C.P. No. 647

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by

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LONDON: HER MAJESTY'S STATIONERY OFFICE

1963

PRICE 5s 6d NET

U.D.C. No. [A.I.] Avro 707B : 533.6.013.15/17 : 532.595 : 533.6.07

C.P. No.647

August, 1961

MEASUREMENTS OF THE MOMENTS OF INERTIA OF THE AVRO 707B AIRCRAFT

by

D. H. Perry, M.A.

SUMMARY

Measurements have been made of the pitching, rolling and yawing moments of inertia of the Avro 707B aircraft using the spring constrained oscillation method. Satisfactory results were obtained for the empty aircraft, but some interference effects were observed when the fuel tank was partially filled. These were traced to the presence of a resonant frequency of fuel motion in the tank which lay close to the aircraft oscillation frequency. Attempts to measure the orientation of the principal inertia axes were unsuccesful because the measuring frequency lay close to the frequency of one of the natural rolling modes of the equipment.

The inertia measurements are compared with estimates which had been made from the weight schedule of the aircraft. Dynamic flight measurements of the lateral derivative n_v, which had been analysed using these estimated inertias, showed a discrepancy with steady state flight measurements. This discrepancy was largely resolved when the measured moments of inertia were used in the analysis.

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*Replaces R.A.E. Tech. Note No. Aero 2778 - A.R.C. 23,439

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1 INTRODUCTION

The measurement of an aircraft's aerodynamic derivatives in flight forms an important branch of flight testing, allowing earlier wind tunnel measurements and estimates to be checked, and building up a body of full scale data which may be used in future designs. One method[1] of making these measurements, which is both rapid and easy to use, lies in the analysis of suitable dynamic flight manoeuvres, but the accuracy which may be obtained by this method depends to a large extent on the precision with which the aircraft's moments of inertia are known. In recent flight measurements[2] made at the R.A.E. on the Avro 707B aircraft, a discrepancy was found between the value of the lateral derivative n_v as measured in straight sideslipping tests, and the value which was calculated from measurements of the period of the lateral oscillation. The measurements made from the sideslipping tests depended on a very direct and accurate method of measuring the aircraft's rudder power, that of towed wing tip parachutes, so they were thought to be reliable. The discrepancy could therefore arise, either from a genuine difference between the value of the derivative in the static and dynamic cases, or from errors in the values of the aircraft's moments of inertia used in analysing the oscillatory tests. These had been estimated[3] from the aircraft weight schedule. Equipment for actually measuring the rolling and yawing moments of inertia of the full size aircraft was not available at the R.A.E. when the original flight tests were made, so that no data, other than the inertia estimates by the design firm, were available. Equipment for measuring the moments of inertia about all three axes, following closely the methods developed at the Cornell Aero Laboratory[4] and the N.A.C.A.[5,6], has since been built and has been used in the present tests.

The techniques which were originally used in aircraft moment of inertia measurement[7,8,9,10,11,12] were almost direct adaptations of classical laboratory methods, such as the compound pendulum and the bifilar suspension, which had originally been developed for measuring the moment of inertia of much smaller masses. Some success was achieved with these methods, but the handling difficulties associated with larger and heavier aircraft, coupled with an inherent limitation in the accuracy of the pendulum method, due to the large corrections for axis transfer[5], led to the search for an alternative technique. A suitable method was found[4,5,6] in the simple oscillatory system formed by pivoting the aircraft about a knife edge fulcrum, or suspending it from a single torsionless cable, and constraining it by coil springs. The equipment which is needed for this method is relatively simple and inexpensive, only one overhead support point being required, and the corrections for axis transfer are comparatively small.

The methods used in the present measurements followed closely those described in the N.A.C.A. references 5 and 6, and no difficulties were experienced in using the equipment to measure the moments of inertia of the empty aircraft. Measurements made with the fuel tank full or partly filled were less satisfactory due to interference caused by fuel sloshing. Attempts were made to measure[6] the orientation of the aircraft's principle inertia axes but these were unsatisfactory because of interference between the various modes of rolling oscillation.

The measurements of the aircraft's inertia showed that the discrepancy between the two values of n_v obtained by different flight test methods could be largely accounted for by errors in the moment of inertia estimates.

2 TEST METHODS

2.1 Measurement of the centre of gravity position

The position of the aircraft's centre of gravity was found by measuring the ground reactions of the main wheels and nose wheel on separate weighbridges. These measurements were made with the longitudinal axis of the aircraft inclined at various attitudes to the horizontal between 10° nose up and 7° nose down. The attitude variation was obtained by altering the height of the nose wheel weighbridge. The method of analysis to determine both the longitudinal and the vertical position of the centre of gravity is shown in Fig.1.

With the fuel tank only partly full the longitudinal position of the aircraft's centre of gravity varied slightly with attitude as fuel ran to the front or back of the tank, but for the squat shape of tank fitted to this aircraft the effect was small enough to be neglected.

The measured positions of the centre of gravity are shown in Table 1.

The position of the centre of gravity of the aircraft with the undercarriage retracted could not be measured directly, but its estimated position, based on the known weight of the undercarriage and the measurements with the undercarriage down, is shown in Table 2.

The accuracy of the determination of the longitudinal centre of gravity position is believed to be within ±0.25", and that of the vertical centre of gravity position, within ±0.5".

2.2 Measurement of the moment of inertia in pitch

Figs.2 and 5 show the general arrangement of the aircraft on the rig used for measuring the moment of inertia in pitch. An axis of oscillation parallel to the aircraft's pitching axis was established by supporting the aircraft on knife edges at the two rear jacking points. The nose of the aircraft rested in a wooden cradle which was suspended on six vertical coil springs, these forming the elastic element of the oscillatory system. The knife edge supports at the rear and the spring support at the nose were carried on standard aircraft serviceing jacks, but heavier jacks than are usual for this class of aircraft were used in order to eliminate rocking of the supports which had occurred during previous tests. The knife edges and springs were similar to those used in previously reported[13] R.A.E. tests.

The motion of the aircraft was measured by a spring constrained rate gyro, mounted in the fuselage of the aircraft, which registered on a continuous trace galvanometer recorder. The specially calibrated time base of this recorder provided the timing reference for measuring the period of the rig's oscillation.

The equation of motion of the aircraft for small amplitude oscillations about the knife edge supports, (and assuming that the aircraft and its contents may be treated as a rigid body), is given by:-

$$I \frac{d^2\theta}{dt^2} = -\left[C_y L_y^2 - W h_y\right]\theta \quad . \qquad (1)$$

The first term on the right hand side of this equation represents the restoring moment provided by the coil springs, while the second term takes account of the varying moment of the aircraft's weight about the knife edges for small angular displacements.

By solution of equation (1) a relationship may be established between the period of oscillation of the rig and its moment of inertia about the knife edge support:-

$$I_{y_{\text{knife edge}}} = \left(C_y L_y^2 - W_y h_y\right)\left(\frac{p}{2\pi}\right)^2 . \tag{2}$$

The moment of inertia measured in this way includes, of course, the moment of inertia of the measuring equipment as well as that of the aircraft. It is also referred to an axis passing through the knife edge supports, whereas the moment of inertia used in evaluating the aerodynamic derivatives from flight tests is that about an axis through the aircraft's centre of gravity. These effects must be accounted for by corrections to the measured values of the moment of inertia. Small corrections must also be made for the effect of the mass of the air surrounding the aircraft, which is set in motion by the oscillation, ("additional mass effect"), and for the bouyancy of the aircraft. These corrections are shown in the following expression:-

$$I_{y_{\text{C.G.}}} = I_{y_{\text{knife edge}}} - I_{y_{\text{equipment}}} - \left(\frac{W}{g} + U\rho\right)Z_y^2 - I_{y_{\text{add. mass}}} . \tag{3}$$

The correction for the moment of inertia of the experimental equipment includes the inertia of those parts of the rig, such as the cradles, which moved with the aircraft, and the inertia contribution due to one third of the true mass of the constraining springs. This assumption follows from the theory of heavy springs. It was found that the more complicated effects arising from the inertia of the springs, such as those described in Ref.17, could be ignored under the conditions of these tests.

The data contained in the standard references[15,16] to additional mass corrections did not cover the present case of the highly tapered delta shape of wing. It was found however, that the data given[16] for the less tapered shapes varied in the way which might have been expected from the moment of inertia of thin laminae, and this feature was used to extrapolate the existing data to the values of taper of the delta wing. Since the correction was, in any case fairly small, the accuracy of the experiment would not be appreciably affected by errors in this assumption, but model test data would have been more satisfactory.

2.3 Measurement of the moment of inertia in roll

Figs.3 and 6 show the general arrangement of the aircraft on the rig used for measuring the moment of inertia in roll. The equipment was very similar in principle to that used for measuring the moment of inertia in pitch, but in this case there were no jacking points on the fuselage fore and aft centre line which could be used to form a rolling axis. The aircraft was therefore mounted in a cradle, with the fuselage supported on bulky wooden formers which coincided with the position of load bearing frames in the fuselage structure. This cradle was supported, in turn, on a pair of knife edges below the fuselage centre line so that the whole assembly of cradle and aircraft was free to rotate about a longitudinal axis some way below the aircraft. The rolling constraint was applied by bunches of four coil springs which stretched between the floor and a strong point on each wing at about the mid span position.

