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Estimation of Stability Derivatives (State of the Art)

by

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ESTIMATION OF STABILITY DERIVATIVES (STATE OF THE ART)

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H. H. B. M. Thomas

SUMMARY

The methods at present available for the estimation of the usual longitudinal and lateral stability derivatives of an aircraft are briefly discussed for each derivative in turn.

This is preceded by an introductory section dealing with trends in aircraft geometry and their implications regarding the stability derivatives. To illustrate this further the general discussion of methods is followed by a rather more detailed consideration of the estimation of these derivatives for a slender-wing type aircraft, mainly at low speeds, when incidence effects are shown to be important.

Replaces R.A.E. Tech. Note No. Aero 2776 - A.R.C. 23,160

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1 INTRODUCTORY REMARKS

It is evident that the above topic is too broad to be dealt with adequately within the scope of a single paper. The plan followed is to avoid detailed discussion of each derivative by reference to a fairly extensive bibliography.

We shall see later that the task of meeting the aerodynamic needs of the flight dynamicist has changed considerably over the years. The problem has become more integrated in that particular derivatives can no longer be regarded as mainly arising from the action of some single component of the aircraft.

Lifting surface theories and more extensive use of computing machinery, which enables the more elaborate forms of these theories to be applied with a reasonable time outlay, have contributed greatly to the estimation of wing characteristics at subsonic and supersonic speeds. Although the sonic flight condition has been treated theoretically on a linearised basis, this is clearly unlikely to provide more than an indication of trends. Whilst this state of affairs may not be too serious a shortcoming as regards the dynamics of the rigid aircraft modes it is of greater importance in flutter and other aeroelastic problems, for which the transonic speed range may often be the most critical flight condition.

In as much as it permits of treatment of wing, body, tail combinations the subsonic and supersonic counterpart of the sonic theory, the slender body theory, has proved useful in overcoming some of the problems of derivative estimation and may well become increasingly so.

2 TRENDS IN AIRCRAFT GEOMETRY AND THEIR IMPLICATION REGARDING DERIVATIVES

Before proceeding to a consideration of the position in respect of each derivative it is instructive to examine the trends in aircraft geometry over the years and see how these have reacted back on the problem of derivative estimation, including changes in emphasis of particular derivatives.

At one end of the scale (see Fig.1) we have the aircraft with large aspect ratio, unswept wing and with tail surfaces on a long tail arm. In fact an aircraft which could be broken down to largely independent aerodynamic components, each making its contribution to the derivatives, but with the contributions from one or two parts dominating in each derivative.

As the speed range of the aeroplane opened out to embrace the transonic and supersonic, wings have become progressively more highly swept and/or of much lower aspect ratio (see Fig.1). The body has tended to become relatively larger and so interference between the various components much more important. This implies more incidence dependence of some of the lateral as well as some of the longitudinal derivatives, an effect that can be further emphasised by occurrence of shock-induced separations.

Smaller wing aspect ratio reduced the damping-in-roll; an effect which is further emphasised by high sweepback. The same geometric features make the rolling moment due to sideslip become numerically much larger.

To fully appreciate the changes and their significance we must look at the dynamics of the aircraft. For many applications we can consider the longitudinal and lateral motions separable.

The derivatives involved in the longitudinal short period mode are m_q, $m_{\dot{w}}$, z_w and m_w. With respect to these derivatives it has become necessary to pay increasing attention to the contribution of the wing to the moment derivatives and at transonic speeds, in particular, to the oscillatory character of the flow in the estimation of $m_{\dot{w}}$. The incidence dependence of the derivative m_w can be marked at some flight conditions, leading to the well-known pitch-up problem.

The lateral modes of motion are usually more complicated but we can gain some insight into their nature and implication regarding the importance of various derivatives by use of an approximate solution of the lateral stability equation. According to this approximation the damping factor (-R = real part of root), R, of the lateral oscillation is given by,

$$2R \approx - y_v - \left\{\frac{i_C \ell_p + i_A(n_r - n_{\dot{v}}) - i_E \ell_{\dot{v}} + i_E \ell_r + i_E n_p}{i_A i_C - i_E^2}\right\}$$

$$+ \left\{\frac{n_v\left(\ell_p + i_E \frac{C_L}{2}\right) + \ell_v\left(i_C \frac{C_L}{2} - n_p\right)}{i_A n_v + i_E \ell_v}\right\}$$

and the imaginary part of the root and hence the frequency of the oscillation by

$$J^2 \approx \frac{\mu(i_A n_v + i_E \ell_v)}{i_A i_C - i_E^2}.$$

At one end of the scale we have the aeroplane with high aspect ratio unswept wing for which i_A and i_C were of the same order and i_E small. There emerges in consequence the well-known approximations

$$2R \approx - y_v - \frac{(n_r - n_{\dot{v}})}{i_C}; \qquad J^2 \approx \frac{\mu n_v}{i_C}$$

provided $-\ell_v$ and C_L are not large.

As sweepback increased, aspect ratio became less, and bodies comparatively bigger, the importance of ℓ_v and the dependence of ℓ_v and n_v on incidence became more marked as is clearly indicated. Accompanying the changes in geometry there was a tendency for the mass distribution to change. For the conditions $i_C >> i_A$ and appreciable incidence further consequences emerge as we may ignore terms in i_A and write $i_E \approx i_C \sin \varepsilon \approx - i_C \sin \alpha$, if the difference between α and ε (the inclination of forward principal axis of inertia to the flight path) is small enough to be neglected. This yields

$$2R \approx - y_v - \frac{\ell_p + (\ell_{\dot{v}} - n_p - \ell_r) \sin \alpha}{i_A - i_C \sin^2 \alpha} - \frac{1}{\sin \alpha}\left(\frac{C_L}{2} - \frac{n_p}{i_C}\right) + \frac{1}{2} \frac{n_v}{\ell_v} C_L.$$

When in addition $\ell_v >> n_v$, as is often the case, the last term may be neglected.

These considerations show that the changes in aircraft layout are reflected in changing requirements regarding estimations of certain derivatives. The larger $-\ell_v$ and its increase with incidence for swept wings as contrasted with n_v and its tendency to decrease with incidence are obvious examples. Again in our final form for the damping it is clear that, as ℓ_p becomes numerically small in consequence of the geometric trends outlined, other less well documented (n_p, ℓ_r) and sometimes neglected ($\ell_{\dot{v}}$) derivatives assume greater importance.

Dependence of the derivatives on incidence is bound to be more marked when the fin is subjected to the influence of a strong vortex system cast off by the body, foreplane or the wing, in the extreme case of a slender wing aircraft.

Representation of the incidence dependence of derivatives require even more careful consideration in problems involving coupling of the lateral and longitudinal motions.

Some of these trends are found to a certain extent to be reflected in the theoretical and experimental work. In the next section each derivative will be considered in turn and the only evidence of any such trend will be in the number of references associated with it, although it must be pointed out that other considerations enter into the extent to which a particular derivative is documented, not least of which is ease of calculation or measurement.

3 DISCUSSION OF INDIVIDUAL DERIVATIVES*

3.1 Derivatives due to change in forward velocity

$$x_u = \frac{X_u}{\rho S V} = \frac{V}{2}\frac{\partial C_x}{\partial V} .$$

The derivative may be written as

$$x_u = -C_D - \frac{M}{2}\left(\frac{\partial C_D}{\partial M} - \frac{\partial C_T}{\partial M}\right)$$

so that if the engine characteristics are known the estimation of the derivative involves the drag coefficient, and its variation with Mach number. For a particular aircraft, drag measurements are often made in the early stages of the design, as they are important from the performance point of view. However, if experimental results are not available, the Aerodynamic Data Sheets of the Royal Aeronautical Society[1] give fairly comprehensive methods for estimating drag. At subsonic speeds, the profile drag at low C_L for wings of various cross-section are given, but the drag due to lift is more difficult to assess. That part due to the presence of trailing edge vortices may be calculated from one of the lifting surface theories e.g. Refs.14, 15, 16 but the additional lift-dependent drag due to the boundary layer has to be estimated from experimental data on similar wings. Drag to body, nacelles etc. is given in Ref.1, but drag due to interference may

* This Section prepared in collaboration with Dr. A. J. Ross.

may also be appreciable. The transonic drag rise depends on wing thickness distribution, sweepback, taper-ratio etc. and is best obtained from experiment; results for rectangular wings of the NACA 65 series section shapes are given in Ref.1. For supersonic speeds, the wave drag of wings, and of bodies, and the lift dependent drag of wings are presented in chart form in Ref.1. Area rule is used to estimate the wave drag of wing body combinations, and the relevant literature is discussed in Ref.17.

$$z_u = \frac{Z_u}{\rho S V} = \frac{V}{2} \frac{\partial C_z}{\partial V}$$

z_u is determined by the lift characteristics of the aircraft, and is given by

$$z_u = -C_L - \frac{M}{2} \frac{\partial C_L}{\partial M}$$

if aeroelastic effects and engine, jet and slipstream effects are neglected. The lift coefficient C_L is known from the flight conditions under consideration, and so only its variation with Mach number is required for z_u. The theory is essentially restricted to the similarity laws for flows at different Mach numbers. These, used in conjunction with the lifting surface theories discussed under the derivative z_w, yield the variation with Mach number.

$$m_u = \frac{M_u}{\rho S V \ell} = \frac{V\bar{c}}{2\ell} \frac{\partial C_m}{\partial V}.$$

Under the same assumptions as for z_u, the derivative m_u may be written as

$$m_u = \frac{M\bar{c}}{2\ell} \frac{\partial C_m}{\partial M}$$

and is estimated in the process of evaluating m_w at different Mach numbers, as discussed in the following section.

3.2 Derivatives due to change in incidence

$$x_w = \frac{X_w}{\rho S V} = \frac{1}{2} \frac{\partial C_x}{\partial \left(\frac{w}{V}\right)}.$$

x_w may be written as

$$x_w = \frac{1}{2}\left(C_L - \frac{\partial C_D}{\partial \alpha}\right).$$

The lift coefficient is known from the flight conditions, and the variation of drag with incidence may be estimated from experimental data, if available, or by the methods discussed for the derivative x_u.

$$z_w = \frac{Z_w}{\rho S V} = \frac{1}{2}\,\frac{\partial C_z}{\partial\left(\frac{w}{V}\right)}.$$

In terms of lift and drag coefficients, z_w becomes

$$z_w = -\frac{1}{2}\left(\frac{\partial C_L}{\partial \alpha} + C_D\right).$$

The estimation of the drag coefficient is discussed under the derivative x_u, and it remains to evaluate the lift curve slope. The main contribution is from the wing, and much work has been done to obtain reliable estimates. A semi-empirical method by Collingbourne, which accounts for wing-thickness, viscosity and compressibility effects at subsonic Mach numbers, has been used to derive the charts in Ref.1. For thin wings, various sets of charts, based on various lifting surface theories, have been prepared, e.g. by R.A.E. (18, 19, 20) NACA (21 to 25, 31 to 33), although values of $\partial C_L/\partial\alpha$ for wings of aspect ratio of the order of 2 are not reliable. Slender body theory has been extended by Adam and Sears[8], and applied to wings with curved leading edges by Squire[26], giving satisfactory agreement with experiment for small angles of incidence and wings of aspect ratio of the order of 1. For such wings, non-linear effects caused by leading edge separation are treated in Refs.27, 28, 29 for narrow delta wings, and Ref.30 for slender wings with curved leading edges. Another fairly recent development, the use of high lift devices for VTOL and STOL aircraft, involves further parameters in the estimation of the lift curve slope, and theoretical results for full-span jet-flaps on an unswept wing are given in Ref.34.

Summing up, under subsonic and supersonic conditions, the theoretical values of $dC_L/d\alpha$ are not too unreliable, but at transonic speeds the effects of planform, section shape etc. on the flow conditions, which involve break-away, make the problem of estimation virtually intractable.

For the lift on the fuselage, slender body theory[7] may be used at small incidences, but non-linearities due to viscous effects should be considered at high incidences. Although the theory of Ref.35 is not physically justifiable, it appears from experimental evidence that the results are reasonable. Semi-empirical methods have been used to obtain the charts in Ref.1.

Wing-body interference can affect the lift, and has been studied using various theories. These are reviewed in Ref.38 and further work is reported in Refs.39 to 45.

The tailplane contribution to z_w depends on its lift-curve slope, and the variation of downwash with wing incidence. In principle, the downwash may be evaluated from any of the lifting surface theories, but the computation required is often lengthy.* The relevant references are:- (i) Subsonic, 1, 46, 47, 48, (ii) Slender wings, 49 to 54, (iii) Supersonic 47, 55 to 61.

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* Many of these theories are now programmed for the available digital computers, which relieves the tediousness of the numerical work.

Spreiter and Sacks[62] have discussed the effect of the rolling up of the trailing vortex sheet, and give a criterion for deciding on the type of flow to be expected at the tailplane position. At low speeds, an analysis of experimental data, and comparison with theory, is given in Ref.63.

$$m_w = \frac{M_w}{\rho S V \ell} = \frac{\bar{c}}{2\ell} \frac{\partial C_m}{\partial \left(\frac{w}{V}\right)} .$$

The pitching moment coefficient due to incidence is obtained in a similar way to the lift coefficient, and so the references given in the discussion on z_w apply. Further information on pitching moment is also given in Refs.64 to 66.

3.3 Derivatives due to rate of pitch

$$x_q = \frac{X_q}{\rho S V \ell} = -\frac{1}{2} \frac{\partial C_D}{\partial \left(\frac{q\ell}{V}\right)} .$$

This derivative is small, having a small effect on the longitudinal stability, and so is usually neglected.

$$z_q = \frac{Z_q}{\rho S V \ell} = -\frac{1}{2} \frac{\partial C_L}{\partial \left(\frac{q\ell}{V}\right)} .$$

Although the derivative z_q may usually be neglected when assessing the dynamic stability of an aircraft, its value may be required in evaluating the change in the pitching moment derivative due to a change in centre-of-gravity (i.e. reference axis) position. The wing contribution may be calculated using Multhopp's lifting surface theory (Refs.14, 67) at subsonic speeds, and Mangler has considered the sonic and supersonic cases in Refs.19 and 68. Charts for various planforms covering a range of supersonic Mach numbers are given in Ref.10, being based on linearised supersonic theories[24,33].

Where it is reasonable to assume that the body contribution is additive, slender body theory[7] may be used to calculate it, but interference effects ought usually to be considered. For wings with A.R. < 3, the allowance for body can be applied as a factor based on the slender-body theory, as suggested by Henderson[69], but in other cases we are forced to a treatment on the lines of Multhopp or Schlichting[65]. At high incidences viscous effects have also to be considered[35,36].

