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# A Theoretical Investigation of the Effect of Change in Axial Velocity on the Potential Flow through a Cascade of Aerofoils

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# A Theoretical Investigation of the Effect of Change in Axial Velocity on the Potential Flow through a Cascade of Aerofoils

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## SUMMARY

An analysis is given in which the method of singularities, used for the determination of cascade performance, is modified to account for a change in axial velocity across the cascade. Changes in axial velocity across blade rows occur frequently in turbo-machines and solid wall wind tunnel cascade tests.

Sources or sinks are distributed in limited regions of the potential flow field so that the axial velocity distribution through the cascade may be controlled.

Calculations show that the blade lift and the air outlet angle decrease when the axial velocity increases through the cascade.

## List of Contents



Notation/

Replaces A.R.C.23,815.

Notation

| | |
|---|---|
| $c$ | blade chord length |
| $C_p$ | pressure coefficient |
| $d$ | cascade axial thickness |
| $K$ | $\tan \varepsilon = V_{my}/V_{mx}$ |
| $m$ | source strength per unit length in tangential direction |
| $n$ | number of strips per unit length in axial direction |
| $p_1$ | inlet static pressure |
| $p_2$ | outlet static pressure |
| $Q'$ | total strip singularity strength |
| $q(x)$ | singularity strength distribution along chord line |
| $q'_d$ | flux in the downstream direction |
| $q'_u$ | flux in the upstream direction |
| $s$ | blade spacing |
| $s/c$ | space chord ratio |
| $u$ | perturbation velocity in x-direction due to singularities along chord |
| $u'$ | perturbation velocity in x-direction due to strip singularities |
| $u_n$ | axial velocity factor $= mnd/2V_{mx}$ |
| $v$ | perturbation velocity in y-direction due to singularities along chord |
| $v'$ | perturbation velocity in y-direction due to strip singularities |
| $v'_a$ | perturbation in axial direction due to strip singularities |
| $V_a$ | axial velocity component |
| $V_L$ | blade surface local velocity |
| $V_m$ | cascade vector mean velocity |
| $V_{mx}$ | component of $V_m$ in x-direction |
| $V_{my}$ | component of $V_m$ in y-direction |
| $V_R$ | axial velocity ratio $V_{a_2}/V_{a_1}$ |
| $V_t$ | tangential velocity component |
| $V_1$ | cascade inlet velocity |
| $V_2$ | cascade outlet velocity |
| $x,y$ | co-ordinates of rectangular axes |
| $x'$ | ordinate in axial direction |
| $y_s$ | slope line ordinate |
| $y_t$ | thickness ordinate |
| $\alpha_1$ | inlet air angle |
| $\alpha_2$ | outlet air angle |
| $\alpha_m$ | vector mean flow angle |

$\gamma(x)$/

$\gamma(x)$ vorticity distribution on x-axis

$\lambda$ cascade blade stagger angle (positive for compressor cascade)

$\varepsilon$ angle between vector mean flow angle and x-axis

Subscripts

$_1$ inlet condition

$_2$ outlet condition

$'$ refers to source strength and velocity perturbation due to strip singularities

## 1. Introduction

In general the design of a stage in an axial flow compressor results in a change of axial velocity through the stage at any given radius. The change may occur along all the blade length, due to a change in annulus area, or may be due to a three-dimensional redistribution of the flow with radius. Thus it would be useful for the designer to know the way in which cascade performance varies with the axial velocity ratio across the blades.

In the testing of two-dimensional cascades in wind tunnels the axial velocity increases on the centreline when separation occurs in the corner between the blade and the tunnel wall. British cascade data has been obtained using a solid side wall technique so that the value of $V_{a_2}/V_{a_1}$ has in general been greater than unity. N.A.C.A. tests have been made in a porous wall tunnel, and by variation of the amount of air sucked away it has been possible to derive a set of results at an axial velocity ratio of unity. Rolls-Royce, using the same porous wall method, have tabulated results for varying axial velocity ratios.

Two basic methods of calculating cascade performance have been used in Ref. 1, the first consisting of a series of conformal transformations and the second a method of singularities. The latter has been modified to include the effect of changing the axial velocity across the cascade. Liverpool University's "Deuce" digital computer has been used to perform the necessary calculations.

## 2. Analysis

### 2.1 Method of singularities

The method of singularities applied to cascade performance and developed by Schlichting[2] has been used by Schneider[3] in the calculation of the performance of cascades of N.A.C.A. profiles. Further development of the method is described in Ref. 1, and a modification of the analysis to include changes in axial velocity across the cascade is given here.

