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# The Boundary Layer Drag of Bodies with Swept Trailing Edges in Supersonic Flow

*by*

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# THE BOUNDARY LAYER DRAG OF BODIES WITH SWEPT TRAILING EDGES IN SUPERSONIC FLOW

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J. C. Cooke, D.Sc.

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## SUMMARY

An earlier paper[1] gave expressions for the drag of a body with its trailing edge lying in a plane normal to the direction of flow at infinity. The present Note extends these results to a body with its trailing edge in the form of any smooth closed twisted curve. In particular it covers swept trailing edges whose angle of sweep may be variable.

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Replaces R.A.E. Tech. Note No. Aero 2867 - A.R.C. 24,739.

# LIST OF CONTENTS



# LIST OF ILLUSTRATIONS

1 INTRODUCTION

In Ref.1 the drag of a body with a straight trailing edge was found to be equal to the drag of a body enlarged by the displacement thickness $\delta^*$ (called the displacement body or sometimes the distorted body) together with a term which was expressed as an integral taken round the trailing edge. For thin wings or slender bodies this integral was written as the sum qN + qE where

$$N = 2 \int \Theta \left(\frac{u_D}{U_\infty}\right)^{2+H-M^2} d\sigma$$

$$E = \int \frac{v_D^2 + w_D^2}{U_\infty^2} \Delta^* \, d\sigma \quad .$$

The meanings of the symbols is the same as in Ref.1 and will be given again in the List of Symbols.

In this Note we obtain the corresponding results for a trailing edge which is swept by an angle $\Lambda$ (which need not be constant along the span). The values of N and E are

$$N = 2 \int \Theta \left(\frac{u_D}{U_\infty}\right)^{2+H-M^2-J\tan\Lambda} \cos\Lambda \, d\sigma$$

$$E = \int \frac{v_D^2 + w_D^2}{U_\infty^2} \Delta^* \cos\Lambda \, d\sigma$$

where

$$H = \Delta^*/\Theta \ , \quad J = \Theta_{12}/\Theta \ , \quad \bar{\rho}_e \bar{u}_e^2 \, \Theta_{12} = \int_0^\delta \rho_b u_b (\bar{u}_e - u_b) \, d\zeta \ ,$$

the subscript b referring to values inside the boundary layer and e to values outside it. The subscript D refers to inviscid flow about the original undistorted body and a bar over a quantity means that the value due to inviscid flow over the displacement body is to be taken.

The result is obtained by a momentum balance method as before. If the wing is thin and the cross-flow is small it is approximately true that the inviscid drag of the displacement body is equal to the inviscid drag of the body itself; to the approximation involved in this assumption we may write

$$N = 2 \int \Theta \cos \Lambda \, d\sigma ,$$

$$E = 0 .$$

This is the extension of the simple formula of Ref.1 to swept trailing edges.

## 2 THE CONTROL SURFACE

We surround the body by a cylinder of radius r, sufficiently large to include all of the body, with a plane end normal to the main stream at the front or sufficiently far upstream of any shocks there may be at the front. At the back the end of the cylinder is closed by a surface S' generated by lines L normal to the trailing edge and to the direction of flow at infinity, where the velocity is $U_\infty$. (We shall write "normal to $U_\infty$" when we mean "normal to the direction of flow at infinity".) These lines are taken all round the trailing edge. This edge need not necessarily be a curve in one plane but it may be a smooth curve in space but not curved in such a way that the lines L intersect in points inside the boundary layer. Outside the boundary layer the surface S' may be any simple surface containing the curves A and B in Fig.2.

An attempt is made in Fig.1 to draw this surface, but it is not easy to represent this three-dimensional figure on paper. All the straight lines on the rear surface are not only normal to the trailing edge but also normal to $U_\infty$. In Fig.2 we show the surface S' projected on to a plane normal to $U_\infty$. The curve A is the outside of the cylinder, B is the section of S' by the edge of the boundary layer, C is the section by the displacement surface and D is the trailing edge. These curves are to be considered to be on S'; we only show their projections for ease of drawing. In Fig.2 the surfaces $S_3$, $S_4$ and $S_5$ (the parts between the curves) are also surfaces on S'. Velocity components will be taken as follows:- u parallel to $U_\infty$, w parallel to a line L, and v normal to u and w, in a right handed system. The flow outside the boundary layer is taken to be the same as inviscid flow outside the displacement surface. The local angle of sweep $\Lambda$ at a point P is defined to be the angle between the tangent plane T to the surface S' at P and the plane normal to $U_\infty$. The surface S' is not a developable surface and so the tangent plane to it is different at different points of L. Hence $\Lambda$ varies not only as we move along the trailing edge (because of varying sweep) but also as we travel across the boundary layer. The last variation in the value of $\Lambda$ will be of order $\delta$, the boundary layer thickness.

