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by

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UNSTEADY LIFT SLOPE VALUES OBTAINED FROM FLIGHT MEASUREMENTS IN GUSTS

by

D. M. Ridland

SUMMARY

Flight measurements have been made of the unsteady lift due to gusts on a Meteor 7 aircraft. Values of the lift slope obtained from these measurements show a marked reduction with increasing frequency, and in the range 1 to 5 c.p.s., $\frac{da(f)}{df} = -0.2$ per c.p.s.; at 5 c.p.s. the value of the lift slope is reduced to three quarters of its steady state value. The main experimental errors inherent in the measurements are shown to be small over the frequency range considered and the experimental values of lift slope compare reasonably well with theory.

Replaces R.A.E. Tech. Note No. Aero 2829 - A.R.C.24,189.

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1 INTRODUCTION

A number of experiments have been made with the object of establishing routine methods, using spectral techniques, for the study of aircraft response to atmospheric turbulence. The results so far analysed indicate that the effects of unsteady flow could be significant as far as aircraft response to gusts is concerned. Should it prove necessary to calculate the magnitude of such effects on future aircraft designs, unsteady (frequency dependent) aerodynamic derivatives will clearly have to be used in place of the conventionally used quasi-steady derivatives.

Much theoretical work has been done on unsteady flows, the earlier studies being concerned with the growth of circulation round an aerofoil due to a sudden change of incidence in a smooth, undisturbed fluid. Wagner's function, probably the most widely known result in this field, was verified experimentally by Walker[1] who propelled his model, a four inch chord aerofoil, through water in a tank specially designed for the purpose. Later work by Küssner resulted in a function describing the lift on a wing due to penetration of a sharp-edged vertical gust. Attempts have been made by Hakkinen and Richardson[2] and Lamson[3] to measure in wind tunnels both the Küssner function and the lift on a wing in a turbulent field. More recently, Zbrozek[4,5] has produced a theory for the motion of a rigid aircraft through turbulent air, which provides also expressions for the main unsteady aerodynamic derivatives. Full scale verification of any part of this theory is difficult and no flight measurements appear to have been published so far. The object of this Note is to present some full scale measurements of the unsteady lift slope of a Meteor 7 aircraft.

The unsteady lift slope has been defined as

$$a(f) = \frac{\text{acceleration response}}{\text{incidence response}} \; \frac{W}{\frac{1}{2}\rho S V^2} \qquad (1)$$

and the response ratio as the square root of the ratio of the power spectra of acceleration and incidence,

$$\sqrt{\frac{\phi_n(f)}{\phi_\alpha(f)}} \; .$$

Before continuing, it should be noted that not only are these measurements of unsteady effects functions of aspect ratio and flight Mach number, they also include the effect of variation in atmospheric turbulence across the wing span, which tends to average the total lift on the aircraft. A comparison is made with theory and the magnitudes of the various effects are estimated, but because of the interpolations, etc. made to obtain the theoretical curve, the agreement, or lack of it, should be taken as indicative only.

2 MEASUREMENTS

The incidence was sensed by a windvane mounted on the nose boom of the aircraft (boom natural frequency 14 c.p.s.) at a position $21\frac{1}{2}$ feet forward of the aircraft C.G., while the normal acceleration was measured at a point $2\frac{1}{2}$ feet forward of the C.G. The measurements were confined to the pitching plane and, for the present case, the analysis was restricted to the frequency range below 5 c.p.s. so that aircraft rigid body modes only were involved.

The results are based on a three minute record of 'rectilinear' flight through atmospheric turbulence at approximately 534 ft/sec T.A.S. and 2500 ft altitude. Principal dimensions and approximate (measured) quasi-steady derivatives are given for the test aircraft, a Meteor 7, in Table 1.

3 ACCURACY OF RESULTS

The main sources of error in the measurements are considered in Appendix 1. The approximate magnitudes of the errors are assessed and the total effect is found to be small below 5 c.p.s. It is estimated that due to these errors the measured values of lift slope exceed the true values by an amount varying from zero at zero frequency to about 3% at 5 c.p.s. Because of some uncertainty about their exact values and because their overall effect has been shown to be small, no corrections have been made for these dynamic errors.

The one correction which has been applied amounted to increasing the acceleration/incidence response ratios by 10% to allow for the static position error of the incidence vane. As this is, in effect, a scaling factor it can have no bearing on the dynamic aspects of the measurements.

4 RESULTS

The basic experimental results are the measured spectra of acceleration, $\phi_n(f)$, and incidence $\phi_\alpha(f)$, and these are given in Figs.1 and 2 respectively. They were obtained from time histories of the aircraft normal acceleration and incidence response to the random vertical velocity component of the atmospheric turbulence, by digital computation of the autocorrelation functions[6] which were then Fourier transformed into the frequency plane to give the required spectra[7].

The primary results, the measured values of unsteady lift slope, were obtained according to the definition

$$a(f) = \sqrt{\frac{\phi_n(f)}{\phi_\alpha(f)}} \; \frac{W}{\frac{1}{2}\rho S V^2} ,$$

by taking, at each frequency, the square root of the ratio of the spectral densities and multiplying by the constant

$$1.1 \; \frac{W}{\frac{1}{2}\rho S V^2} ,$$

the factor 1.1 being the allowance for the static position error of the windvane mentioned in Section 3. These values of unsteady lift slope are plotted in Fig.3. The scatter is normal for experimental results involving the determination of power spectra and no mean curve has been drawn. Further, the points on the incidence and acceleration spectra were used when deriving the lift slope values.

It will be seen that the lift slope decreases progressively with increasing frequency. Over the range 1 to 5 c.p.s. the measured effect of frequency, assuming linearity, is approximately

$$\frac{d\,a(f)}{df} = -0.2 \text{ per c.p.s.}$$

or in terms of reduced frequency, $\nu_1 = \frac{\omega \bar{c}}{V}$,

$$\frac{d\,a(\nu_1)}{d\,\nu_1} = -1.8.$$

At 5 c.p.s. the value of the lift slope is reduced to three quarters of its steady state value.