Basically the method consists of replacing the blades of a given cascade by a singularity distribution of sources, sinks and vortices along each blade chord. A uniform flow, parallel to the mean cascade air direction, is superimposed on these singularities and the magnitudes of the latter are chosen such that the resulting flow has one streamline identical to each of the replaced profiles. The flow direction of this streamline is matched with the profile gradients at a number of chordwise positions. The uncambered base profile, and the camber line are considered separately. With $y_u$, $y_\ell$ the profile upper and lower ordinates respectively, the base profile ordinate $y_t$ is given by,

$$y_t = \tfrac{1}{2}(y_u - y_\ell) \qquad \dots (1)$$

and/

and the camber line ordinate $y_s$ is given by,

$$y_s = \tfrac{1}{2}(y_u+y_\ell) \quad \ldots (2)$$

Consider the continuity equation for an element of base profile as shown in Fig.(1a),

$$(V_{m_x}+u)y_t + \tfrac{1}{2}q(x)dx = \left(V_{m_x}+u+\frac{\partial u}{\partial x}dx\right)\left(y_t+\frac{dy_t}{dx}dx\right)$$

(the variation of u with y is neglected). If $\partial u/\partial x$ is considered small, then

$$\frac{\tfrac{1}{2}q(x)}{V_{m_x}+u} = \frac{dy_t}{dx} \quad \ldots (3)$$

The camber line is a streamline and the flow direction is, Fig.(1b)

$$\frac{dy_s}{dx} = \frac{V_{m_y}+v}{V_{m_x}+u} \quad \ldots (4)$$

The source distribution $q(x)$, and vortex distribution $\gamma(x)$ are related to the induced velocities u and v by,

$$u-iv = \pm\frac{\gamma(x)-iq(x)}{2} + \frac{c}{s}\frac{e^{i\lambda}}{2}\int'_{\frac{\bar{x}}{c}=0}\left\{\left[q\left(\frac{\bar{x}}{c}\right)+i\gamma\left(\frac{\bar{x}}{c}\right)\right]\coth \pi e^{i\lambda}\frac{(x-\bar{x})}{s}\right\}d\left(\frac{\bar{x}}{c}\right) \quad \ldots (5)$$

(see Refs. 1, 2 and 3), and $q(x)$ and $\gamma(x)$ are defined in terms of a Fourier series. For each aerofoil matching point chosen a pair of simultaneous equations (3) and (4) are produced. Thus if n matching points are taken a matrix of 2n simultaneous equations is formed and n Fourier coefficients of each series for $q(x)$ and $\gamma(x)$ may be derived. The induced velocity close to the chord line on either side is given by

$$V_x = V_{m_x}+u\pm\tfrac{1}{2}\gamma(x) \quad \ldots (6)$$

and the velocity on the blade surface by a transformation (Ref. 1).

### 2.2 Change in axial velocity

In the analysis of §2.1 the net fluid produced by the sources and sinks is zero so that the axial velocity does not change across the cascade. To effect a change in axial velocity a further system of sources and sinks is superimposed on the existing flow conditions. These new singularities are strip sources and sinks, along the cascade tangential direction $y'$ and stretching from $y' = -\infty$ to $y' = +\infty$. The flow produced is along the cascade axial direction $x'$ as shown in Fig. 2.

For a uniform distribution of singularities, with

m = source strength per unit length in cascade direction

n = number of strips per unit length in axial direction,

where m and n are constant,

$x'$ = distance from origin in cascade axial direction,

d = cascade axial thickness,

the/

the total source strength across the cascade is,

$q' = mnd$ per unit length in the cascade tangential direction $y'$.

The velocity in the upstream direction

$$v'_{a_1} = -\frac{mnd}{2} \qquad \ldots (7)$$

and in the downstream direction

$$v'_{a_2} = +\frac{mnd}{2} . \qquad \ldots (8)$$

At any point $x'$ within the cascade, the flux in the downstream direction is

$$q'_d(x') = \frac{mnx'}{2}$$

and the flux in the upstream direction is

$$q'_u(x') = \frac{mn(d-x')}{2} .$$

The net flux at $x'$ is thus,

$$d\ q'_d(x')-q'_u(x') = \frac{mnx'}{2} - \frac{mn(d-x')}{2} \qquad \ldots (9)$$

and the velocity in the axial direction is

$$v'_a(x') = \frac{mnd}{2}\left(2\frac{x'}{d} - 1\right) . \qquad \ldots (10)$$

From Fig. 2 $x'/d = x/c$, so that the components of $v'_a(x')$ in the $x$ and $y$ directions, $u'$ and $v'$ are given by

$$u' = \frac{mnd}{2}\left(2\frac{x}{c} - 1\right) \cos\lambda \qquad \ldots (11)$$

$$v' = -\frac{mnd}{2}\left(2\frac{x}{c} - 1\right) \sin\lambda . \qquad \ldots (12)$$

2.3 Flow conditions

The two basic equations used in the method of singularities (§2.1) are applied to the systems shown in Figs. (3a) and (3b), which include the strip sources and the velocities induced by them. For the base profile (Fig. (4a) therefore

$$(V_{m_x}+u+u')y_t+\tfrac{1}{2}q(x)dx+q'(x)y_t dx = \left(V_{m_x}+u+u'+\frac{\partial u}{\partial x}dx+\frac{\partial u'}{\partial x}dx\right)\left(y_t+\frac{dy_t}{dx}dx\right) .$$

