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Power Spectra of Low Level Atmospheric Turbulence Measured from an Aircraft

by

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POWER SPECTRA OF LOW LEVEL ATMOSPHERIC TURBULENCE MEASURED FROM AN AIRCRAFT

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SUMMARY

The note presents 30 power spectra of the vertical component of atmospheric turbulence and 12 power spectra of the horizontal component measured on a Canberra aircraft while flying at low altitude. The spectra relate to various types of terrain in U.K. and N. Africa. Comparative spectra are given for different heights above ground within the height band 200 to 1000 ft. Some information is given on the associated meteorological conditions.

The results are discussed briefly with a view to generalising the basic shape of the spectra for the purpose of predicting fatigue loads. Comparison is made with proposed analytical expressions defining the basic shape.

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1 INTRODUCTION

The note presents power spectra of the vertical component of atmospheric turbulence obtained from measurements made on an aircraft while flying in the lowest 1000 ft of the atmosphere*. In some cases power spectra of the horizontal component of turbulence in the direction of flight are included. The range of wavelength covered extends from 50 to 8000 ft.

The spectra, which were obtained during an experimental investigation into power spectral methods of determining gust loads in aircraft, were not intended to form a comprehensive set of data on their own, but rather to supplement more extensive work going on in this field[2,3]. They do, however, indicate some interesting trends - particularly with regard to the variation of spectral shape with height. They are also of interest in as much as they provide a direct comparison of atmospheric turbulence in temperate and sub-tropical conditions in U.K. and N. Africa respectively.

In order to use power spectral methods for determining gust loads it is first necessary to specify the power spectrum of atmospheric turbulence in general terms, preferably in as simple a form as possible. With this purpose in mind consideration is given to the reduction of the experimental power spectra to a common form and an examination is made of the fit of curves derived from analytical expressions, some of which are in current use[4].

The note is confined to information on atmospheric turbulence. Information was also obtained on the resulting response of the aircraft in terms both of acceleration power spectra and of counts of acceleration levels exceeded. It is thus possible to study the relation between the turbulence input and the aircraft response expressed in various forms. It is proposed to treat this matter in a separate note.

2 OUTLINE OF METHOD

The aircraft used was a standard Canberra B6. The velocities of the atmospheric airflow were measured in flight by the "direct method" in which the velocity is determined from differences between the airflow past the aircraft and the aircraft motions relative to the ground. The velocity of airflow along the normal, fore-and-aft and lateral flight axes of the aircraft was measured by means of a nose-probe with pressure sensing orifices at its head (see Figs.1 and 2). Lateral measurements were not, however, analysed owing to suspected inaccuracies in the corrections associated with the Dutch roll. The motions of the aircraft were measured by free and rate gyros and by accelerometers, the signals from which were integrated numerically to give velocity. Further details of the instrumentation and method of measurement are given in Appendix 1.

3 FLIGHT CONDITIONS

The duration of sample chosen was $2\frac{1}{2}$ minutes (in a few cases it was cut to 2 minutes) which corresponded to a run of $14\frac{1}{2}$ statute miles for flight at

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*A physical description of the power spectrum of atmospheric turbulence and definitions of terms relating to its properties will be found in Ref.1.

300 kts E.A.S., the basic speed used for the trials. The measurements were made over set tracks in U.K. and N. Africa, the aircraft being flown at approximately constant height above ground. For comparative purposes the heights chosen were either 200, 400 and 600 ft above ground to correspond to "Operation Swifter" heights[5], or 200, 500 and 1000 ft corresponding to the heights of the turbulence measurements on the Cardington balloon cable[6]. In one flight the height was reduced to 100 ft over the flat desert. Further details of the method of measurement are given in Appendix 2. Details of the flight conditions are listed in Table 1.

3.1 Topography

The runs in U.K. were mostly made in the vicinity of Cardington over moderately flat agricultural land; the track avoided towns and villages but passed over occasional farms and trees. Runs made in Sussex covered the same type of agricultural terrain but the ground was considerably more rolling and hilly. Runs in E. Anglia were over flat fenland and farming country.

In N. Africa runs were made from El Adem either over flat desert or over hilly desert near the coast at Derna; the hilly desert was of rather an unusual formation consisting of a plateau sharply eroded by wadis: these wadis were mostly too narrow for the aircraft to follow their contour.

Runs were also made from Idris over flat cultivated desert. The N. African runs were over selected parts of the routes used in "Operation Swifter".

3.2 Meteorological conditions

Observations of surface wind-speed and direction, cloud type and amount, visibility, screen temperature and relative humidity, as made on an hourly basis for synoptic charts, were obtained for the hours closest to the time of flight from the nearest meteorological stations. These observations are listed in Table 2. Besides these observations, outside air temperature was recorded on the aircraft in continuous trace form throughout the runs. An extra "temperature run" at 1000 ft was included when not already part of the programme, so that an average lapse rate could be determined from the differences in temperature between adjacent runs at 200 ft and 1000 ft. The averages obtained are listed in Table 2.

4 DATA REDUCTION

It is not proposed to discuss this matter in detail. The original recordings, which were in continuous trace form, were digitised at discrete intervals of 1/20 sec. After combining and integrating the digitised information to give the required velocities of airflow, the power spectra were obtained as the Fourier transforms of the auto-correlation functions, the computation being carried out on DEUCE and Mercury. A number of processes were included to refine the accuracy of the spectral estimates; these included "pre-whitening" and "hanning"[7]. Forty estimates were obtained with a resolution of 0.25 c.p.s. The analysis was then repeated with the digitised readings added in non-overlapping groups of four to obtain more refined data at the longer wavelengths. The resolution was then 0.0625 c.p.s.

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4.1 Accuracy of method

The accuracy of the method is discussed in Appendix 3. For wavelengths less than 2500 ft the 90% confidence limits determined on the lines suggested by Tukey[7] lie 26% to 29% either side of the estimated power spectra. At longer wavelengths the accuracy deteriorates.

Check comparisons at different aircraft airspeeds showed that the power spectra did not vary significantly with aircraft speed; this indicates that the motions of the aircraft, which have different characterisitics at different speeds, had been successfully eliminated. Checks were also made that the response of the aircraft to the movement of the controls by the pilot was not affecting the spectra. A check comparison made between spectra obtained from concurrent measurements on the aircraft and on a captive balloon cable showed reasonable agreement[6]. These checks are discussed more fully in Appendix 3.

5 RESULTS

5.1 Presentation of results

Estimates for the individual power spectra of the vertical and horizontal components of turbulence are plotted in Figs.3 to 11. Log-log paper is used in conformity with usual aeronautical practice. The estimates show a certain amount of scatter and faired curves have been drawn for comparative purposes. Comparative spectra for different heights, terrain and wind direction are shown in Figs.12 to 19. Vertical and horizontal components of turbulence are compared in Figs.20 to 23. Spectra for the aircraft flown "hands on" and "stick free" are compared in Fig.24 and spectra for different airspeeds in Fig.25.

Characteristics of the turbulence obtainable from the power spectra such as the root mean square velocity, the scale parameter of the turbulence and the index of the frequency with which the power spectral density decreases at short wavelengths are listed in Table 3. The r.m.s. values are obtained by integrating the power spectra over the waveband 10,000 to 10 ft. Some extrapolation has been necessary to cover this waveband, which has been proposed as a standard for the evaluation of r.m.s. velocities of turbulence. The extrapolation has involved only small additions of the order of 3% to the r.m.s. values for the waveband covered by the measurements. The scale parameter of the turbulence ℓ is determined from the formula $\ell = \bar{u}/2\pi \, n_m$, where $\bar{u}$ is the mean airspeed and n_m is the frequency at which the product of the frequency and the power spectral density is a maximum[18]. In physical terms the scale parameter as defined here is the most common wavelength or eddy size divided by 2π. Faired curves were used both in the determination of ℓ and of the frequency index p with which the spectral density decreases at short wavelengths; original spectral density estimates were used in the determination of the r.m.s. values.

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*In a number of cases, particularly when the spectra refer to horizontal turbulence, no maximum occurs within the waveband covered by the spectrum, implying that the scale parameter exceeds $8000/2\pi \simeq 1300$ ft.

In the discussion which follows it is sometimes convenient to treat the ordinate of the power spectra as defining the energy content of the turbulence. The reason underlying this usage is that the ordinate is proportional to the square of the velocity and hence to the energy if the density is fixed.

5.2 Normalised spectra

5.2.1 Variation of basic shape with height

In order to study the basic shape of the spectra with a view to generalisation, the spectra have been normalised by reducing them to a common r.m.s. value of 1 ft/sec. The spectra are classified in 3 classes of height:- 200 ft, 500 ft (this includes spectra obtained at 400 and 600 ft) and 1000 ft. Considering first the vertical component (Fig.26) it is immediately apparent that there is a change in basic shape between 200 ft and 1000 ft, the spectra relating to 500 ft lying in between and to some extent overlapping the other two classes. At long wavelengths the spectral density decreases with decreasing height and at short wavelengths it increases, the crossover occurring at a wavelength of about 1500 ft. In general the effect of such a shift of energy from long to short wavelength would be to increase the number of gust loads predicted by the power spectral method, the exact amount depending on the frequency characteristics of the aircraft. Rough calculations indicate that the amount is likely to be significant and that a generalised form of power spectrum which varies with height is desirable.

The effect of height on the horizontal component of turbulence in the direction of flight is similar to that on the vertical component but much less marked (Fig.27). The need here for a generalised form of spectrum which takes account of height is less apparent.

5.2.2 Comparison of basic shape of U.K. and N. African spectra

At any one height the normalised spectra of the vertical component of turbulence are remarkably alike despite the variation in topographical and meteorological conditions under which they were obtained (see Fig.26). The agreement is particularly close for the U.K. spectra at 1000 ft (spectra were not obtained at 1000 ft in N. Africa). At 500 ft slight differences can be detected between U.K. and N. African spectra: the latter are just starting to flatten at long wavelengths in a manner similar to spectra at 200 ft. (The only N. African spectra of this class which does not show this flattening relates to 600 ft.) At 200 ft the effect of terrain and its interaction with the meteorological environment is becoming evident and the spectra are more variable in shape particularly at long wavelengths. Within this variation it is impossible to distinguish between U.K. and N. African spectra.

5.2.3 Comparison with analytical expressions

Figs. 28 to 30 show spectra of the vertical component of turbulence derived from two analytical expressions compared with the experimental results. The band covered by the normalised experimental spectra is shown hatched. The first analytical expression which is in current use is[9]

$$G(\Omega) \;=\; \frac{\sigma_w^2 \, L \, (1 + 3\,\Omega^2\,L^2)}{\pi(1 + \Omega^2\,L^2)^2} \tag{1}$$

where L the scale length is taken equal to the height above ground, Ω is the angular wave number $(= 2\pi/\lambda$ where λ is the wavelength), and the expression is normalised to give a r.m.s. value of 1 ft/sec over the same waveband as the experimental results.

The second analytical expression

$$G(\Omega) \;=\; \frac{2.32 \; \sigma_w^2 \, L \, \tanh \Omega L}{(\Omega L)^{5/3}} \tag{2}$$

is rewritten here in a form slightly different from the original suggested by Henry[10]. Scale lengths of 620, 540 and 160 ft have been chosen to give a good fit at heights of 1000, 500 and 200 ft respectively, and the expression is normalised as above.

It is apparent that the sharply defined "knee" of expression (1) is not in general justified by experimental results although it can provide a good fit in a few cases; nor is there any justification for the frequency index of -2 with which the expression decreases at short wavelengths. Average values of the slopes defining the frequency index of the experimental spectra are:-

Ht above ground	Average slope	Range	Standard deviation
1000 ft	-1.65	-1.53 to -1.80	0.16
500 ft	-1.68	-1.27 to -1.92	0.23
200 ft	-1.58	-1.20 to -2.03	0.32

These values are much nearer to the index of -5/3 of Kolmogoroff's similarity theory than to an index of -2.

Expression (2) gives a much better fit than expression (1) both because the curvature is more gradual and because it conforms with a frequency index of -5/3. This expression may not prove altogether satisfactory though, since it gives increasingly large values of spectral density at long wavelengths contrary to expectation. Further experimental observations of spectral density at longer wavelengths are required to resolve this matter.

Houbolt, Steiner and Pratt[11] have suggested a modified version of expression (1) such that the spectral density varies with a frequency index of -5/3 at short wavelengths. At long wavelengths the expression is, however, still very like expression (1) with the same rather pronounced "knee", which does not in general fit the experimental results.

It is not considered worthwhile to discuss the fitting of analytical curves to the power spectra of the horizontal component of turbulence, since these spectra obey a simple frequency law throughout the waveband observed. The result is that any analytical curve, which decreases with an appropriate frequency index at short wavelengths, can be made to fit if the scale of turbulence is chosen long enough.

5.3 Unnormalised spectra

So far only normalised spectra have been considered; a more complete picture is obtained if the spectra are considered in their unnormalised form which contains information on the intensity or r.m.s. velocity of the turbulence as well as on the basic shape.

5.3.1 Comparison of vertical and horizontal turbulence

The relation between the power spectra of the vertical and of the horizontal component of turbulence is strongly dependent on height above ground. Figs.20 to 22 show a remarkably similar pattern in the effect of height on this relation. At a height of 1000 ft the vertical spectra lie above the horizontal; at 500 ft they tend to come together at long wavelengths, and at 200 ft and below they cross over. Since the r.m.s. velocities are determined mainly by the long wavelength energy this effect is reflected in a decrease in the ratio of s_w to s_u with decreasing height, where s_w and s_u are respectively the r.m.s. velocities of the vertical and horizontal components of turbulence. This ratio changes from a value greater than 1 at 1000 ft to less than 1 at 200 ft (see Fig.31).

In general, the scale parameter of the horizontal component of turbulence is greater than that of the vertical component. This is in agreement with earlier experimental results[8]. Quantitative comparison of the scale parameters is not possible since that of the horizontal component is too large to be determined from the experimental observations. For turbulence over flat and slightly hilly terrain the scale parameter of the vertical component approximates to the height above ground, except for some one-third of the spectra when it is too large to be determinate (see Fig.32). There is no apparent explanation for these large scales in terms of the environmental parameters: the influence of convective activity on the scale of turbulence is, however, suspected to be complex[12]. Over hilly desert the scale of the vertical component is somewhat larger than over flat or slightly hilly terrain (see Fig.32).

5.3.2 Effect of environmental parameters

The emphasis so far has been on the similarity of spectra at any one height. At 200 ft, however, some variation due to environmental parameters is becoming apparent and the effect at this height of certain parameters on the unnormalised spectra is now discussed. It should be noted that the cases considered are few in number and results are therefore only indicative.

(i)   Terrain roughness

Fig.17 shows comparative spectra of the vertical component of
turbulence for hilly desert and flat desert. As might be expected the hilly
desert spectrum is the more severe with a r.m.s. velocity of 3.8 ft/sec compared
with 2.2 ft/sec for the flat desert. The increase in spectral density occurs at
all wavelengths but is most marked at long wavelengths. Other spectra obtained
over the hilly desert also show high values of spectral density at long wave-
lengths (see Fig.13).

(ii)   Wind direction relative to flight path

Comparative spectra from runs made up- and downwind in E. Anglia, surface
wind 17 kts, and in Sussex, surface wind 11 kts, show little difference due to
wind direction (Figs.18 and 19). The vertical spectra would have lain even
closer together if ground speed instead of airspeed had been used in converting
from time to space media. This suggests that the frequency characteristics of
the vertical component of turbulence are influenced by the spacing of topo-
graphical features. Differences are too small and data too few, however, for
this result to be conclusive.

Crosswind spectra on the other hand show a marked difference from up- and
downwind spectra (see Fig.19 - results for E. Anglia only since low flying
restrictions prevented crosswind runs in Sussex). The energy at long wave-
lengths is considerably less for the crosswind spectra. It would be unwise,
however, to attribute this to the effect of wind direction without further
evidence; the runs were of necessity made over different tracks and, although
the terrain was chosen to be as homogeneous as possible, this could have
influenced the results.

6   CONCLUSIONS

Power spectra of atmospheric turbulence have been obtained from aircraft
measurements in the height band 200 to 1000 ft above ground, over various types
of terrain in U.K. and N. Africa, and in conditions of neutral to moderate
atmospheric instability; a study of these spectra leads to the following
conclusions:-

6.1   Normalised spectra

(i)   The basic shape of the power spectra of the vertical component of
turbulence varies with height above ground, the proportion of long wave
(> 1500 ft) to short wave energy decreasing with decreasing height. The
horizontal component shows the same trend but to a much less marked degree.

(ii)   At any one height there is considerable similarity in the basic
shape of the spectra despite the variation of topographical and meteorological
conditions. Generalisation of the shape of the spectrum in a form which takes
account of height appears practical. Variation in the basic shape of the
spectrum of the vertical component, due to the effect of environmental parameters,
is becoming more apparent, however, as the height decreases to 200 ft.

two methods was well within the confidence limits. This is not a conclusive check, however, since some differences, especially at long wavelengths, might have been expected; small differences were in fact indicated such as have been observed by earlier experimenters when comparing fixed point spectra with spectra obtained travelling through the medium[13].

INSTRUMENTAL AND STATISTICAL ACCURACY

The above exercises provide a rough overall check of the accuracy of the method. Particular sources of inaccuracy are now discussed in rather more detail.

Inaccuracies of two kinds are likely to occur: inaccuracies in the measurements due to transducer, recorder and trace-reader limitations and statistical inaccuracies due to the finite duration of the sample. Both kinds of inaccuracy tend to be greatest at the low frequency end of the spectra. If wavelengths longer than 2500 ft are excluded then the accuracy of measurement is probably well within 10%, while the 90% confidence limits determined on the lines suggested by Tukey lie 26% to 29% either side of the estimated power spectra.

At longer wavelengths it is difficult to assess the accuracy. Instrument errors arise through gyro drift and small variations in the 1g datum level of the accelerometer which accumulate when integrated. Another source of error is that the sample length is somewhat short to justify the extension of the analysis to such long wavelengths. Nevertheless the results at long wavelengths appear to indicate trends in a region which is becoming of increasing interest as aircraft speeds increase, and they have therefore been included although quantitatively they should be treated with reserve.

LIST OF SYMBOLS

G( )		generalised function for expressing power spectra of turbulence
L	ft	scale of turbulence used in analytical representation
ℓ	ft	scale parameter derived from spectral estimates, $= \bar{u}/2\pi \, n_m$
n	c.p.s.	frequency
n_m	c.p.s.	frequency at which the product of the frequency and the power spectral density is a maximum
p		frequency index with which power spectral density decreases in sub-inertial range
u, w		suffices denoting fore-and-aft and vertical components of turbulence
$\bar{u}$	ft/sec	mean airspeed of aircraft
s	ft/sec	root mean square velocity of turbulence obtained from spectral estimates for waveband 10,000 ft to 10 ft
Ω	rad/ft	angular wave number $= 2\pi/\lambda$
λ	ft	wavelength
σ	ft/sec	root mean square velocity of turbulence over waveband ∞ to 0, used in analytical representation.

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APPENDIX 1

INSTRUMENTATION AND METHOD OF MEASUREMENT

INSTRUMENTATION

The velocity of the airflow relative to the aircraft was measured by means of a nose-probe mounted on a 10 ft boom. The head of the probe consisted of a steel hemisphere with five pressure sensing orifices, the pressures from which, together with the pressure from a static ring, activated inductive differential pressure transducers, S.E. Laboratory type 70, fitted behind the orifices (see Fig.2). The pressure transducers were kept as close as possible to the orifices in order to minimise organ pipe resonances in the connecting tubes.

The accelerations of the aircraft at a point near the aircraft c.g. were measured with a Structures type 4 accelerometer sensitive to normal acceleration, and with a specially designed combined accelerometer sensitive to lateral and fore-and-aft accelerations. Lateral and fore-and-aft accelerations were, however, found to be small and were not included in the analysis; nor were they recorded except during the early stages of the experiment. A standard type IT 3-7-15, 10°/sec rate gyro was used to measure rate of pitch and yaw and a type IT 1-5-8 Pitch and Roll Indicator (free gyro) to measure angle of pitch. This gyro was modified so that the gravity erection could be switched off prior to recording since it was thought that the erection device might feed in spurious signals in turbulent conditions. Initially a second gyro of special design, with no pendulum erection, was used to check the IT 1-5-8 gyro.

Standard methods of amplification and recording were used. Initially the pressure transducers were used in conjunction with S.E. Laboratory modulators but later they were used with Films and Equipment amplifiers. Signals were recorded in continuous trace form on a Films and Equipment recorder and on a Beaudouin A13 recorder.

METHOD OF MEASUREMENT

Measurement of vertical turbulence

In deducing the vertical velocity of the turbulent airflow account was taken of the vertical translation of the aircraft and of the angle of pitch and its rate of change. Means were provided for correcting both for deflection of the probe boom under 'g' and for oscillatory flexure of the boom; in practice these corrections were found to be unnecessary, the probe proving very stiff in flexure, (natural still air frequency 36 c.p.s.). Rate of pitch was neglected in the final analysis since excluding this quantity made no significant difference.

Measurement of horizontal turbulence

Although it was originally intended to obtain the power spectra of the component of turbulence in the lateral direction, this undertaking was abandoned because of the prevalence of the Dutch roll. The yawing oscillations in this mode, which occurred at a frequency of about 1 cycle every $3\frac{1}{2}$ seconds at an airspeed of 300 kts E.A.S., produced such large differential pressures

at the probe tip that they dwarfed the required differential pressures from the lateral turbulence. Although in theory a correction could have been made for this effect, in practice it would have implied obtaining the velocity of the turbulent airflow as a small difference of large quantities and the method was considered invalid. Power spectra were obtained, however, for the horizontal component of turbulence along the fore-and-aft axis of the aircraft by means of a sensitive pitot-static arrangement feeding into an inductive pressure transducer. The large signal due to the steady airspeed was balanced out electrically and only changes in airspeed recorded. It was assumed that all changes were due to variation in the horizontal airflow and that the aircraft ground speed remained sensibly constant. Because of the inertia of the aircraft this assumption is justified at all but very low frequencies such as that of the phugoid motion. At this frequency, which is of the order of 1 cycle every 50 seconds at 300 kts E.A.S., sea level, there is likely to be considerable error; the corresponding wavelength of 25,000 ft, however, lies well outside the waveband of 8000 ft to 50 ft considered in this note. Special steps were taken in the analysis to minimise the spurious diffusion of energy from very long wavelengths into the waveband of interest.

APPENDIX 2

FLIGHT CONDITIONS

Measurements were taken during each run while the aircraft was flown on a constant heading and at as near a constant airspeed as possible. Height above ground was maintained approximately constant with the aid of the radio altimeter: in cases of hilly or undulating terrain this involved some manoeuvring in order to follow the contours of the ground. No special instructions were given to the pilot as regards control movements except for a few runs over moderately flat terrain when he was asked to remove his hands and feet from the controls altogether. It was found that, provided the aircraft was carefully trimmed beforehand, it could be flown hands and feet off quite successfully for the entire run with only very occasional touches on the stick to steady any phugoid motion which developed.

When comparative spectra were obtained at different heights the runs were repeated in the same direction over the same track in quick succession to minimise variation in meteorological conditions. Three such runs could be made in about 20 minutes. Comparative runs upwind, downwind and crosswind were made as far as possible over homogeneous terrain; these runs could be carried out in somewhat quicker succession than those at different heights. The comparative runs for flat and hilly desert were made within 30 minutes of each other.

Although most runs were made at 300 kts E.A.S., on one occasion runs were repeated at airspeeds of 200, 300, 400 and 475 kts E.A.S. The duration in time of the sample was kept constant so that the runs at the higher speeds covered more terrain than those at lower speeds. The terrain was flat desert and appeared to be homogeneous so that the same turbulence spectrum was expected from all runs.

ACCURACY OF METHOD

Checks were made that the spectra of vertical turbulence did not vary significantly either with airspeed - to some extent this checks that the motions of the aircraft are being eliminated - or with the amount of control exercised by the pilot.

EFFECT OF VARYING AIRSPEED

Fig.25 shows comparative power spectra of the vertical component of turbulence obtained at different airspeeds. The agreement between spectra is on the whole quite good and within the confidence limits for wavelengths of less than 2500 ft. At larger wavelengths, where rather more discrepancy might be expected due to the finite lengths of the samples the run at 400 kts has given rather high values of spectral density compared with the other runs. This does not appear to be due to a systematic error with airspeed since the long wave-length values at 475 kts are in close agreement with those at 200 and 300 kts. Taken as a whole these results indicate that the motions of the aircraft are being successfully eliminated.

EFFECT OF PILOT'S CONTROL MOVEMENTS

In theory the response of the aircraft due to the pilot's movements of the controls should not affect the turbulence spectra. Because of possible inaccuracies, however, in the measurement of the aircraft motions and its analytical treatment, it was considered advisable to check that the pilot was not feeding energy into the spectra. Comparative spectra, relating to flight in which the aircraft was flown "hands on" and "stick free" are shown in Figs.24(a) and (b). There is little evidence to suggest that the pilot is contributing in a positive sense to the turbulence spectra; at 500 ft the "hands on" and "hands off" spectra are comparable, while at 1000 ft the "stick free" spectrum shows a slightly greater energy content at all wavelengths than the "hands on" spectrum. This may be due to incomplete elimination of the rather marked phugoid motion developed during the "stick free" run. Although the wave-length of this motion lies well outside the range of interest, some energy from this wavelength could, despite preventative processes in the analysis, have diffused into the waveband of interest.

In conclusion it appears that the pilot is not feeding in any significant energy to the power spectra of the turbulence; on the contrary by checking the phugoid motion it is probable that he is improving the accuracy of the turbulence measurements.

COMPARISON OF AIRCRAFT AND BALLOON CABLE MEASUREMENTS

When comparative measurements of atmospheric turbulence were made on a captive balloon cable at Cardington and on the Canberra aircraft flying past the cable, agreement between the power spectra of vertical turbulence obtained by the

two methods was well within the confidence limits. This is not a conclusive check, however, since some differences, especially at long wavelengths, might have been expected; small differences were in fact indicated such as have been observed by earlier experimenters when comparing fixed point spectra with spectra obtained travelling through the medium[13].

INSTRUMENTAL AND STATISTICAL ACCURACY

The above exercises provide a rough overall check of the accuracy of the method. Particular sources of inaccuracy are now discussed in rather more detail.

Inaccuracies of two kinds are likely to occur: inaccuracies in the measurements due to transducer, recorder and trace-reader limitations and statistical inaccuracies due to the finite duration of the sample. Both kinds of inaccuracy tend to be greatest at the low frequency end of the spectra. If wavelengths longer than 2500 ft are excluded then the accuracy of measurement is probably well within 10%, while the 90% confidence limits determined on the lines suggested by Tukey lie 26% to 29% either side of the estimated power spectra.