5 COMPARISON WITH THEORY

The basis of the theoretical method used for comparison with the experimental results is as follows.

Steady state lift is defined as

$$L = -\rho S V^2 z_w \alpha \tag{2}$$

where z_w is the lift derivative and α is the aircraft incidence.

$$z_w = -\left(\frac{a}{2} + C_D\right) \simeq -\frac{a}{2} . \tag{3}$$

Unsteady lift is similarly defined as

$$L(f) = -\rho S V^2 z_{w_t}(f)\, \alpha_t(f) \tag{4}$$

where $L(f)$ is a function of frequency, f, $z_{w_t}(f)$ is the total unsteady lift derivative and $\alpha_t(f)$ is the total unsteady incidence. For flight through gusts the total unsteady incidence, $\alpha_t(f)$, consists of two parts, namely

$\alpha_g(f)$ the incidence due to gusts, and

$\alpha_w(f)$ the incidence due to the aircraft response.

Thus

$$\alpha_t(f) = \alpha_g(f) + \alpha_w(f).$$

The corresponding partial derivatives are

$z_g(f)$ the lift derivative associated with $\alpha_g(f)$, and

$z_w(f)$ the lift derivative associated with $\alpha_w(f)$.

The unsteady lift, equation (4), may therefore be written as

$$L(f) = -\rho S V^2 [z_g(f)\ \alpha_g(f) + z_w(f)\ \alpha_w(f)] \ . \qquad (6)$$

To determine the relationship of $z_g(f)$ and $z_w(f)$ to $z_{w_t}(f)$, use is made of the incidence function, i.e. the normalised incidence per unit gust, K_i, (Ref.5). This is defined as

$$K_i = \frac{\text{total unsteady incidence}}{\text{gust induced incidence}}$$

$$= \frac{\alpha_g(f) + \alpha_w(f)}{\alpha_g(f)} = 1 + \frac{\alpha_w(f)}{\alpha_g(f)}$$

$$= \frac{V\ \alpha_t(f)}{w_g} \qquad (7)$$

where w_g is the vertical gust velocity. Hence

$$\alpha_w(f) = (K_i - 1)\ \alpha_g(f) \ . \qquad (8)$$

From equations (4) and (6)

$$z_{w_t}(f)\ \alpha_t(f) = z_g(f)\ \alpha_g(f) + z_w(f)\ \alpha_w(f). \qquad (9)$$

Using equations (7) and (8), the total unsteady lift derivative at each frequency can therefore be written

$$z_{w_t}(f) = \frac{1}{K_i}\ z_g(f) + \frac{K_i - 1}{K_i}\ z_w(f), \qquad (10)$$

where $z_g(f) \neq z_w(f)$ for $f > 0$ .... unsteady lift,

$z_g(f) = z_w(f) = -\frac{a}{2}$ for $f = 0$, ... steady state lift.

When flying through gusts, at high impressed frequencies, well above the aircraft natural frequency, the aircraft will not respond and the unsteady incidence will be due to gusts alone; K_i will thus be unity and the total unsteady lift derivative will simply be $z_g(f)$. At zero frequency, however, the aircraft has time to follow the gust; the unsteady incidence contributions from gust and aircraft heaving motion are thus equal and of opposite sense, giving $K_i = 0$, and there is no unsteady lift.

In the course of the present experiments the incidence function, K_i, was measured[8] and is given in Fig.4, showing the relative contributions of gust and heaving motion to the unsteady lift. It can be seen that from say 0.8 c.p.s. upwards, the incidence and hence the lift, is, with small error, due to gusts alone, i.e. $K_i \simeq 1$.

Estimates of the effects of gusts alone at frequencies from 1 c.p.s. upwards on the lift slope for the wing alone have been made using the theoretical method of Ref.5. The values for the required aspect ratio of 4 were obtained by interpolation of the curves of Ref.5 which cover aspect ratios of ∞, 6 and 3. In addition allowance has been made for compressibility effects at the experimental Mach number of 0.48, using Refs.9 and 4.

A curve showing the theoretical values for the frequency dependent lift curve slope is given in Fig.3 and is basically the modulus of Zbrozek's unsteady lift function due to gusts for aspect ratio four, corrected to $M = 0.48$ and multiplied by the steady state lift slope, $a = 4$.

6 DISCUSSION

It should be stressed that the measured values of unsteady lift slope are for the complete aircraft in atmospheric (three dimensional) turbulence, whereas the theoretical curve is for the wing alone, with allowances only for the effects of aspect ratio and Mach number (assuming these effects are independent of each other). No allowance has been made for the presence of the fuselage or nacelles or for the effect of turbulence variation across the wing span. As unsteady effects are much smaller for fuselage and nacelles than for wings, an allowance for the presence of the fuselage would increase the values of the theoretical curve, while an allowance for the effects of the spanwise distribution of turbulence, which reduce the overall lift, particularly at high frequencies, would decrease them.

A theoretical study of the effects of variation in turbulence across the aircraft wing span has been made by Diederich[10,11] and, on the basis of his results, the magnitude of this effect in the present case has been estimated and found to be about one quarter of the unsteady lift effect. Applying this correction to the theoretical estimates of Fig.3 would result in the theoretical estimate of the effect of frequency on the lift curve slope appearing slightly larger than experimentally determined, particularly at the higher frequencies; however, the effect of the fuselage and nacelles, which it has not been possible to estimate, will as noted be in the opposite sense and tending to restore agreement.

With regard to the theoretical estimates of the effects of turbulence variation across the span, it should be noted that these estimates depend on the shape of the correlation function of the vertical component of turbulence velocity. Diederich assumes that the correlation function is of the form

$$g(r) = \left(1 - \frac{1}{2}\,\frac{r}{L}\right) e^{-r/L},$$

where r is the lag and L is the so called turbulence scale. In the present estimation L was taken as the currently accepted 1000 ft. (An increase in L would reduce these effects). As neither the correlation function shape nor the 'turbulence scale' are well established for atmospheric turbulence, the estimate given above must be regarded as indicative only.

