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By

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SIDE FORCE ON A WING BODY COMBINATION DUE TO TRAILING VORTICES

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J.R. Barnes

SUMMARY

A long cylindrical body has two wings attached symmetrically but set at equal and opposite angles relative to its axis. When this system is placed at incidence in a uniform flow a side force is experienced. The purpose of this note is to make some estimate of the magnitude of this side force.

The vortices shed by the wings are assumed to have rolled up at the trailing edge and their paths are calculated using slender body theory. Expressions are given for the forces experienced by the body aft of the wings and some numerical calculations made.

Replaces R.A.E. Tech. Note No. Aero. 2834-A.R.C. 24, 221.

(82975)

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1 INTRODUCTION

It is the purpose of this note to determine the order of magnitude of the side force experienced by a slender wing-body combination with asymmetric lift when it is placed at incidence to a uniform stream in an inviscid fluid.

The model consists of a long circular cylindrical body to which are attached symmetrically two slender low aspect ratio pointed wings. These wings can assume any arbitrary incidence relative to the body. The body is at incidence to the uniform stream (Fig. 1).

We may assume that the nose of the body lies sufficiently far upstream to have no effect upon the flow over the wings.

We will assume also that the vortices generated by the model have rolled up into two vortices at the wing trailing edge, with initial semi-span s_0 and that these will constitute the wake. The strengths of these vortices are taken to be proportional to the incidences of the wings behind which they trail.

Slender body theory is used so that the flow about the model may be treated as a two-dimensional potential problem in planes perpendicular to the x-axis. Thus we can use the mathematical representation employed by Owen and Maskell[1] and later by Owen and Anderson[2].

We therefore consider the body to be infinitely long and to have initially vortices at equal distances s_0 away on either side. Using the method of images we can write down the complex potential from which we obtain four ordinary differential equations for the velocity components of the two trailing vortices. Integration of these equations gives the coordinates of the vortices in planes perpendicular to the axis of the body downstream of the trailing edge.

Having obtained these coordinates we can use Ward's theory[3] to calculate the lateral forces upon any given length of the body downstream. This method is then applied to two examples, the differential equations being integrated numerically.

2 METHOD OF ANALYSIS

The coordinate system is that shown in Fig. 1, with the x-axis in the direction of the uniform stream, the xz-plane vertically upwards and the y-axis orthogonal to the x- and z-axes forming a left-handed system. The origin is at the centre of the body in the line of the wing trailing edge.

For an inviscid fluid the linearised equation for the velocity potential ϕ is

$$(1 - M^2) \frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} + \frac{\partial^2 \phi}{\partial z^2} = 0 \qquad (1)$$

where M is the Mach No. of the undisturbed flow.

Assuming that slender body conditions are satisfied equation (1) reduces to

$$\frac{\partial^2 \phi}{\partial y^2} + \frac{\partial^2 \phi}{\partial z^2} = 0 . \qquad (2)$$

Thus the flow can be treated as a two-dimensional potential problem in the yz-plane with a parameter x. We may therefore introduce the complex variable

$$\zeta = y + i z$$

and the usual complex variable methods in two dimensional flow can be applied.

2.1 Position of the trailing vortices

Consider the positions of the vortices in a plane perpendicular to the direction of motion downstream of the wing trailing edge (see Fig. 2).

Let the positions of the free vortices in the plane under consideration be ζ_1 (y_1, z_1) and ζ_2 (y_2, z_2). The boundary condition on the body is satisfied by inserting bound image vortices at the inverse points

$$\zeta_1' = \frac{a^2}{\bar{\zeta}_1}, \quad \zeta_2' = \frac{a^2}{\bar{\zeta}_2}$$

where a is the radius of the body.

The flow in this plane will be that due to these four vortices together with the cross flow Uα in the direction of the positive z-axis, α being the body incidence to the free stream of velocity U. The complex potential for the system is given by

$$\omega = \phi + i\psi = -\frac{i K_1}{2\pi} \log (\zeta - \zeta_1) - \frac{i K_2}{2\pi} \log (\zeta - \zeta_2)$$

$$+ \frac{i K_1}{2\pi} \log (\zeta - \zeta_1') + \frac{i K_2}{2\pi} \log (\zeta - \zeta_2') - i U\alpha \left(\zeta - \frac{a^2}{\zeta}\right) \quad (3)$$

where K_1 and K_2 are the strength of the vortices associated with ζ_1 and ζ_2 respectively.

Now each free vortex will have a motion induced by the remaining vortices and the cross flow. In the yz-plane being considered the vortex of strength K_1 at ζ_1 will move with the local stream velocity which is obtained by differentiating the complex potential with respect to ζ and omitting the term giving the singularity when $\zeta = \zeta_1$.

