C.P. No. 708

C.P. No. 708

MINISTRY OF AVIATION

AERONAUTICAL RESEARCH COUNCIL

CURRENT PAPERS

The Measured Response of an Aircraft to the Vertical Velocity Component of Atmospheric Turbulence

By

D.M. Ridland

LONDON: HER MAJESTY'S STATIONERY OFFICE

1964

SEVEN SHILLINGS NET

U.D.C. No. AI(42)Meteor 7 : 551.55 : 533.6.013.47

C.P. No. 708

February, 1963

THE MEASURED RESPONSE OF AN AIRCRAFT TO THE VERTICAL VELOCITY COMPONENT OF ATMOSPHERIC TURBULENCE

by

D. M. Ridland

SUMMARY

Measurements have been made of the rigid body response in the pitching plane of an aircraft to the vertical velocity component of atmospheric turbulence. Spectral methods were used to evaluate the aircraft frequency response functions, which agreed very well with theoretical predictions. Limited analysis of the measured elevator movement suggests that the pilot is the dominant factor in aircraft response at very low frequencies during flight through atmospheric turbulence.

Replaces R.A.E. Tech. Note No. Aero 2866 - A.R.C. 24,817.

(62981)

LIST OF CONTENTS

Page

1 INTRODUCTION 4

2 AIRCRAFT AND INSTRUMENTATION 4

3 MEASUREMENTS MADE 5

4 ANALYSIS OF THE MEASURED DATA 5

5 RESULTS 8

- 5.1 Measured spectra 8
- 5.2 Measured pilot + rigid aircraft frequency response functions 9
- 5.3 Pilot - elevator - gust relationship 9
- 5.4 Spectra of acceleration, incidence and rate of pitch due to elevator displacement 10
- 5.5 Measured rigid aircraft frequency response functions 10

6 COMPARISON WITH THEORETICAL RIGID AIRCRAFT FREQUENCY RESPONSE FUNCTIONS 11

7 SIGNIFICANCE OF THE EFFECT OF THE PILOT 13

8 ACCURACY OF RESULTS 13

9 CONCLUSIONS 14

LIST OF SYMBOLS 15

LIST OF REFERENCES 15

17

TABLE 1 - Principal dimensions of Meteor 7 (VW 412) 18

APPENDICES 1 AND 2 20 - 21

ILLUSTRATIONS - Figs.1 - 18 -

DETACHABLE ABSTRACT CARDS -

LIST OF APPENDICES

Appendix			Page
1	-	Weather conditions during test on 22nd January 1959 for the route Wratting Common to near Bedford	20
2	-	Comment on instrumentation accuracies for spectral measurements	21

LIST OF ILLUSTRATIONS

	Fig.
Photograph of Meteor 7 VW 412	1
Positions of instruments	2
Transfer functions of instruments	3
Measured spectrum of vertical gust velocity	4
Measured spectrum of aircraft C.G. normal acceleration	5
Measured spectrum of aircraft incidence	6
Measured spectrum of aircraft rate of pitch	7
Measured spectrum of elevator movement	8
Measured frequency response functions showing response of pilot-aircraft combination to unit vertical gust	9
Pilot response in terms of elevator displacement to unit vertical gust input	10
Frequency response characteristics of test aircraft (Meteor 7) to elevator input	11
Spectra of acceleration, incidence and rate of pitch due to elevator displacement	12
Maximum possible effects of elevator displacement on spectra of acceleration, incidence and rate of pitch	13
Comparison of non-dimensional measured and theoretical normal C.G. acceleration/gust input frequency response functions for rigid aircraft	14
Comparison of non-dimensional measured and theoretical incidence/gust input frequency response functions for rigid aircraft	15
Comparison of measured and theoretical rate of pitch/gust input frequency response functions for rigid aircraft	16
Comparison of non-dimensional measured normal C.G. acceleration/gust input frequency response functions obtained from different samples	17
Comparison of the mean acceleration/gust input frequency response functions from Fig.17	18

---

1 INTRODUCTION

Spectral analysis is now an accepted technique for the investigation of atmospheric turbulence and of aircraft behaviour in turbulence. As distinct from the discrete gust concept, spectral analysis allows the aircraft dynamics to be fully accounted for in assessments of aircraft response to turbulence. Also, it is theoretically feasible to determine the aircraft transfer functions by measuring spectra of the gust input and the aircraft response parameters during flight through atmospheric turbulence. Spectral techniques, however, are statistical in character and the accuracy of a measured spectrum is very difficult to assess. As an exploratory exercise therefore, measurements were made of the response of an aircraft in the pitching plane to the vertical velocity component of atmospheric turbulence, and the aircraft transfer functions (frequency response functions) were evaluated. They are discussed and compared with theory in this note.

Measurements of elevator displacement, made during the investigation give a limited indication of the part played by the pilot during this type of flight test.

2 AIRCRAFT AND INSTRUMENTATION

The test aircraft was a Meteor 7 of approximately 15,000 lb A.U.W. It had neither auto-pilot nor power controls, the stick being geared directly to the elevator. During the test the aircraft was flown with low static stability (static margin, $K_n = 0.027$ and manoeuvre margin, $H_m = 0.071$) and at the test height and airspeed, 2500 ft and 534 ft/sec T.A.S. respectively, the natural frequency of the short period pitching oscillation was 0.54 c.p.s. A photograph of the aircraft is given in Fig.1 and the main aircraft characteristics, including approximate values of the quasi-steady derivatives (measured previously on the test aircraft) are given in Table 1.

The instrumentation consisted of an incidence vane, a normal accelerometer, a pitch rate gyroscope and an elevator position transmitter. The vane was positioned $21\frac{1}{2}$ feet forward of the aircraft C.G. on a nose boom (boom structural frequency 14 c.p.s.) to reduce aerodynamic position errors, while both the accelerometer and rate gyroscope were $2\frac{1}{2}$ feet forward of the C.G. Recording was by means of a galvanometer recorder which had a tuning-fork-maintained time base giving 1/100th second timing marks. Paper width was six inches and speed five inches per second.

The positions of the instruments are shown in Fig.2 and the main instrument characteristics are tabulated below.

Instrument	Range	Undamped natural frequency	Damping ratio	Sensitivity 1 cm equivalent to
Incidence vane	±4 degs	48 c.p.s.*	0.06 approx*	0.807 degs
Accelerometer	0.2 to 1.8g	12 c.p.s.	0.65	0.131g
Rate gyroscope	±5 degs/sec	$10\frac{1}{2}$ c.p.s.	0.70	0.657 degs/sec
Elevator transmitter	+5 to -7 degs	-	-	1.510 degs

*at 304 knots E.A.S. and 2500 ft altitude

The natural frequency and damping ratio of the incidence vane were determined in flight by suddenly releasing the vane from one end of its travel in smooth air at the test height and airspeed.

Approximate transfer functions of the instrument systems are given in Fig.3, that of the incidence vane being based on the measured values of frequency and damping and the assumption that the vane was a linear second order system. The transfer functions of the accelerometer[1] and rate gyroscope were measured in the laboratory and shown to be independent of amplitude. The transfer function of the elevator position transmitter (a potentiometer) was simply that of the recording galvanometer.

Where necessary the instrument transfer functions have been used to correct the results.

3. MEASUREMENTS MADE

All measurements were made during one test flight in which the aircraft was flown on approximately a constant course, with the pilot making, as far as possible, only slow adjustments to trim. The object of this was to restrict pilot interference with the aircraft response to gusts, to low frequencies. A continuous record was taken over a period of 206 seconds, of the four variables, incidence, normal acceleration, rate of pitch and elevator displacement and from this a time history of the vertical gust velocity was computed.

During the test the true airspeed was 534 ft/sec and pressure altitude about 2500 ft. The prevailing meteorological situation is given in Appendix 1.

4. ANALYSIS OF THE MEASURED DATA

The first step in the analysis of the measured data was to obtain the time history of the vertical gust velocity. Spectral methods were then used for all subsequent analysis of the data, i.e. of the five random time histories. These two stages in the analysis are considered briefly below.

4.1 Computation of the vertical gust velocity

The computed gust velocities were obtained from the vane readings by correcting them for aircraft heaving and pitching motions[2,3,4]. As corrections were thus applied implicitly for any pilot induced pitching motions, the resulting gust velocities were independent of any controlling action taken by the pilot. The time history of the vertical gust velocity was computed on the basis of the equation

$$w_g = V\alpha - V\theta - w + \ell_1 \dot{\theta} \tag{1}$$

where

w_g	vertical gust velocity, ft/sec
V	mean true airspeed, ft/sec
α	windvane reading, rads
w	aircraft normal velocity, ft/sec
ℓ_1	distance from accelerometer to windvane, ft
$\dot{\theta}$	rate of pitch, rads/sec
θ	aircraft attitude, rads.

The first term in Equation (1) is the basic vane reading, which is corrected by the second term for the instantaneous attitude of the aircraft (a datum correction), while the third and fourth terms are corrections for the aircraft heaving and pitching velocities respectively; the normal velocity, w, was derived from integration of the normal acceleration. It is assumed in Equation (1) that the airspeed is constant and equal to the mean value and that all angular displacements are small enough for their cosines to be taken as unity. Aircraft normal velocity may then be taken as equal to aircraft vertical velocity, which strictly is the variable required.

4.2 Spectral analysis

In the present context the general heading of spectral analysis covers three main computational stages; these are the determination of respectively, the auto-correlation coefficients, the power spectral densities and the aircraft frequency response or transfer functions.

For the spectral analysis it was decided that a convenient reading interval* would be 0.05 secs and that the spectra would be examined up to the

---

*The reading interval, Δt secs is defined by the Nyquist frequency, $f_N = \frac{1}{2\Delta t}$ c.p.s., which should be chosen so as to avoid contamination (aliasing) of the required spectrum by the high frequency content of the record (see Ref.5). In practice the reading interval is a compromise between the need for a long enough sample for satisfactory extraction of the lowest frequency required, the need to avoid aliasing, and the reading and computational effort available.

maximum frequency (Nyquist frequency) compatible with this interval, 10 c.p.s. All time histories were therefore read at 0.05 second intervals for the total record length of 206 seconds. From the first 150 seconds of this digitised record, auto-correlation functions[6] were calculated up to 127 lags, i.e. up to a maximum lag of 6.35 seconds (4.2 per cent of the 150 second sample length) for each of the variables including vertical gust velocity. The values of 150 seconds (a rounded off value) and 6.35 seconds were related to limits in the computing programme used. (Apart from that for the gust velocity computation, no special programmes were written for this investigation.)

The auto-correlation functions were then transformed to the frequency plane to give the required spectral density functions[7] of,

vertical gust velocity,
normal acceleration,
incidence,
rate of pitch,
elevator displacement.

These spectra constitute the basic experimental results.

It may be noted that the value of the auto-correlation function for a lag of T seconds (or the equivalent distance) gives the average relationship, over the whole run, between the measured quantity and its value T seconds previously. The auto-correlation function thus contains information on the time (or spatial) structure of the process. The power spectrum, which is the Fourier cosine transform of the auto-correlation function, gives information on the frequency structure of the process and is the more convenient function for this type of work.

Finally, the required frequency response or transfer functions were obtained by means of the fundamental spectral equation[8,9],

$$\phi_{OUT} = |T_s|^2 \phi_{IN}, \qquad (2)$$

where

ϕ_{OUT} spectrum of the output,

T_s transfer function of the system,

ϕ_{IN} spectrum of the input.

The acceleration, incidence and rate of pitch spectra (outputs) were divided by the vertical gust velocity spectrum (input) and the square root taken of the ratios. The acceleration and incidence response functions were non-dimensionalised to facilitate subsequent comparison with theory.

These frequency response functions were determined up to 5 c.p.s. only. The natural frequencies of the first two structural modes of the aircraft were approximately 7.7 and 10.3 c.p.s. and their damping was typically low. Up to 5 c.p.s. therefore, the aircraft response was for practical purposes unaffected by structural oscillations and the measured frequency response functions represent the response of the combination of pilot and rigid aircraft to vertical gusts.

The spectrum of elevator displacement was also divided by that of the vertical gust velocity and the square root taken, but, because of the low sensitivity of the elevator position transmitter, (quoted in Section 2 - see also Appendix 2), this function was only determined up to $2\frac{1}{2}$ c.p.s.

During the test flight the incidence vane was subject to aerodynamic position errors, that is the distortion of the airflow direction by the bulk of the aircraft and of the boom itself, caused the vane to over read. The static position error was estimated as ten per cent and subsidiary flight tests confirmed that this was of the right order. Corrections were therefore made to the spectra of vertical gust velocity and incidence for the effects of static position error. It should be noted, however, that position errors are not independent of frequency. There must, fundamentally, be a dynamic position error due to the lag in build up of circulation and acting so as to reduce the static position error. This effect, however, was considered to be negligibly small over the present range of frequencies.

5 RESULTS

5.1 Measured spectra

The measured spectra of vertical gust velocity, normal acceleration, incidence, rate of pitch and elevator displacement are given in Figs.4 to 8 for the frequency range 0.83 to 10 c.p.s.

Three frequency scales are given in each figure:

(i) time frequency, f, in cycles per second,

(ii) space frequency, $\frac{1}{\lambda}$, in cycles per foot, and

(iii) reduced frequency, $\nu_1 = \frac{\omega\bar{c}}{V}$, which is non-dimensional.

These are related by

$$\frac{1}{\lambda} = \frac{f}{V} = \frac{\nu_1}{2\pi\bar{c}}$$

and in the present case

$$\frac{1}{\lambda} = 0.00187\ f, \quad \nu_1 = 0.111\ f \text{ and } V = 534 \text{ ft/sec.}$$

Mean curves have been drawn by eye up to 5 c.p.s. on each of the spectra, except that for elevator displacement where, because of the previously mentioned low sensitivity of the transmitter, the curve has been stopped at $2\frac{1}{2}$ c.p.s.

No corrections were made to the results for the effects of airframe elasticity and peaks corresponding to the 7.7 and 10.3 c.p.s. aircraft structural modes can be seen on each of the spectra. It would appear therefore that the resolution of the measured spectra is good almost up to the Nyquist frequency (10 c.p.s.) The large magnitude of the peaks on the rate of pitch spectrum (Fig.7) indicates that the rate gyroscope was positioned near a nodal point and so exaggerated the effect of the structural oscillations.