If $\partial u/\partial x$ is considered small then

$\dfrac{\frac{1}{2}q(x)}{V_{m_x}}$ /

$$\frac{\frac{1}{2}q(x)}{V_{m_x}} - \frac{u}{V_{m_x}}\frac{dy_t}{dx} = \left(1 + \frac{u'}{V_{m_x}}\right)\frac{dy_t}{dx} + \frac{y_t}{V_{m_x}}\frac{du'}{dx} - \frac{q'(x)}{V_{m_x}} . \qquad \dots (13)$$

For the camber line

$$\frac{V_{m_y}+v+v'}{V_{m_x}+u+u'} = \frac{dy_s}{dx}$$

and

$$\frac{V_{m_y}}{V_{m_x}} + \frac{v}{V_{m_x}} - \frac{u}{V_{m_x}}\frac{dy_s}{dx} = \left(1 + \frac{u'}{V_{m_x}}\right)\frac{dy_s}{dx} - \frac{v'}{V_{m_x}} . \qquad \dots (14)$$

Now $u' = \frac{mnd}{2}\left(2\frac{x}{c} - 1\right)\cos\lambda$ , $v' = -\frac{mnd}{2}\left(2\frac{x}{c} - 1\right)\sin\lambda$

$$\frac{\partial u'}{\partial x} = \frac{mnd}{c}\cos\lambda$$

$$q'(x) = mn .$$

Writing $u_n = \frac{mnd}{2V_{m_x}}$ and substituting for $u'$, $\frac{\partial u'}{\partial x'}$, and $q'(x)$ in (13) and (14)

$$\frac{\frac{1}{2}q(x)}{V_{m_x}} - \frac{u}{V_{m_x}}\frac{dy_t}{dx} = \left\{1+u_n\cos\lambda\left(2\frac{x}{c} - 1\right)\right\}\frac{dy_t}{dx} + 2u_n\frac{y_t}{c}\left(\cos\lambda - \frac{1}{\cos\lambda}\right) \quad (15)$$

$$\frac{V_{m_y}}{V_{m_x}} + \frac{v}{V_{m_x}} - \frac{u}{V_{m_x}} = \left\{1+u_n\cos\lambda\left(2\frac{x}{c} - 1\right)\right\}\frac{dy_s}{dx} + u_n\sin\lambda\left(2\frac{x}{c} - 1\right) . \quad (16)$$

As for the method of singularities the matrix of simultaneous equations is built up by matching the gradients $dy_t/dx$, $dy_s/dx$ at given values of $x$ with the induced velocities in equations (15) and (16). The solution of the matrix reveals the Fourier coefficients of the series for $q(x)$ and $\gamma(x)$, from which $u$, $v$ can be calculated.

The velocity induced close to the chord line on either side is given by

$$V'_x = V_{m_x}+u+u'\pm\tfrac{1}{2}\gamma(x) \qquad \dots (17)$$

and the local profile velocity at $x/c$ by

$$\frac{V_L}{V_{m_x}} = \frac{V'_x}{V_{m_x}}\frac{1}{\sqrt{1+\left(\frac{dy_s}{dx}\pm\frac{dy_t}{dx}\right)^2}} . \qquad \dots (18)$$

In/

In equations (17) and (18) the positive sign refers to the upper surface the negative sign to the lower surface (see Ref. 1). The pressure coefficient $C_p$ is

$$C_p = \frac{p_L - p_1}{\frac{1}{2}\rho V_1^2} = 1 - \left(\frac{V_L}{V_1}\right)^2 . \qquad \dots (19)$$

The velocity triangles are shown in Fig. 4, from which it can be seen that

$$\tan \alpha_1 = \frac{V_{m_x} \sin \lambda + V_{m_y} \cos \lambda + \Delta V_t}{V_{m_x} \cos \lambda - V_{m_y} \sin \lambda - V_{m_x} u_n} .$$

Writing $V_{m_y}/V_{m_x} = K$, $= \tan(\alpha_m - \lambda)$

$$\tan \alpha_1 = \frac{\sin \lambda + K \cos \lambda + \dfrac{\Delta V_t}{V_{m_x}}}{\cos \lambda - K \sin \lambda - u_n} \qquad \dots (20)$$

and similarly

$$\tan \alpha_2 = \frac{\sin \lambda + K \cos \lambda - \dfrac{\Delta V_t}{V_{m_x}}}{\cos \lambda - K \sin \lambda + u_n} . \qquad \dots (21)$$

The axial velocity at inlet to the cascade $V_{a_1}$ is given by

$$V_{a_1} = V_{m_x}(\cos \lambda - K \sin \lambda - u_n)$$

and the axial velocity at outlet from the cascade $V_{a_2}$,

$$V_{a_2} = V_{m_x}(\cos \lambda - K \sin \lambda + u_n) .$$

Thus the axial velocity ratio across the cascade $V_R$ is

$$V_R = \frac{V_{a_2}}{V_{a_1}} = \frac{\cos \lambda - K \sin \lambda + u_n}{\cos \lambda - K \sin \lambda - u_n} . \qquad \dots (22)$$

The blade forces may also be obtained using the velocity triangles of Fig. 4.