A relationship, similar to that at (2) for the pitching inertia, may be obtained relating the period of oscillation of the rig to the rolling inertia about the knife edge axis:-

$$I_{x_{\text{knife edge}}} = \left(C_x L_x^2 - W_x h_x\right)\left(\frac{P}{2\pi}\right)^2 \qquad (4)$$

leading, with similar corrections to those for the pitching inertia, to an expression for the rolling moment of inertia about a longitudinal axis through the aircraft's centre of gravity:-

$$I_{x_{C.G.}} = I_{x_{\text{knife edge}}} - I_{x_{\text{equipment}}} - \left(\frac{W}{g} + U\rho\right)Z_x^2 - I_{x_{\text{add mass}}} \quad . \qquad (5)$$

2.4 Measurement of the moment of inertia in yaw

In the bifilar suspension system which has been used in the past for measuring the moment of inertia in yaw, the yawing restoring moments were inherently supplied by the suspension itself. In the present tests this system was discarded in favour of a single torsionless suspension cable and the yawing constraint was supplied by horizontal coil springs attached to the nose and tail of the aircraft. Apart from the advantage of requiring only one overhead support, this system could be easily adapted to measuring the inclination of the principal inertia axes by the N.A.C.A.[6] method.

Figs.4 and 7 show the general arrangement of the aircraft in position on the rig. The aircraft was suspended from a single point at the crane hook by a four wire sling. Since the useful flying life of this aircraft had ended, the sling could be attached directly to lugs bolted onto the aircraft in the most convenient positions. In other instances it might be necessary to build a cradle[12] to distribute the loads to suitable strong points on the aircraft, while maintaining the sling attachments in their proper positions.

Experiment showed that the form of suspension adopted was virtually torsionless, and that the upper end of the jib of the 20 ton Coles crane could be treated as a fixed point when the aircraft was moving through small amplitudes. The height of the hanger roof was only just sufficient to allow the crane jib to be elevated to the minimum angle for the weight of the aircraft, and difficulties might occur in testing heavier aircraft in this situation. The relationship between the period of oscillation of the rig and the yawing moment of inertia about the suspension axis is obtained in a similar manner to that for the other axes:-

$$I_{z_{\text{suspension}}} = C_z L_z^2 \left(\frac{P}{2\pi}\right)^2 \quad . \qquad (6)$$

Because the aircraft was suspended from a single point, the experimental yawing axis necessarily passed through the aircraft's centre of gravity. The axis transfer corrections needed for the pitching and rolling inertia measurements were therefore unnecessary here. Small corrections were made for additional mass effects and for the inertia of the experimental equipment in the manner previously described:-

$$I_{z_{C.G.}} = I_{z_{suspension}} - I_{z_{equipment}} - I_{z_{add\ mass}} \qquad (7)$$

The position of the aircraft's centre of gravity varied with fuel loading and it was necessary to make slight adjustments to the geometry of the lifting sling in order to maintain the aircraft's longitudinal axis in a horizontal attitude. A simple system of remote indicating lamps operated by a mercury level switch was used when levelling the aircraft.

2.5 Measurement of the inclination of the principal axes of inertia

The method of measuring the orientation of the principal inertia axes proposed by the N.A.C.A.[6] used a simple modification of the yawing equipment described in the previous section. It relies on measuring the inertially induced rolling motion which occurs when the yawing axis of the test does not coincide with the yawing principal inertia axis. One method of making this measurement would be to vary the attitude of the aircraft on the rig until there was zero rolling motion accompanying the yawing oscillations. The test yawing axis would then lie along the principal inertia axis. A simpler method used in Ref.6, which avoids the need to vary the aircraft attitude, lies in placing the nose and tail restraining springs at different heights. This leads to rolling moments being produced by the springs during the yawing oscillations, and these moments can be made to either oppose or reinforce the inertially induced moments by adjusting the relative heights of the springs. The advantages of a null method may be obtained by measuring the amplitude of the rolling motion at various spring positions, and interpolating for the position at which the rolling moments due to the springs exactly balance those due to the inertia effects. The relationship which then exists between the various parameters is derived in Ref.6.

The success of the method depends on the precise measurement of the rolling motion which results from the spring and inertial moments. Unfortunately it was found that it was impossible, in the present tests, to set up the yawing oscillation without at the same time stimulating the rolling natural frequencies of the equipment. As one of these frequencies lay very close to that of the yawing oscillation, it proved impossible to separate the rolling motion which it was desired to measure from that due to excitation of the rolling natural modes. These effects are discussed in Section 3.4.

3 RESULTS

The measurements of moment of inertia given in this note are referred to an axis system through the centre of gravity, having the longitudinal axis parallel to the aircraft datum line and the other two axes forming the usual mutually perpendicular system.

Attempts to measure the orientation of the principal inertia axes during these tests were unsuccesful, but it is believed that the longitudinal principal axis lay very nearly parallel to the aircraft datum line. The results given should therefore correspond very closely to the values of the principal moments of inertia of the aircraft.

In using these results in studies of flight dynamics it will normally be necessary to convert them to the wind body axis system appropriate to the flight condition. The necessary relationships are given in Ref.14.

3.1 Moment of inertia in pitch

Measurements of the pitching moment of inertia for various undercarriage positions and with various quantities of fuel are shown in Table 3. The moment of inertia for the empty aircraft with the undercarriage up is 15,109 slug-feet^2. The probable error of this measurement, calculated in the error analysis of Table 6, is ±310 slug-feet^2 (i.e. ±2%).

The variation of the measured moment of inertia with different quantities of fuel was within the probable error of measurement for the empty aircraft. The expected[19] increase in inertia due to full fuel was 45 slug-feet^2. In practice all the measurements with fuel were slightly lower than that for the empty aircraft, and this may be partly accounted for by the assumption, made when calculating the axis transfer from the measuring to the reference axes, that the fuel behaved as a frozen solid. It is known that the frozen solid analogy is incorrect[18,19] for motion consisting predominantly of rotation, but it was thought to be reasonably accurate for translational motion, such as that considered in the axis transfer. The small overall effect of fuel load on the pitching moment of inertia arises from the proximity of the pitching reference axis to the centre of the tank, and from the squat shape of the tank, (a mean length : depth ratio of about unity).

The measured increase in pitching moment of inertia due to lowering the undercarriage was 305 slug-feet^2, a change of 2%. This increase again lay within the probable error of measurement, so that the discrepancy between it and the estimated increase, 18 slug-feet^2, is not unreasonable.

3.2 Moment of inertia in roll

Measurements of the rolling moment of inertia for various undercarriage positions and with various quantities of fuel are shown in Table 4. The moment of inertia of the empty aircraft with the undercarriage up is 4247 slug-feet^2. The probable error of this measurement, calculated in the error analysis of Table 6, is ±132 slug-feet^2, (i.e. ±3%).

The variation in the rolling moment of inertia with quantity of fuel was expected to be small, as in the pitching case, since the rolling axis passed close to the centre of the fuel tank. The actual measurements again gave values for the inertia at intermediate fuel states which were smaller than those for the empty aircraft, but in this case the discrepancies were much larger than those of the pitching measurements. For one case, 157 gallons, the discrepancy was 13%; much larger than could reasonably be accounted for by experimental error; while for 70 gallons of fuel the discrepancy of 3½% was larger than would be expected from the probable error of measurement. Examination of the actual records for the tests at 157 gallons revealed the probable reason for this. The time history reproduced in Fig.8a shows a marked beating effect, such as that which might occur if a resonant frequency for fuel motion in the tank lay close to the natural frequency of the aircraft on the inertia rig. This effect was investigated more closely by removing one of the four constraining springs on either side of the rig, so that the period of the aircraft oscillation was increased from 1.0 seconds to about 1.25 seconds. As may be seen from Fig.8b, the beating effect became more severe.

Calculation of the lowest resonant frequency of this quantity of fuel in the tank, using the relationship[19]:-

$$\omega_R = \sqrt{\frac{\pi g}{2\ell} \tanh \frac{\pi h_F}{2\ell}} \qquad (8)$$

gave a value of 1.2 seconds, and there can thus be little doubt that the observed phenomena was due to fuel resonance effects.

A similar effect could not be detected on the records for the tests with only 70 gallons of fuel, but the fuel resonant frequency of 1.4 seconds in this case would have been further removed from that of the aircraft rolling oscillation, and a smaller mass of fuel was involved. The effects may, nevertheless, have been sufficiently important to cause part, at least, of the measured discrepancy.