The contribution from the tail depends on its location relative to the wing; if it is sufficiently far from the wing trailing edge, the effect of downwash may be neglected, and the tail is considered to be at an effective angle of incidence of $q\ell/V$. In present-day designs, the interference effects become important, and it is necessary to evaluate the downwash using the appropriate wing loading[60], as is done in Ref.70 (for delta wings) or Ref.59 (for rectangular wings) in supersonic flow. Similar calculations for subsonic speeds and for sonic speed, have been made in an unpublished Ministry of Aviation report by Thomas and Spencer.

Oscillatory motion in pitch has also been studied quite extensively, e.g. Refs.67, 68, 71 to 82. If $\dot{\vartheta}$ is the angular velocity in pitch, in the space fixed system of axes, we have the relationship, $z_{\dot{\vartheta}} = z_q + z_{\dot{w}}$. The majority of the theoretical papers use the assumption of low frequency, so that resulting derivatives are independent of frequency in subsonic and supersonic flows, but not in the transonic region. Effects of frequency are considered in Ref.75 (incompressible flow) and 77 (supersonic flow) for triangular wings, and the method given by Richardson[79] for all speeds, can be used for all planforms. The unpublished paper by Thomas and Spencer mentioned above compares the various theories (excluding Richardson's method at supersonic speeds) and experimental results. It can be concluded that at subcritical subsonic speeds the lifting surface theory gives a reasonable basis for estimation and at supersonic speeds the theoretical results are generally acceptable for thin wings. Thickness effects can become important however, but available theoretical methods[83,84,85] are all essentially two-dimensional, and so have to be applied either by a strip analysis or as a correction factor to the thin-wing result. The same remarks apply for the tail contribution to $z_{\dot{\vartheta}}$ as for z_q and $z_{\dot{w}}$ (see below), but at transonic speeds the usual quasi-steady approximation does not apply and frequency effects become large. The downwash has been evaluated for delta wings oscillating in sonic flow using as a basis Mangler's theory[68], and other planforms could be dealt with in the same way.

$$m_q = \frac{M_q}{\rho S V \ell^2} = \frac{\bar{c}}{2\ell} \frac{\partial C_m}{\partial\left(\frac{q\ell}{V}\right)} .$$

m_q is the most important of the derivatives due to rate of pitch, and may be estimated by the methods given for z_q, although a few papers (Refs.86 to 90) deal more directly with damping-in-pitch. Experimental work on the oscillatory damping $(m_{\dot{\vartheta}} = m_q + m_{\dot{w}})$ has shown the importance of further parameters to those discussed for $z_{\dot{\vartheta}}$ e.g. mean incidence of the wing, and amplitude of the oscillation. The former problem was considered by Jones[91] some time ago, and is currently being investigated with reference to non-linear effects due to leading edge separation on slender wings. Amplitude effects are not amenable to calculation, but should not be important for the range associated with longitudinal stability problems.

3.4 Linear acceleration derivatives

$$z_{\dot{w}} = \frac{Z_{\dot{w}}}{\rho S \ell} = -\frac{1}{2} \frac{\partial C_L}{\partial\left(\frac{\dot{w}\ell}{V^2}\right)} .$$

The force due to acceleration in direction of z-axis is usually considered in conjunction with the oscillatory motion in pitch (see above), but quasi-steady results for wing contributions have been evaluated for supersonic flow[92,93], and the results are given in chart form in Ref.10. For tailed aircraft, the tail contribution is the more important however, and some attempts have been made to improve Glauert's approximation, which considers only the time lag of the steady downwash. The subsonic and sonic theories are given by

Thomas and Spencer, and results for supersonic wing-tail combinations are given in Refs.70 and 94, the former containing charts for the derivatives due to configurations with triangular tails behind triangular or rectangular wings.

Other effects, such as wing-body interference and wing thickness, have been discussed in the preceding paragraph, and Ref.95 gives additional information on thickness effects.

$$m_{\dot{w}} = \frac{M_{\dot{w}}}{\rho S \ell^2} = \frac{\bar{c}}{2\ell} \frac{\partial C_m}{\partial\left(\frac{\dot{w}\ell}{V^2}\right)} .$$

The information given for $z_{\dot{w}}$ also applies for the pitching moment.

3.5 Derivatives for longitudinal controls

$$x_\eta = \frac{X_\eta}{\rho S V^2} = -\frac{1}{2} \frac{\partial C_D}{\partial \eta} .$$

The estimation of the drag due to elevators is difficult and experimental data has to be used, if available. Ref.1 gives some information for low speed conditions.

$$z_\eta = \frac{Z_\eta}{\rho S V^2} = -\frac{1}{2} \frac{\partial C_L}{\partial \eta} .$$

For aircraft with a tail or canard, the lift due to elevator deflection can usually be neglected, but for tailless aircraft, the lift associated with the required pitching moment can be considerable. Subsonic lifting surface theories[14,96] or the experimental data which has been reduced to chart form in Refs.1 and 97, form a satisfactory basis for the estimation of z_η due to elevators for all wing and tail planforms, and wing-tip controls have been studied by Thomas and Mangler[98]. At transonic speeds, little information is available, and slender wing theory[99] takes no account of flap chord. At supersonic speeds, charts are given in Ref.100, for rectangular and triangular tip controls, based on linearized lifting surface theory.

$$m_\eta = \frac{M_\eta}{\rho S V^2 \ell} = \frac{\bar{c}}{2\ell} \frac{\partial C_m}{\partial \eta} .$$

The same references as for z_η apply.

The derivatives due to rate of change of elevator angle, $z_{\dot{\eta}}$ and $m_{\dot{\eta}}$, are important for control-free stability calculations, and may be obtained from appropriate lifting surface theories.

3.6 Derivatives due to sideslip

$$y_v = \frac{Y_v}{\rho V S} = \frac{1}{2} \frac{\partial C_Y}{\partial\left(\frac{v}{V}\right)} .$$

The wing contribution to y_v is small, of order α^2, and so its accurate estimation is not vital. Theoretical values are given in Refs.2, 3, 11, which cover the speed range and most current planforms, or experimental values for similar wings may be used if available.

The estimation of the sideforce on the body alone is exactly the same as for lift (z_w), and so the same Refs.1, 35 apply. The interference factor between the wing and body has to be determined from slender body theory[7], where applicable, or from experimental data.

The main contribution to y_v is that of the fin, and its estimation reduced to evaluating the lift-curve-slope of the fin, allowing for interference effects due to the presence of the body, tailplane and wing. At subsonic speeds, some progress has been made towards an acceptable method of estimation for conventional aircraft designs (Refs.97, 101 to 105). Jacobs[106,107] has discussed the effects of sidewash due to delta wings, but for fins mounted directly on the wing, satisfactory results are obtained by assuming that the wing acts as a total reflection plate. At supersonic speeds, the isolated fin, and some fin-tail configurations, have been treated theoretically[108 to 111], giving fin planform effects. The fuselage will influence these values at all speeds, and corrections may be made as for the fuselage interference on wing lift. Further interference from vortices shed by canard, wing or fuselage will affect the sideforce on the fin, as discussed in Ref.112. Corrections to the fin contribution may be made on the basis of slender body theory, following the techniques suggested for the incidence case in Ref.54, where interference factors are evaluated due to a vortex and its body image. Spahr[113] has extended the method to include sideslip for wing (or fin) panels in supersonic flow, and at large combined angles of attack and sideslip, the sideforce becomes non-linear, since the angle of inclination is given by $(\alpha^2 + \beta^2)^{\frac{1}{2}}$.

$$\ell_v = \frac{L_v}{\frac{1}{2}\rho S V b} = \frac{\partial C_\ell}{\partial\left(\frac{v}{V}\right)} .$$

The main contribution to ℓ_v comes from the wing, with wing-body interference also being important. The wing contribution itself is a function of planform, Mach number and dihedral angle. At subsonic speeds Ref.1 give charts for the estimation of ℓ_v/C_L for sweptback wings with taper ratio 0.5 and 1.0, and other planforms are considered in Refs.5,114,115. Transonically, slender wing theory may be used, or Ref.115 for sweptback wings. Jones and Alksne[3] have presented the supersonic results for a number of planforms, but different results have been obtained by Harman[116] for rectangular wings (due to his assumption that the Kutta-Joukowski condition does not hold at the trailing tip). Refs.117, 118 give a more general treatment for sweptback wings with streamwise tips, but the analysis is complicated and only a few results have been computed as far as is known. The effect of dihedral has also been studied theoretically, and the results obtained by De Young[119] and Levacic[121] are given in Ref.1 for subsonic speeds. The approximate relationship between ℓ_v due to dihedral and ℓ_p suggested by Purser[122] is supported by available experimental data for wings at transonic and supersonic speeds. Direct methods of calculation are available for delta wings with dihedral (Refs.5, 123).

The wing-body interference is determined largely by the vertical height of the wing on the body, and results are given in Ref.1 based on the theories of Multhopp[124] and Levacic[125]. These results, or those from slender body theory[7,126] must also be used for the supersonic case, as no theoretical analysis has been published, as far as is known. The effect of wing position is underestimated for a wing-body configuration which has been tested at supersonic speeds.

For fins located at some distance from the wing, it is sufficient to take the moment of the sideforce on the fin as an estimate of fin effect at all speeds, and the wing-fin interference is estimated as for y_v. The horizontal tail also contributes to ℓ_v, and is treated subsonically in Refs.104, 105, 125, and supersonically (for triangular horizontal tails) in Ref.109.

$$n_v = \frac{N_v}{\frac{1}{2}\rho S V b} = \frac{\partial C_n}{\partial\left(\frac{v}{V}\right)} .$$

The n_v of most aircraft configurations is difficult to assess, in that it is the balance between two large contributions, an unstable moment contributed by the fuselage, and a stabilizing one from the fin. The wing contribution is important only at large incidences, and may be estimated for most planforms from Refs. 2, 3 and 11.

The fuselage contribution may be estimated from the same references as given for the pitching moment due to incidence on bodies, i.e. slender-body theory[7] for small incidences and sideslip, or the charts in Ref.1 for large combined angles.

The fin contribution is readily obtained from the sideforce on the fin if the tailarm is large, but if the fin is near the wing, more care must be taken in the estimation of the position of the centre of pressure on the fin. Again, the references quoted for y_v are relevant, and the same interference effects must be taken into consideration.

Other effects on n_v which have been investigated experimentally, such as wing height, tail height, and presence of a canard, are found to be small, but propeller slipstream and jet exhaust may cause considerable changes in n_v.

3.7 Derivatives due to rate of roll

$$y_p = \frac{Y_p}{\frac{1}{2}\rho V S b} = \frac{1}{2}\,\frac{\partial C_Y}{\partial\left(\frac{pb}{2V}\right)} .$$

For most aircraft, it is sufficient to assume that the sideforce due to rate of roll may be neglected, from the dynamic stability point of view, but its value may be required for transformation of other derivatives to a different system of reference axes.

The wing contribution is given in Refs.2, 3, 11, and the fin contribution may be obtained from Refs.105, 110, 111, 130, 131.

$$\ell_p = \frac{L_p}{\rho V S \left(\frac{b}{2}\right)^2} = \frac{\partial C_\ell}{\partial \left(\frac{pb}{2V}\right)} .$$

The main contribution to damping-in-roll comes from the wing, and a great deal of theoretical work has been published. The charts of Ref.1 have been derived from De Young[119] for the subsonic case, and from linearized supersonic flow theory (Refs.19, 23, 25, 90). At low speed, effects of dihedral, incidence etc. are given in Refs.132 to 138, and the effect of thickness in supersonic flow has been estimated by Martin and Gerber[139]. Thickness effects may also cause a loss in damping in the transonic region, due to shock-induced separation, but this is difficult to predict theoretically.

The effect of the presence of a body has been estimated from slender wing theory[135,140,141], and results for delta and rectangular wings are given in Ref.1. For a body diameter less than a quarter of the wing span, the interference effect can be ignored.

The fin and tail contributions of ℓ_p are usually small[110,111], but they may be appreciably affected by sidewash due to the rolling wing. An empirical factor of $\frac{1}{2}$ is suggested for estimating the tailplane contribution from its damping as an isolated surface. For the fin, a method of estimating sidewash effects subsonically is given in Ref.130, and sidewash in supersonic flow has been evaluated by Bobbitt for triangular and swept wings[60,142]; both general methods require a knowledge of the wing loading in roll, e.g. Refs.119, 120, 143. Slender body theory has also been applied, but there are a few algebraic mistakes in the published paper[131].

Addition of tip fins and fuel tanks increase the damping of the wing at moderate incidences, and their effect is best assessed from available data. For cylindrical tanks, the potential flow distribution of velocity may be evaluated and used with strip theory to calculate the increment in damping.

$$n_p = \frac{N_p}{\rho S V \left(\frac{b}{2}\right)^2} = \frac{\partial C_n}{\partial \left(\frac{pb}{2V}\right)} .$$

The wing contribution to n_p arises from the drag forces. At subsonic speeds the contribution due to the induced drag, which may be calculated from lifting surface theory, used to be more important, but for highly swept wings with sharp leading edges there is also an appreciable contribution due to the variation of profile drag with incidence. There are often experimental data for the profile drag, and the yawing moment has to be calculated from strip theory. Charts for untapered wings are given in Ref.144. At high subsonic speeds, the semi-empirical method suggested by Wiggins[145,146] works well for the planforms tested. The supersonic theories[128,129] are based on the assumption that the theoretical leading edge suction force is attained. Investigations for drag estimation, based on experimental data have shown that this is not so in practice, and a correction factor of $\frac{1}{2}$ is suggested. For slender wings with leading edge separation, it seems best to assume that the suction force is negligible, so that the yawing moment arises from the component of normal force.

The only other aircraft component to contribute significantly to n_p is the fin. The isolated fin is considered in supersonic flow in Refs.110,111, but sidewash effects are again important, and are evaluated as for ℓ_p.

Interference effects between the fuselage and wing, and fuselage and fin, have to be assessed from slender body theory[7].