5.2 Measured pilot + rigid aircraft frequency response functions

The measured frequency response functions of acceleration, incidence and rate of pitch are given in Fig.9 for the frequency range 0.083 to 5 c.p.s. These functions represent the response of the combination of pilot + rigid aircraft to a unit vertical gust input.

Above about 1 c.p.s. the incidence response is independent of frequency and has unit value; this indicates that the aircraft is not responding significantly at frequencies much above its natural frequency (0.54 c.p.s.) and the measured incidence is almost entirely due to gusts.

The acceleration response shows quite clearly the effects of unsteady aerodynamic flow. With increasing frequency there is an increasing lag in the build up of circulation and full incremental lift has less and less chance of being established. This is reflected in the progressive decrease (above about 1 c.p.s.) in the measured acceleration per unit gust with increasing frequency.

The measured rate of pitch response is almost independent of frequency. The small average value of the rate of pitch, 0.1 deg/sec for 1 ft/sec gust input should be noted.

5.3 Pilot-elevator-gust relationship

During the test flight the pilot interfered as little as possible with the elevator other than gently maintaining trim. Fig.8, however, indicates that some elevator movements occurred over the full range of frequencies. Apart from the deliberate control exercised by the pilot these elevator movements could have been caused by the pilot inadvertently reacting to a sudden gust, by turbulence acting directly on the elevator or by the elevator reacting to the motion of the aircraft (e.g. the peak at 7.7 c.p.s. in Fig.8 showing elevator reaction to a structural oscillation). The spectral densities of the elevator movement at frequencies above 1 c.p.s. are, however, less than 8 per cent of the maximum measured value so that the principal effect of the elevator can be said to be restricted to low frequencies only (below 1 c.p.s.). It may be assumed that in this low frequency range the above mentioned inadvertent causes of elevator movement are sufficiently small to be completely masked by the pilot's conscious control. Consequently, the significant part of the elevator spectrum can be mainly attributed to the pilot.

The 'pilot transfer function' in Fig.10, may therefore be interpreted as indicating that the pilot applied an average over the frequency range 0.083 to 1 c.p.s., about 0.028 degrees of elevator to control the response of the aircraft to a unit vertical gust input. The maximum elevator displacement recorded during the test was approximately 0.37 degrees.

It should be remembered that the pilot was deliberately attempting to make slow control movements only and the present results are not therefore expected to be typical of flight in turbulence.

It should be noted also that the aircraft had manual controls, and the elevator displacements were probably larger than could be expected in similar conditions for an aircraft with irreversible power controls.

5.4 Spectra of acceleration, incidence and rate of pitch due to elevator displacement

The spectrum of the elevator displacement which occurred during the test flight is given in Fig.8. By regarding the elevator displacement as an input to the aircraft system, and knowing the aircraft transfer functions with respect to elevator, the equivalent response of the aircraft in terms of acceleration, incidence and rate of pitch can be obtained using Equation (2).

Moduli of the functions representing the aircraft response to a unit elevator input at constant speed are given in Fig.11. They are based on the aerodynamic derivatives in Table 1 and the constant speed (short period) approximation for the aircraft response to elevator[10]. The assumption of constant speed is valid as the phugoid oscillation (approximate period 74 seconds) is outside the range of frequencies being considered. The values of the derivatives in Table 1 were obtained from transient response tests made at the same height and airspeed as the present test.

The spectra of acceleration, incidence and rate of pitch due to the elevator displacements measured in the turbulence test are given in Fig.12 and were obtained by multiplying the modulus squared of each elevator input response function, Fig.11, by the elevator displacement spectrum, Fig.8. Mean curves of the total measured spectra (from Figs.5, 6 and 7) are also given in Fig.12 for comparison.

It is clear from Fig.12 that at the lowest frequencies considered the spectra of the aircraft response parameters associated with elevator displacement, are approximately equal in magnitude to the total spectra obtained in the tests. The low frequency motion of the aircraft could therefore be due entirely to elevator movement i.e. to the pilot.

5.5 Measured rigid aircraft frequency response functions

It is only possible to correct the total spectra for the effects of elevator displacement when the phase relationships between elevator displacement and the relevant variables are known. Such phase information can only be obtained by cross-spectral analysis and, while a limited amount of work has been done with promising results, a full cross-spectral analysis of the present data has not been made.

The spectra of acceleration, incidence and rate of pitch due to elevator displacement shown in Fig.12 and compared with the total spectra for these quantities, do, however, define the maximum possible effect of elevator in each case. By adding and subtracting the spectra due to elevator to and from the total spectra, limits can be established within which the corrected spectra (representing the response of the rigid aircraft alone to gusts) must lie. This has been done in Fig.13.

In the cases of normal acceleration and incidence the effect of elevator is negligible above 0.5 c.p.s., and of rate of pitch above about 4 c.p.s. Above these frequencies, therefore, the total measured spectra are practically uncontaminated by elevator movement. It is to be remembered that an allowance for phase could only narrow these limits and reduce the range of frequencies at which elevator was significant.

At first sight there is an apparent discrepancy between the effects of elevator on the three aircraft response parameters. The elevator input ceases to have an effect on the measured response in incidence and normal acceleration at a frequency of approximately 0.5 c.p.s., whereas it affects the rate of pitch response significantly up to 3 c.p s. This is due to the fact that at the higher frequencies at least, the measured values of incidence and normal acceleration are not reflecting an actual aircraft response but rather the gust input directly. The actual contribution of the aircraft response would appear, for instance, as a difference from unity of the 'incidence response function' and this is obviously very small, Fig.15. The recorded rate of pitch on the other hand, is a measure of the actual dynamic response of the aircraft and, in correspondence with the incidence response of the aircraft, is a very small quantity which could be easily swamped by even relatively small extraneous effects such as inadvertent elevator movements. On this basis and knowing that corrections for phase must reduce the effect of elevator, it is reasonable to take 3 c.p.s. as the maximum frequency for significant elevator effects on rate of pitch.

Having established that above certain frequencies the measured frequency response functions represent the response of the rigid aircraft alone to unit vertical gusts, the measurements can now be compared with theory.

6 COMPARISON WITH THEORETICAL RIGID AIRCRAFT FREQUENCY RESPONSE FUNCTIONS

Zbrozek in Ref.11 gives two alternative methods of estimating the longitudinal response of an aircraft to vertical gusts. In the complete theory, unsteady aerodynamic effects are allowed for by calculating first unsteady aerodynamic derivatives and using them in the appropriate transfer functions. In a much simpler approximation Zbrozek suggests that quasi-steady derivatives be used in a conventional transfer function and the result multiplied by the unsteady lift functions where appropriate.

As almost prohibitive effort would be required to calculate the correct solution, only the approximate solution has been compared with the present experimental results in Figs.14 to 16. However, in order to indicate the possible error in this approximation, comparisons are also given in each of the figures between results obtained with the correct theory and approximate results calculated by Zbrozek for an aircraft very similar to the present test aircraft. In view of the similarity between the test aircraft and Zbrozek's example it is reasonable to assume that the differences are representative of those between the correct solution and the approximation for the test aircraft.

Comparing the results on this basis, the measured acceleration response function, Fig. 14, is in excellent agreement with the complete theory. In particular, the effects of unsteady lift which are clearly shown by the measurements in turbulence, i.e. the decrease in the value of the function with increasing frequency, are faithfully predicted by the theory.

Similar remarks apply to Fig. 15 for the incidence function. Both the measured and theoretical unsteady functions attain unit value around 1 c.p.s. Unit value of the incidence function is significant in that it indicates that the aircraft is not responding to the gusts[11,12]. For practical purposes, therefore, the aircraft is not responding significantly above 1 c.p.s.

The rate of pitch frequency response function is given in Fig. 16. Above 3 c.p.s., where pilot interference may be discounted, the absolute values of rate of pitch are extremely small, as has already been indicated. Furthermore, the rate of pitch frequency response function itself is highly sensitive to changes in static margin. Zbrozek's calculations were made for two values of static margin, $K_n = 0$ and 0.1, whereas the present aircraft was flown with a static margin of $K_n = 0.027$. In addition to extrapolating from the results obtained by Zbrozek's approximation to the correct theory therefore, interpolation is now necessary between the two C.G. positions considered by Zbrozek. However, it can be seen from Fig. 16 that the required interpolation appears possible only at frequencies well above the natural frequency of the aircraft when resonance effects have largely subsided. Such interpolated values are indicated in Fig. 16 and as far as can be judged from this comparison, the complete theory and experiment are again in satisfactory agreement.

It can be concluded that, in general, at frequencies where the comparison is valid, agreement between the measurements in turbulence and Zbrozek's theory is very good if unsteady derivatives are used. It is likely that the theory will satisfactorily predict rigid aircraft response down to the lowest frequencies necessary for design purposes, but this has not been demonstrated. In the rate of pitch case, where the comparison must be made above 3 c.p.s. because of pilot interference, the measured rates of pitch are very small; in fact, they are too small above 1 c.p.s. to make a significant contribution to the aircraft C.G. acceleration or incidence.

The effects of unsteady derivatives on the incidence response function are comparatively small. This is fortunate in that, for practical purposes, the incidence function could be calculated using quasi-steady derivatives; simple multiplication of the incidence function by the Küssner function (the unsteady lift function due to penetration of a sharp edged gust) would then yield the gust load (C.G. acceleration) function, with a considerable saving in computational effort.

The overall motion of the aircraft e.g. pitching is, however, very much affected by unsteady flow, at least at very small static margins, as was actually predicted by the theory. Satisfactory estimates of aircraft rate of pitch response to turbulence will thus only be obtained if the complete theory, which includes unsteady derivatives, is used.

7 SIGNIFICANCE OF THE EFFECT OF THE PILOT

In flight tests of the type reported in this note it is desirable to suppress as far as possible pilot's interference with the aircraft motion, if evaluation of the aircraft transfer functions is the main objective. This is why during the test the pilot attempted to just hold the aircraft in trim with the minimum of elevator control. To maintain the basic flight condition, i.e. speed and height, the pilot would obviously suppress the phugoid motion of the aircraft, which by itself is virtually undamped. The phugoid period, approximately 74 seconds, is, however, well outside the range of frequencies analysed and consequently would not affect the present results. Nevertheless, the actual elevator displacements recorded made quite significant contributions to the overall aircraft response at least over part of the frequency range of interest.

The present results do not provide an assessment of the natural behaviour of the pilot in flight through turbulence when he is not restrained by specific test instructions and the measured elevator control function cannot be interpreted as a true transfer function in the generally accepted sense. However, it is still useful to consider the contributions of the pilot to the aircraft response recorded in the present test as they may represent something like minima to be expected in such tests.

Figs.12 and 13 suggest that in fact the pilot was responsible for virtually all the recorded aircraft response at frequencies below 0.1 c.p.s. With rate of pitch the pilot may have been responsible for most of the recorded response even up to frequencies of 3 c.p.s. As the general tendency for aircraft speeds to increase implies an extension of interest to longer wavelengths (i.e. future tests will have to be made at lower frequencies) full allowance must be made for the pilot's contribution by cross-spectral analysis.

8 ACCURACY OF RESULTS

A detailed assessment of the accuracy of the present results would be very difficult and has not been attempted, but the following discussion suggests that the individual spectra (excepting that for elevator displacement) are fairly reliable between $\frac{1}{4}$ and 5 c.p.s.

A simple check was made on the measurements. This consisted of computing the acceleration/gust input frequency response function from two additional samples taken from the original 206 second record (Section 3) each of 100 seconds duration and separated by six seconds. The first of these 100 second samples commenced at the same time as the basic 150 second sample used for the principal analysis (see Fig.17). The point of this is that, while the individual measured spectra are functions of sample length and possibly of the sample itself, the aircraft transfer functions, assuming they are linear, are dependent solely on the particular aircraft configuration tested. Any differences between the three resulting acceleration/gust transfer functions may thus be taken as a measure of the overall accuracy of the experiment. Ideally, the measured transfer functions should be identical.

Comparison is made of the three computed results in Fig.17 and of their mean values, which represent the measured transfer functions, in Fig.18. Noting that the scatter in Fig.17 is normal for work involving spectral analysis and that, in view of the many possible sources of discrepancy, the two to three per cent difference between the curves of Fig.18 is considered negligibly small, the agreement is very good indeed; it extends even to the lowest frequency, which represents only about eight full wavelengths in the 100 second sample.

A further general comment on accuracy is made in Appendix 2. This, coupled with experience gained during the investigation, suggests that an above average standard of instrumentation is necessary if reasonable accuracy is to be maintained, particularly at the higher and very low frequencies, where the gust input may be of small amplitude. (In this connection, higher frequencies with respect to the present investigation, where V = 534 ft/sec T.A.S., may be taken as say 4 c.p.s. and above, and very low frequencies as $\frac{1}{4}$ c.p.s. and below.)

9 CONCLUSIONS

Three conclusions may be drawn from this work. The first is that spectral techniques do in practice enable satisfactory measurements to be made of aircraft response to atmospheric turbulence. The indications are, however, that an above average standard of instrumentation is necessary if reasonable overall accuracy is to be attained.

The second is that Zbrozek's complete theory, including unsteady derivatives, for the response of a rigid aircraft to oscillatory vertical gusts predicted very accurately the load (C.G. acceleration) and incidence frequency response functions at all frequencies where comparison with experiment was valid, that is, over almost all the range of frequencies not affected by aeroelastic modes. In the rate of pitch case comparison with experiment could only be made at the higher frequencies, but again agreement was satisfactory. The usefulness of the theory may well extend beyond these limits, but this has not been demonstrated due to limitations in the measurements.

The third conclusion is that more must be learned about the actions of the pilot before flight through atmospheric turbulence at long wavelengths is fully understood. There is strong enough evidence from the present measurements to suggest that the pilot's contribution to the aircraft motion in the pitching plane increases with decreasing frequency and probably becomes the dominant factor at very low frequencies.

