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A Crude Theory of Hovercraft Performance at Zero Tilt

by

S. B. Gates

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A CRUDE THEORY OF HOVERCRAFT PERFORMANCE AT ZERO TILT

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S. B. Gates

SUMMARY

The pioneer British hovercraft are presumably being designed on a model of the flow which is naturally a very crude one; little evidence is available as to the accuracy of the performance estimates that follow from it; and no critical appraisal of the aerodynamics of the problem at its present level seems to have been published. In this situation the analysis given below may serve as a basis of research discussion in three respects:-

(1) to give a rather clearer view of the assumptions and parameters involved in the crude theory,

(2) to encourage a stricter comparison between prediction and ad hoc test results as they become available,

(3) as a point of departure in planning basic experiments that would lead most economically to a better understanding of the matter.

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1 INTRODUCTION

It often happens in the earliest stage of the development of a useful machine that a few are designed and made to work on a combination of crude theory, sketchy experimental data, and guesses that may or may not be inspired. It is only much later that a full understanding of the principles of the matter is worked out. The aeroplane itself round about 1910 was a case in point. The hovercraft is now another and a rather more difficult one, since its basic problem - to determine the flow field when an annular jet issues from the perimeter of a base moving close to the ground - is essentially more complex. It is also rather more difficult for the research worker who starts to explore the hovercraft field because, for various reasons connected with the organisation of design and production, much of the work actually being done is not published. Thus if there is at the moment a generally accepted hovercraft theory by which its performance can be roughly estimated, I know of no British paper that gives it adequate treatment*.

Consequently it may help to work out in some detail what I take to be the crude theory of hovercraft performance in the simplest case, that of horizontal flight over level ground at zero incidence. In the course of this I shall try to

(1) keep an eye on the various simplifying assumptions leading to the crude model of the flow,

(2) choose the various parameters in such forms as are most tractable to basic experimental work.

2 STEPS FROM THE REAL FLOW TO ITS CRUDE MODEL

In the real flow, whether there is forward motion or not, the issuing annular jet entrains air from both sides on its way to the ground. It thus surrounds itself by two layers of turbulent flow, in which vorticity and total head are varying, in addition to boundary layers below the base and on the ground. Even in the hovering condition there is flow within the cushion. This situation being much too intractable we replace it by inviscid flow of a special kind, having the following features:-

(1) There is no flow, but a constant pressure p_c, in the cushion.

(2) Whatever the shape of the perimeter, and whatever the forward speed, the jet flow is axisymmetric in the sense that it is the same in any vertical plane perpendicular to an element of the perimeter of the base. In loose terms the jet flow is the same all round the perimeter, just as it would be in the truly axisymmetric flow out of a hovering circular base. Each element of the jet defined in this way has the same total head, mass flow and momentum flow.

(3) But in forward motion the pressure will in general vary over the whole of the outer boundary of the jet. This being again too difficult, we replace it by a constant pressure p_o averaged over the whole of the outer boundary. It seems that p_o has usually been neglected.

Thus by evading several important issues we have reduced the problem to that of a two-dimensional inviscid jet sustaining a constant pressure difference and ending up horizontally with the ground as a streamline. This can

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*Since this was written Stanton Jones's IAS Paper No.61-45[1], which covers some of the same ground, has reached me.

be solved by conformal mapping[2], but the solution is rather elaborate, involving jet boundaries that are not circular. We therefore abandon the ground streamline condition and look for solutions in which the jet boundaries are circular and the jet flow only 'touches' the ground.

3 TWO-DIMENSIONAL CIRCULAR JET FLOW APPROXIMATIONS

The notation is shown in the sketch of Fig.1. Across the jet of thickness t at exit the total head H is constant and the pressure p, with atmospheric as datum, varies from p_o to p_c, v being the variable velocity. The energy equation across the jet at exit is

$$p + \tfrac{1}{2}\rho v^2 = H, \quad \rho \text{ being constant.} \tag{1}$$

Consider an element $d\tau$ of the jet thickness across which the pressure rise is dp. If R is its radius of curvature at exit we have

$$dp = \frac{\rho v^2 \, d\tau}{R} . \tag{2}$$

We now assume that the element's horizontal momentum is changed from $-\rho v^2 d\tau \cos\theta$ to $\rho v^2 d\tau$ in height h by the constant pressure increment dp, so that

$$h \, dp = \rho v^2 d\tau (1 + \cos\theta). \tag{3}$$

Then from (2),(3)

$$R = \frac{h}{1 + \cos\theta} \tag{4}$$

and so the circle of curvature is constant across the jet at exit and touches the ground.

It follows from (1),(2) that

$$p + \frac{R}{2}\frac{dp}{d\tau} = H . \tag{5}$$

3.1 Solution A

A common approximation, apparently introduced by Chaplin[3], is to replace the differentials in (5) by the finite quantities already defined. In what follows a bar denotes mean value across the jet. We assume that the pressure variation across the jet of thickness t is linear and so

$$p = \bar{p} = \tfrac{1}{2}(p_c + p_o)$$

$$dp = p_c - p_o$$

$$d\tau = t\ .$$

If now we write $x = t/R$, (5) reduces to

$$\frac{p_c + p_o}{2} + \frac{p_c - p_o}{2x} = H\ . \tag{6}$$

Let q be the dynamic pressure of the forward speed V. Then with the substitutions

$$\sigma = \frac{q}{p_c}\ , \qquad p_o = bq\ ,$$

we have

$$\frac{p_o}{p_c} = b\sigma$$

and equation (6) becomes

$$\frac{p_c}{H} = \frac{2x}{(1 + x) - b\sigma(1 - x)} \tag{7}$$

and so

$$\frac{\bar{p}}{H} = \frac{(1 + b\sigma)x}{(1 + x) - b\sigma(1 - x)}\ . \tag{8}$$

From (1) we have

$$\tfrac{1}{2}\rho\overline{v^2} = H - \bar{p}$$

and so from (8)

$$\frac{\rho\overline{v^2}}{p_c} = \frac{1 - b\sigma}{x}\ . \tag{9}$$

To this approximation we must also have $\bar{v} = (\overline{v^2})^{\frac{1}{2}}$, and so

$$\left(\frac{\rho}{p_c}\right)^{\frac{1}{2}} \bar{v} = \left(\frac{1 - b\sigma}{x}\right)^{\frac{1}{2}} . \tag{10}$$

It is useful to introduce the speed u defined by $p_c = \frac{1}{2}\rho u^2$ so that (10) becomes

$$\frac{\bar{v}}{u} = \left(\frac{1 - b\sigma}{2x}\right)^{\frac{1}{2}} . \tag{10a}$$

It is commonly assumed that b = 0, in which case we have the familiar formulae

$$\left.\begin{aligned} \frac{p_c}{H} &= \frac{2x}{1 + x} \\ \frac{\bar{p}}{H} &= \frac{x}{1 + x} \\ \frac{\rho\overline{v^2}}{p_c} &= \frac{1}{x} \\ \text{or} \quad \frac{\bar{v}}{u} &= \frac{1}{(2x)^{\frac{1}{2}}} . \end{aligned}\right\} \tag{11}$$

3.2 Solution B

An alternative approach, introduced by Stanton Jones, is to integrate (5) across the jet, with the boundary conditions

$$\begin{aligned} p &= bq \quad \text{at} \quad \tau = 0 \\ &= p_c \qquad\quad \tau = t . \end{aligned}$$

The result is

$$\frac{p}{H} = 1 - \left(1 - \frac{bq}{H}\right) e^{-\frac{2\tau}{R}} \tag{12}$$

and so

$$\frac{p_c}{H} = 1 - \left(1 - b\sigma \frac{p_c}{H}\right) e^{-2x} .$$

or
$$\frac{p_c}{H} = \frac{1 - e^{-2x}}{1 - b\sigma\, e^{-2x}} . \tag{13}$$

The mean values across the jet may now be obtained from (1) and (12) by integration. The result for the momentum flow $\rho \overline{v^2}$ must be the same as for solution A, equation (9), since it follows straight from the momentum equation (3) in both cases. The other mean values are different:-

$$\frac{\bar{p}}{H} = 1 - \frac{1 - b\sigma}{2x} \; \frac{1 - e^{-2x}}{1 - b\sigma\, e^{-2x}} \tag{14}$$

$$\frac{\bar{v}}{u} = \left(\frac{1 - b\sigma}{1 - e^{-2x}}\right)^{\frac{1}{2}} \frac{1 - e^{-x}}{x} \tag{15}$$

and when b = 0 these become

$$\left.\begin{aligned} \frac{\bar{p}}{H} &= 1 - \frac{1 - e^{-2x}}{2x} \\ \frac{\bar{v}}{u} &= \frac{1 - e^{-x}}{x(1 - e^{-2x})^{\frac{1}{2}}} . \end{aligned}\right\} \tag{16}$$

3.3 Discussion of solutions A,B

It will be realised that these solutions are very loose approximations to the jet flow near the ground, since the jet boundaries are two equal circles touching the ground (Fig.1). (The infinitely thin jet is the only one that really satisfies the flow conditions.) They are however rough shots at determining the conditions at the exit in terms of the thickness there and the basic radius R, solution B being the more exact.

