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Some Studies of Pressure Distributions on the Windward Surfaces of Conical Bodies at High Supersonic Speeds.

By

A. Akers

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SOME STUDIES OF PRESSURE DISTRIBUTIONS ON THE WINDWARD SURFACES OF CONICAL BODIES AT HIGH SUPERSONIC SPEEDS

by

A. Akers*

SUMMARY

This paper reports a study of the results of pressure measurements on conical lifting bodies, of various cross-sections, at high supersonic Mach numbers. The work was done during a Vacation Consultancy period at the R.A.E. Farnborough, from 7th July to 18th August 1962.

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1 INTRODUCTION

Previous studies[1,2,3] of pressure distributions on conical bodies of non-circular cross-section, for the Mach number range $1{\cdot}3 < M < 4$, have shown that while linear theory predicts quite well the pressure distribution on bodies with subsonic edges at zero lift, it was inadequate for predicting the pressures on the windward surfaces of such shapes for all but very small angles of incidence. However, it was noted by Squire[1], in regard to bodies of rhombic cross-section at $M = 4$, that the empirical Newtonian and tangent-cone methods gave good predictions of the increase of the mean pressure on the windward surface with increase of incidence, although the agreement with the actual pressure distribution was less good, particularly near the leading edges. Both of these methods are "local" theories, which give the pressure at a point on a body surface in terms of the local incidence of the surface at that point to the free stream. The Newtonian approximation gives the pressure coefficient as $C_p = 2 \sin^2\theta$, where θ is the local incidence; in the tangent-cone method, the pressure at a point where the local incidence is θ, is assumed to be the same as that on a circular cone at zero incidence, whose semi-apex angle is θ. Both of these methods rely on the assumption that at hypersonic speeds the pressure on the windward surfaces of a body is determined principally by the local angle between the surface and the free stream flow[4].

The aim of this paper is therefore to investigate the applicability of the Newtonian and tangent-cone approximations in predicting pressures on the windward surfaces of conical lifting bodies, over a range of incidence and Mach number above which linear theory has been found to be inadequate, but below the range of Mach number where hypersonic approximations are usually expected to be valid.

2 RESULTS AND DISCUSSION

2.1 General

Many calculations of overall aerodynamic coefficients have been made by the Newtonian method (for example Ref.5), the justification for this being the agreement of experimental overall force measurements with the values calculated by this method. However, it has been found[6] with circular cones, that pressure distributions calculated by the Newtonian method often differ significantly from experimental distributions at supersonic and low hypersonic Mach numbers. Thus agreement in overall coefficients can be fortuitous, due to discrepancies between experimental and calculated distributions cancelling out during integration. For other body shapes, it seems likely that agreement of experimental and calculated values of overall forces might also be fortuitous, which leads to the conclusion that the only reliable way of checking the various empirical methods is by comparison of predicted and measured pressure distributions.

The conical body shapes considered in this Note have the following cross-section shapes:-

(i) Elliptic
(ii) Biconvex circular arc
(iii) Circular cone segment
(iv) Rhombic.

In the above family it can be seen that there is a progressive deterioration of simplicity of shape in the order in which they are listed. Although flow patterns have not been studied in detail, there is some evidence to show that there is also a decreasing correspondence between the cross-sectional shapes of the bodies and their enveloping shocks, on going from shape (i) to shape (iv), which would probably lead to an increasing complication in flow pattern and pressure distribution. It can also be argued that the concept of a thin shock layer, which is an essential and fundamental assumption of the Newtonian and tangent-cone approximations is increasingly violated in going from shape (i) to shape (iv).

Details of wings for which pressure distribution results are available are given in Table 1. The geometric details given are cross-section shape, leading-edge angle in cross-section, and aspect ratio; the aerodynamic properties given are Mach number, and slenderness ratio $\beta s/\ell$.

All results are plotted in the form of "impact coefficients", K, where $K = C_p/\sin^2\theta$. Expressions for calculating the local incidence, θ, for wings of biconvex and rhombic cross-section, respectively, are given in Appendices 1 and 2. The advantage of plotting results in this form is that the percentage difference between calculated and experimental values is clearly shown for all values of the local incidence, rather than differences being obscured at low values of θ by the scale used for plotting results, as would be the case in a conventional plot of C_p vs. θ.

2.2 Elliptic cones

Results are available for two elliptic cones with ratios a/b of 1·39 and 1·78, at Mach numbers of 3·09 and 6·0 [7]; in this report, the experimental pressure coefficients were compared with values estimated from the Newtonian approximation (i.e. $C_p = 2 \sin^2\theta$). From this comparison it was possible to calculate values of the impact coefficient, K, and the local incidence, θ, and these values are plotted in Figs.3 and 4.

The experimental results show less scatter, and closer agreement with the tangent-cone method, for the higher Mach number (i.e. Fig.4), and for the cone with the lower value of a/b (Figs.3(a) and 4(a)). This is perhaps to be expected, as it has been shown elsewhere[6] that an approximately defined single curve of K versus θ is obtained with results from circular cones at high Mach numbers.

2.3 Cones of biconvex cross-section

Results for this body shape are available for the Mach number range $1{\cdot}3 < M < 4$, aspect ratios of 2/3, 1 and 4/3, and edge angles of 30, 60 and 120 degrees[1,2]. An expression for the local incidence on the surface of such shapes is derived in Appendix 1, and plots of impact coefficient against local incidence are given in Fig.5.

It is found that pressures on only about the central 60% of the lower surface of these bodies (i.e. $-0{\cdot}6 < \eta < 0{\cdot}6$), are correlated by plotting them against local incidence. Outboard of this region, the proximity of the leading edges and the leeward surface of the body reduces the pressures substantially. For the higher angles of incidence, a typical value of impact coefficient in areas close to the leading edges is about half the value of K over the central 60% of the wing. Clearly, the Newtonian approximation of a constant value for K is quite unrealistic in this case. It should be noted though, that agreement between experimental and tangent-cone values of K improves with increase of Mach number (Fig.5(b)), except at very low local angles of incidence.

2.4 Cone segments

Results are available[8] for 1/8 and $\frac{1}{2}$-cone segments at a Mach number of 4·3, and are plotted in Fig.6(a) and (b).

For the $\frac{1}{2}$-cone segment, an approximately defined curve of K vs θ is obtained, as was the case with complete cones[6]. At first sight, a reasonable correlation is obtained also with the results for the 1/8-cone segment, but closer inspection reveals that distinct curves of K vs θ are obtained for each value of $\eta(= y/s)$, K decreasing with increase of η in a way similar to that found with wings of biconvex cross-section. Thus the remarks made in section 2.3 regarding the use of the Newtonian and tangent-cone approximations to biconvex wings, apply also to cone segments where the edge-angle in cross-section is less than 90 degrees.