It is not considered that the fuel resonant effects encountered here are of direct significance in relation to the flight measurements, since the aircraft oscillatory frequencies and the translational motion of the fuel tank in flight would be quite different. Fuel motion effects are, of course, sometimes important in aircraft dynamics and with this particular aircraft they were noticeable in the longitudinal plane, following abrupt disturbances such as those due to extending the air brakes.

The increase in the rolling moment of inertia due to lowering the undercarriage was measured at two different fuel states. The results, of 468 slug-feet^2 for the empty aircraft and 516 slug-feet^2 for the 70 gallon condition, were reasonably consistent and agreed fairly well with the estimated increase of 446 slug-feet^2. These values represent an increase in the rolling inertia of about 12% due to lowering the undercarriage.

3.3 Moment of inertia in yaw

Measurements of the yawing moment of inertia for the aircraft with various undercarriage positions and quantities of fuel are shown in Table 5. The moment of inertia of the empty aircraft with the undercarriage up was 19,720 slug-feet^2. The probable error of this measurement shown in the error analysis of Table 6 is ±297 slug-feet^2, (i.e. ±1.5%).

As in the previous cases the expected increase in the moment of inertia due to fuel was small, but the actual measurements showed unsystematic variations similar to those found in the pitching and rolling cases. With 70 gallons of fuel, the measured moment of inertia rose by 1%, while with 157 gallons and 217 gallons it fell by 2.5% and 1.5% respectively. These changes are rather larger than the expected variation due to errors in measurement.

Similarly, the measured increase in inertia due to lowering the undercarriage, 90 slug-feet^2, was much smaller than the estimated value of 360 slug-feet^2. This measurement, in contrast to those in the pitching and rolling cases, was obtained from tests with the fuel tank partially filled, so that some of the errors which appear to be inherent in the partially full tests may have contributed to the discrepancy. The estimated increase in yawing inertia due to lowering the undercarriage represents about 2% of the total inertia.

3.4 Measurement of the inclination of the principal axes

The difficulties which were encountered in trying to measure the orientation of the principal axes have been outlined in section 2.5. The type of suspension used in the present tests, consisting of a four wire sling suspended from a crane hook lowered some way below the jib, allowed two natural modes of oscillation in the rolling plane. These are shown in Fig.9. The first consisted predominantly of a pendulum like motion, centered on the crane jib and having a period of 2.7 seconds. The second was a rocking motion which involved little sideways movement of the aircraft and had a period of 1.1 seconds. It will be seen that the longer of these periods, that of the pendulum motion, coincided with the period of the natural yawing oscillations. The rolling motion which resulted from the combined effects of these natural modes with the rolling moments produced by the yawing springs is shown in Fig.10. The higher frequency rocking mode is very apparant.

In future tests it is hoped to overcome these difficulties by varying the proportions and stiffnesses of the rig so that the various natural frequencies are well separated, or by designing a suspension that eliminates some of the natural modes entirely.

4 COMPARISON OF MEASURED AND ESTIMATED MOMENTS OF INERTIA

Estimates of the pitching, rolling and yawing moments of inertia were made by the design firm from the aircraft weight schedule[3], and these values had been used in analysing the flight measurements[2]. Recently it has become apparent that the original estimation procedure tended to give values for the inertias which were too large, and new estimates by the firm[20], based on a revised procedure have been produced. Both sets of values are compared in Table 7 with the results of the measurements.

As had been anticipated, the discrepancies between the measurements and the original estimates were quite large; 23% in the case of the pitching inertia, 4% in the rolling case and 16% for the yawing inertia. The discrepancies between the measurements and the revised estimates were much smaller for the pitching (1.3%) and yawing (3%) inertias, but the revised estimate for the rolling inertia was now some 10% below the measured value.

5 FLIGHT MEASUREMENTS OF THE AERODYNAMIC DERIVATIVE n_v USING MEASURED INERTIA VALUES

In the flight measurements described in Ref.2 the lateral derivative, n_v, was measured by two independent methods. The first relied on measurements of the rudder angle needed to trim straight sideslips and depended on a knowledge of the aircraft's rudder power. This had been reliably established in another part of the experiment. These were essentially steady state measurements. The second method relied on a theoretical relationship which had been established[21] between the period of the aircraft's lateral oscillation and various aerodynamic and inertial parameters:-

$$T = \frac{2\pi S}{V}\sqrt{\frac{\mu_2}{\left(\frac{n_v}{i_C} + \frac{i_E}{i_C}\frac{\ell_v}{i_A}\right)}} \qquad (9)$$

One of the predominant aerodynamic parameters in this relationship is the lateral derivative n_v, so that provided the equally important inertial properties, i_C, i_A and i_E have been accurately established, values of n_v may be calculated from measurements of the oscillatory period.

Slight adjustments to the estimated and measured values of the moments of inertia were necessary to make them representative of the condition of the aircraft during the flight tests, and these adjusted values are shown in Table 8.

Values of n_v obtained from the two flight test methods, but using the earlier estimated values of the moments of inertia, are compared in Fig.11.

The difference between them is rather larger than could be accounted for by purely experimental inaccuracies. Revised calculations of n_v, using the same oscillatory data, but substituting the measured values for the inertias are shown in Fig.12. Here the agreement is much improved and the discrepancies, at least at low lift coefficients, are probably wholly accounted for by experimental inaccuracies. At the higher lift coefficients the discrepancies may be partly due to the effects of flow separation.

6 CONCLUSIONS

The moments of inertia of the Avro 707B aircraft about the pitching, rolling and yawing axes have been measured by the spring constrained oscillation method. The measured inertias for the empty aircraft with the undercarriage up, (and the probable errors in these measurements) were:-

moment of inertia in pitch: 15,109 (±310) slug-feet^2
moment of inertia in roll : 4,247 (±132) slug-feet^2
moment of inertia in yaw : 19,720 (±297) slug-feet^2.

The methods used in the present tests proved to be simple to operate and fairly reliable for the empty aircraft, but severe interference effects due to fuel sloshing were encountered in some of the rolling tests. Other unexplained variations in the measured inertias with different quantities of fuel suggested that care was needed in accepting the results of measurements with the fuel tank completely or partially full.

It was not possible to measure the orientation of the principal inertia axes by the method which had originally been proposed because of interference between the various possible natural modes of oscillation of the aircraft on the rig. Redesign of the equipment would be needed to eliminate these effects.

Differences which had occurred between values of the lateral derivative n_v for this aircraft when measured in flight by two different methods, one dynamic and the other steady state, could now be largely attributed to errors in the estimates of the aircraft inertia which were used in the former. This underlines the importance of moment of inertia measurements.

LIST OF SYMBOLS

Symbol	Unit	Definition
C_x, C_y, C_z	lb wt/foot	Spring stiffness for springs used in the rolling, pitching and yawing tests.
g	feet/sec^2	Acceleration due to gravity.
h_x, h_y	feet	Height of centre of gravity of aircraft and equipment above rolling and yawing knife edges.
h_f	feet	Depth of fuel in the fuel tank.

LIST OF SYMBOLS (Cont'd)

Symbol	Unit	Definition
I_x, I_y, I_z	slug-feet^2	Moments of inertia about the rolling, pitching and yawing axes defined in section 3.
i_A, i_C, i_E	-	Non dimensional inertia coefficients, see Ref.21.
ℓ_x, ℓ_y, ℓ_z	feet	Arm of the constraining springs used in rolling, pitching and yawing tests.
ℓ_v	-	Non dimensional aerodynamic derivative of rolling moment due to sideslip.
ℓ	feet	half-length of the fuel tank.
n_v	-	Non dimensional aerodynamic derivative of yawing moment due to sideslip.
P	secs	Period of oscillation of the aircraft on the inertia rig.
S	feet	Aircraft semispan.
T	secs	Period of the aircraft lateral oscillation in flight.
U	feet^3	Total volume of the aircraft.
V	feet/sec	Aircraft true airspeed.
W	lb wt	Weight of the aircraft.
W_x, W_y	lb wt	Weight of the aircraft and attached equipment during inertia tests.
Z_x, Z_y	feet	Distance from the rolling and yawing knife edges to the aircraft centre of gravity.
θ	radians	Attitude of the aircraft on the pitching rig.
ρ	slugs/cubic foot	Air density.
μ_2		Relative density of the aircraft, see Ref.21.
ω_R	radians/sec	Lowest angular frequency of fuel oscillations.