The force on the fluid in the tangential direction $T_F$ is

$$T_F = \rho s V_{a_2} V_{t_2} - \rho s V_{a_1} V_{t_1}$$

and the force on the fluid in the axial direction $A_F$ is

$$A_F = s(p_2 - p_1) + \rho s V_{a_2}^2 - \rho s V_{a_1}^2 .$$

The blade lift in a direction normal to the chord line $L_F$ is

$$L_F = T_F \cos \lambda + A_F \sin \lambda .$$

Now $V_{t_1} = V_{a_1} \tan \alpha_1$, $V_{t_2} = V_{a_2} \tan \alpha_2$, $V_1 = V_{a_1} \cos \alpha_1$ and $V_R = \dfrac{V_{a_2}}{V_{a_1}}$

and/

and

$$C_L = 2\frac{s}{c}\cos^2\alpha_1 \cos\lambda(\tan\alpha_1 - V_R^2 \tan\alpha_2) + \frac{s}{c}\cos^2\alpha_1 \sin\lambda$$

$$\sec^2\alpha_1 \qquad - V_R^2 \sec^2\alpha_2$$

$$\times \{(1+\tan^2\alpha_1) - V_R^2(1+\tan^2\alpha_2) + 2(V_R^2-1)\}$$

where
$$C_L = \frac{L_F}{\frac{1}{2}\rho V_1^2 c} .$$

## 3. Results

A calculation of the potential flow through a cascade of 10C4-30C50 profile blades set at +36° (compressor cascade) stagger with space chord ratio $s/c = 1.0$ and inlet angle $\alpha_1 = 52.83°$ is described in detail in Ref. 1. The effect of changes in axial velocity on the performance of the above cascade is shown in Figs. 5 and 6, with the original pressure distribution and outlet angle deviation for comparison. As the axial velocity increases through the cascade both the lift and the deviation decrease. Table 1 shows the lift coefficients obtained from the pressure distribution and from the calculated turning angle. The term $V_R \tan\alpha_2$ is also shown.

Table 1

| $u_n$ | $V_R$ | $V_R \tan\alpha_2$ | $C_L$ Calculated from Turning Angle | $C_L$ from Pressure Distribution |
|---|---|---|---|---|
| -0.053 | 0.862 | 0.5344 -.054 | 0.763 | 0.735 |
| 0 | 1.0 | 0.5914 +.0615 | 0.717 | 0.715 |
| +0.053 | 1.154 | 0.6529 | 0.686 | 0.691 |

Over this variation in axial velocity the change in outlet angle is 2.2°.

In a further calculation the terms arising from $\partial u'/\partial x$ and $q'(x)$ in equations (13) and (15) were considered small and were neglected. The omission of these terms had negligible effect on either the pressure distribution or the outlet angle.

## 4. Conclusions

A method is given for calculating pressure distribution and flow angles through a cascade where the axial velocity is changing. The range of axial velocity ratio considered includes values which may occur in turbo-machine design, and values ($V_R > 1$) which arise from cascade tests in solid wall tunnels.

Results show a small but significant change in performance of the cascade. In the example quoted, over a range of axial velocity ratio from $V_R = 0.862$ to $V_R = 1.154$, the lift decreases by just over 5% and outlet angle decreases by 2°.

References/

## References


| No. | Author(s) | Title, etc. |
|---|---|---|
| 1 | D. Pollard and J. Wordsworth | A comparison of two methods for predicting the potential flow around arbitrary aerofoils in cascade. A.R.C. C.P.618. June. 1962. |
| 2 | H. Schlichting | Berechnung der reibungslosen inkompressiblen strömung für ein vorgegebenes ebenes schaufelgitter. VDI Forschungshaft 447, 1955. |
| 3 | K. H. Schneider | Potential flow through a cascade of known aerofoils. M.I.T. Report No.32, August, 1955. |