2.5 Cones of rhombic cross-section

Results for this body shape are available for the Mach number range $1 \cdot 3 < M < 4$, aspect ratios of 2/3, 1 and 4/3, and edge angles of 30 and 60 degrees[1,2]. An expression for the local incidence on the surface of such shapes is derived in Appendix 2. It should be noted that since the body shape is made up of four flat facets, the local incidence is constant over each facet. The Newtonian and tangent-cone approximations therefore predict a constant pressure over each facet, for a given body attitude. However, experiments show that this is not so, higher pressures being obtained on the central region of the body than near the leading edges. The comparison of experimental results with values calculated by the tangent-cone method in Fig.7 is therefore restricted to particular spanwise stations, rather than a region of the body surface as previously. Even under these restrictions, it is found that a reasonable correlation of K vs θ is only obtained for bodies with nominally subsonic leading edges (Fig.7), with poorer agreement with tangent-cone values than was obtained in the case of bodies of circular-arc section.

3 CONCLUSIONS

Since both the Newtonian and tangent-cone approximations are "local" methods, which give the pressure at a point on a body in terms of the local incidence of the body to the free stream, θ, the most revealing way of comparing experimental and calculated pressure distributions is to reduce pressure coefficients to "impact coefficients, K" by dividing them by $\sin^2\theta$.

For all the body shapes considered, it was found that plotting impact coefficients against local incidence gave reasonable correlations of the results, the best correlations being obtained at the higher Mach numbers and local angles of incidence, and for body cross-section shapes near to circular. However, the correlations obtained for the various body shapes all differed from each other, i.e. no single curve of K vs θ could be defined which was applicable to all the body shapes considered, at a particular Mach number.

It was found that the Newtonian approximation of K = 2 was only adequate for local angles of incidence above about 25 degrees, and then only for the higher Mach numbers ($M > 4$) and for regions of the body where there were no rapid changes in surface shape.

For local angles of incidence less than 25 degrees, the tangent-cone method gave better estimates of impact coefficient than the Newtonian method, but generally failed to predict accurately the high values of K obtained at small local angles of incidence, i.e. for $\theta < 10$ degrees. This failing might not always be of great consequence, since pressures on surfaces where $\theta < 10$ degrees would be small, typically $C_p < 0{\cdot}1$.

Clearly then, for the supersonic Mach number range, and for most body cross-section shapes, the indiscriminate use of these empirical methods is unrealistic. Use of the Newtonian method can only be expected to give predictions of reasonable accuracy when the local incidence is greater than about 25 degrees, and then only in regions where there are no rapid changes in body shape; the tangent-cone method should only be used for body shapes not far removed from circular cross-section, and then not in regions close to sharp edges.

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APPENDIX 1

DERIVATION OF LOCAL INCIDENCE FOR DELTA WINGS OF CIRCULAR-ARC CROSS-SECTION AT COMBINED PITCH AND YAW

(a) Wing of circular-arc section in the Oyz plane

Diagrams of this type of wing are given in Figs.1(a) and (b).

Plane ADCFB is tangential to the wing surface at the generator DF, and intersects the Ox,Oy,Oz axes at A, B and C respectively. Let OF be of length R and make an angle ϕ with the Oz axis. with a maximum value of ϕ_1 at the wing tip, and let DE be of length r. For a wing of root-chord c, semi-span s, and semi-thickness t, the angles δ and ε are defined by

$$\varepsilon = \tan^{-1}\frac{s}{c} \qquad \delta = \tan^{-1}\frac{t}{c}$$

It follows that

$$s = R\sin\phi_1 = c\tan\delta \quad \text{thus} \quad R = \frac{c\tan\delta}{\sin\phi_1} \tag{1}$$

and

$$r = R\cos\phi_1 = \frac{c\tan\delta}{\tan\phi_1} \tag{2}$$

Triangles AED and AOC are similar and thus

$$\frac{AE}{AO} = \frac{x_A - c}{x_A} = \frac{DE}{CO} = \frac{r}{R\sec\phi}$$

$$\left.\begin{aligned} \text{i.e.} \quad x_A &= \frac{cR\sec\phi}{R\sec\phi - r} \\ \text{also} \quad y_B &= R\,\mathrm{cosec}\,\phi \\ \text{and} \quad z_C &= R\sec\phi \end{aligned}\right\} \tag{3}$$

Thus from (1), (2) and (3)

$$x_A = \frac{c\sec\phi}{\sec\phi - \cos\phi_1} \qquad y_B = \frac{c\tan\delta}{\sin\phi_1\sin\phi} \qquad z_C = \frac{c\tan\delta}{\sin\phi_1\cos\phi} \tag{4}$$

From this we obtain the equation of plane ABC in intercept form as

$$\frac{x}{x_A} + \frac{y}{y_B} + \frac{z}{z_C} = 1$$

and the direction cosines of a perpendicular to this plane as

$$\ell = \frac{1/x_A}{\sqrt{\frac{1}{x_A^2} + \frac{1}{y_B^2} + \frac{1}{z_C^2}}}, \quad m = \frac{1/y_B}{\sqrt{\frac{1}{x_A^2} + \frac{1}{y_B^2} + \frac{1}{z_C^2}}}, \quad n = \frac{1/z_C}{\sqrt{\frac{1}{x_A^2} + \frac{1}{y_B^2} + \frac{1}{z_C^2}}} \qquad (5)$$

For an angle of incidence α, and an angle of yaw β, the direction cosines of the free stream direction are

$$\ell' = \cos\alpha \qquad m' = \cos\beta \qquad n' = \sqrt{\sin^2\alpha - \cos^2\beta} \qquad (6)$$

and

$$\sin\theta = \cos(90^\circ - \theta) = \ell\ell' + mm' + nn' \qquad (7)$$

where θ is the local angle of incidence of the generator DF on the wing surface.

Thus from (5), (6) and (7)

$$\sin\theta = \frac{\frac{(\sec\phi - \cos\phi_1)}{\sec\phi}\cos\alpha + \frac{\sin\phi_1 \sin\phi}{\tan\delta}\cos\beta + \frac{\sin\phi_1 \cos\phi}{\tan\delta}\sqrt{\sin^2\alpha - \cos^2\beta}}{\sqrt{\frac{(\sec\phi - \cos\phi_1)^2}{\sec^2\phi} + \frac{\sin^2\phi_1}{\tan^2\delta}}}$$

... (8)

which for zero yaw, when $m' = 0$ and $n' = \sin\alpha$ reduces to

$$\sin\theta = \frac{\frac{(\sec\phi - \cos\phi_1)\cos\alpha}{\sec\phi} + \frac{\sin\phi_1 \cos\phi \sin\alpha}{\tan\delta}}{\sqrt{\frac{(\sec\phi - \cos\phi_1)^2}{\sec^2\phi} + \frac{\sin^2\phi_1}{\tan^2\delta}}} \qquad (9)$$

For a circular cone, $\phi_1 = 90°$, and equation (9) reduces to

$$\sin\theta = \sin\delta\cos\alpha + \cos\delta\sin\alpha\cos\phi \tag{10}$$

(b) Cone segment

In Fig.1(c), VQP is a segment cut from a cone of semi-apex angle ε, but VPR is the segment which we must consider. By doing this we may obtain ϕ_1 measured in the plane containing OPR (see Figs.1(a) and (c)) and so be able to use directly the expressions derived in section (a) of this Appendix.

$$\cos\phi_1' = \frac{O'P}{O'Q} \tag{11}$$

where ϕ_1' is measured in the plane containing O'PQ

$$\cos\phi_1 = \frac{OP}{OR} = \frac{OP}{OP + PR} \tag{12}$$

But

$$OP = \frac{O'P}{\cos\sigma'}, \tag{13}$$

therefore

$$\cos\phi_1 = \frac{1}{1 + \frac{PR}{O'P}\cos\sigma'} \tag{14}$$

from (12) and (13).