3.8 Derivatives due to rate of yaw

$$y_r = \frac{Y_r}{\rho S V \left(\frac{b}{2}\right)} = \frac{1}{2} \frac{\partial C_y}{\partial \left(\frac{rb}{2V}\right)} .$$

As for y_p, the accurate estimation of y_r is usually unnecessary. Wing contributions are given in Refs.1, 3, 11, and body contributions may be obtained from slender wing theory[7]. As far as is known, no experimental evidence exists to help in the estimation of interference effects, and so at low speeds the fin contribution must be obtained from results for an equivalent surface in pitching motion or from the sideslip derivatives due to the fin. At supersonic speeds, charts are given in Refs.110 and 111 for isolated fins, but these results should be corrected for interference effects from wing, fuselage and tailplane on the basis of slender body theory.

$$\ell_r = \frac{L_r}{\rho S V \left(\frac{b}{2}\right)^2} = \frac{\partial C_\ell}{\partial \left(\frac{rb}{2V}\right)} .$$

Lifting surface theories will give the subsonic wing contribution to ℓ_r, for wings which are not too highly swept, as described in Refs.125, 147 148. Charts for unswept wings are presented in Ref.1, including the effects of wing twist, and corrections for sweepback, compressibility and dihedral may be obtained from Refs.2, 3, 121 respectively. A semi-empirical method, using experimental results of ℓ_v, suggested by Campbell and Goodman[149] gives satisfactory results for the planforms tested experimentally. At supersonic speeds, linearized theory does not, in general, give a solution for a wing in steady yawing motion. A modified strip theory has been used for delta and rectangular planforms, from which it appears that the loading due to yawing is related to that due to rolling for the two-dimensional flow region. This result has been applied to other planforms in Ref.11, but probably gives inaccurate results for low aspect ratio wings and at transonic speeds. Slender planforms may be treated using slender body theory[7].

The only other contribution which needs to be considered is that of the fin. Results for fins with an appreciable tailarm are given in Refs.1, 125 (subsonic) and 110 (supersonic), but if the tailarm is small the only available method of estimating interference effects is again slender body theory[7].

$$n_r = \frac{N_r}{\rho S V \left(\frac{b}{2}\right)^2} = \frac{\partial C_n}{\partial \left(\frac{rb}{2V}\right)} .$$

The major part of the damping-in-yaw arises from the body and the fin. The wing contribution is small, being dependent on the drag, and may be obtained from Refs.2, 3, or more accurately from Refs.147, 148 at subsonic speeds, and from Ref.11 at supersonic speeds.

Slender body theory[7] gives satisfactory results for the contribution of the body to n_r at small incidences, independent of Mach number, but loss in damping may occur at large angles, especially for bodies with flattened cross-sections. As for the damping-in-pitch, Refs.35 and 36 give some information.

The damping of the fin may be estimated from a knowledge of the sideslip derivatives when the tailarm is large (c.f. estimation of the damping in pitch due to tailplane), or by evaluating the damping in pitch of an effective wing surface derived by reflecting the fin about its root. Refs.110 and 111 give results for isolated fins in supersonic flow, and interference factors should be applied from slender body theory. Sidewash effects have not been investigated thoroughly, but do not seem to be as important as for rolling wings.

3.9 Derivatives due to acceleration in sideslip

These derivatives, $y_{\dot{v}}$, $\ell_{\dot{v}}$ and $n_{\dot{v}}$, have in the past often been neglected in stability calculations, but for present day designs their effect of the lateral motion may be appreciable. Methods of estimation have not been developed to any great extent; slender body theory may be used for suitable configurations, although Sacks[7] restricts his results for acceleration derivatives to zero incidence. The motion of the fin will be analogous to a wing in vertical acceleration, so that lifting surface theories could be used for the estimation of the force and moments, as is done for isolated fins in supersonic flow in Ref.110. There will also be a contribution from the lag in sidewash[150], which can be evaluated provided that the spanwise lift distribution in sideslip is known (see Refs.114, 117 to 120) and charts have been prepared for supersonic flow[60].

Since the acceleration in sideslip is associated with the oscillatory motion in yaw, frequency and amplitude effects may also have to be considered. As far as is known, no theoretical work on such topics exists.

3.10 Derivatives due to lateral controls

Ailerons

$$y_\xi = \frac{Y_\xi}{\rho V^2 S} = -\frac{1}{2}\frac{\partial C_y}{\partial \xi}.$$

The side force due to aileron deflection is small, and may usually be neglected.

$$\ell_\xi = \frac{L_\xi}{\frac{1}{2}\rho V^2 S b} = \frac{\partial C_\ell}{\partial \xi}.$$

As for the estimation of m_η, lifting surface theories, e.g. Multhopp[14], De Young[119], may be used to estimate ℓ_ξ for ailerons, and are the basis of the charts in Ref.1 for flap type controls at low speed. Tip controls are considered in Refs.98, 100 and 151, and some work on spoiler ailerons is given in Ref.152. Results for controls on slender wings are given in Figs.11(a & b).

$$n_\xi = \frac{N_\xi}{\frac{1}{2}\rho S V^2 b} = \frac{\partial C_n}{\partial \xi}.$$

The yawing moment arises from the drag due to the ailerons, and so is best estimated from experimental data.

Rudder

$$y_\zeta = \frac{Y_\zeta}{\rho V^2 S} = -\frac{1}{2}\frac{\partial C_y}{\partial \zeta}.$$

The sideforce due to rudder deflection is derived from a knowledge of its lift curve, and may again be obtained from lifting surface theory, as for the elevator derivatives.

$$\ell_\zeta = \frac{L_\zeta}{\frac{1}{2}\rho V^2 S b} = \frac{\partial C_\ell}{\partial \zeta}.$$

This derivative is usually small, and involves estimating the height of the centre of pressure on the fin from the spanwise loading due to control deflection.

$$n_\zeta = \frac{N_\zeta}{\frac{1}{2}\rho\, V^2 S\, b} = \frac{\partial C_n}{\partial \zeta} .$$

If the tailarm is large, an accurate estimate of the position of the centre of pressure on the fin is not necessary, but for short tail-arms lifting surface theories must be used, as for m_η.

3.11 Derivatives associated with varying density

In certain problems the aircraft's speed and flight condition may be such that it is necessary to account for the variation of atmospheric density with altitude. These problems require, in addition to the above aerodynamic derivatives, the evaluation of derivatives with respect to height, or, more appropriately, with respect to a non-dimensional form of this, see for example Refs.154 and 156.

4 DERIVATIVE ESTIMATION FOR A SLENDER WING AIRCRAFT

In this section we shall consider, by way of illustration of the methods outlined and the trends in the derivatives, the case of a tailless slender wing aircraft.

4.1 Longitudinal derivatives

To confine the discussion we restrict ourselves to low speed conditions. To obtain the derivatives z_w we require the relation between the lift coefficient and incidence. Peckham (R.A.E.) has given an empirical collapse of a large number of test results, which is reproduced here in Fig.2 and which can be represented with good approximation by the relationship,

$$C_L = K_o \sqrt{\frac{s}{c_o}} \cdot \alpha + K_1 \sqrt{\frac{s}{c_o}} \cdot \alpha^{5/3}$$

where K_o and K_1 are essentially constant, independent of planform (delta, gothic and ogee are included in experimental points shown).

In view of the slender wing theory result that the linear term would be expected to vary in proportion to $\frac{s}{c_o}$ (strictly aspect ratio) rather than $\sqrt{\frac{s}{c_o}}$ the success of this empirical law is at first sight puzzling. If, however, the results from the various lifting surface theories, which tie in well with experiment, and slender wing theory, for the delta type wing are plotted as in the inset figure of Fig.2 (only one result was available for other shapes) the reason, underlying the success of what, on the basis of slender wing theory, would be considered an inept parameter, is now clear. Within the slenderness ratios $\left(\frac{s}{c_o}\right)$ covered by the test results $K_o\sqrt{\frac{s}{c_o}}$ is a fair approximation to the rate of change of the lift coefficient with incidence for small incidences as indicated by results covering a much wider range of aspect ratio.

Theoretical results extending the slender wing theory to include the non-linear contribution from the leading edge vortex sheets are available and do not tend to confirm the form taken by the second term. The difference in order of α involved is explicable on expected trends with finite thickness wing (finite edge angle). However, since at large aspect ratio a return to an almost linear relation is to be expected, one would expect this to be reflected in the form of the second term.

To summarise, the empirical relationship could be modified so that the linear term is taken from lifting surface theory results or semi-empirical analysis based thereon. Thereby it could gain something in generality. In view of the fact that the slender wing theory tends to over emphasise the effect of planform (in terms of $\frac{s}{c_o}$, since $A = \frac{2}{p} \cdot \frac{s}{c_o}$)* on the linear term, as compared with the effect for more practical values of $\frac{s}{c_o}$, it may be equally misleading regarding the non-linear term. It is thus clear that there exists less theoretical backing for a plausible form for this term.

To estimate the stiffness derivative, m_w, we need in addition to the lift the centres of the two lift contributions. The linear part of the pitching moment curve or strictly the aerodynamic centre at near zero incidence is considered first. In Fig.3 the available experiment data are plotted against three quantities (1) the centre of area (plain symbols), (2) the aerodynamic centre calculated on a slender wing theory basis (half-filled symbols), and finally (3) the aerodynamic centre from lifting surface theory calculations, where these were available. The degree of correlation as indicated by deviation from the line of perfect correlation improves as we pass from (1) to (3). The other two plots are of interest in that it seems that the aerodynamic centre trend seems to be simply related to the trends of (1) and (2). For the correlation with centre of area (first proposed by workers at Messrs. Handley-Page) there would seem to be no theoretical explanation. There is indeed little more reason for expecting the success of the second plot either, since it is known that the aerodynamic centre deviates fairly quickly from the value for $A \to 0$.

Some residual discrepancy remains which, though small, is of considerable practical importance. At present this difference, which is probably a thickness effect, must be allowed for on an empirical basis.

Returning to the basically non-linear character of the pitching moment curve with incidence the data seemed to separate naturally into two groups, those referring to delta wings and those referring to those wings having streamwise curved tips. A simple plot of the displacement of the centre of non-linear lift relative to its linear counterpart against the aerodynamic centre at zero incidence is shown in Fig.4. The latter property of the wing was chosen as one embodying the effects of planform shape in a single variable. In so far as the limited data allow one to judge there is an approximately linear relationship between the two quantities. An interesting feature is that alleviation of pitch-up tendency by rounding the tips is clearly indicated, as well as the fact that a wing designed to have a linearized aerodynamic centre of about 57 per cent of the wing root chord (or what is approximately equivalent from Fig.3 a centre of area of around 66 per cent of the chord) is expected to have an approximately linear relation between pitching moment and lift.

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* In this expression, p is the ratio of wing area to that of the circumscribed rectangle.

We next turn our attention to damping-in-pitch derivatives m_q and $m_{\dot{w}}$. From what has been said previously we have available methods of calculating these, which have given encouraging results for a wide range of wing plan-forms within a limited range of incidence and amplitude. It is seen from Figs.5 and 6, (where experiment and theory are compared for the combined derivative) that this is equally true for the type of wing we are considering at small values of incidence. However and not unexpectedly, we have a marked incidence effect on the derivatives as indicated by variation of $m_{\dot{\vartheta}}$. This cannot be reliably accounted for within the known theoretical treatments at present. We may investigate the extent to which the knowledge of incidence effects on z_w and m_w can help us. The transformation from one axis to another is effected by means of the relationship, c.f. Ref.153,

$$m_{\dot{\vartheta}} = m_{\dot{\vartheta}_o} - \left(z_{\dot{\vartheta}_o} + m_{w_o}\right)(H - H_o) + z_w(H - H_o)^2$$

which can be rewritten as

$$m_{\dot{\vartheta}} = \left\{m_{\dot{\vartheta}_o} - z_{\dot{\vartheta}_o}(H - H_o)\right\} + \left\{z_w(H - H_o)(H - H_{a.c})\right\}$$

where H is distance of axis to which $m_{\dot{\vartheta}}$ refers from the wing apex in terms of mean chord,

H_o is the corresponding quantity for the datum axis chosen.

To make the most use of knowledge of the non-linear character of z_w and m_w we may attempt to fix H_o such that the contribution from first term is small. It is immediately obvious that this procedure fails completely near $H = H_{a.c.}$ Furthermore we still rely on our theoretical method in so far as the value of H_o and first bracket are concerned. For this reason we resist the natural urge to define H_o by setting the theoretical value of the first bracket zero. This would result in often quite extreme and unreliable values of H_o. We thus content ourselves to choose H_o according to the more general condition previously mentioned. To define the values of $m_{\dot{\vartheta}}$ for all axis positions we require only two specific values, since then all coefficients of the parabolic relationship quoted above can be determined. In view of the remarks made we choose axes on either side of $H_{a.c.}$

Writing $a = - m_{\dot{\vartheta}_o}$, $b = - z_{\dot{\vartheta}_o}$, $c = - z_w$ we have the conditions,

$$H - H_o \gg \frac{a}{c\,|H - H_{a.c.}| + b} \quad \text{with } H \gtrless H_{a.c.},\ H > H_o$$

and

$$H_o - H \gg \frac{a}{c\,(H_{a.c.} - H) - b} \quad \text{with } H < H_{a.c.},\ H < H_o.$$

Using such a device Figs.5 and 6 were constructed, in which the variation with incidence of $m_{\dot{\theta}}$ at three axis positions for each of two wings, a delta and a gothic planform respectively, as estimated on the lines just outlined, is compared with the experimentally determined variation.* Although there are significant differences the comparison shows that allowing for non-linear incidence effects in this way gives an indication of trends. At the same time it shows that some well worthwhile gains may result from efforts to calculate these effects directly albeit for slender delta wings with conical type flow (c.f. Ref.29).

4.2 Lateral derivatives

We commence a discussion of the lateral derivatives of such a layout by considering the data available on the rolling moment and yawing moment produced by the wing alone in sideslip. There are tests of a good number of such wings, some flat plate, others with thickness and covering a range of wing shapes - delta, gothic and ogee - available. It is known from flow observation that sideslip does affect the vortex pattern both as regards strength and position relative to the wing. On the face of it there would seem to be little hope that any theory not accounting for these features would be at all useful at any except the very small incidences. However, the assembled experimental evidence in Fig.7 does suggest that in spite of such misgivings the slender wing theory** (attached flow) yields results in remarkably good agreement with experiments over a considerable incidence range, thus indicating that counter effects are present. Their mechanism is not well understood at present and the matter is receiving further attention.

Turning to the yawing moment we may, since the pressure distribution is substantially normal to such wings, seek an approximate estimate of n_v by writing

$$\frac{\partial n_v}{\partial \alpha} \approx \ell_v \tan \alpha \approx \ell_v \alpha .$$

The validity of this p ocedure can be assessed from Fig.7, where the experimental results are displayed. The curve marked $\pi\alpha^2/2$ corresponds to the above equation.