At longer wavelengths it is difficult to assess the accuracy. Instrument errors arise through gyro drift and small variations in the 1g datum level of the accelerometer which accumulate when integrated. Another source of error is that the sample length is somewhat short to justify the extension of the analysis to such long wavelengths. Nevertheless the results at long wavelengths appear to indicate trends in a region which is becoming of increasing interest as aircraft speeds increase, and they have therefore been included although quantitatively they should be treated with reserve.

TABLE 1 - Flight conditions

Date	Flight & Run No.	Location	Terrain	Height ft a.g.l.	Airspeed kts E.A.S.	Direction of run degrees	Surface wind		Lapse rate °C/1000 ft	Remarks
							Direction degrees	Speed kts		
25/5/60	64 02	El Aden	hilly desert	200	300	246	310	17	1.25-6.25	Comparative runs over flat and hilly desert
	05		Flat desert	200	300	276	360	15	2.5	
29/6/60	77 01	El Aden	Flat desert	200	200	276	350	16	2.5	Comparative runs at 4 airspeeds
	02			200	300	"	"	"	"	
	03			200	400	"	"	"	"	
	04			200	475	"	"	"	"	
27/7/60	90 02	Cardington	Flat farmland	500	300	246	220	05	3.2	Comparative runs at 3 heights
	03			1000	"	"	"	"	"	
	04			200	"	"	"	"	"	
30/8/60	94 01	El Aden	hilly desert	200	300	246	330	14	3.5	Comparative runs at 3 heights
	02			400	"	"	"	"	"	
	03			600	"	"	"	"	"	
21/9/60	99 02	Sussex	Rolling farmland	200	300	90	320	11	3.75	Upwind and downwind runs. Wind slightly across track.
	03			200	"	270	"	"	"	
12/10/60	106 01	Cardington	Flat farmland	1000	300	240	320	13	3.45	Runs at 3 heights into wind and crosswind. Into wind and crosswind runs not strictly comparable
	02			500	"	"	"	"	"	
	03			200	"	"	"	"	"	
	04			1000	"	305	"	"	"	
	05			500	"	"	"	"	"	
	06			200	"	"	"	"	"	
9/7/61	152 02	Idris	Cultivated desert	200	300	Not recorded	Not recorded	06	3.75	Comparative runs at 3 heights
	03			100	"				"	
	04			500	"				3.10	
14/8/61	160 01	Sussex	Rolling farmland	1000	300	270	270	12	Not recorded	Comparative runs 'hands on' and 'stick free'
	02			1000	"	90	"	"		
	03			500	"	270	"	"		
	04			500	"	90	"	"		
18/8/61	162 01	E. Anglia	Flat fenland	200	300	280	280	17	Not recorded	Direction of runs relative to wind varied
	02			200	"	100	"	"		
	03			200	"	010	"	"		

Vertical spectra only

Vertical and horizontal spectra except Flight 160

<div align="center">TABLE 2 – Meteorological observations</div>

Flight No.	Surface wind		Visibility n.m.	Screen temperature °C	Humidity %	Lapse rate °C/1000 ft	Cloud amount type and base	Nearest meteorological station
	Speed kts	Direction degrees						
64.02	15	330	11	24	62	1.25-6.25*	3/8 alto cu	Cyrene
05	13	360	11	28	56	2.5*	1/8 alto cu	El Adem
77	16	350	5½	30	30	2.8	Nil	El Adem
90	5	220	10	18	73	2.8	3/8 cu increasing to 6/8 cu 2800' 7/8 alto stratus 25,000'	Cardington
94	14	330	20	28	73	3.5	3/8 strato cu	El Adem
99	11	320	11	13	72	3.75	1/8 cu 1800' 1/8 alto cu 18,000'	Gatwick
106	13/14	320	8	11	54	3.45	2/8 cu increasing to 4/8 cu 3000' 1/8 strato cu 5000' 4/8 alto cu 25,000'	Cardington
152	6	Not recorded	Not recorded		Not recorded	3.75 decreasing to 3.1	Nil	Idris
160	12	270	22	17	63	Not recorded	6/8 cu 2500' 6/8 strato cu 22,000'	Gatwick
162	17	280	25	16	63	Not recorded	2/8 cu 3000' 7/8 strato cu 4500'	Coningsby & Waterbeach

*Spot readings of temperature only.

TABLE 3 - Power spectra characteristics

	Vertical component of turbulence			Horizontal component of turbulence			Ratio $\frac{s_w}{s_u}$	Flight conditions			
	s_w (ft/sec)	ℓ (ft)	p	s_u (ft/sec)	ℓ (ft)	p		Location	Height ft agl	Surface wind kts	Wind speed at height of run ft/sec
64.02	3.81	700	-2.03					El Adem	200	17	
05	2.19	70	-2.00						200	15	
77.02	3.13	200	-2.02					El Adem	200	16	
90.02	2.78	>1300	-1.92					Cardington	500	5	10.02
03	2.94	>1300	-1.77						1000	"	10.03
04	2.84	200	-1.83						200	"	10.02
94.01	4.31	450	-1.80					El Adem	200	13	
02	4.68	480	-1.76						400	"	
03	4.54	1000	-1.92						600	"	
99.02	2.97	80	-1.39					Sussex	200	11	
03	2.93	130	-1.38						200	"	
106.01	3.23	>1300	-1.48	2.70	>1300	-1.52	1.20	Cardington	1000	13/14	31.3
02	3.42	600	-1.59	2.78	"	"	1.23		500	"	29.8
03	3.31	200	-1.46	3.27	"	-1.34	0.99		200	"	27.4
04	3.86	950	-1.80	2.72	"	-1.68	1.46		1000	"	33.1
05	3.10	>1300	-1.55	2.87	"	-1.66	1.08		500	"	31.2
06	1.95	70	-1.20	2.55	"	-1.40	0.73		200	"	28.5
152.02	3.34	270	-1.59	2.59	"	-1.48	1.29	Idris	200	6	
03	2.64	95	-1.36	2.94	"	-1.53	0.90		100	"	
04	3.87	500	-1.76	2.91	"	-1.57	1.33		500	"	
160.02	2.81	>1300	-1.53					Sussex	1000	12	
04	2.80	400	-1.27						500	"	
162.01	2.30	>1300	-1.24	3.34	"	-1.45	0.67	E. Anglia	200	17	
02	2.81	>1300	-1.21	3.19	"	-1.42	0.88		"	"	
03	2.01	320	-1.40	2.78	"	-1.38	0.72		"	"	
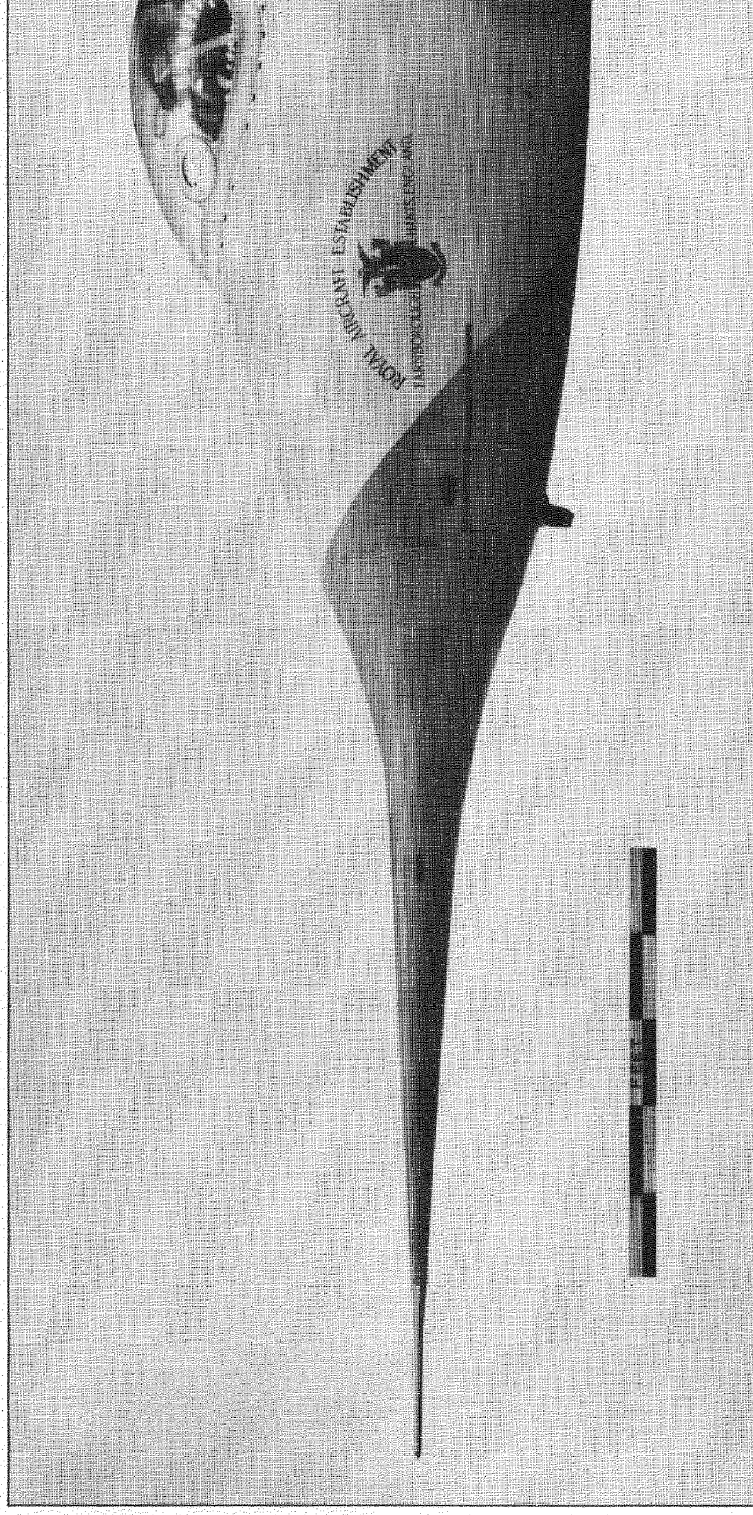
FIG.1. NOSE-PROBE AND BOOM MOUNTED ON CANBERRA AIRCRAFT