The measured quantities are ϕ_1', thickness PR and length PV. Thus O'P may be calculated from (11) and deduction of σ' from PR and PV yields, on substitution in (14) the value for ϕ_1.

It should be noted that as $\sigma \to 0$,

$$PQ \to PR \to O'Q$$

and

$$O'P \to 0$$

therefore

$$\cos\phi_1 \to \frac{1}{\infty}$$

i.e.

$$\phi_1 \to \frac{\pi}{2}$$

APPENDIX 2

DERIVATION OF LOCAL INCIDENCE FOR DELTA WINGS OF RHOMBIC CROSS-SECTION AT COMBINED PITCH AND YAW

A diagram of this type of wing is given in Fig.2. The analysis is similar to that in Appendix 1, but is simplified by the local incidence being constant over each of the facets which make up the body shape.

The equation of plane ABC in intercept form is

$$x + y \cot \delta + z \cot \varepsilon - 1 = 0 \tag{1}$$

and the direction cosines of a perpendicular to this plane are

$$\ell = \frac{1}{\sqrt{1 + \cot^2\delta + \cot^2\varepsilon}}, \quad m = \frac{\cot \delta}{\sqrt{1 + \cot^2\delta + \cot^2\varepsilon}}, \quad n = \frac{\cot \varepsilon}{\sqrt{1 + \cot^2\delta + \cot^2\varepsilon}} \qquad \ldots (2)$$

As in Appendix 1, the direction cosines of the free stream direction are

$$\ell' = \cos \alpha , \quad m' = \cos \beta , \quad n' = \sqrt{\sin^2\alpha - \cos^2\beta} \tag{3}$$

From (1), (2) and (3) the local incidence, θ, is obtained as

$$\sin \theta = \frac{\cos \alpha + \cos \beta \cot \delta + \sqrt{\sin^2\alpha - \cos^2\beta} \cot \varepsilon}{\sqrt{1 + \cot^2\delta + \cot^2\varepsilon}} \tag{4}$$

which for zero yaw reduces to

$$\sin \theta = \frac{\cos \alpha + \sin \alpha \cot \varepsilon}{\sqrt{1 + \cot^2\delta + \cot^2\varepsilon}} \tag{5}$$

TABLE 1

Details of body shapes

Wing number	Reference	Section shape	L.E. angle in cross-section (degrees)	A.R.	M	$\frac{\beta s}{\ell}$
1	7	ellipse	$\frac{a}{b}$ = 1·39		3·09 6·0	
2	7	ellipse	$\frac{a}{b}$ = 1·78		3·09 6·0	
3	1	biconvex	60	2/3	4·0	0·65
4	1	biconvex	60	4/3	4·0	1·28
5	2	biconvex	60	1	1·3 2·0 2·8	0·21 0·33 0·65
6	2	biconvex	120	1	1·3 2·0 2·8	0·21 0·33 0·65
7	8	1/8-cone	29	1·10	4·3	1·15
8	8	1/2-cone	90	1·07	4·3	1·12
9	1	rhombic	30	2/3	4·0	0·65
10	1	rhombic	30	4/3	4·0	1·28
11	1,2	rhombic	60	2/3	1·3 1·6 2·1 2·8 4·0	0·14 0·21 0·31 0·43 0·65
12	1,2	rhombic	60	1	1·3 2·0 2·8 4·0	0·21 0·33 0·65 0·96
13	1	rhombic	60	4/3	4·0	1·28

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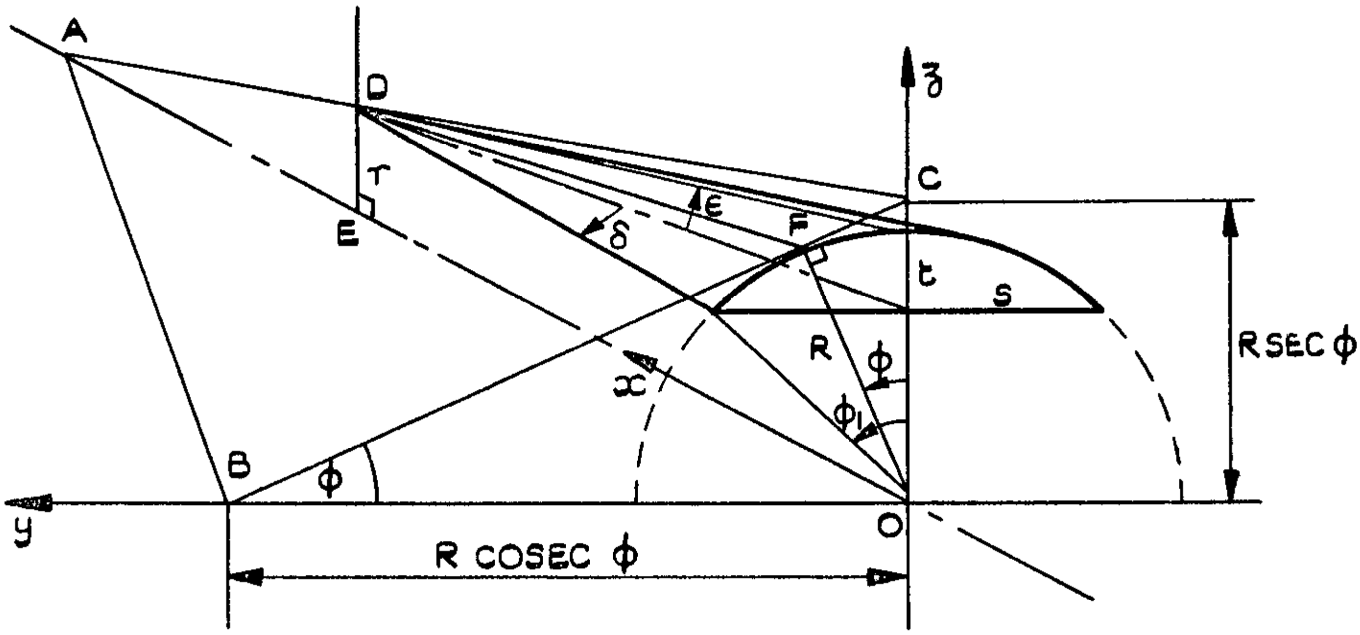
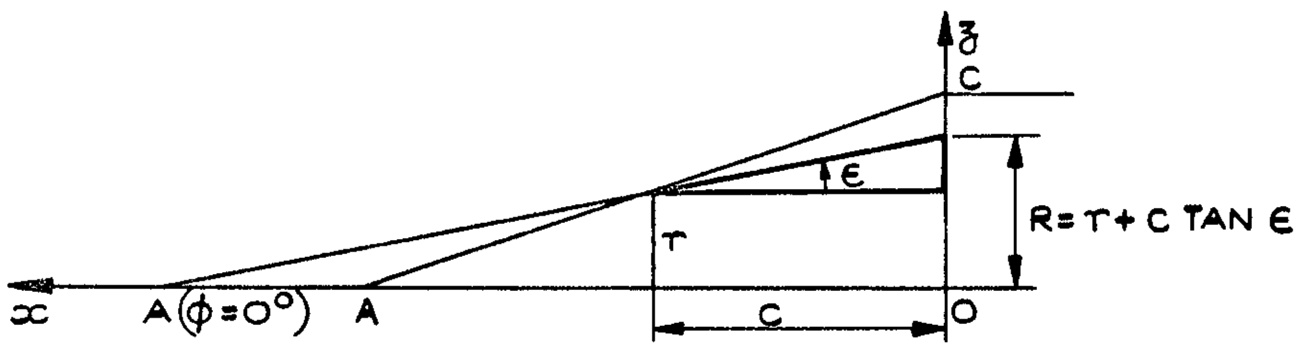
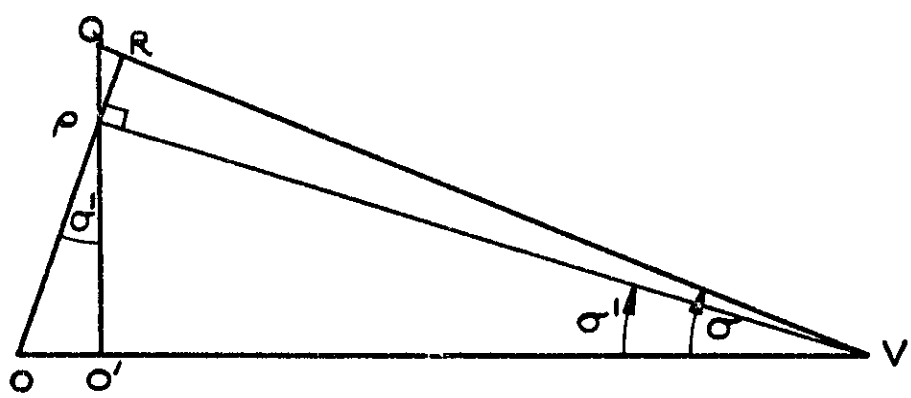