LIST OF SYMBOLS

$\bar{c}$	mean aerodynamic chord (reference chord)
f	frequency
n	normal acceleration increment above 1 g
q	rate of pitch
V	aircraft forward velocity
w_g	vertical gust velocity
α	aircraft incidence
η	elevator displacement
λ	wavelength
ν_1	reduced frequency $\left(\nu_1 = \frac{\omega\bar{c}}{V}\right)$
ϕ_x	spectrum of x
ω	circular frequency

LIST OF REFERENCES

No.	Author(s)	Title, etc.
1	MacLaren, I.	The design and testing of a low range acceleration transducer with predictable response characteristics. A.R.C. C.P.575, March 1961.
2	Notess, C.B., Eakin, G.J.	Flight test investigation of turbulence spectra at low altitude using a direct method for measuring gust velocities. Rep No.VC-839-F-1 Cornell Aero Lab Inc, also WADC Tech Rep 54-309, July 1954
3	Chilton, R.G.	Some measurements of atmospheric turbulence obtained from flow-direction vanes mounted on an airplane. NACA TN.3313, 1954

LIST OF REFERENCES (CONTD)

No.	Author(s)	Title, etc.
4	Crane, H.L., Chilton, R.G.	Measurements of atmospheric turbulence over a wide range of wavelength for one meteorological condition. NACA TN.3702, ARC 18,699, June 1956
5	Press, H., Tukey, J.W.	Power spectral methods of analysis and their application to problems in airplane dynamics. AGARD Flight Test Manual, Vol.IV, Part IVC, June 1956
6	Priestley, M.B., Samet, P.A.	The analysis of stationary time series I. Computation of correlation coefficients on a high speed computer. Unpublished M.O.A. Report
7	Priestley, B.A.	The analysis of stationary time series II. Estimation of power spectra. Unpublished M.O.A. Report
8	James, H.M., Nichols, N.B., Phillips, R.S.	Theory of servo mechanisms. McGraw - Hill Book Co. Inc., New York, 1947
9	Bendat, J.S.	Principles and applications of random noise theory. John Wiley and Sons Inc. New York, 1958
10	Ridland, D.M.	The longitudinal frequency response to elevator of an aircraft over the short period frequency range. A.R.C. C.P.476, September 1959
11	Zbrozek, J.K.	Longitudinal response of aircraft to oscillatory vertical gusts (frequency analysis including the effect of unsteady aerodynamics). Unpublished M.O.A. Report
12	Ridland, D.M.	Unsteady lift slope values obtained from flight measurements in gusts. A.R.C. C.P.651, June 1962

TABLE 1

Principal dimensions of Meteor 7 (VW.412)

Mainplane	
Span	37 ft 2 in.
Gross area	350 sq ft
Aerofoil section EC 1240 at root to EC 1040 at tip	
Incidence	1°
Dihedral	
centre plane spar datum	$0^\circ\ 52\frac{1}{2}'$
outer plane spar datum	6°
Tailplane	
Span	15 ft 8 in.
Gross area	61 sq ft
Fin	
Gross area	33.3 sq ft
General	
Total length	43.5 ft
Tail arm, ℓ	23.3 ft
Reference chord, $\bar{c}$	9.4 ft
Weight, W	15,200 lb

Quasi-steady derivatives, etc.

Lift slope of the aircraft $a = 4.0$

Pitching moment derivative due to rate of pitch $m_q = \frac{\bar{c}}{2\ell}\,\frac{\partial C_m}{\partial\left(\frac{q\ell}{V}\right)} = -0.47$

Pitching moment derivative due to heaving velocity $m_w = \frac{\bar{c}}{2\ell}\,\frac{\partial C_m}{\partial\left(\frac{w}{V}\right)} = -0.022$

Pitching moment derivative due to heaving acceleration $m_{\dot{w}} = \frac{\bar{c}}{2\ell}\,\frac{\partial C_m}{\partial\left(\frac{\left(\frac{\dot{w}}{V}\right)\ell}{V}\right)} = -0.16$

Pitching moment derivative due to elevator deflection $m_\eta = \frac{\bar{c}}{2\ell}\,\frac{\partial C_m}{\partial\eta} = -0.14$

TABLE 1 (CONTD)

Short period damping stability coefficient $B = \frac{a}{2} - \frac{m_q}{i_B} - \frac{m_{\dot{w}}}{i_B} = 8.3$

Short period stiffness stability coefficient $C = -\frac{\mu m_w}{i_B} - \frac{a}{2}\,\frac{m_q}{i_B} = 15.16$

Auxiliary coefficient in response equations $F = -\frac{a}{2} - \frac{m_{qT}}{i_B} + \frac{m_{\dot{w}}}{i_B} = 0$ (assumed),

where m_{qT} is tailplane contribution only to m_q (see Ref.11)

Coefficient of pitching moment of inertia $i_B = \left(\frac{k_B}{\ell}\right)^2 = 0.1$

Aircraft density parameter $\mu = \frac{W}{g\rho S\ell} = 26.2$

Manoeuvre margin $H_m = K_m - \frac{\ell}{\bar{c}}\,\frac{m_q}{\mu} = 0.071$

Restoring margin (equal to static margin when there are no Mach number effects)

$$K_m = -\frac{\partial C_m}{\partial C_L} = 0.027$$

Short period damping ratio, ratio of actual to critical damping

$$\zeta = \frac{B}{2\sqrt{C}} = 1.05$$

Aircraft natural frequency, $f_o = 0.54$ c.p.s. at $V = 534$ ft/sec T.A.S. and pressure altitude 2500 ft.

APPENDIX 1

WEATHER CONDITIONS DURING TEST ON 22ND JANUARY 1959 FOR THE ROUTE WRATTING COMMON TO NEAR BEDFORD - PERIOD 1500-1515 GMT

Meteorological Situation:	Depression centred over North Sea moving quickly Northeast. Trough of low pressure with occlusion extending from centre to Skegness to Birmingham to Cardiff at 1500 hours moving Southeast at 15 kt. Strong unstable Southwesterly airstream over area.
Surface Wind:	210-220 deg (true) speed 22-25 kt. Gusty with gusts reported 34-37 kt.
Weather:	Cloudy. Frequent showers of rain, moderate at times.
Visibility:	10-15 miles but 4-5 miles in showers.
Cloud: (Heights above ground level)	Variable Cu mainly 4/8-6/8 base 2000-2500 ft with occasional Cumulonimbus, base 1800 ft. In showers patches of Stratus base 1000 ft. Variable Sc., mainly 3/8-5/8 base 2500-3000 ft. No reported information on cloud tops but it is estimated that cloud tops would be above 6000 ft and in the case of large Cu and Cumulonimbus well above 10,000 ft. Little or no medium cloud. 7/8 Ci in the region 20,000 to 25,000 ft.
Surface temperature.	50 deg F. (Plus 10 deg C.)
Surface Dew Point:	44 deg F. (Plus 6.7 deg C.)
Wind at 2,500 ft:	230 deg (true) 55-60 kt.
Wind at 5,000 ft:	230 deg (true) 60-65 kt.
Temperature at 2,500 ft:	Plus 06 deg C.
Temperature at 5,000 ft:	Plus 01 deg C.
Freezing level:	5,500 ft.

APPENDIX 2

COMMENTS ON INSTRUMENTATION ACCURACIES FOR SPECTRAL MEASUREMENTS

Some idea of the standard of instrumentation necessary for the satisfactory measurement of spectra at the higher frequencies may be obtained from the magnitudes of the variables in the present test. By taking mean values of each of the spectra at 5 c.p.s. (Figs.5 to 8) and integrating over the frequency bandwidth relevant to each of the points in the spectra, 0.083 c.p.s., the RMS^2 of each variable over the frequency range 4.96 to 5.04 c.p.s. was obtained. These '5 c.p.s. RMS' and total RMS values are tabulated below

Total RMS		5 c.p.s. RMS	$\frac{\text{Total RMS}}{\text{5 c.p.s. RMS}}$	mms on record ≡ 5 c.p.s. RMS
Vertical gust vel., ft/sec	3.94	0.10	39	-
Normal acceleration, g	0.133	0.0043	31	0.33
Incidence, degs.	0.266	0.011	24	0.16
Rate of pitch, degs/sec	0.383	0.012	32	0.18
Elevator displacement, degs.	0.099	0.003*	33	0.02

*by straight line extrapolation of mean curve to 5 c.p.s.

The total RMS values were obtained from the recorded time histories and each can be represented by a corresponding reading in millimetres on the record. Analogous measurements for the 5 c.p.s. case are those given.

When considering instrument resolution, it is important to note that the total RMS values are roughly thirty times greater than the corresponding values at 5 c.p.s. and maximum values of the variables were found to be approximately four times greater than total RMS values. Assuming that measurements to within half, at most, of the RMS values are necessary to establish the RMS, then resolution and accuracy of better than 0.5 per cent is necessary to measure accurately both maximum and 5 c.p.s. RMS values. On this basis it would be necessary in the present case to read the record to half the values given in the last column of the table. This could be done for acceleration, incidence and rate of pitch, but would be impossible for elevator displacement.

It could be expected, therefore, that the accelerometer, incidence vane and rate gyroscope, which approximately meet the 0.5 per cent requirement, would give, in the relevant spectra, satisfactory frequency response functions up to 5 c.p.s., while the elevator transmitter, which fell far short of the requirement, would not. This is, in fact, so (Figs.14, 15, 16 and 10) and would suggest that instrumentation of relatively high quality is necessary for such measurements.