$\alpha_1 = 62.83$ $\tan \alpha_1 = 1.318885$

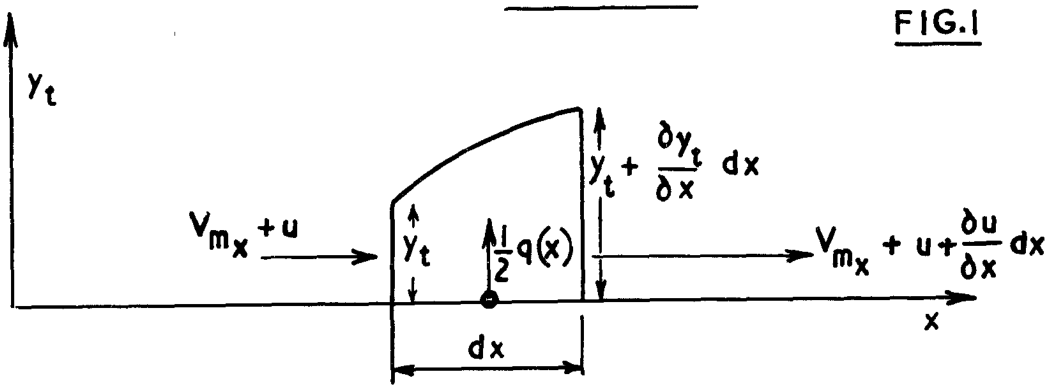
$\alpha_2' = 36 - 15 = 21°$

| $\alpha_2$ | $\delta$ | $\tan \alpha_m$ | $\cos \alpha_m$ | $G_m$ | |
|---|---|---|---|---|---|
| 31.80 | 10.80 | .9453 | .4088 | 1.1945 | −.142 |
| 30.60 | 9.60 | .95515 | .4231 | 1.052 | |
| 29.50 | 8.50 | .9154 | .4846 | .912 | +.160 |

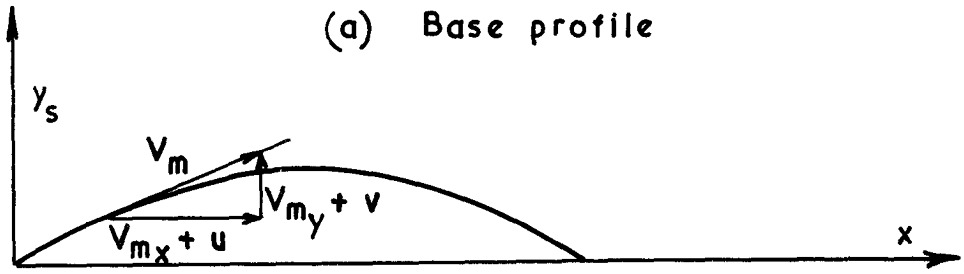
$$\Delta(V_R \tan\alpha_2) \doteq \frac{V_R - 1}{2} \tan\bar{\beta} = \frac{V_R - 1}{4}(\tan\beta_1 + \tan\beta_2)$$

$$G = \frac{4s}{c}\frac{(\tan\alpha_1 - \lambda\tan\alpha_2)}{1+\lambda}\cos\alpha_m$$ for contracting annulus (constant mass flow)