(a) BICONVEX WING SHOWING TANGENT PLANE ADCFB INTERSECTING AXES Ox, Oy, Oz.



(b) THE Oxz PLANE.



(c) ROTATION OF AXES FOR CONE SEGMENT.

FIG. 1.(a - c) WINGS OF CIRCULAR-ARC SECTION. GEOMETRY AND NOTATION.

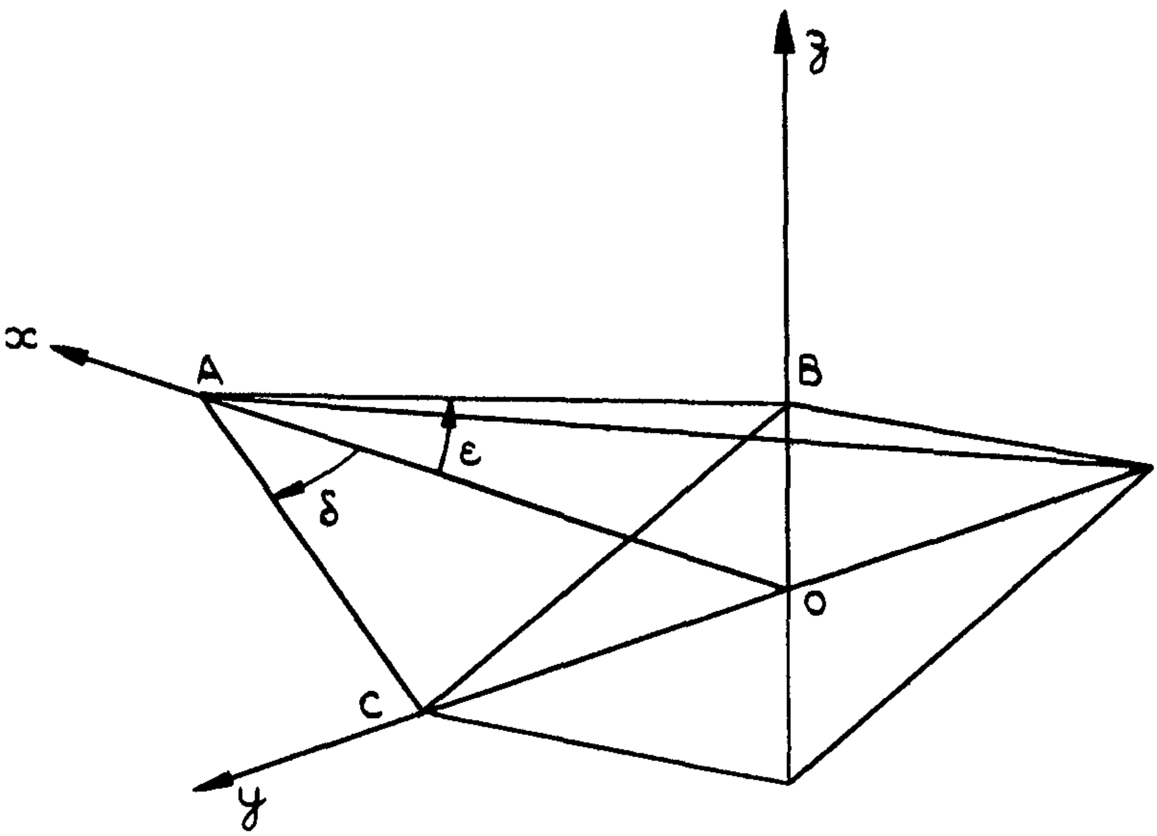
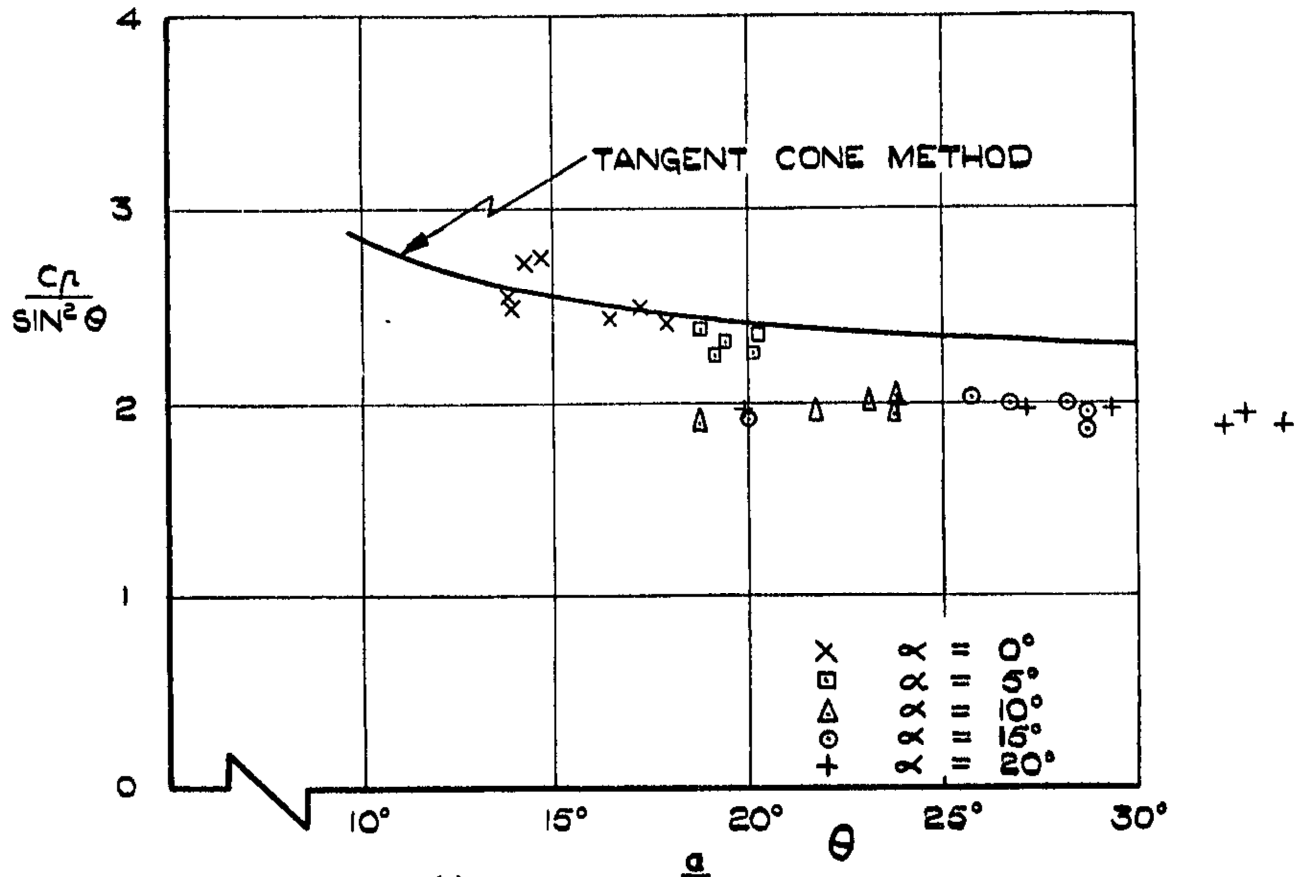
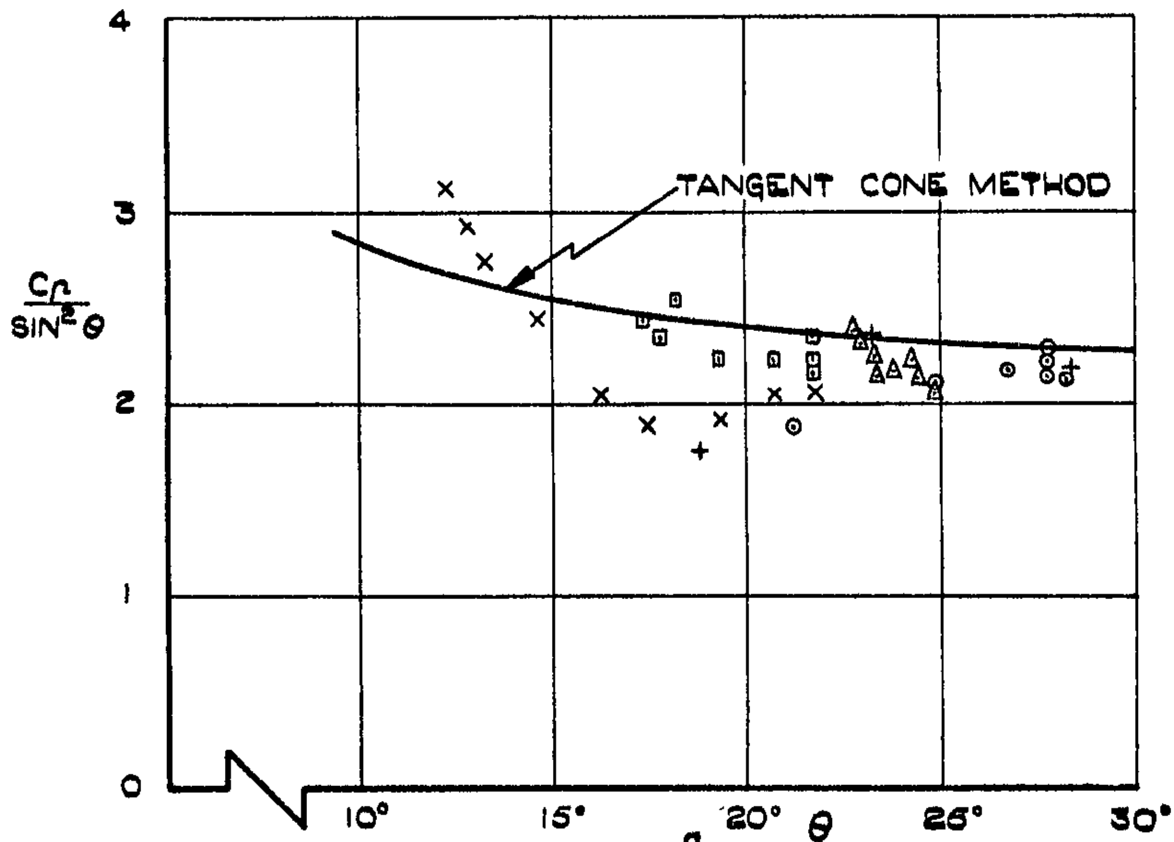


FIG. 2. WINGS OF RHOMBIC CROSS-SECTION.
GEOMETRY AND NOTATION.