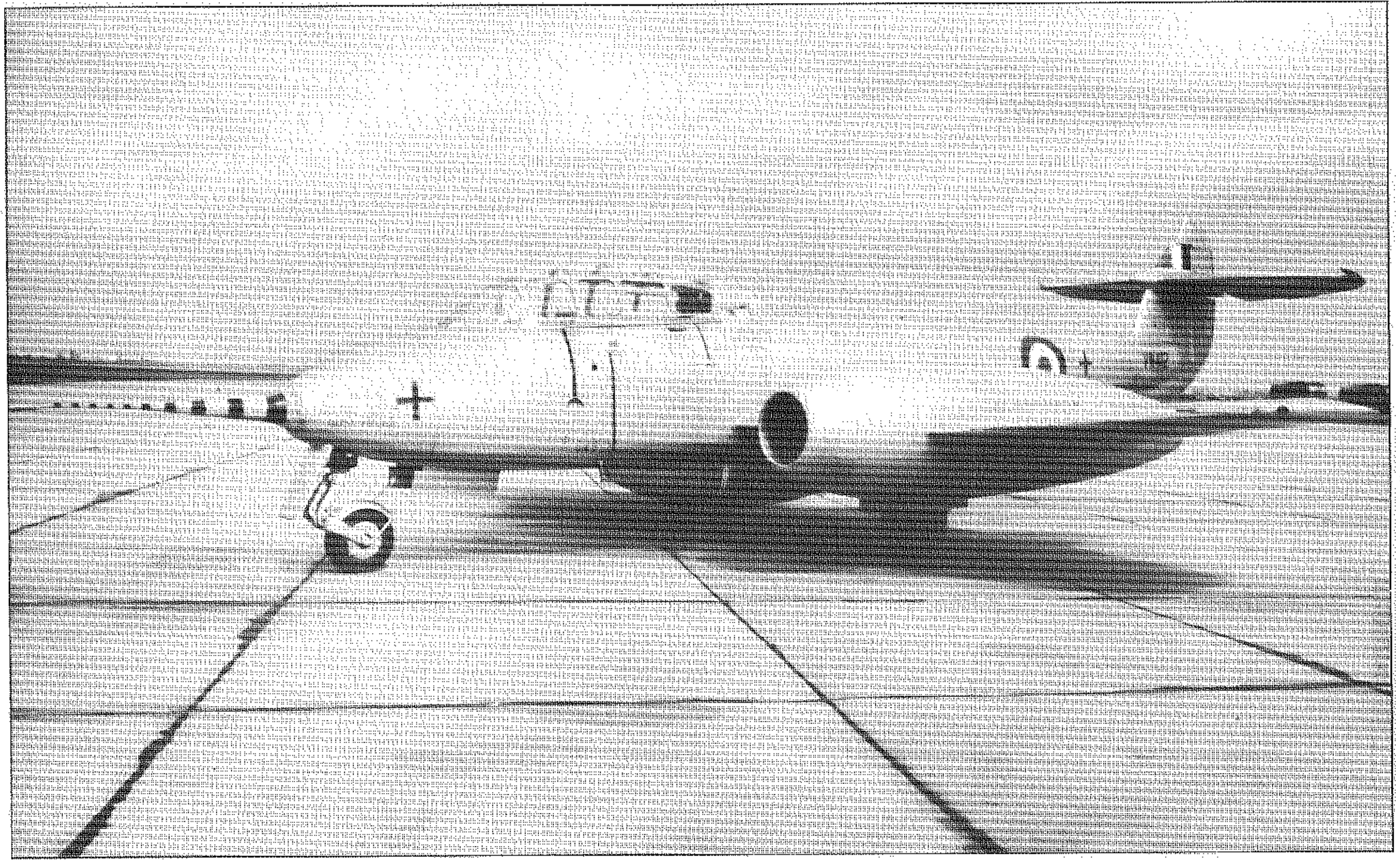


FIG.1. METEOR 7 VW412

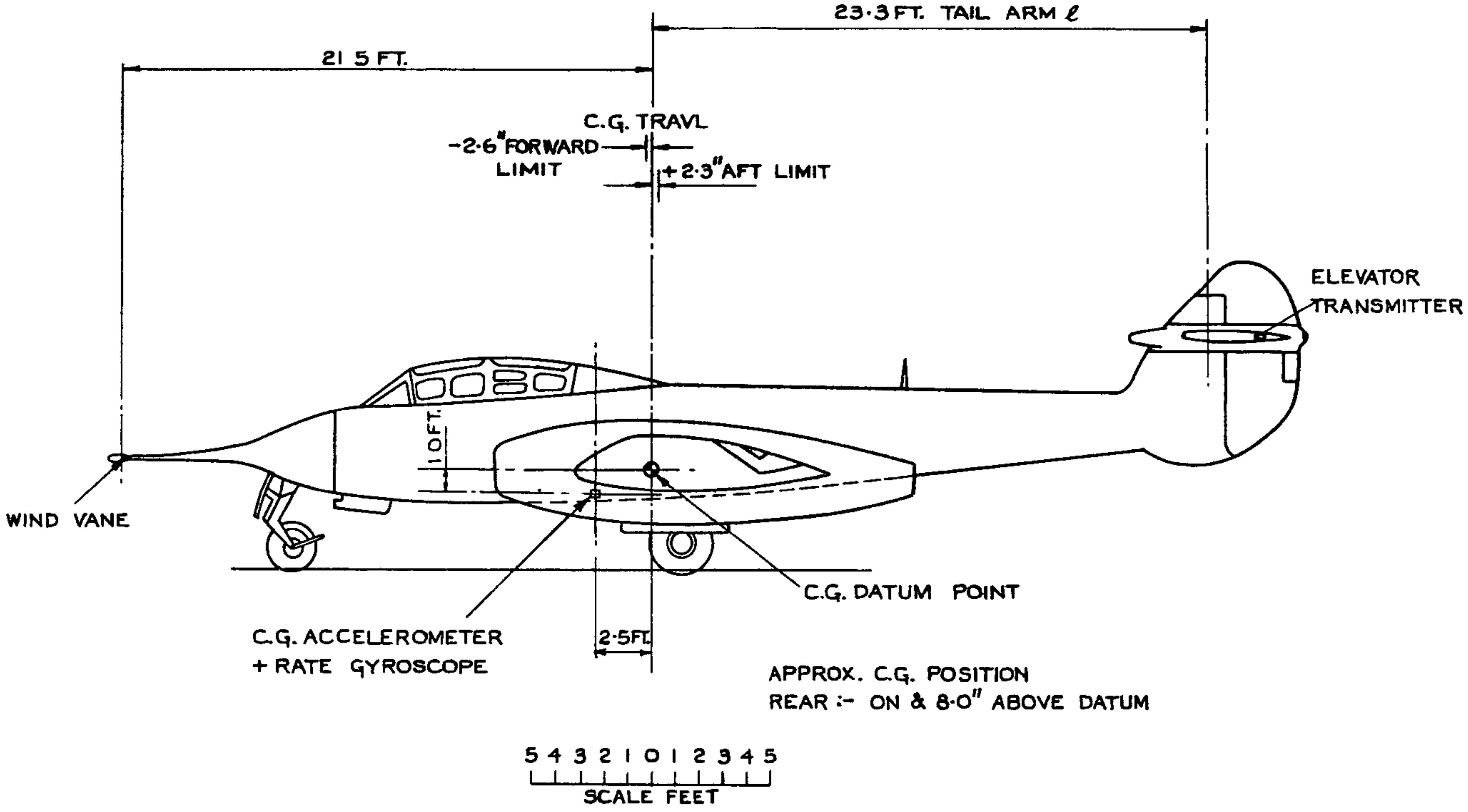


FIG. 2. POSITION OF INSTRUMENTS.

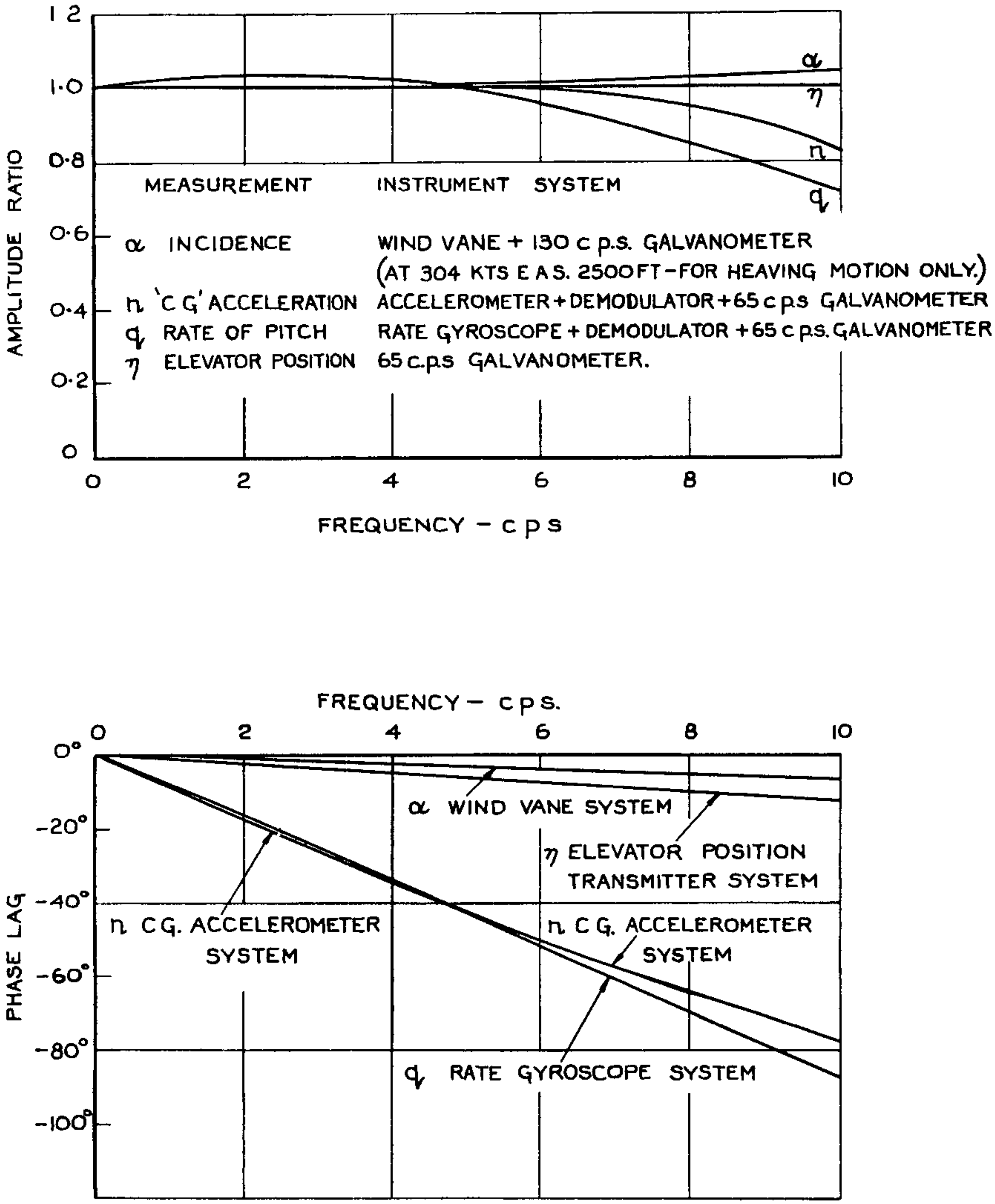


FIG. 3. TRANSFER FUNCTIONS OF INSTRUMENTS.

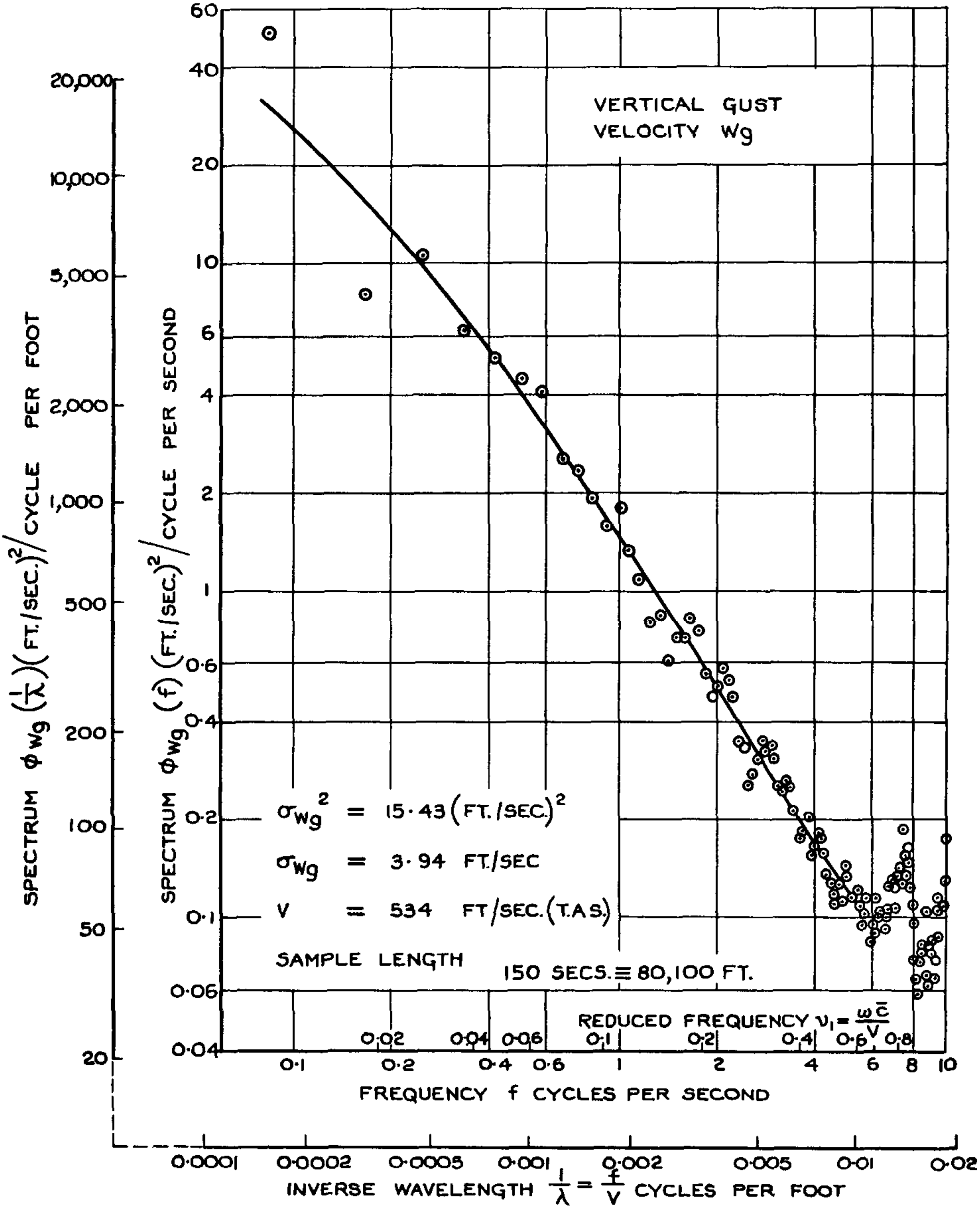


FIG.4. MEASURED SPECTRUM OF VERTICAL GUST VELOCITY.

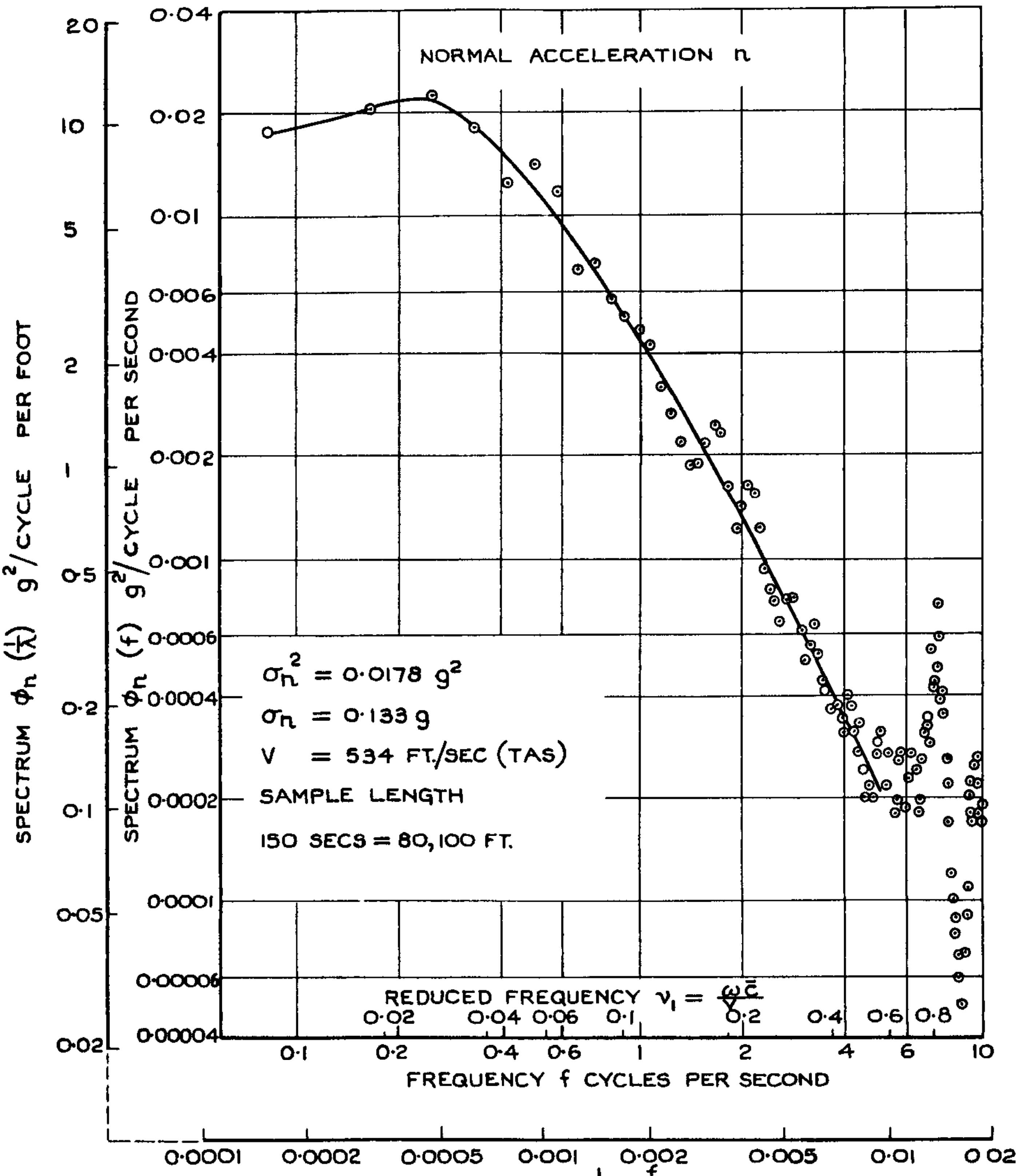


FIG.5. MEASURED SPECTRUM OF AIRCRAFT C.G. NORMAL ACCELERATION.