Provided a suitable convention as to the sign of $\Lambda$ is employed it can be shown that the velocity component normal to the surface, that is, normal to T, is $u \cos \Lambda - v \sin \Lambda$. It may be pointed out that on the trailing edge $90^\circ - \Lambda$ is the angle between the tangent to the trailing edge and $U_\infty$. This is the easiest way to find $\Lambda$ when the trailing edge is a twisted curve in space.

As in Ref.1 we assume that the flow outside the boundary layer is the same as inviscid flow over a body thickened by an amount $\delta^*$, the displacement thickness. This body we call the "displacement body" or the "distorted body".

## 3 MOMENTUM BALANCE

We denote by the subscript b values inside the boundary layer, and a bar over any quantity denotes its value for flow over the distorted body, that is the displacement body.

Conservation of mass through the cylinder gives

$$\int_{S_1} \rho_\infty U_\infty \, dS - \int_{S_2} \bar{\rho} \, U_\infty \frac{\partial \bar{\phi}}{\partial r} \, dS - \int_{S_3} \bar{\rho}(\bar{u} \cos \Lambda - \bar{v} \sin \Lambda) \, dS$$

$$- \int_{S_4+S_5} \rho_v (u_b \cos \Lambda - v_b \sin \Lambda) \, dS = 0 . \qquad (1)$$

The drag force on the body will balance the flux of momentum in the $U_\infty$ direction of the fluid leaving the cylinder. Hence the drag $\bar{D}$ is given by

$$\bar{D} = \int_{S_1} (p_\infty + \rho_\infty U_\infty^2) \, dS - \int_{S_2} \bar{\rho} \, U_\infty \frac{\partial \bar{\phi}}{\partial r} \bar{u} \, dS$$

$$- \int_{S_3} \left\{ \bar{p} \cos \Lambda + \bar{\rho}(\bar{u} \cos \Lambda - \bar{v} \sin \Lambda) \, \bar{u} \right\} dS$$

$$- \int_{S_4+S_5} \left\{ p_b + \rho_b (u_b \cos \Lambda - v_b \sin \Lambda) \, u_b \right\} dS - p_B S(B)$$

where $p_B$ is the base pressure and $S(B)$ the projected area of the base on a plane normal to $U_\infty$. $U_\infty \bar{\phi}$ is the velocity potential.

Multiply equation (1) by $U_\infty$ and subtract from equation (2) and we have, writing $\bar{u}' = \bar{u} - U_\infty$

$$\bar{D} = \int_{S_1} p_\infty \, dS - \int_{S_2} \bar{\rho} \, U_\infty \frac{\partial \bar{\phi}}{\partial r} \bar{u}' \, dS - \int_{S_3} (\bar{p} \cos \Lambda + \bar{\rho} \bar{u} \bar{u}' \cos \Lambda - \bar{\rho} \bar{u} \bar{u}' \sin \Lambda) \, dS$$

$$- \int_{S_4+S_5} (p_b \cos \Lambda + \rho_b u_b u_b' \cos \Lambda - \rho_b v_b u_b' \sin \Lambda) \, dS$$

$$- p_B \, S(B) .$$

Now

$$\int_{S_1} p_\infty \, dS = \int_{S_3} p_\infty \cos \Lambda \, dS + \int_{S_4+S_5} p_\infty \cos \Lambda \, dS + p_\infty S(B)$$

and hence

$$\bar{D} = I_2 + J_3 + J_{45} + (p_\infty - p_B) \, S(B) ,$$

where

$$I_2 = - \int_{S_2} \bar{\rho} \frac{\partial \bar{\phi}}{\partial r} \bar{u}' \, dS , \tag{2}$$

$$J_3 = - \int_{S_3} (q \, \bar{c}_p \cos \Lambda + \bar{\rho} \bar{u} \bar{u}' \cos \Lambda - \bar{\rho} \bar{v} \bar{u}' \sin \Lambda) \, dS = - \int_{S_3} K dS , \tag{3}$$

$$J_{45} = - \int_{S_4+S_5} (q \, \bar{c}_p \cos \Lambda + \rho_b u_b u_b' \cos \Lambda - \rho \, v_b u_b' \sin \Lambda) \, dS , \tag{4}$$

$$q \bar{c}_p = \bar{p} - p_\infty , \quad q = \tfrac{1}{2} \rho_\infty U_\infty^2 ,$$

assuming that the pressure remains constant throughout the boundary layer, that is, $p_b$ is equal to $\bar{p}$, or differs from it by a quantity of order $\delta$.