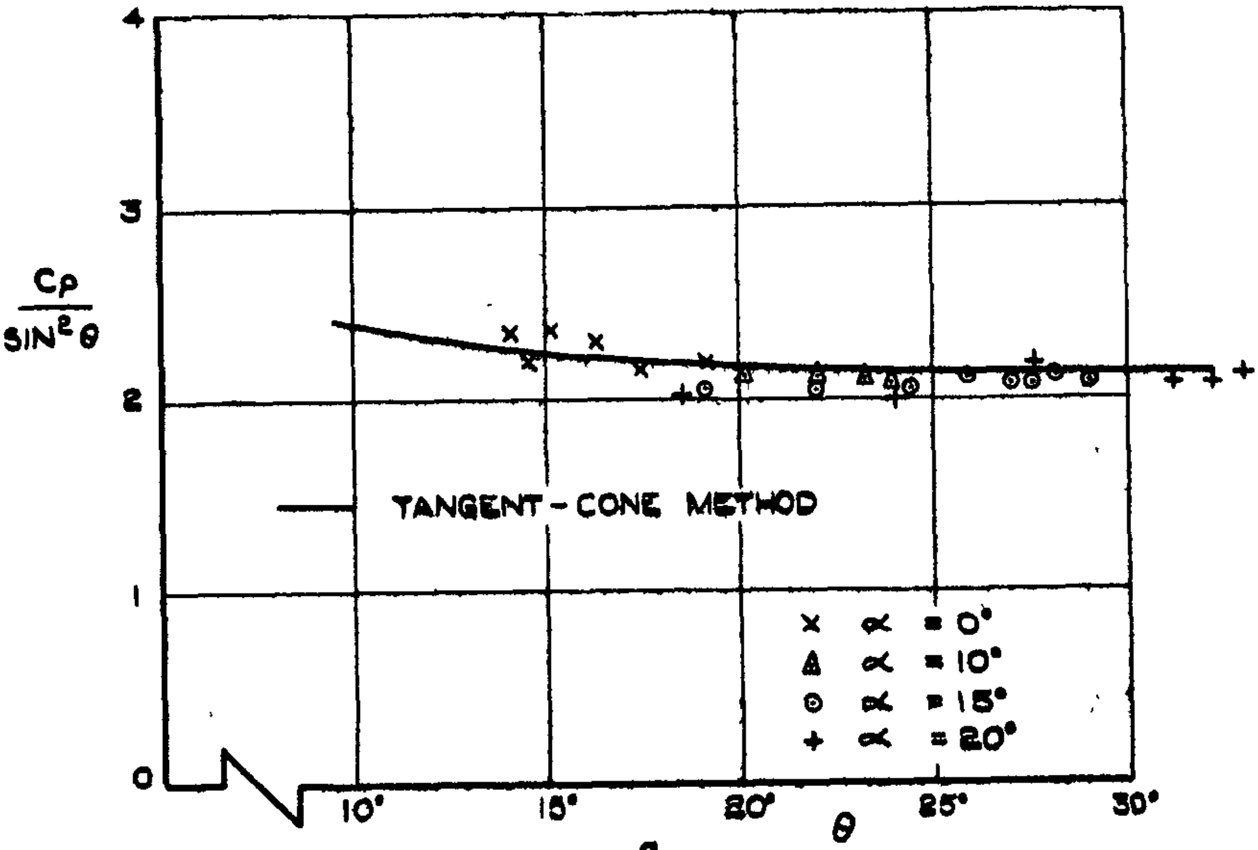


(a) BODY 1. $\frac{a}{b} = 1{\cdot}39$

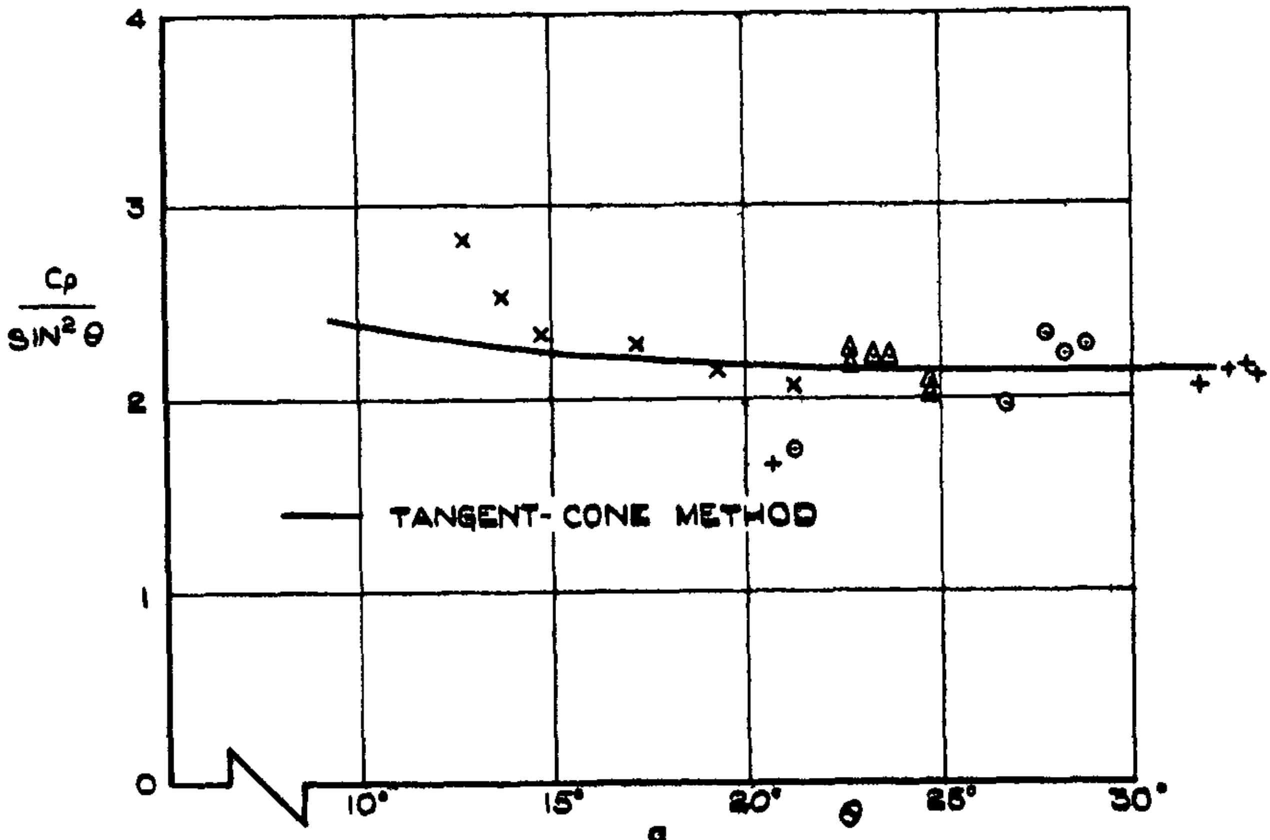


(b) BODY 2. $\frac{a}{b} = 1{\cdot}78$