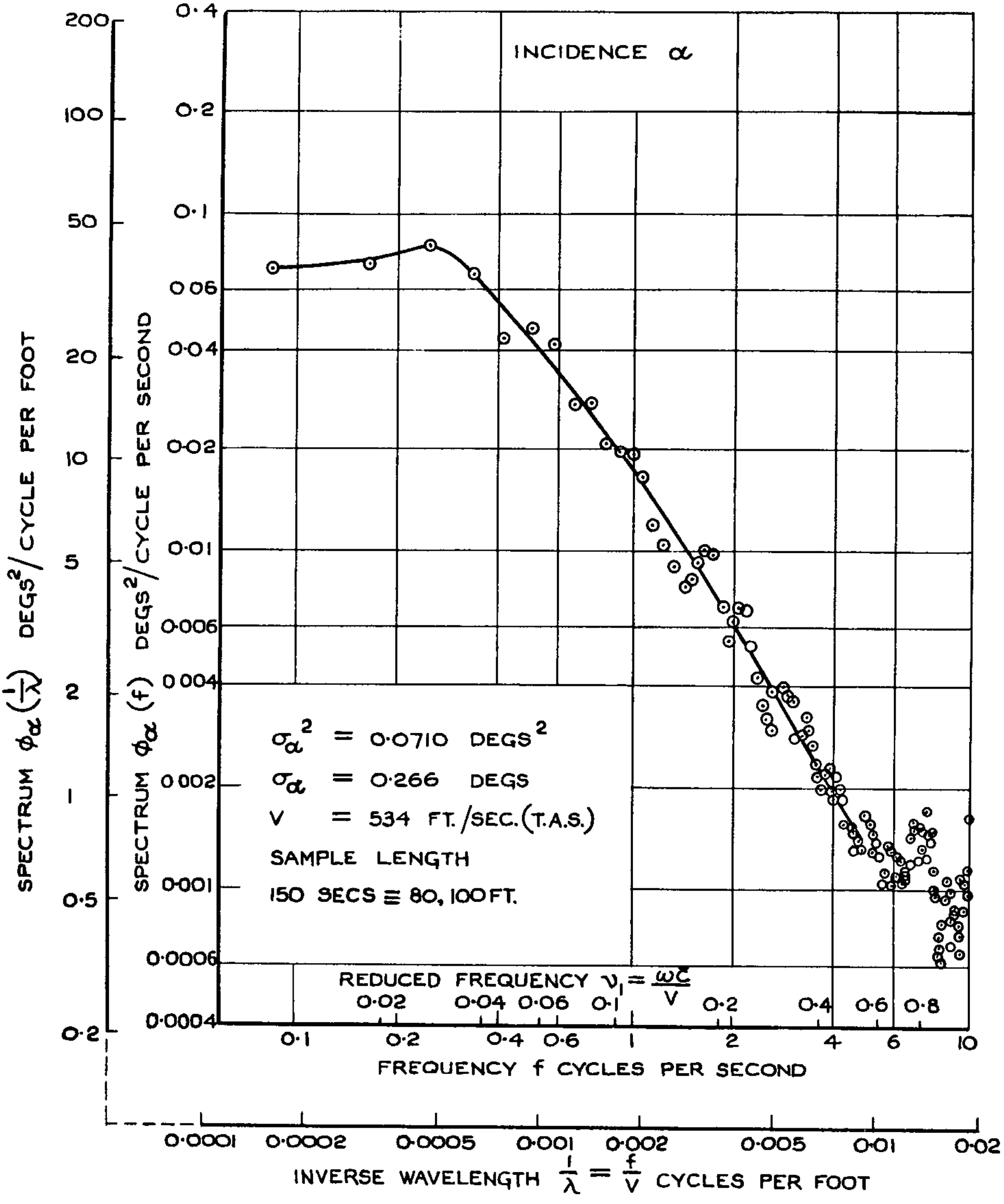


FIG. 6. MEASURED SPECTRUM OF AIRCRAFT INCIDENCE.

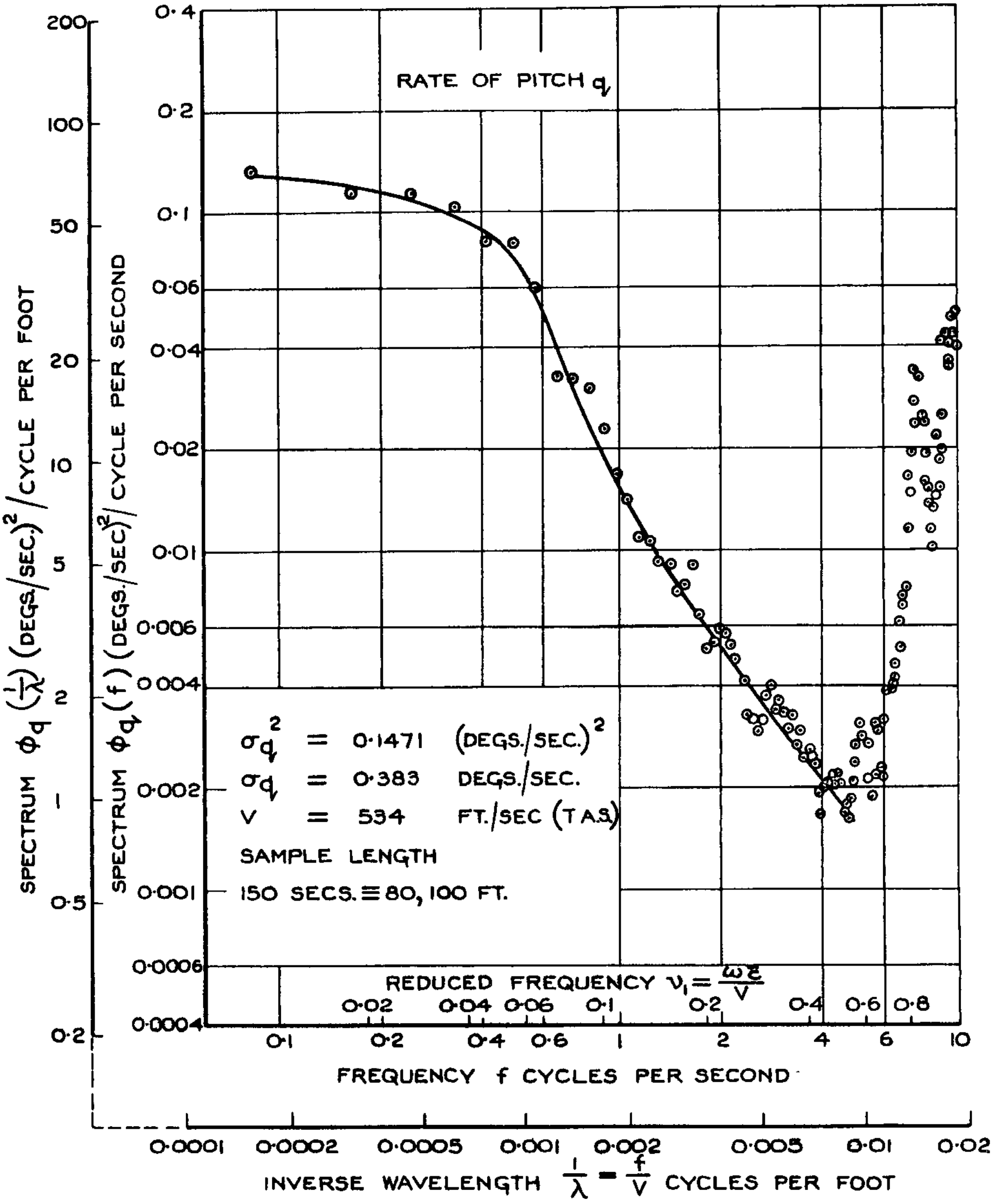


FIG. 7. MEASURED SPECTRUM OF AIRCRAFT RATE OF PITCH.

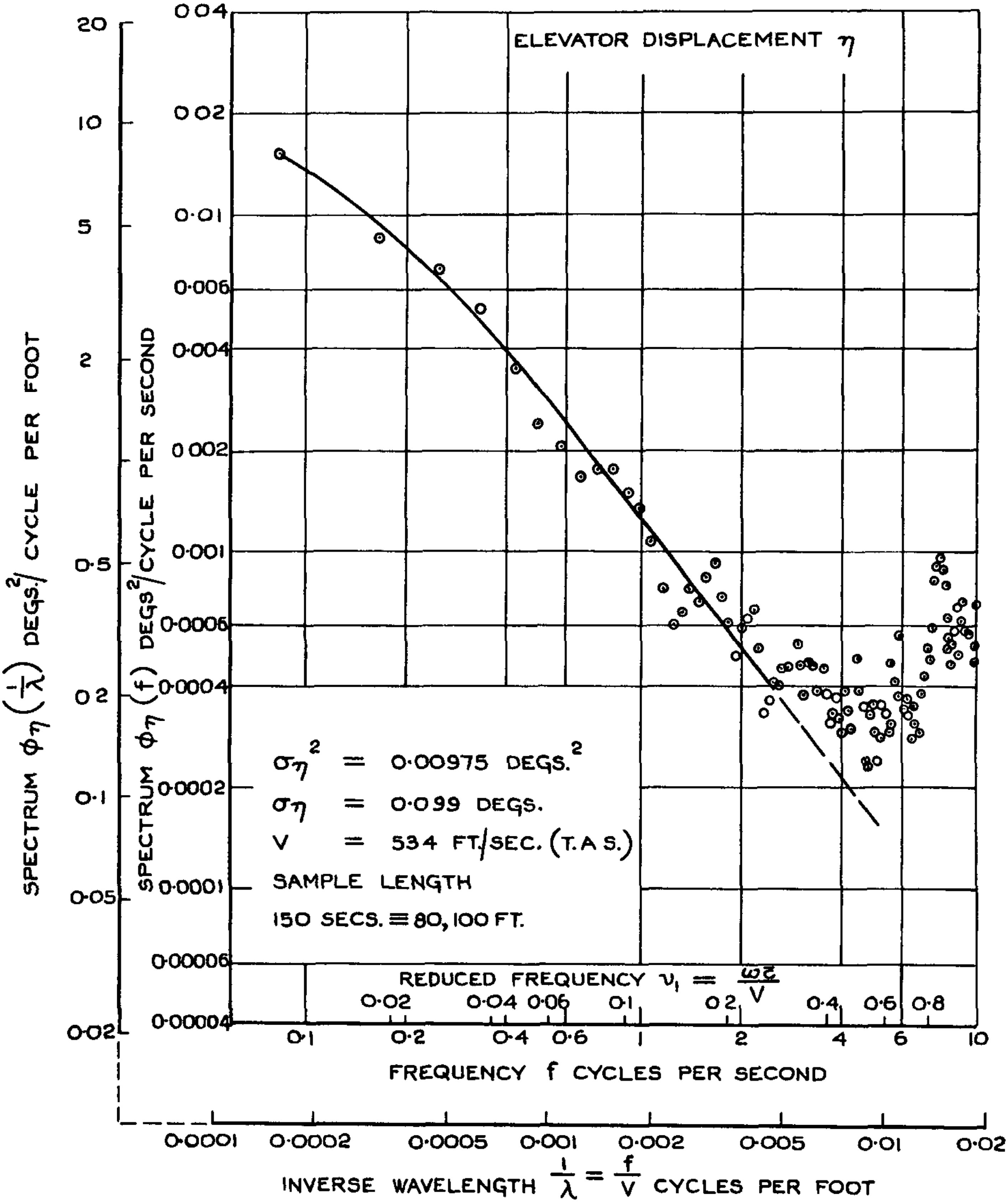
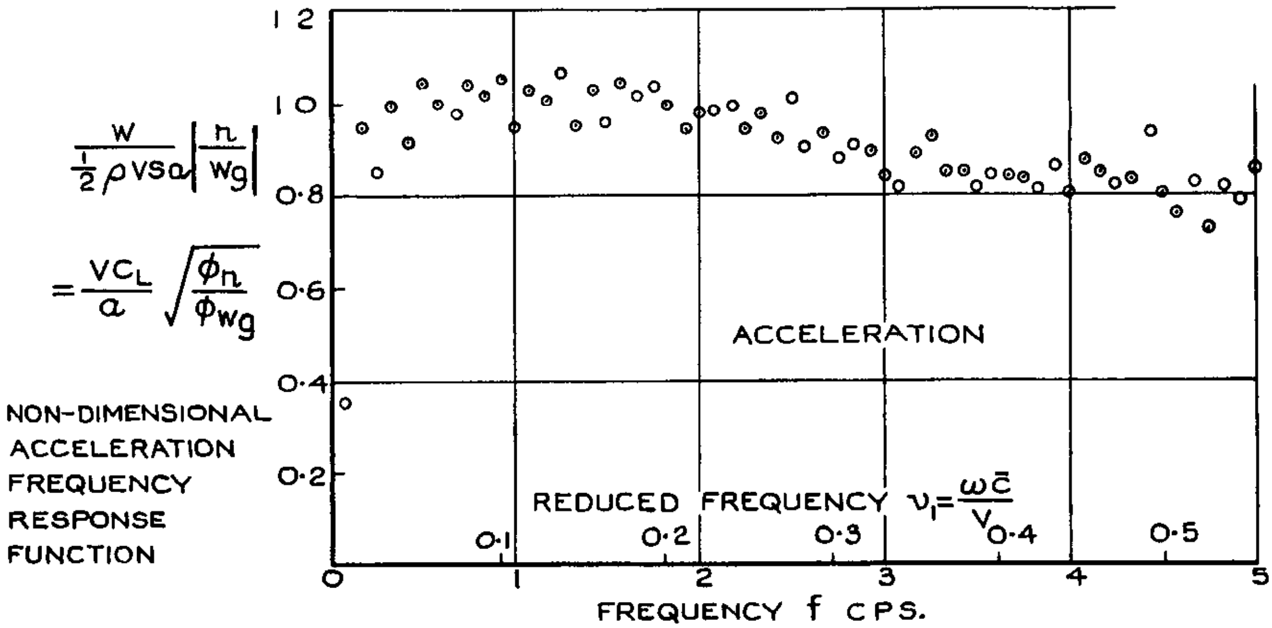
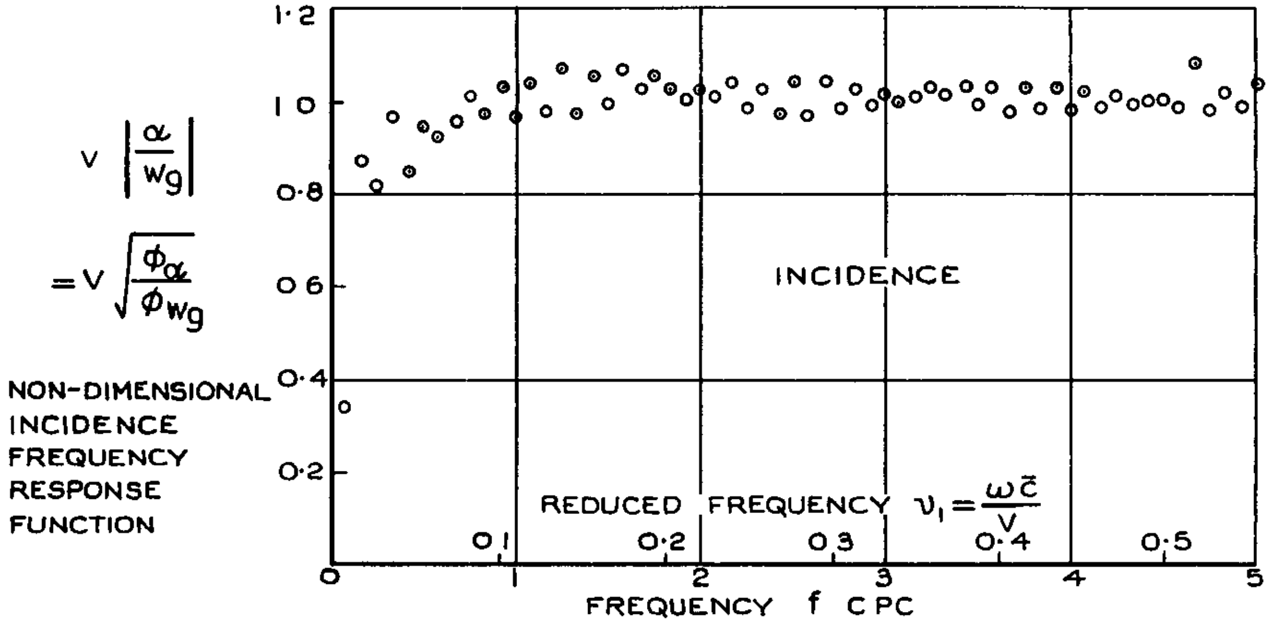