No extensive tests have been made of the damping-in-roll (ℓ_p) derivative. Such results as there are indicate that slender wing theory gives reasonably reliable estimates at small incidence. At incidence the wing-chord body axes derivatives were estimated and transformed to the usual wing-body axes without neglect of higher order incidence terms. This yields the fall off of damping with increase of incidence in qualitative agreement with experiment, see Fig.8. Agreement with experiment is not materially improved by attempts to allow for the presence of the leading edge vortices.

To estimate n_p, the yawing moment due to rolling derivative, we again write

$$n_p \approx \ell_p \tan \alpha$$

which reproduces the experimental variation of n_p with α for incidences up to about 15°.

* The experimental points are given for three values of the frequency parameter ν.
** The parameter p in Fig.7 is as defined in the foot-note on p.18 and F is a function of planform shape and thickness distribution.

Some test results are also available for wings fitted with fins. Fin contribution tends to dominate in certain derivatives, in particular, damping-in-yaw, side force and yawing moment due to sideslip derivatives (in the absence of an aerodynamically significant body).

Consider the damping derivatives n_r and $n_{\dot{v}}$, which for our present purpose we shall take in the form of the combined derivatives $(n_r - n_{\dot{v}})$. Here we essentially apply the methods discussed under the damping-in-pitch of the wing. A rough assessment indicated that refined attempts to allow for wing-fin interference as outlined in the previous section of the paper introduce only a small correction to y_v and n_v due to fin, as calculated on basis of total reflection in the wing. Accordingly because of the very limited nature of the theory-experiment comparison only those effects indicated on Fig. 9(a) were taken into account.

The experimental technique used to obtain the values of $n_r - n_{\dot{v}}$ shown in Fig. 9(a) was a free oscillation covering a range of frequencies. No marked frequency effects were noticed except at very large incidence.

Estimates and experimental results for fin contribution to ℓ_p and n_p are shown in Figs. 9(b) and (c). In all the fin contributions the leading edge vortices will undoubtedly play an important part but as yet we have insufficient experimental data for fins of different height to wing semi-span ratio to display this effect satisfactorily.

With this in mind we pass on to the side force derivative with respect to sideslip. On the left-hand side of Fig. 10(b) is shown the variation of y_v with incidence for the model configuration shown in Fig. 10(a) (with zero anhedral). This is compared with the values calculated assuming: (1) the fin effective aspect ratio to correspond to total reflection in the wing, (2) on the slender-body theory for the wing-fin combination.

It is seen that as the fin aspect ratio decreases the two estimates come together and are in good agreement with experiment throughout the incidence range. This demonstrates two things:

(1) The wing-fin interference is small.

(2) The vortex-induced sidewash on the fin has a small nett effect.

If (1) is in fact estimated on the lines suggested in the previous section of the paper we do find that it would be of the order of a tenth of y_v as given by calculation on the basis of reflection in the wing. Insufficient data exist at present to make a reliable estimate of (2). For the accompanying set of figures, which refer to the same layout with the wings set at 20° anhedral, we have a rather different situation. Here both (1) and (2) are significant effects. Three estimated values of y_v are displayed alongside the experimental results. They correspond to the two basic calculations referred to above and to the correction of the first of these to allow for (1) on the lines suggested earlier. Of these the last mentioned gives the generally best estimate.

Note the effect of vortex flow as indicated by the rapid increase of y_v above $\alpha = 5^\circ$. No estimate of this effect was made.

Fig.10(c) shows the corresponding comparisons for the yawing moment derivative, n_v, whose estimation is naturally linked to that of y_v for a fin. Here, therefore, much the same remarks apply.

From the y_v and n_v of fins on the aspect ratio 1 delta wing we pass on to the fin contribution to ℓ_v (rolling moment derivative w.r.t. side-slip) for the same set of fin-wing combinations. Here comparison is made again with the two basic calculations and the allowance of interference again brings the estimates into closer agreement with experiment (see Fig.10(d)).

Let us now consider the position regarding the estimation of the characteristics of flap type control surfaces fitted to the type of wing under consideration. The lift and moment derivatives due to control deflection, mentioned in Sections 3.5 and 3.10, can be predicted adequately for a wide range of wing and control geometry on the basis of various lifting surface theories. Slenderness ratios $\left(\frac{s}{c_o}\right)$ in the range 0.25 - 0.50 implies, however, that we are working near the limit of applicability of many of these methods. On the other hand slender body theory yields the physically unacceptable result that the effectiveness of a control is independent of its chord-ratio. This result is in fact the direct consequence of applying the slenderness concepts to both wing and control surface or more strictly for that part of the wing over which deflection of the control induces loads. This argument would apply in the true limiting case of vanishing span, but for wing of small but finite s/c_o and for which the wing may be regarded as aerodynamically slender, the other area mentioned is not slender, see Fig.11. These thoughts suggested a reformulation of the theory. According to this we regard the wing as being slender and so deflection of the control does not produce load on parts forward of the control surface. Since we no longer look upon the area affected by control as slender the problem is in fact equivalent to that of a control fitted to a wing defined by the shaded area of inset figure of Fig.11, in many cases a large aspect ratio wing. As an example, we consider the calculation of rolling moment due to aileron deflection.

In Figs.11(a) and (b) charts are prepared for two outboard control planforms giving the rolling moment derivative, ℓ'_ξ, the dash denoting that it is based on the area and span of the shaded portion of the wing. From these charts the rolling moment in its usual derivative form (ℓ_ξ) can be readily estimated. This is done for two cases, in which the control plan-form is rectangular, and for which there were some free flight test results. A comparison of estimated and measured values is made in Fig.12.

It is of interest to note that, as the control shape becomes slender, the true slender wing theory result is approached.

5 CONCLUDING REMARKS

No discussion of the present position on derivative estimation would be complete without some examination of their adequacy as a representation of the aerodynamics required in current flight dynamic work. There are two aspects of this question that seem to call for comment.

The first concerns the inability of derivatives to give an exact representation of the instantaneous aerodynamic forces, as they are independent of the history of the motion. Etkin (see Ref.155) has proposed the use of aerodynamic transfer functions as an alternative presentation, which would overcome the shortcomings of the derivative approach. The method is restricted to a linearized representation of the aerodynamic forces, i.e. we assume that the differential equations governing the unsteady pressures over the aerodynamic surfaces are linear in the usual wing theory sense. The presence of vortices of considerable strength above the wing and, as the configurations become even more slender, in proximity to the tail surfaces could be taken as pointers to the need for a reassessment of the situation. On the other hand no marked frequency effects of a consistent nature have been noted.

The matter is, nevertheless, important in certain conditions and so needs to be kept under constant review. The possibility of operating in other than a constrained mode (necessary for particular derivatives) which is offered by some experimental equipment for derivative measurement, should be exploited to give a direct check on the extent to which the derivatives fall short.

The second aspect of the question concerns representation of the aerodynamics at large incidences and amplitudes. This may indicate presentation in coefficient form as functions of the variables, which is, of course, the normal presentation of experimental results. Such a representation of the aerodynamics is particularly suited to problems in which the static forces and moments dominate, as for example in missile dynamics, where the aerodynamic damping is relied upon only to a very small extent. The usual derivative form may be sufficient approximation for these damping terms.

It is understandable that in the interest of simplicity the theory is usually linearized, but there are possibly circumstances in which this could have been avoided thereby giving the results the desired generality. It is suggested that a certain amount of effort should be directed towards this generalization.

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DAMPING

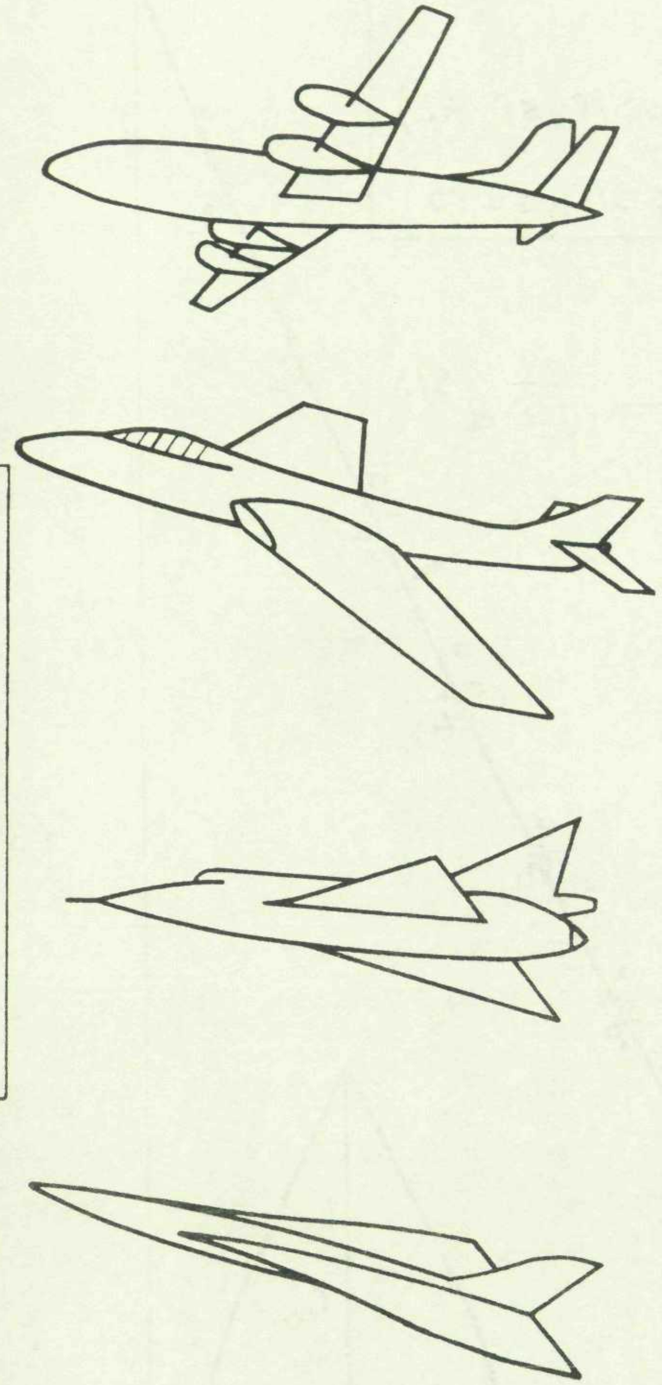
$$2R \approx -y_v - \frac{n_r - n_{\dot v}}{i_C}$$

$$2R \approx -y_v - \frac{\left\{ i_C l_p + i_A \left(n_r - n_{\dot v} \right) - i_E l_{\dot v} + i_E n_p + i_E l_r \right\}}{i_A i_C - i_E^2}$$

$$+ \frac{\left\{ n_v \left(l_p + \frac{1}{2} i_E C_L \right) + l_v \left(\frac{1}{2} i_C C_L - n_p \right) \right\}}{i_A n_v + i_E l_v}$$

$$2R \approx \frac{1}{i_A - i_C \sin^2 \alpha} \left\{ -l_p + \left(l_{\dot v} + n_p - l_r \right) \sin \alpha \right\}$$

$$- \left\{ \frac{\frac{C_L}{2} - \frac{n_p}{i_C}}{\sin \alpha} \right\}$$



FREQUENCY

$$J^2 \approx \frac{\mu n_v}{i_C}$$

$$J^2 \approx \frac{\mu \left(i_A n_v + i_E l_v \right)}{i_A i_C - i_E^2}$$

$$J^2 \approx \frac{\mu \left(i_A n_v + i_E l_v \right)}{i_A i_C - i_E^2}$$

FIG. 1 APPROXIMATIONS FOR DAMPING AND FREQUENCY OF THE LATERAL OSCILLATION APPLYING TO THE VARIOUS AIRCRAFT SHAPES.

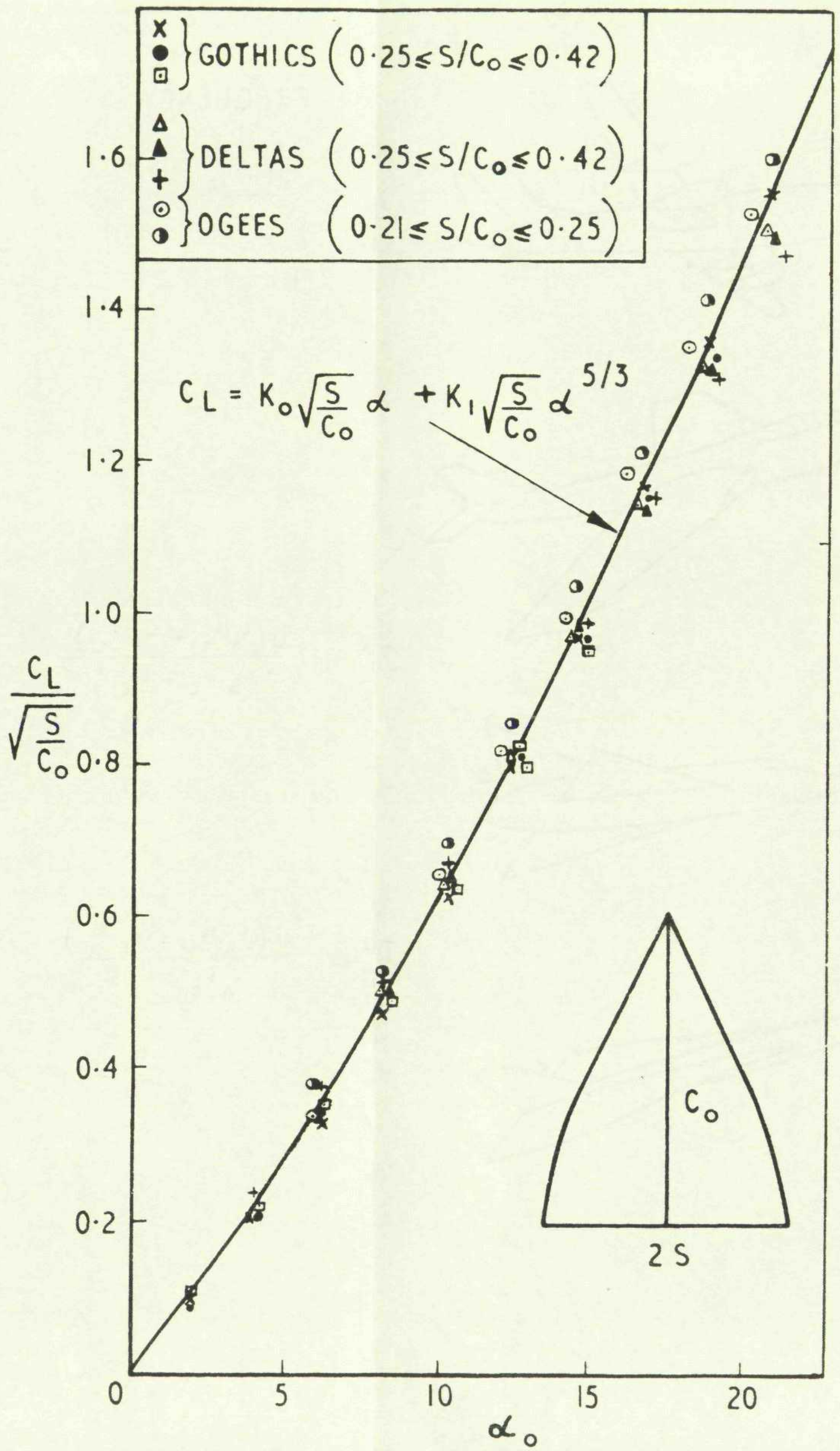


FIG. 2(a) COLLAPSE OF LIFT CURVES FOR SLENDER WINGS.