FIG. 3. (a & b) ELLIPTIC CONES. $M = 3{\cdot}09$



(a) BODY 1. $\frac{a}{b} = 1{\cdot}39$



(b) BODY 2. $\frac{a}{b} = 1{\cdot}78$

FIG. 4 (a & b) ELLIPTIC CONES M = 6·0

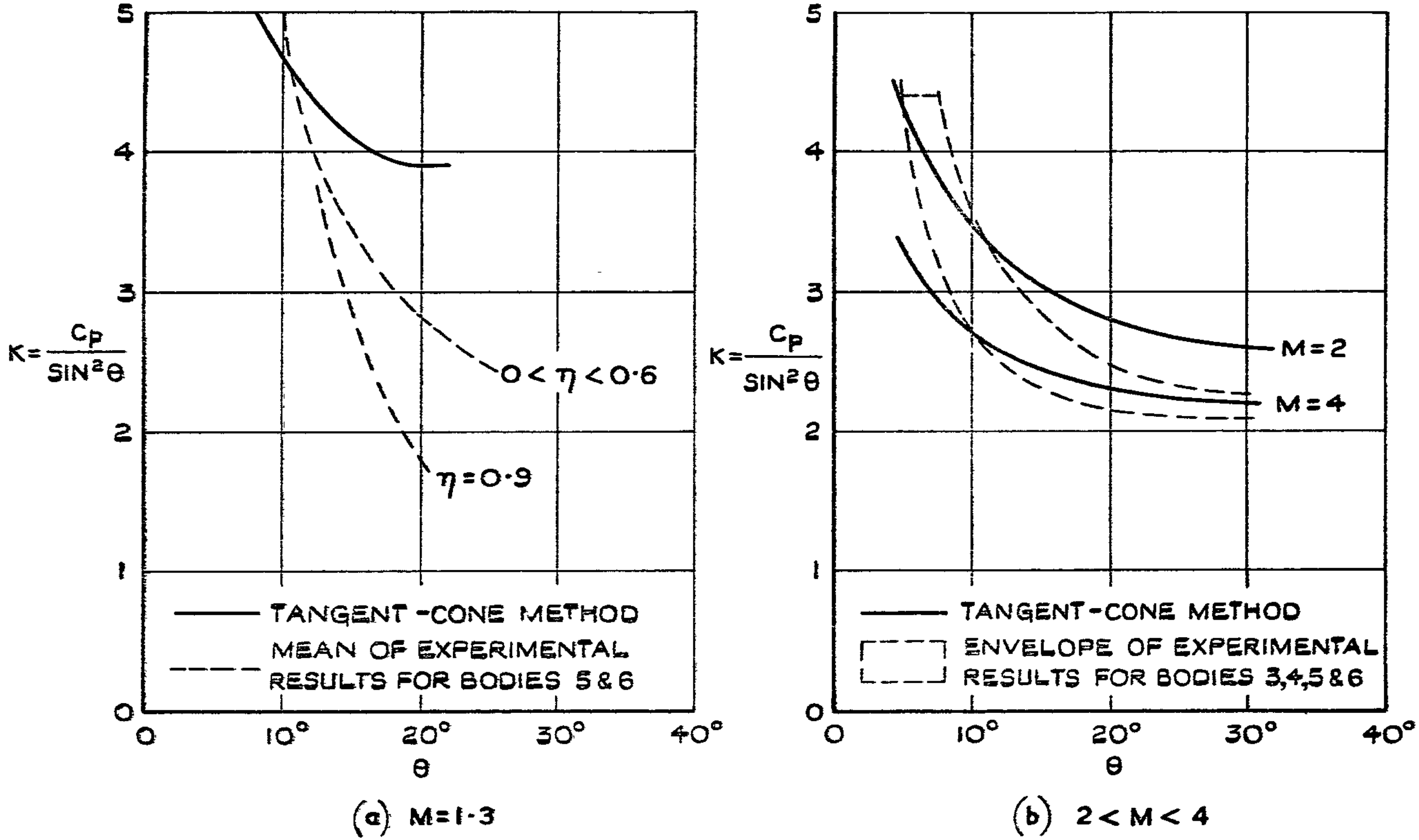
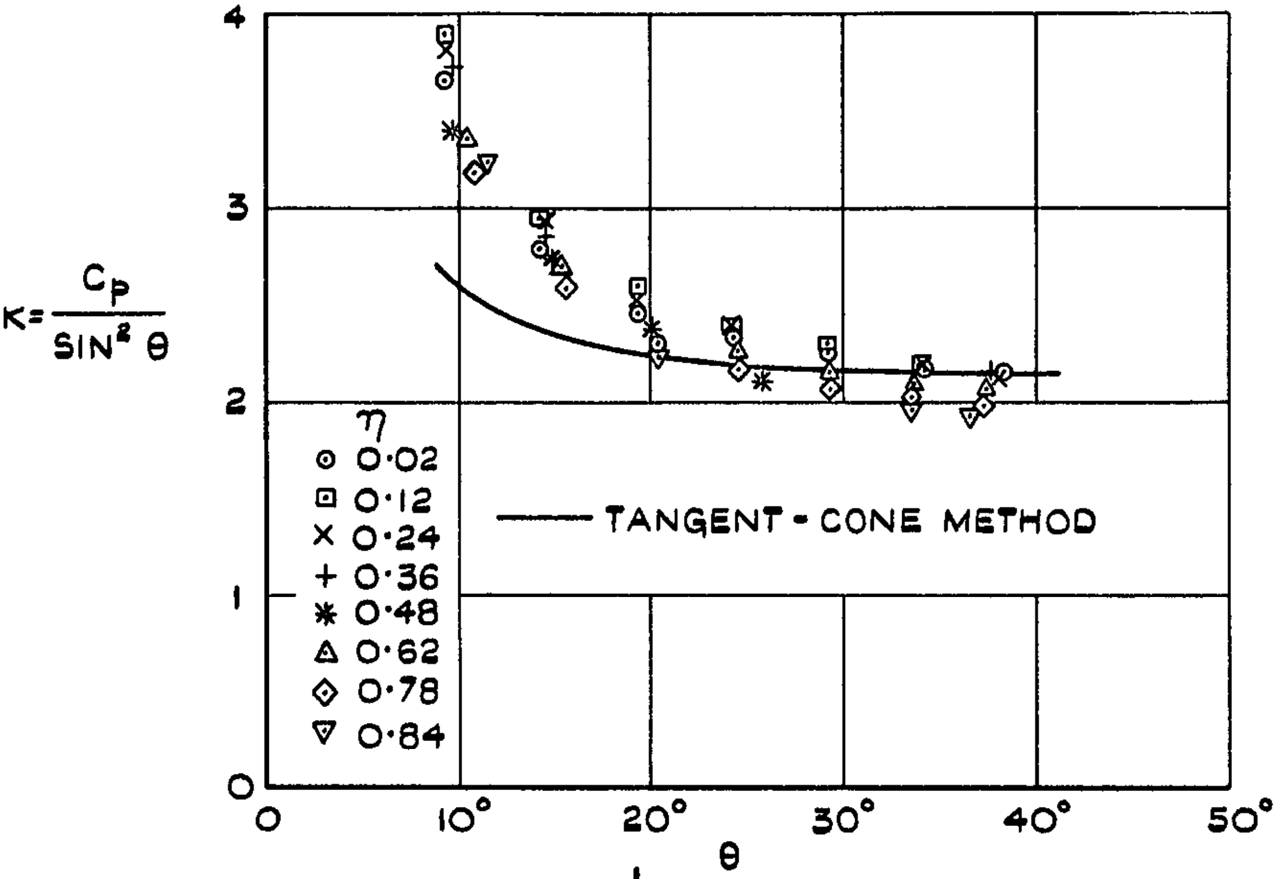
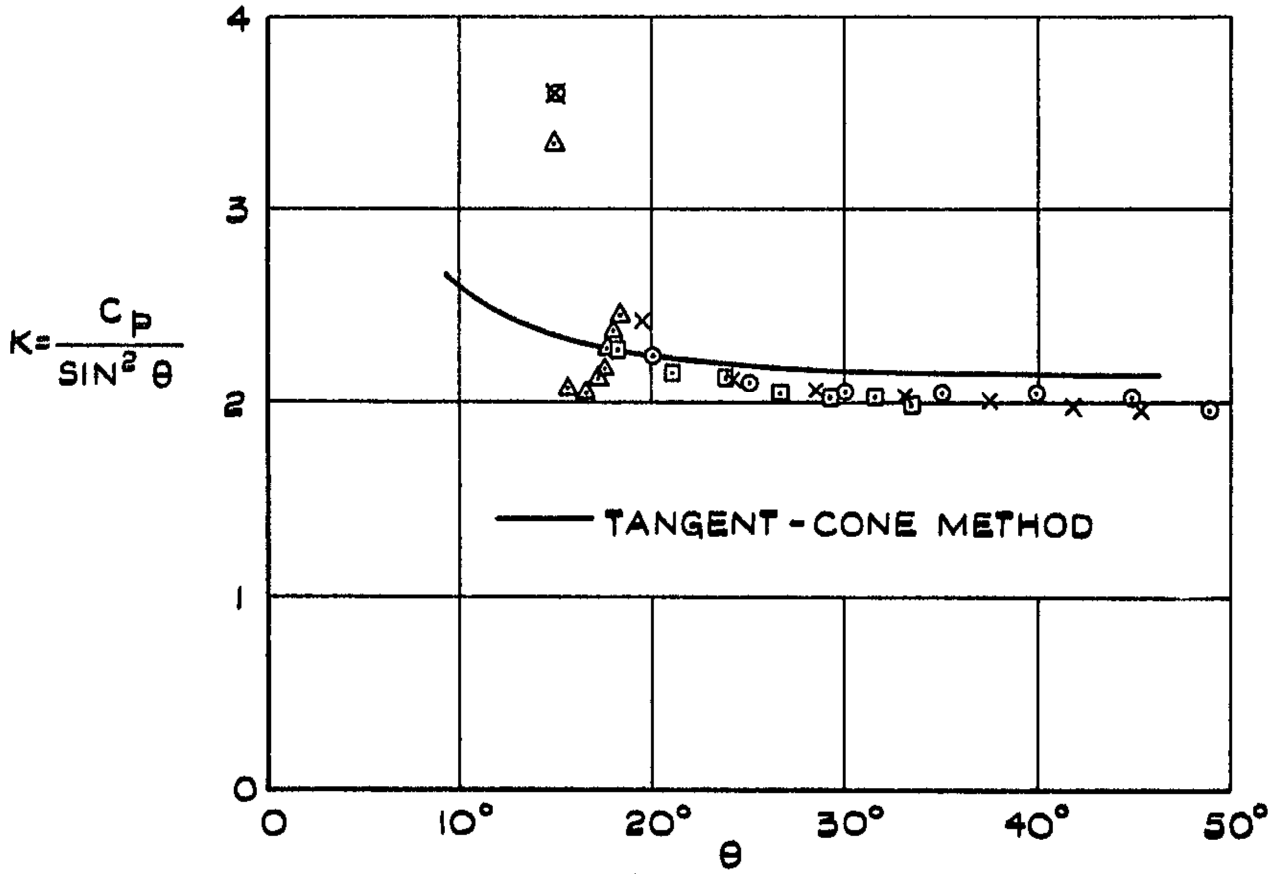