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TABLE 1

Measured centre of gravity positions. Undercarriage down

Fuel Gallons	Aircraft weight lb	Longitudinal C of G position inches A.O.D.	Vertical C of G position inches below fuselage datum
0	7376	105.1"	3.3"
157	8647	106.5"	3.0"
217 (Full)	9134	106.7"	2.2"
Accuracy	±15 lb	±0.25"	±0.5"

TABLE 2

Calculated centre of gravity positions. Undercarriage up

Fuel Gallons	Aircraft weight lb	Longitudinal C of G position A.O.D.	Vertical C of G position below fuselage datum
0	7376	104.8"	1.1"
157	8647	106.25"	1.2"
217 (Full)	9134	106.5"	0.5"
Accuracy	±15 lb	±0.25"	±0.5"

TABLE 3

Analysis of pitching moment of inertia measurements

Fuel gallons	Under-carriage position	Weight of aircraft lb	Period of oscillation secs	Measured moment of inertia of aircraft + cradle about the knife edges. Slug/ft^2	M of I of cradle about knife edges Slug/ft^2	Additional mass and entrapped air	Moment of inertia of aircraft about knife edges Slug/ft^2	Axis transfer	Moment of inertia of aircraft about reference axes Slug/ft^2
0	Down	7376	0.692	21,648	2567	271	18,810	3396	15,414
0	Up	7376	0.689	21,444	"	"	18,606	3497	15,109
70	Up	7942	0.689	21,435	"	"	18,597	3666*	14,931
157	Up	8647	0.692	21,666	"	"	18,828	3855*	14,973
217	Up	9134	0.696	21,906	"	"	19,068	4058*	15,010
* Fuel treated as a frozen mass									Probable error ±310

TABLE 4

Analysis of rolling moment of inertia measurements

Fuel gallons	Under-carriage position	Weight of aircraft lb	Period of oscillation secs	Measured moment of inertia of aircraft + cradle about the knife edges. Slug/ft^2	M of I of cradle about knife edges Slug/ft^2	Additional mass and entrapped air	Moment of inertia of aircraft about knife edges Slug/ft^2	Axis transfer	Moment of inertia of aircraft about reference axes Slug/ft^2
0	Down	7376	1.013	6999	119	320	6560	1845	4715
0	Up	7376	0.999	6774	"	"	6335	2088	4247
70	Down	7942	1.014	6982	"	"	6543	1931**	4612
70	Up	7942	0.998	6723	"	"	6284	2188**	4096
157	Up	8647	0.992*	6579	"	"	6140	2429**	3711
217	Up	9134	1.056	7410	"	"	6971	2676**	4295
									Probable error ±132

* Fuel sloshing made this determination unreliable (see section 3.2)
** Fuel treated as a frozen mass

TABLE 5

Analysis of yawing moment of inertia measurements

Fuel gallons	Undercarriage position	Weight of aircraft lb	Period of oscillation secs	Measured moment of inertia of aircraft + equipment about suspension. Slug/ft^2	M of I of equipment about suspension	Additional mass	Moment of inertia of aircraft about reference axis. Slug/ft^2
0	Up	7376	2.807	19.912	81	111	19,720
70	Down	7942	2.829	20,181	"	"	19,989
70	Up	7942	2.820	20,091	"	"	19,899
157	Up	8647	2.773	19,384	"	"	19,192
217	Up	9134	2.781	19,553	"	"	19,361
							Probable error ±297

TABLE 6

Error analysis

Source of error	Units	Pitch measurement		Roll measurement		Yaw measurement	
		Possible error	Error in inertia $Slug/ft^2$	Possible error	Error in inertia $Slug/ft^2$	Possible error	Error in inertia $Slug/ft^2$
Period of oscillation	Secs	± 0.002	± 127	± 0.003*	± 41	± 0.015	± 214
Spring constant	lb/ft	± 138(2%)	± 443	± 141	± 160	± 18	± 385
Spring arm	ft	± 0.015	± 42	± 0.015	± 34	± 0.04	± 3
Weight of aircraft	lb	± 25	± 11	± 25	± 9	–	–
Vertical height of C.G.	ft	± 0.04	± 4	± 0.04	± 63	–	–
Longitudinal position of C.G.	ft	± 0.02	± 35	–	–	–	–
Weight of experimental equipment	lb	± 5	± 40	± 2	± 2	± 2	± 6
Radius of gyration "	ft	± 0.05	neg	± 0.05	± 1	± 0.05	± 1
Additional mass	$Slug/ft^2$		± 50		± 75		± 50
Probable error = $0.675 \sqrt{\Sigma \text{(possible error)}^2}$	$Slug/ft^2$		± 310		± 132		± 297

* Only applies to empty aircraft

TABLE 7

Comparison of measured and estimated moments of inertia. Full fuel. Undercarriage down

Source of data	Aircraft weight	Centre of gravity position	Moment of inertia		
			Pitching	Rolling	Yawing
Present tests	9240 lb	105.6 ins A.O.D.	15,390 Slugs/ft^2	4790 Slugs/ft^2	19,790 Slugs/ft^2
Estimates from Ref. 20	9290 lb	105.1 ins A.O.D.	15,650 Slugs/ft^2	4280 Slugs/ft^2	19,180 Slugs/ft^2
Estimates from Ref.3 (as used in Ref.2)	9290 lb	105.1 ins A.O.D.	18,990 Slugs/ft^2	4980 Slugs/ft^2	22,860 Slugs/ft^2

TABLE 8

Comparison of estimates used in reducing the flight measurements of Ref.2 with revised values based on the present tests

Source of data	Aircraft weight	Centre of gravity position	Moment of inertia	
			Yawing	Rolling
Estimates based on Ref.3 (as used in Ref.2)	8700 lb (half fuel)	102.8	23,600 Slug/ft^2	4500 Slug/ft^2
Present tests	8700 lb	102.7	20,680 Slug/ft^2	4250 Slug/ft^2

TABLE 9

Table of miscellaneous data

Pitching equipment			
C_y,	Spring stiffness used in pitching test	6993.0	lb wt/foot
h_y,	Height of centre of gravity above knife edge	0.65*	feet
ℓ_y,	Moment arm of constraining springs	16.0	feet
	Weight of nose cradle	290.25	lb wt
	Weight of constraining springs	98	lb wt
Rolling equipment			
C_x,	Spring stiffness used in rolling tests	7055	lb wt/foot
h_x,	Height of centre of gravity above knife edges	2.8*	feet
ℓ_x,	Moment arm of constraining springs	6.42	feet
	Weight of cradle	526	lb wt
	Weight of constraining springs	189	lb wt
Yawing equipment			
C_z,	Spring stiffness used in yawing tests	926	lb wt/foot
ℓ_z,	Moment arm of constraining springs	10.37	feet
	Weight of constraining springs	12	lb wt

* Varies with fuel load and undercarriage position

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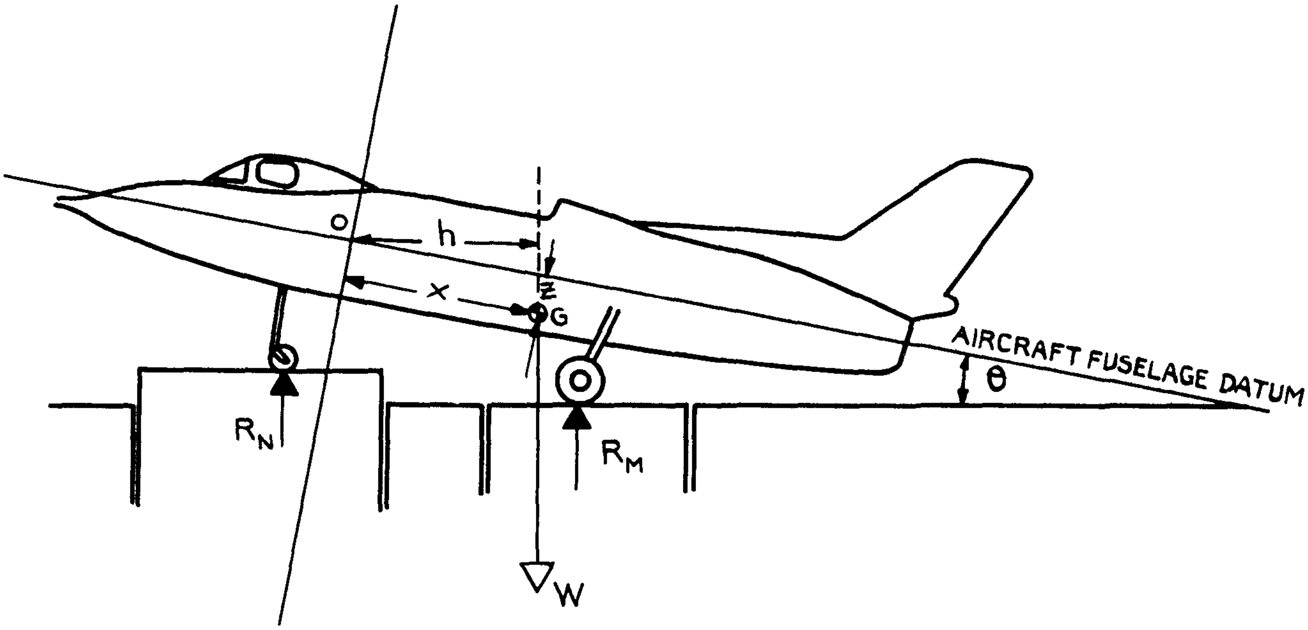