ASSEMBLED PROBE

OUTER BODY

INSTRUMENT
SLEEVE

3 INCHES

END CAP

AND PLUG
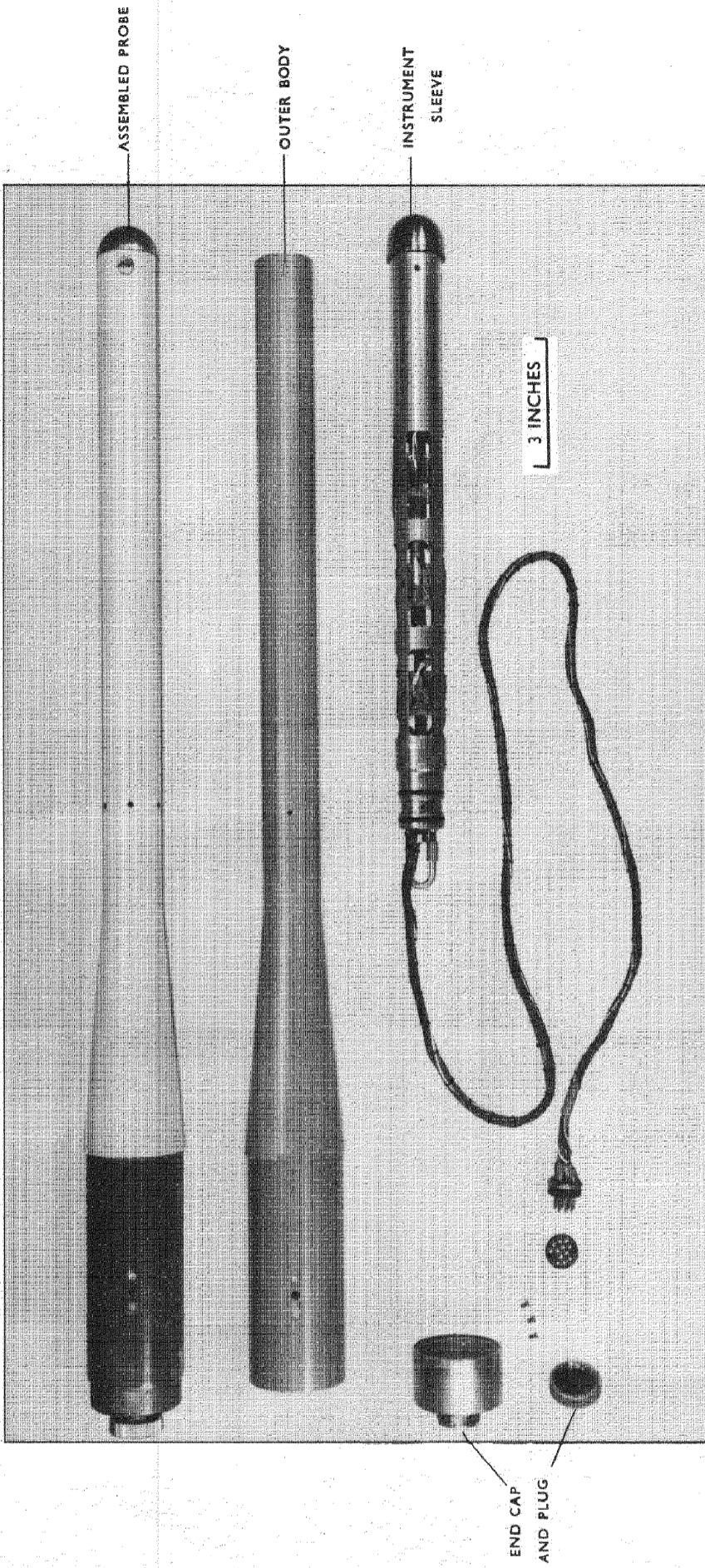
FIG.2. DETAILS OF NOSE-PROBE
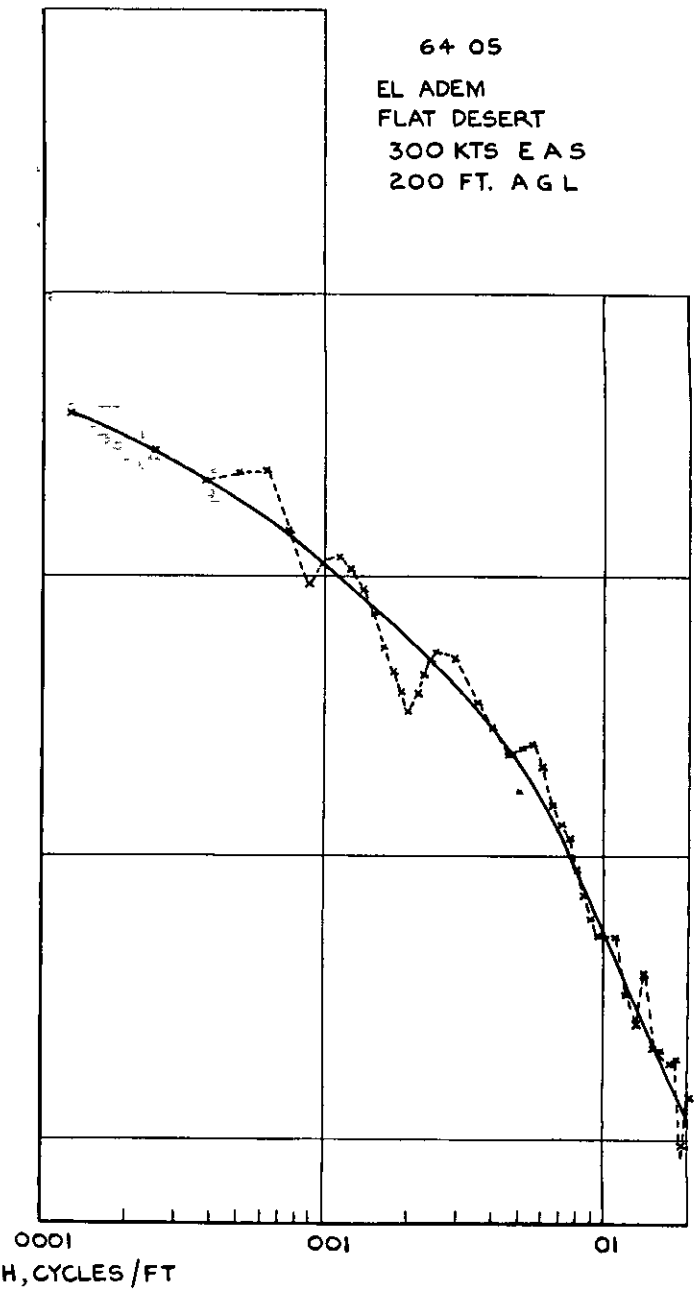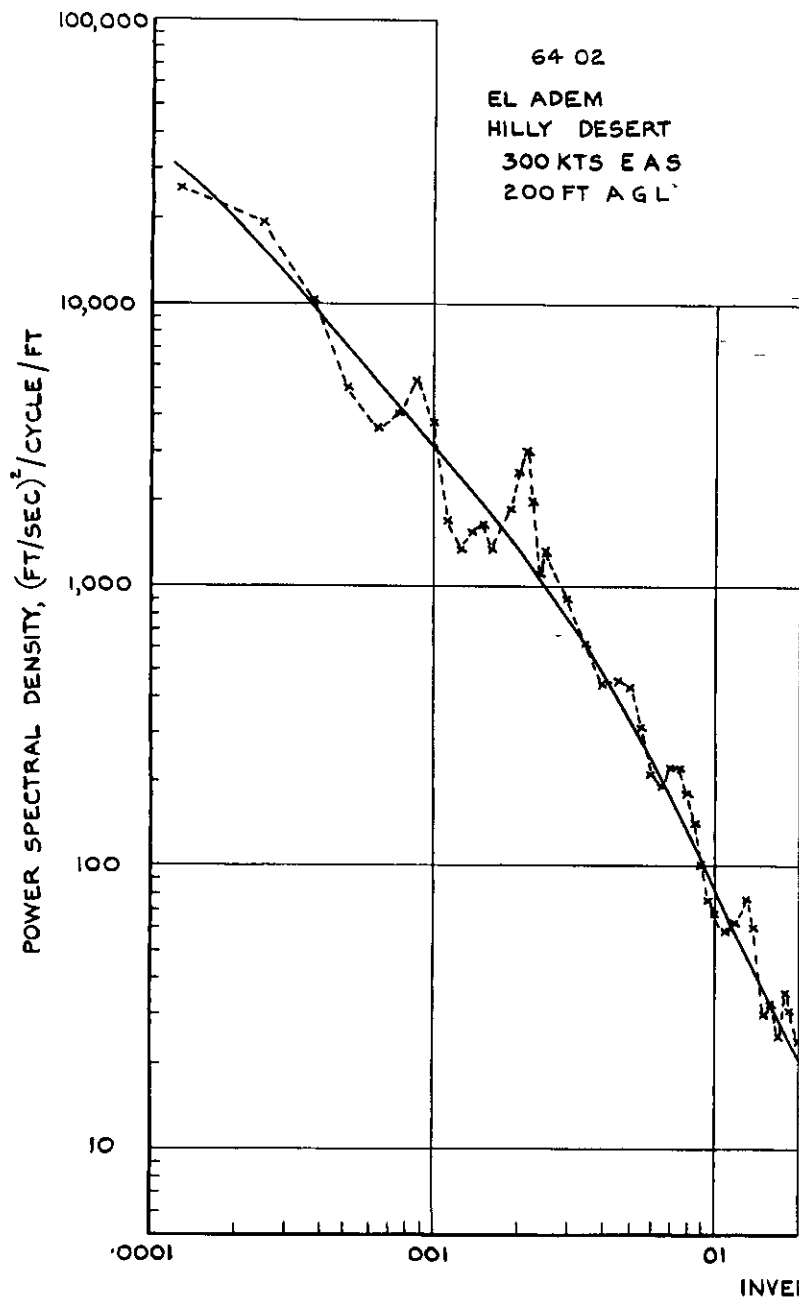
FIG. 3. POWER SPECTRA — FLIGHT 64 — VERTICAL TURBULENCE VELOCITY.
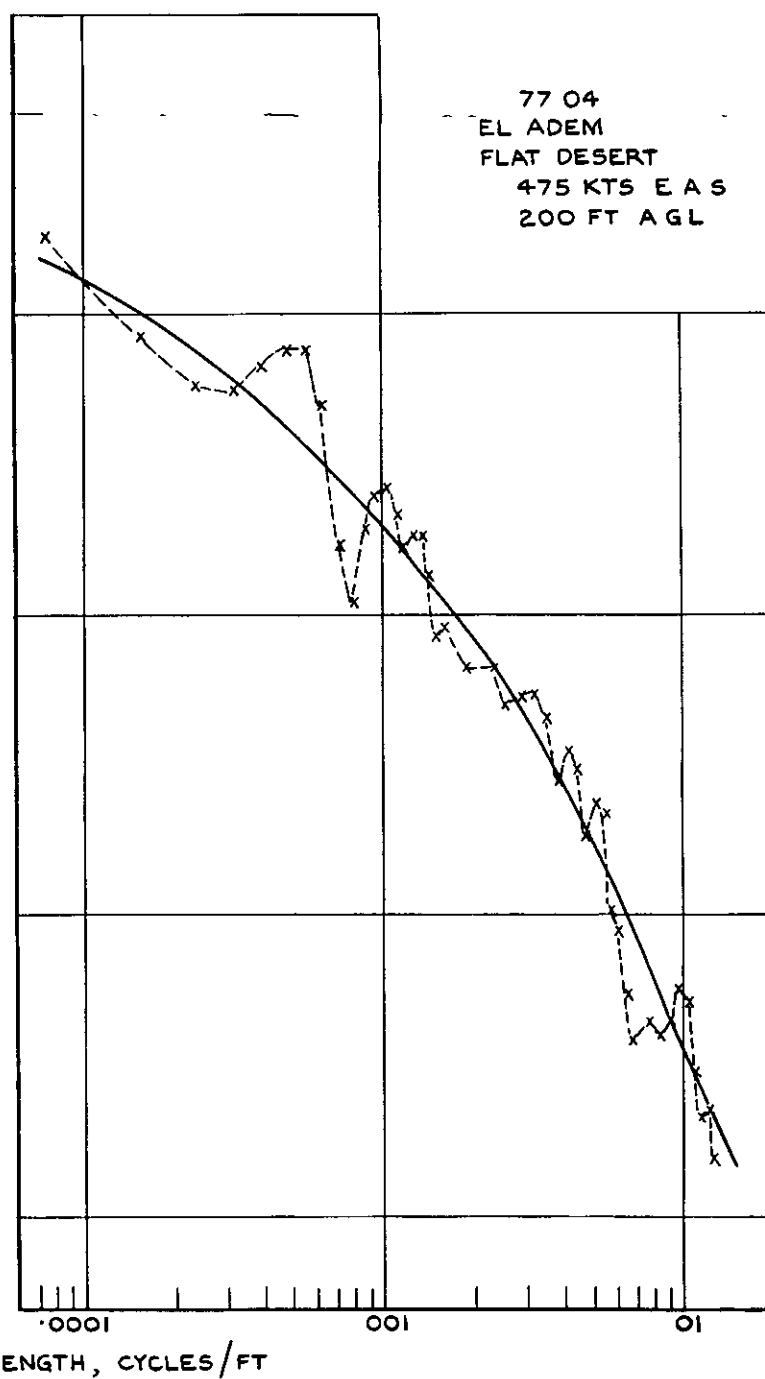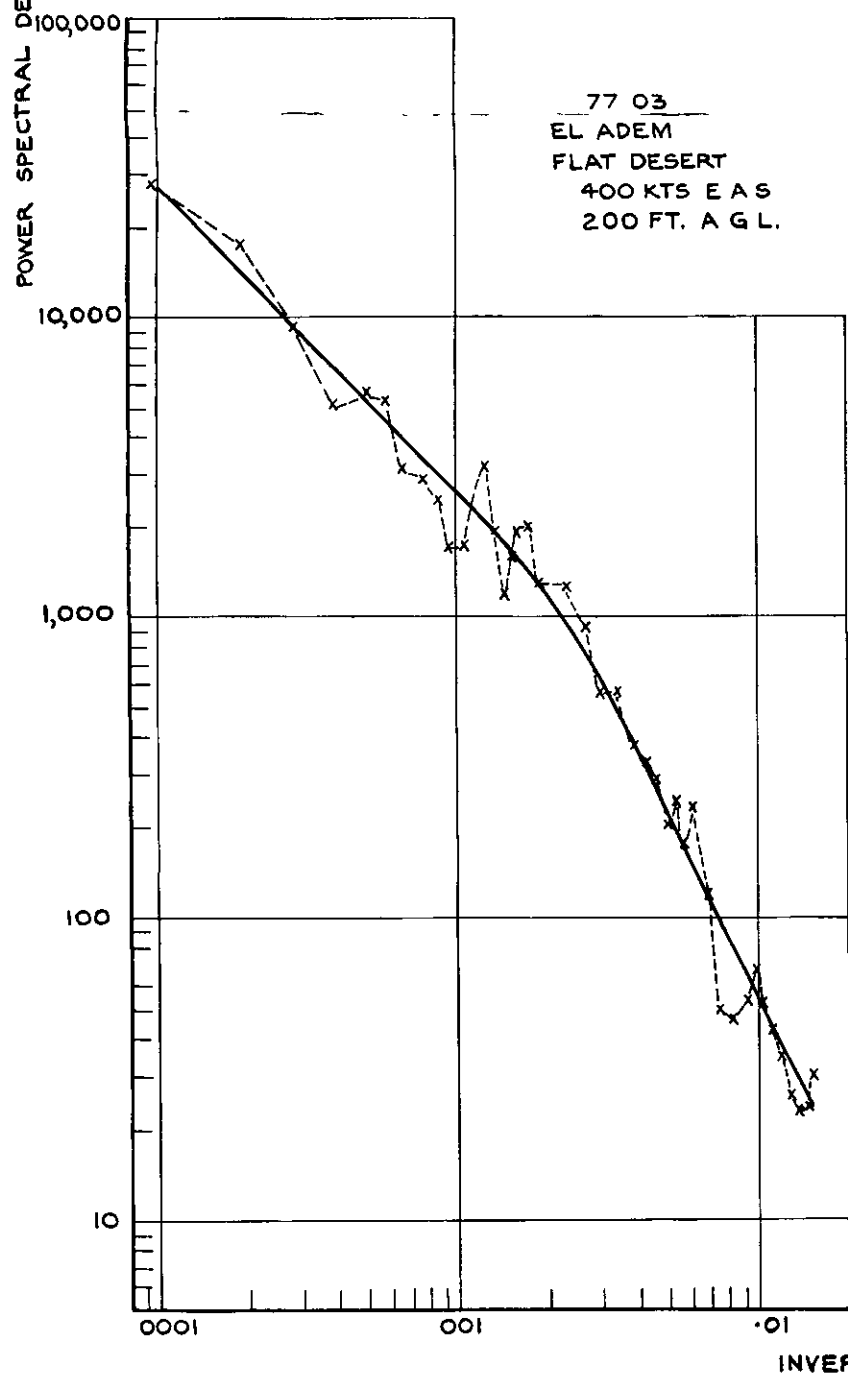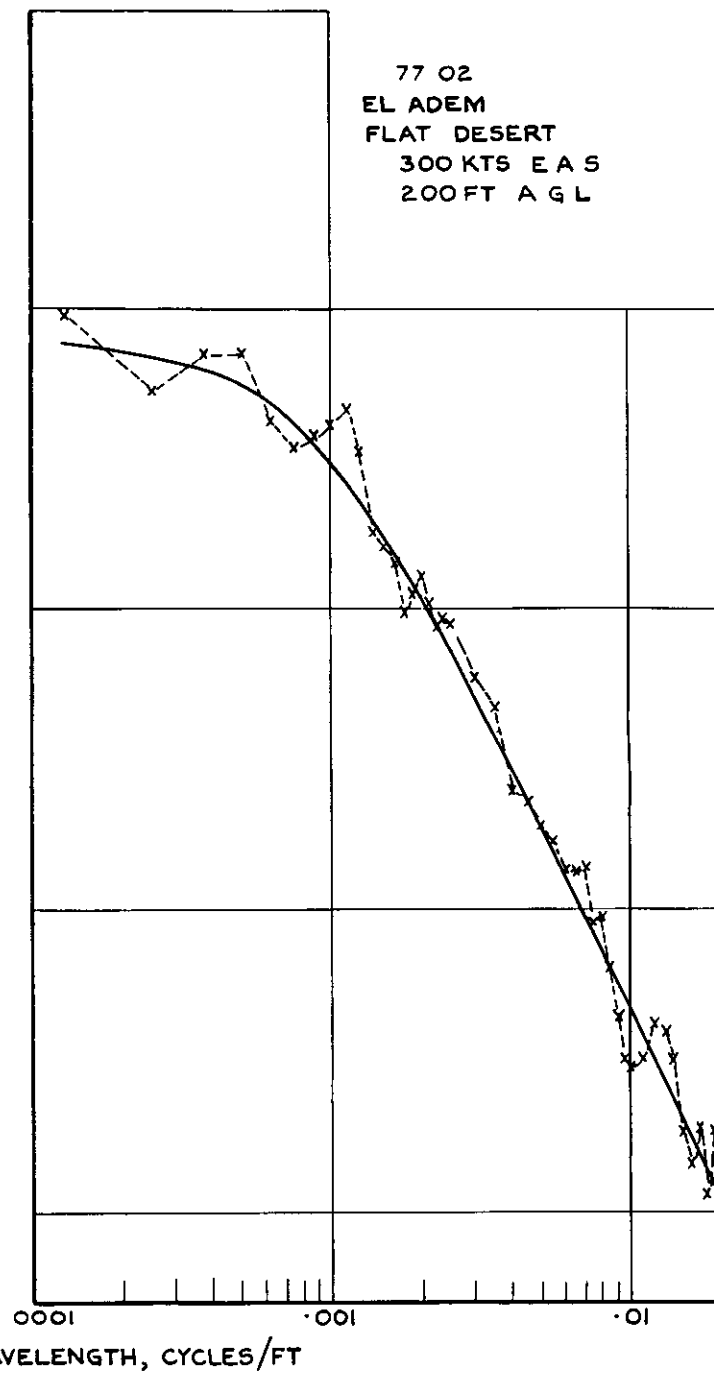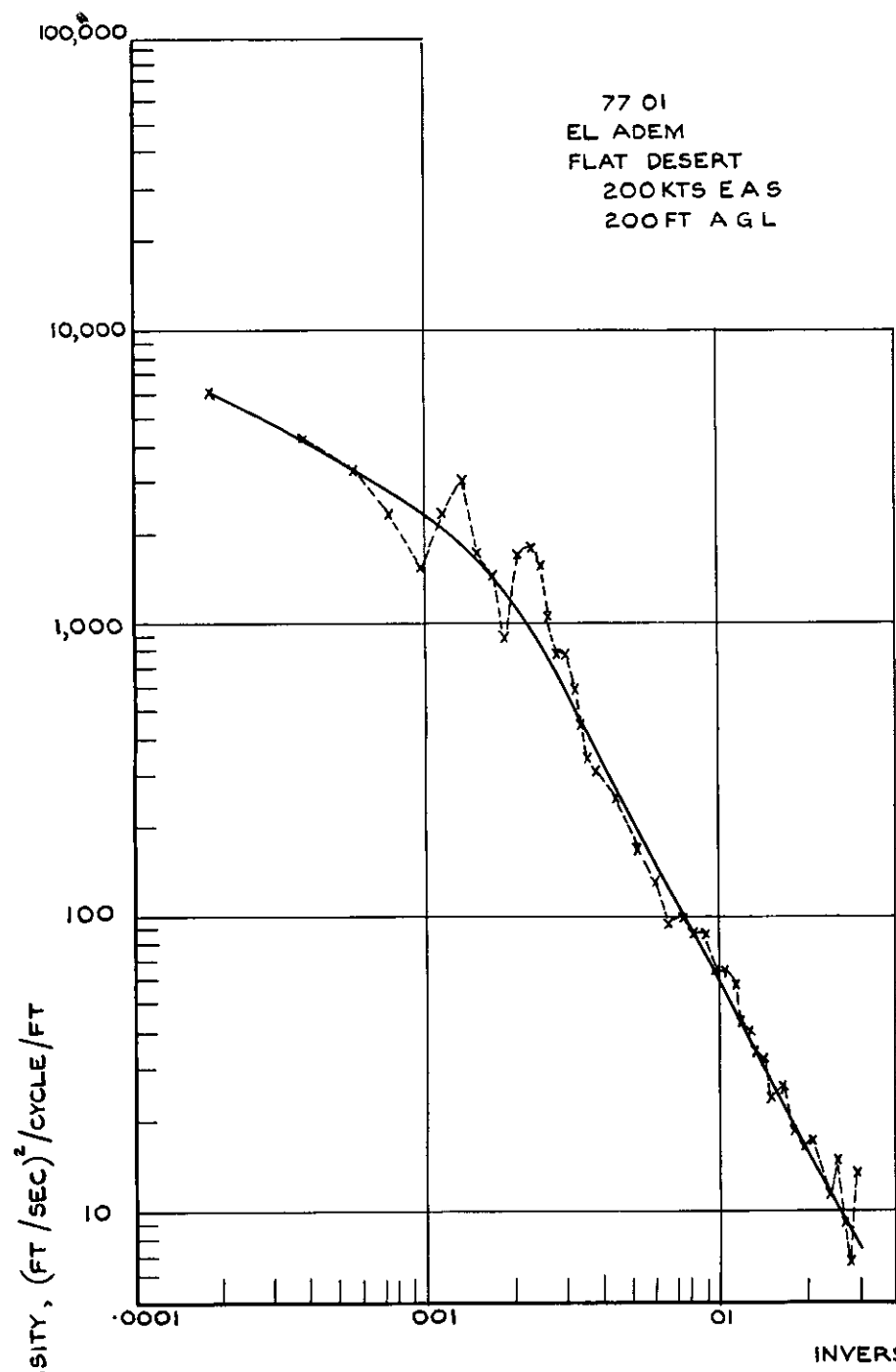
FIG. 4. POWER SPECTRA-FLIGHT 77-VERTICAL TURBULENCE VELOCITY.
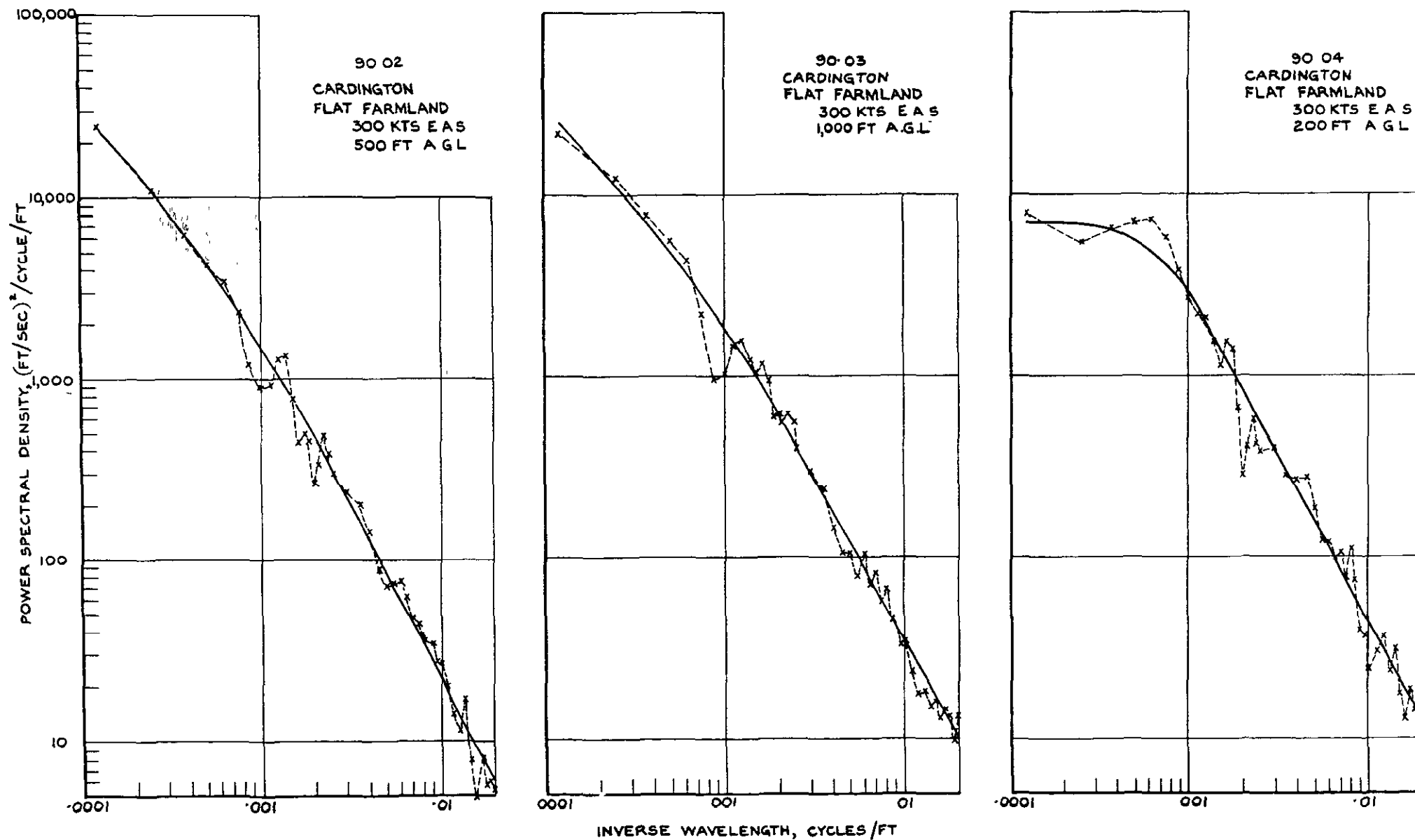
FIG. 5. POWER SPECTRA – FLIGHT 90 – VERTICAL TURBULENCE VELOCITY
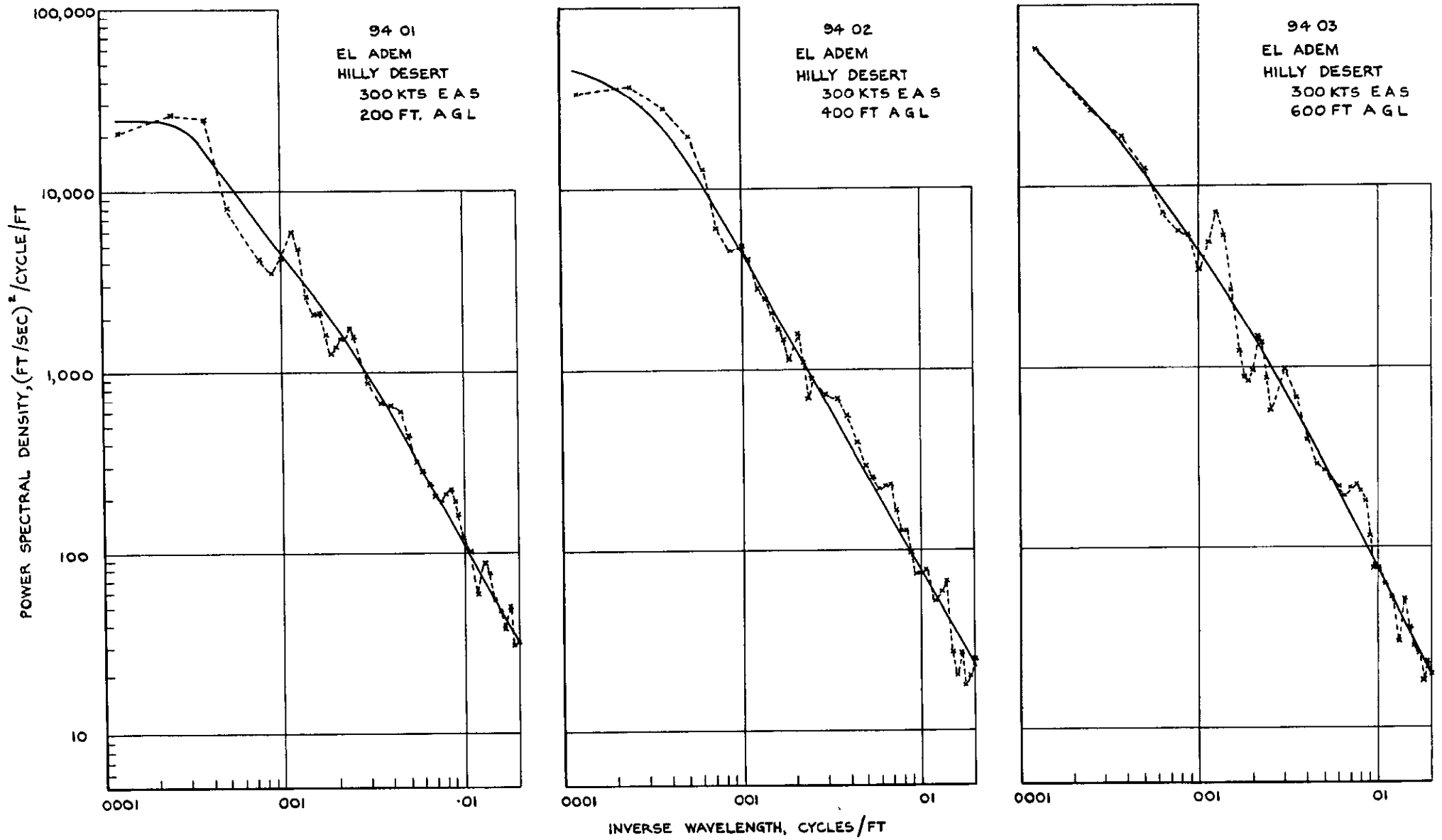
