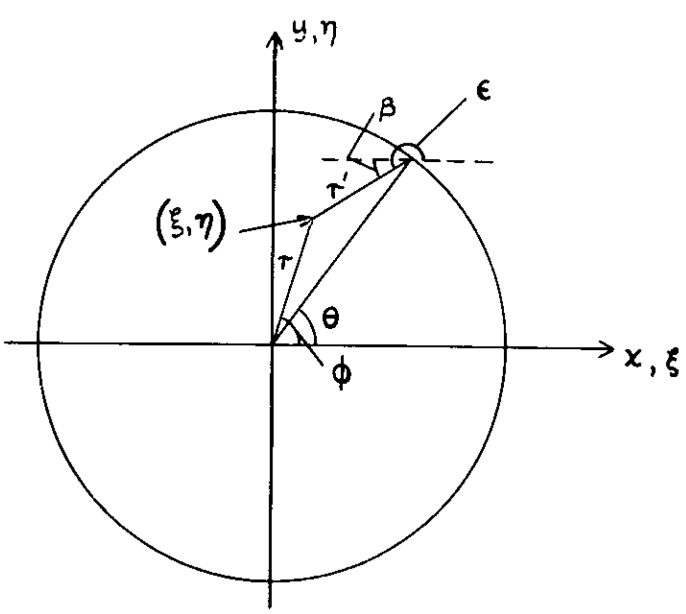

FIG. 4 GEOMETRY FOR THE FIELD AT THE POINT (ξ, η)

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