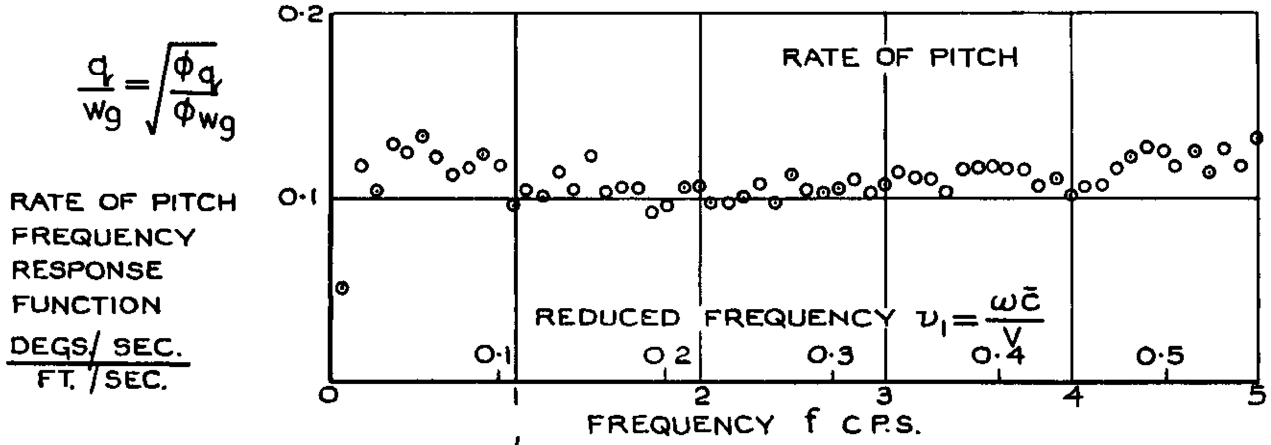
FIG.8. MEASURED SPECTRUM OF ELEVATOR DISPLACEMENT.



MEASURED ACCELERATION/VERTICAL GUST FREQUENCY RESPONSE FUNCTION



MEASURED INCIDENCE/VERTICAL GUST FREQUENCY RESPONSE FUNCTION



MEASURED RATE OF PITCH/VERTICAL GUST FREQUENCY RESPONSE FUNCTION

FIG. 9. MEASURED FREQUENCY RESPONSE FUNCTIONS SHOWING RESPONSE OF PILOT-AIRCRAFT COMBINATION TO UNIT VERTICAL GUST.

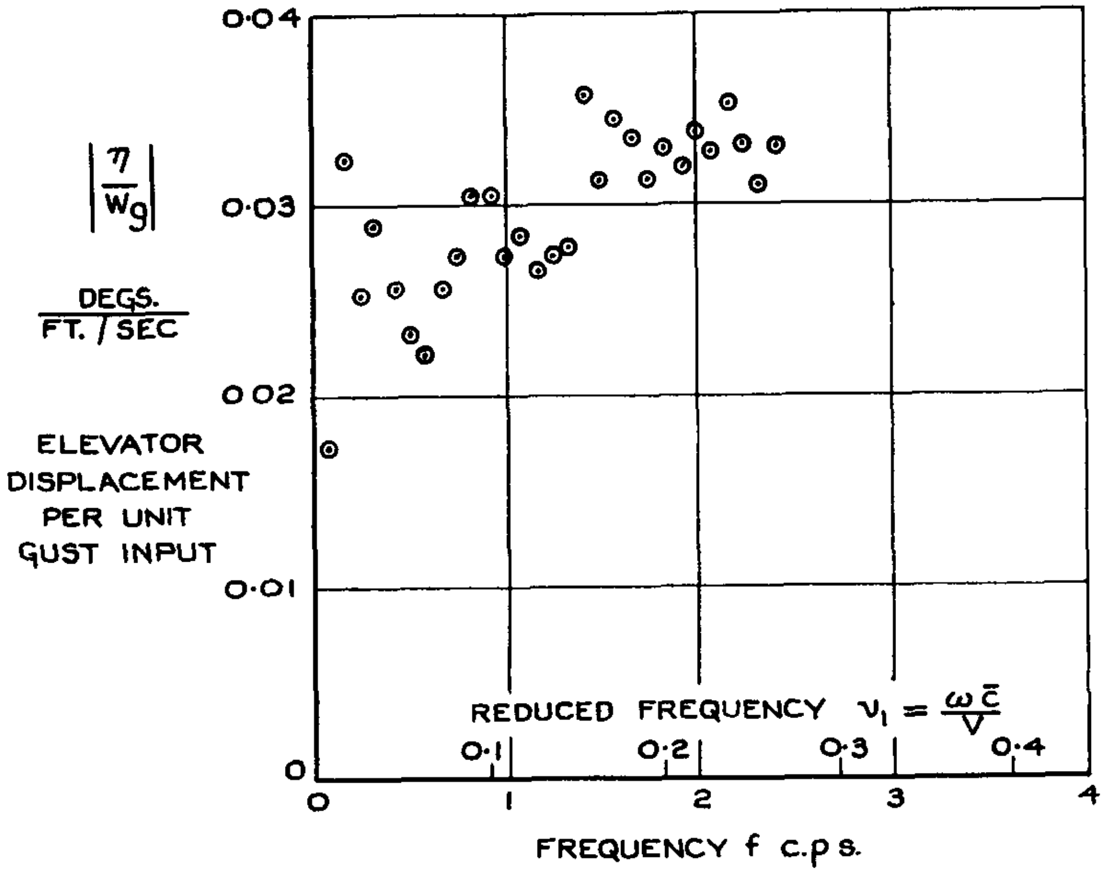


FIG. 10. PILOT RESPONSE IN TERMS OF ELEVATOR DISPLACEMENT TO UNIT VERTICAL GUST INPUT.

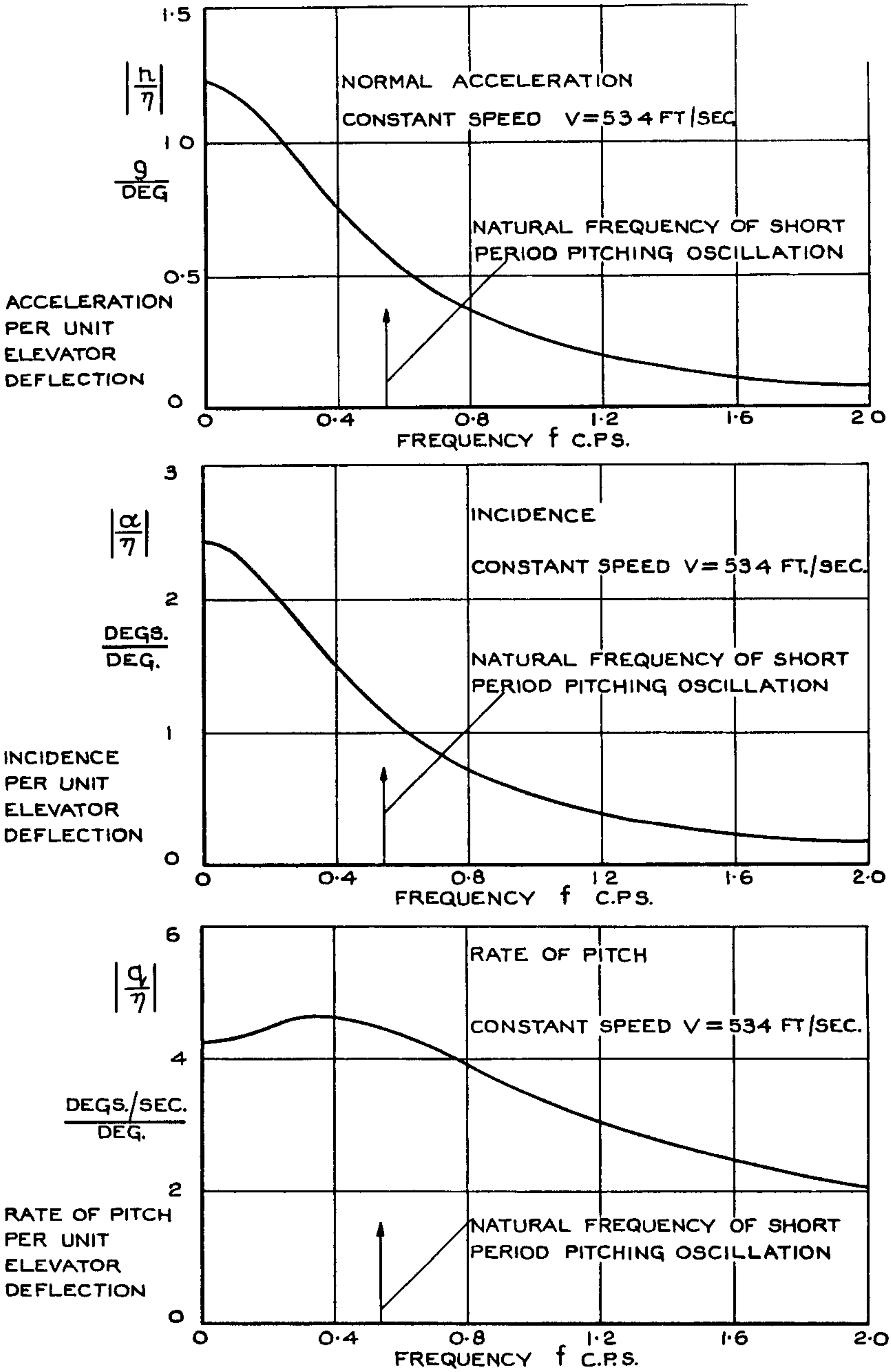


FIG. 11. FREQUENCY RESPONSE CHARACTERISTICS OF TEST AIRCRAFT (METEOR 7) TO ELEVATOR INPUT (MEAN MEASURED VALUES)

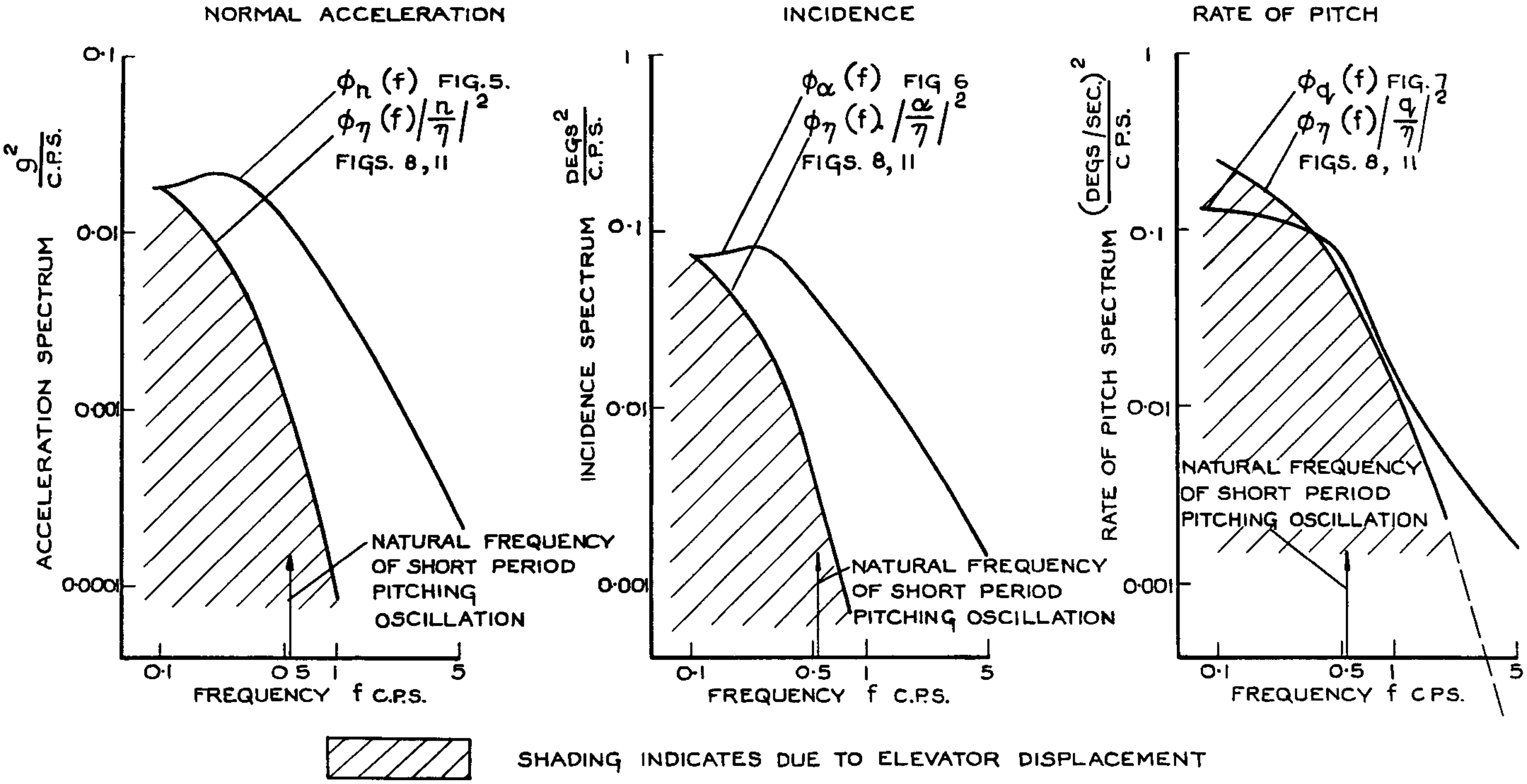


FIG.12. SPECTRA OF ACCELERATION, INCIDENCE AND RATE OF PITCH DUE TO ELEVATOR DISPLACEMENT.