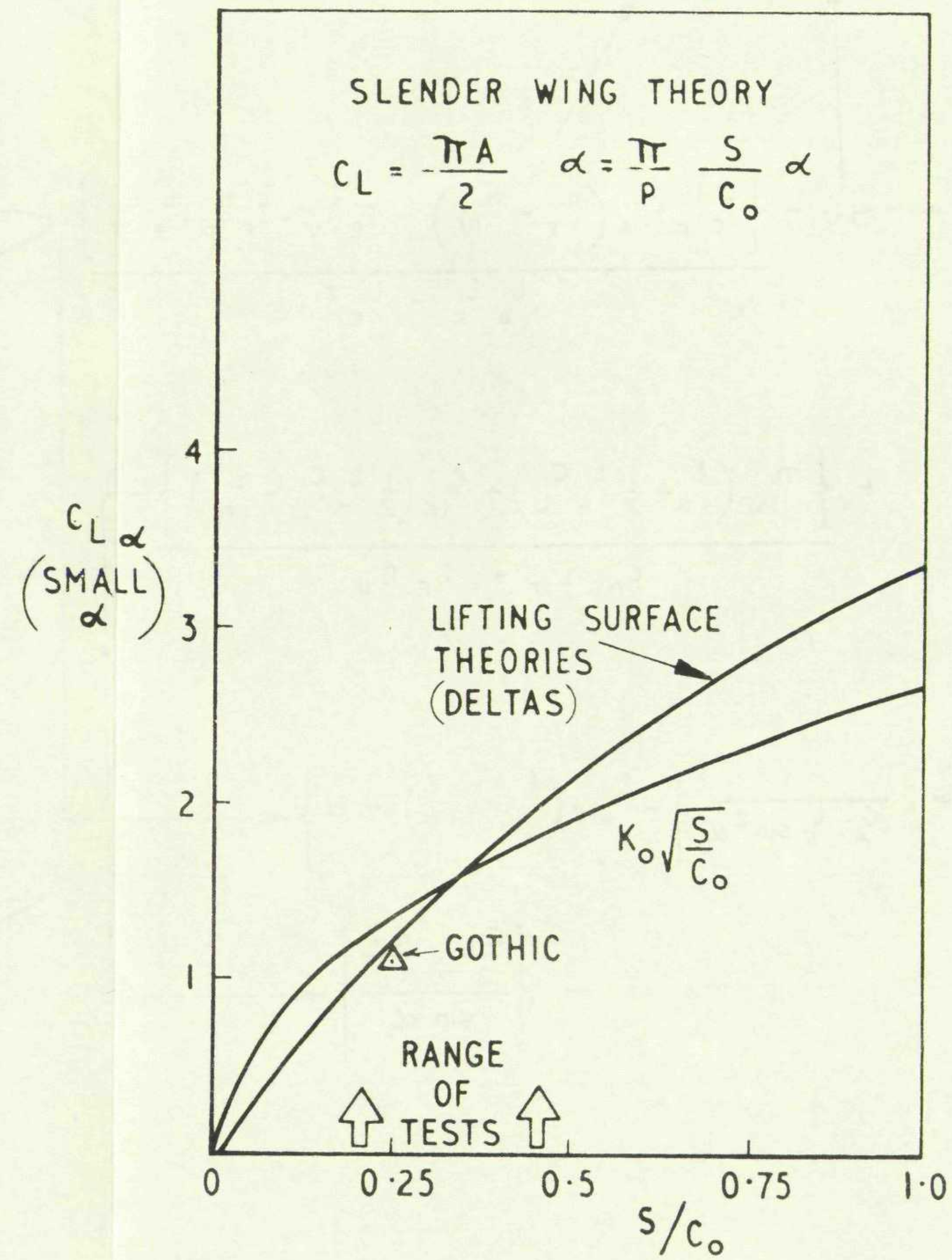


FIG. 2(b) COMPARISON OF LIFTING SURFACE THEORIES WITH EMPIRICAL CURVE FOR $C_{L\alpha}$ AT SMALL INCIDENCES.

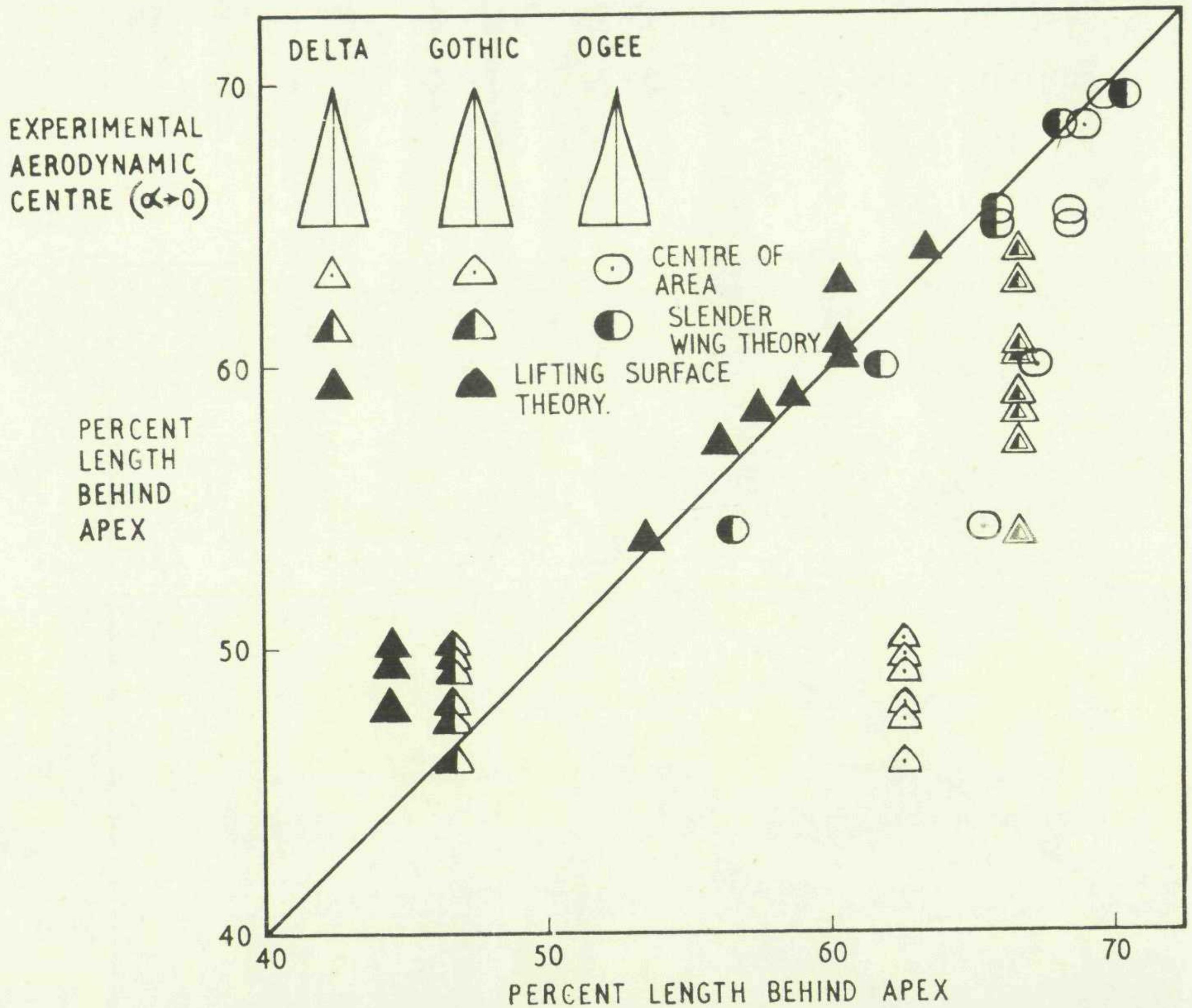


FIG. 3 COMPARISON OF EXPERIMENTAL AND ESTIMATED AERODYNAMIC CENTRES AT SMALL INCIDENCES.

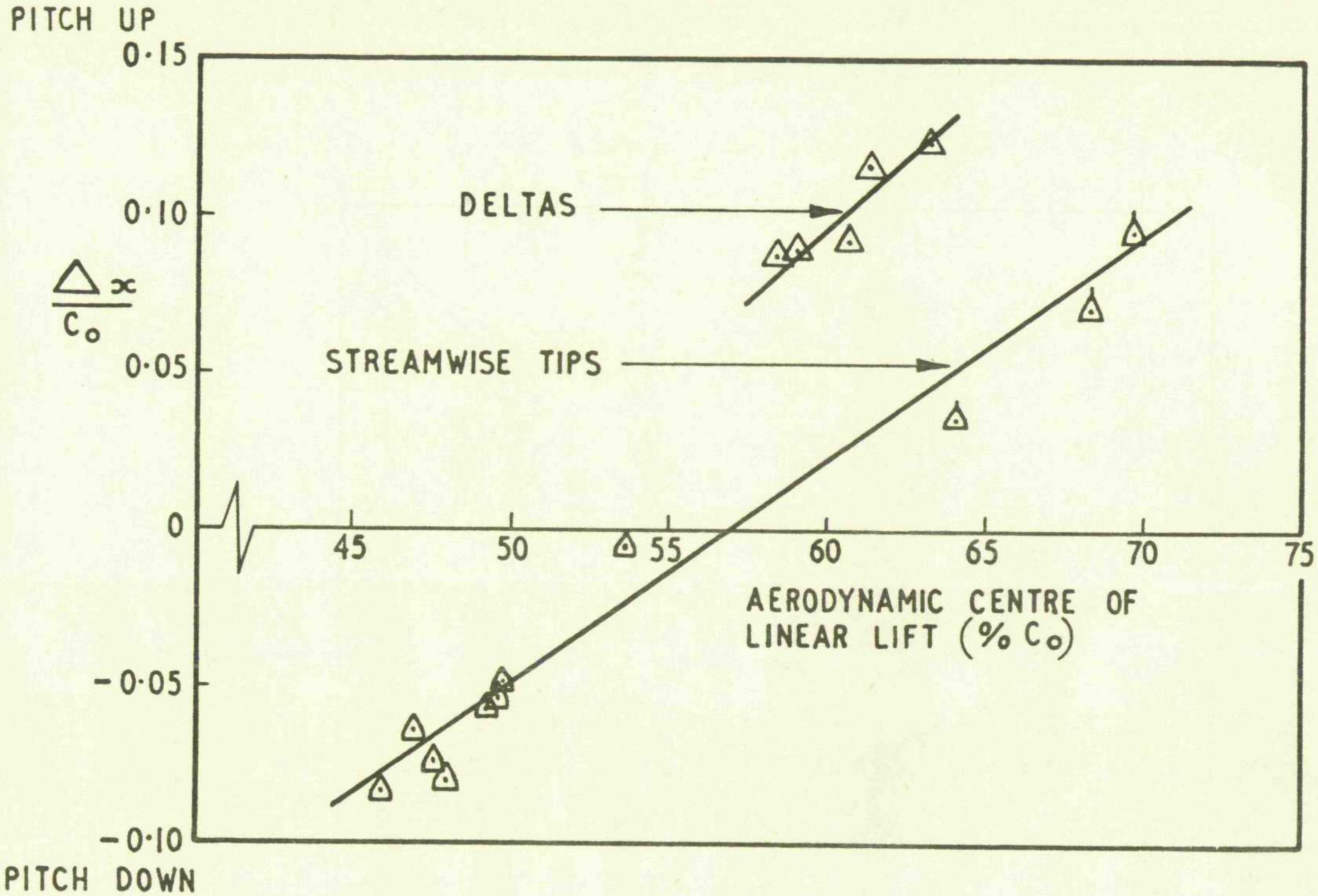


FIG. 4 DISTANCE OF CENTRE OF NON-LINEAR LIFT FROM AERODYNAMIC CENTRE AT LOW INCIDENCE. (POSITIVE FORWARD)