This assumption will be discussed in Section 6. For convenience we shall drop the term $(p_\infty - p_B)\, S(B)$. It may be inserted at any stage if required, that is, if the body has a base.

## 4 THE VALUE OF $I_3 + J_{45}$

We may write

$$J_3 = -\int_{S_3} K dS = -\int_{S_3+S_4} K dS + \int_{S_4} K dS \,.$$

In this equation we may give the part of the integrand over $S_4$ any value we wish; we shall give $\bar{p}$, $\bar{\rho}$, $\bar{u}$ and $\bar{v}$ their values due to inviscid flow over the distorted body. These are not their true values since $S_4$ is inside the boundary layer. Thus we have

$$\int_{S_4} K dS = \int_{S_4} (q\, \bar{c}_p \cos\Lambda + \bar{\rho}\,\bar{u}\,\bar{u}' \cos\Lambda - \bar{\rho}\,\bar{v}\,\bar{u}' \sin\Lambda)\, dS \,.$$

If on $S_4$ we replace $\bar{u}$ by $\bar{u}_e$, $\bar{\rho}$ by $\bar{\rho}_e$ and $\bar{v}$ by $\bar{v}_e$ the error is of order $\delta$. Since $S_4$ is of order $\delta$ we may make this substitution in the integral with error of order $\delta^2$. Hence we have

$$J_3 + J_{45} = -\int_{S_3+S_4} K dS + q\,\bar{N} + O(\delta^2)$$

where

$$q\,\bar{N} = \int_{S_4+S_5} (\bar{\rho}_e \bar{u}_e \bar{u}_e' \cos\Lambda - \bar{\rho}_e \bar{v}_e \bar{u}_e' \sin\Lambda - \rho_b u_b u_b' \cos\Lambda + \rho_b v_b u_b' \sin\Lambda)\, dS$$

$$- \int_{S_5} (q\, \bar{c}_p \cos\Lambda + \bar{\rho}_e \bar{u}_e \bar{u}_e' \cos\Lambda - \bar{\rho}_e \bar{v}_e \bar{u}_e' \sin\Lambda)\, dS$$

$$= \int_{S_4+S_5} \left\{ \rho_b \, u_b(\bar{u}_e - u_b) + \bar{u}'_e(\bar{\rho}_e \, \bar{u}_e - \rho_b \, u_b) \right\} \cos \Lambda \, dS$$

$$- \int_{S_4+S_5} \left\{ \rho_b \, w_b(\bar{u}_e - u_b) - \bar{u}'_e(\bar{\rho}_e \, \bar{v}_e - \rho_b \, w_b) \right\} \sin \Lambda \, dS$$

$$- \int_{S_5} (q \, \bar{c}_p \cos \Lambda + \bar{\rho}_e \, \bar{u}_e \, \bar{u}'_e \cos \Lambda - \rho_e \, \bar{v}_e \, \bar{u}'_e \sin \Lambda) \, dS \, . \qquad (5)$$

We shall neglect all quantities of order $\delta^2$.

At this stage we ignore the change in $\Lambda$ across the boundary layer, and give it the value it has on the surface itself. Here the error in $\Lambda$ is of order $\delta$ and so gives an error of order $\delta^2$. We obtain

$$\bar{N} = 2 \int_D \frac{\bar{\rho}_e \bar{u}_e^2}{\rho_\infty U_\infty^2} \Theta \cos \Lambda \, d\sigma + 2 \int_D \frac{\bar{\rho}_e \bar{u}_e \bar{u}'_e}{\rho_\infty U_\infty^2} (\Delta^* - \delta^*) \cos \Lambda \, d\sigma$$

$$- 2 \int_D \frac{\rho_e \bar{u}_e \bar{u}'_e}{\rho_\infty U_\infty^2} \Theta_{12} \sin \Lambda \, d\sigma - 2 \int_D \frac{\bar{\rho}_e \bar{u}_e \bar{u}'_e}{\rho_\infty U_\infty^2} \left( \Delta'^* - \frac{\bar{v}_e}{\bar{u}_e} \delta^* \right) \sin \Lambda \, d\sigma$$