Hence the complex velocity is given by

$$\left(\frac{d\omega}{d\zeta}\right)_{\zeta=\zeta_1} = v_1 - i w_1 = -\frac{i K_2}{2\pi}\left(\frac{1}{\zeta_1 - \zeta_2}\right) + \frac{i K_1}{2\pi}\left(\frac{\bar{\zeta}_1}{\bar{\zeta}_1 \zeta_1 - a^2}\right)$$

$$+ \frac{i K_2}{2\pi}\left(\frac{\bar{\zeta}_2}{\bar{\zeta}_2 \zeta_1 - a^2}\right) - i U\alpha \left(1 + \frac{a^2}{\zeta_1^2}\right) \quad (4)$$

and similarly for $\zeta = \zeta_2$

$$\left(\frac{d\omega}{d\zeta}\right)_{\zeta=\zeta_2} = v_2 - i\, w_2 = -\frac{i\, K_1}{2\pi}\left(\frac{1}{\zeta_2 - \zeta_1}\right) + \frac{i\, K_1}{2\pi}\left(\frac{\bar{\zeta}_1}{\zeta_2\, \bar{\zeta}_1 - a^2}\right)$$

$$+ \frac{i\, K_2}{2\pi}\left(\frac{\bar{\zeta}_2}{\bar{\zeta}_2\, \zeta_2 - a^2}\right) - i\, U\alpha\left(1 + \frac{a^2}{\zeta_2^2}\right) \qquad (5)$$

where v_1, v_2 are the velocity components in the y direction, and w_1, w_2 the components in the z direction.

Since the vortices follow the local streamlines we may write,

$$v_1 = \frac{dy_1}{dt} = U\frac{dy_1}{dx} \quad ; \quad w_1 = U\frac{dz_1}{dx}$$

$$v_2 = U\frac{dy_2}{dx} \quad ; \quad w_2 = U\frac{dz_2}{dx} \; .$$

For convenience we also define the non-dimensional quantities:-

$y_1 = Y_1\, s_o$; $z_1 = Z_1\, s_o$; $y_2 = Y_2\, s_o$; $z_2 = Z_2\, s_o$; $x = X\, s_o$; $a = A\, s_o$;

where s_o is the initial semi-span of the trailing vortices.

Simplifying and separating real and imaginary parts in equations (4) and (5) we have

$$\frac{dY_1}{dX} = \kappa_1\left\{\frac{Z_1}{Y_1^2 + Z_1^2 - A^2}\right\} + \kappa_2\left\{\frac{Z_1\, Z_2^2 + Z_1\, Y_2^2 - Z_2 A^2}{(Y_1\, Y_2 + Z_1\, Z_2 - A^2)^2 + (Y_1\, Z_2 - Z_1\, Y_2)^2}\right.$$

$$\left. - \frac{(Z_1 - Z_2)}{(Y_1 - Y_2)^2 + (Z_1 - Z_2)^2}\right\} - \frac{2\alpha A^2 Y_1\, Z_1}{(Y_1^2 + Z_1^2)^2} \qquad (6)$$

$$\frac{dZ_1}{dX} = -\kappa_1\left\{\frac{Y_1}{Y_1^2 + Z_1^2 - A^2}\right\} - \kappa_2\left\{\frac{Y_1\, Y_2^2 + Y_1\, Z_2^2 - Y_2\, A^2}{(Y_1\, Y_2 + Z_1\, Z_2 - A^2)^2 + (Y_2\, Z_1 - Z_2\, Y_1)^2}\right.$$

$$\left. - \frac{(Y_1 - Y_2)}{(Y_1 - Y_2)^2 + (Z_1 - Z_2)^2}\right\} + \alpha\left\{\frac{(Y_1^2 - Z_1^2 + A^2)(Y_1^2 - Z_1^2) + 4\, Y_1^2\, Z_1^2}{(Y_1^2 + Z_1^2)^2}\right\} \qquad (7)$$

$$\frac{dY_2}{dX} = \kappa_1 \left\{ \frac{Z_1^2 Z_2 + Z_2 Y_1^2 - Z_1 A^2}{(Y_1 Y_2 + Z_1 Z_2 - A^2)^2 + (Z_2 Y_1 - Z_1 Y_2)^2} + \frac{(Z_1 - Z_2)}{(Y_1 - Y_2)^2 + (Z_1 - Z_2)^2} \right\}$$

$$+ \kappa_2 \left\{ \frac{Z_2}{Y_2^2 + Z_2^2 - A^2} \right\} - \frac{2 \alpha A^2 Y_2 Z_2}{(Y_2^2 + Z_2^2)^2} \tag{8}$$

$$\frac{dZ_2}{dX} = - \kappa_1 \left\{ \frac{Y_1^2 Y_2 + Y_2 Z_1^2 - Y_1 A^2}{(Y_1 Y_2 + Z_1 Z_2 - A^2)^2 + (Z_2 Y_1 - Z_1 Y_2)^2} + \frac{(Y_1 - Y_2)}{(Y_1 - Y_2)^2 + (Z_1 - Z_2)^2} \right\}$$

$$- \kappa_2 \left\{ \frac{Y_2}{Y_2^2 + Z_2^2 - A^2} \right\} + \alpha \left\{ \frac{(Y_2^2 - Z_2^2 + A^2)(Y_2^2 - Z_2^2) + 4 Y_2^2 Z_2^2}{(Y_2^2 + Z_2^2)^2} \right\} \tag{9}$$

where $\kappa_1 = \frac{K_1}{2 \pi U s_o}$, $\kappa_2 = \frac{K_2}{2 \pi U s_o}$.

The solutions of equations (6) - (9) give the motions of the two trailing vortices downstream of the wing trailing edge.