It is clear from their derivation that the two solutions become identical as $x \to 0$ and diverge when x is large. For example it can be shown by expanding the exponentials that $p_c/H \to 0$ in the same manner to $0(x^2)$ as $x \to 0$, but when $x \to \infty, p_c/H \to \frac{2}{1 + b\sigma}$ in solution A and $\to 1$ in solution B. Now an essential physical condition is that p_c must be less than H. It therefore follows from equation (7) that solution A becomes invalid when $x > 1$ and from equation (13) that there is no such limitation in solution B.

It will be seen later that practical values of x are small enough for the difference between the two solutions to be comparatively small, and so, in view of the large errors probably occurring in other parts of the theory, the use of the simpler solution A may be justifiable.

4 RESULTS FOR UNIT LENGTH OF THE JET ANNULUS

Using the mean values already obtained we can now calculate thrust T_1, mass flow m_1, momentum drag D_{m_1}, and power required P_1, <u>for unit length of the jet annulus</u>*. The basic relations are

$$\left.\begin{aligned} m_1 &= \rho \bar{v} t \\ T_1 &= \rho \overline{v^2} t \\ D_{m_1} &= m_1 V = m_1 \left(\frac{2q}{\rho}\right)^{\frac{1}{2}} . \end{aligned}\right\} \quad (17)$$

P_1 is the power required to produce the jet and to overcome the momentum drag. Thus if the fraction aq of the dynamic pressure is recovered in the duct we have

$$\left.\begin{aligned} P_1 &= \bar{v}t(H - aq) + D_{m_1} V \\ &= \frac{m_1}{\rho}(H - aq) + \frac{2m_1 q}{\rho} . \end{aligned}\right\} \quad (18)$$

The algebra necessary to reduce these quantities to non-dimensional forms expressed in terms of the basic parameters x, σ, a, b is tedious but straightforward. The results are as follows:-

*There is a buried assumption in making this step, for in using a two-dimensional jet analysis for the element of the annular jet we assume that the radius of curvature of the perimeter of the base is everywhere much greater than R. Thus we should expect the solution to break down at the bow and stern of a very slender craft, and at the corners of a rectangular one.

Solution A

$$\frac{m_1}{\rho uR} = \left[(1 - b\sigma)\,\frac{x}{2}\right]^{\frac{1}{2}} \equiv j$$

$$\frac{T_1}{p_c R} = 1 - b\sigma\,, \text{ (the same for B)}$$

$$\frac{D_{m_1}}{p_c R} = [2\sigma(1 - b\sigma)x]^{\frac{1}{2}} \equiv g$$

$$\frac{P_1}{p_c Ru} = \frac{1}{2^{3/2}}(1 - b\sigma)^{\frac{1}{2}}\left[(1 + k\sigma)x^{\frac{1}{2}} + (1 - b\sigma)x^{-\frac{1}{2}}\right] \equiv f \qquad (19)$$

where $k = 2(2 - a) + b \div 2(2 - a)$

$$\frac{P_1}{p_c RV} = f\sigma^{-\frac{1}{2}}\,.$$

Solution B

$$\frac{m_1}{\rho uR} = (1 - b\sigma)^{\frac{1}{2}}\,\frac{1 - e^{-x}}{(1 - e^{-2x})^{\frac{1}{2}}} \equiv J$$

$$\frac{D_{m_1}}{p_c R} = 2[\sigma(1 - b\sigma)]^{\frac{1}{2}}\,\frac{1 - e^{-x}}{(1 - e^{-2x})^{\frac{1}{2}}} \equiv G$$

$$\frac{P_1}{p_c Ru} = (1 - b\sigma)^{\frac{1}{2}}\,\frac{1 - e^{-x}}{(1 - e^{-2x})^{\frac{1}{2}}}\left[\frac{1 - b\sigma\, e^{-2x}}{1 - e^{-2x}} + (2 - a)\sigma\right] \equiv F \qquad (20)$$

$$\frac{P_1}{p_c RV} = F\sigma^{-\frac{1}{2}}\,.$$

It may also be useful to tabulate the results for hovering, by putting $\sigma = 0$.

Hovering

	Solution A	Solution B
$\frac{p_c}{H}$	$\frac{2x}{1 + x}$	$1 - e^{-2x}$
$\frac{\overline{v^2}}{u^2}$	$\frac{1}{2x}$	$\frac{1}{2x}$
$\frac{\bar{v}}{u}$	$\frac{1}{(2x)^{\frac{1}{2}}}$	$\frac{1 - e^{-x}}{x(1 - e^{-2x})^{\frac{1}{2}}}$
$\frac{T_1}{p_c R}$	1	1
$\frac{m_1}{\rho u R}$	$\left(\frac{x}{2}\right)^{\frac{1}{2}}$	$\frac{1 - e^{-x}}{(1 - e^{-2x})^{\frac{1}{2}}}$
$\frac{P_1}{p_c u R}$	$\frac{1}{2^{3/2}} (x^{\frac{1}{2}} + x^{-\frac{1}{2}}) \equiv f_o$	$\frac{1 - e^{-x}}{(1 - e^{-2x})^{3/2}} \equiv F_o$

(20a)

5 PERFORMANCE ESTIMATE

We can now make a rough shot at the lift L, drag D, and power required P, of the whole system.

Let the plan area be S, the perimeter s, and write $\ell = S/s$.

Lift. This is derived from the major source p_c, but there is also the vertical component of the thrust and the suction if any on the upper surface of the craft.

Thus

$$L = p_c S + T_1 \sin \theta . s + \delta S q$$

where δ is a coefficient which must depend on some such quantity as h/ℓ and can only be got from experiment.

We now have

$$\frac{L}{p_o S} = 1 + \frac{T_1}{p_o \ell} \sin\theta + \delta\sigma$$

$$= 1 + y \sin\theta(1 - b\sigma) + \delta\sigma \qquad (21)$$

where $y = R/\ell$ is another basic parameter of the system.

We cannot reduce this equation further except to write it

$$\frac{L}{p_o S} = 1 + \varepsilon \qquad (22)$$

where ε is expected to be <<1 because as will be seen later the second and third terms of the above equation will each be of this order in practical cases.

The first approximation to the lift, which is always used in what follows, is therefore

$$\frac{L}{p_o S} = 1 \, . \qquad (23)$$

Drag. This is made up of the momentum drag D_m and the rest, which we may call the profile drag D_o but cannot calculate.

For the momentum drag we have

$$D_m = D_{m_1} s$$

and so

$$\left.\begin{aligned} \frac{D_m}{p_o S} &= yg \, , \quad \text{solution A} \\ &= yG \, , \quad \text{solution B} \end{aligned}\right\} \qquad (24)$$

where g,G are given in equations (19,20).

D_o should be related to some easily measured drag, for instance the profile drag of the craft without jet and far from the ground. If this is C_{D_o} Sq we therefore write

$$D_o = \lambda C_{D_o} S q \tag{25}$$

where λ is an unknown function of x,y.

Thus $$\frac{D_o}{p_o S} = \lambda C_{D_o} \sigma$$

and so
$$\left.\begin{aligned} \frac{D}{p_o S} &= yg + \lambda C_{D_o} \sigma , \quad A \\ &= yG + \lambda C_{D_o} \sigma , \quad B . \end{aligned}\right\} \tag{26}$$

Lift drag ratio

It follows from (22),(26) that this is given by

$$\left.\begin{aligned} \frac{D}{L} &= (yg + \lambda C_{D_o} \sigma)\big/(1 + \varepsilon) , \quad A \\ &= (yG + \lambda C_{D_o} \sigma)\big/(1 + \varepsilon) , \quad B \end{aligned}\right\} \tag{27}$$

where for the first approximation ε = 0.

Power required

This is given by

$$P = P_1 s + D_o V$$

and so
$$\frac{P}{p_o S u} = \frac{P_1}{p_o R u} \frac{Rs}{S} + \frac{D_o}{p_o S} \frac{V}{u} .$$

But weight $W = L = p_o S$ to first approximation

Hence
$$\left.\begin{aligned} \frac{P}{Wu} &= yf + \lambda C_{D_o} \sigma^{3/2} , \quad A \\ &= yF + \lambda C_{D_o} \sigma^{3/2} , \quad B \end{aligned}\right\} \tag{28}$$

and $$\frac{P}{WV} = \sigma^{-\frac{1}{2}} \frac{P}{Wu}$$

$$\left.\begin{aligned} &= yf\sigma^{-\frac{1}{2}} + \lambda C_{D_o}\sigma \quad , \quad A \\ &= yF\sigma^{-\frac{1}{2}} + \lambda C_{D_o}\sigma \quad , \quad B \end{aligned}\right\} \quad . \qquad (29)$$

where f,F are given in equations (19),(20).