Overall, it is considered that the agreement between experiment and theory is fairly good. Independent of the comparison, however, the important fact remains that the experimental results show a pronounced effect of frequency on the values of the lift slope.

7 CONCLUSIONS

Measurements have been made in turbulent air of values of the unsteady lift slope of a Meteor 7 aircraft and these experimental results have been compared with existing theory.

Over the frequency range 1 to 5 c.p.s. the decrease in the value of lift slope with increasing frequency is marked, being approximately

$$\frac{d\,a(f)}{df} = -0.2 \text{ per c.p.s.}$$

or, in terms of reduced frequency,

$$\frac{d\,a(\nu_1)}{d\,\nu_1} = -1.8.$$

At 5 c.p.s. the lift slope is reduced to three quarters of its steady state value.

Theoretical estimates have been made, for a wing without fuselage, of the effect of frequency on the lift slope, including the effect of turbulence variation across the span. In view of the approximations involved, the agreement between theory and experiment is considered reasonably good.

LIST OF SYMBOLS

a per rad	quasi-steady lift slope of aircraft
$a(f)$ per rad	unsteady (or frequency dependent) lift slope of aircraft
a_T per rad	lift slope of tailplane
a_2 per rad	lift slope of elevator
B slug ft^2	pitching moment of inertia of aircraft
$B = \frac{a}{2} + \nu + \chi$ $= \frac{a}{2} - \frac{m_q}{i_B} - \frac{m_{\dot{w}}}{i_B}$	short period damping stability coefficient
$C = \omega + \frac{a}{2}\nu$ $= -\frac{\mu m_w}{i_B} - \frac{a}{2}\frac{m_q}{i_B}$ $= \frac{a}{2}\frac{\bar{c}}{\ell}\frac{\mu}{i_B}H_m$	short period stiffness stability coefficient

$\sqrt{C} = 2\pi f_o \hat{t}$	natural undamped short period frequency of the aircraft in aerodynamic time.
C_L	lift coefficient
$\bar{c}$ ft	mean aerodynamic chord (reference chord)
$F = -\frac{a}{2} + \frac{1}{\mu}\omega_T$ $= -\frac{a}{2} + \nu_T - \chi$	auxiliary coefficient in response equations (see Ref.5)
f c.p.s.	frequency
f_o c.p.s.	natural undamped short period frequency of the aircraft in real time
$g(r)$	autocorrelation function of vertical component of turbulence velocity
$H_m = K_m - \frac{\ell}{\bar{c}}\frac{m_q}{\mu}$ $= \frac{2\ell\, i_B}{a\,\bar{c}\,\mu} C$	manoeuvre margin
$i_B = \left(\frac{k_B}{\ell}\right)^2$	coefficient of pitching moment of inertia
$K_m = -\frac{\partial C_m}{\partial C_L}$ $= -\frac{2\ell}{a\bar{c}} m_w$	restoring margin (restoring margin is equal to static margin when there are no Mach number effects)
$k_B = \sqrt{\frac{gB}{W}}$ ft	radius of gyration in pitch
$L = 2\int_0^\infty g(r)\,dr$	turbulence scale
ℓ ft	reference length, usually tail arm length
$m_w = -\frac{a\bar{c}}{2\ell} K_m$ $= \frac{\bar{c}}{2\ell}\frac{\partial C_m}{\partial \alpha}$	pitching moment derivative due to heaving velocity, w (i.e. due to incidence)
$m_{w_T} = -\frac{a_T}{2}\frac{S_T}{S}\left(1 - \frac{d\varepsilon}{d\alpha}\right)$	tailplane contribution to m_w derivative (see Ref.5)

$m_{\dot{w}} = -\frac{d\varepsilon}{d\alpha}\frac{a_T}{2}\frac{S_T}{S}$	pitching moment derivative due to heaving acceleration (i.e. due to rate of change of incidence)
$m_q = -\frac{a_T}{2}\frac{S_T}{S}(1+\varphi)$	pitching moment derivative due to rate of pitch, q
$m_{q_T} = -\frac{a_T}{2}\frac{S_T}{S}$	tailplane contribution to m_q derivative (see Ref.5)
$m_\eta = \frac{\bar{c}}{2\ell}\frac{\partial C_m}{\partial\eta}$ $= -\frac{a_2}{2}\frac{S_T}{S}$	pitching moment derivative due to elevator deflection, η
n g	normal acceleration increment above 1g
q rads/sec	rate of pitch
S ft^2	wing area
S_T ft^2	tail area
$t = \hat{t}\,\tau$ sec	time
$\hat{t} = \frac{W}{g\rho S V}$ $= \frac{\mu\ell}{V}$	unit of aerodynamic time
V ft/sec	forward velocity of aircraft in undisturbed flight
W lb	weight of aircraft
w_g ft/sec	vertical gust velocity, positive upwards
$z_g(f)$	unsteady lift derivative due to gusts (see Ref.5)
$z_w(f)$	unsteady lift derivative due to aircraft heaving (see Ref.5)
$z_{w_t}(f)$	total unsteady lift derivative (see equation (10))
$\zeta = \frac{B}{2\sqrt{C}}$	short period damping ratio, ratio of actual to critical damping
$\mu = \frac{W}{g\rho S\ell}$	aircraft density parameter
$\nu = -\frac{m_q}{i_B}$	rotary damping coefficient
$\nu_1 = \frac{\omega\bar{c}}{V}$	reduced frequency (see Ref.5)

LIST OF SYMBOLS (CONTD.)