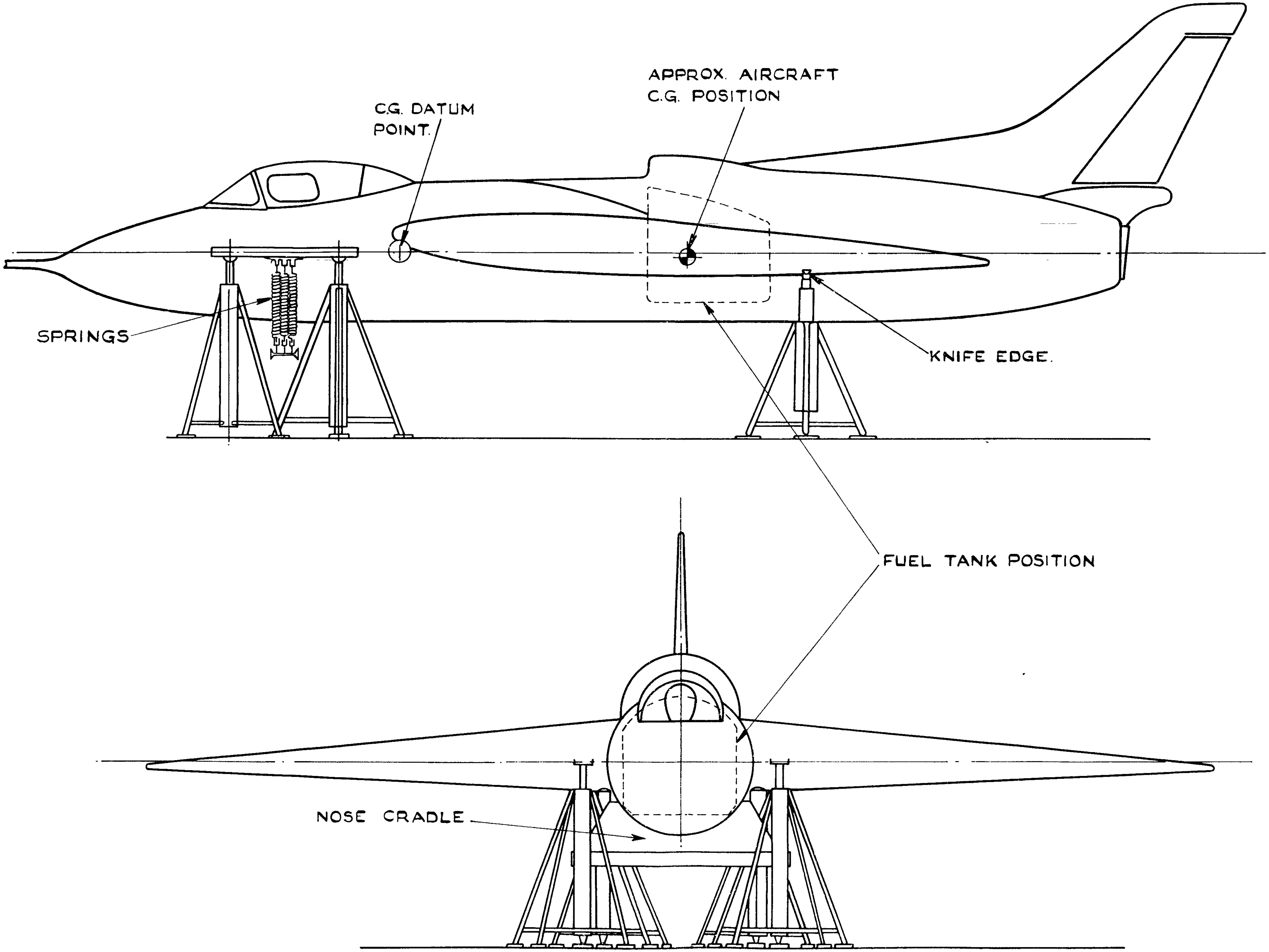
THE HORIZONTAL DISTANCE h OF THE CENTRE OF GRAVITY FROM THE C.G. ORIGIN, O, MAY BE FOUND BY THE USUAL WEIGHING PROCEEDURES TO DETERMINE THE NOSE WHEEL AND MAIN WHEEL REACTIONS, R_N AND R_M.

THE CO-ORDINATES X AND z OF THE C. OF G WITH RESPECT TO THE C.G. ORIGIN MAY BE EXPRESSED IN TERMS OF h AND THE AIRCRAFT ATTITUDE θ :–

$$x = h \sec\theta + z \tan\theta$$

IF MEASUREMENTS OF h ARE MADE FOR A NUMBER OF DIFFERENT ATTITUDES θ, THE SLOPE OF THE PLOT OF $h \sec\theta$ AGAINST $\tan\theta$ IS z AND THE ORDINATE AT $\theta = 0$ IS X.

FIG. 1. DETERMINATION OF THE CENTRE OF GRAVITY POSITION.



SCALE $\frac{1}{36}$

FIG. 2. AIRCRAFT ON THE PITCHING MOMENT OF INERTIA RIG.

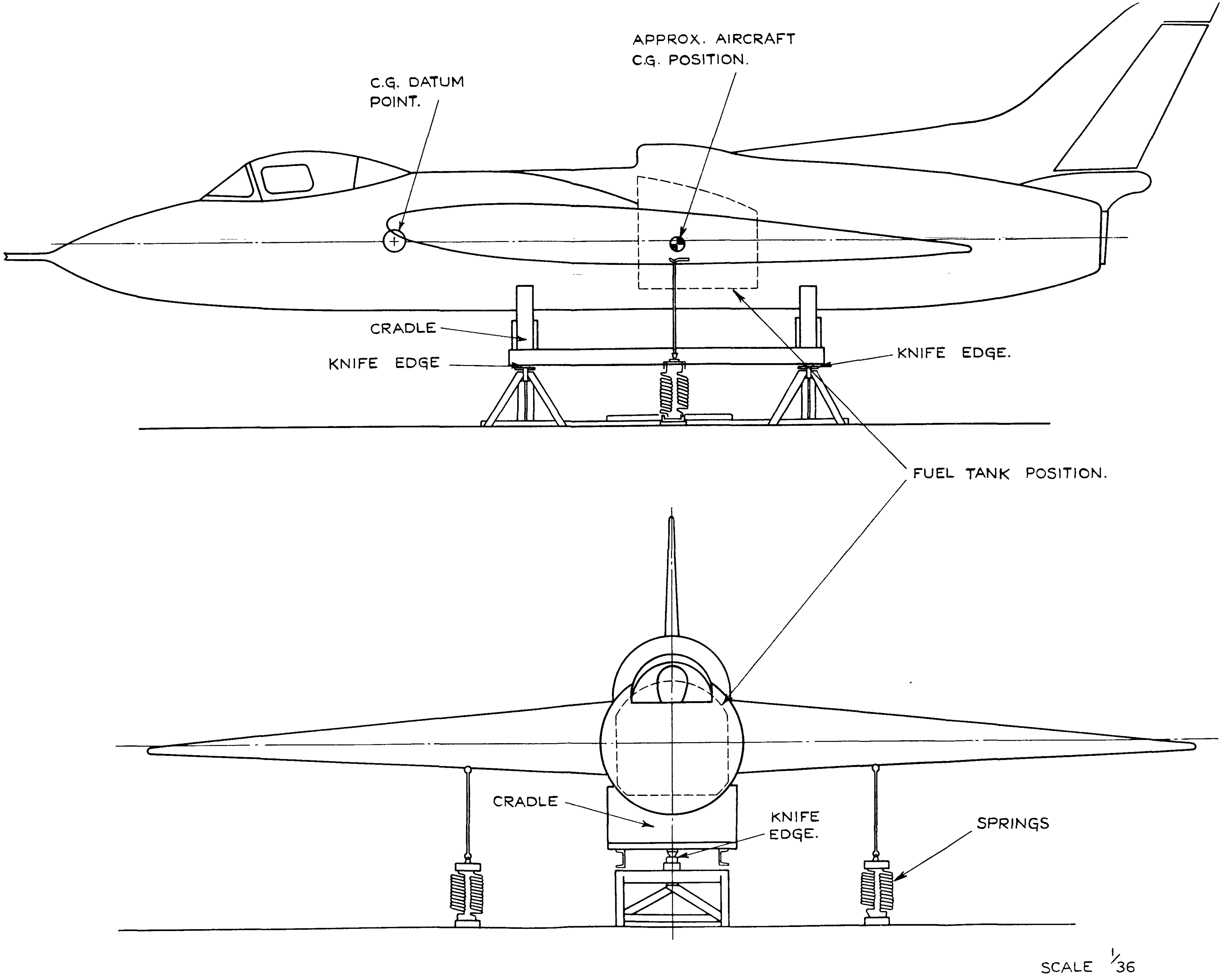


FIG. 3. AIRCRAFT ON THE ROLLING MOMENT OF INERTIA RIG.