FIG. 6. POWER SPECTRA – FLIGHT 94 – VERTICAL TURBULENCE VELOCITY.
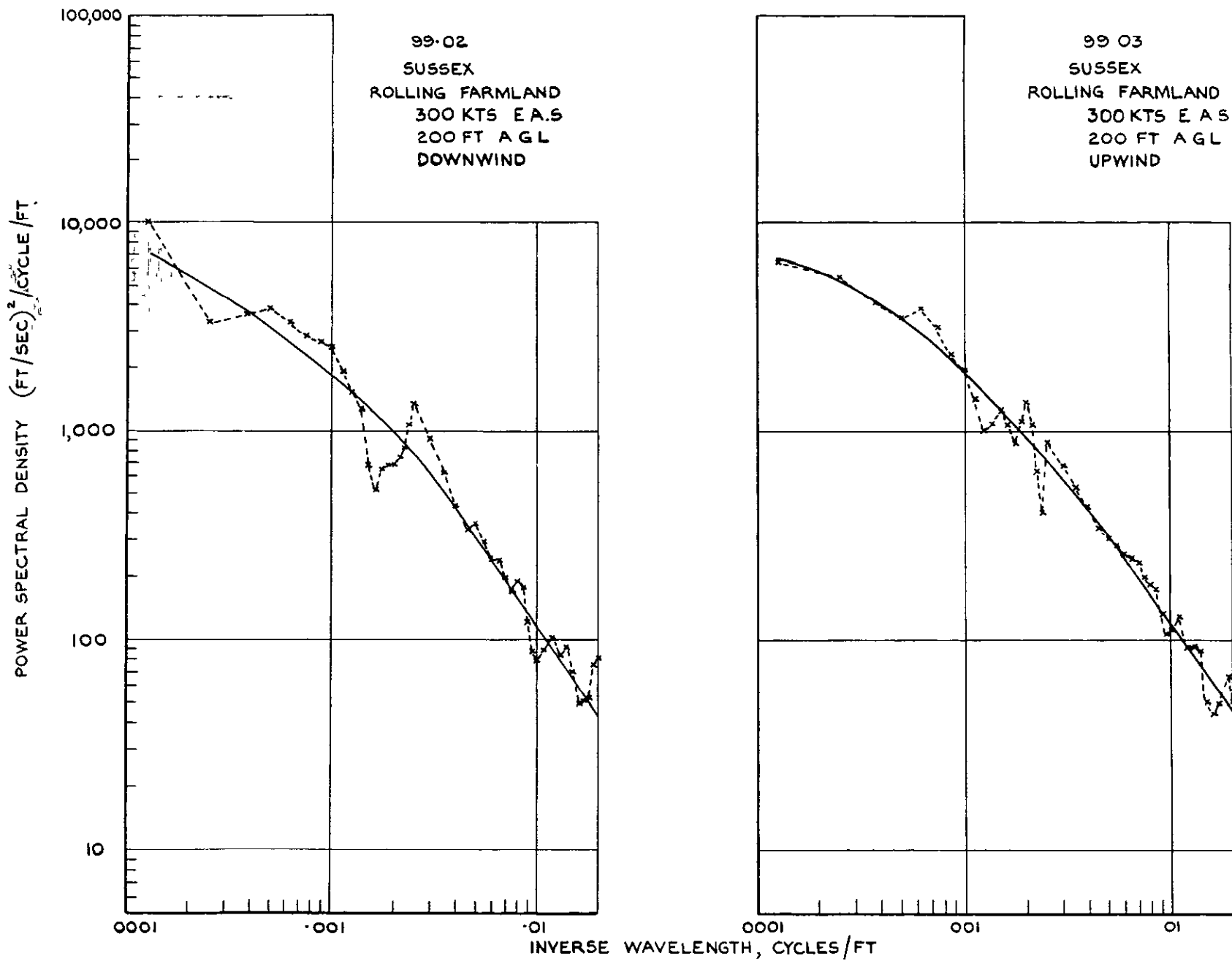
FIG.7. POWER SPECTRA—FLIGHT 99 — VERTICAL TURBULENCE VELOCITY.
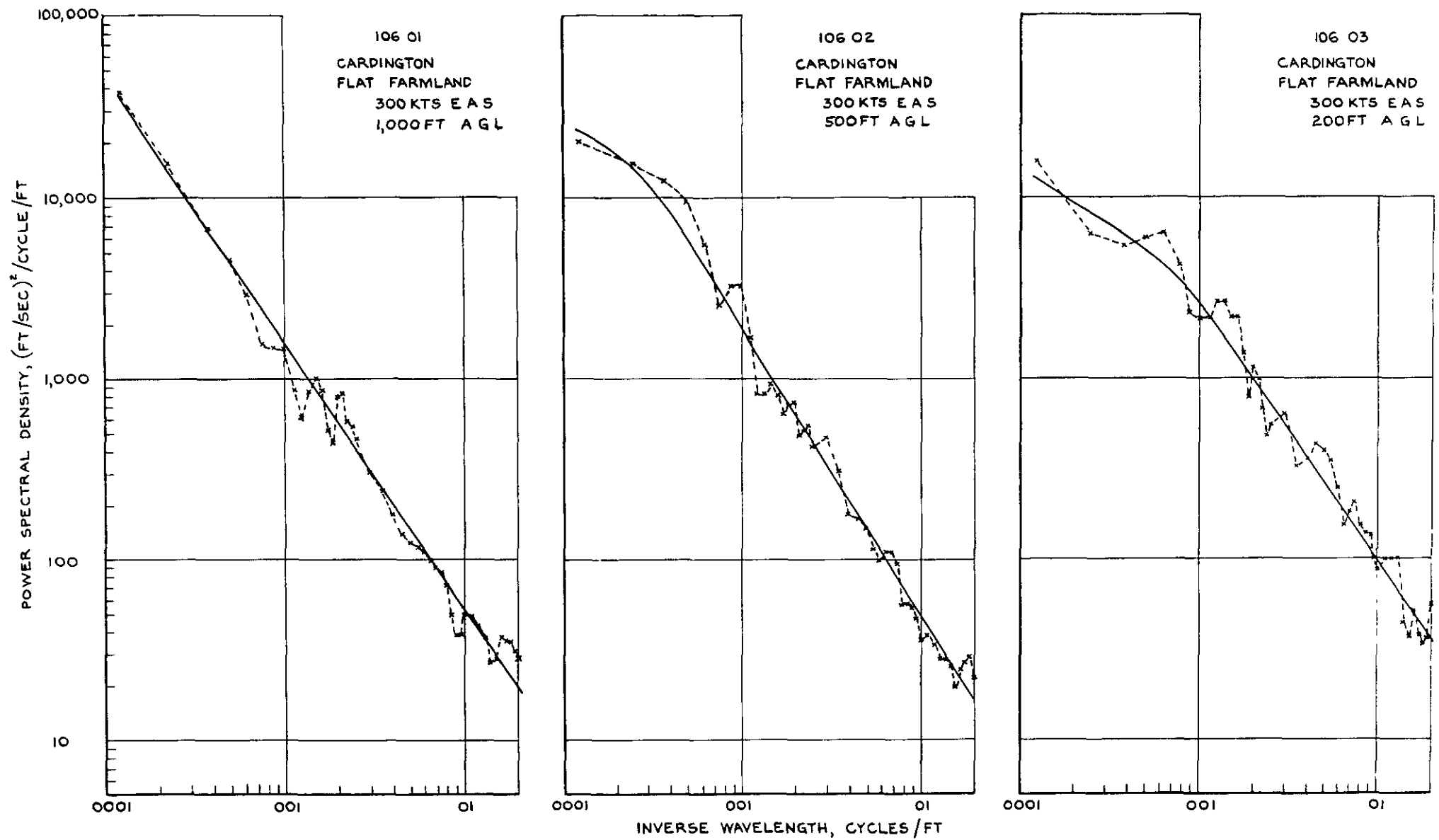
FIG.8 (a). POWER SPECTRA – FLIGHT 106 – VERTICAL TURBULENCE VELOCITY
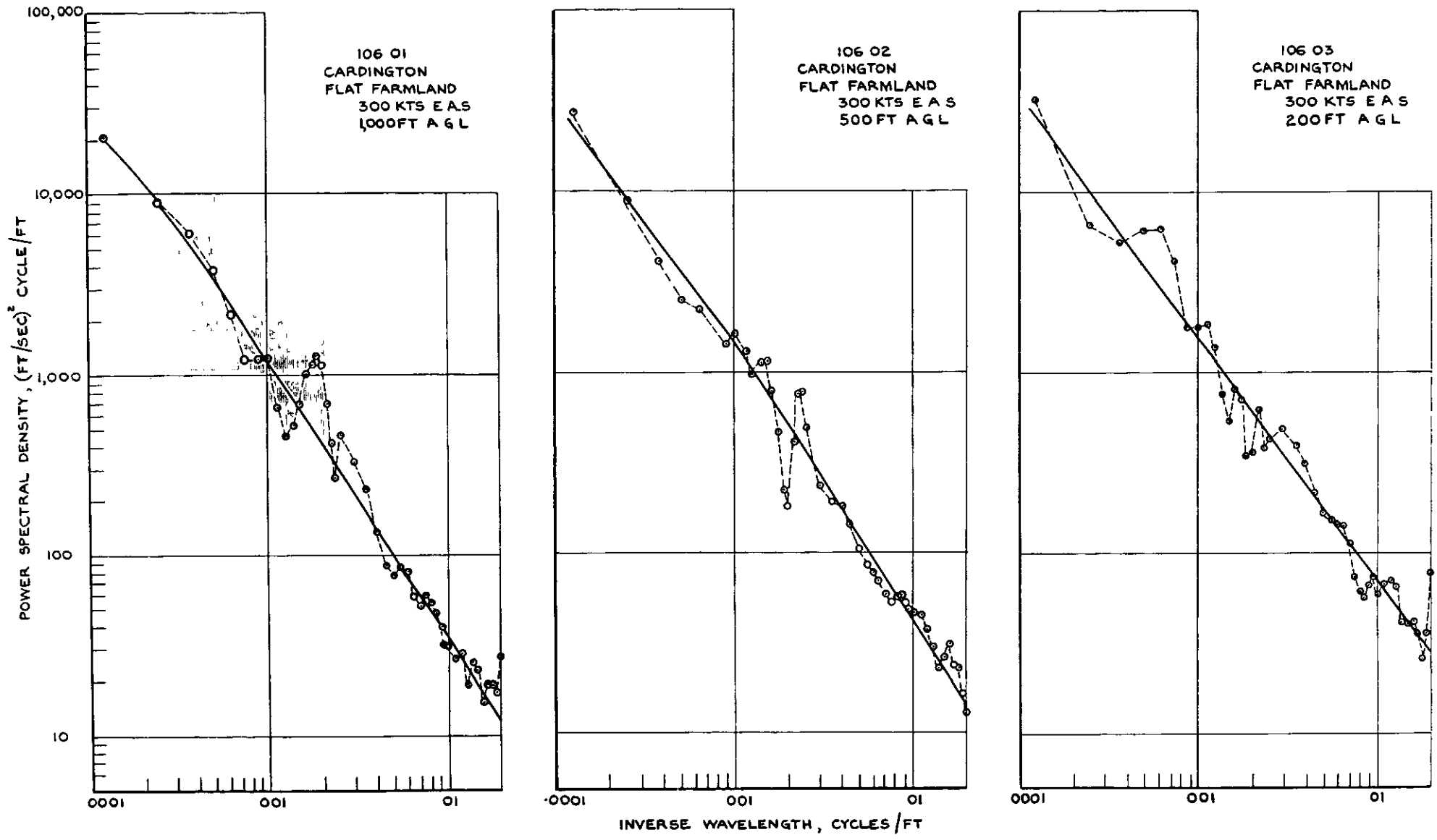
FIG.8(b) POWER SPECTRA-FLIGHT 106-FORE-AND-AFT TURBULENCE VELOCITY
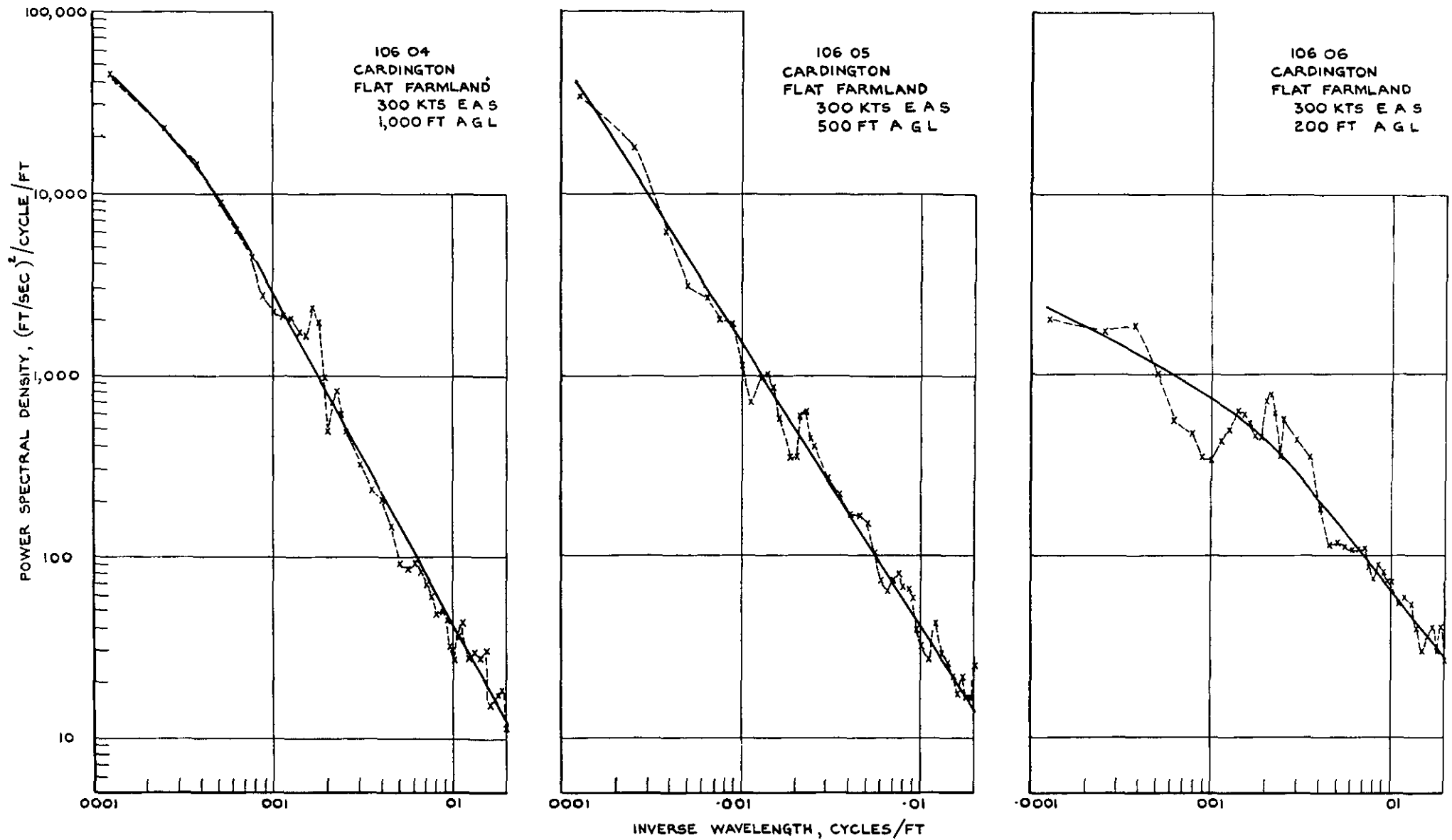
FIG. 8.(c) POWER SPECTRA - FLIGHT 106 - VERTICAL TURBULENCE VELOCITY
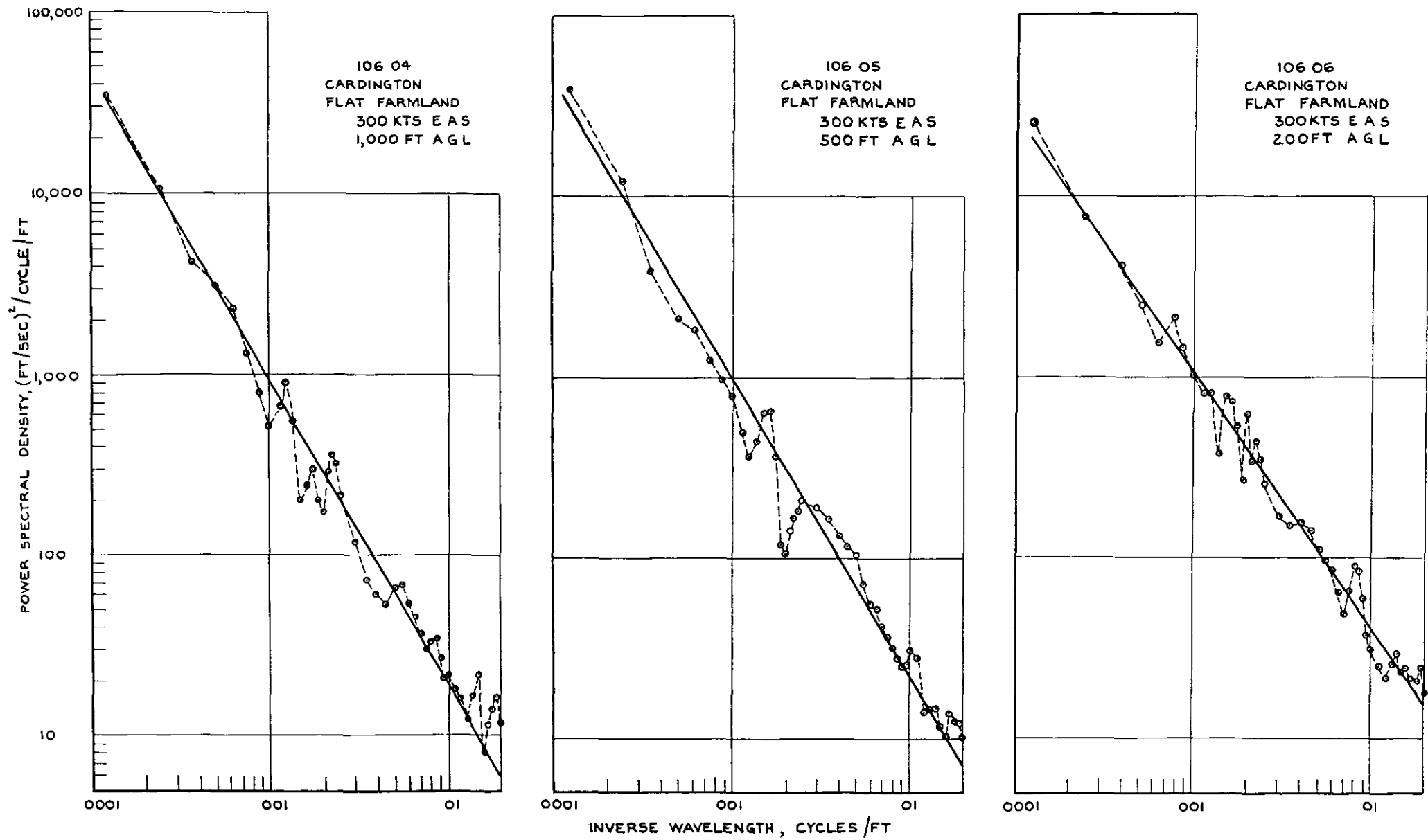
FIG. 8.(d) POWER SPECTRA – FLIGHT 106 – FORE-AND-AFT TURBULENCE VELOCITY
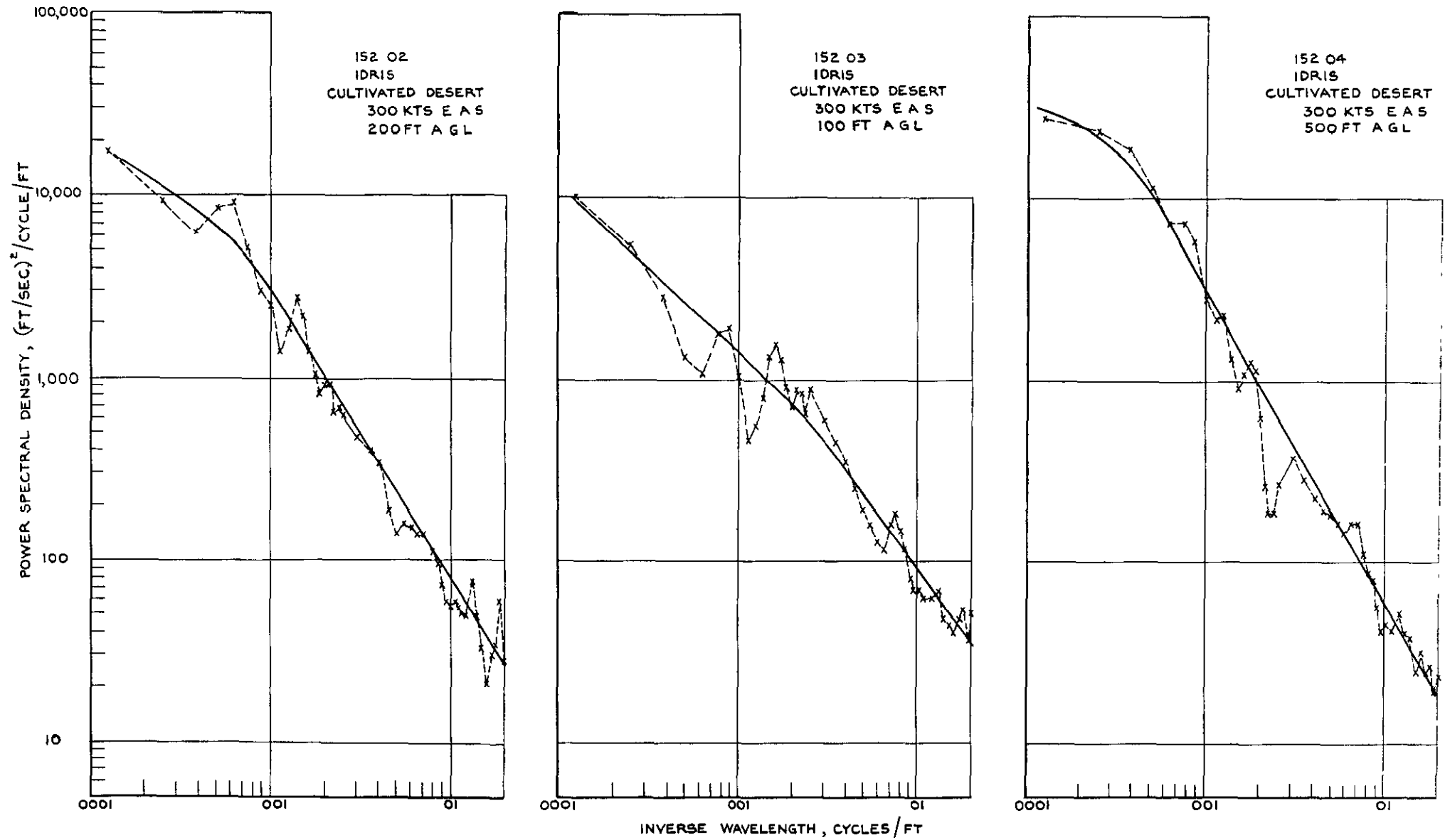
FIG.9.(a) POWER SPECTRA — FLIGHT 152 — VERTICAL TURBULENCE VELOCITY.