FIG. 5. (a & b) CONES OF BICONVEX CROSS-SECTION $0 < \eta < 0 \cdot 6$



(a) BODY 7 $\frac{1}{8}$- CONE SEGMENT.



(b) BODY 8 $\frac{1}{2}$- CONE SEGMENT.

FIG. 6. (a & b) CONE SEGMENTS M = 4·3.

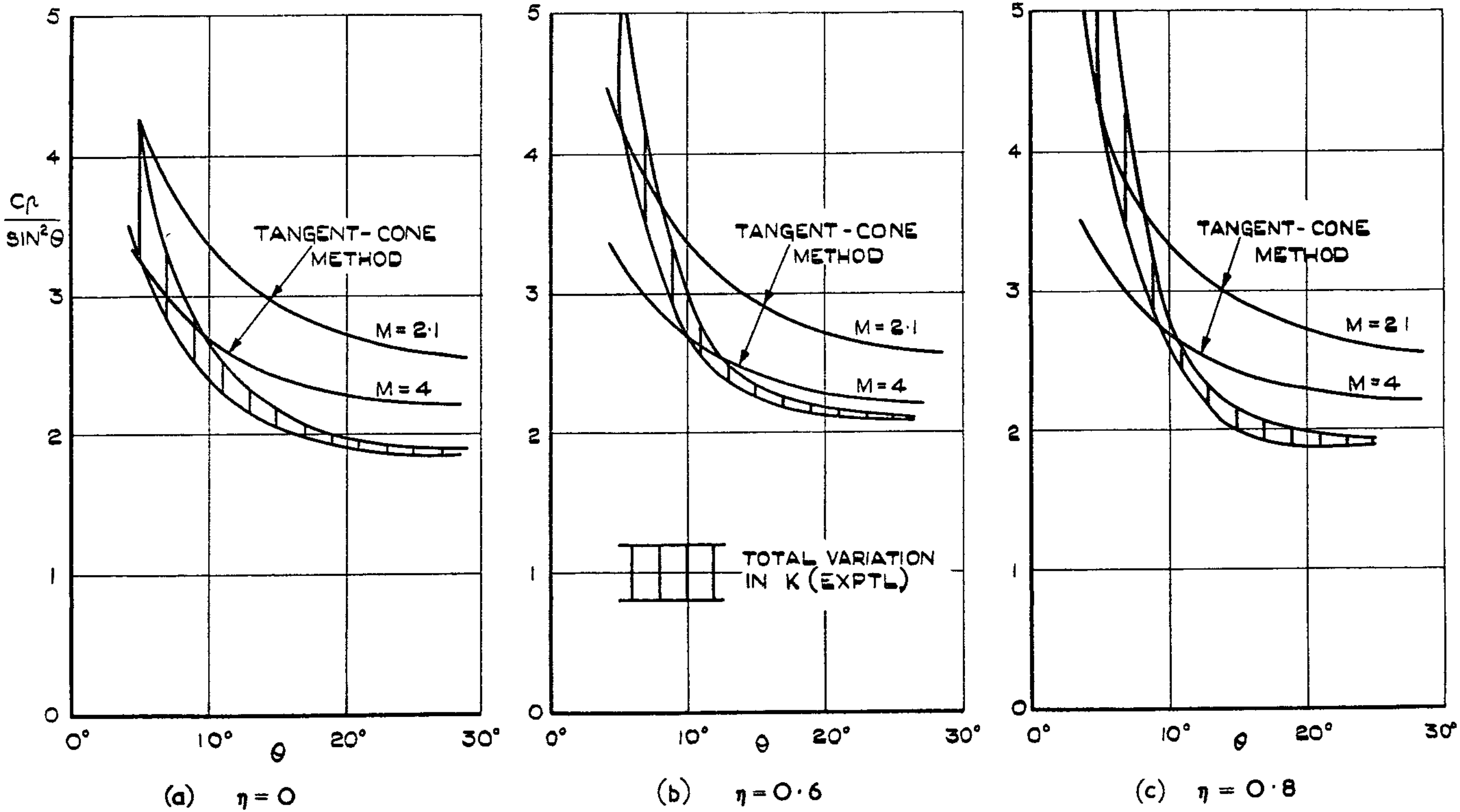


FIG. 7. (a–c) CONES OF RHOMBIC SECTION, BODIES 9 & 11 $2{\cdot}1 \leq M < 4$ A.R. = $2/3$

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