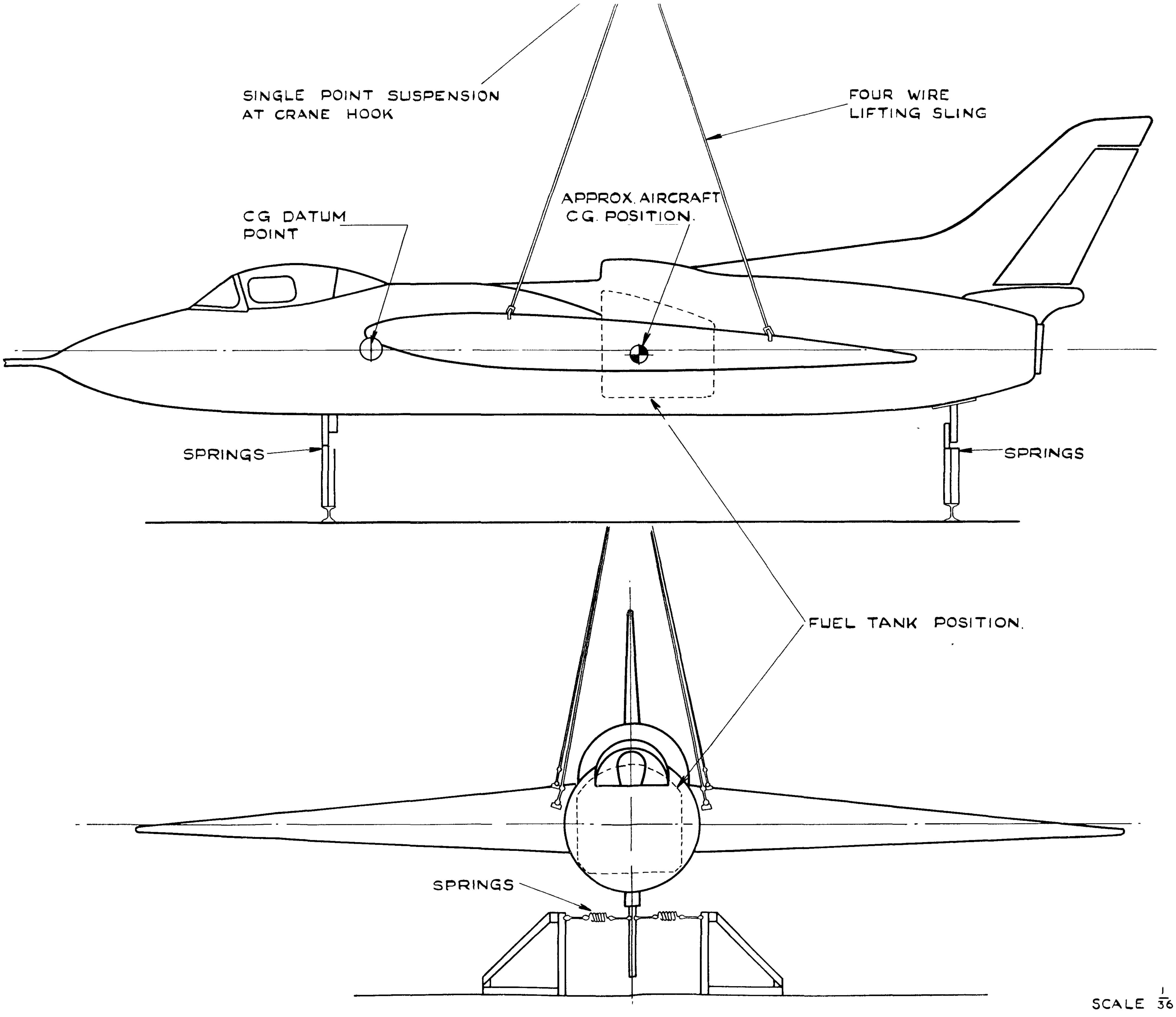

SINGLE POINT SUSPENSION AT CRANE HOOK
FOUR WIRE LIFTING SLING
APPROX. AIRCRAFT C.G. POSITION.
CG DATUM POINT
SPRINGS
SPRINGS
FUEL TANK POSITION.
SPRINGS
SCALE 1/36

FIG.5 VIEW OF AIRCRAFT ON THE PITCHING RIG

FIG.6 VIEW OF AIRCRAFT ON THE ROLLING RIG

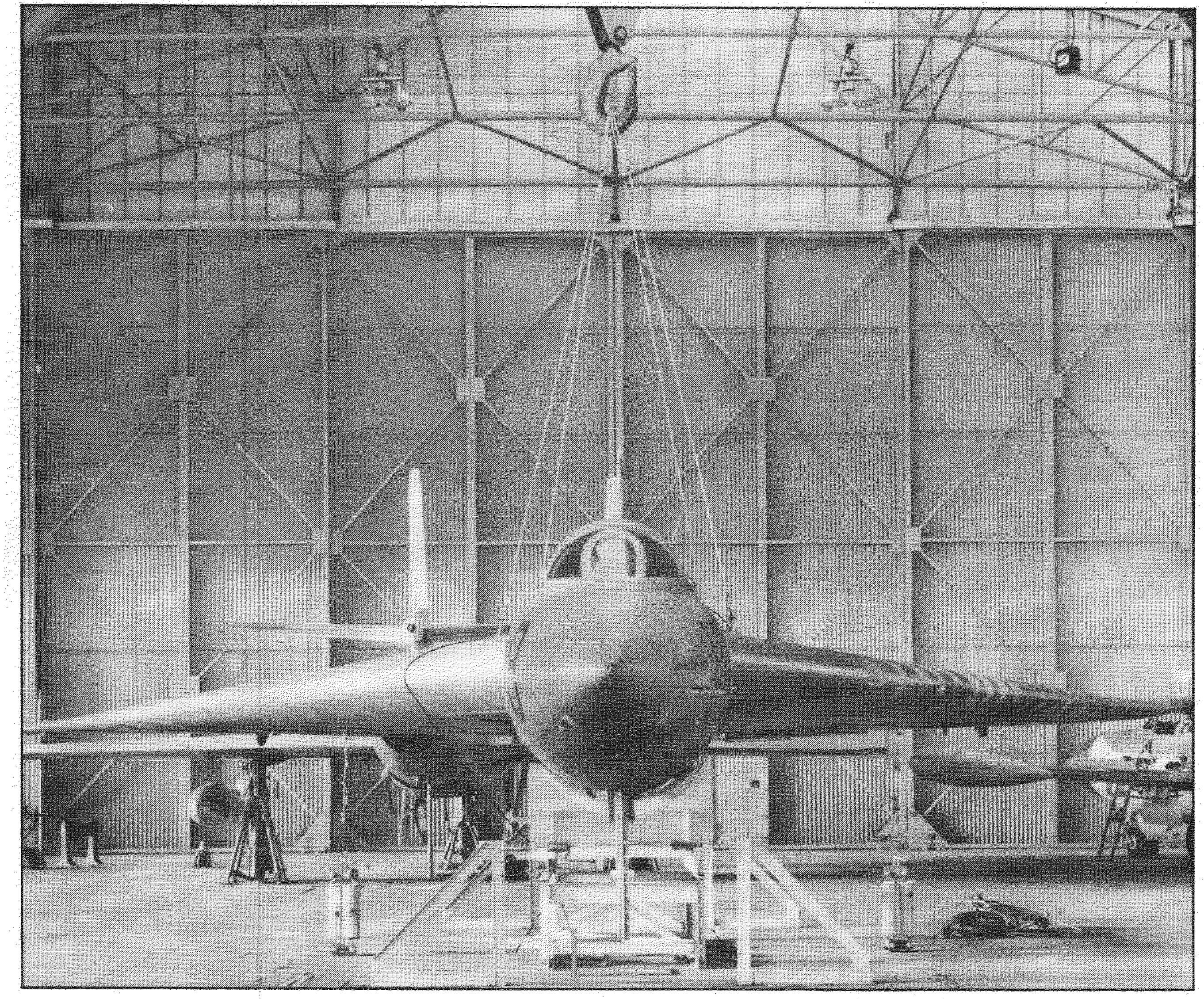
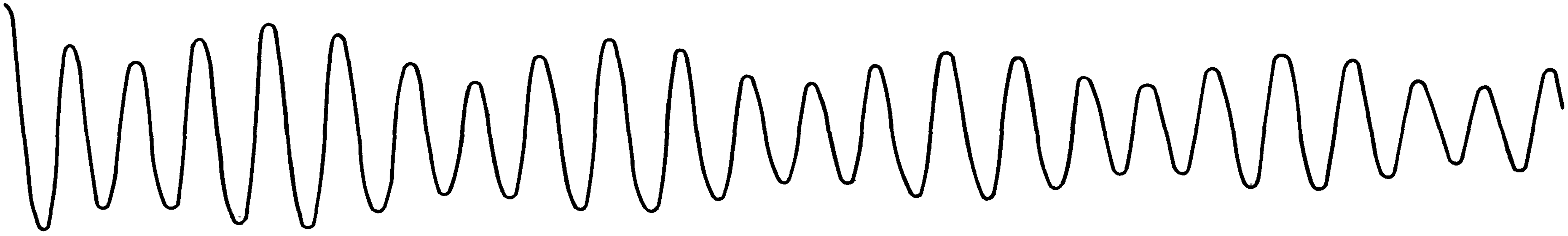
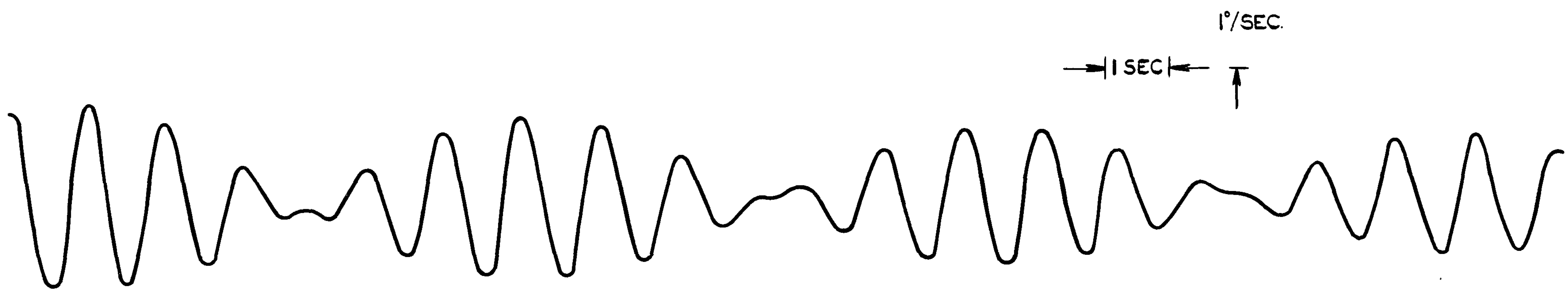