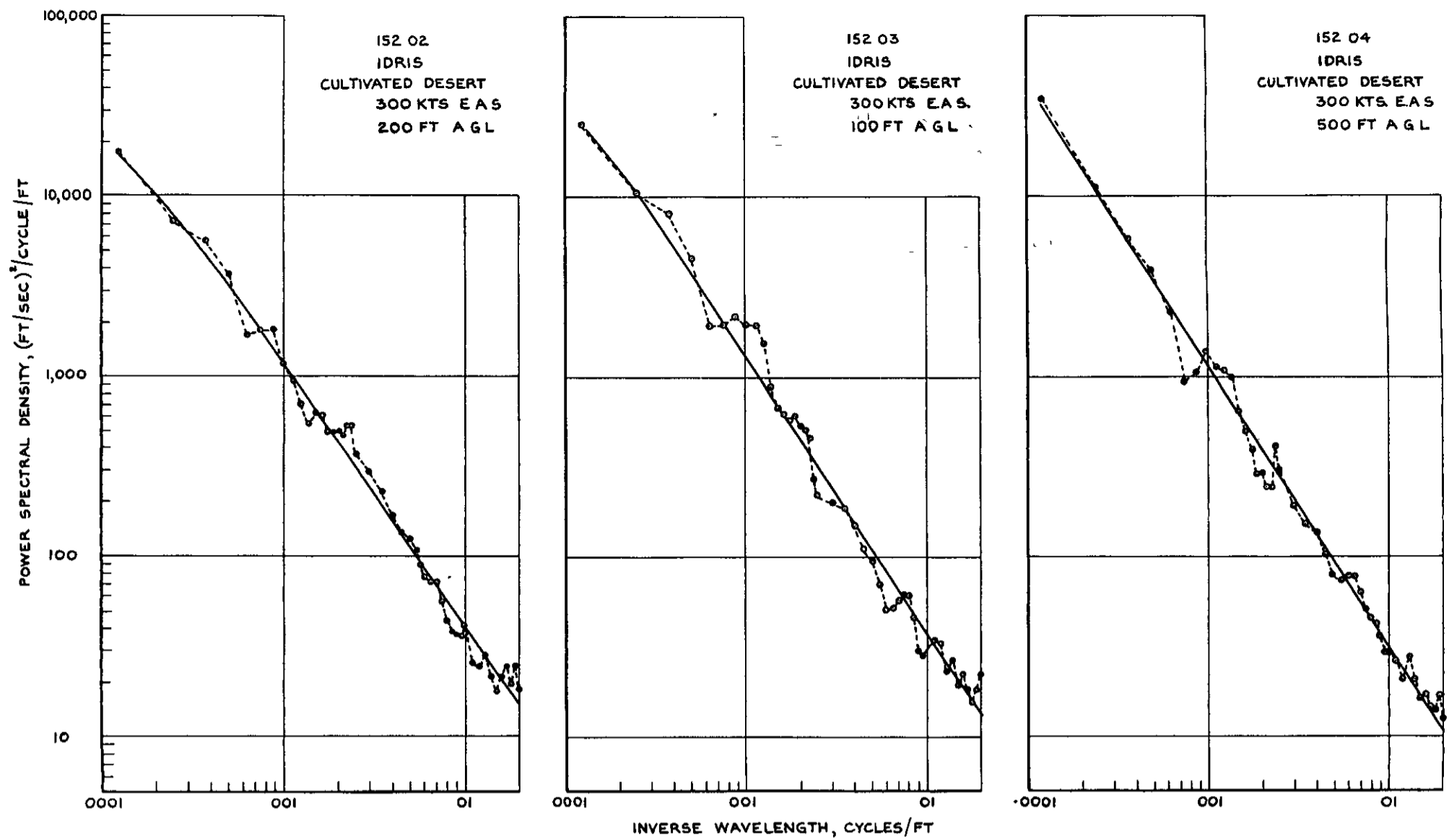
FIG. 9 (b). POWER SPECTRA–FLIGHT 152 – FORE-AND-AFT TURBULENCE VELOCITY.
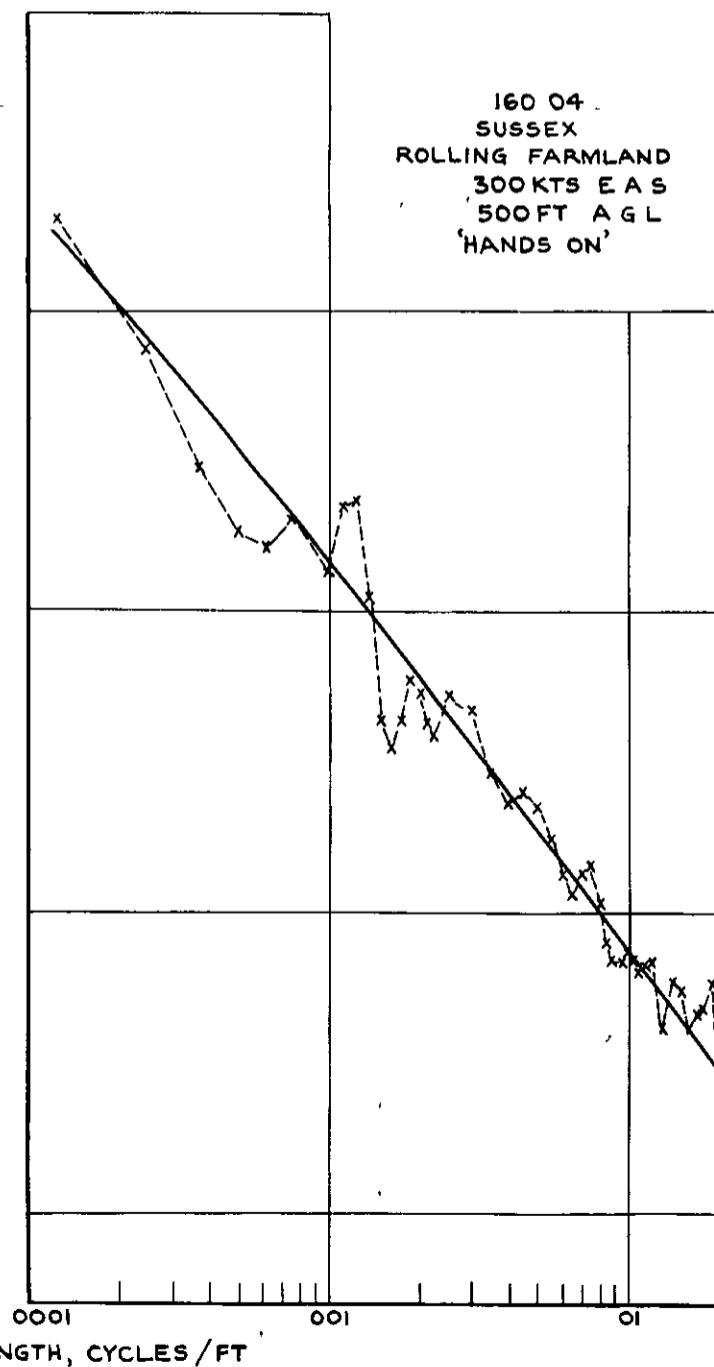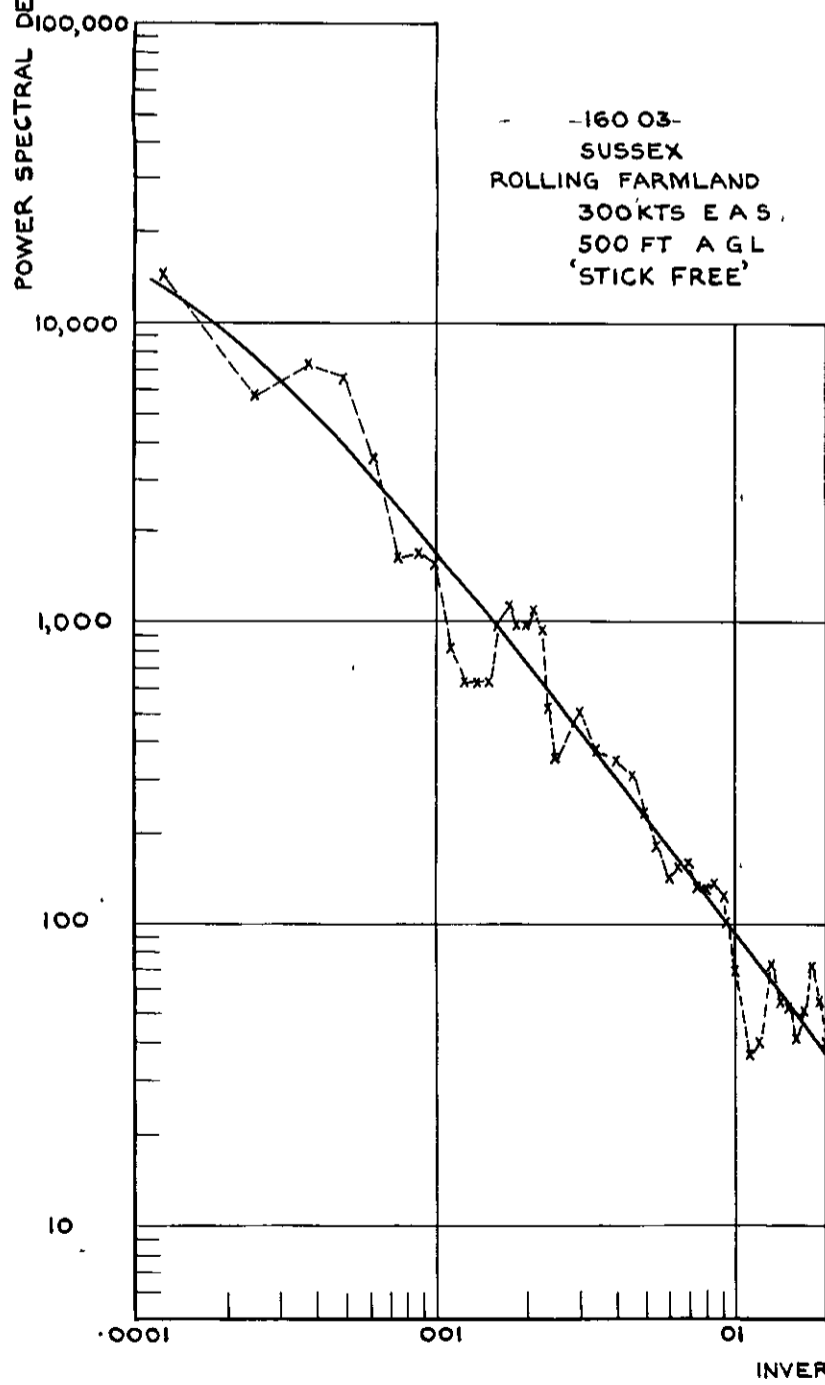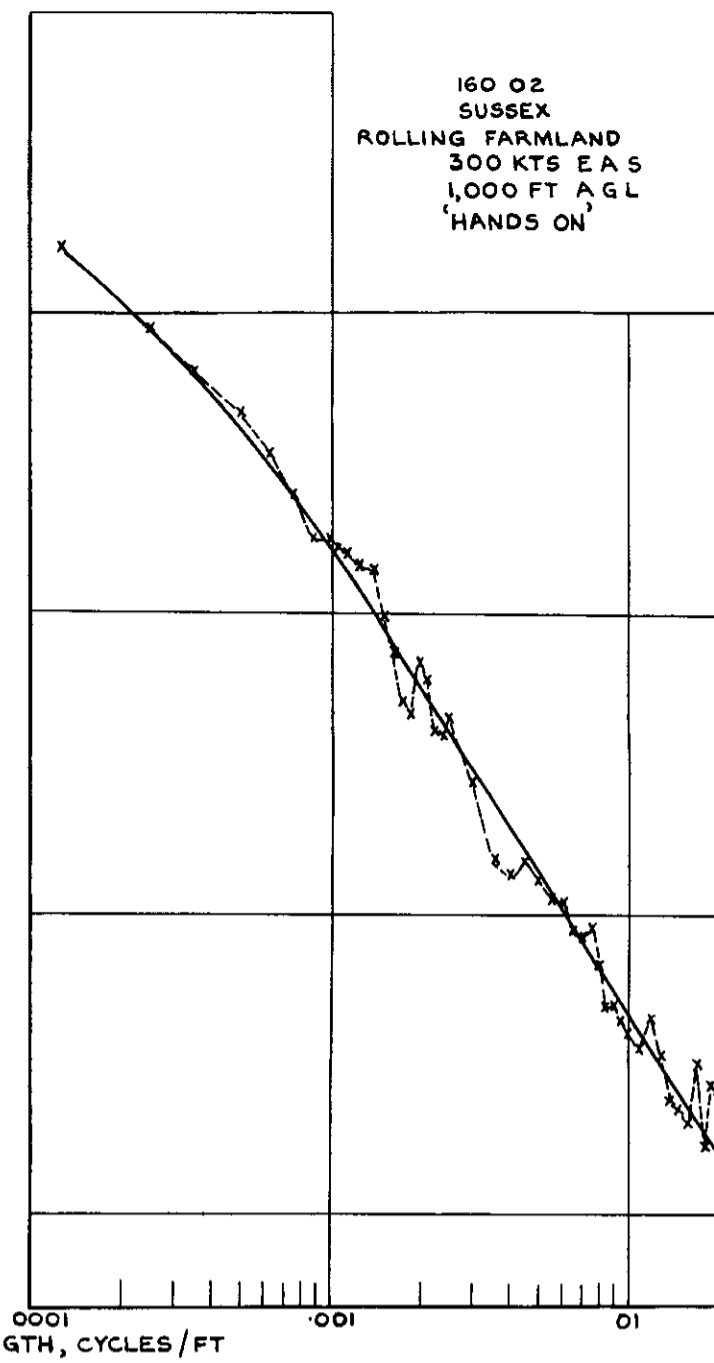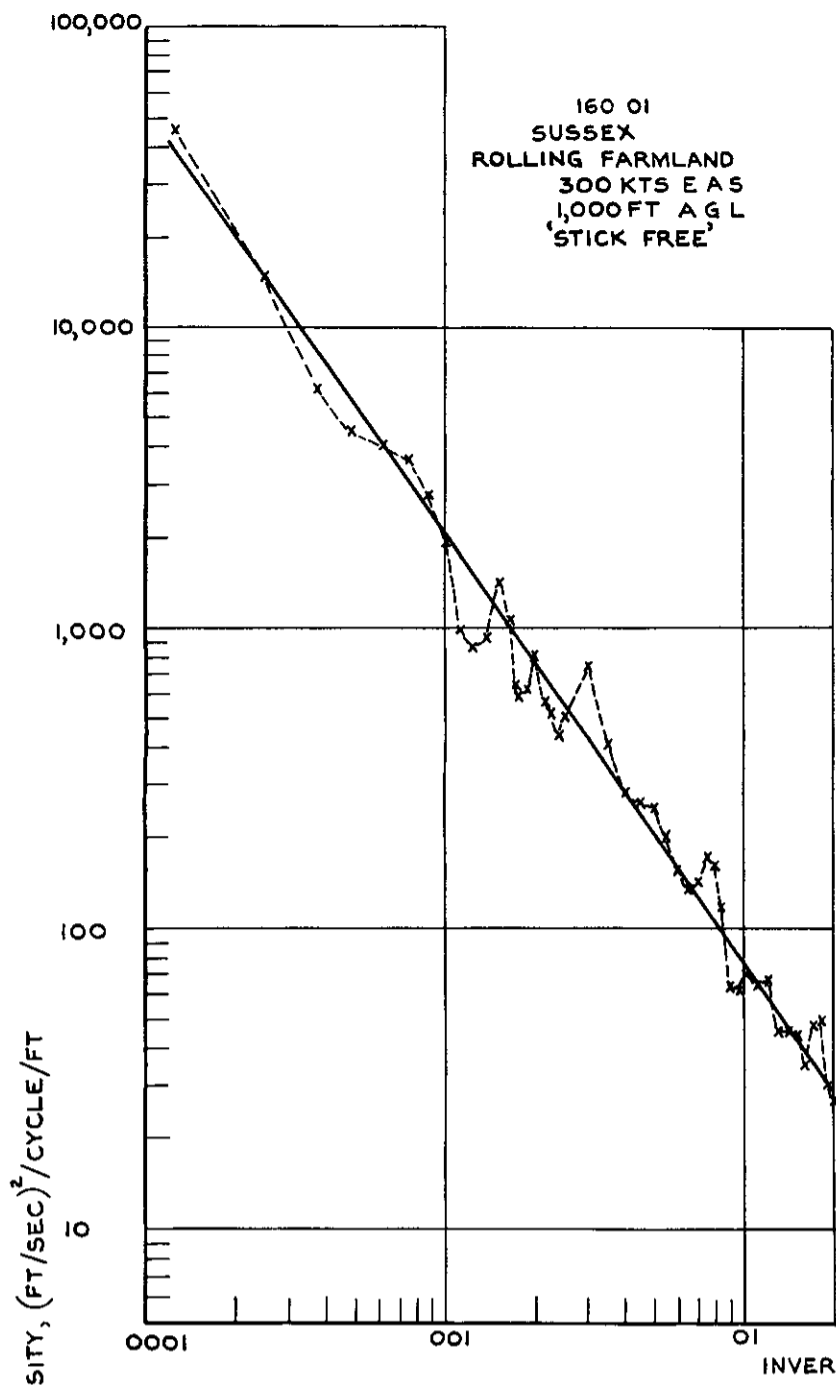
FIG. IO. POWER SPECTRA – FLIGHT 160 – VERTICAL TURBULENCE VELOCITY.
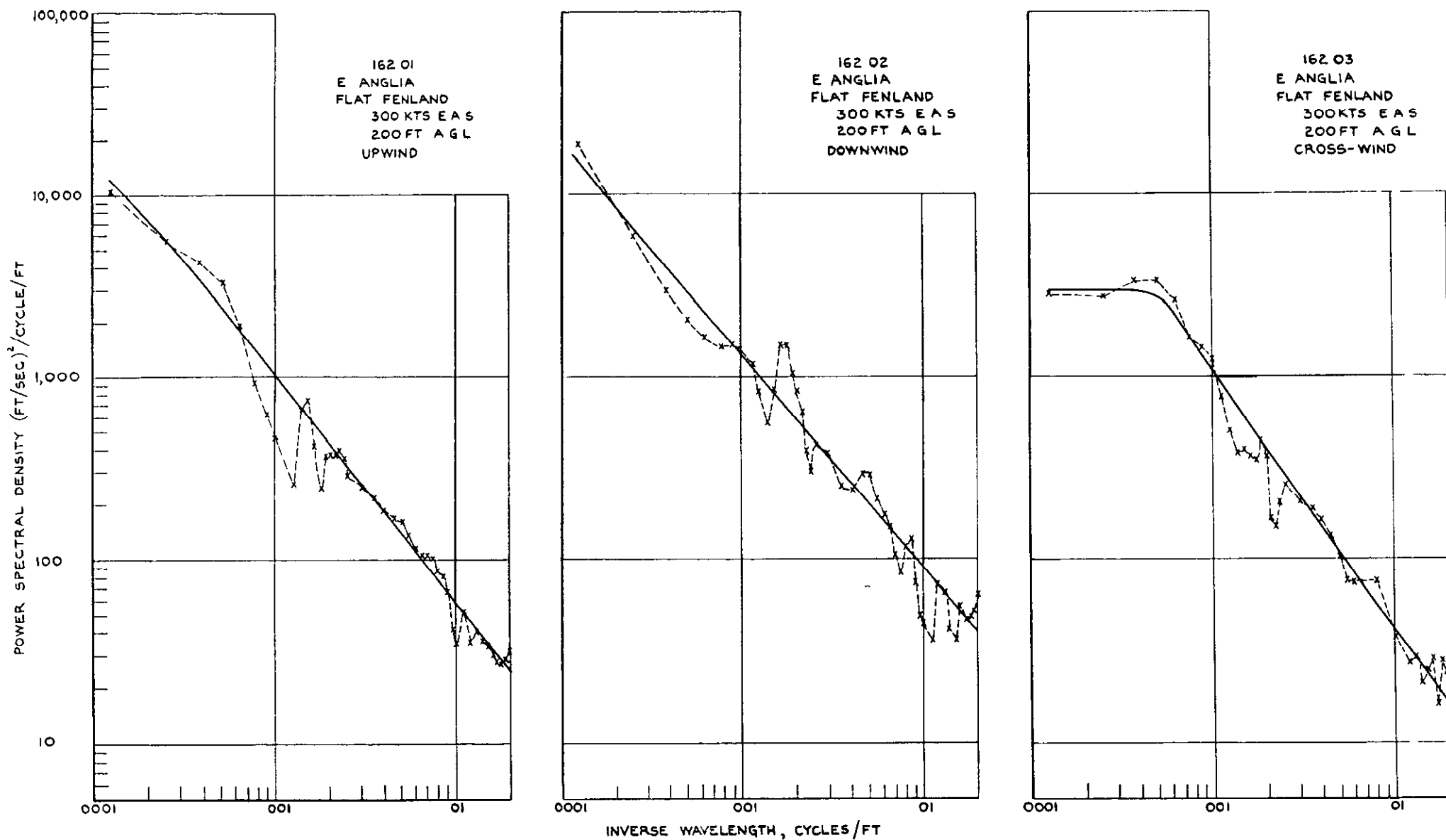
FIG. II.(a) POWER SPECTRA—FLIGHT 162—VERTICAL TURBULENCE VELOCITY.
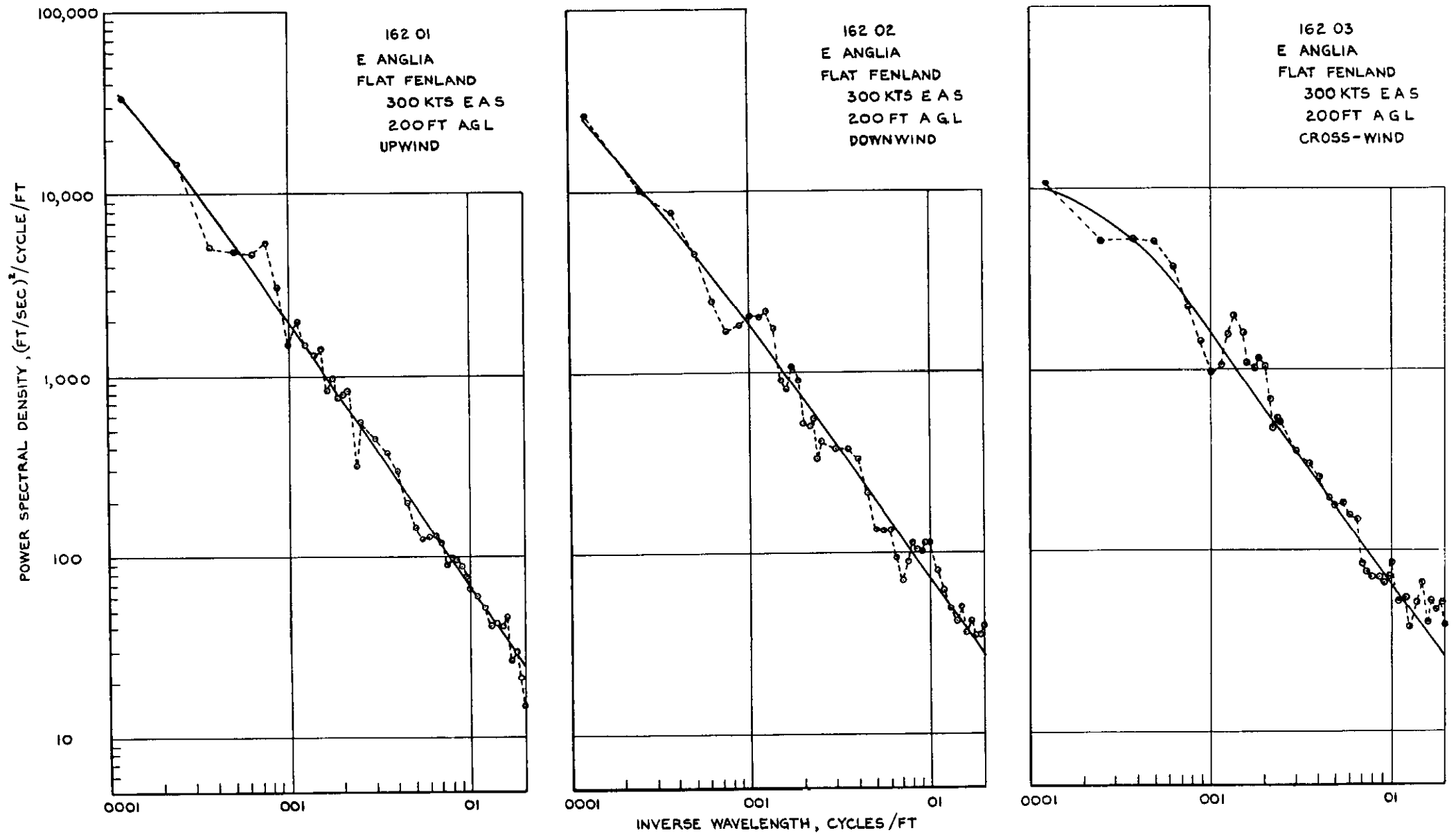
FIG.II(b). POWER SPECTRA-FLIGHT 162-FORE-AND-AFT TURBULENCE VELOCITY.
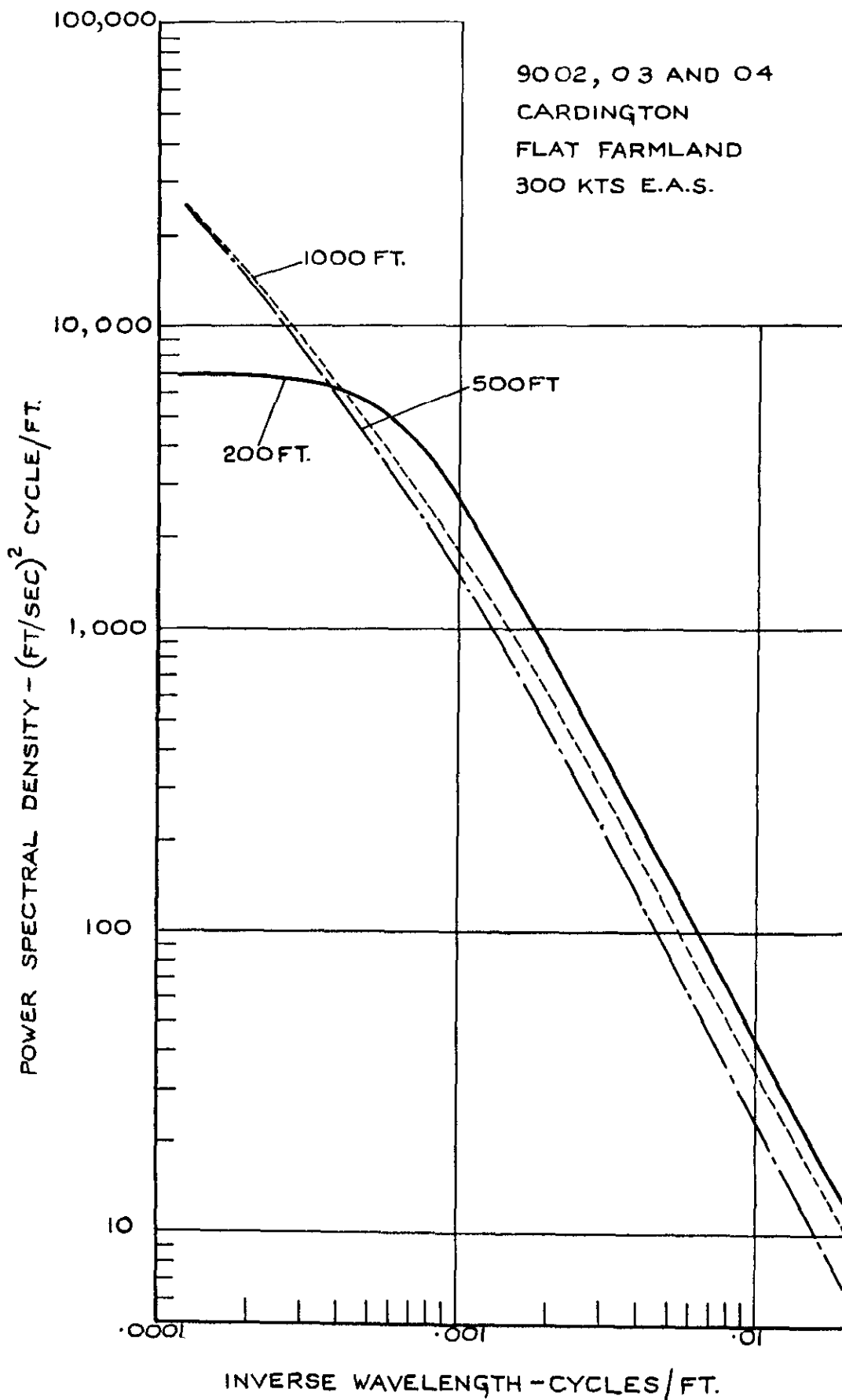
FIG. 12. COMPARATIVE POWER SPECTRA AT DIFFERENT HEIGHTS