ρ slugs/ft^3	air density
$\tau = \dfrac{t}{\hat{t}}$	aerodynamic time
$\chi = -\dfrac{m_{\dot{w}}}{i_B}$	rate of change of incidence damping coefficient
$\omega = -\dfrac{\mu\, m_w}{i_B}$	static stability coefficient
$= \dfrac{a}{2}\dfrac{\bar{c}}{\ell}\dfrac{\mu}{i_B} K_m$	
$\omega_T = -\dfrac{\mu\, m_{w_T}}{i_B}$	tailplane contribution to static stability coefficient, ω, (see Ref.5)
$\omega_g = \dfrac{\mu \ell \lambda}{\bar{c}}$	non-dimensional gust frequency, related to aircraft (see Ref.5)
$= \mu\, \ell\, \dfrac{2\pi f}{V}$	
λ ft	wavelength

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APPENDIX

SOURCES OF ERROR IN THE MEASUREMENTS OF UNSTEADY LIFT SLOPE

There are considered to be four sources of error in the measurements which might be significant over the range of frequencies considered. These are errors due to the dynamics of the instruments used, the unsteady aerodynamic position error of the incidence vane, errors due to the geometrical positions of the instruments (i.e. due to the instruments not being positioned at the aircraft C.G.) and errors arising from the flexibility of the aircraft. The magnitudes of these errors are considered in the following sections.

1 ERRORS DUE TO INSTRUMENT DYNAMICS

(a) Windvane - By means of a step input in flight through smooth air at the test height and airspeed the windvane was found to have an undamped natural frequency of 48 c.p.s. and a damping ratio of approximately 0.06. As the windvane relative motion was predominantly a heaving rather than a pitching one, the windvane could be regarded as a second order spring-mass system[10]. Maximum errors in the range 0 - 5 c.p.s., including those due to the recording galvanometer, occurred at 5 c.p.s. and were approximately one per cent (increase) in modulus and three degrees in phase lag. Wind tunnel measurements of the windvane response to a step input, given in Ref.11, were in agreement with the flight results.

(b) Accelerometer - The accelerometer was found experimentally to have an undamped natural frequency of 12 c.p.s. By means of sinusoidal inputs over ranges of frequencies and amplitudes it was also found that the accelerometer was dynamically linear and had a damping ratio of approximately 0.65. Over the range 0 - 5 c.p.s. the response modulus of the accelerometer system, including demodulator and galvanometer, was flat and phase lags increased almost linearly from zero to 43 degrees. Measurements of the dynamic response of the accelerometer and its demodulator are given in Ref.12.

The joint effect of the dynamics of the two instruments therefore was to increase the value of the measured lift slope by about one per cent at 5 c.p.s. These errors were effectively cancelled by the unsteady position error, which was of approximately equal magnitude but opposite sign (see following section).

2 AERODYNAMIC POSITION ERROR OF THE INCIDENCE VANE

This term is used in the conventional sense and means the error arising from distortion of the airflow direction by the aircraft. It affects the windvane and may be considered in two parts.

(a) Static position error - This was estimated theoretically as approximately 10 per cent i.e. due to upwash the incidence as recorded by the vane was 10 per cent greater than the actual incidence of the aircraft, and flight experiments confirmed that this was of the right order. The acceleration/incidence response ratios were therefore increased by 10 per cent to correct for this.

(b) Unsteady position error - This error is basically due to lag in the growth of lift. Immediately after a sudden increase in incidence the full incremental lift is not generated as the disturbed airflow takes a finite time to re-establish itself. The full upwash will likewise not be established and the windvane will under-read with respect to the uncorrected static values. This effect, which is the unsteady position error, will clearly be zero at zero frequency, i.e. the total position error will equal the static position error, and it will increase in some manner with frequency, reducing the total position error ultimately to zero at some very high frequency.

Some idea of the magnitude of the present unsteady position error may be gained in the following manner. The ten per cent static position error can be apportioned thus: two per cent from the nose boom, five from the fuselage and three from the wings. The nose boom and fuselage may be said to be of near zero aspect ratio and will cause almost no lag in the re-establishment of the airflow, whereas the wings are fully effective in causing lag. The wing contribution of three per cent, plus say one per cent because the fuselage is not strictly of zero aspect ratio, gives four per cent of static error which is frequency dependent or which is modified by the unsteady position error. As the upwash is directly related to the generation of lift, it can be argued that the unsteady position error will be reasonably described by the unsteady lift function due to gusts, for the correct aspect ratio, in this case four (see Section 5). This function has been obtained by interpolation from Fig.12 of Ref.5 and the order of each contribution to the total position error is shown in Fig.5. In the range 0 to 5 c.p.s. the maximum value of the unsteady position error is one per cent and occurs at 5 c.p.s. This error reduces the value of the lift slope as measured at 5 c.p.s. by about one per cent in comparison with the actual lift slope and as previously mentioned, counters the error from the dynamics of the instruments.

3 ERRORS DUE TO POSITIONS OF INSTRUMENTS (GEOMETRICAL)

For correct results the measurements of incidence and acceleration should have been made at the aircraft C.G., but, as stated in Section 2 of the paper the windvane was at a distance $\ell_1 = 21.5$ ft and the accelerometer at $\ell_2 = 2.5$ ft forward of the C.G. Errors due to the rate of pitch of the aircraft, q, were thus incurred, their respective values being $\Delta\alpha = \frac{q\,\ell_1}{V}$ for incidence and $\Delta n = q\,\ell_2\,\frac{2\pi f}{g}$ for acceleration, $\Delta\alpha$ and Δn being vectors. As no phase angles are obtained by spectral analysis and a cross spectral analysis of the present data has not been completed, corrections could not properly be applied. However, using the measured spectra of α, n and q, moduli of the errors (i.e. the maximum possible errors) in the measured incidence and acceleration were determined at five frequencies. The corresponding phase angles were then approximated by using the theory of Ref.5 and the measured derivatives in Table 1. The resulting percentage errors are

frequency c.p.s.	1	2	3	4	5
error in measured incidence %	-2.7	-3.3	-3.2	-3.4	-3.5
error in measured acceleration %	-1.1	-1.1	-0.5	+0.4	+1.6

The negative sign indicates that the measured value is below the value at the aircraft C.G. These errors are small and their joint effect is to reduce the slope of the true lift curve by about $3\frac{1}{2}$ per cent, i.e. the measured curve in Fig.1 is more horizontal than it should be because of these errors.