P_o the power required for hovering is given by

$$\left.\begin{aligned} \frac{P_o}{Wu} &= yf_o \quad , \quad A \\ &= yF_o \quad , \quad B \end{aligned}\right\} \qquad (30)$$

where f_o,F_o are given in (20a).

Mass flow

This is given by $m = m_1 s$, and so

$$\begin{aligned} \frac{m}{\rho uS} &= yj \quad , \quad A \\ &= yJ \quad , \quad B \end{aligned}$$

when j,J are given in equations (19),(20).

It follows that

$$\left.\begin{aligned} \frac{m}{W} &= \frac{2y}{u} j \quad , \quad A \\ &= \frac{2y}{u} J \quad , \quad B \, . \end{aligned}\right\} \qquad (31)$$

Jet velocity

$\frac{\bar{v}}{u}$ has already been given by equation (10a) for solution A and by equation (15) for solution B.

6 DISCUSSION OF PARAMETERS

In what follows we shall be mainly concerned with drag and power. These have been expressed in terms of a number of parameters in the general functional forms:-

$$\left.\begin{aligned}
\frac{D}{W} &= y\,\frac{g}{G}\,(x, \sigma, b) + C_{D_o}\lambda\sigma \\
\frac{P}{Wu} &= y\,\frac{f}{F}\,(x, \sigma, a, b) + C_{D_o}\lambda\sigma^{3/2} \\
\frac{P}{WV} &= \sigma^{-\frac{1}{2}}\,\frac{P}{Wu}
\end{aligned}\right\} \qquad (32)$$

$$\text{where} \quad p_o = \tfrac{1}{2}\rho u^2 .$$

The functions f,g are for solution A and F,G for solution B. g,G account for momentum drag, and f,F for the power required to produce the jet and overcome the momentum drag.

It must be admitted of course that a,b,λ are not constant but themselves may depend on both x and y. In what follows a,b,λ will be treated as functions of y only, that is, as dependent on height but not on jet thickness.

6.1 Power parameters

The power functions P/WV, P/Wu, besides being in what seems the simplest practical form, can be used to compare hovercraft performance with that of other aircraft operating out of ground effect.

For example, P/WV can be related to its value for an ordinary aircraft, which is simply its cruising D/L. Noting that $\sigma = 1/C_L$ to the hovercraft approximation adopted here, we have for the ordinary aircraft

$$\frac{D}{L} = C_{D_o}\sigma + \frac{2}{\pi A^x}\,\frac{1}{\sigma}$$

C_{D_o} being the profile drag coefficient and A^x the effective aspect ratio, and so

$$\left(\frac{D}{L}\right)_{min} = 2^{3/2}\left(\frac{C_{D_o}}{\pi A^x}\right)^{\frac{1}{2}} .$$

Similarly P/Wu at hovering can be related to the ideal case of a ducted fan in which there is no contraction. If the mass flow through the disc of area S is m at speed w we have

$$\left.\begin{aligned} m &= \rho S w \\ W &= m w \\ P &= \tfrac{1}{2} m w^2 \end{aligned}\right\} \qquad (33)$$

and so $\frac{W}{S} = \rho w^2 = \frac{1}{2}\rho u^2$ from the definition of u

$$\frac{P}{Ww} = \frac{1}{2} .$$

It follows that $\frac{P}{Wu} = 2^{-3/2}$ for the ducted fan. (34)

Now for the hovercraft solution A

$$\frac{P_o}{Wu} = 2^{-3/2}\, y(x^{\frac{1}{2}} + x^{-\frac{1}{2}}) .$$

Hence $y(x^{\frac{1}{2}} + x^{-\frac{1}{2}})$ is a measure of the hovering efficiency compared with a ducted fan. Its minimum value is 2y at x = 1. This comparison was introduced by Chaplin.

6.2 Geometrical parameters

$x = t/R$ and $y = R/\ell$ are two ratios of the 3 lengths ℓ,R,t. $\ell = S/s$ is a linear dimension of the base area. It depends on the planform and can be related to the greatest dimension d by means of the fineness ratio n. To illustrate this it will suffice to consider ellipses and rectangles. For the ellipse of axes d, nd $(n < 1)$

$$S = \frac{\pi}{4} n d^2$$

$$s = 2dE$$

where $$E = \int_0^{\pi/2} [1 - (1 - n^2)\sin^2\theta]^{\frac{1}{2}}\, d\theta$$

and so

$$\left.\begin{aligned} \frac{\ell}{d} &= \frac{\pi n}{8E} \\ \text{and for the circle} \qquad & \\ \frac{\ell}{d} &= \frac{1}{4}. \end{aligned}\right\} \qquad (35)$$

For the rectangle of sides d, nd

$$\left.\begin{aligned} \frac{\ell}{d} &= \frac{n}{2(1 + n)} \\ &= \frac{1}{4} \text{ for the square, as for the circle.} \end{aligned}\right\} \qquad (36)$$

$\frac{\ell}{d}$ is plotted against n for these two families in Fig.2. There is little difference between the two curves.

$R = \frac{h}{1 + \cos \theta}$ is the radius of the circle cutting the base at angle θ and touching the ground. It should be noticed that in the approximation used here, when the contribution of the thrust component to the lift is neglected, the jet exit angle θ enters the problem only through R. For instance, given R, any base up to height 2R will when equipped with a constant jet strength at the appropriate exit angle θ, produce the same $p_c - p_o$, and therefore the same p_c and the same lift if p_o remains the same as the height changes, see Fig.3. This seems valid for hovering, but not for forward speed, when p_o must depend on θ. One of the crudities of the analysis is thus exposed. In the real flow the jet exit angle must have a more powerful influence than it assumes here.

$y = \frac{R}{\ell}$ is very important because the power required for lift and momentum drag is directly proportional to it. We have therefore to decide how small it can be in practice, a question which depends on the tolerable lower limit to the height. This may turn out to be an operational problem. In some applications it may be possible to fix a lower limit to the height which is independent of the size of the craft. In this case y is inversely proportional to ℓ, and leads to the familiar claim that the efficiency of the vehicle will increase with its size, but the argument is not a very convincing one.

In other cases the lower limit may appear as the angular ground clearance to give the angles of pitch and roll necessary either for manoeuvre, for the production of thrust by pitch in steady flight, or for clearance of combinations of surface roughness and waviness. We may define this angle in relation to the maximum dimension d of the base. If the angular clearance is β radians, assumed small, then

$$\beta = \frac{2h}{d}$$

and we have

$$y = \frac{R}{\ell} = \frac{h}{(1 + \cos\theta)\ell}$$

or

$$\frac{y}{\beta} = \frac{1}{2(1 + \cos\theta)} \frac{d}{\ell} \tag{37}$$

where d/ℓ is given in Fig.2.

If β is fixed, y decreases as the jet angle decreases and as the planform approaches a circle or a square. For example, if $\theta = 45^\circ$

$$\frac{y}{\beta} \doteqdot 1.2 \quad \text{for} \quad n = 1 \text{ (circle or square)}$$

$$\doteqdot 1.8 \qquad n = \tfrac{1}{2} .$$

The least tolerable β is anyone's guess at the moment. If it is of the order 0.1, the order of minimum y is between 0.1 for n = 1 and 0.3 for $n = \frac{1}{2}$.

The parameters x,y have arisen naturally in the analysis and have the virtue of producing drag and power functions that are linear in y (eqns. 32). On the other hand they do not separate the basic variables R and t, which can be done by using $y = R/\ell$ and $z = xy = t/\ell$ at the expense of losing linearity in y. This form is useful for studying the performance of a given design for then z is constant. The transformed expressions for solution A are

$$\left.\begin{aligned}
\frac{P}{Wu} &= 2^{-3/2}(1 - b\sigma)^{\frac{1}{2}}\left\{(1 + k\sigma)z^{\frac{1}{2}}y^{\frac{1}{2}} + (1 - b\sigma)z^{-\frac{1}{2}}y^{3/2}\right\} + \lambda C_{D_o}\sigma^{3/2} \\
\frac{D}{W} &= [2\sigma(1 - b\sigma)yz]^{\frac{1}{2}} \\
\frac{mu}{W} &= [2(1 - b\sigma)yz]^{\frac{1}{2}} \\
\frac{\bar{v}}{u} &= \left[(1 - b\sigma)\frac{y}{2z}\right]^{\frac{1}{2}} .
\end{aligned}\right\} \tag{38}$$

6.3 Aerodynamic parameters

The speed parameter $\sigma = q/p_c = \frac{V^2}{u^2}$ is simply $\frac{1}{C_L}$ as usually defined if the first approximation to the lift is used, but in this problem the use of C_L merely confuses the issue.

The parameter a expresses how much work we get from the forward speed dynamic pressure in producing the jet. It depends mainly on intake and duct design, and little, we may hope, on the geometrical parameters x,y.