2.2 Strength of the vortices

As has been stated, the trailing vortices are assumed to have rolled up at the wing trailing edge with initial semi-span s_o.

Using the approximation given in Ref. 1 the strength of a vortex may be written:-

$$K = 2 U s_o \gamma \left(k - \frac{a^2}{k s_o^2} \right), \tag{10}$$

where $k = \frac{b}{s_o}$, b being the semi wing span, and γ is the wing incidence to the free stream.

In terms of the non-dimensional quantities given in section 2.1, equation (10) reduces to

$$K = 2 U s_o \gamma \left(k - \frac{A^2}{k} \right), \tag{11}$$

so that

$$\kappa = \frac{\gamma}{\pi} \left(k - \frac{A^2}{k} \right). \tag{12}$$

2.3 Lateral forces on the body

According to Ward[3] the lateral forces F_y, F_z experienced by the body forward of any station x = c are given by

$$\frac{F_y + i F_z}{\frac{1}{2}\rho U^2} = 4\pi (d)_{s=c} + 2S'(c)\,\zeta_g(c) + 2S(c)\,\zeta_g'(c) \tag{13}$$

where d is the coefficient of $1/\zeta$ in the series expansion of the complex potential ω, $S(x)$ is the cross-sectional area of the body and $\zeta_g (= y_g + i z_g)$ is the position of the centre of area of the body. Primes denote partial derivatives with respect to x.

In the problem considered here $S(x)$ is constant and ζ_g is zero. Hence the force on a length c of the body aft of the trailing edge is given by

$$\frac{F_y + i F_z}{\frac{1}{2}\rho U^2} = 4\pi \left\{ (d)_{x=c} - (d)_{x=o} \right\} \tag{14}$$

Separating real and imaginary parts and writing in terms of the non-dimensional quantities defined above we have

$$\frac{F_y}{2\pi\rho U^2 s_o^2} = -\left\{ \left[\frac{\kappa_1 Z_1 (Y_1^2 + Z_1^2 - A^2)}{(Y_1^2 + Z_1^2)} + \frac{\kappa_2 Z_2 (Y_2^2 + Z_2^2 - A^2)}{(Y_2^2 + Z_2^2)} \right]_{x=c} - \left[\frac{\kappa_1 Z_1 (Y_1^2 + Z_1^2 - A^2)}{(Y_1^2 + Z_1^2)} + \frac{\kappa_2 Z_2 (Y_2^2 + Z_2^2 - A^2)}{(Y_2^2 + Z_2^2)} \right]_{x=o} \right\} \tag{15}$$

and

$$\frac{F_z}{2\pi\rho U^2 s_o^2} = \left\{ \left[\frac{\kappa_1 Y_1 (Y_1^2 + Z_1^2 - A^2)}{(Y_1^2 + Z_1^2)} + \frac{\kappa_2 Y_2 (Y_2^2 + Z_2^2 - A^2)}{(Y_2^2 + Z_2^2)} + \alpha A^2 \right]_{x=c} - \left[\frac{\kappa_1 Y_1 (Y_1^2 + Z_1^2 - A^2)}{(Y_1^2 + Z_1^2)} + \frac{\kappa_2 Y_2 (Y_2^2 + Z_2^2 - A^2)}{(Y_2^2 + Z_2^2)} + \alpha A^2 \right]_{x=o} \right\} \tag{16}$$

3 NUMERICAL EXAMPLES

We take the particular case with the wing-body angle equal to $\pm\beta$ (Fig. 3). Then from equation (12) the strengths of the two vortices are

$$\kappa_1 = \frac{(\beta + \alpha)}{\pi} \left(k - \frac{A^2}{k} \right) \tag{17}$$

and

$$\kappa_2 = \frac{(\beta - \alpha)}{\pi} \left(k - \frac{A^2}{k} \right). \tag{18}$$

From Ref. 1 a suitable value for k is approximately 1.15.

We will take the two cases

(a) $\alpha = 1^\circ, \beta = 7^\circ \quad A = \frac{1}{3}$

and (b) $\alpha = 5^\circ, \beta = 7^\circ \quad A = \frac{1}{3}$.

For the initial conditions

$$Y_1 = 1, \quad Z_1 = 0, \quad Y_2 = -1, \quad Z_2 = 0 ,$$

the motions of the vortices are given by the solutions of equations (6) - (9). These solutions are shown in Figs. 4 and 5. All values are non-dimensional to the length s_0 and x is measured along the axis of the body.

The lateral forces on the body are calculated for different lengths of body, using equations (15) and (16) and are shown in Figs. 6 and 7.

4 DISCUSSION

From calculations made during the course of this work there appear to be three different types of vortex motion, depending upon the relative strengths of the vortices and cross-flow. These are when

(i) both vortices rotate around the body axis,

(ii) only one vortex rotates about the axis and the other leaves the body completely,

and (iii) both vortices leave the body completely.

(i) and (iii) represent limiting types of motion and are obtained when the following conditions exist.

(i) the body is at zero incidence to the free stream and the wings have equal and opposite incidence relative to its axis,

and (iii) the body is at incidence to the free stream and the wing-body incidence is zero.

Of the two cases presented in section 3 case (a) gives rise to motion of type (ii) and (b) gives a variation of type (iii).