FIG. 1



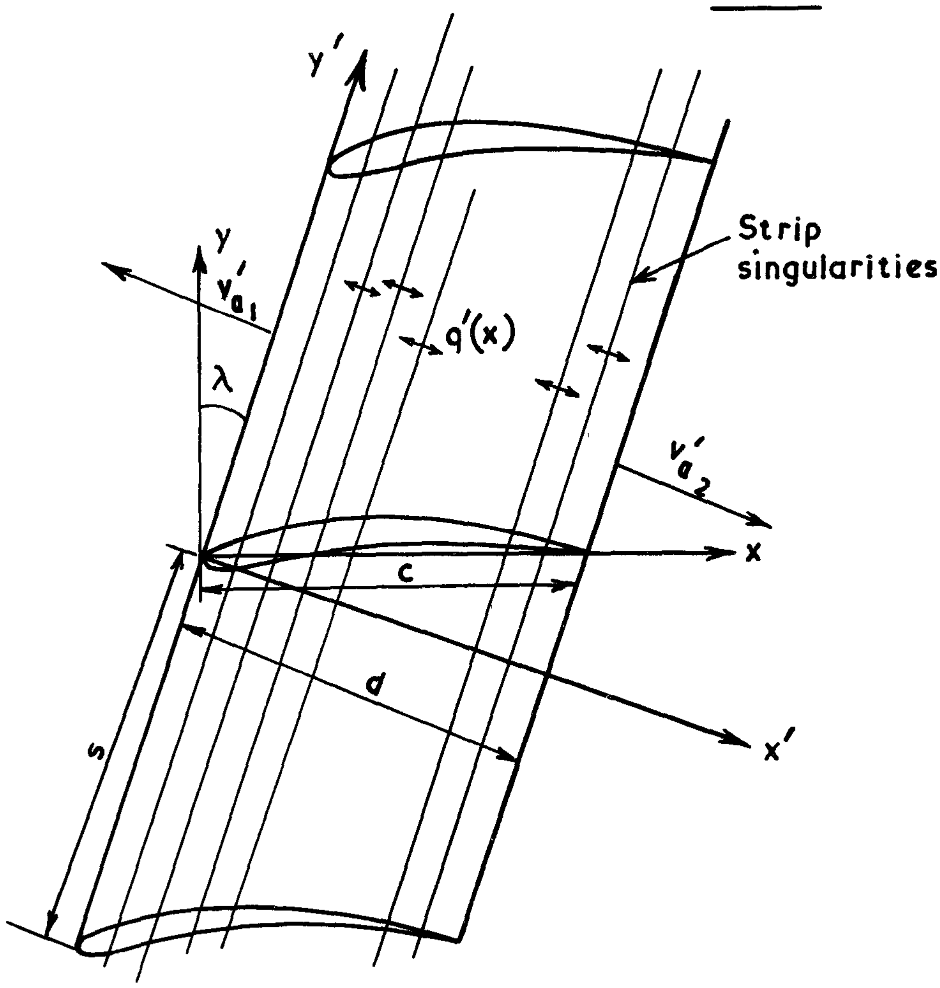

(a) Base profile




(b) Camber line

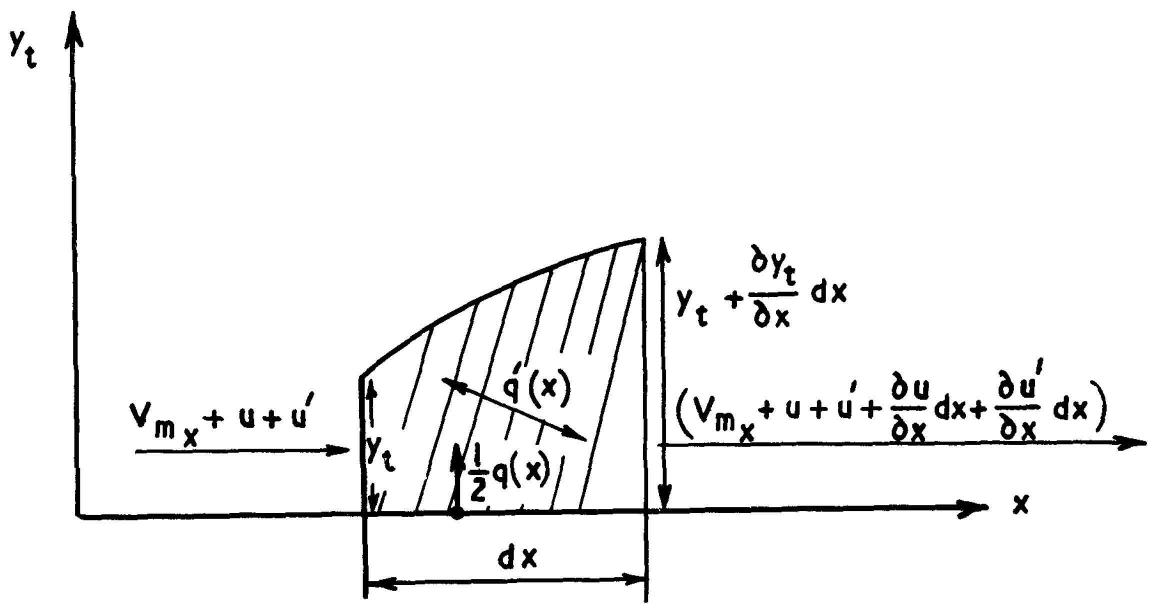
Flow conditions $V_R = 1$

FIG. 2

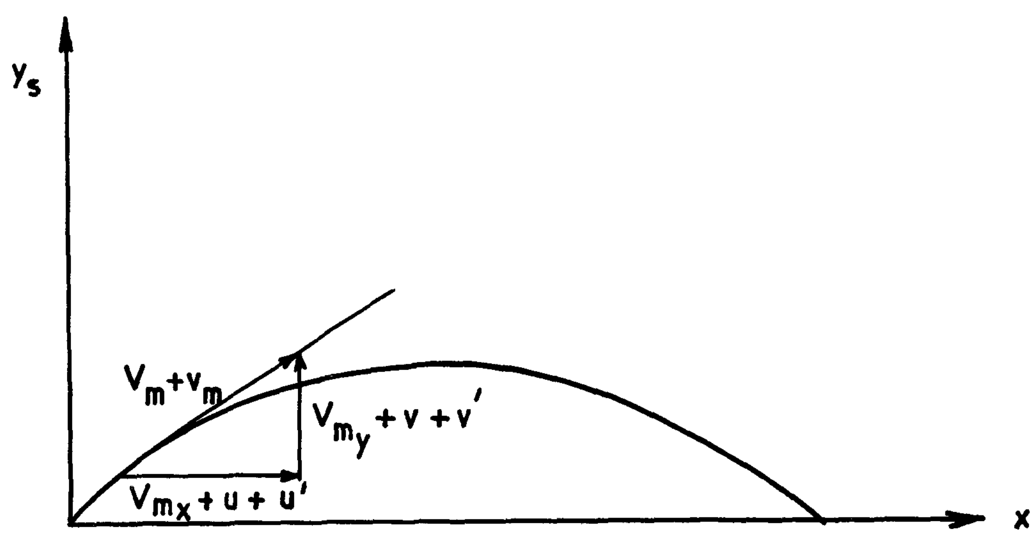



Distribution of strip singularities