The parameter b is the mean pressure coefficient (referred to forward speed) over the whole of the outer surface of the jet. It is introduced to allow for the fact that if for example the base is elliptical, then the cushion pressure for flight along the major axis may be very different from that for flight along the minor axis, everything else being supposed equal. Its magnitude and sign are unknown, and can only be obtained indirectly from experiment. We may guess that

(1) it depends strongly on planform, decreases with aspect ratio and might be negligible for slender shapes,

(2) it varies strongly with y (the height parameter), but not with x.

C_{D_o}, the profile drag coefficient of the craft out of ground effect and without jet, depends of course on the cleanness of its superstructure, which may be expected to vary greatly with the job it is designed for. The associated parameter λ is primarily a function of y and may be less than unity if the loss of most of the base friction predominates, but it can only be found from experiment. It seems worth while isolating C_{D_o} in this way, but for some purposes it is better to work with $\lambda C_{D_o} = c$ say.

Summarising, it is good enough as a first step to consider a,c as functions of y only, and b as depending on both y and planform.

7 COMPARISON OF SOLUTIONS A,B

With the above as background, the solutions A and B can be compared by drawing a few curves of the functions f,F and g,G (equations 19,20).

The power functions f,F are shown in Figs.4 and 5 as functions of x for $b = 0$, $a = \frac{1}{2}$ and 1, and σ ranging from 0 to 2. f and F tend to infinity in the same way as $x \to 0$, they have well defined minima at values of $x < 1$, and diverge for large x, f tending to ∞ as $x^{\frac{1}{2}}$ and F having the asymptotic value $(1 - b\sigma)^{\frac{1}{2}}\{1 + (2 - a)\sigma\}$. The minima of f are given by

$$\left.\begin{aligned} x &= \frac{1 - b\sigma}{1 + k\sigma} \\ f &= (1 - b\sigma)\left(\frac{1 + k\sigma}{2}\right)^{\frac{1}{2}} . \end{aligned}\right\} \qquad (39)$$

The momentum drag functions g,G are shown in Fig.6 for b = 0.* They tend to 0 as $x \to 0$ in the same way, but g tends to ∞ as $x^{\frac{1}{2}}$ and G has the asymptotic value $2[\sigma(1 - b\sigma)]^{\frac{1}{2}}$.

In these diagrams, which cover the practical range of x, σ and a, the difference between the two solutions is usually much less than 10%. For this reason the more tractable solution A will be used henceforward. The influence of b in the range ±0.2 on the functions f and g is shown in Figs.7 and 8. It may reach 20%.

8 OPTIMA OF P/WV

The minima of P/WV as a function of σ,x can be found as follows, assuming that a,b,c are functions of y only, and that $(b\sigma)^2$ can be neglected, i.e. $p_0 \ll p_c$.

We then have

$$\left.\begin{aligned} \frac{P}{WV} &= \sigma^{-\frac{1}{2}} fy + c\sigma \\ \text{where}\quad 2^{3/2}\,\sigma^{-\frac{1}{2}} f &= \left(\sigma^{-\frac{1}{2}} + k\sigma^{\frac{1}{2}} - \frac{bk}{2}\,\sigma^{3/2}\right) x^{\frac{1}{2}} + \left(\sigma^{-\frac{1}{2}} - \frac{3b}{2}\,\sigma^{\frac{1}{2}}\right) x^{-\frac{1}{2}} . \end{aligned}\right\} \qquad (40)$$

The stationary conditions

$$\frac{\partial(P/WV)}{\partial\sigma} = \frac{\partial(P/WV)}{\partial x} = 0$$

reduce respectively to

$$(1 - k\sigma)x + 1 + \frac{3}{2}\,b\sigma(1 - k\sigma x) = 2^{5/2}\,\sigma^{3/2}\,x^{\frac{1}{2}}\,\frac{c}{y} \qquad (41)$$

$$(1 + k\sigma)x - 1 + \frac{b\sigma}{2}\,(3 - k\sigma x) = 0. \qquad (42)$$

The minima of $\frac{P}{WV}$, and the values of σ and x at which they occur, are given by (40), (41),(42), as functions of a,b,c and y.

<u>When b = 0</u> the equations take the comparatively simple form:-

---

*The curves are drawn up to x = 2 for the sake of comparison, but the A curves cease to be significant at x = 1.

$$x^4 - \left(\frac{2}{k}\right)^3 \left(\frac{c}{y}\right)^2 (1 - x)^3 = 0 \qquad (43)$$

$$(1 + k\sigma)x = 1 \qquad (44)$$

$$\frac{1}{y}\ \frac{P}{WV} = (2\sigma x)^{-\frac{1}{2}} + \sigma\frac{c}{y}\ . \qquad (45)$$

It should be noted from (43) to (45) that as $\frac{c}{y} \to 0$,

$$\left.\begin{aligned}
&x \to 0 \quad \text{like} \quad \left(\frac{c}{y}\right)^{\frac{1}{2}} \\
&\sigma \to \infty \quad \text{like} \quad \left(\frac{c}{y}\right)^{-\frac{1}{2}} \\
&k\sigma x \to 1 \\
\text{and so} \quad &\sigma\frac{c}{y} \to 0 \quad \text{like} \quad \left(\frac{c}{y}\right)^{\frac{1}{2}} \\
\text{and} \quad &\frac{1}{y}\ \frac{P}{WV} \to \left(\frac{k}{2}\right)^{\frac{1}{2}} .
\end{aligned}\right\} \qquad (46)$$

The solution is plotted against c/y for a = 0 and a = 1 in Fig.9.

The strength of the minima in x and σ are shown respectively in Figs.10 and 11, by curves of

(a) $\frac{1}{y}\ \frac{P}{WV}$ against x under the condition $\frac{\partial}{\partial\sigma}\left(\frac{P}{WV}\right) = 0$.

(b) $\frac{1}{y}\ \frac{P}{WV}$ against σ under the condition $\frac{\partial}{\partial x}\left(\frac{P}{WV}\right) = 0$.

The troughs are shallow between c/y = 0.25 and 1.0, but the x strength becomes large when c/y is small and the σ strength large when c/y is large.

It is useful also to know the values of the jet velocity $\bar{v}$ and the mass flow m at the power minima.

For b = 0 these are given by

$$\frac{\bar{v}}{u} = (2x)^{-\frac{1}{2}}$$

and

$$\frac{1}{y}\ \frac{mu}{W} = (2x)^{\frac{1}{2}}$$

and are plotted in Figs.12 and 13.

Finally $\frac{1}{y}\ \frac{P}{Wu} = \sigma^{\frac{1}{2}}\frac{1}{y}\ \frac{P}{WV}$ is shown in Fig.14.

8.1 The effect of b

A rough approximation to σ,x from the general equations (41) and (42) can be obtained by treating the terms in b as small quantities of the first order which produce increments $\delta\sigma$, δx in the solution for b = 0.

We then have from (41) and (42)

$$\left.\begin{aligned} \delta\{(1 - k\sigma)x\} - 2^{5/2}\frac{c}{y}\,\delta(\sigma^{3/2}\,x^{1/2}) + \frac{3}{2}\,b\sigma(1 - k\sigma x) &= 0 \\ \delta\{(1 + k\sigma)x\} \qquad\qquad + \frac{b\sigma}{2}\,(3 - k\sigma x) &= 0 \end{aligned}\right\} \quad (47)$$

where σ,x satisfying (43) and (44) are to be used after differentiation.

This calculation yields the following values for a = 1

c/y	$\delta x/b$	$\delta\left(\frac{1}{y}\ \frac{P}{WV}\right)\Big/\,b$
0.25	-0.41	-1.37
0.5	-0.34	-0.98
1	-0.27	-0.71
2	-0.22	-0.52 ,

the increments $\delta\sigma$ being small throughout.

These results are plotted in Fig.15.

The calculation clearly breaks down for small values of c/y.

9 FURTHER ANALYSIS OF POWER REQUIRED

Having studied the total power required, we can now go on to dissect it. The simple hovercraft analysed here must have two jets*, the annular one providing the power P_J to produce the lift and another straight one producing:-

P_M to overcome the momentum drag

P_D to overcome the profile drag.

But as we have seen the forward speed modifies P_J required for hovering by producing a pressure recovery aq at the annular jet exit and an additional mean pressure bq over the outer surface of the annular jet which may be either positive or negative. Thus we may write

$$P_J = P_{J_o} + P_{J_\sigma}$$

suffix o denoting hovering and σ the increment for the forward speed condition. For the total power P we have

$$P = (P_{J_o} + P_{J_\sigma}) + (P_M + P_D)$$

the first bracket being supplied by the annular and the second by the straight jet.

9.1 Power dissection for a given design

We first calculate these four components for a given design flying at a given height. In this case it is convenient to use the power function P/Wu since u is constant.