VERTICAL VELOCITY OF TURBULENCE,
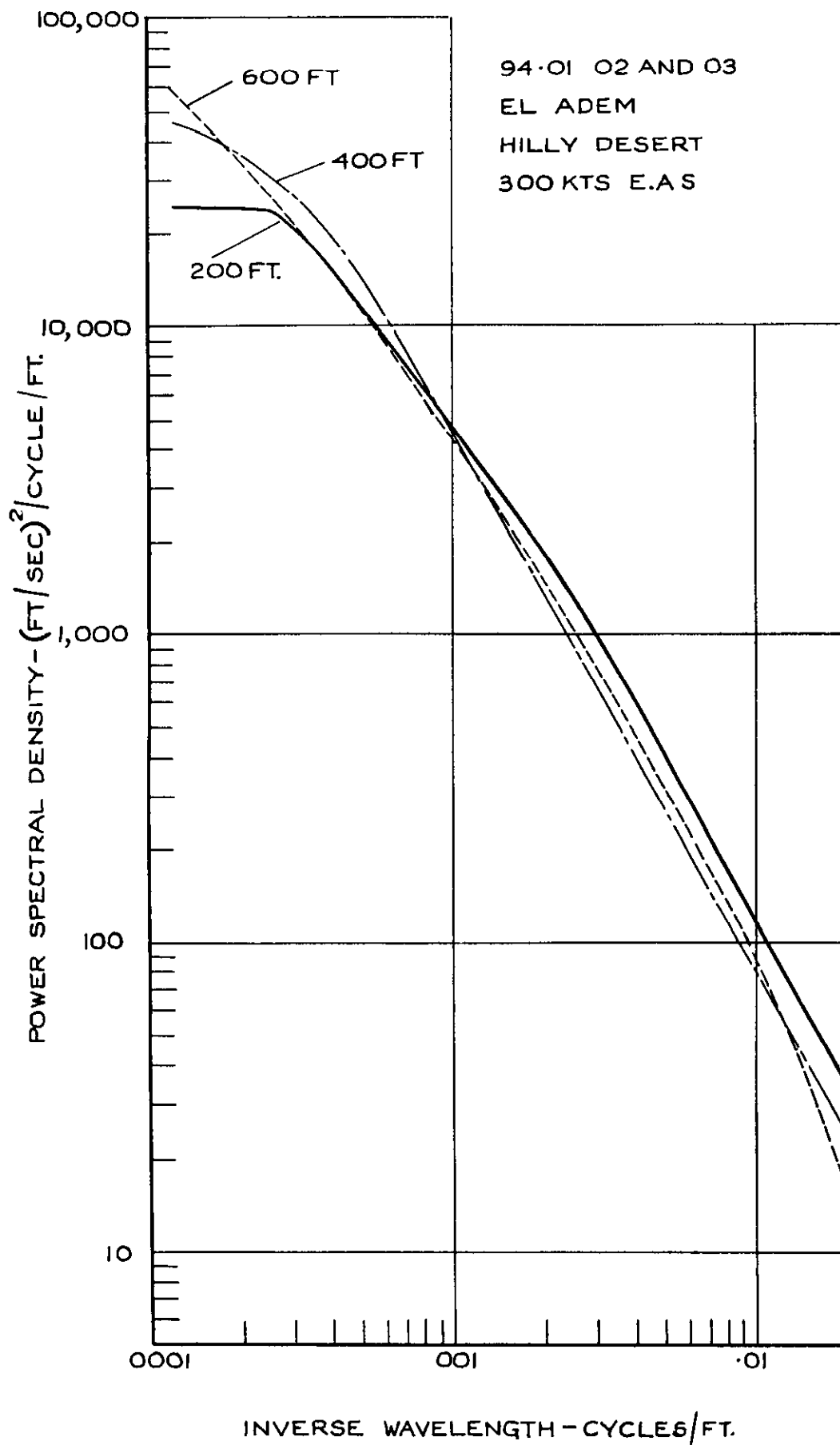
FIG.13 COMPARATIVE POWER SPECTRA AT DIFFERENT HEIGHTS
VERTICAL VELOCITY OF TURBULENCE.
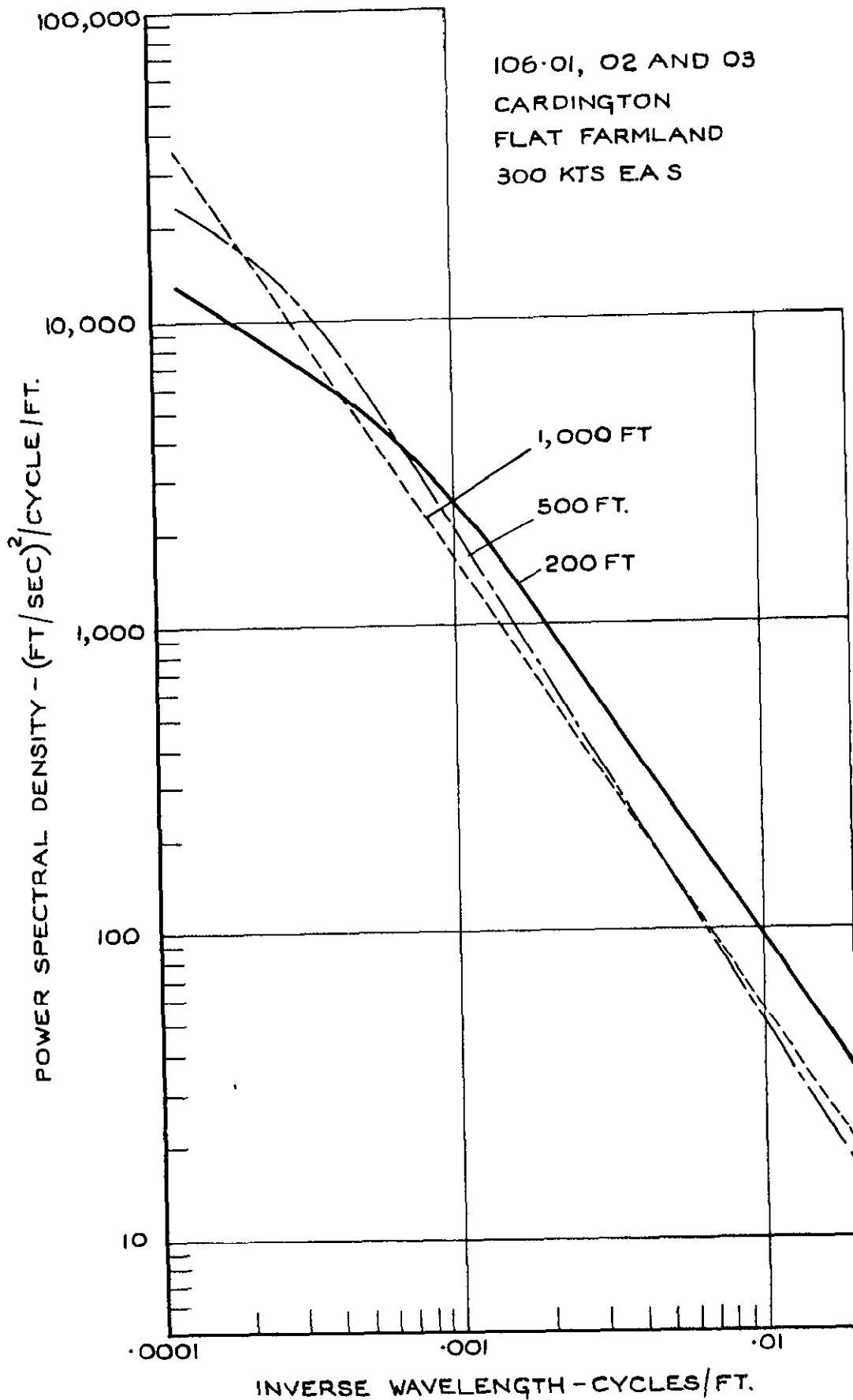
FIG.14.(a) COMPARATIVE POWER SPECTRA AT DIFFERENT HEIGHTS
VERTICAL VELOCITY OF TURBULENCE.
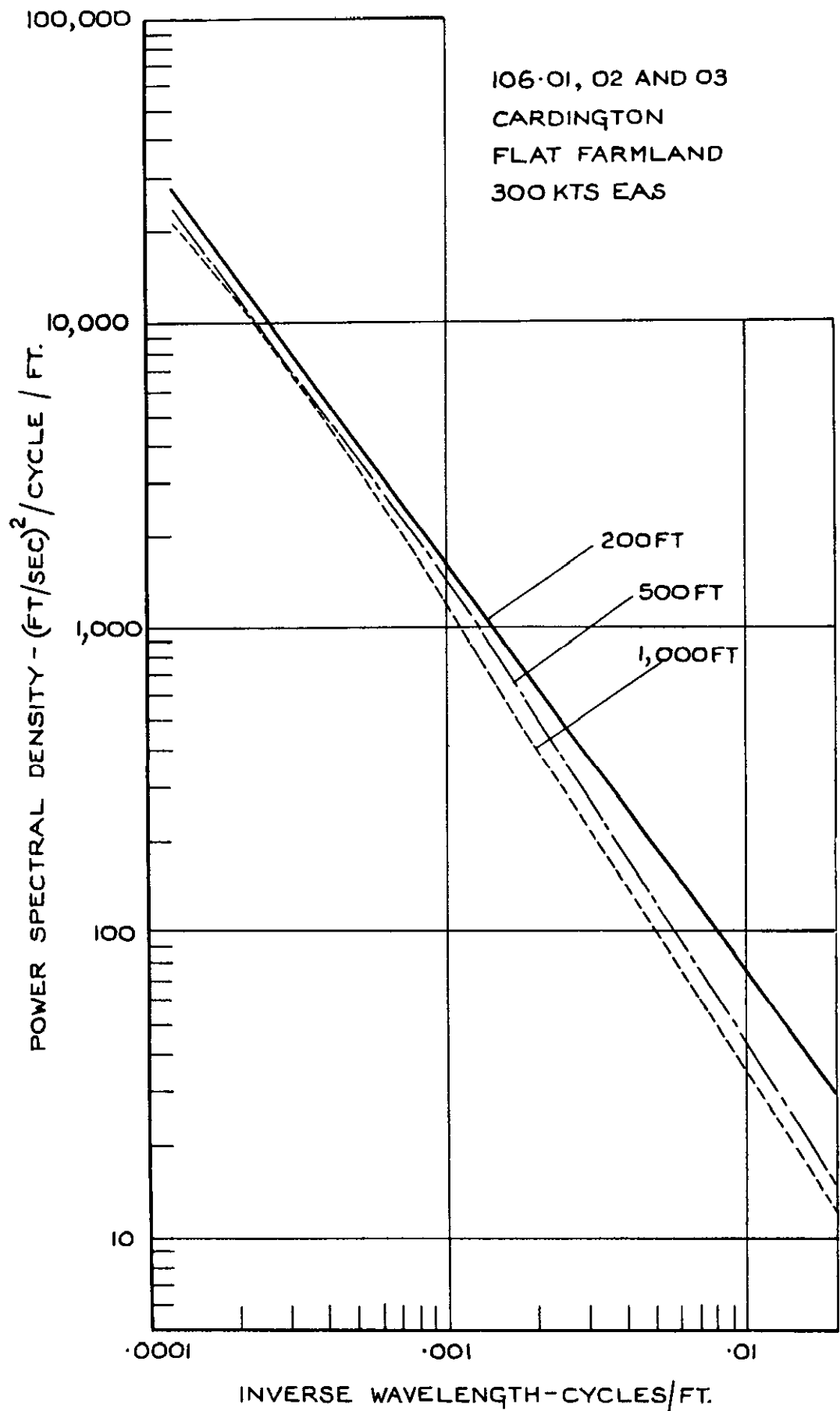
FIG 14.(b) COMPARATIVE POWER SPECTRA AT DIFFERENT HEIGHTS
FORE-AND-AFT VELOCITY OF TURBULENCE.
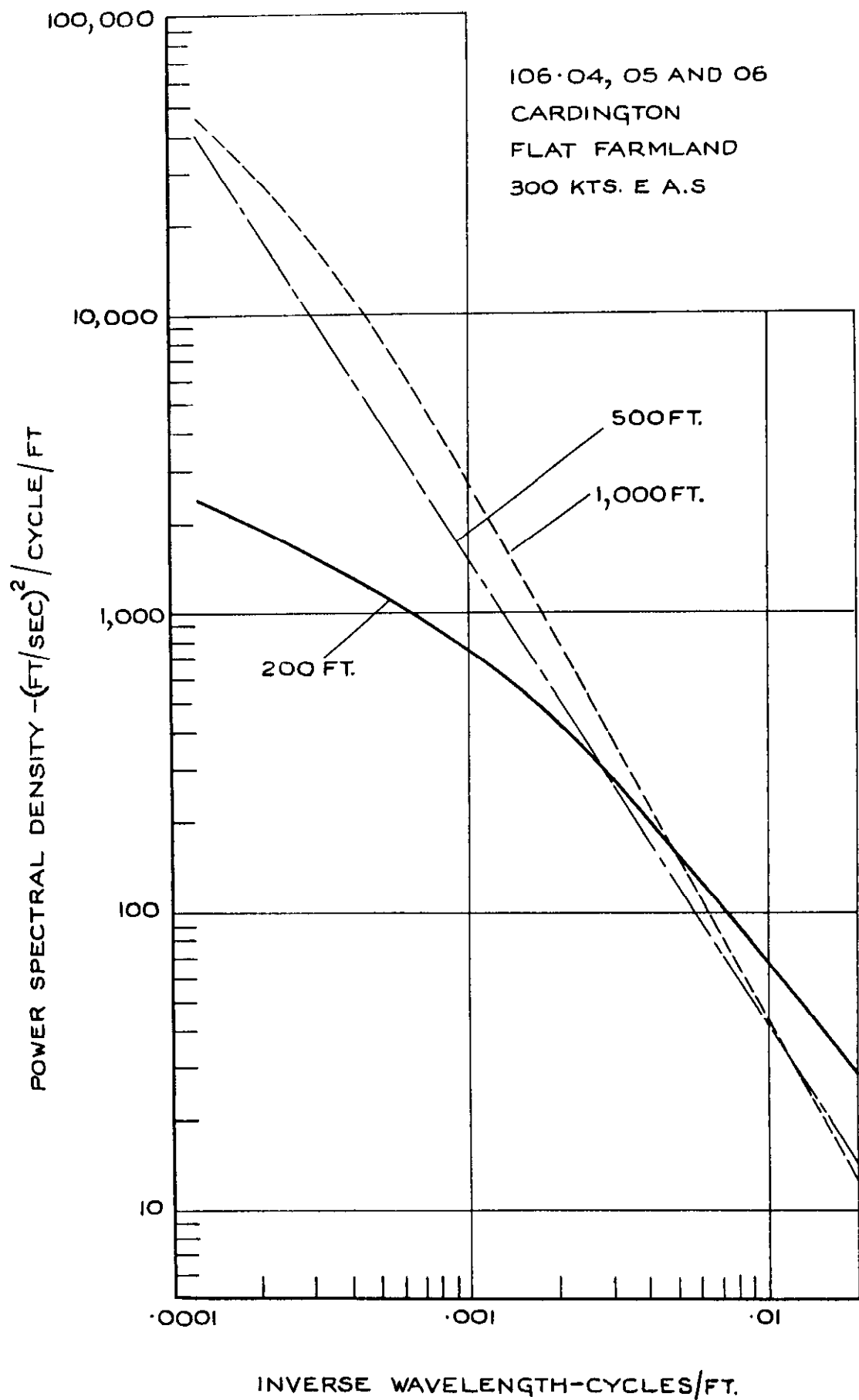
FIG.15(a) COMPARATIVE POWER SPECTRA AT DIFFERENT HEIGHTS
VERTICAL VELOCITY OF TURBULENCE.
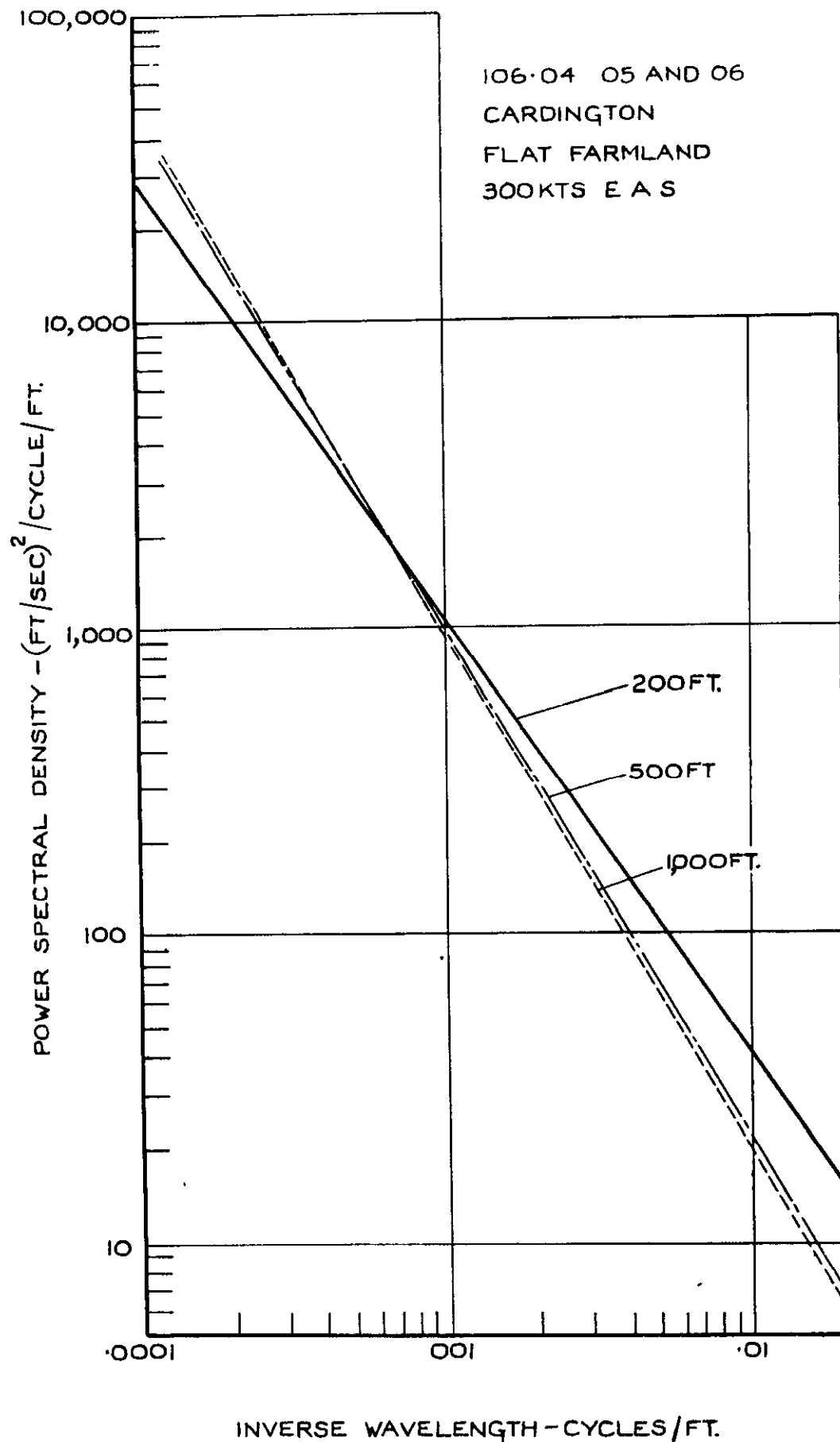
FIG. 15.(b) COMPARATIVE POWER SPECTRA AT DIFFERENT HEIGHTS
FORE-AND-AFT VELOCITY OF TURBULENCE.