FIG. 3



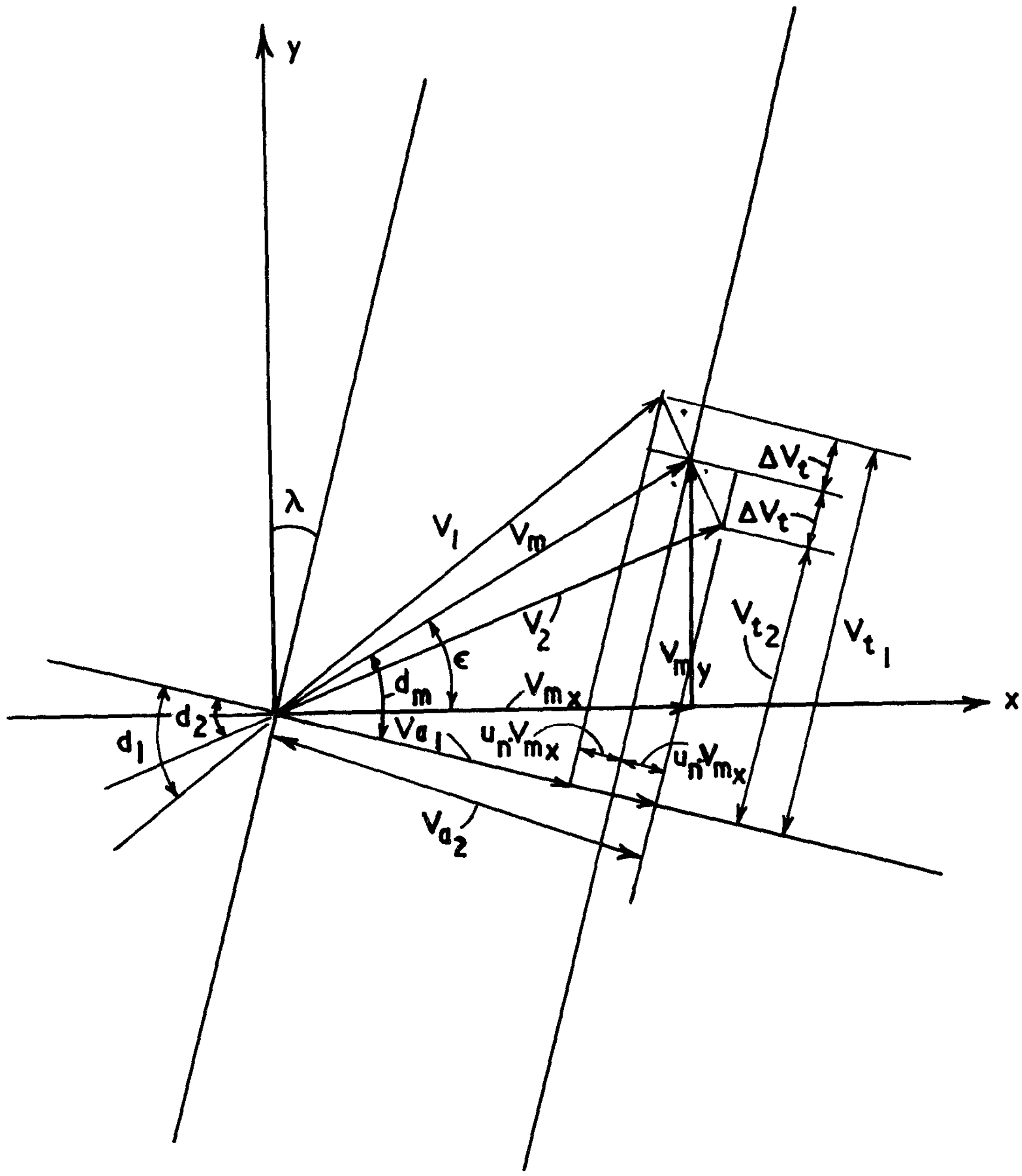

(a) Base profile




(b) Camber line

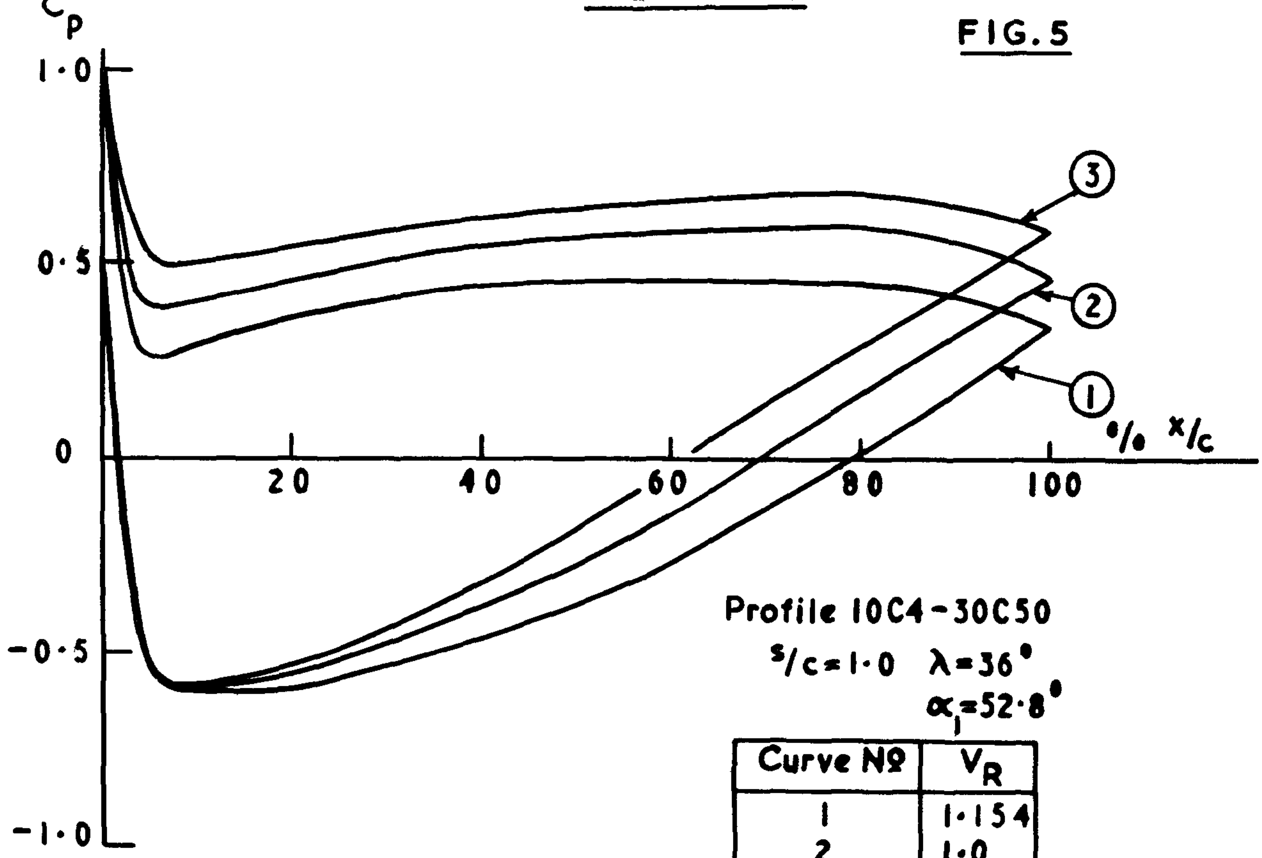
Flow conditions with strip singularities

FIG. 4




Velocity triangles showing change in axial velocity