$$- \int_D \bar{c}_p \, \delta^* \cos \Lambda \, d\sigma \qquad (6)$$

where

/Equation (7)

$$\bar{\rho}_e \, \bar{u}_e^2 \, \Theta = \int_D^{\delta} \rho_b \, u_b (\bar{u}_e - u_b) \, d\zeta$$

$$\bar{\rho}_e \, \bar{u}_e \, \Delta^* = \int_D^{\delta} (\bar{\rho}_e \, \bar{u}_e - \rho_b \, u_b) \, d\zeta$$

$$\bar{\rho}_e \, \bar{u}_e^2 \, \Theta_{12} = \int_D^{\delta} \rho_b \, v_b (\bar{u}_e - u_b) \, d\zeta \qquad (7)$$

$$\bar{\rho}_o \, \bar{u}_e \, \Delta'^* = \int_D^{\delta} (\bar{\rho}_e \, \bar{v}_e - \rho_b \, v_b) \, d\zeta \, .$$

In calculating these "thicknesses" we may if we wish, replace $\bar{u}_e$ by $u_D$ and similarly for other symbols with subscript e. This is in accordance with usual boundary layer practice; although it is not satisfactory to do this at the trailing edge in subsonic flow it is probably sufficiently accurate in supersonic flow, when there is not such a great difference between $\bar{u}_e$ and $u_D$.

We show in Appendix 1 that if the cross-flow is small $\Delta^* - \delta^*$ and $\Delta'^* - (\bar{v}_e/\bar{u}_e)\, \delta^*$ are small. Both of these are multiplied by $\bar{u}'_e = U - \bar{u}_e$ which is also small for thin wings and slender bodies. We shall ignore these terms. $\Delta^* - \delta^*$ was taken as zero in Ref.1 without comment. We shall also replace $\bar{u}_e$ etc. by $u_D$ etc. with error of order $\delta$. Hence the error in $\bar{N}$ will be of order $\delta^2$. We finally obtain

$$\bar{N} = 2 \int_D \frac{\rho_D u_D^2}{\rho_\infty U_\infty^2} \Theta \cos \Lambda \, d\sigma - 2 \int_D \frac{\rho_D u_D u'_D}{\rho_\infty U_\infty^2} \Theta_{12} \sin \Lambda \, d\sigma - \int_D c_p \, \Delta^* \cos \Lambda \, d\sigma \, . \qquad \dots (8)$$

In the last term we have replaced $\delta^*$ by $\Delta^*$.

For slender bodies we have approximately

$$\frac{\rho_D}{\rho_\infty} = 1 - M^2 \frac{u'_D}{U_\infty} ,$$

$$c_p = - \frac{2 u'_D}{U_\infty} - \frac{v_D^2 + w_D^2}{U_\infty^2} ,$$

and so we may write in such a case

$$\bar{N} = N + E$$

where

$$N = 2 \int \Theta \left\{ 1 + \frac{u'_D}{U_\infty} (2 + H - M^2 - J \tan \Lambda) \right\} \cos \Lambda \, d\sigma \qquad (9)$$

$$= 2 \int \Theta \left( \frac{u_D}{U_\infty} \right)^{2+H-M^2-J\tan\Lambda} \cos \Lambda \, d\sigma ,$$

$$E = \int_D \frac{v_D^2 + w_D^2}{U_\infty^2} \Delta^* \cos \Lambda \, d\sigma . \qquad (10)$$

In these equations we have written

$$H = \frac{\Delta^*}{\Theta} , \qquad J = \frac{\Theta_{12}}{\Theta} .$$

$J$ is small if the cross-flow is small, and indeed $J \tan \Lambda$ is likely to be small, even if the sweep is large, since $\Theta_{12}$ may be expected to behave like $\varepsilon \sin \Lambda \cos \Lambda$, where $\varepsilon$ is a small quantity related to the cross-flow, especially in the case of a swept wing. In such a case we may write

$$N = 2 \int \Theta \left( \frac{u_D}{U_\infty} \right)^{2+H-M^2} \cos \Lambda \, d\sigma \qquad (11)$$

and if we are content with a larger error term we may write

$$N = 2 \int \Theta \cos \Lambda \, d\sigma \tag{12}$$

and ignore E; this result is obtained from equations (9) and (10) by ignoring terms of order $\delta u_D'$, that is of order $\delta(u_D - U_\infty)$.