4 ERRORS DUE TO FLEXIBILITY EFFECTS

In general, aeroelasticity will affect the lift slope, but in the present case it is considered that the effects of aircraft flexibility on the frequency dependent lift slope are negligible.

The I.A.S. of the aircraft during the experiment was comparatively low, 300 knots, and the first two fundamental modes which could be excited by turbulence had frequencies of 7.7 and 10.3 c.p.s. respectively; they were thus well above the maximum frequency of 5 c.p.s. considered here. Ground resonance tests by the firm show that the two modes were in bending with little or no torsion. There was negligible change therefore in geometrical incidence across the span and no lift increment due to structural deflection.

5 TOTAL EFFECT OF ERRORS

The most important characteristic of Fig.3 is the measured effect of frequency on lift slope. By summing the effects of the errors for which no corrections were made, the order of the total error in $\frac{d\,a\,(f)}{df}$ can be found. Letting +ve denote an increase (more negative) in slope, the contributions are

	percentage
instrument dynamic errors	+1
dynamic position errors	-1
errors due to position of instruments	$-3\frac{1}{2}$
total error in slope	$-3\frac{1}{2}$

Because of these errors the measured values of unsteady lift slope are greater than they should be, the error increasing progressively from zero at zero frequency to about 3 per cent at 5 c.p.s.

TABLE 1

Principal dimensions of Meteor 7 (VW.412)

Mainplane	
Span	37 ft 2 in
Gross area	350 sq ft
Aerofoil section EC.1240 at root to EC.1040 at tip	
Incidence	1°
Dihedral	
centre plane spar datum	$0^\circ\ 52\frac{1}{2}'$
outer plane spar datum	6°
Tailplane	
Span	15 ft 8 in
Gross area	61 sq ft
Fin	
Gross area	33.3 sq ft
General	
Total length	43 ft 6 in
Tail arm, ℓ	23.3 ft
Reference chord, $\bar{c}$	9.4 ft
Weight, W	15,200 lb

Quasi-steady derivatives, etc.

$a = 4.0$	$i_B = 0.1$	$\nu = 4.7$
$m_q = -0.47$	$\mu = 26.2$	$\chi = 1.6$
$m_w = -0.022$	$C = 15.16$	$\omega = 5.76$
$m_{\dot{w}} = -0.16$	$B = 8.3$	$\nu_T = 3.6$
$m_\eta = -0.14$	$F = 0$ (assumed)	
$m_{q_t} = -0.36$	$\zeta = 1.05$	
$H_m = 0.07$	$C_L = 0.138$	
$K_m = 0.025$	$H_m = \dfrac{2\ell\, i_B}{a\,\bar{c}\,\mu}\, C$	$K_m = -\dfrac{2\ell}{a\,\bar{c}}\, m_w$

aircraft natural frequency, $f_o = 0.54$ c.p.s.

non-dimensional gust frequency, $\omega_g = 7.19\ f$

reduced frequency, $\nu_1 = 0.111\ f$

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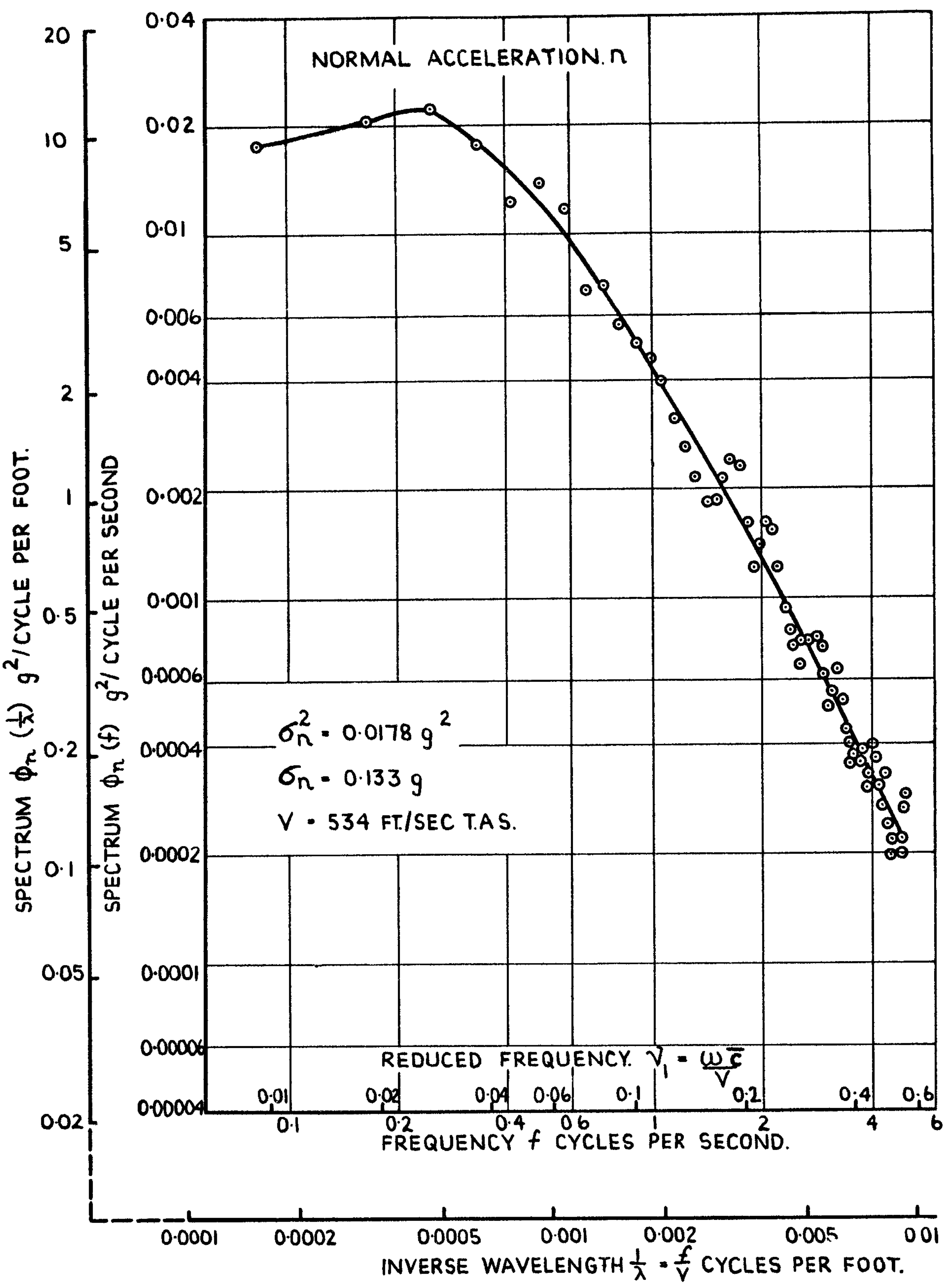