In motion of type (i) the vortices spiral about the body axis on a cylinder of constant radius concentric with the body, always remaining at the ends of a diameter. This motion will give zero lateral forces on the body.

In the other limiting case (iii) the problem is the same as that considered by Owen and Maskell[1]. The vortices trail from their initial positions ultimately becoming parallel to one another. This gives a lift force but no side force.

These two sets of conditions were used as partial checks of the method of solution used in the cases presented in section 3.

In case (a) of section 3 the incidence of the body is small giving a weak cross-flow, whilst the vortices are relatively strong and in the ratio 4:3. The paths of the vortices are shown in Fig. 4. Numbers attached to points of the curves are distances downstream expressed in terms of the semi-span s_0. The vortices start to rotate around one another as in case (i), until the effect of the cross-flow is sufficient to overcome the spiral motion of the weaker vortex and take it away from the body. The stronger vortex appears to remain with the body downstream. The rather surprising irregularity in the path of the weaker vortex has been checked by graphical methods.

The forces experienced by the body as a result of this motion are shown in Fig. 6. As might be expected by observing the vortex motion in Fig. 4, the lift force changes sign, corresponding to the cyclic motion of the stronger vortex. The side force remains in the same sense for different body lengths but increases rapidly as the length increases.

In case (b) the body is at greater incidence giving a relatively stronger cross-flow and vortex strengths in the ratio 6:1. Here the motion is tending towards that of type (iii), the rotation being negligible (Fig. 5). There is a side force on the body which amounts to about three times the lift force for a body length of ten vortex semi-spans aft of the wing trailing edge (Fig. 7).

The analysis shows that there exists a considerable side force on a body due to the motion of the vortices and that its magnitude and direction depend upon the relative strengths of the vortices and cross-flow. It should be emphasised that the forces calculated are solely those experienced by the body as a result of the vortices and cross-flow. No account is taken of the lift forces exerted by the wing panels themselves, or on the body forward of the trailing edges of the wings.

LIST OF SYMBOLS

a	body radius
A	a/s_o
b	wing semi-span
c	length of body
F_y	side force
F_z	lift force
K	strength of vortex
k	b/s_o
M	Mach No.
s_o	initial vortex semi-span
x,y,z	cartesian coordinates
X,Y,Z	x/s_o, y/s_o, z/s_o
U	free stream velocity
v,w	velocity components
α	body incidence to free stream
β	wing-body incidence
γ	incidence of wing to free stream
ζ	complex variable

LIST OF SYMBOLS (Contd)

κ	$K/2\pi U s_0$
ρ	air density
ϕ	velocity potential
ψ	stream function
ω	complex potential

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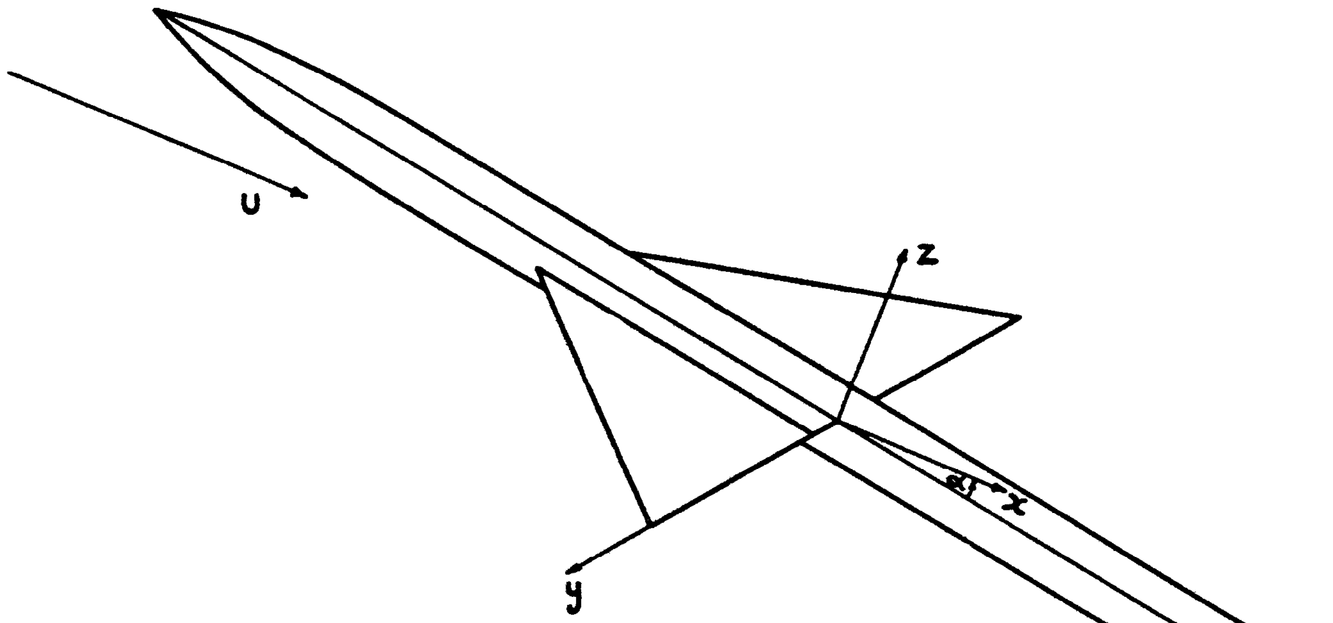

FIG. 1. SKETCH OF WING-BODY COMBINATION AND THE COORDINATE SYSTEM.