Starting from equations (40) we have

$$\frac{1}{y}\,\frac{P}{Wu} = 2^{-3/2}\left[\left(1 + k\sigma - \frac{bk}{2}\sigma^2\right)x^{\frac{1}{2}} + \left(1 - \frac{3b}{2}\sigma\right)x^{-\frac{1}{2}}\right] + \frac{c}{y}\sigma^{3/2}$$

and by separating the various terms it is easy to show that

*In more advanced designs parts of the annular jets can be inclined backward to reduce the momentum drag, and it may even become possible to arrange the annulus to supply all the power required.

$$\left.\begin{aligned}
\frac{1}{y}\,\frac{P_{J_o}}{Wu} &= 2^{-3/2}\left(x^{\frac{1}{2}} + x^{-\frac{1}{2}}\right) \\
\frac{1}{y}\,\frac{P_{J_\sigma}}{Wu} &= -\,2^{-3/2}\left(2ax^{\frac{1}{2}} + \frac{3b}{2}\,x^{-\frac{1}{2}}\right)\sigma \\
\frac{1}{y}\,\frac{P_M}{Wu} &= (2x)^{\frac{1}{2}}\left(1 - \frac{b\sigma}{2}\right)\sigma \\
\frac{1}{y}\,\frac{P_D}{Wu} &= \frac{c}{y}\,\sigma^{3/2}\,.
\end{aligned}\right\}\qquad(48)$$

When $b = 0$

$$\left.\begin{aligned}
\frac{1}{y}\,\frac{P_{J_\sigma}}{Wu} &= -\,a\left(\frac{x}{2}\right)^{\frac{1}{2}}\sigma \\
\frac{1}{y}\,\frac{P_M}{Wu} &= (2x)^{\frac{1}{2}}\sigma
\end{aligned}\right\}\qquad(49)$$

so that

$$\frac{1}{y}\,\frac{P_{J_\sigma} + P_M}{Wu} = \left(1 - \frac{a}{2}\right)(2x)^{\frac{1}{2}}\sigma \qquad(50)$$

and this is always positive.

Fig.16 gives some idea of the relative value of the various items of power over the speed range, using as an illustration the values

$$x = 0.4,\quad a = 0.5,\quad b = 0,\quad \frac{c}{y} = 0.4\,.$$

9.2 Optimum power dissection

As another illustration we can use the above equations, multiplied by $\sigma^{-\frac{1}{2}}$, to dissect the minimum power values shown in Fig.7. The results are plotted in Fig.15. One feature of these diagrams, which has already been pointed out by Stanton Jones, is the ratio of P_J the power to produce the lift to P_D the profile drag power.

It follows from equation (48) that when b = 0

$$\frac{P_J}{P_D} = \frac{y}{c} (2\sigma)^{-3/2} [(1 - 2a\sigma)x^{\frac{1}{2}} + x^{-\frac{1}{2}}] .$$

Using the optimum conditions of equations (43) and (44), this reduces to

$$\frac{P_J}{P_D} = 2 + 4(1 - a)\sigma$$

and so

$$\frac{P_J}{P_D} = 2 \quad \text{when} \quad a = 1$$

$$= 2 + 4\sigma \qquad a = 0$$

where σ is given as a function of c/y in Fig.9.

Thus at optimum total power parameter, the power for lift is twice the power for profile drag, when a = 1, as can be seen from the upper diagram of Fig.17. This can be contrasted with the aeroplane where the optimum occurs when the drag due to lift is equal to the profile drag. When a = 0 the power for lift is much greater than twice the power for profile drag (lower diagram of Fig.17).

In making these points the large power required to counter the momentum drag, which is in fact part of the mechanism for the production of lift, should not be forgotten. The corresponding ratio at the optimum is

$$\frac{P_M}{P_D} = 4\sigma .$$

These results apply only at b = 0.

10 EXAMPLES OF PERFORMANCE ESTIMATES

10.1 Design for minimum power

To show how this analysis can be used for rough performance estimates, consider a hovercraft with an elliptic planform of fineness ratio $\frac{1}{2}$ and jet angle 45°, with an angular clearance of 0.11 radians. It is to be designed for minimum power at a speed of 100 f.s., i.e. q = 11.9.

From Fig.2, $\ell/d = 0.16$ at $n = \frac{1}{2}$, and so from equation (37), y = 0.2.

Also $S = \frac{\pi}{8} d^2$ and so we have

$$\frac{d}{\sqrt{W}} = \frac{1.6}{\sqrt{p_c}}$$

$$\frac{h}{d} = 0.055$$

$$\frac{t}{h} = 0.585x .$$

We shall consider a large range of profile drag coefficients, c going from 0.025 to 0.2. b is neglected throughout.

Then using the minimum power values of Figs.9, 12 and 13 we have the following tables for a = 1 and a = 0.

a = 1, y = 0.2, V = 100

c	$\frac{c}{y}$	x	σ	$p_c = \frac{W}{S}$	u	$\bar{v}$	$\frac{P}{W}$	$\frac{mg}{W}$	$\frac{d}{\sqrt{W}}$	$\frac{t}{h}$
0.025	0.125	0.26	1.50	7.9	81.8	114	25.6	0.056	0.57	0.152
0.05	0.25	0.36	0.88	13.5	107	126	29.4	0.051	0.44	0.216
0.1	0.5	0.45	0.61	19.5	128	135	33.2	0.048	0.36	0.264
0.2	1	0.55	0.41	29.0	156	149	38.0	0.043	0.30	0.322

a = 0, y = 0.2, V = 100

c	$\frac{c}{y}$	x	σ	$p_c = \frac{W}{S}$	u	$\bar{v}$	$\frac{P}{W}$	$\frac{mg}{W}$	$\frac{d}{\sqrt{W}}$	$\frac{t}{h}$
0.025	0.125	0.16	1.30	9.15	89	156	33.2	0.041	0.48	0.093
0.05	0.25	0.24	0.78	15.3	113	163	36.6	0.039	0.41	0.140
0.1	0.5	0.315	0.53	22.4	137	173	40.0	0.037	0.34	0.183
0.2	1	0.40	0.36	37.0	167	187	44.0	0.034	0.28	0.234

These tables yield the following solution for W = 10,000 (see Fig.18).

a = 1

c	S	d	h	t	Horse Power	mg(lb/sec)
0.025	1260	56.7	3.12	0.47	466	566
0.05	740	43.6	2.40	0.51	535	510
0.1	513	36.2	1.99	0.52	605	475
0.2	345	29.7	1.63	0.53	690	430

a = 0

c	S	d	h	t	Horse Power	mg(lb/sec)
0.025	1095	48.3	2.66	0.25	605	412
0.05	655	40.9	2.25	0.31	665	392
0.1	445	33.8	1.86	0.34	725	373
0.2	303	27.9	1.53	0.36	800	345

Results for any other speed can be got from these tables by noting that

$$\left.\begin{aligned} S &\propto \frac{1}{V^2} \\ d,h,t &\propto \frac{1}{V} \\ u,\bar{v},P,m &\propto V \\ p_c &\propto V^2 . \end{aligned}\right\}$$

Results for a rectangle of the same fineness ratio can be simply deduced by noting from Fig.1 that ℓ/d at $n = \frac{1}{2}$ is practically the same for rectangle and ellipse, and thus y remains at 0.2. Thus if we assume that rectangle and ellipse have the same profile drag at the same y, the solution is exactly the same for the rectangle except as regards d,h,t.

If d' is the longer side of the rectangle we have

$$\frac{\pi}{8} d^2 = \frac{d'^2}{2}$$

since the areas are the same, and so

$$\frac{d'}{d} = 0.89.$$

Thus d,h,t are to be multiplied by this factor to get the rectangle solution.

10.2 Off-design performance

Having optimised the design for minimum power at V = 100 we can go on to calculate the performance in other conditions, for example

(a) for other speeds at the same height,

(b) for other heights at the same speed.

Consider the optimum design for c = 0.1, a = 1 which has been obtained as

$$W = 10{,}000, \quad S = 513, \quad d = 36.2, \quad t = 0.52, \quad u = 128.$$

At V = 100, y = 0.2 we have seen that

$$x = 0.45, \quad \sigma = 0.61, \quad \bar{v} = 135, \quad h = 1.99, \quad m = 14.8, \quad HP = 605.$$

Some of these quantities will change in what follows.

(a) Speed variation at the design height

In this case x and therefore $\bar{v}$ and m remain constant and we seek the variation of P with σ. This is obtained from the equation

$$\frac{P}{Wu} = 2^{-3/2} y\left\{(1 + k\sigma)x^{\frac{1}{2}} + x^{-\frac{1}{2}}\right\} + c\sigma^{3/2} \tag{51}$$

where u = 128, x = 0.45, y = 0.2, k = 2, c = 0.1, W = 10,000.

Thus $$\frac{P}{W} = 19.6 + 12.2\sigma + 12.8\sigma^{3/2}$$

where $$\sigma = \left(\frac{V}{u}\right)^2 .$$

The result is shown in Fig.19 where H.P. is plotted against V.