FIG.7a.



(a) FREQUENCY~1 C.P.S. (4 CONSTRAINING SPRINGS PER SIDE.)



(b) FREQUENCY ~ 0·8 C.P.S. (3 CONSTRAINING SPRINGS PER SIDE.)

FIG. 8. TIME HISTORIES OF ROLLING OSCILLATIONS SHOWING THE EFFECT OF FUEL SLOSHING. 157 GALLONS.

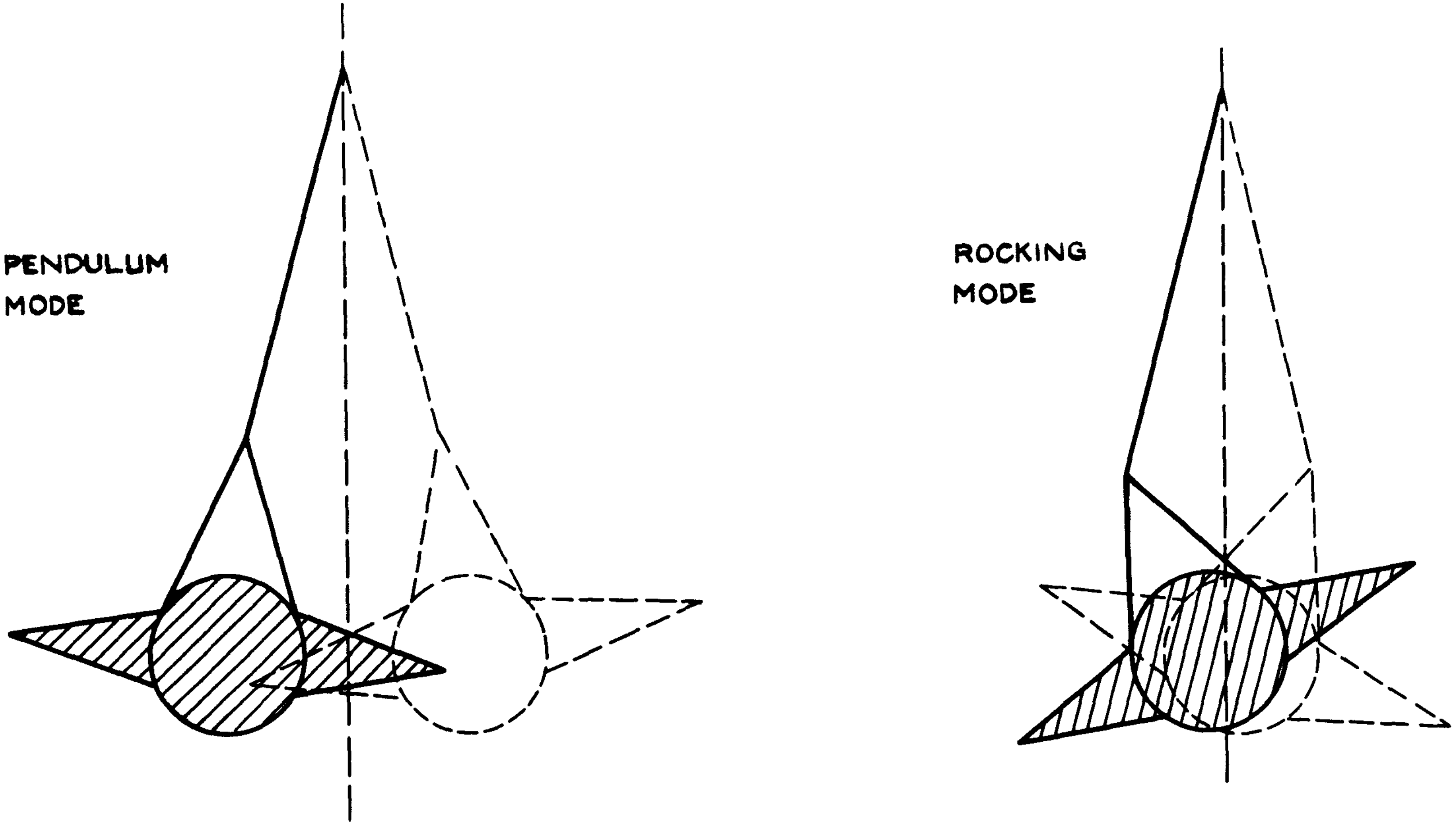


FIG. 9. MODES OF AIRCRAFT ROLLING OSCILLATIONS WHEN SUSPENDED ON THE YAWING RIG.

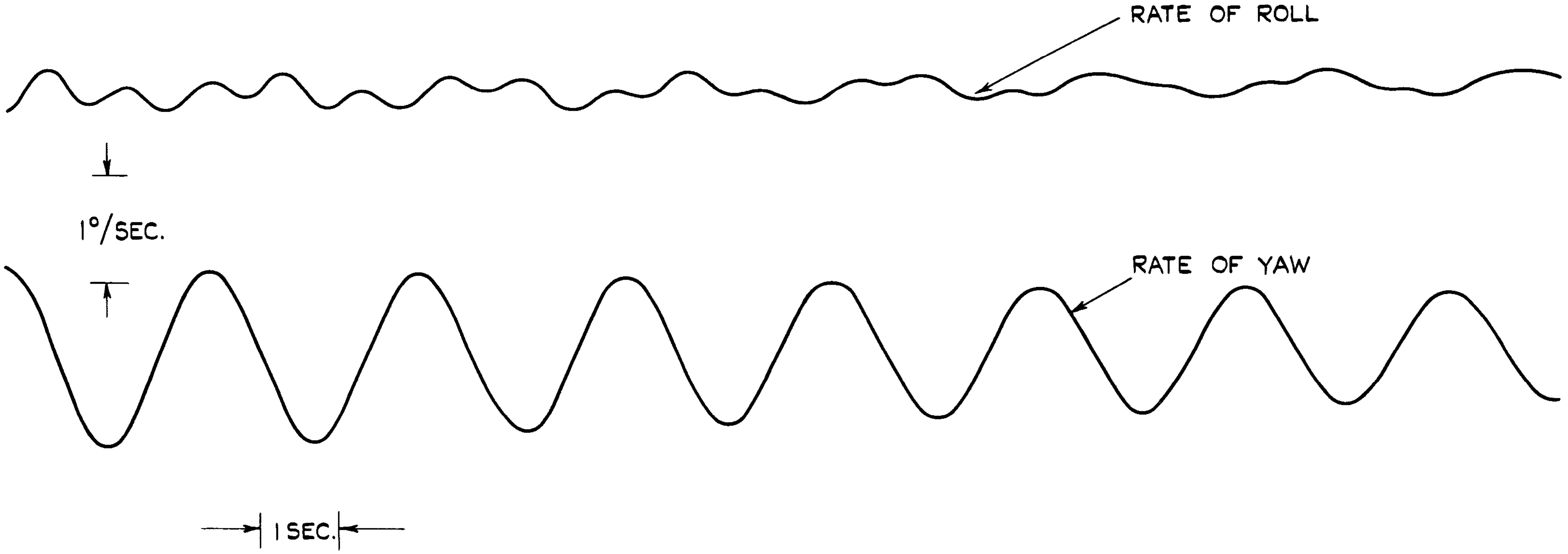


FIG. 10. TIME HISTORY OF ROLLING AND YAWING MOTIONS WHEN MEASURING THE INCLINATION OF THE PRINCIPAL AXES.