152·02, 03 AND 04
IDRIS
CULTIVATED DESERT
300 KTS EA S.

500 FT

200 FT.

100 FT.

POWER SPECTRAL DENSITY - (FT/SEC)² CYCLE/FT

100,000

10,000

1,000

100

10

·0001   ·001   ·01

INVERSE WAVELENGTH - CYCLES/FT.
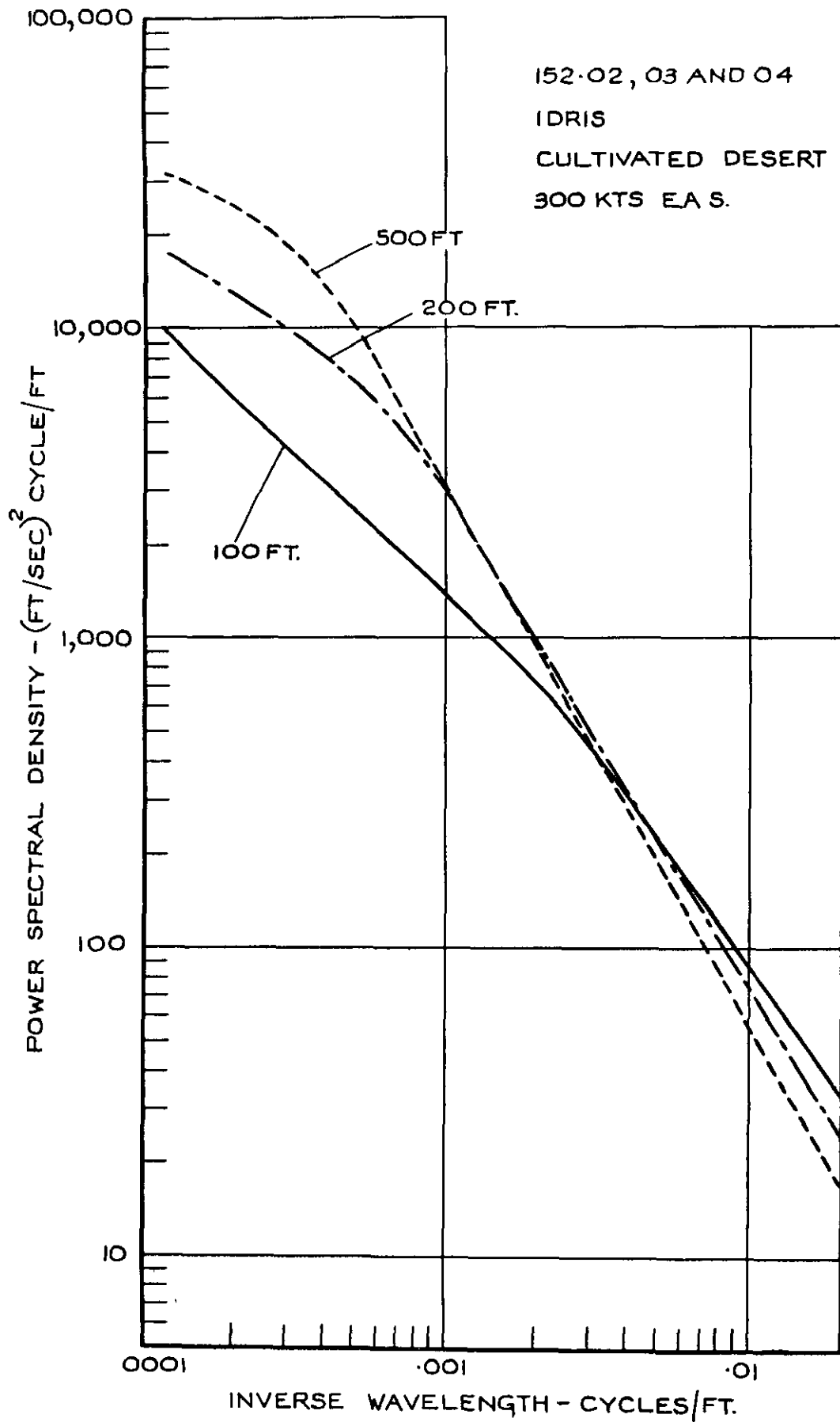
FIG.16.(a)COMPARATIVE POWER SPECTRA AT DIFFERENT HEIGHTS
VERTICAL VELOCITY OF TURBULENCE.
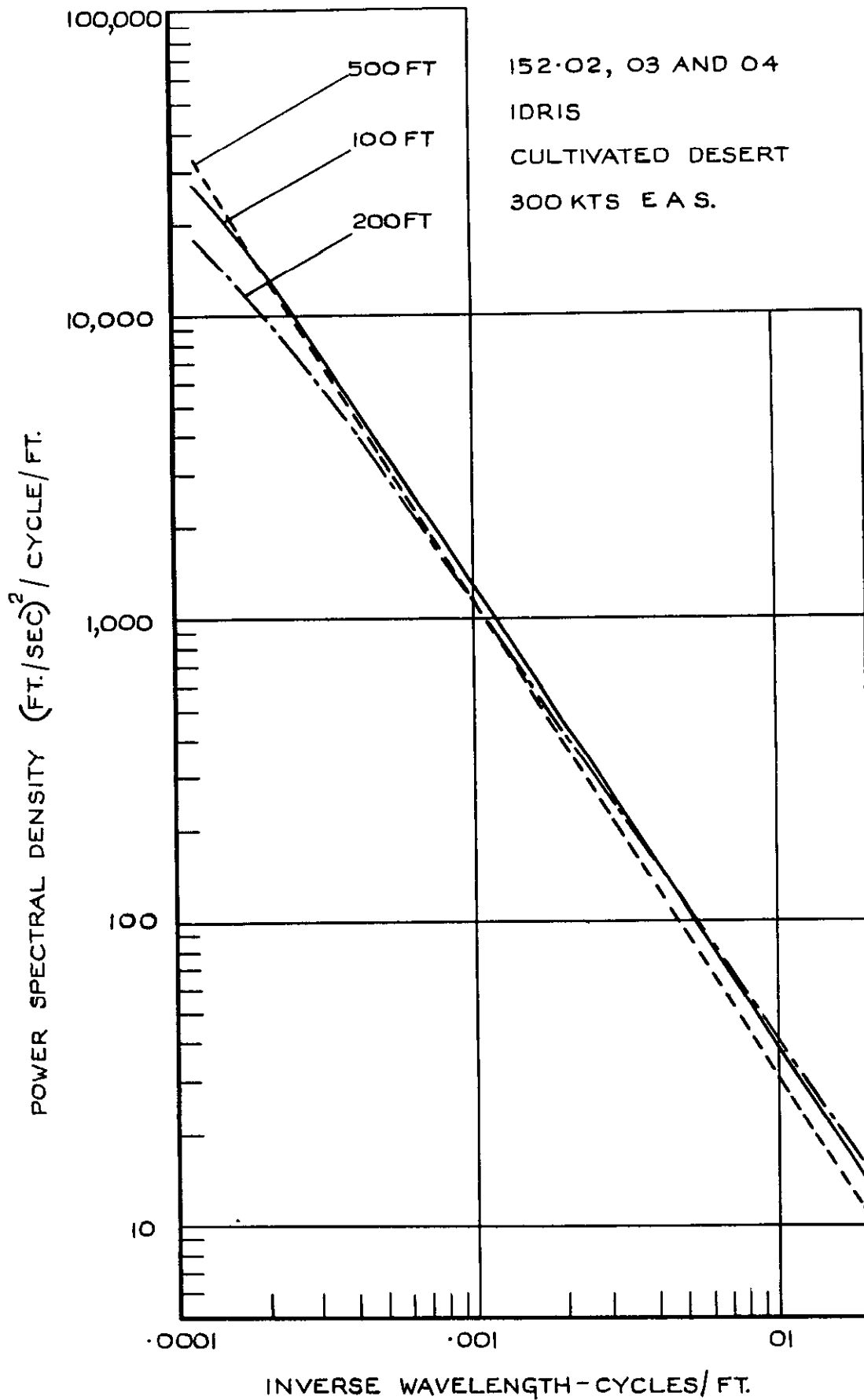
FIG.16.(b) COMPARATIVE POWER SPECTRA AT DIFFERENT HEIGHTS
FORE-AND-AFT VELOCITY OF TURBULENCE
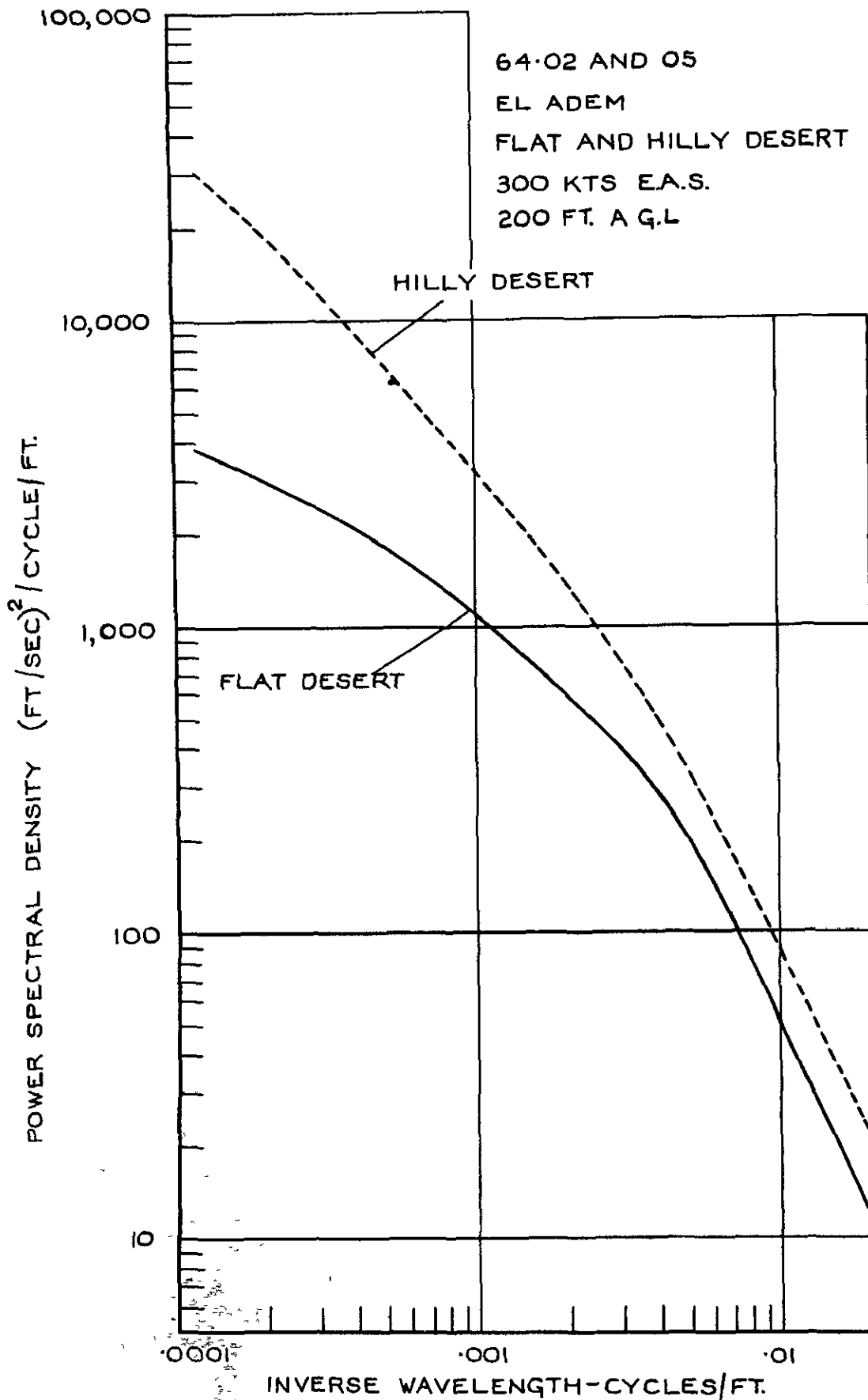
FIG.17. COMPARATIVE POWER SPECTRA OVER FLAT & HILLY DESERT
VERTICAL VELOCITY OF TURBULENCE.
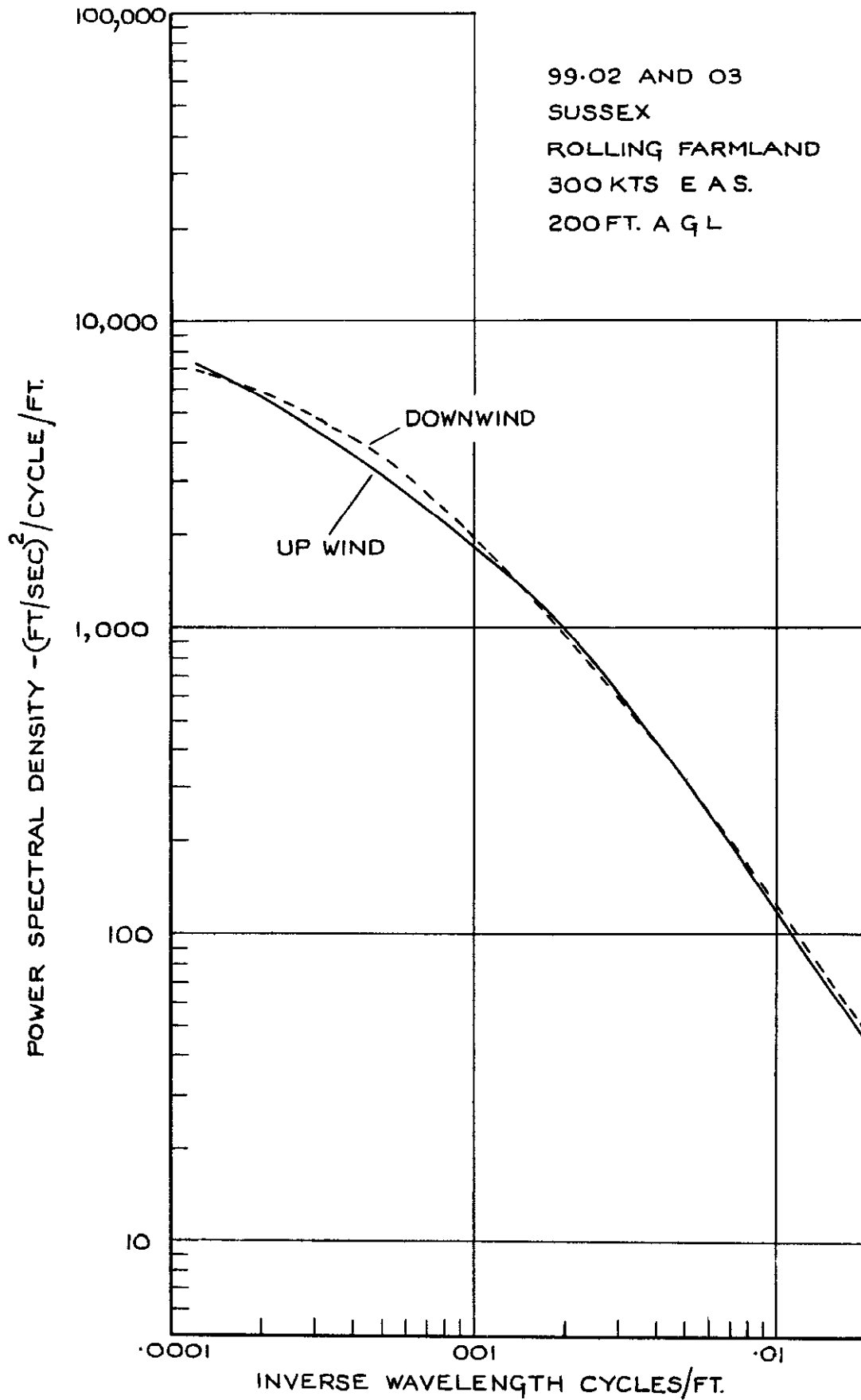
FIG.18. COMPARATIVE POWER SPECTRA UP, AND DOWNWIND
VERTICAL VELOCITY OF TURBULENCE.
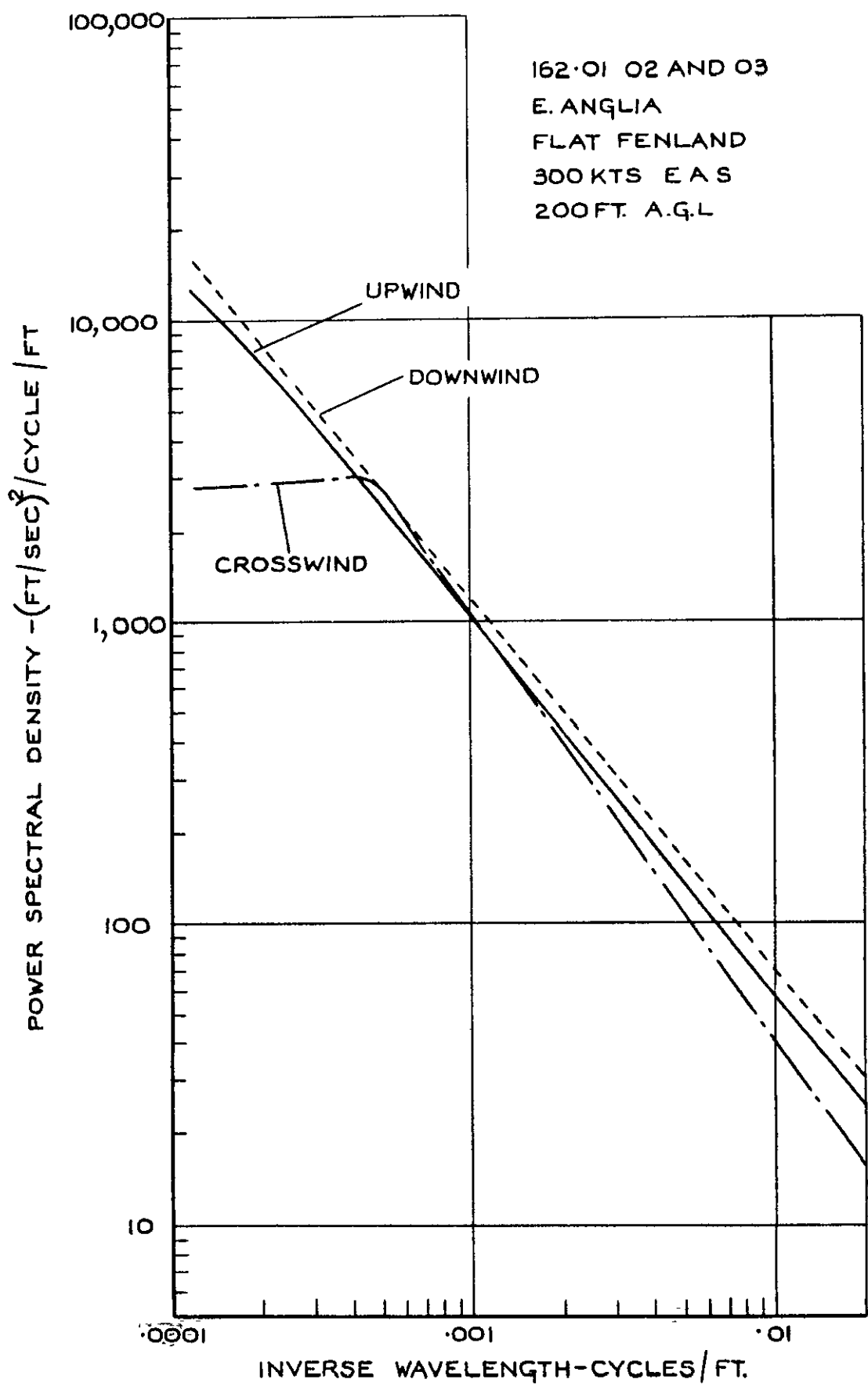
FIG.19. COMPARATIVE POWER SPECTRA UP, DOWN, AND CROSSWIND
VERTICAL VELOCITY OF TURBULENCE.
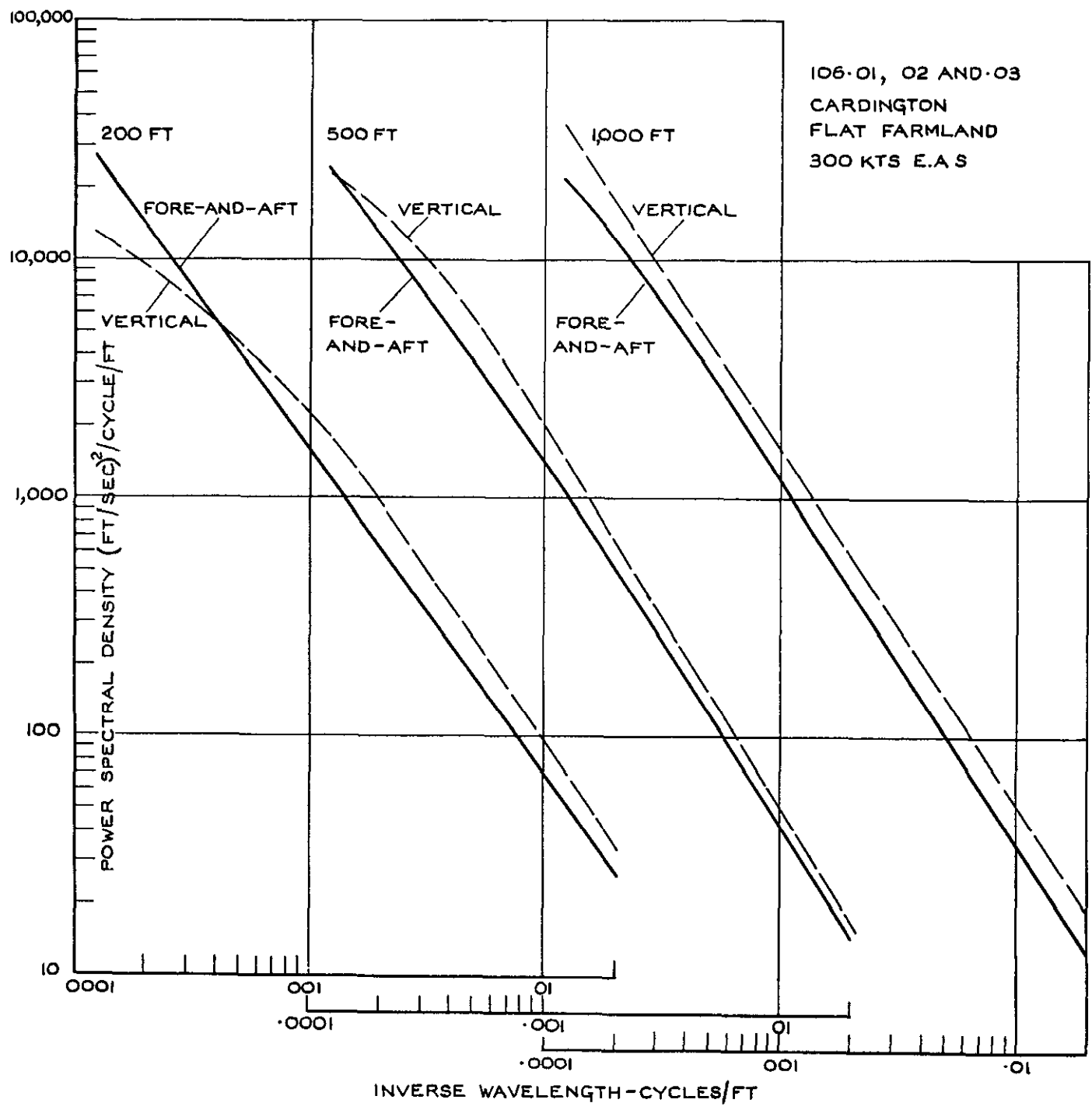
FIG 20. COMPARATIVE POWER SPECTRA OF VERTICAL AND
FORE-AND-AFT TURBULENCE AT DIFFERENT HEIGHTS.
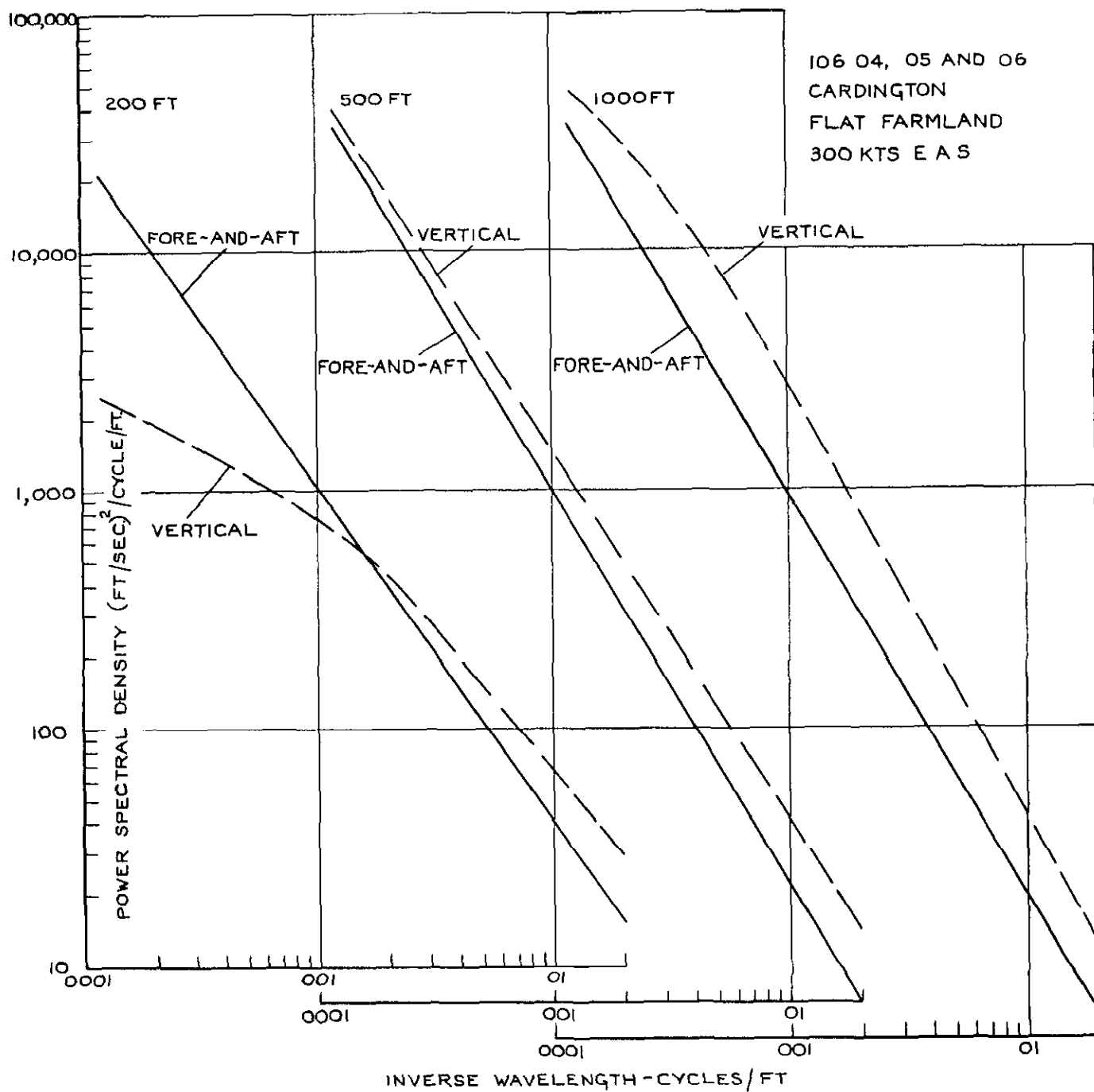
FIG 21. COMPARATIVE POWER SPECTRA OF VERTICAL AND
FORE-AND-AFT TURBULENCE AT DIFFERENT HEIGHTS.
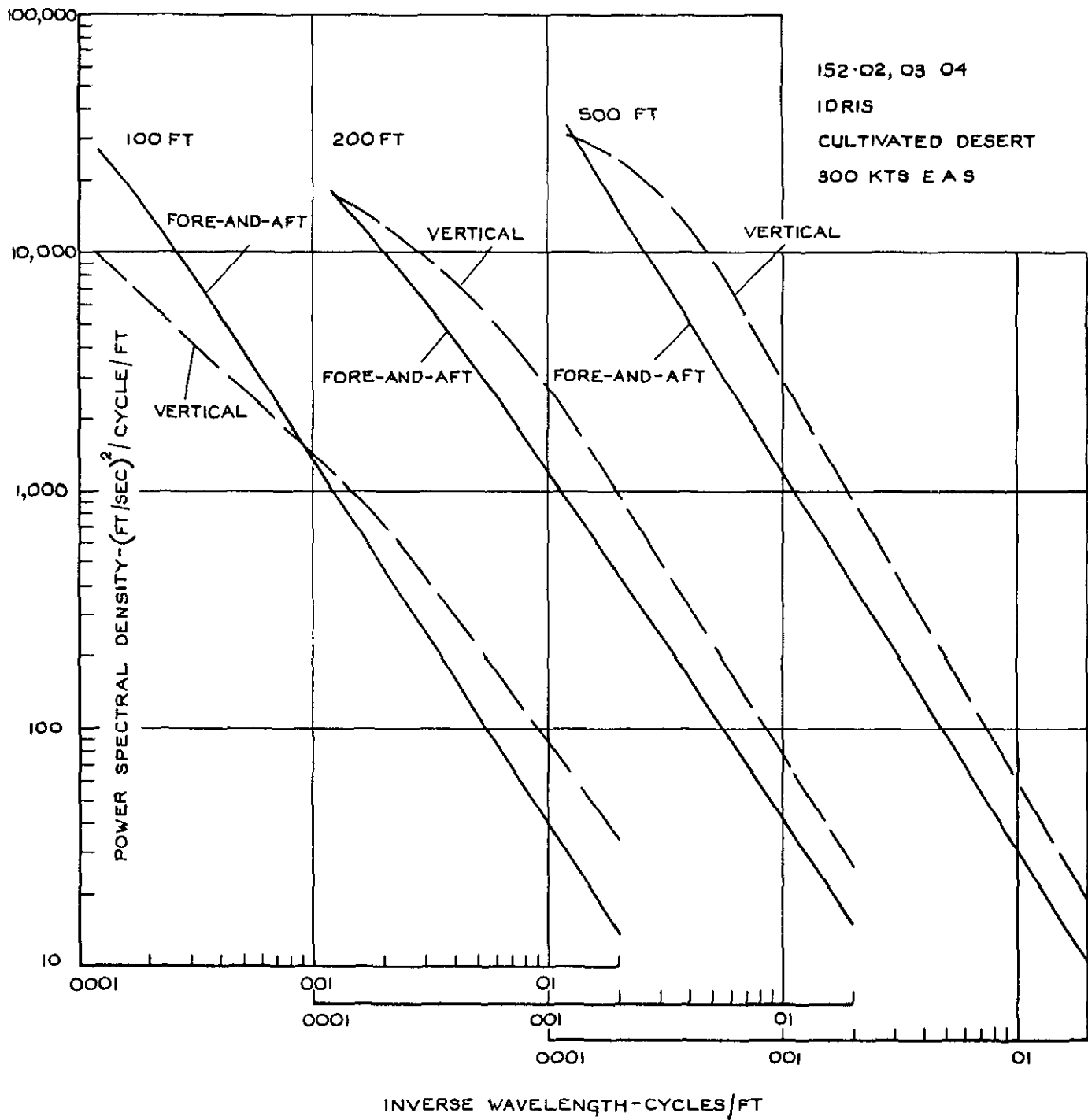
FIG.22. COMPARATIVE POWER SPECTRA OF VERTICAL AND
FORE-AND-AFT TURBULENCE AT DIFFERENT HEIGHTS.
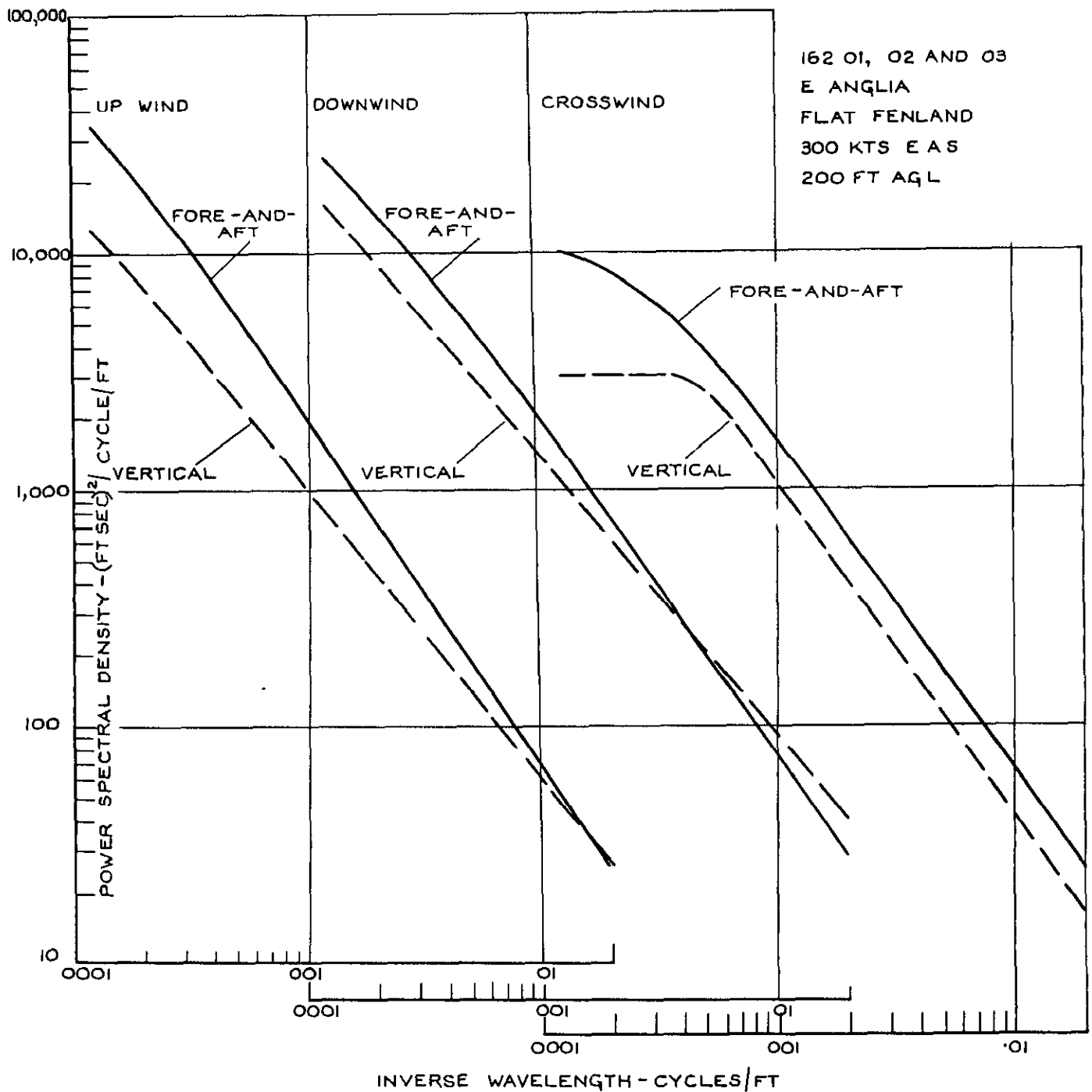
FIG. 23. COMPARATIVE POWER SPECTRA OF VERTICAL AND FORE-AND-AFT TURBULENCE FOR DIFFERENT WIND DIRECTIONS RELATIVE TO FLIGHT PATH.
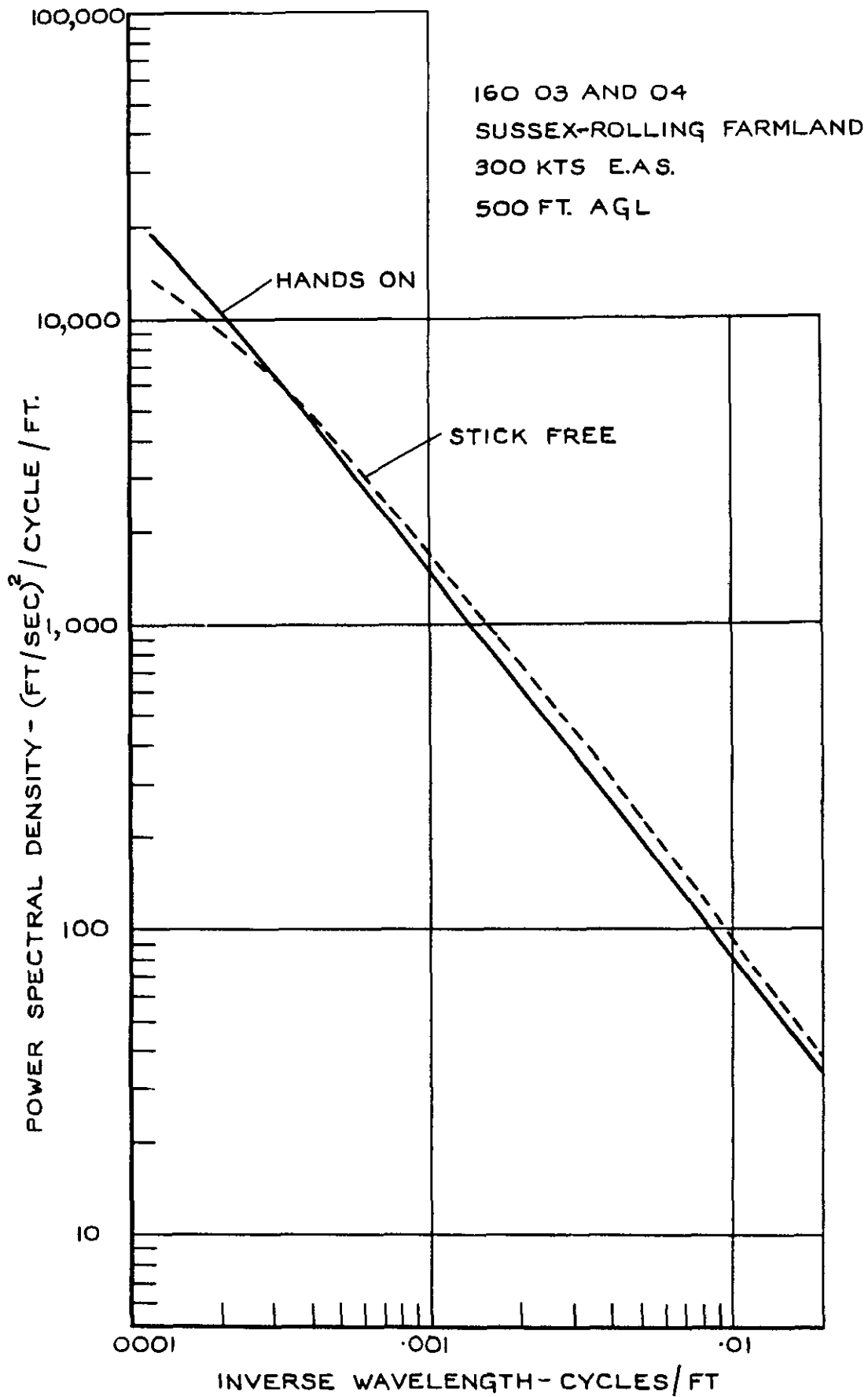
FIG. 24.(a) COMPARATIVE POWER SPECTRA-HANDS ON & STICK FREE
VERTICAL VELOCITY OF TURBULENCE.
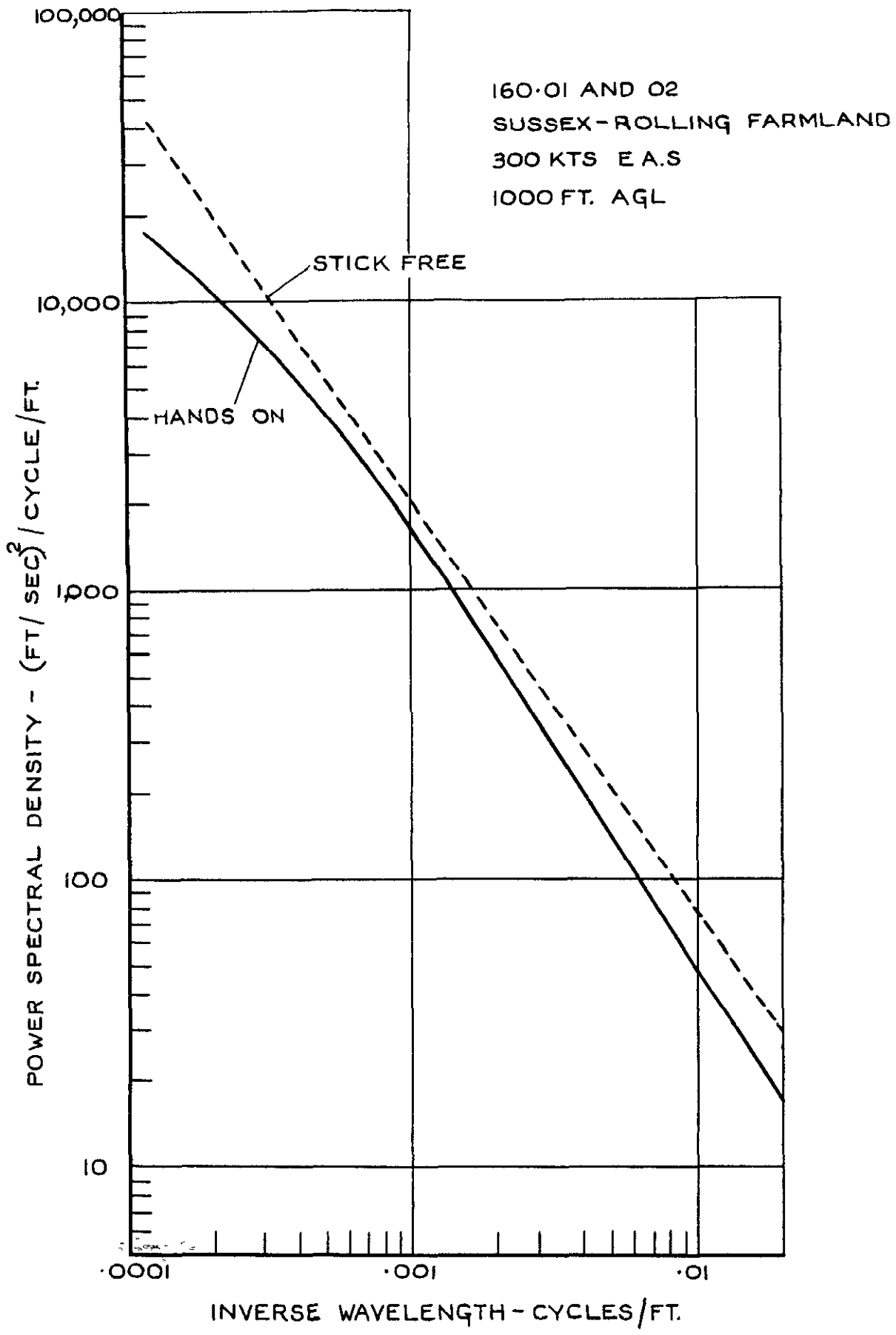
FIG.24.(b)COMPARATIVE POWER SPECTRA-HANDS ON & STICK FREE
VERTICAL VELOCITY OF TURBULENCE.
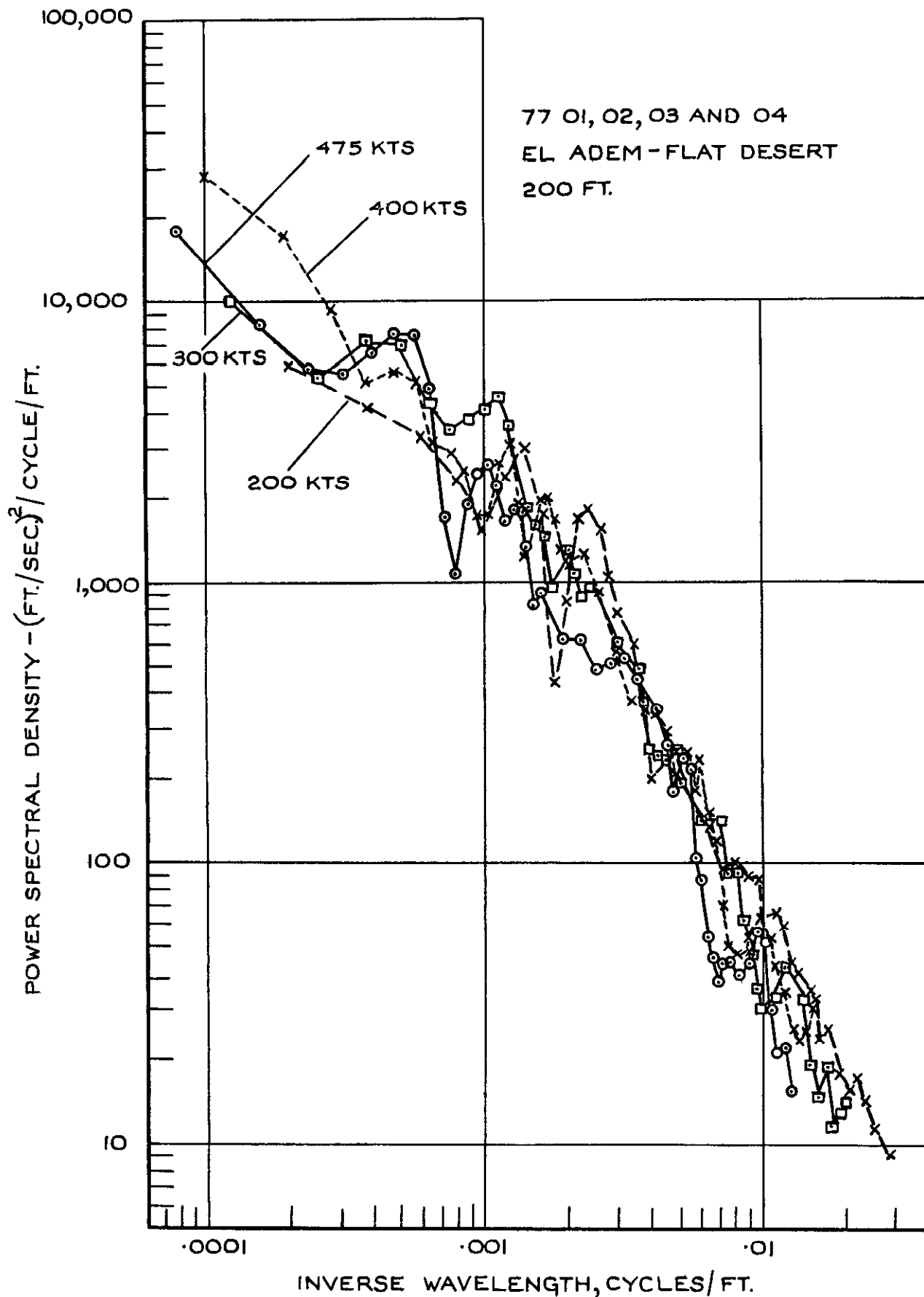
FIG.25. COMPARATIVE POWER SPECTRA AT VARIOUS AIRSPEEDS