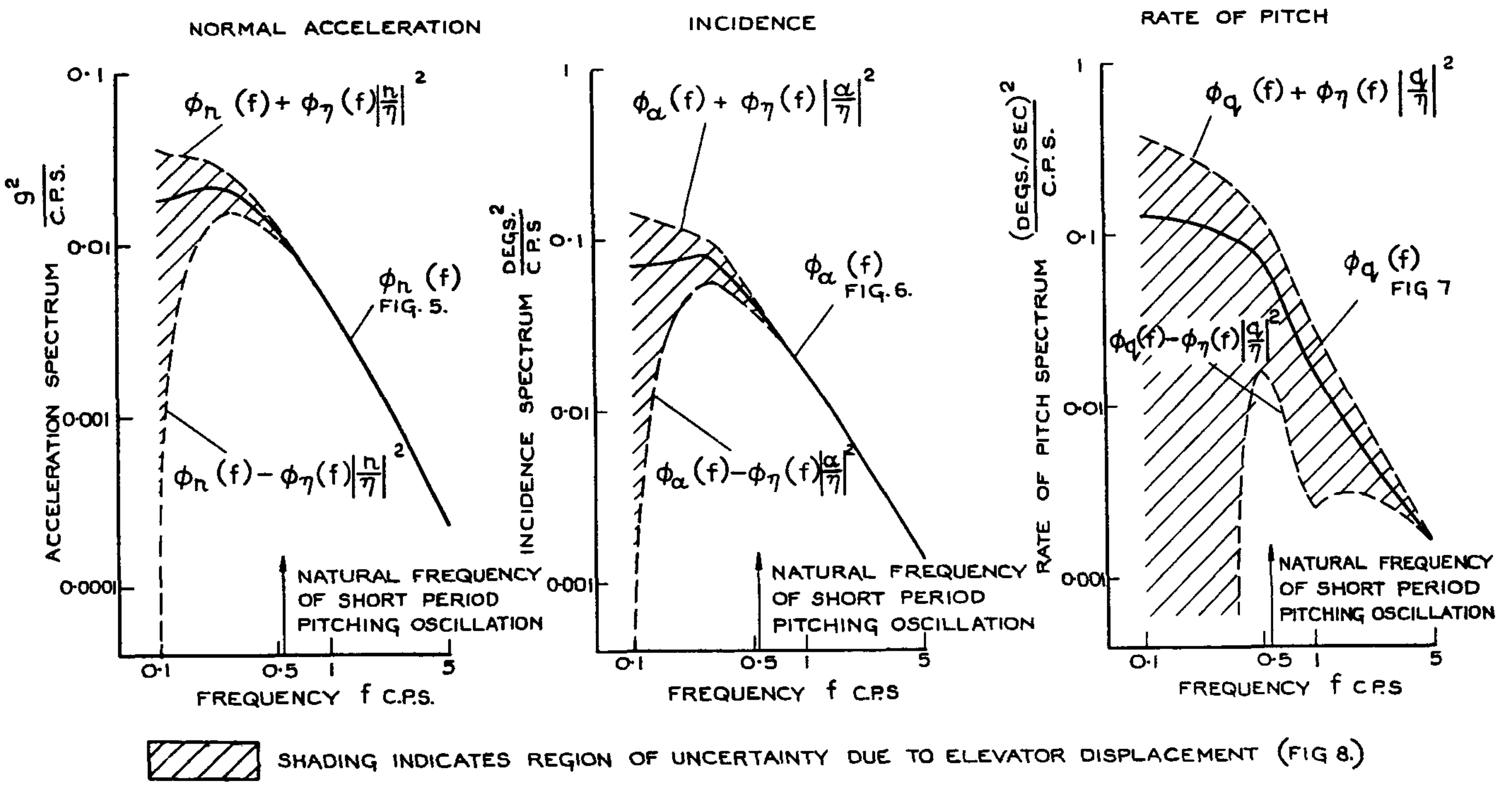


FIG.13. MAXIMUM POSSIBLE EFFECTS OF ELEVATOR DISPLACEMENT ON SPECTRA OF ACCELERATION, INCIDENCE AND RATE OF PITCH.

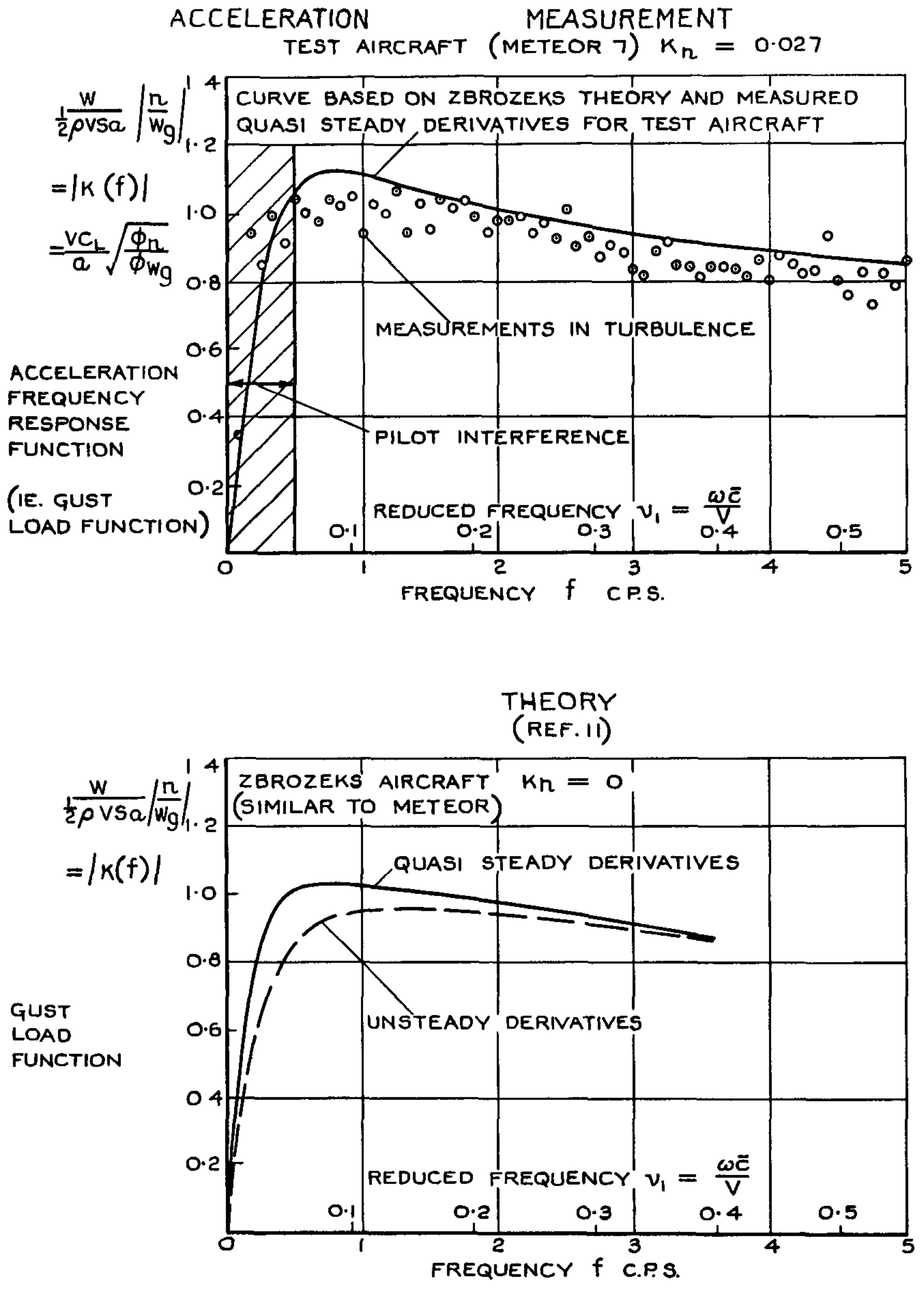


FIG. 14. COMPARISON OF NON DIMENSIONAL MEASURED AND THEORETICAL NORMAL C.G. ACCELERATION / GUST INPUT FREQUENCY RESPONSE FUNCTIONS FOR RIGID AIRCRAFT.

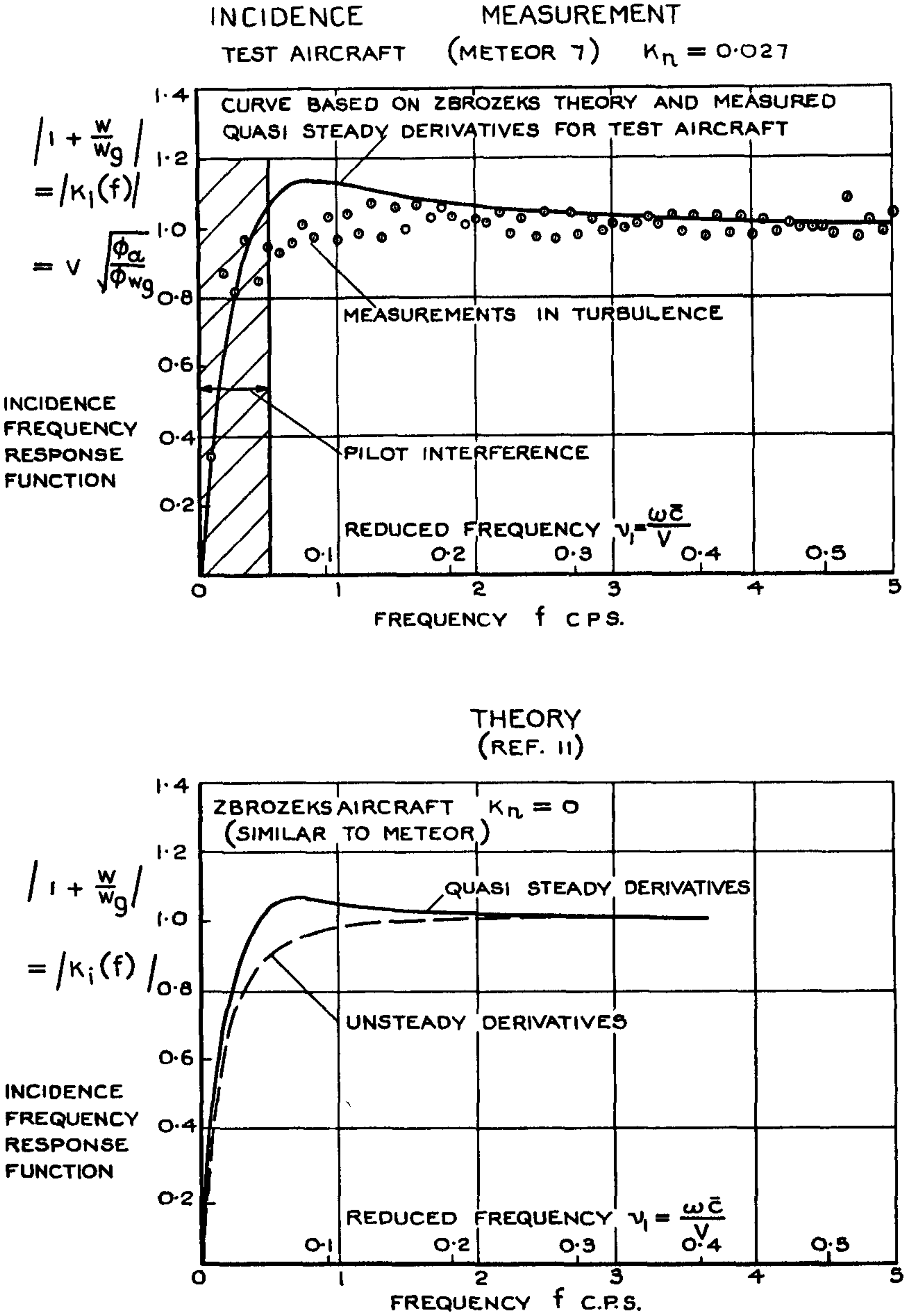


FIG. 15. COMPARISON OF NON DIMENSIONAL MEASURED AND THEORETICAL INCIDENCE/GUST INPUT FREQUENCY RESPONSE FUNCTIONS FOR RIGID AIRCRAFT.