## 5 TOTAL DRAG

We have

$$\bar{D} = I_2 + J_3 + J_{45}$$

$$= I_2 - \int_{S_3+S_4} K dS + q\bar{N} .$$

The sum of the first two terms gives the drag of the displacement surface (assuming that it has a base pressure $p_\infty$) that is, the U component of $p - p_\infty$ integrated over the displacement surface; hence we have the general result that the drag is equal to that of the displacement surface together with an amount $q\bar{N}$, where $\bar{N}$ is given by equation (6). For slender bodies with small cross-flow $\bar{N}$ may be replaced by N + E where N and E are given by equations (9) and (10).

Finally to a rough approximation N may be written as in equation (11) and less accurately still we may ignore the difference between the drag of the body and that of the displacement surface. To the order of approximation involved in this last assumption we may say, in a manner analogous to what we did in Ref.1, that the total drag of the body is equal to the inviscid drag of the body together with a term $q N_1$ where

$$N_1 = 2 \int \Theta \cos \Lambda \, d\sigma \tag{13}$$

taken round the trailing edge. This is the formula corresponding to equation (25) of Ref.1.

## 6 DISCUSSION

Doubts have been expressed about the validity of assuming the pressure to be constant across the boundary layer, as is done in this paper and was done in Ref.1. It has been suggested that this is not true, for instance, in two-dimensional subsonic flow near to the trailing edge because of the result $\partial p/\partial\zeta = \rho_b \kappa u_b^2$, $\kappa$ being the curvature of a streamline. If one assumes that the curvature is finite, then this equation shows that the pressure change across the boundary layer is of order $\delta$ and so may be ignored in our equations, since in these the pressure occurs multiplied by a quantity of order $\delta$. However there is on the surface a sharp turn in direction of flow and so there is an infinite curvature at the trailing edge. This, however, is at a place where $u_b$ is zero. Further out as $u_b$ increases $\kappa$ decreases rapidly so that it may well be true that $\rho_b \kappa u_b^2$ is nowhere large. Moreover, in supersonic flow the fluid near to the wall is hotter and the density lower so that in places where $\kappa$ is large the density is small as well as $u_b$. This should give a reduced effect in supersonic flow compared with that at lower speeds. There seems to be only one attempt to assess the effect of curvature near to the trailing edge and this is due to Spence[2], but unfortunately it does not seem possible to extend this work to flow where the boundary layer is partly subsonic and partly supersonic. In subsonic flow Spence found the pressure difference across a turbulent boundary layer to be

$$\frac{\rho\, u_e^2 \tau}{\omega\pi} (H-1)^{-1}$$

where $\tau$ is the trailing edge angle and $\omega = 2 - (\tau/\pi)$. However it is of course by no means certain that this analysis goes over into supersonic flow. If, however, this result does apply in the supersonic case our expression for N in equation (8) has an appearance of precision which is not justified, since the error in the last term in equation (8) will be of the same order as the term itself.

Thus until the matter of the pressure change across the boundary layer can be cleared up we are not justified in making a statement any more precise than that given at the end of Section 5. This may not at present be a serious limitation in view of the fact that most applications at present involve thin wings and slender bodies, and of the fact that no great precision is yet possible in calculating turbulent boundary layers.

The amount of practical use that may be made of the present theory is in some doubt. In practice shock waves often occur somewhere along the chord of the wing and not merely at the leading and trailing edges. This would not invalidate the general theory but would probably render it of no practical value. If the trailing edge is subsonic it would seem inevitable for shocks to appear on the wing surface, since in such a case the Mach cone at a given point on the trailing edge will include some part of the trailing edge downstream of it.

The method described here may possibly be of some practical use if the Mach number is high enough for the trailing edge to be supersonic, or in a case where the only shocks present are at the leading and trailing edges.

## 7 CONCLUDING REMARKS

The main result of this paper is that, to order $\delta$, the drag of the body is equal to that of the displacement body, together with a term qN, where N is given by equation (6). This form is scarcely of practical value, and may indeed have an appearance of precision which cannot be justified, and so various approximations and assumptions lead to less accurate values; the simplest but least accurate of the results is that the total drag is equal to

$$2\,q \int \Theta \cos \Lambda \, d\sigma$$

taken round the trailing edge. Whether the assumptions are justified or not needs testing by experiment, but it would seem essential that there be no shocks except at the leading and trailing edges. Thus the method will not apply to transonic flow or to wings with subsonic trailing edges.