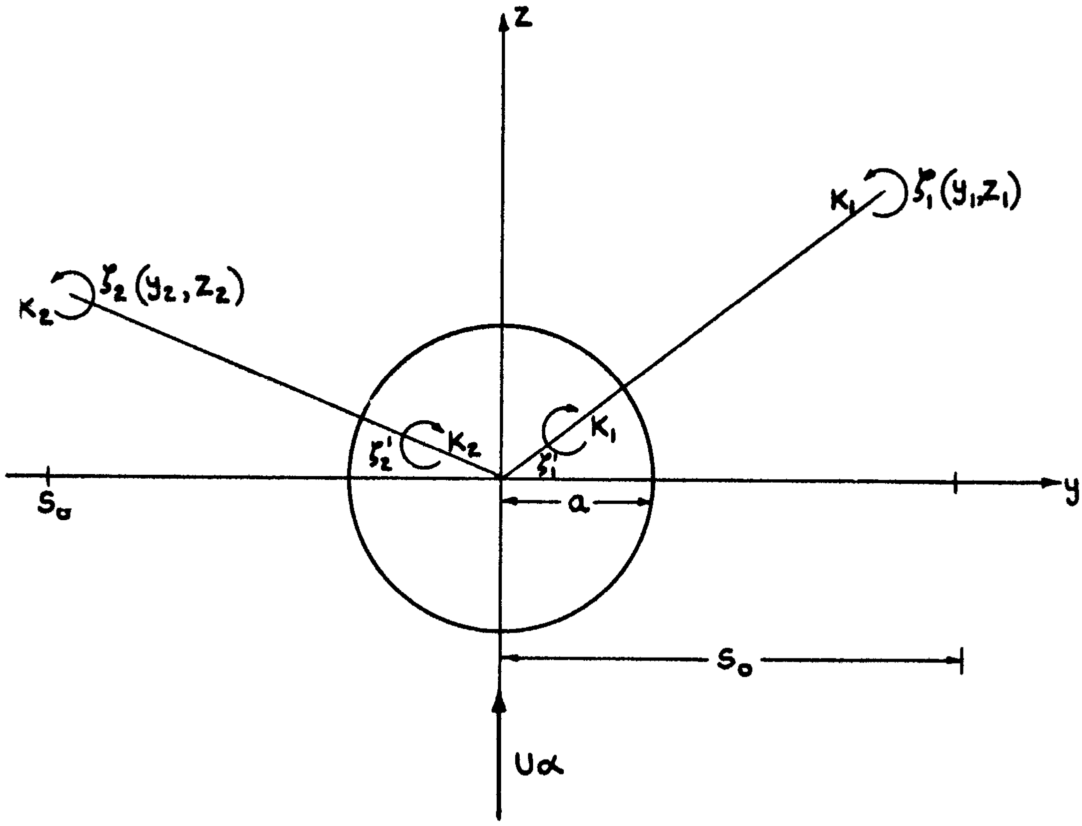

FIG. 2. VORTEX ARRANGEMENT IN PLANE DOWNSTREAM OF WING TRAILING EDGE PERPENDICULAR TO THE DIRECTION OF MOTION (LOOKING DOWNSTREAM).

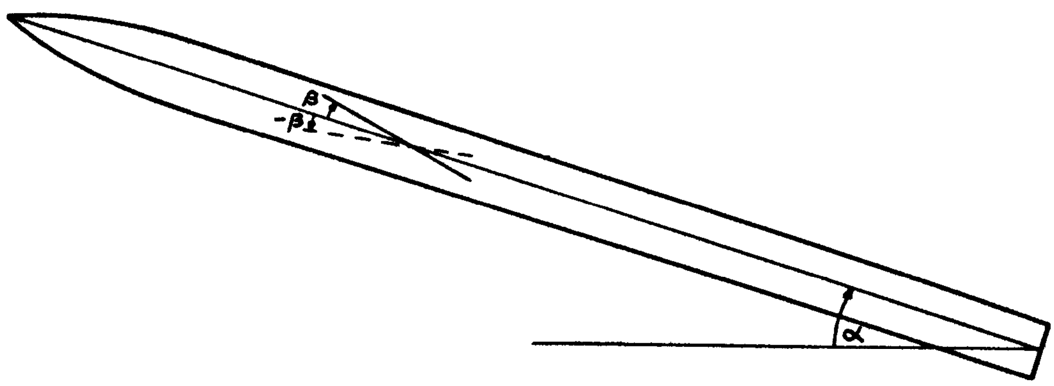

FIG. 3. WING-BODY ARRANGEMENT FOR CASES CONSIDERED IN SECTION 3.