FIG.1. MEASURED SPECTRUM OF AIRCRAFT NORMAL ACCELERATION.

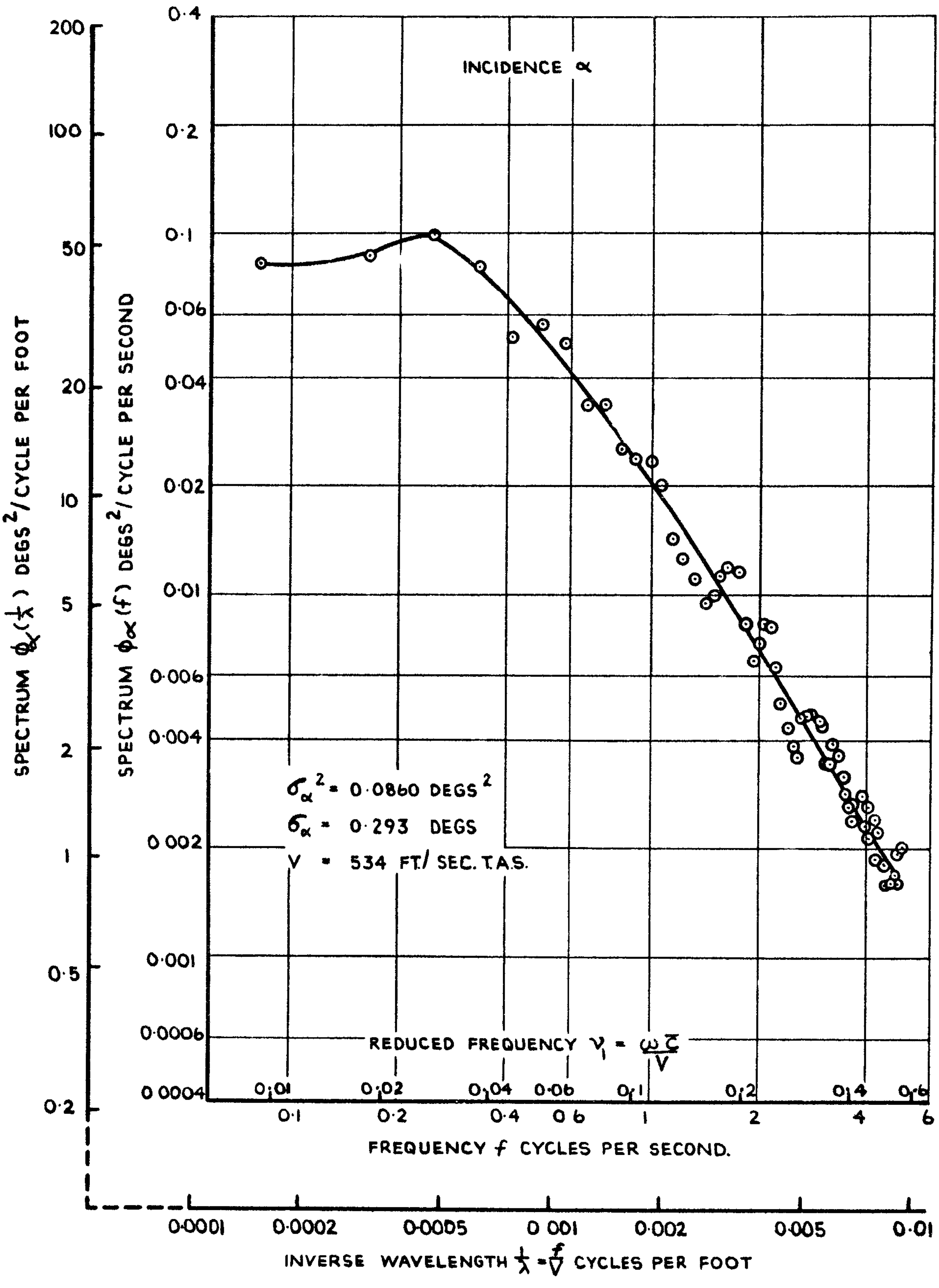

FIG. 2. MEASURED SPECTRUM OF AIRCRAFT INCIDENCE.