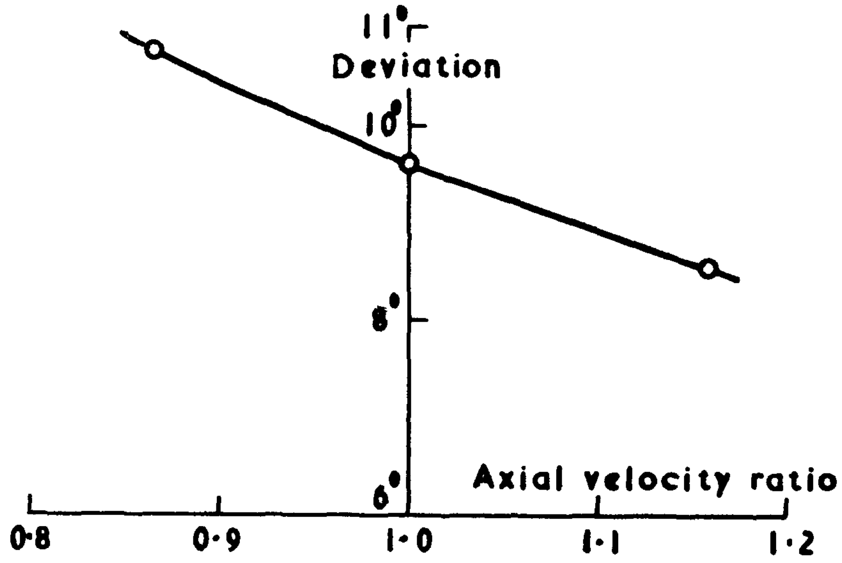
FIG. 5




| Curve Nº | $V_R$ |
|---|---|
| 1 | 1·154 |
| 2 | 1·0 |
| 3 | 0 862 |

Pressure distribution

FIG. 6




Deviation vs. axial velocity ratio.

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