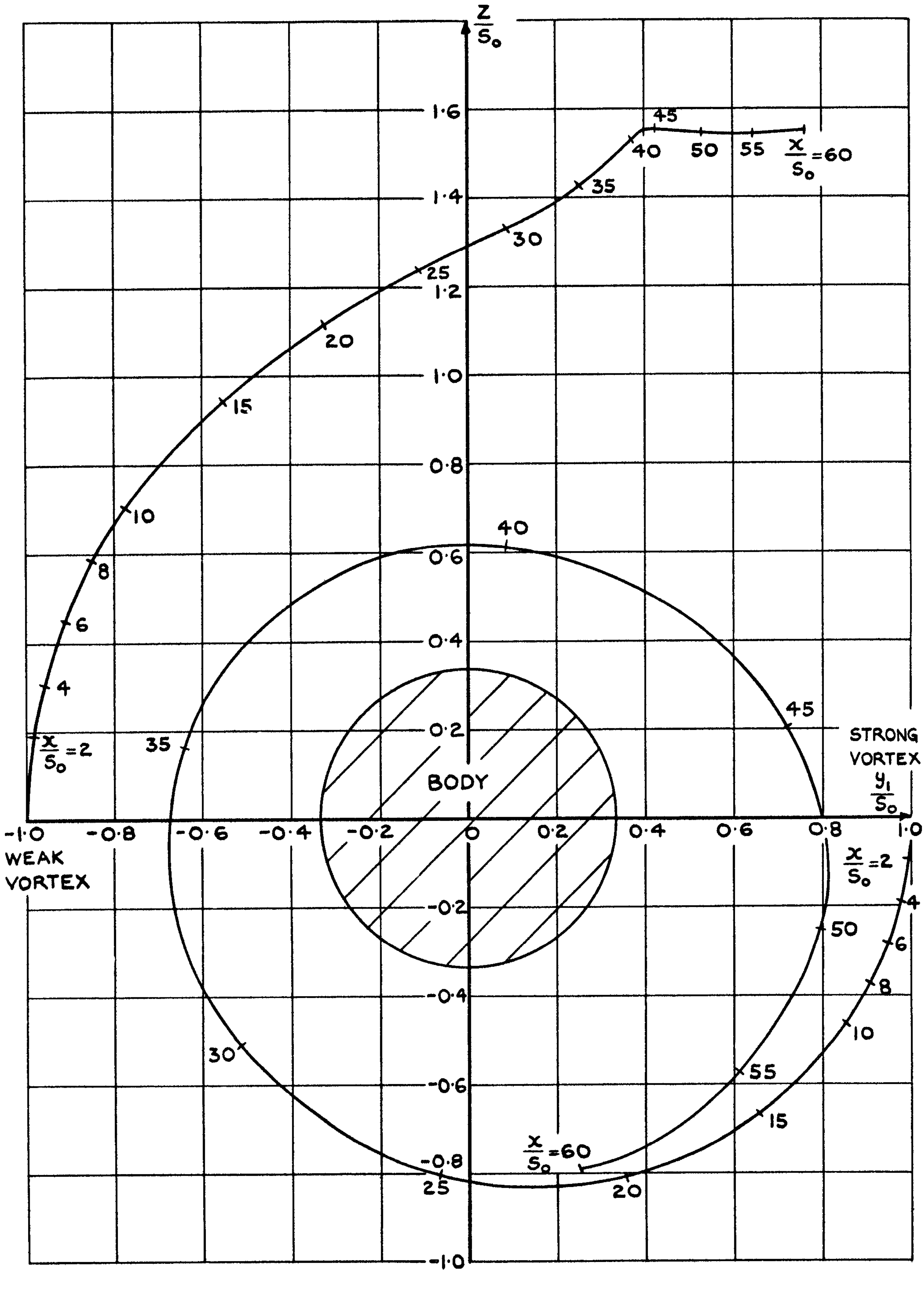

FIG. 4. PATHS OF VORTICES FOR $\alpha = 1°$, $\beta = 7°$.