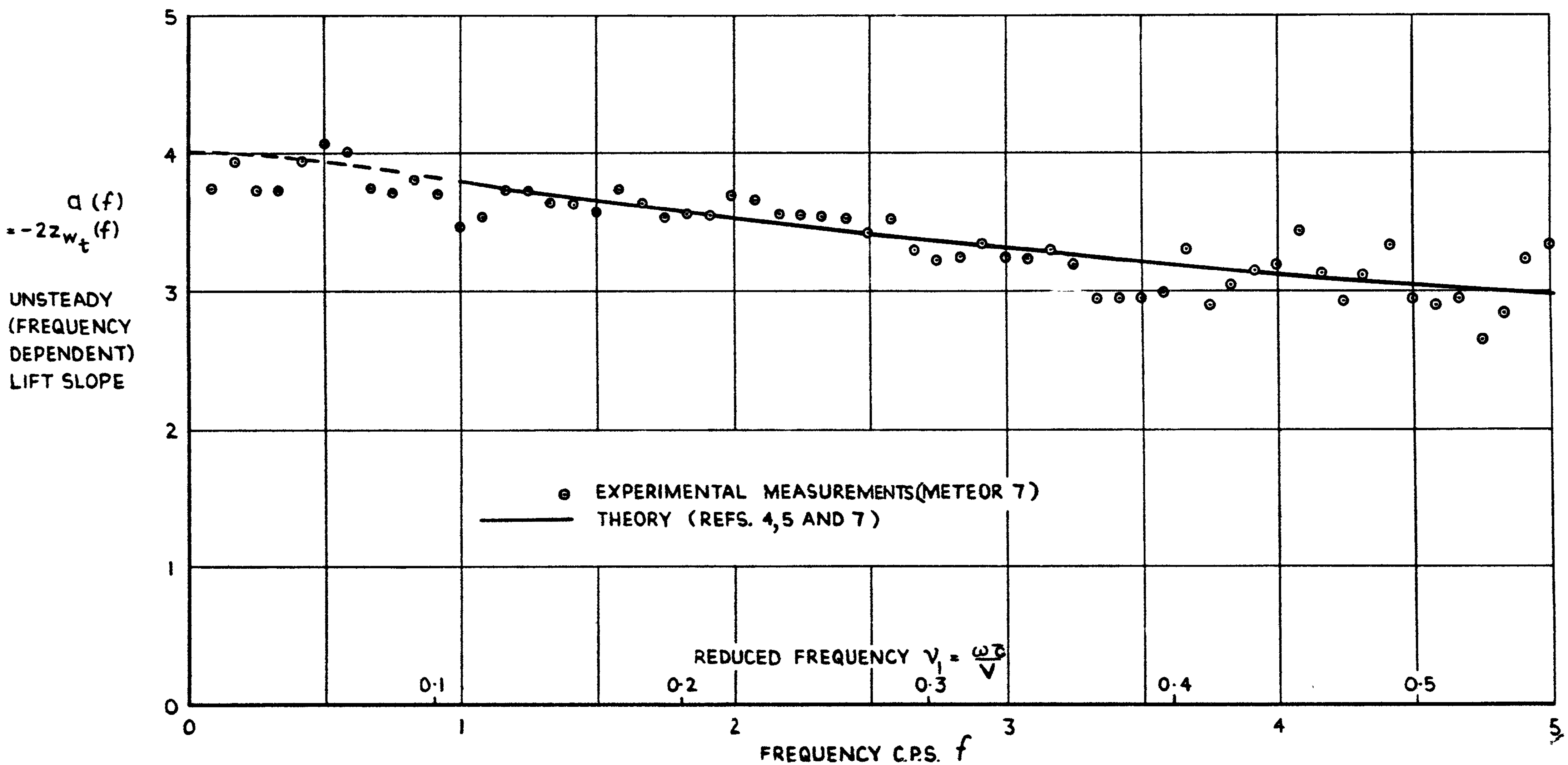

FIG. 3. THE MEASURED FULL SCALE UNSTEADY LIFT SLOPE (DUE TO GUSTS.)