(b) Height variation at the design speed

In this case σ remains constant and the variation of x and y is such that

$$xy = z = t/\ell = \text{constant.}$$

Using this relation to transform equation (51) we have

$$\frac{P}{Wu} = 2^{-3/2}\left\{(1 + k\sigma)z^{\frac{1}{2}}y^{\frac{1}{2}} + z^{-\frac{1}{2}}y^{3/2}\right\} + c\sigma^{3/2} \tag{52}$$

and we have already seen that

$$\frac{\bar{v}}{u} = (2x)^{-\frac{1}{2}} = (2z)^{-\frac{1}{2}}y^{\frac{1}{2}} \tag{53}$$

$$\frac{m}{W} = (2x)^{\frac{1}{2}}\frac{y}{u} = (2z)^{\frac{1}{2}}\frac{y^{\frac{1}{2}}}{u} . \tag{54}$$

The constants in (51) to (53) are

u = 128, k = 2, σ = 0.61, z = 0.45 × 0.2 = 0.09, c = 0.1, W = 10,000.

The equations therefore become

$$\frac{P}{W} = 151\ y^{3/2} + 30.2\ y^{\frac{1}{2}} + 6.15$$

$$\bar{v} = 302\ y^{\frac{1}{2}}$$

$$\frac{m}{W} = 0.00331\ y^{\frac{1}{2}}\ .$$

The results are plotted in Fig.20 where the curves have been continued right down to $y = 0$ although the approximate theory must break down before this. The calculation is also questionable because no allowance has been made for the unknown variation of c with y.

The curves for cases (a),(b) are however useful in giving a rough idea of the power margins that may have to be provided. Expressions for the slopes of the power curves, obtained by differentiating equations (50) and (51), are:-

$$\frac{\partial}{\partial V}\left(\frac{P}{W}\right) = 2^{-\frac{1}{2}}\ kx^{\frac{1}{2}}\ \sigma^{\frac{1}{2}} y + 3\,c\sigma \qquad (55)$$

$$\frac{\partial}{\partial y}\left(\frac{P}{W}\right) = 2^{-5/2}\ u\left\{(1 + k\sigma)x^{\frac{1}{2}} + 3x^{-\frac{1}{2}}\right\}. \qquad (56)$$

11 SECOND APPROXIMATION TO LIFT

The first approximation to lift (equation 23) has been used throughout to simplify the analysis. The second approximation can be added in the form of a correction Δ, using equation (21). For example we have

$$\Delta\left(\frac{P}{WV}\right) = -\frac{P}{WV}\ \{y \sin\theta(1 - b\sigma) + \delta\sigma\} \qquad (57)$$

and if b and δ are neglected, thus including only the vertical component of the thrust, the correction for typical values of y and θ is of the order 10%.

12 CRITICAL SPEED?

There must be forward speeds at which the actual flow ceases to bear any relation to that postulated in this analysis, but the model is so crude that it can hardly lead to a criterion. On the assumptions made here the pressure outside the annulus must approach q in the forward parts of the annulus, and so as q rises toward p_0 ($\sigma = 1$) we should expect the curvature of the leading jets to be reversed. Qualitatively there is clear evidence that the front jets ultimately get blown back as the speed rises. The process seems to be rather a gradual one than a sudden change to a different regime, but its mechanism is of course much more complex than would be suggested by this approach. When this happens the momentum drag would be relieved, but what happens to the lift and the profile drag remains obscure, and so as far as my information goes it remains an open question whether the power required rises or falls, in relation to that calculated here, as the speed rises. It seems that this doubt can only be settled by careful experiment.

13 THE NEED FOR EXPERIMENTAL CHECKS

The analysis given here is defensible, if at all, only after alignment with experimental results. Indeed it has been prepared mainly as a basis of discussion and use by experimenters. This could be done in two ways.

(a) The vehicles now being designed usually have features, such as backward inclined main jets and stability jets, that are omitted in this analysis. Thus measurements of their performance could only be plotted on the diagrams suggested here after a good deal of adaptation. On the other hand there is probably much supporting work on simpler models, not yet published, that could furnish spot checks at various points of this analysis.

(b) Hovercraft performance depends on so many interdependent quantities that basic experiments tend to give way in favour of those in support of a particular design. The parameters used here are intended to give a clear view of the essentials of the crude flow model on which most calculations are, I take it, still based. They can probably be used as the framework for the design of model experiments so chosen as to test the empiricism of current performance estimates in the shortest possible time. We should then begin to know where we are.

14 ACKNOWLEDGEMENT

I should like to record the friendly help given me by M.S. Igglesden in the preparation of this paper.

NOTATION

Geometry	S	area	base
	s	perimeter	base
	ℓ	S/s	base
	d	maximum dimension	base
	n	fineness ratio	base
	h	height above ground	jet exit
	t	thickness	jet exit
	θ	angle to horizontal, inwards	jet exit
	R	radius of circle inclined at θ at jet exit and touching ground	jet exit

Pressures	p_c	cushion pressure
	p_o	mean pressure over outer surface of jet
	H	total head } jet at exit
	p	pressure } jet at exit
Velocities	V	forward speed, dynamic pressure q
	$\bar{v}$	mean jet velocity at exit
	u	given by $p_o = \frac{1}{2}\rho u^2$
Parameters	$x = \frac{t}{R}$, $y = \frac{R}{\ell}$, $z = xy = \frac{t}{\ell}$	
	$\sigma = q/p_c$	
	a	fraction of q recovered at jet exit
	b	p_o/q
	c =	profile drag coefficient referred to S
		$= \lambda C_{D_o}$
	C_{D_o}	drag coefficient out of ground effects, jet off
	k =	$2(2 - a) + b \div 2(2 - a)$
Miscellaneous	L	lift
	T	thrust at jet exit
	m	mass flow at jet exit
	D	drag
	D_m	momentum drag
	D_o	profile drag
	P	total power required
	P_o	total power required hovering
	P_{J_o}, P_{J_σ}, P_m, P_D	parts of P, para.9
	f, g, j / F, G, J	para.4
	β	angular clearance $= \frac{2h}{d}$ radians
	δ, ε	para.5
	W	weight

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No.	Author	Title, etc.
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2	Strand, T.	Inviscid-incompressible-flow theory of static peripheral jets in proximity to the ground. Journal of Aerospace Sciences 28 (1), 27-33, January 1961.
3	Chaplin, H. Stephenson, B.	Preliminary study of the hovering performance of annular jet vehicles in proximity to the ground. U.S. Navy Dept., Taylor Model Basin. Aero Report 947. P.78137. August 1958.

WT.2078.C.P.608.K3 - Printed in England

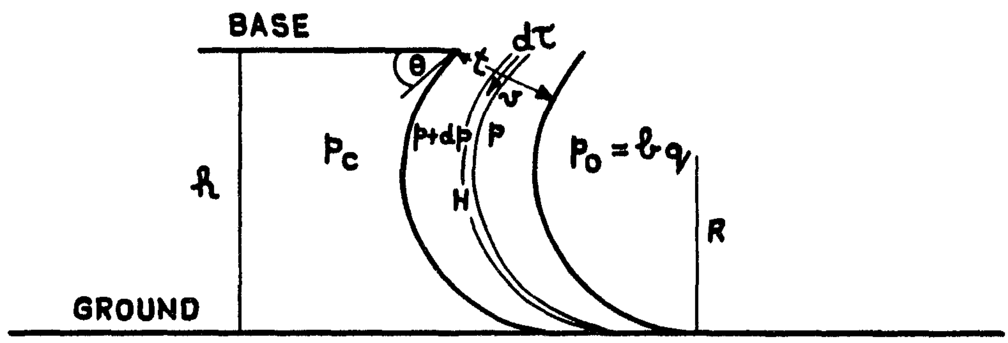


FIG. 1. NOTATION FOR JET.

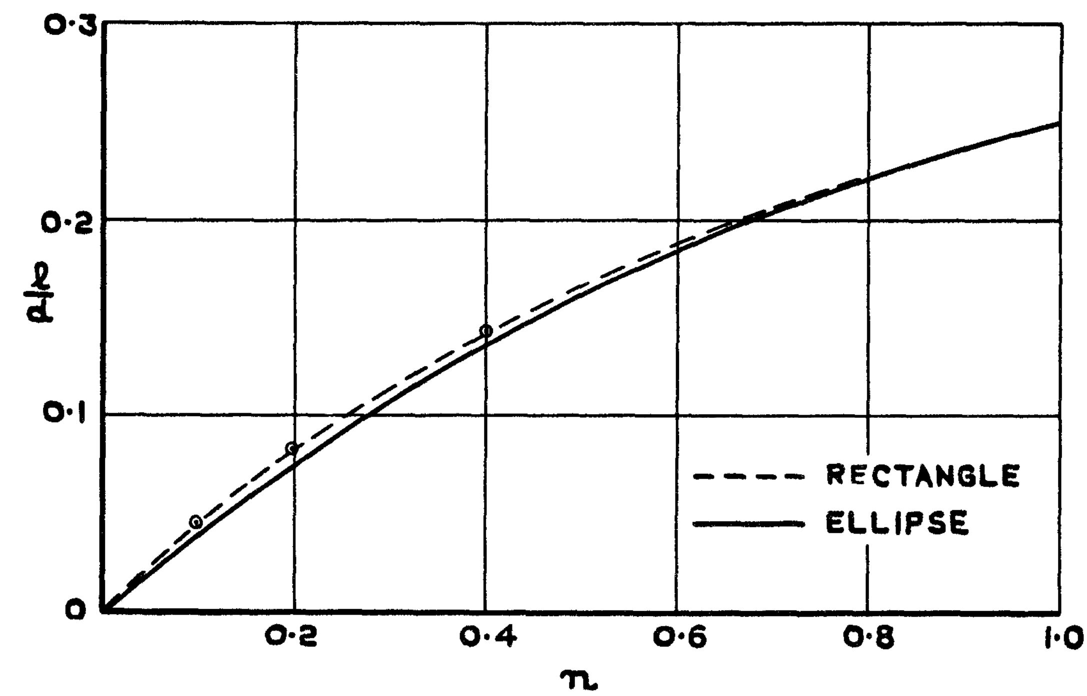


FIG. 2. l/d AGAINST FINENESS RATIO FOR ELLIPSE AND RECTANGLE.