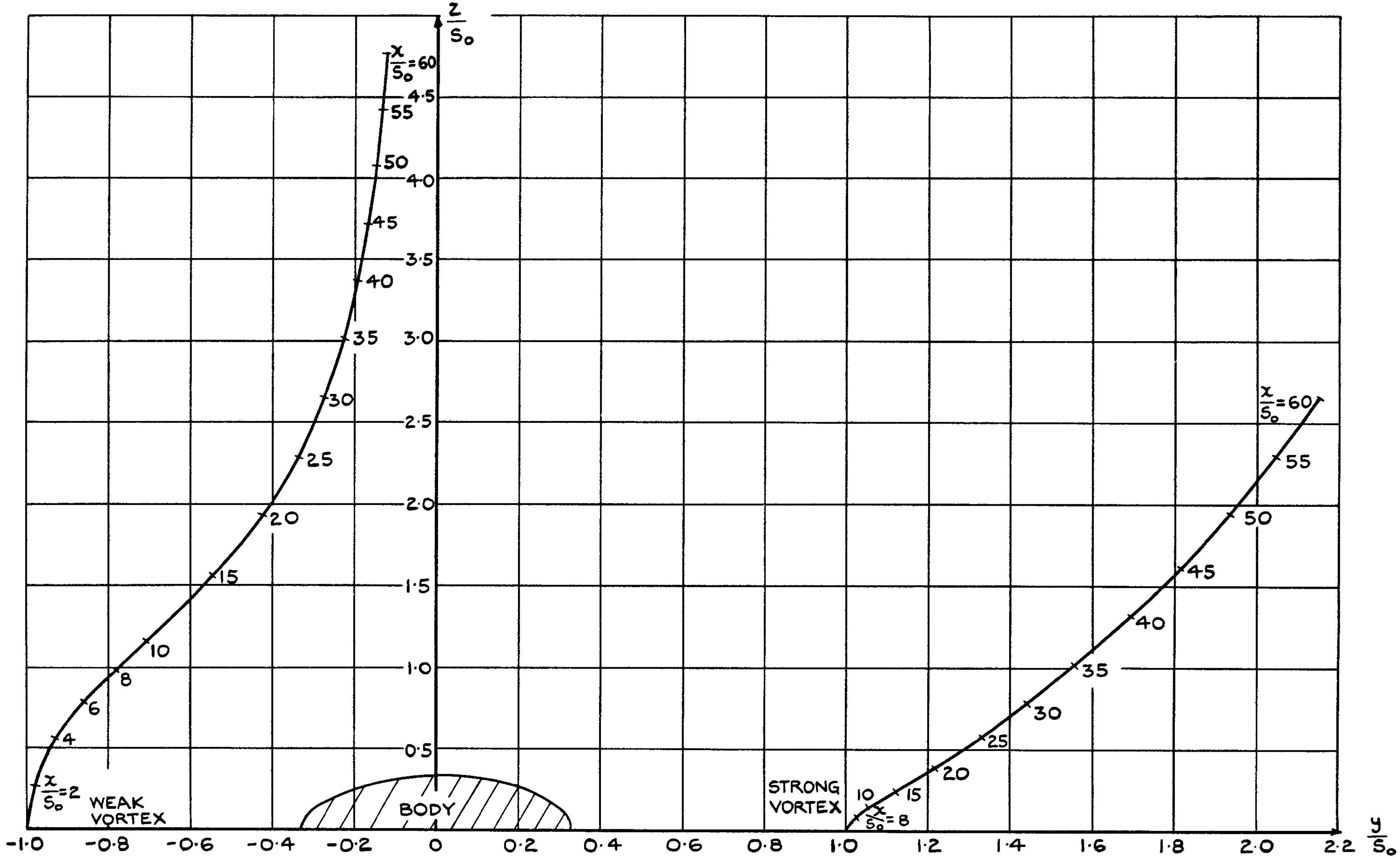

FIG. 5 PATHS OF VORTICES FOR $\alpha = 5^\circ$, $\beta = 7^\circ$

(BODY APPEARS ELLIPTIC DUE TO SCALING).