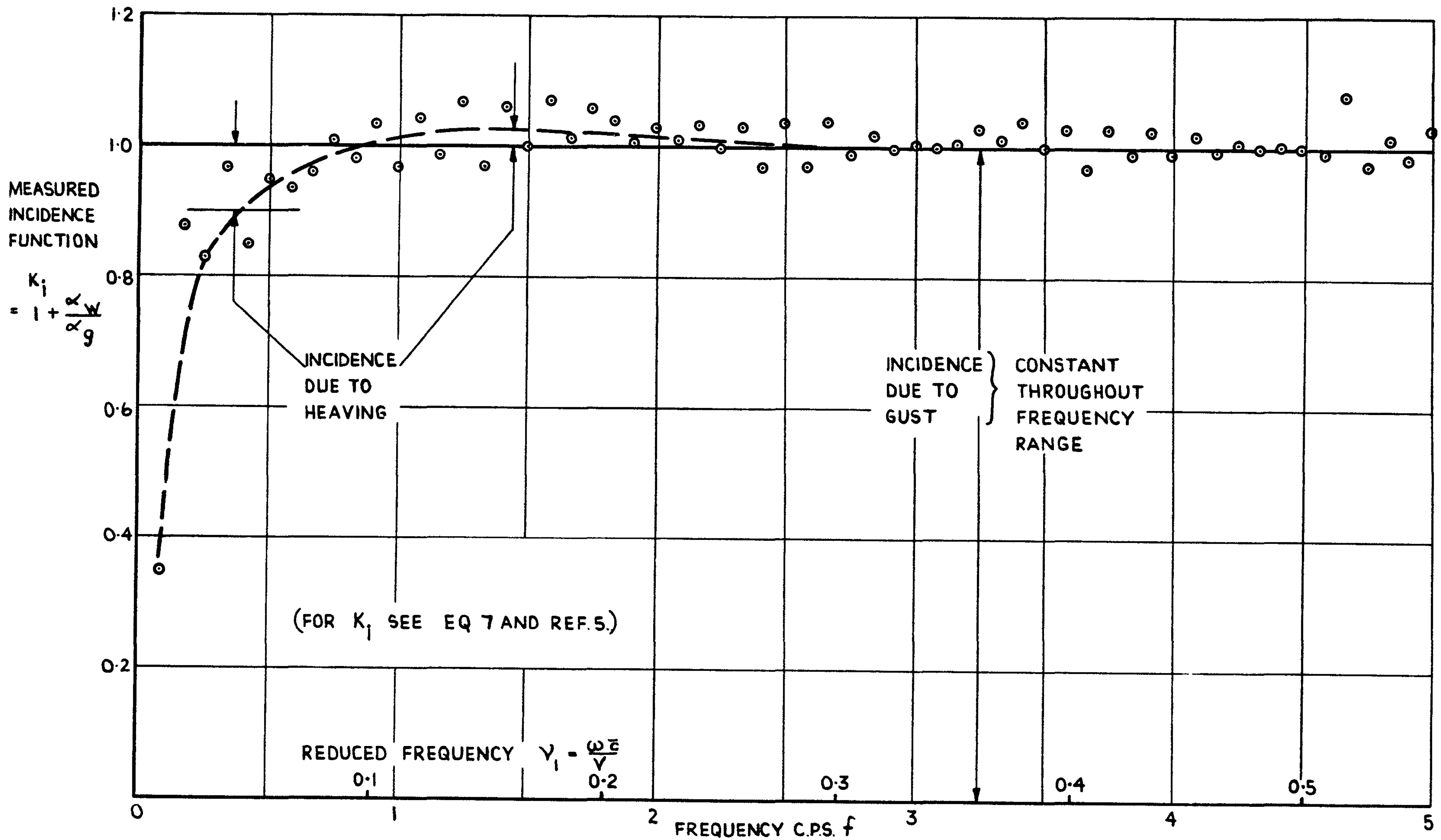

FIG. 4. USE OF THE MEASURED INCIDENCE FUNCTION K_i TO INDICATE THE RELATIVE CONTRIBUTIONS OF HEAVING MOTION AND GUSTS TO UNSTEADY LIFT.

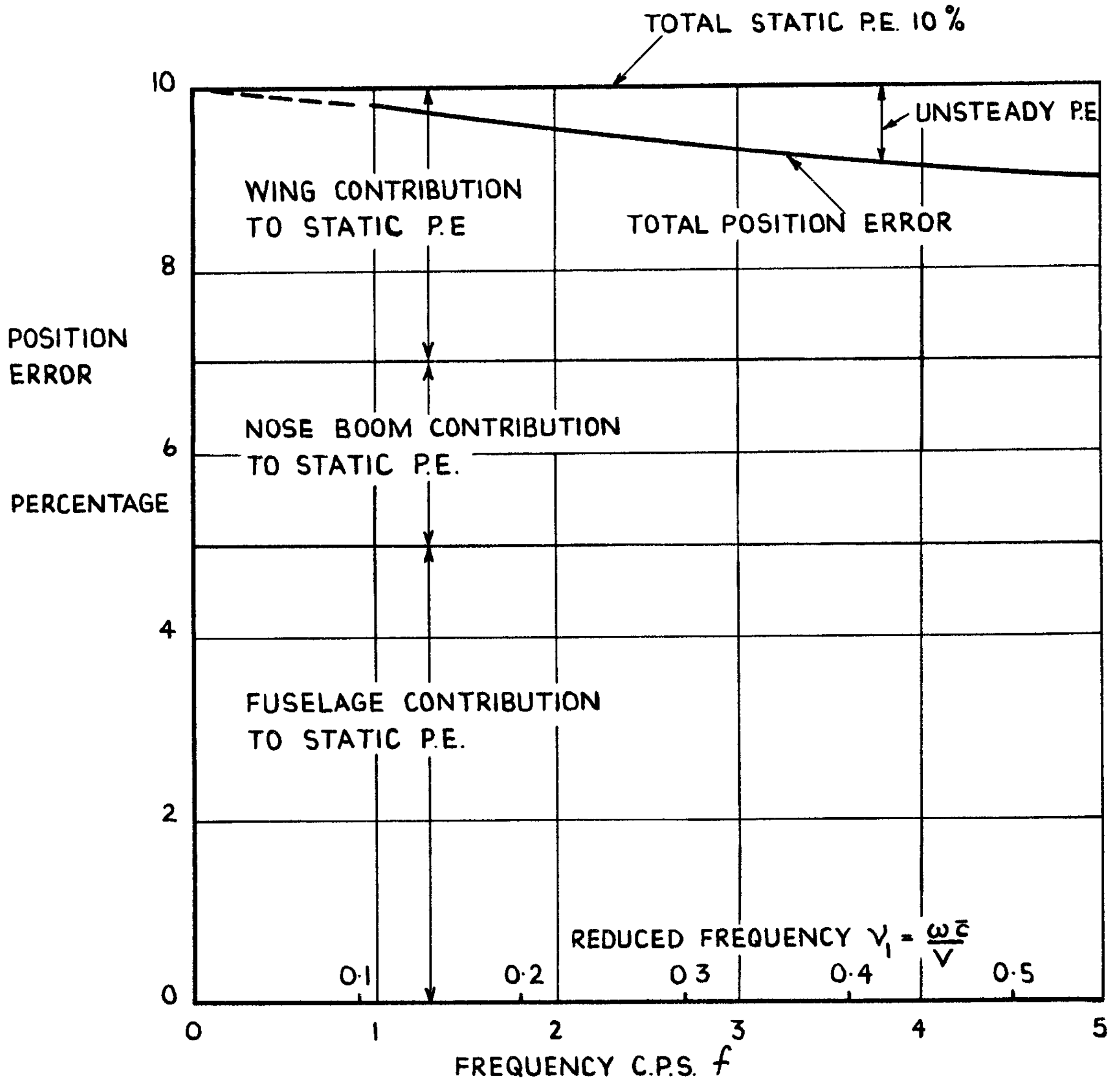

FIG.5. ESTIMATED STATIC AND UNSTEADY POSITION ERRORS.

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Ridland, D. M. June, 1962.

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