VERTICAL VELOCITY OF TURBULENCE.
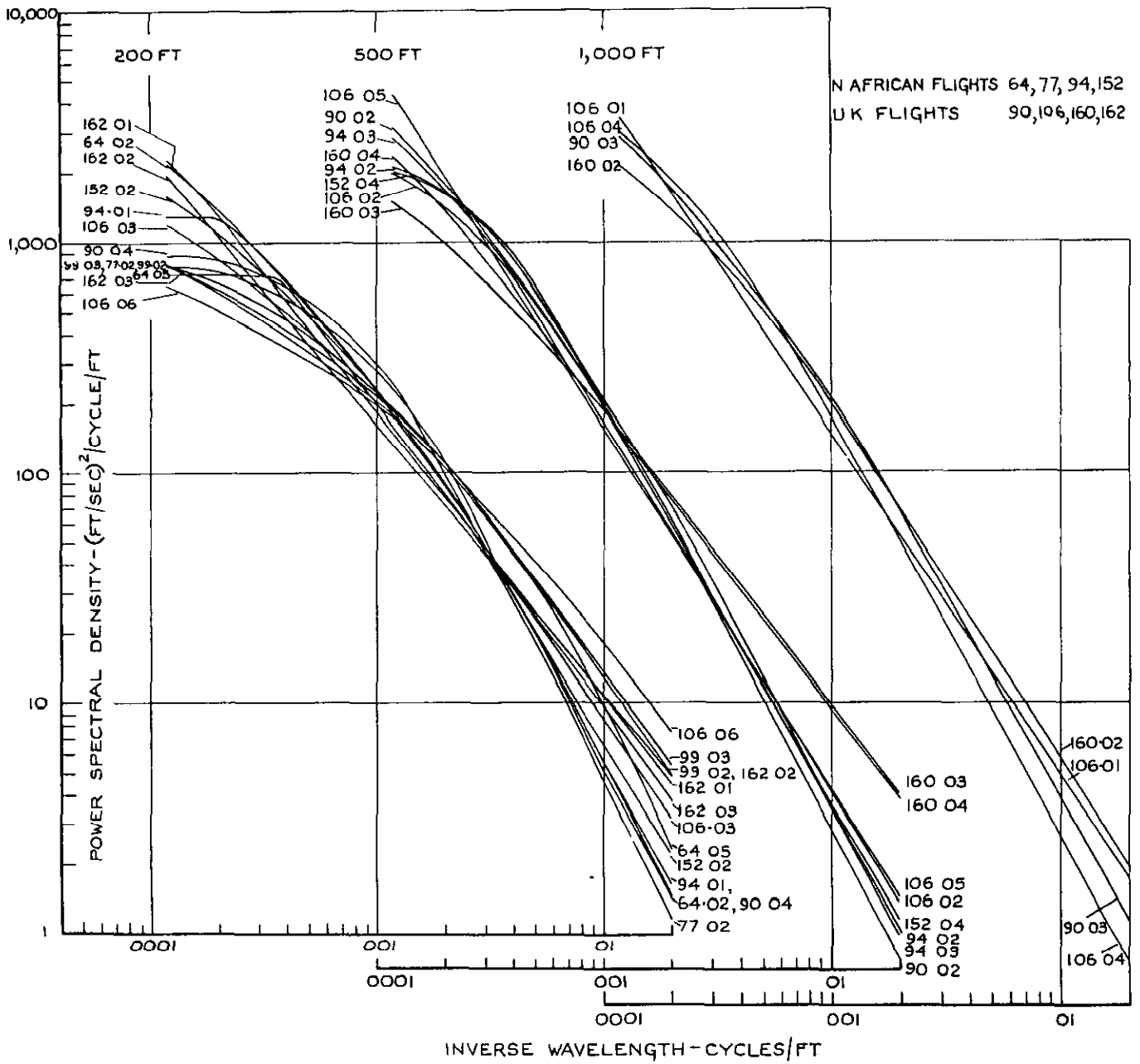
FIG. 26. COMPOSITES OF NORMALISED POWER SPECTRA OF VERTICAL TURBULENCE AT DIFFERENT HEIGHTS.
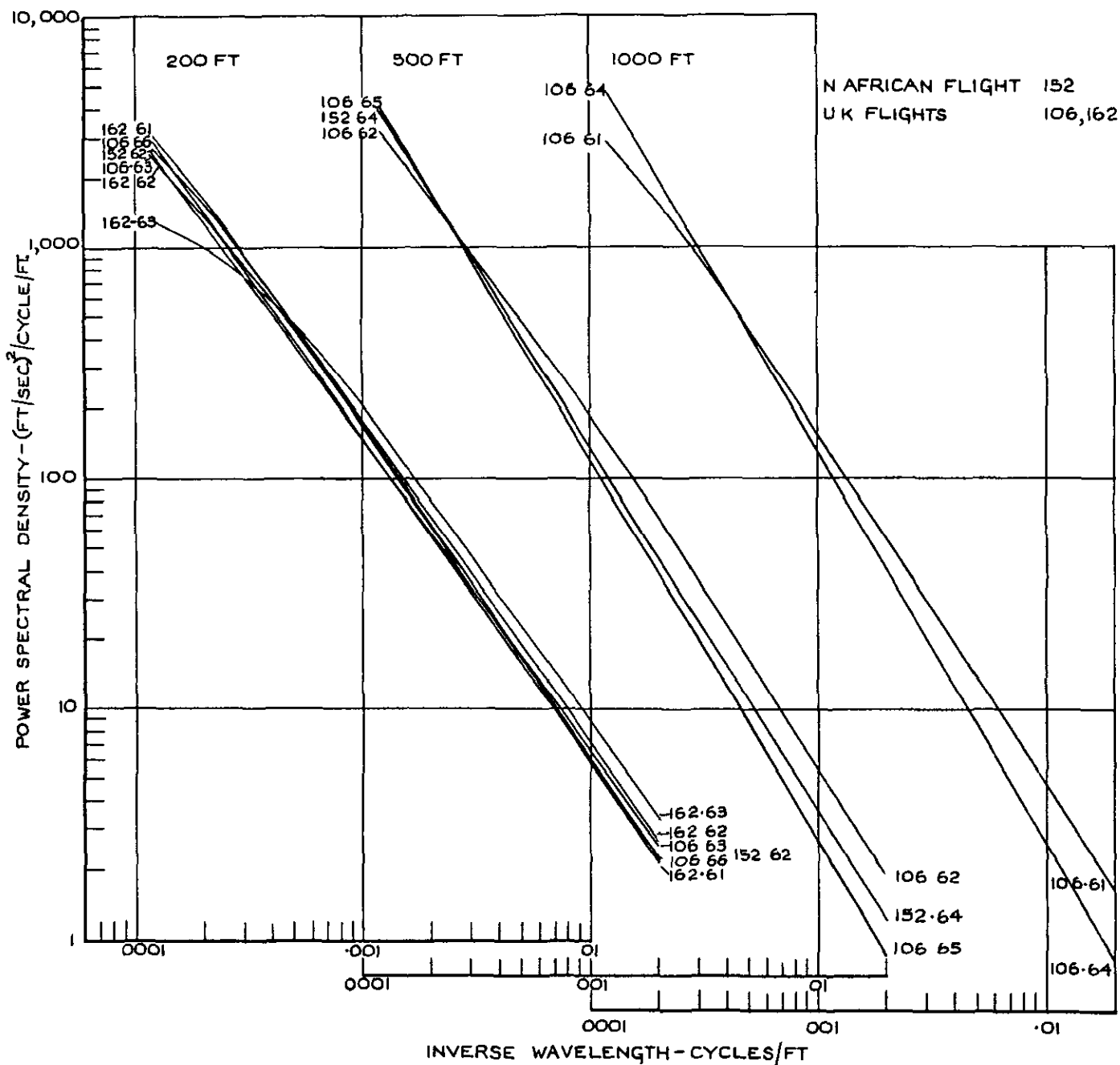
FIG.27. COMPOSITES OF NORMALISED POWER
SPECTRA OF FORE-AND-AFT TURBULENCE
AT DIFFERENT HEIGHTS.
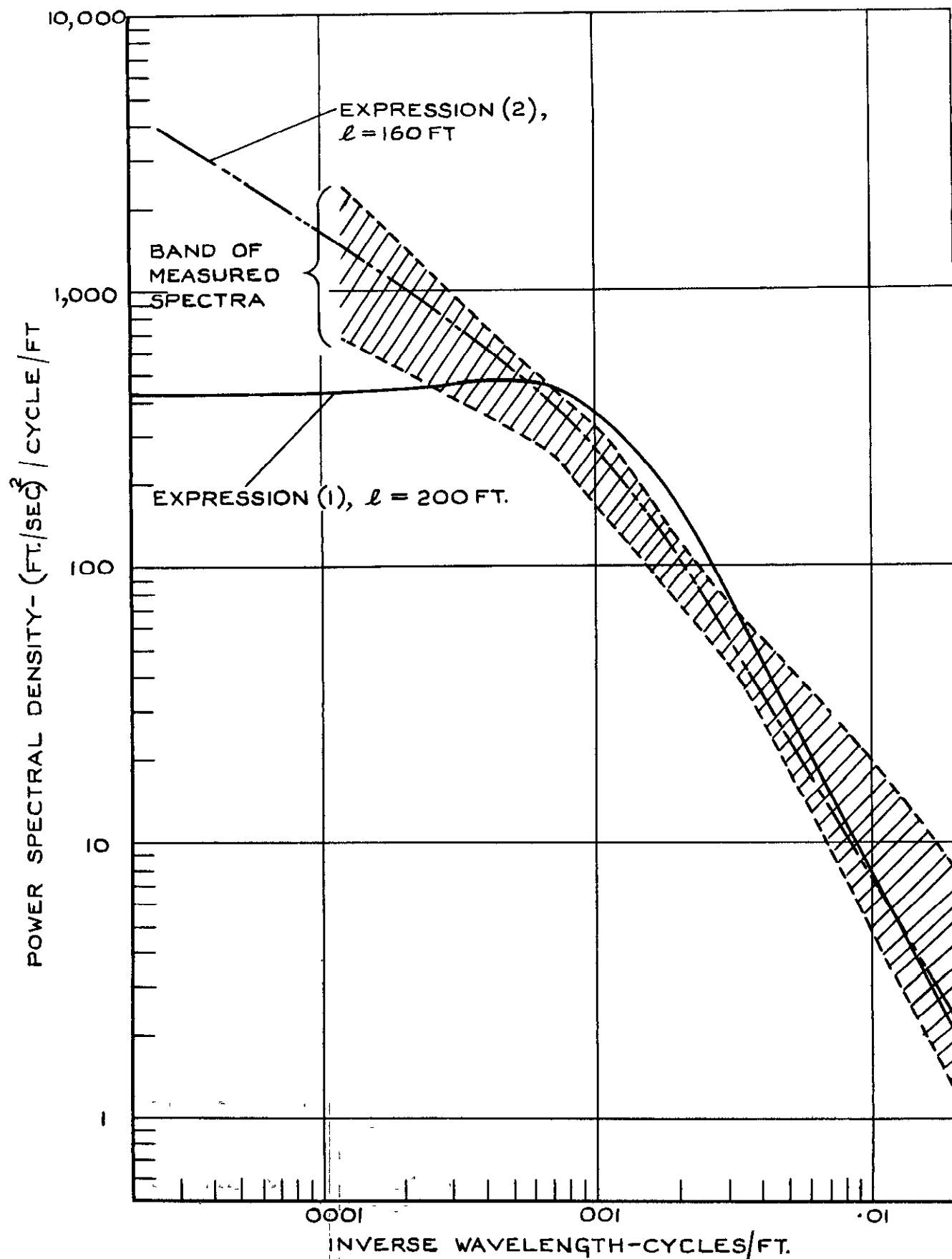
FIG. 28. MEASURED AND FITTED NORMALISED POWER SPECTRA
VERTICAL TURBULENCE AT 200 FT.
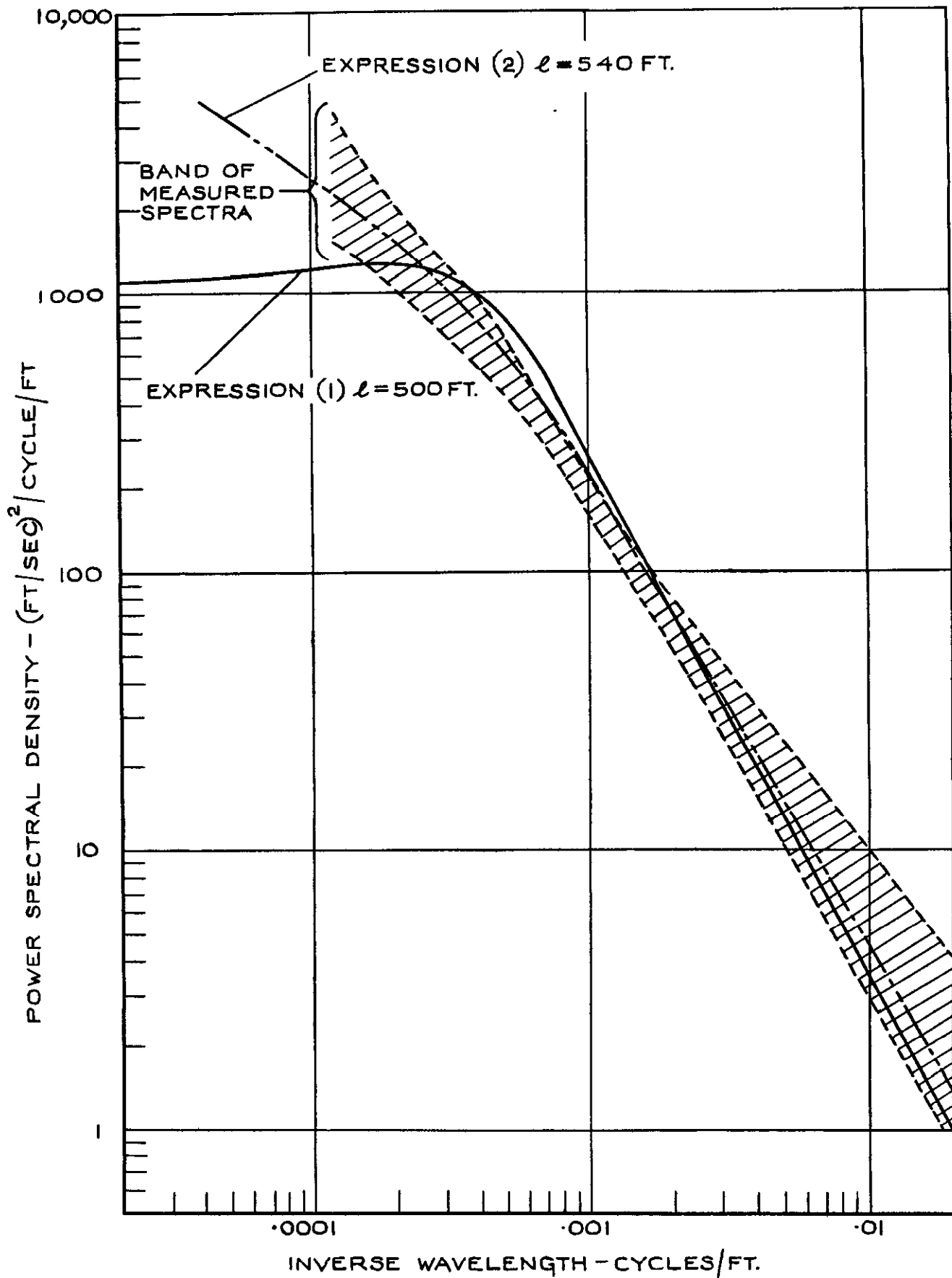
FIG.29. MEASURED AND FITTED NORMALISED POWER SPECTRA
VERTICAL TURBULENCE AT 500 FT.
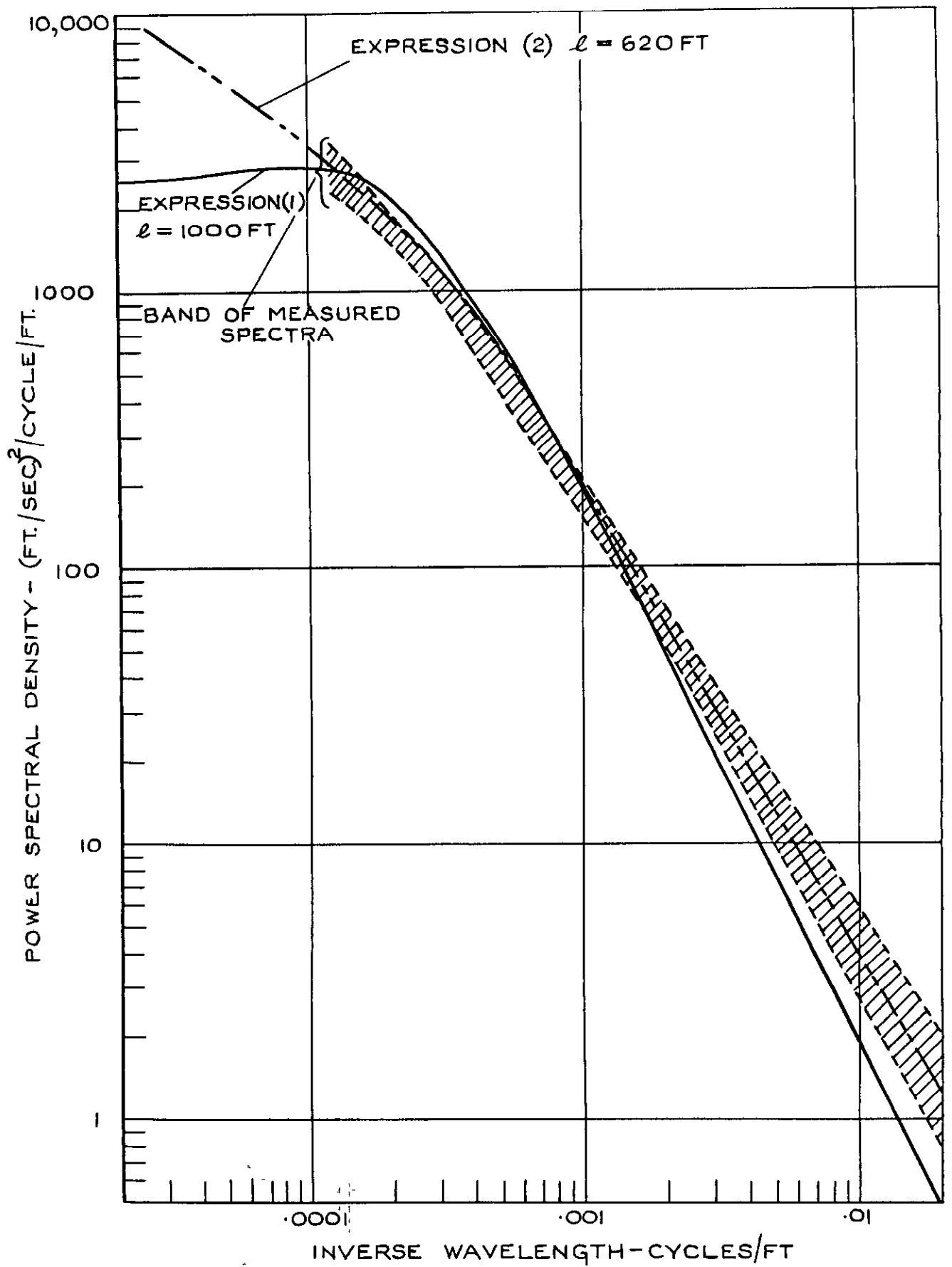
FIG. 30. MEASURED AND FITTED NORMALISED POWER SPECTRA
VERTICAL TURBULENCE AT 1000 FT.
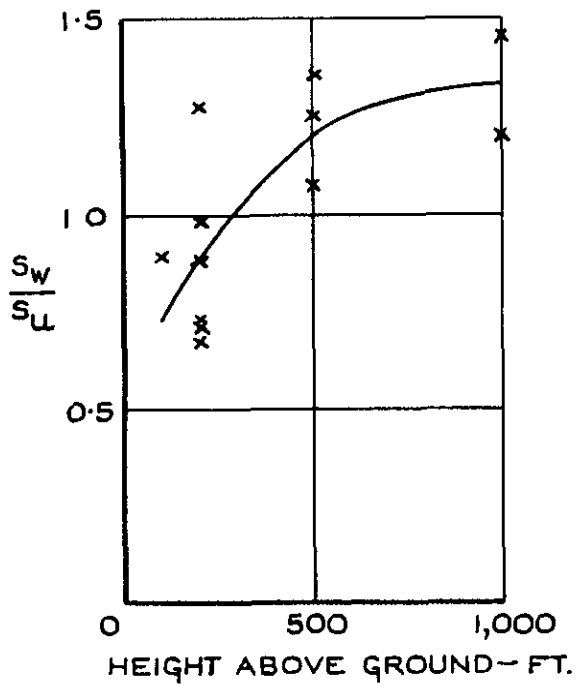
FIG.31. RATIO OF INTENSITY OF VERTICAL TO FORE-AND-AFT
TURBULENCE AT DIFFERENT HEIGHTS.
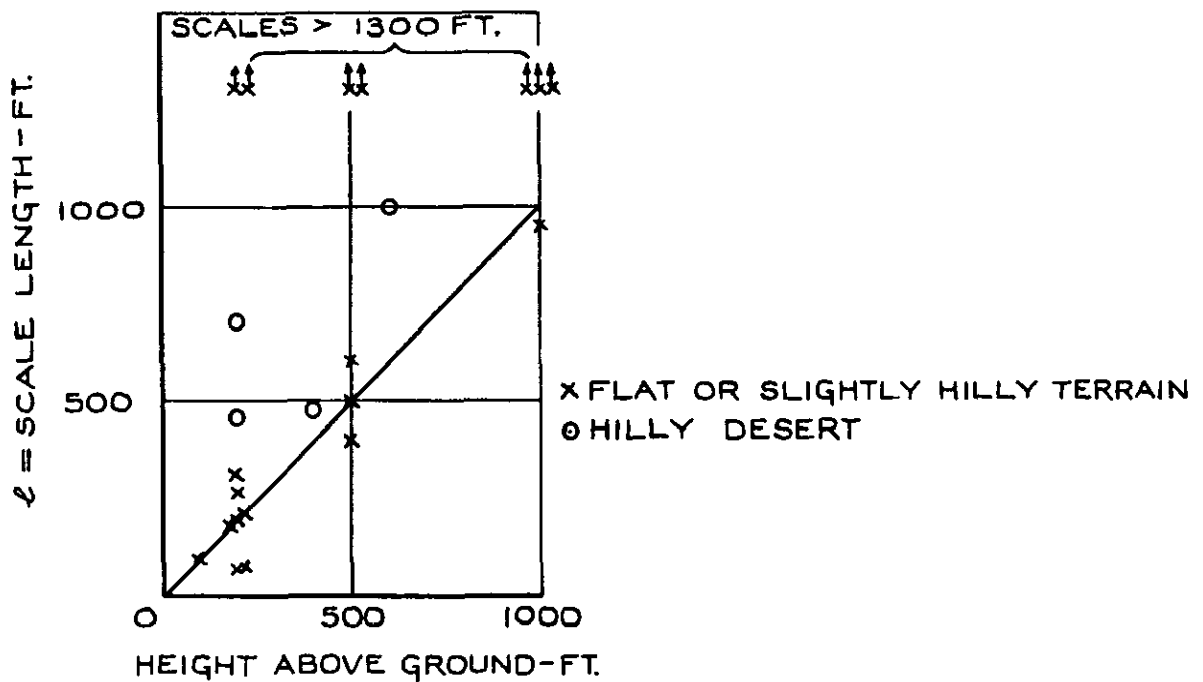
FIG.32. SCALE LENGTH OF VERTICAL TURBULENCE
AT DIFFERENT HEIGHTS.

A.R.C. C.P. No. 733   551.551 :
                                                                   539.431

POWER SPECTRA OF LOW LEVEL ATMOSPHERIC TURBULENCE MEASURED FROM AN
AIRCRAFT.   Burns, A.   April, 1963.

The note presents 30 power spectra of the vertical component of atmos-
pheric turbulence and 12 power spectra of the horizontal component measured
on a Canberra aircraft while flying at low altitude. The spectra relate to
various types of terrain in U.K. and N. Africa. Comparative spectra are
given for different heights above ground within the height band 200 to
1000 ft. Some information is given on the associated meteorological
conditions.
The results are discussed briefly with a view to generalising the basic
shape of the spectra for the purpose of predicting fatigue loads. Compari-
son is made with proposed analytical expressions defining the basic shape.