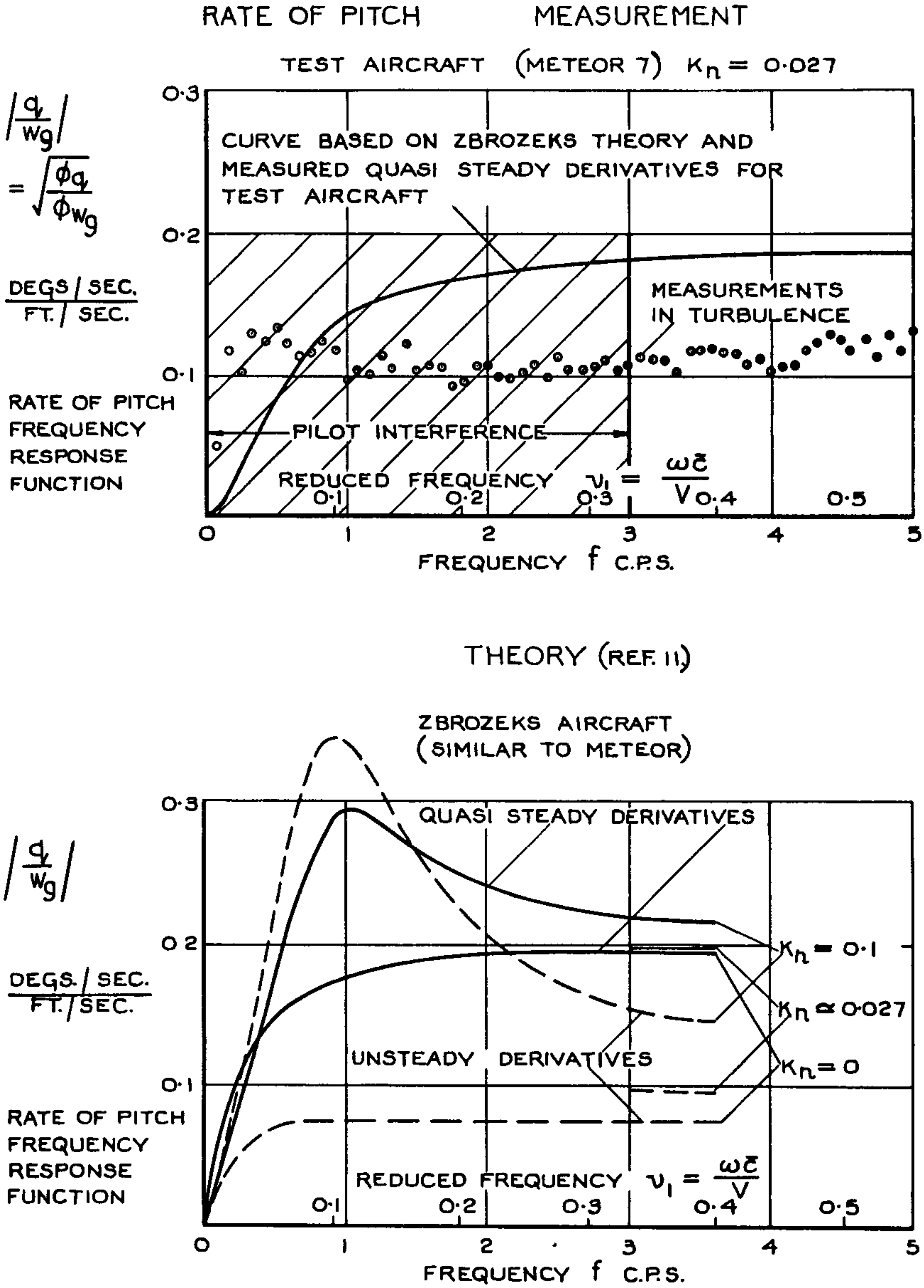


FIG. 16. COMPARISON OF MEASURED AND THEORETICAL RATE OF PITCH / GUST INPUT FREQUENCY RESPONSE FUNCTIONS FOR RIGID AIRCRAFT.

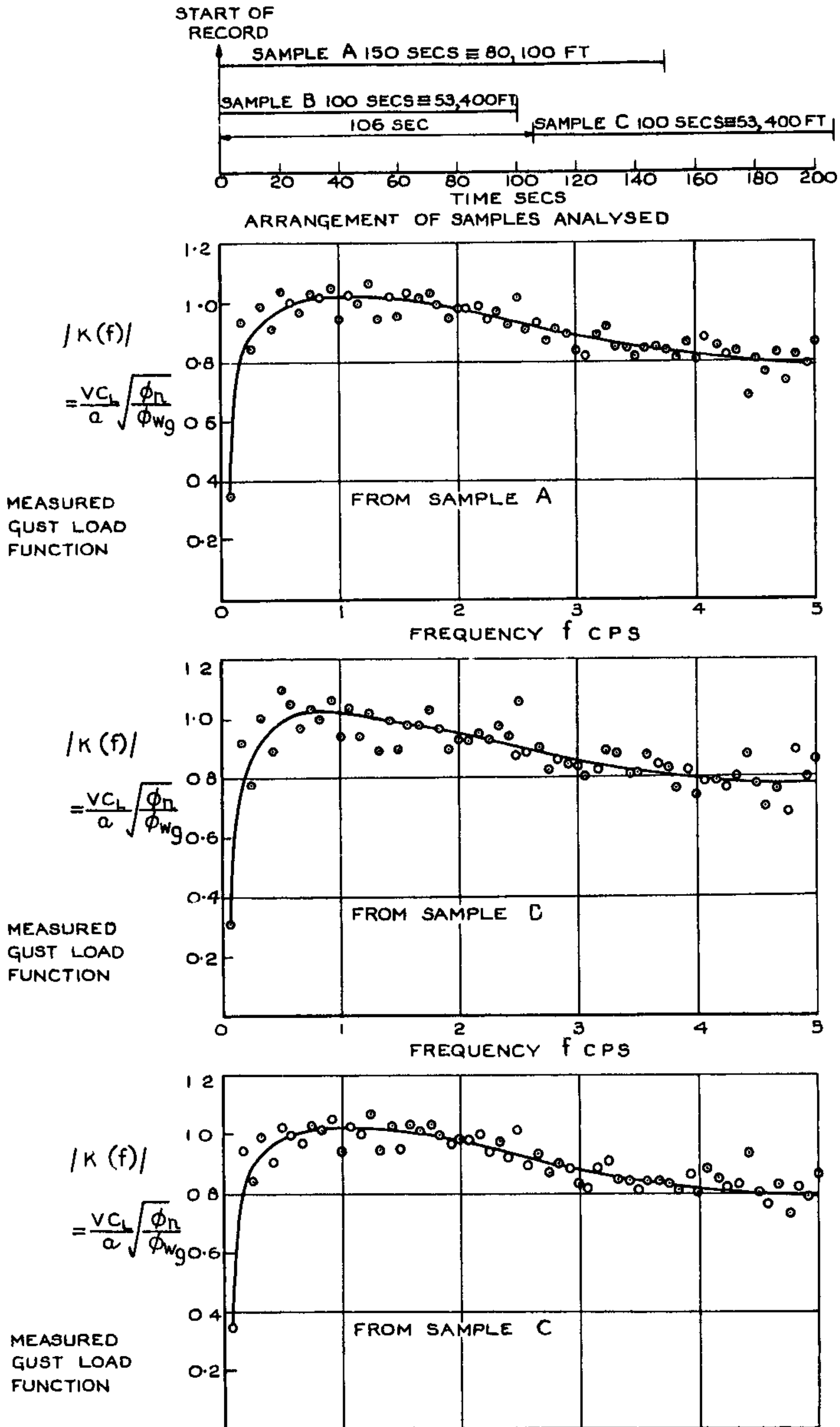
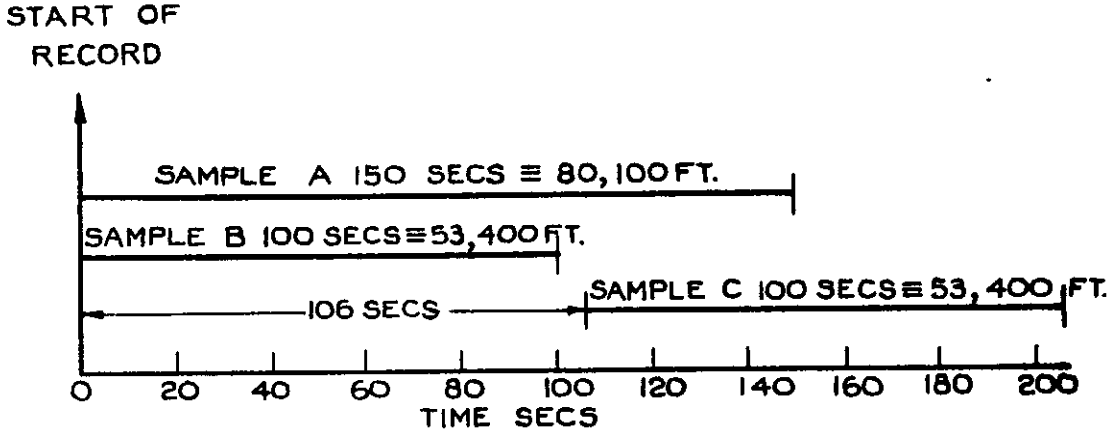


FIG. 17. COMPARISON OF NON-DIMENSIONAL MEASURED NORMAL C.G. ACCELERATION / GUST INPUT FREQUENCY RESPONSE FUNCTIONS OBTAINED FROM DIFFERENT SAMPLES.



ARRANGEMENT OF SAMPLES ANALYSED

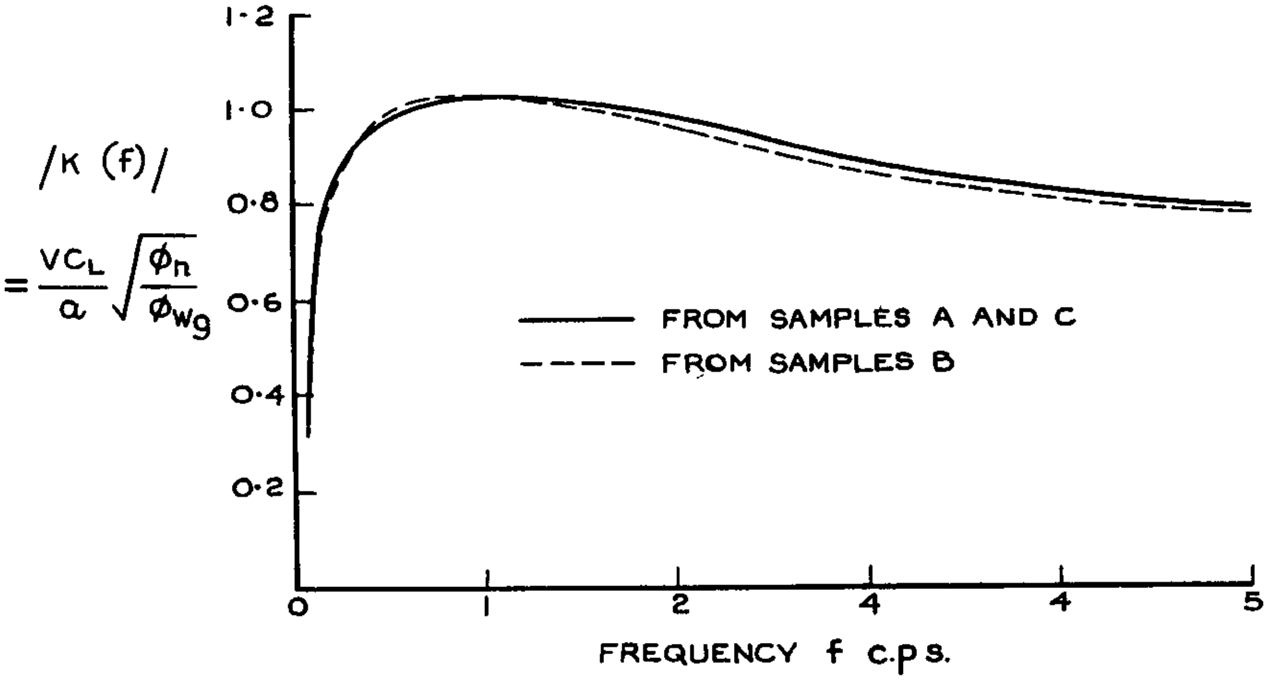


FIG. 18. COMPARISON OF THE MEAN ACCELERATION/GUST INPUT FREQUENCY RESPONSE FUNCTIONS FROM FIG 17.

A.R.C. C.P. No. 708

AI(42)Meteor 7 :
551.55 :
533.6.013.47

THE MEASURED RESPONSE OF AN AIRCRAFT TO THE VERTICAL VELOCITY COMPONENT OF ATMOSPHERIC TURBULENCE. Ridland, D.M. February 1963.

Measurements have been made of the rigid body response in the pitching plane of an aircraft to the vertical velocity component of atmospheric turbulence. Spectral methods were used to evaluate the aircraft frequency response functions, which agreed very well with theoretical predictions. Limited analysis of the measured elevator movement suggests that the pilot is the dominant factor in aircraft response at very low frequencies during flight through atmospheric turbulence.

A.R.C. C.P. No. 708

AI(42)Meteor 7 :
551.55 :
533.6.013.47

THE MEASURED RESPONSE OF AN AIRCRAFT TO THE VERTICAL VELOCITY COMPONENT OF ATMOSPHERIC TURBULENCE. Ridland, D.M. February 1963.

Measurements have been made of the rigid body response in the pitching plane of an aircraft to the vertical velocity component of atmospheric turbulence. Spectral methods were used to evaluate the aircraft frequency response functions, which agreed very well with theoretical predictions. Limited analysis of the measured elevator movement suggests that the pilot is the dominant factor in aircraft response at very low frequencies during flight through atmospheric turbulence.

A.R.C. C.P. No. 708

AI(42)Meteor 7 :
551.55 :
533.6.013.47

THE MEASURED RESPONSE OF AN AIRCRAFT TO THE VERTICAL VELOCITY COMPONENT OF ATMOSPHERIC TURBULENCE. Ridland, D.M. February 1963.

Measurements have been made of the rigid body response in the pitching plane of an aircraft to the vertical velocity component of atmospheric turbulence. Spectral methods were used to evaluate the aircraft frequency response functions, which agreed very well with theoretical predictions. Limited analysis of the measured elevator movement suggests that the pilot is the dominant factor in aircraft response at very low frequencies during flight through atmospheric turbulence.

C.P. No. 708

Printed and published by
HER MAJESTY'S STATIONERY OFFICE
To be purchased from
York House, Kingsway, London W.C.2
423 Oxford Street, London W.1
13A Castle Street, Edinburgh 2
109 St Mary Street, Cardiff
39 King Street, Manchester 2
50 Fairfax Street, Bristol 1
35 Smallbrook, Ringway, Birmingham 5
80 Chichester Street, Belfast 1
or through any bookseller

Printed in England

C.P. No. 708

S O Code No 23-9015-8