---

## LIST OF SYMBOLS

| | |
|---|---|
| A | edge of control surface |
| B | edge of boundary layer |
| C | edge of displacement body |
| $c_p$ | pressure coefficient |
| D | trailing edge |
| E | defined in (9) |
| H | $\Delta^*/\Theta$ |
| $h_\xi, h_\eta$ | coefficients in line element |
| $I_2$ | defined in (2) |
| $J_3, J_{45}$ | defined in (3) and (4) |
| J | $\Theta_{12}/\Theta$ |
| K | defined in (3) |
| L | line normal to the trailing edge and to $U_\infty$ |
| M | Mach number |
| $\bar{N}$ | defined in (5) |
| N | defined in (9) |

## LIST OF SYMBOLS (Contd)

| | |
|---|---|
| $p$ | pressure |
| $q$ | $\frac{1}{2} \rho_\infty U_\infty^2$ |
| $r$ | radius of control cylinder |
| $S_1, S_2, S_3, S_4, S_5, S'$ | surfaces defined in Figs.1 and 2 |
| $S(B)$ | base area projected on a plane normal to $U_\infty$ |
| $U_\infty$ | velocity at infinity |
| $U_\xi$ | external velocity component along streamlines |
| $u, v, w$ | velocity components |
| $u_\xi, u_\eta$ | velocity components along and normal to streamlines |
| $u'$ | $u - U_\infty$ |
| $\alpha$ | angle between streamlines and $U_\infty$ |
| $\delta_\xi, \delta_\eta$ | defined in (14) |
| $\delta^*$ | displacement thickness |
| $\Delta^*, \Delta'^*$ | defined in (7) |
| $\zeta$ | distance normal to surface |
| $\Theta, \Theta_{12}$ | defined in (7) |
| $\Lambda$ | angle of sweep |
| $\rho$ | density |
| $\phi$ | velocity potential |

### Subscripts

| | |
|---|---|
| b | denotes values inside the boundary layer |
| e | denotes values just outside the boundary layer |
| D | denotes values for inviscid flow about the true body |

A bar over a quantity denotes that its value for flow over the displacement body is to be taken.

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## APPENDIX 1

## THE VALUES OF $\Delta^* - \delta^*$ AND $\Delta'^* - (v_e/u_e)\,\delta^*$

Lighthill[3] has shown that if streamline coordinates $\xi$ and $\eta$ are taken such that the line elements along and perpendicular to the external streamlines are $h_\xi\, d\xi$ and $h_\eta\, d\eta$ respectively, then

$$\delta^* = \delta_\xi - \frac{1}{\rho_e U_\xi h_\eta} \frac{\partial}{\partial \eta} \int_0^\xi \rho_e U_\xi h_\xi \delta_\eta \, d\eta$$

where

$$\delta_\xi = \int_0^\delta \left(1 - \frac{\rho_b u_\xi}{\rho_e U_\xi}\right) d\zeta\,, \quad \delta_\eta = \int_0^\delta \frac{\rho_b u_\eta}{\rho_e U_\xi}\, d\zeta\,, \qquad (14)$$

and $U_\xi$, $u_\xi$ are the external and internal components of velocity in the streamline direction and $u_\eta$ is the component normal to the streamlines. $U_\eta$ is of course zero. Now

$$\Delta^* = \int_0^\delta \left(1 - \frac{\rho_b u_b}{\rho_e u_e}\right) d\zeta$$

and we have

$$u_b = u_\xi \cos\alpha - u_\eta \sin\alpha$$

$$u_e = U_\xi \cos\alpha$$

$$v_e = U_\xi \sin\alpha$$

$$v_b = u_\xi \sin\alpha + u_\eta \cos\alpha$$

where $\alpha$ is the angle between the external streamlines and $U_\infty$. Hence

$$\Delta^* = \int_0^{\delta} \left[1 - \frac{\rho_b(u_\xi \cos\alpha - u_\eta \sin\alpha)}{U_\xi \cos\alpha}\right] d\zeta = \delta_\xi + \delta_\eta \tan\alpha ,$$

and so

$$\Delta^* - \delta^* = \delta_\eta \tan\alpha + \frac{1}{\rho_e U_\xi h_\eta} \frac{\partial}{\partial\eta} \int_0^{\xi} \rho_e U_\xi h_\xi \delta_\eta \, d\xi . \tag{15}$$

If the cross-flow is small, $\delta_\eta$ is also small and hence so is $\Delta^* - \delta^*$.