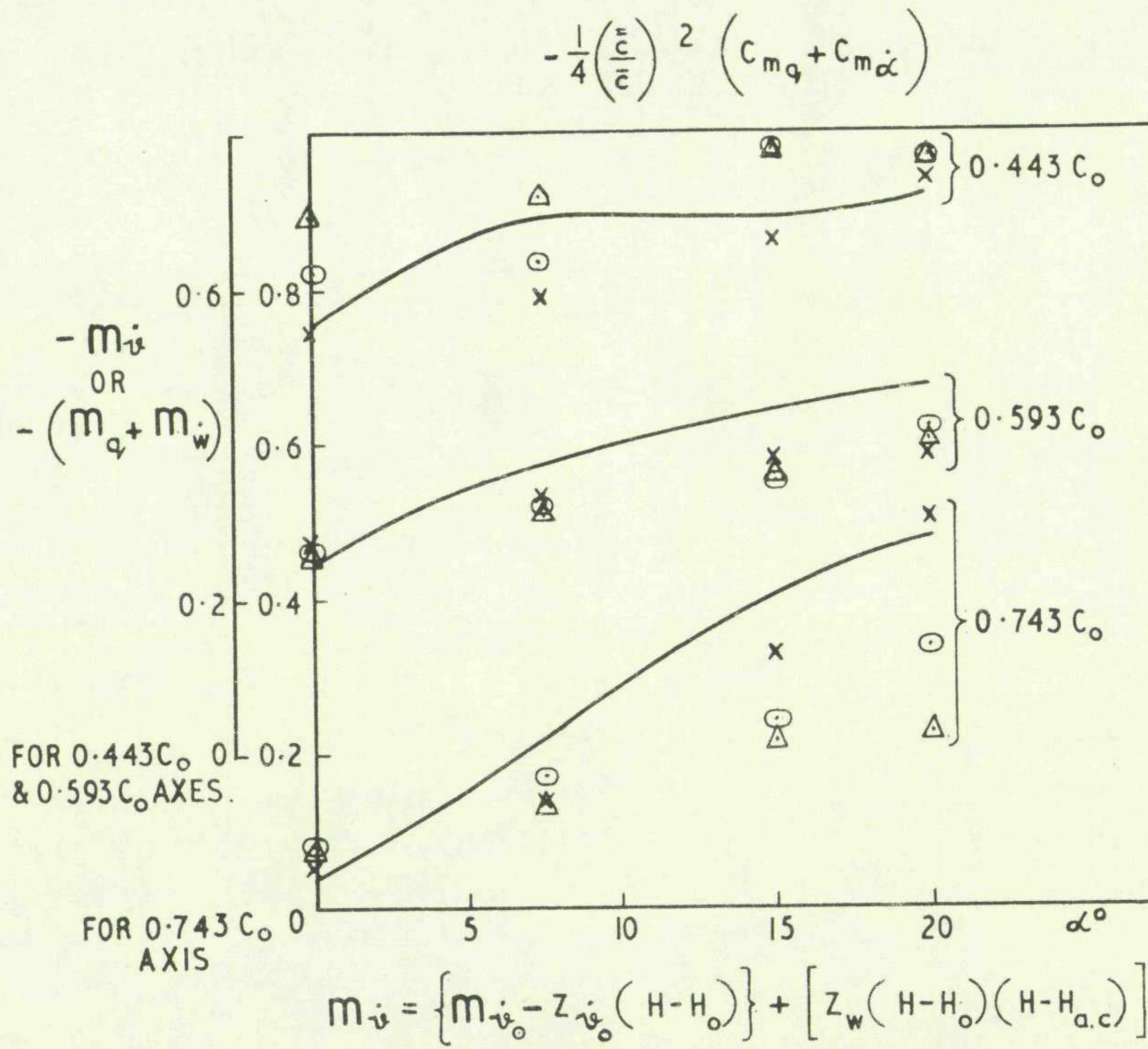
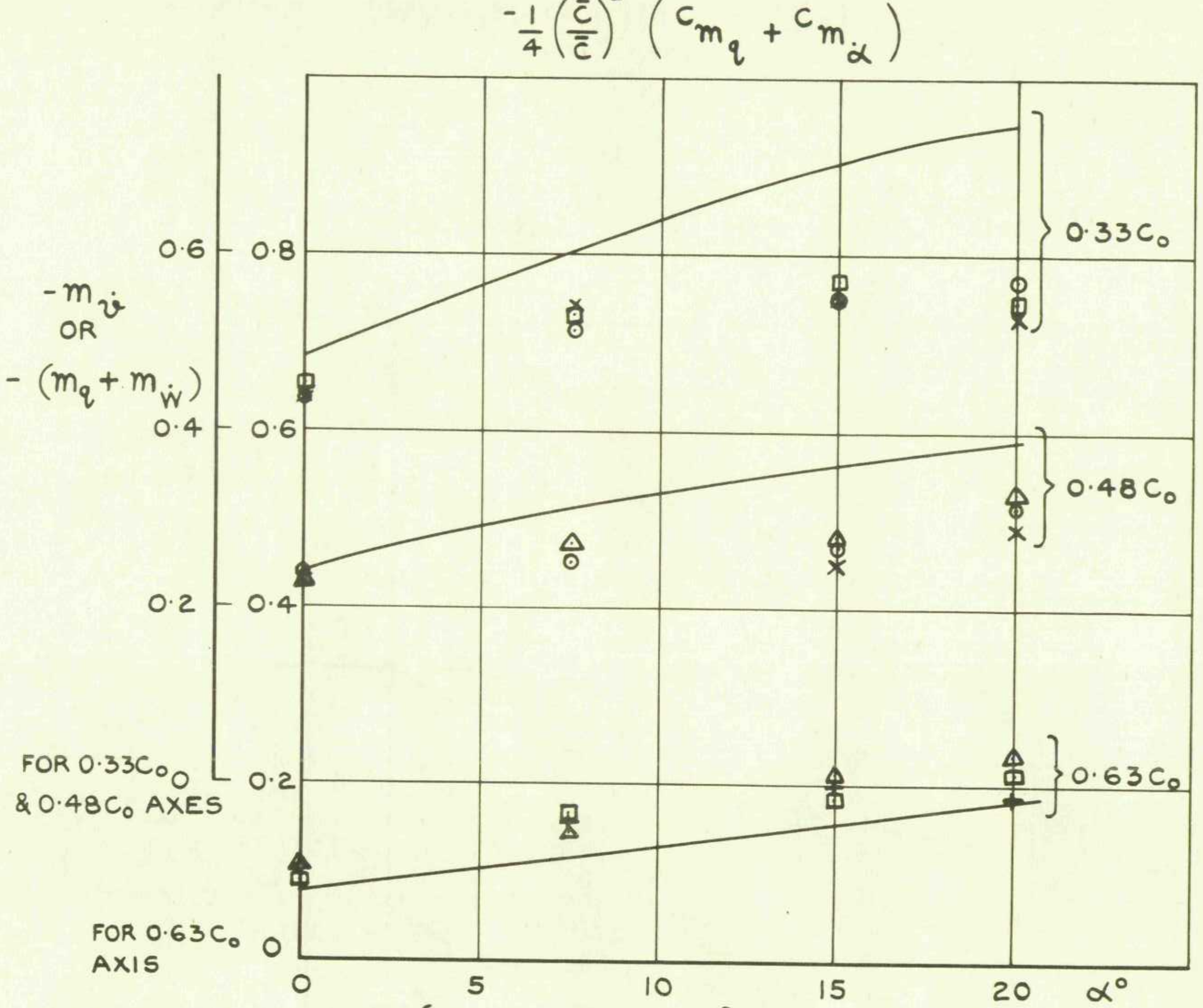


FIG. 5 DAMPING-IN-PITCH OF DELTA WING.



$$m_{\dot{\vartheta}} = \left\{ m_{\dot{\theta}_0} - z_{\dot{\theta}_0} (H - H_0) \right\} + \left[z_w (H - H_0)(H - H_{a.c}) \right]$$

SYMBOL	△	+	⊙	⊡	X
$\nu \approx$	0·6	0·45	0·35	0·25	0·1

FIG. 6. DAMPING – IN – PITCH OF GOTHIC WING.

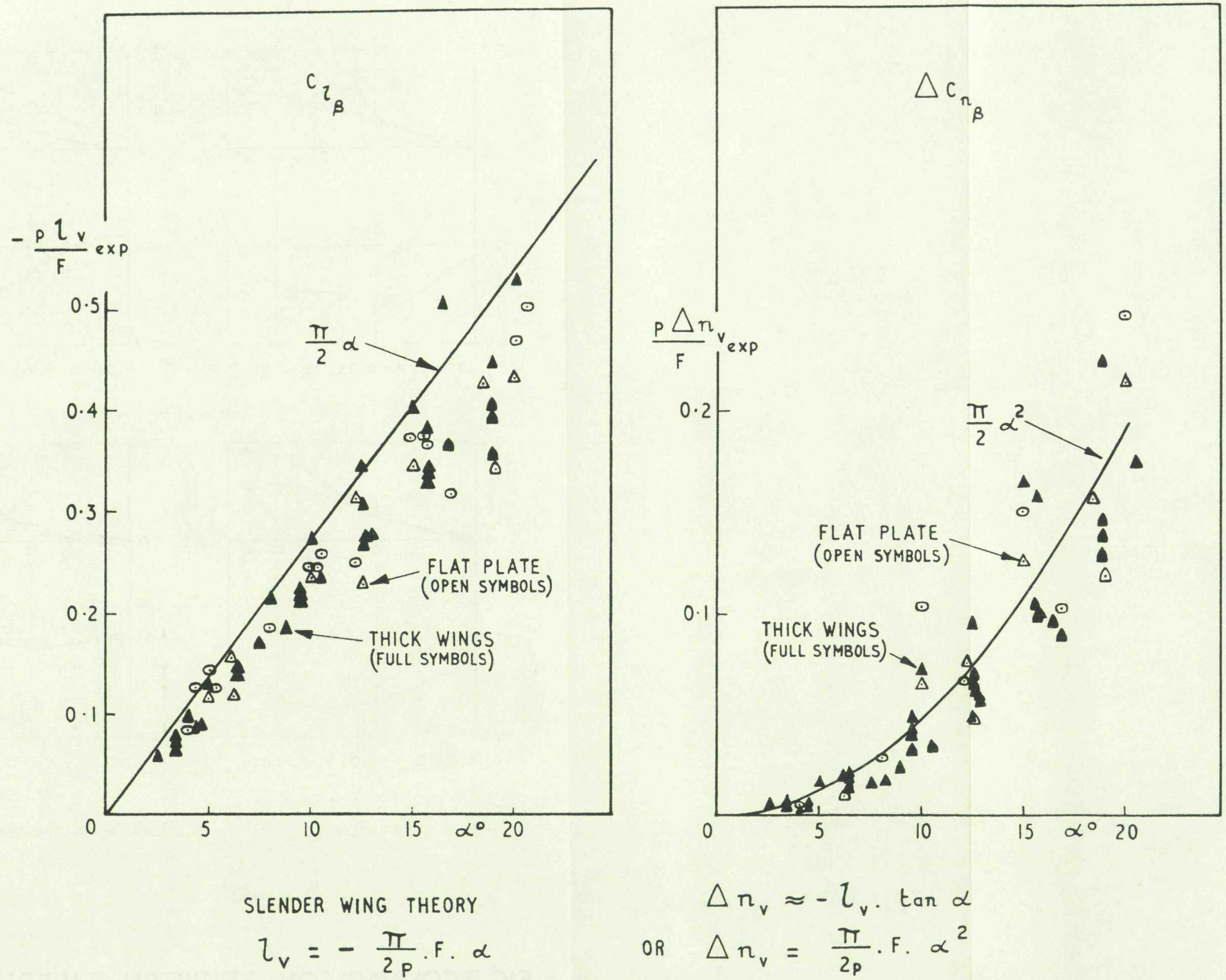


FIG. 7 COMPARISON OF EXPERIMENT AND THEORY FOR SIDESLIP DERIVATIVES.

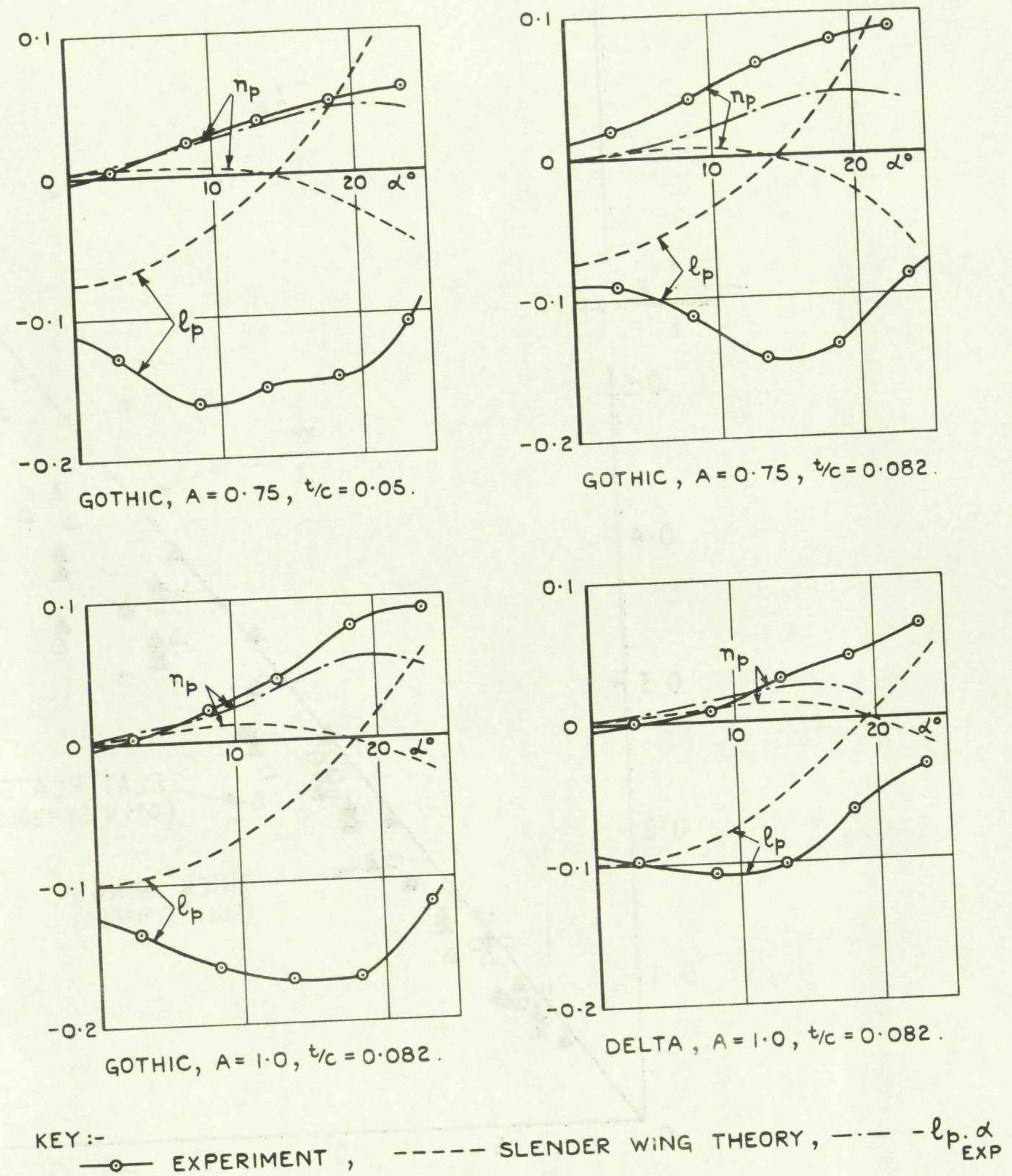
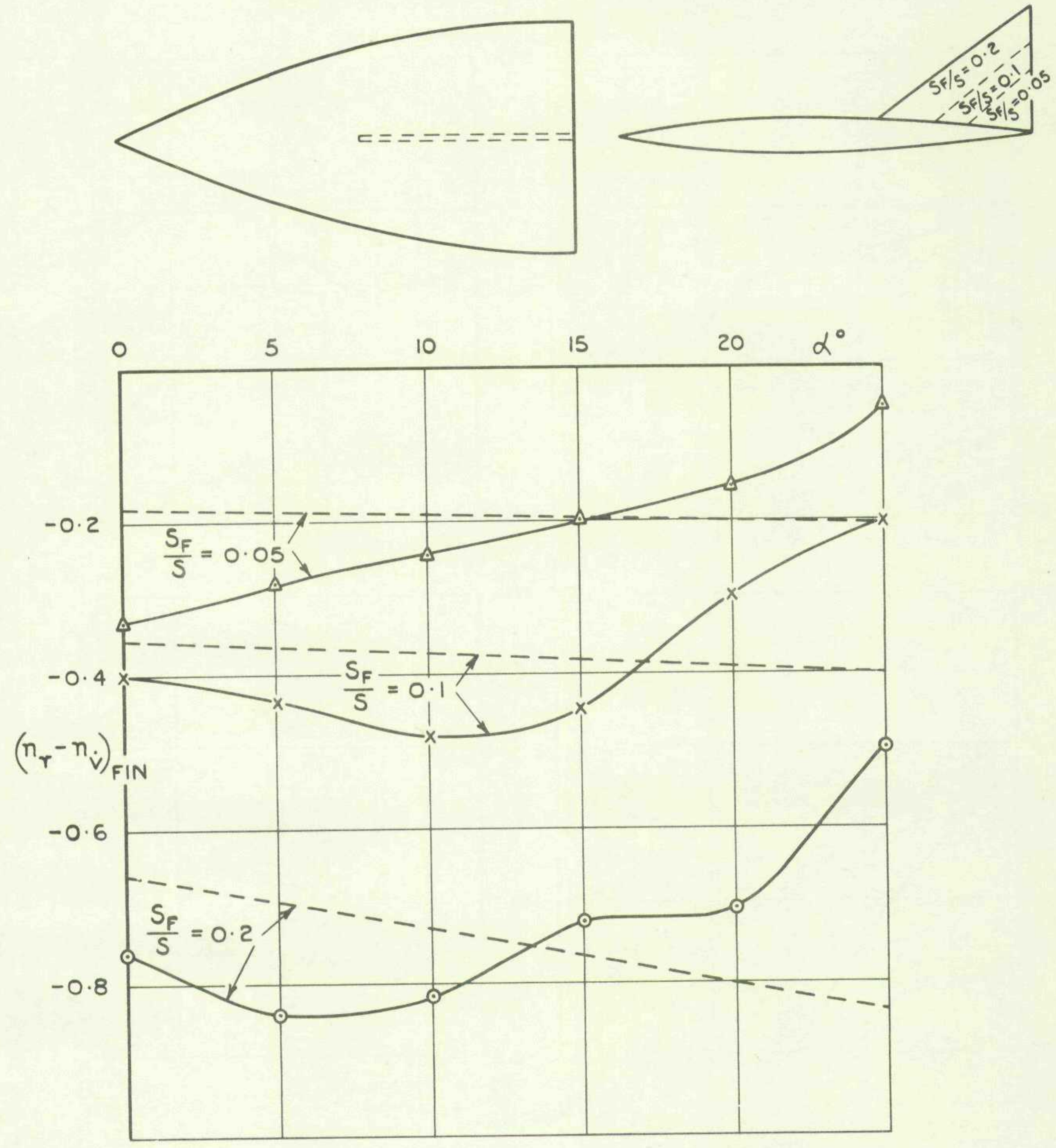