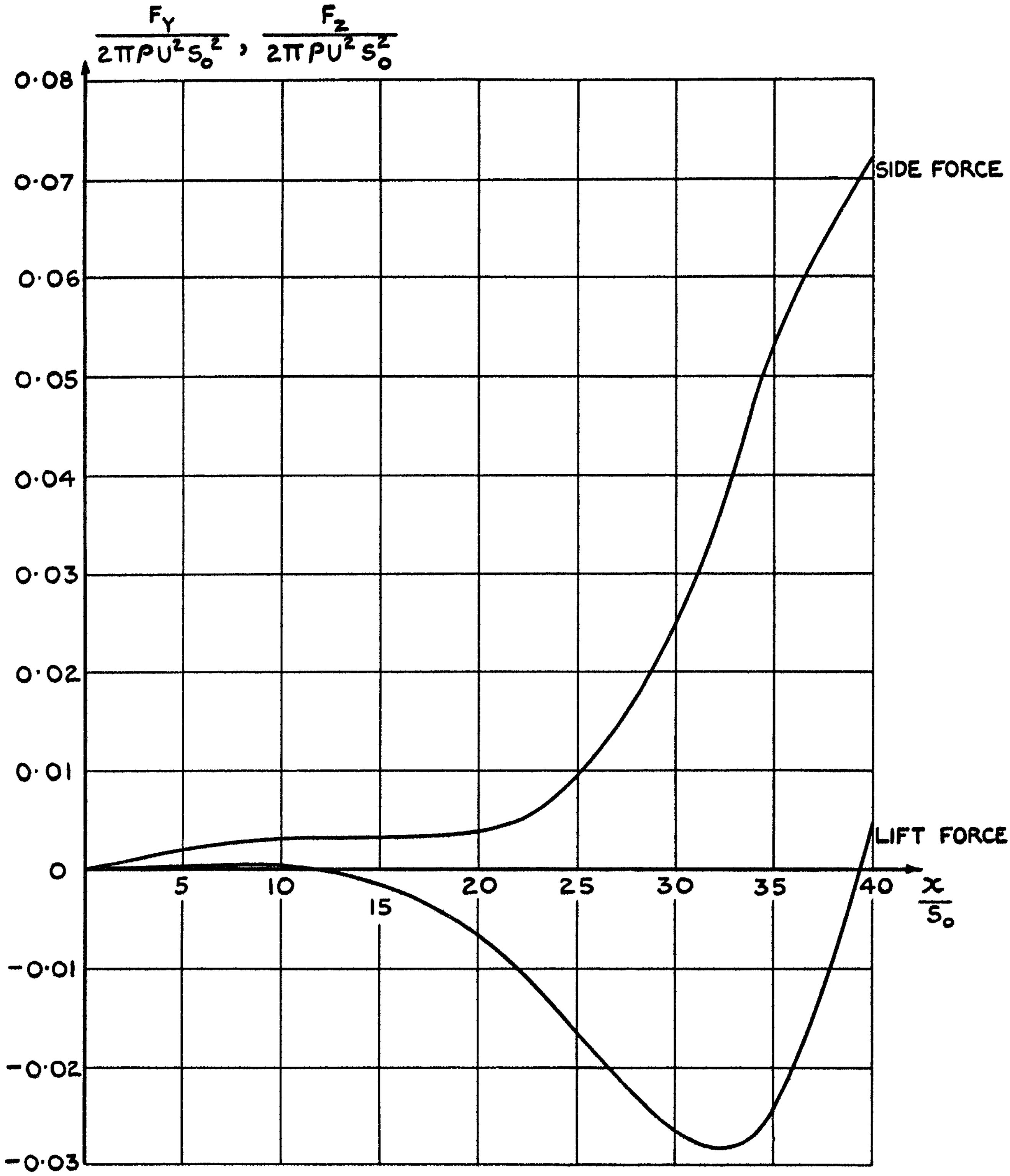

FIG.6. LATERAL FORCES ON BODY $\alpha = 1^\circ$, $\beta = 7^\circ$.

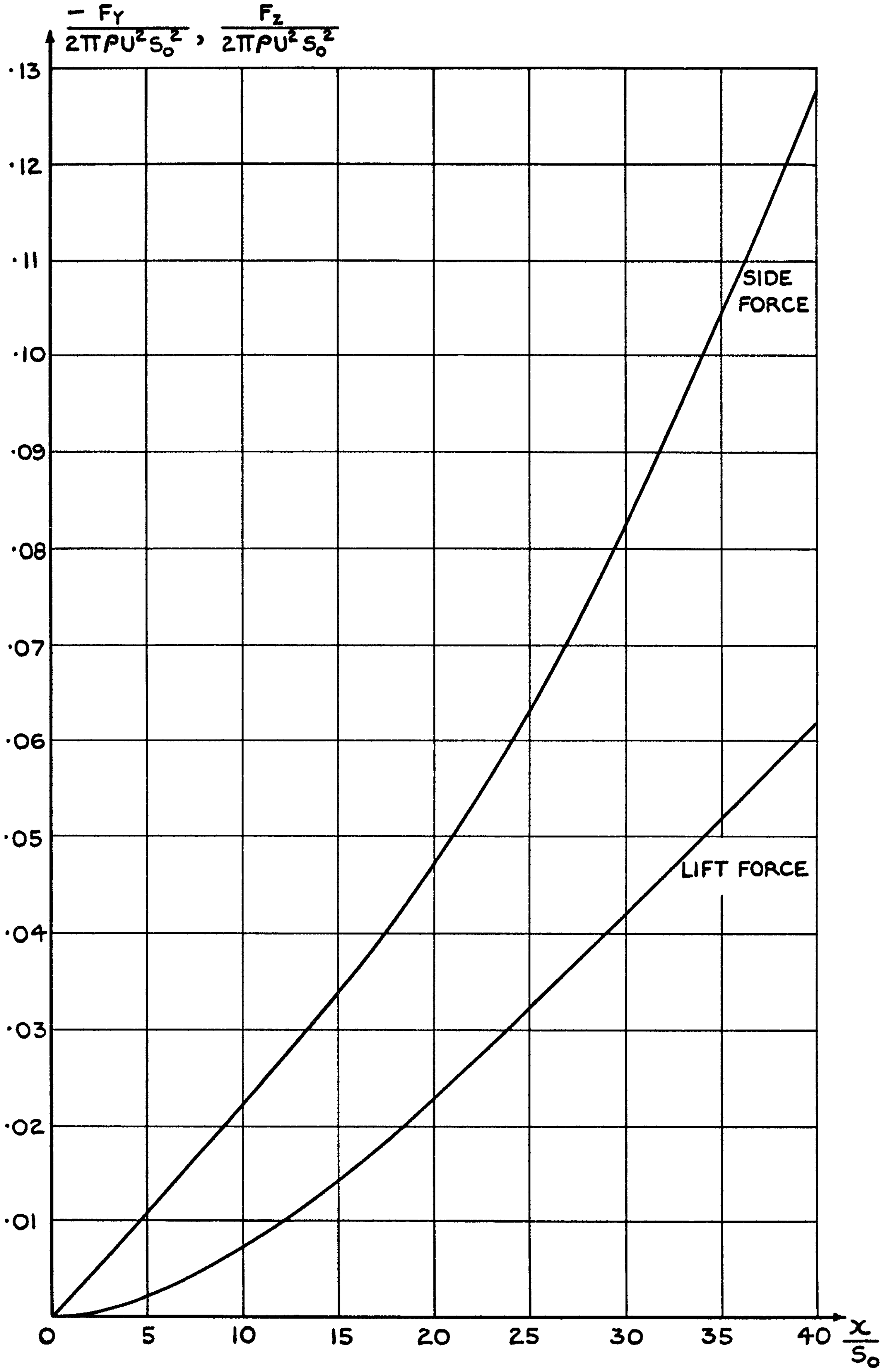

FIG. 7. LATERAL FORCES ON BODY $\alpha = 5°$, $\beta = 7°$

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