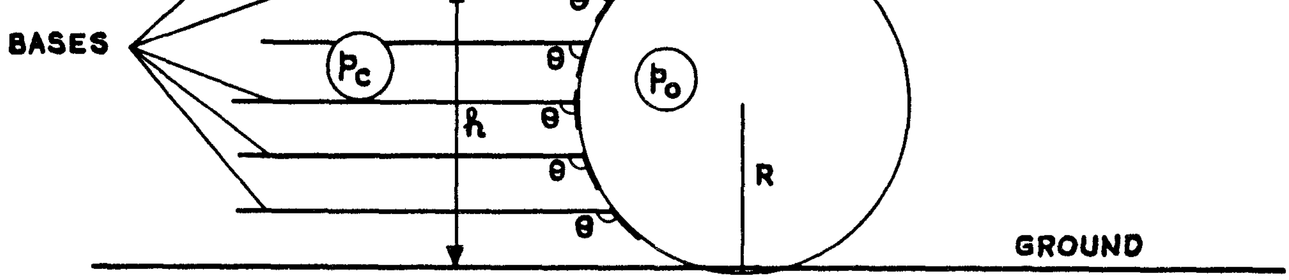


FIG. 3. ARRANGEMENTS PRODUCING THE SAME CUSHION PRESSURE.

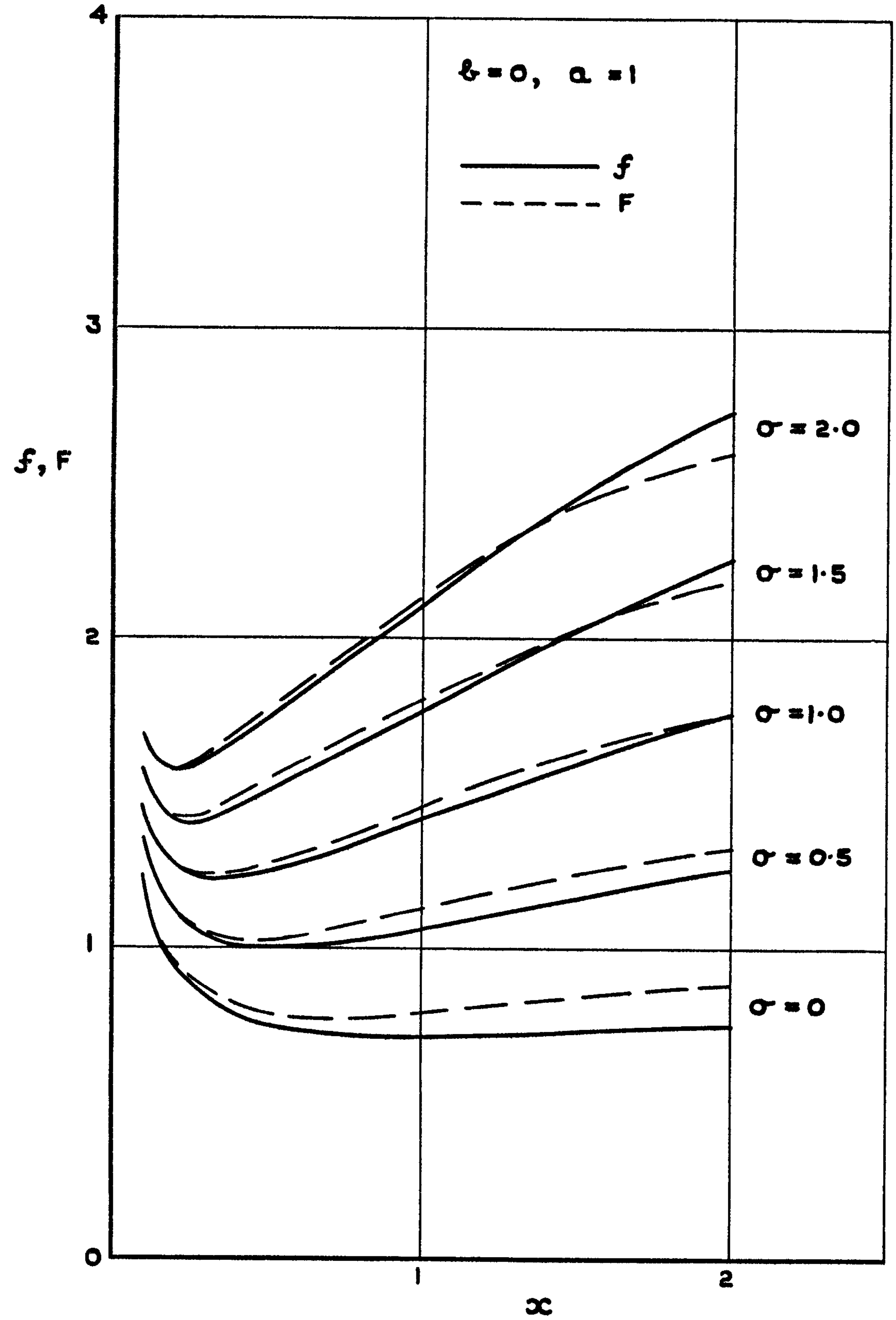


FIG. 4. FUNCTIONS f, F FOR $b = 0$, $a = 1$.

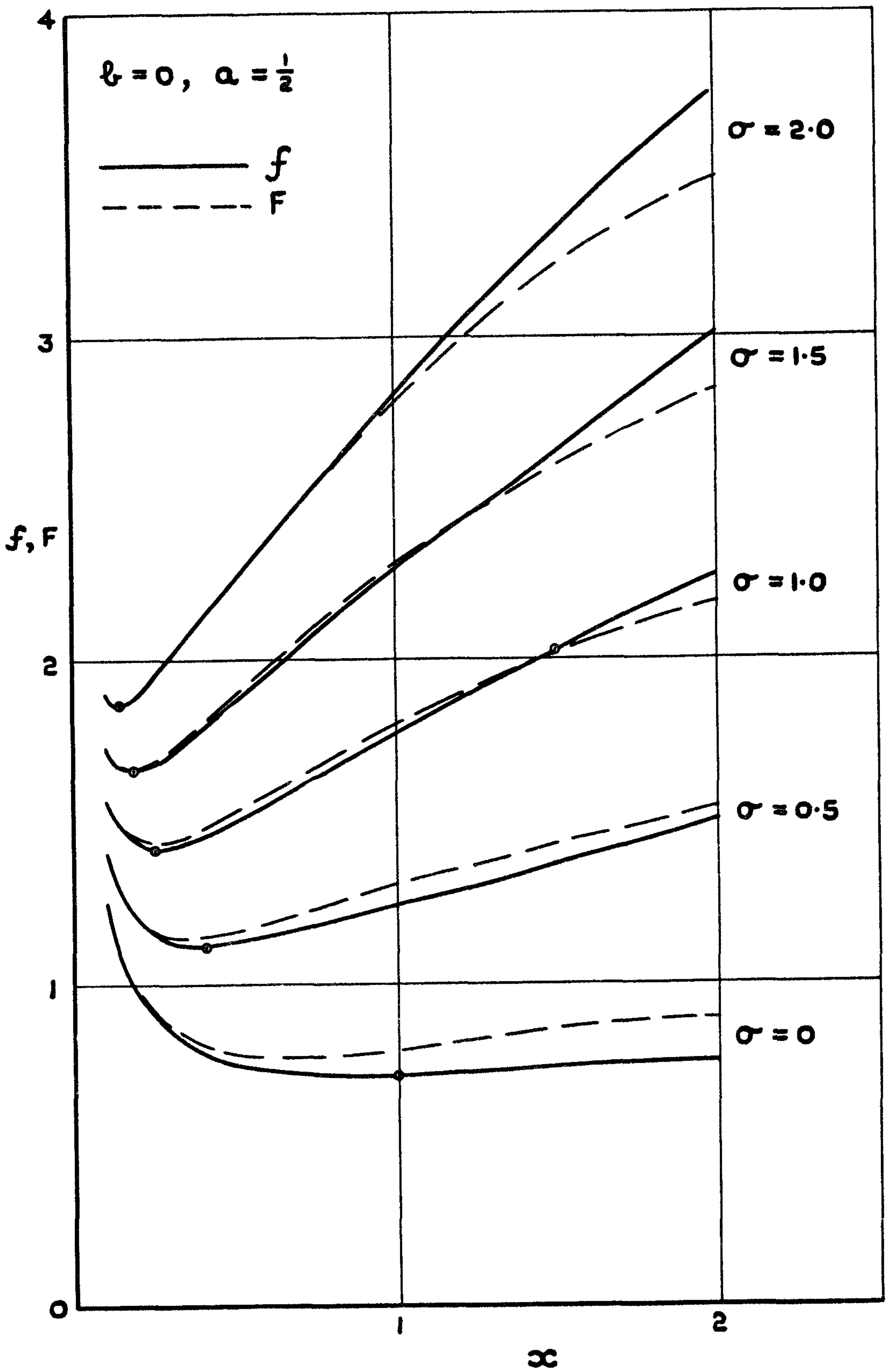


FIG. 5. FUNCTIONS f, F FOR $b = 0$, $a = \frac{1}{2}$.