FIG.8. COMPARISON BETWEEN EXPERIMENT AND ATTACHED FLOW THEORY FOR ℓ_p AND n_p OF WINGS.

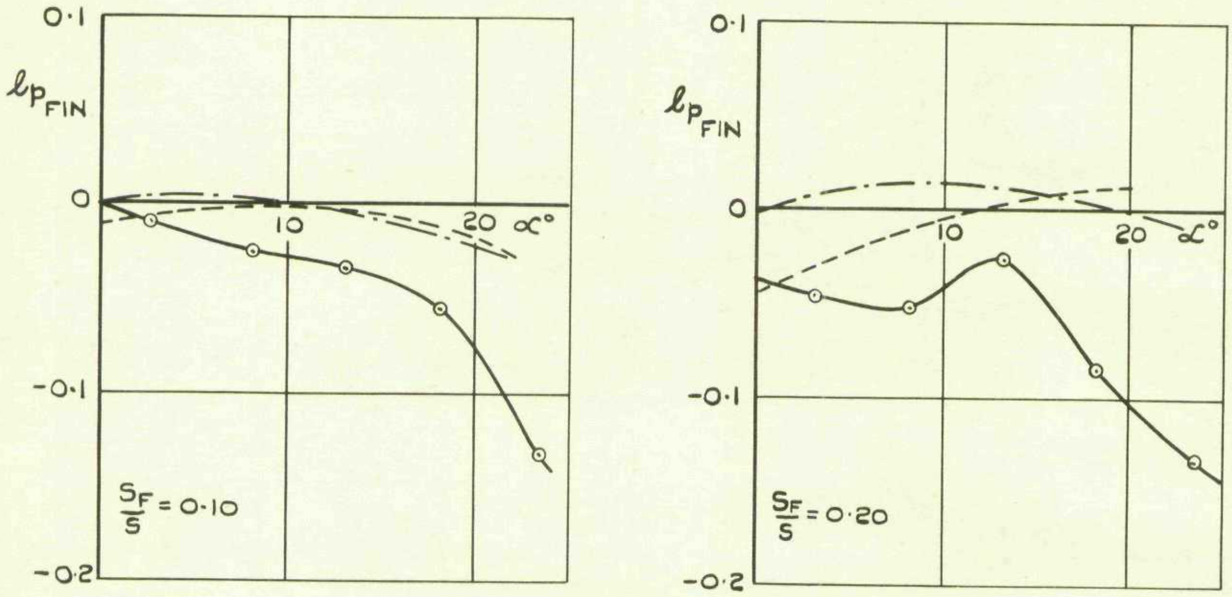


KEY :- ——— EXPERIMENT.

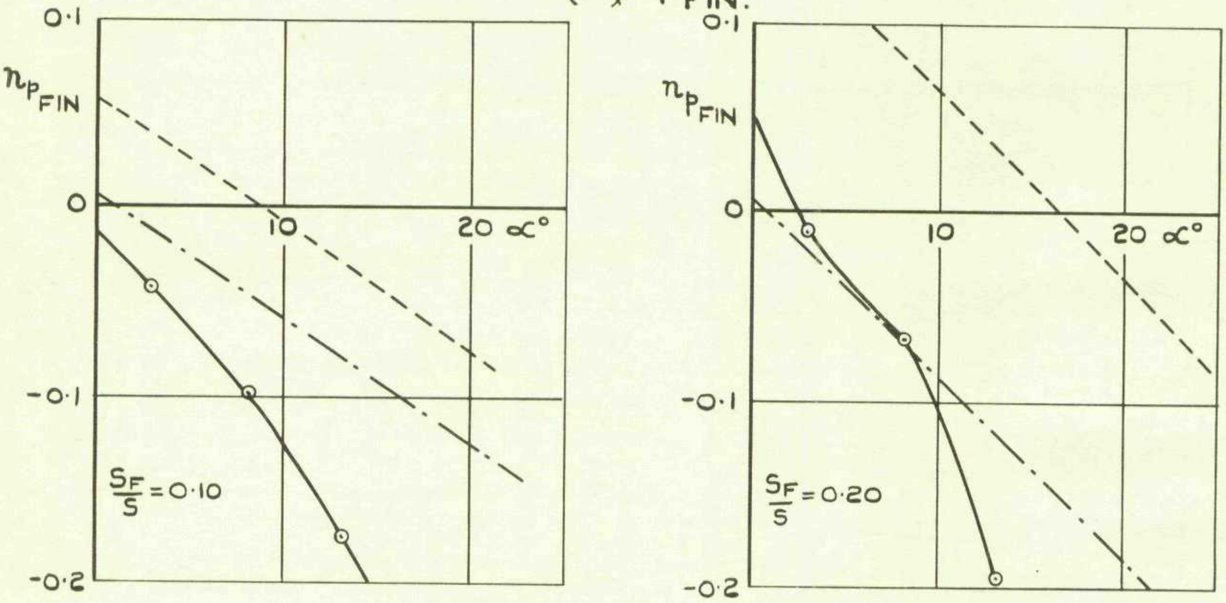
- - - - THEORY (REFLECTED FIN)

FIG. 9(a). COMPARISON BETWEEN EXPERIMENT AND THEORY FOR FIN CONTRIBUTIONS TO $(n_r - n_{\dot{v}})$.

WING AND FINS OF FIG. 9(a)



(b) ℓp_{FIN}.



(c) $n_{p\,FIN}$.

KEY :- —⊙— EXPERIMENT
- - - - THEORY, REFLECTED FIN
—·—·— THEORY, WITH SIDEWASH

FIG. 9(b&c) COMPARISON BETWEEN EXPERIMENT & THEORY FOR FIN CONTRIBUTIONS TO ℓ_p & n_p.

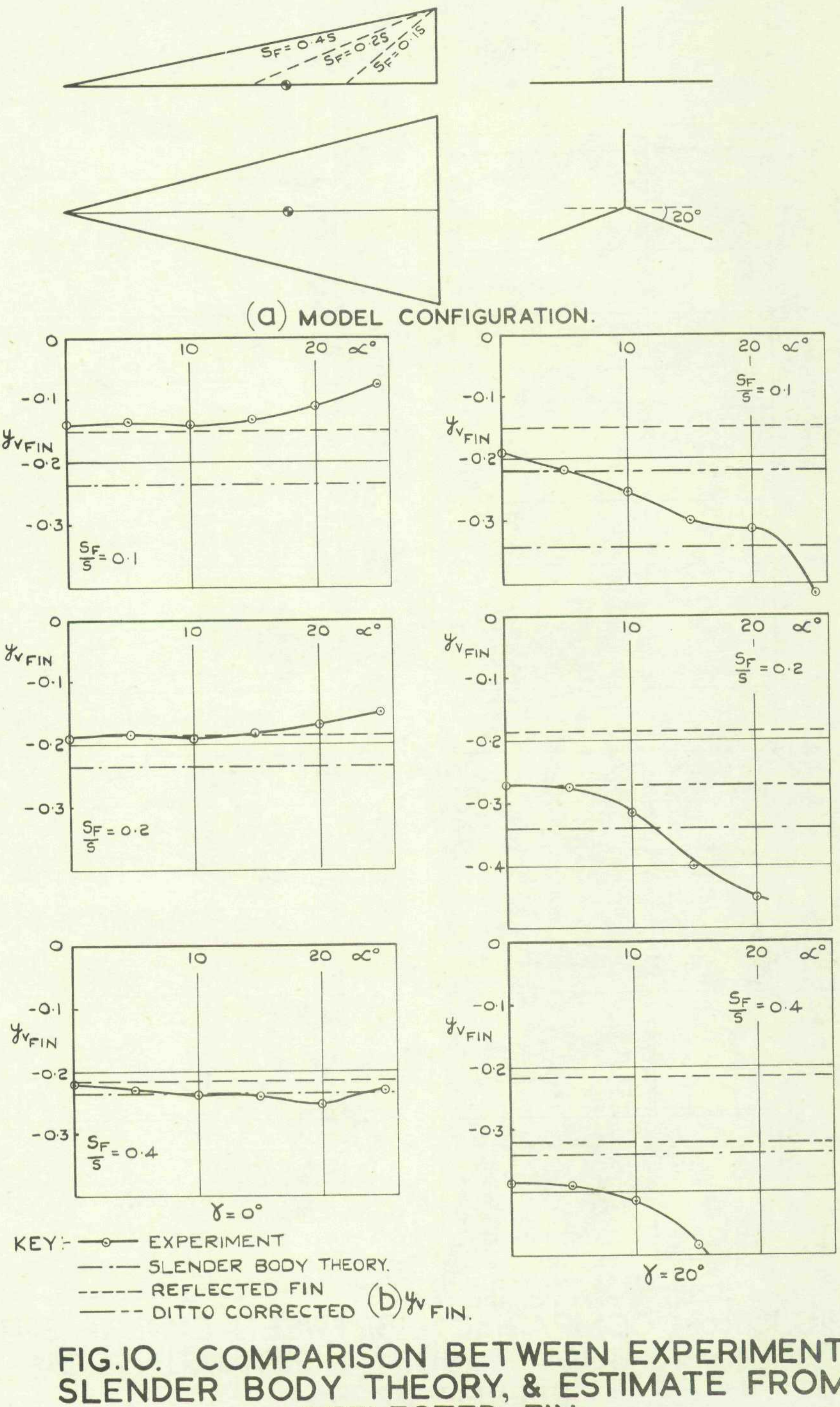


FIG.10. COMPARISON BETWEEN EXPERIMENT, SLENDER BODY THEORY, & ESTIMATE FROM REFLECTED FIN.

FOR FIN CONTRIBUTIONS TO y_v ℓ_v AND n_v.