We also have

$$\Delta'^* = \int_0^{\delta} \left[\frac{v_e}{u_e} - \frac{\rho_b(u_\xi \sin\alpha + u_\eta \cos\alpha)}{\rho_e U_\xi \cos\alpha}\right] d\zeta$$

$$= \tan\alpha \, \delta_\xi - \delta_\eta$$

and so

$$\Delta'^* - \frac{v_e}{u_e} \delta^* = - \delta_\eta + \frac{\tan\alpha}{\rho_e U_\xi h_\eta} \frac{\partial}{\partial\eta} \int_0^{\xi} \rho_e U_\xi h_\xi \delta_\eta \, d\eta . \tag{16}$$

This is small if $\delta_\eta$ is small.

In equations (6) both of the small quantities (15) and (16) are multiplied by $u_e'$ which is small for thin wings or slender bodies. In such a case, therefore, we may ignore the products. If the cross-flow is not small it is still true that $\Delta^* - \delta^*$ is smaller than the boundary layer thickness, and the approximation of equation (12) must hold since it was obtained by ignoring terms of order $\delta u_D'$.

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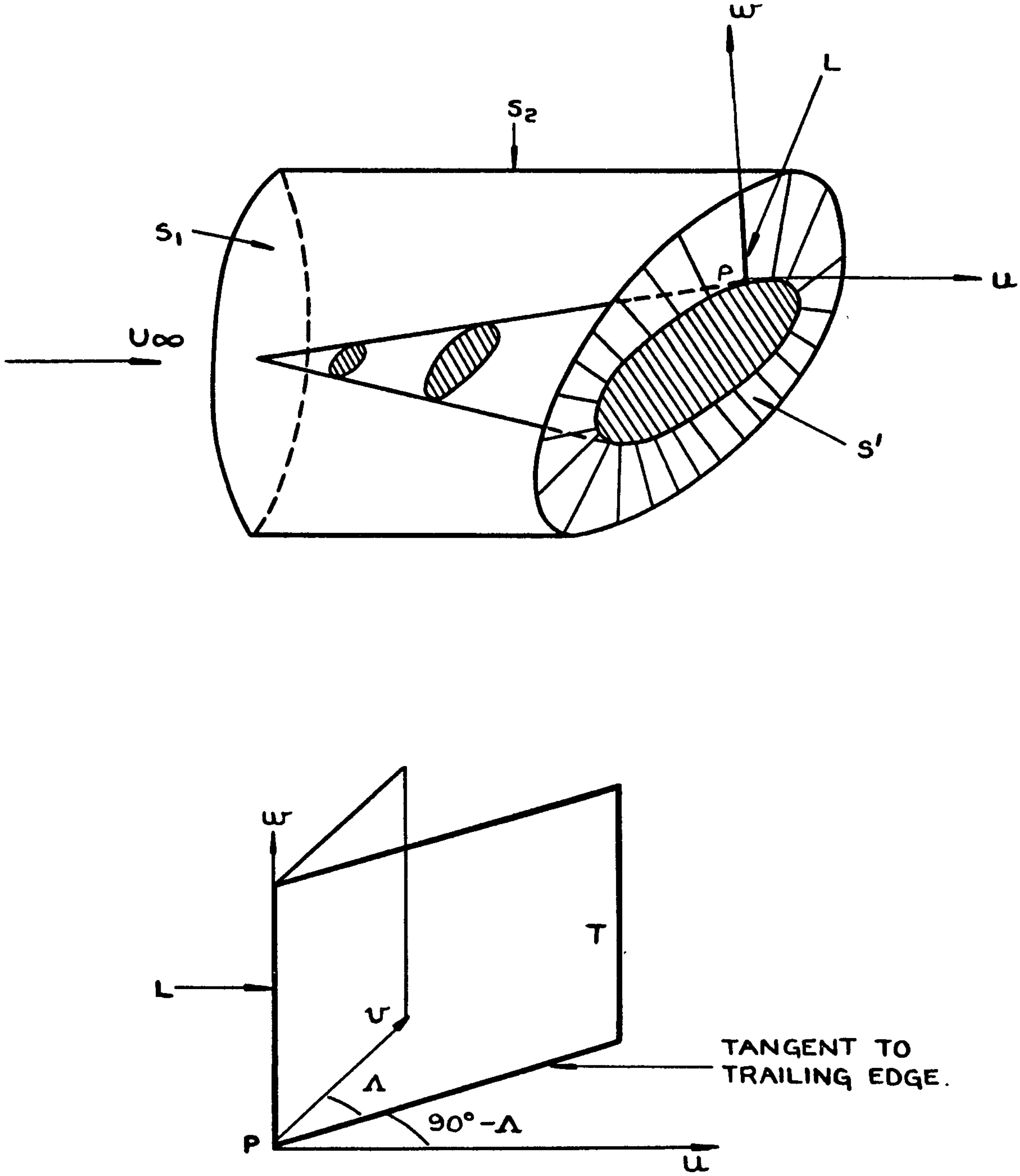