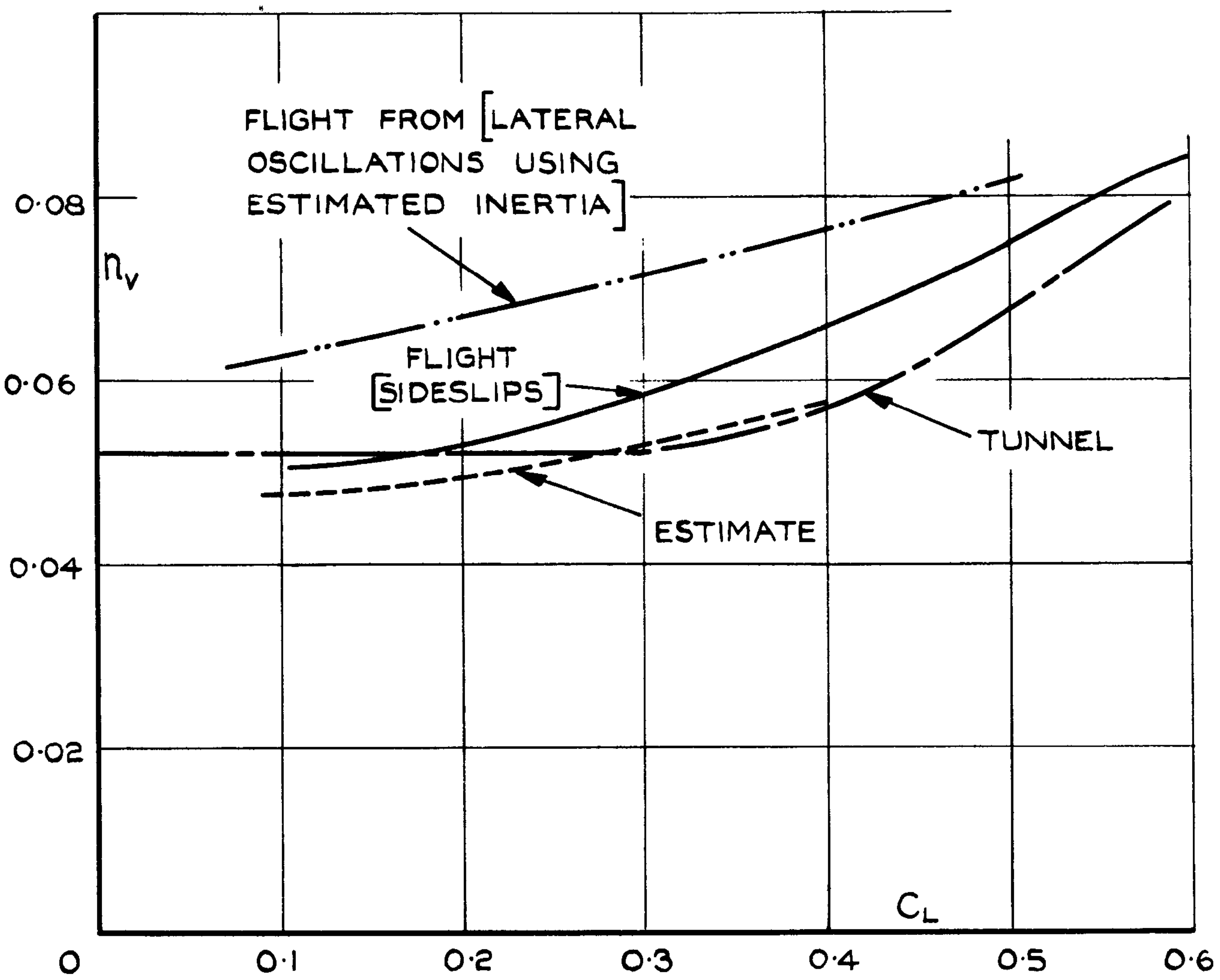


FIG. 11. n_v FROM LATERAL OSCILLATIONS USING ESTIMATED MOMENTS OF INERTIA.

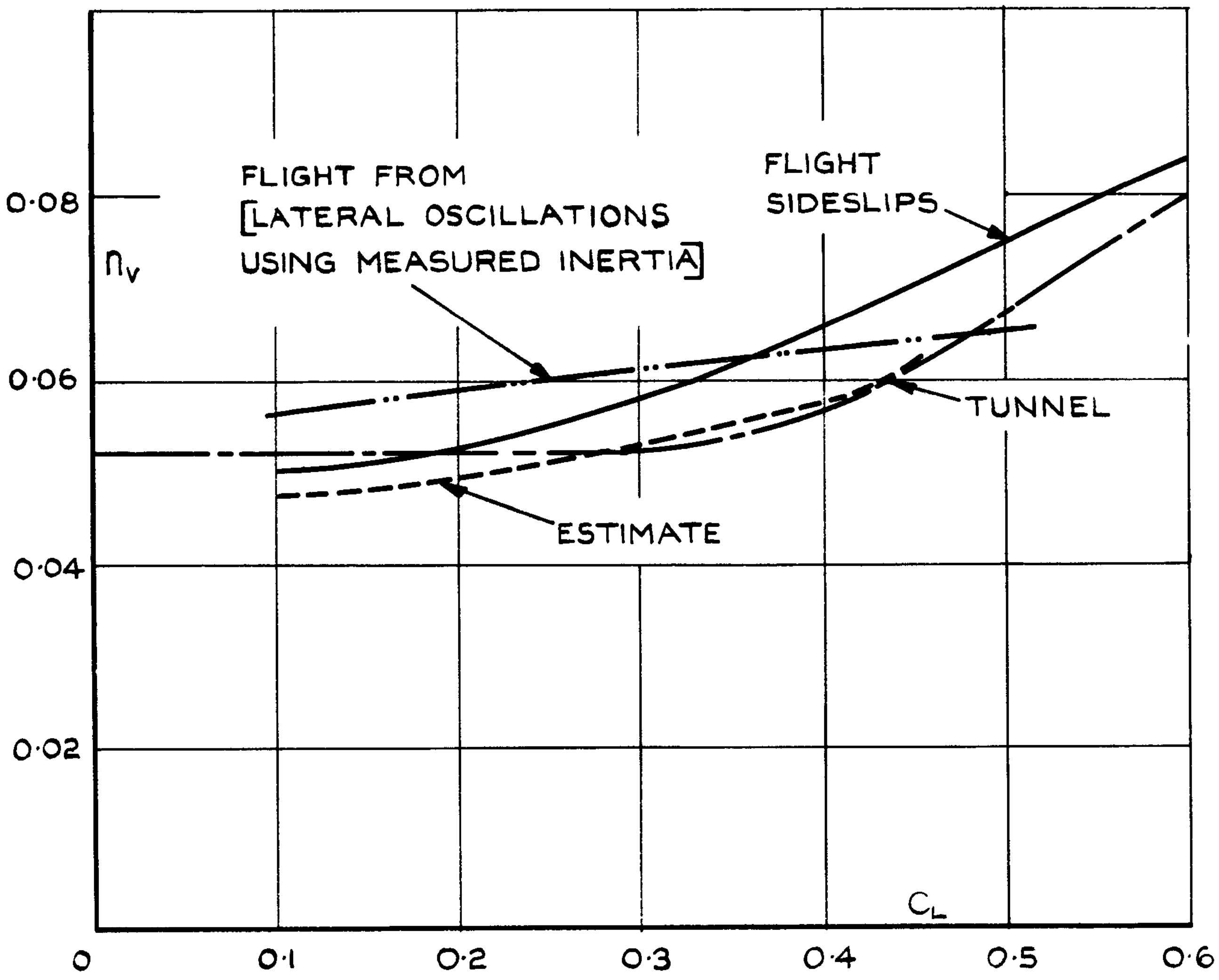


FIG. 12. n_v FROM LATERAL OSCILLATIONS USING MEASURED MOMENTS OF INERTIA.

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MEASUREMENTS OF THE MOMENTS OF INERTIA OF THE AVRO 707B AIRCRAFT. Perry, D.H. August, 1961.

Measurements have been made of the pitching, rolling and yawing moments of inertia of the Avro 707B aircraft using the spring constrained oscillation method. Satisfactory results were obtained for the empty aircraft, but some interference effects were observed when the fuel tank was partially filled. These were traced to the presence of a resonant frequency of fuel motion in the tank which lay close to the aircraft oscillation frequency. Attempts to measure the orientation of the principal inertia axes were unsuccesful because the measuring frequency lay close to the frequency of one of the natural rolling modes of the equipment.

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(Over)

The inertia measurements are compared with estimates which had been made from the weight schedule of the aircraft. Dynamic flight measurements of the lateral derivative n_v, which had been analysed using these estimated inertias, showed a discrepancy with steady state flight measurements. This discrepancy was largely resolved when the measured moments of inertia were used in the analysis.

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