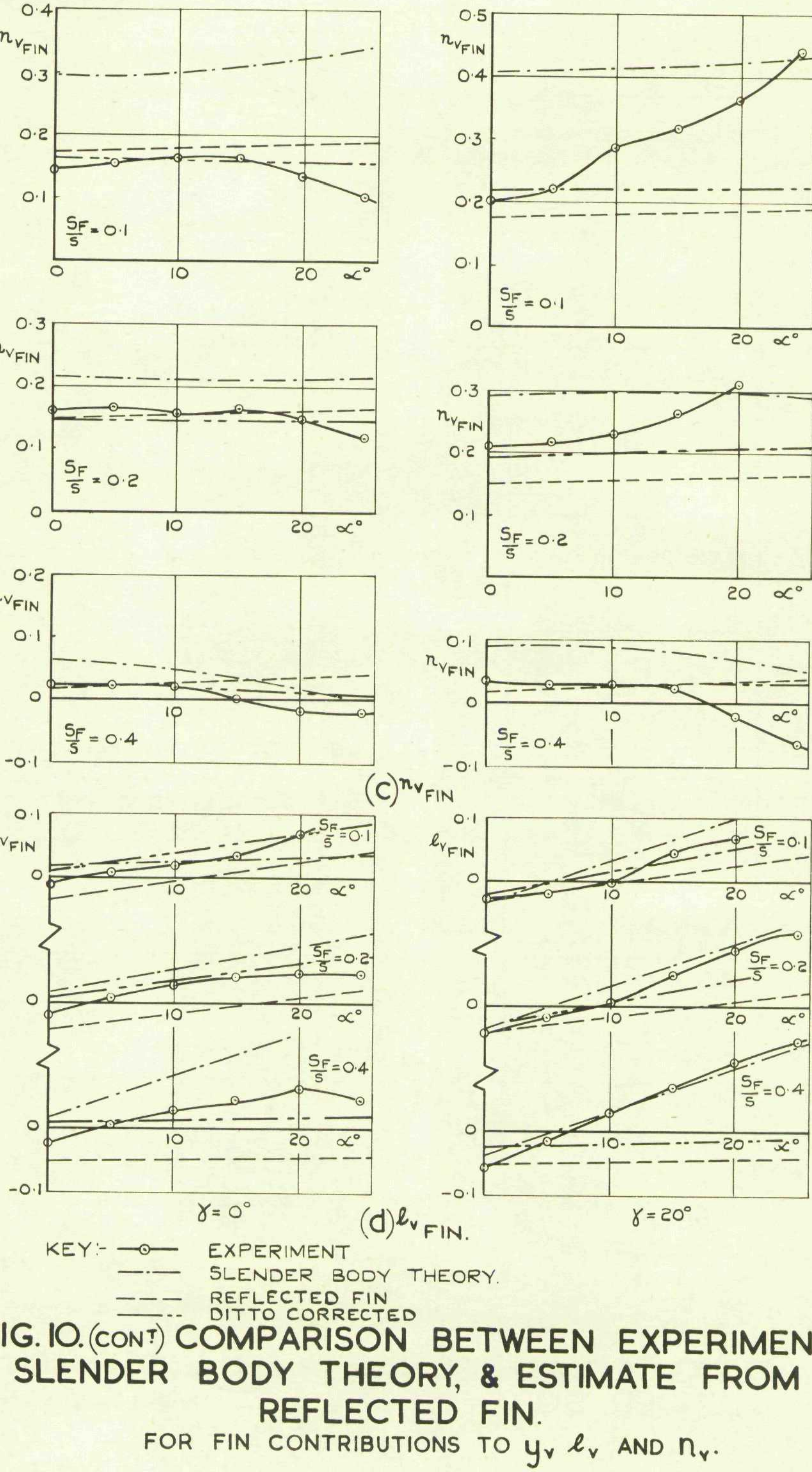


FIG. 10. (CONT.) COMPARISON BETWEEN EXPERIMENT, SLENDER BODY THEORY, & ESTIMATE FROM REFLECTED FIN.

FOR FIN CONTRIBUTIONS TO y_v l_v AND n_v.

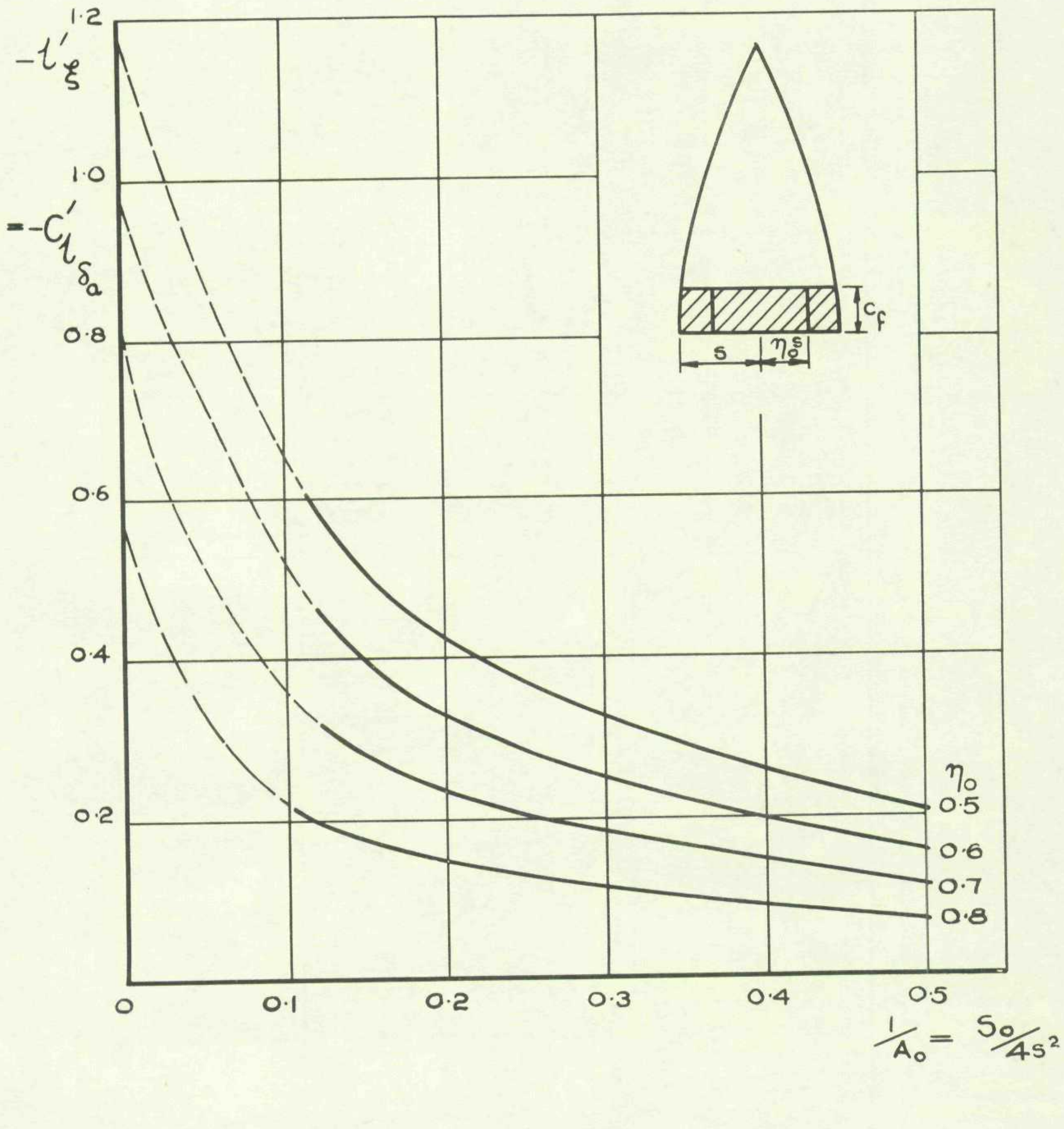


FIG. 11(a) ROLLING EFFECTIVENESS OF RECTANGULAR TIP AILERONS (BASED ON SHADED AREA = S_0)

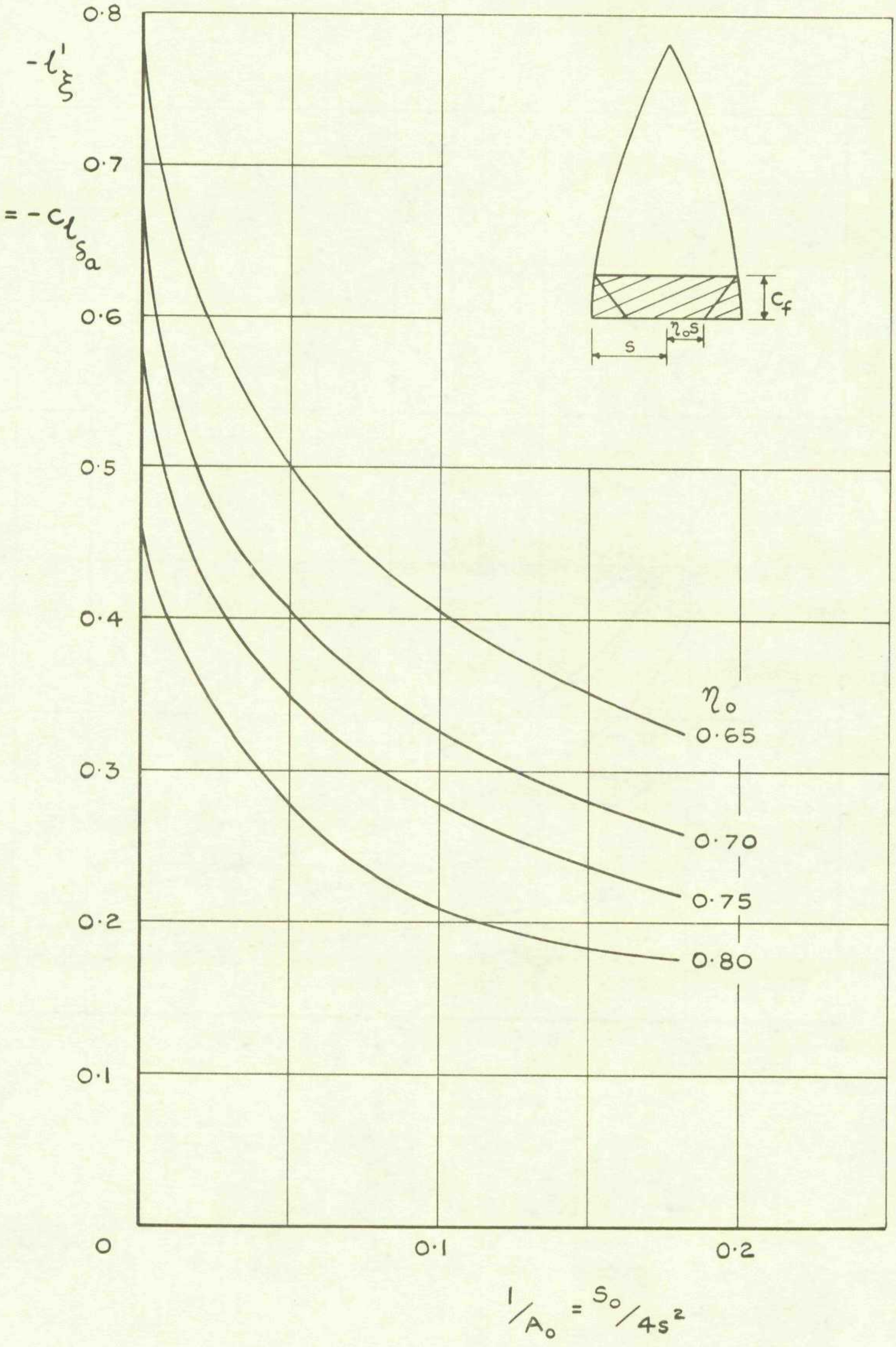


FIG. 11 (b) ROLLING MOMENT EFFECTIVENESS OF TRIANGULAR TIP AILERONS (BASED ON SHADED AREA = S_0)

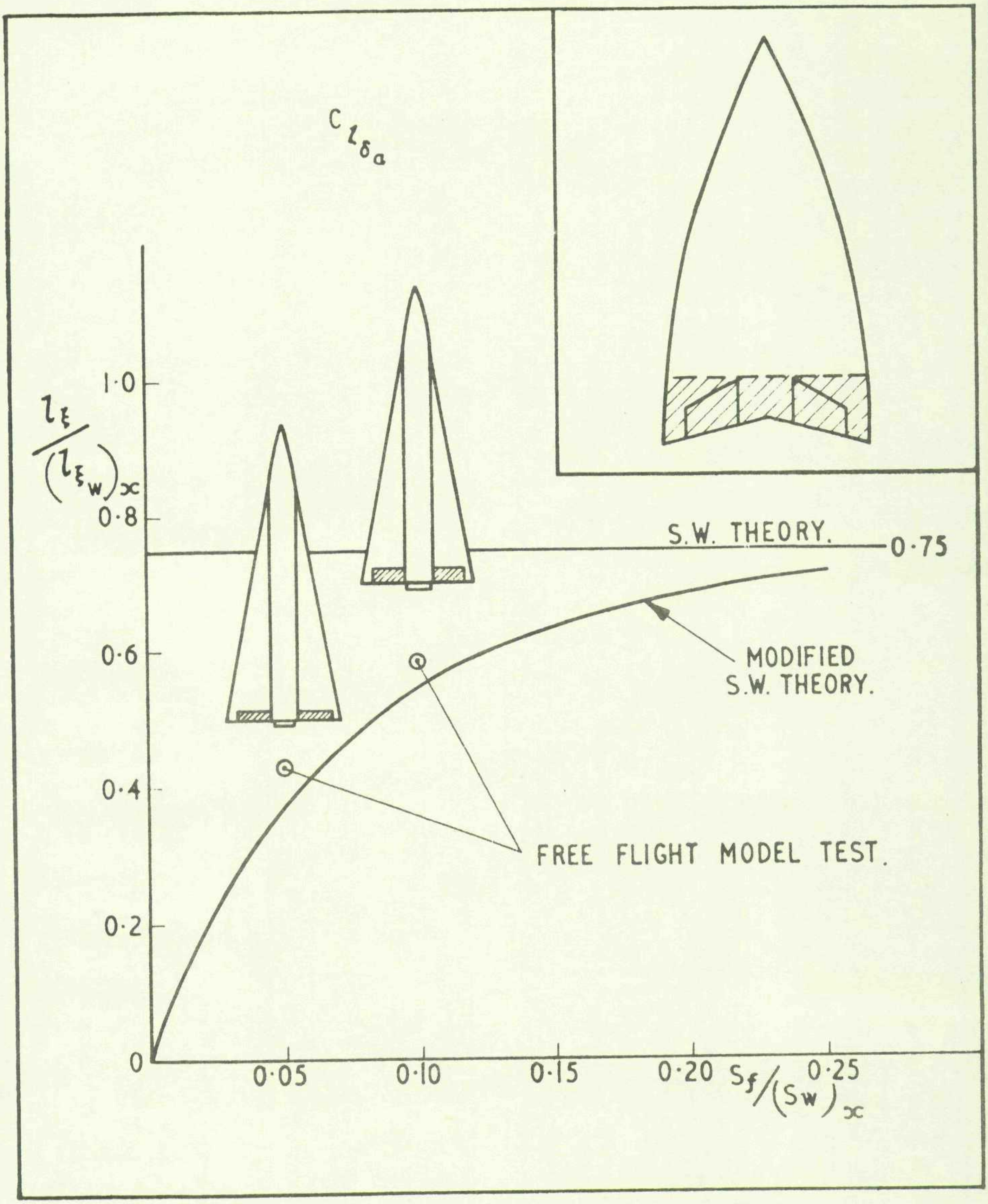


FIG.12 COMPARISON OF EXPERIMENT AND THEORY FOR ROLLING MOMENT DUE TO AILERONS (x DENOTES EXPOSED WING AREA)

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