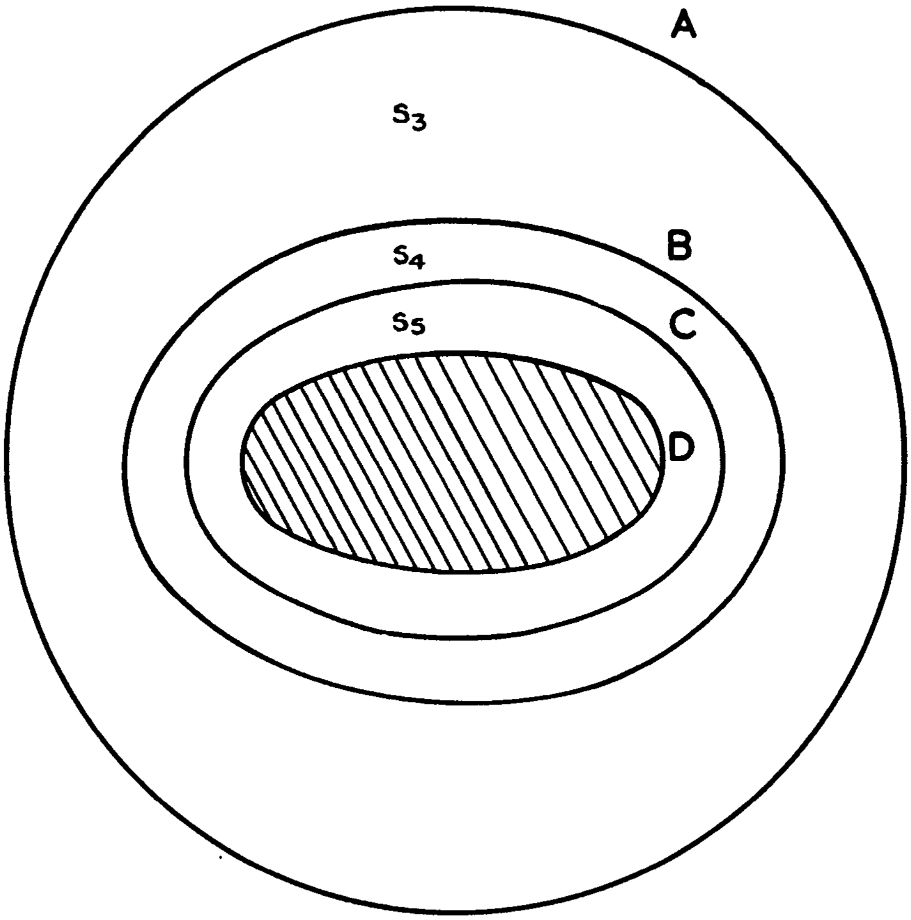



FIG. 1. THE CONTROL SURFACE.
THE LINES L ARE NORMAL TO THE TRAILING EDGE
AND TO THE DIRECTION OF $U_\infty$.




A EDGE OF CONTROL CYLINDER.
B EDGE OF BOUNDARY LAYER.
C EDGE OF DISPLACEMENT SURFACE.
D EDGE OF BODY.

FIG. 2. THE PROJECTION OF $S'$ ON A PLANE NORMAL TO THE DIRECTION OF FLOW AT INFINITY.

A.R.C. C.P. No.699

533.696 :
533.6.013.12 :
533.6.011.5

THE BOUNDARY LAYER DRAG OF BODIES WITH SWEPT TRAILING EDGES IN SUPERSONIC FLOW. Cooke, J. C. February, 1963.

An earlier paper gave expressions for the drag of a body with its trailing edge lying in a plane normal to the direction of flow at infinity. The present Note extends these results to a body with its trailing edge in the form of any smooth closed twisted curve. In particular it covers swept trailing edges whose angle of sweep may be variable.

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