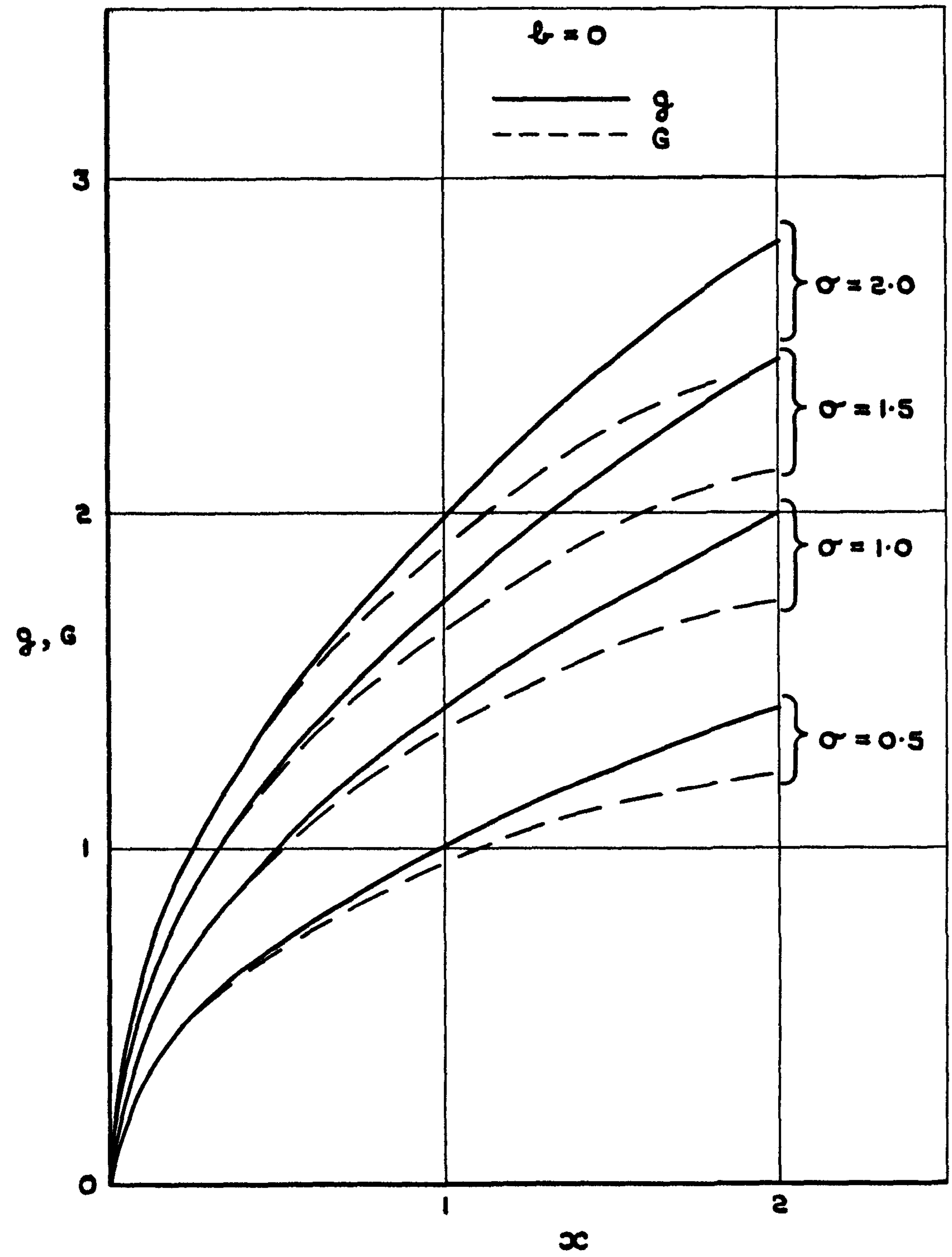


FIG. 6. FUNCTION g,G FOR b=O.

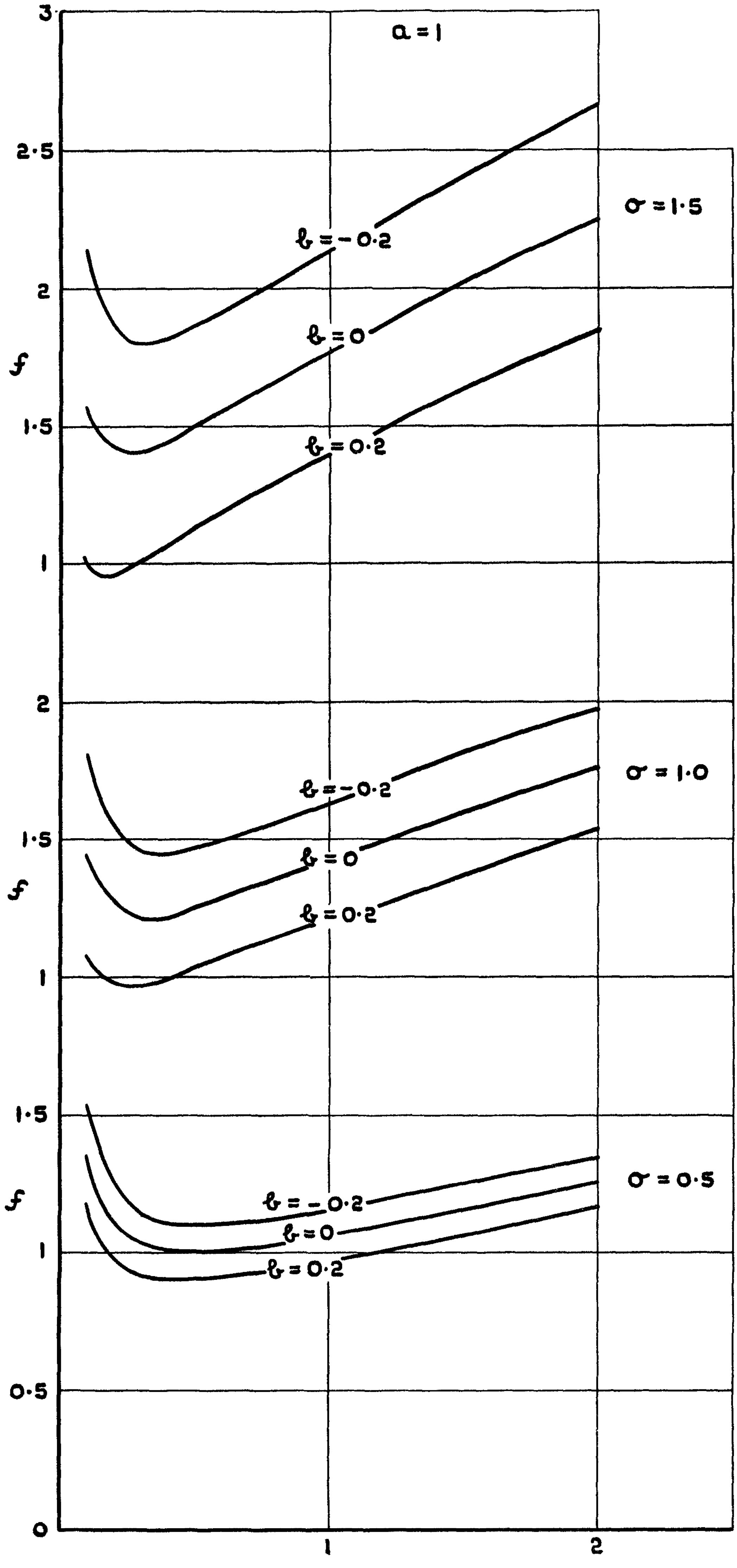


FIG. 7. EFFECT OF b ON FUNCTION f, a=1.

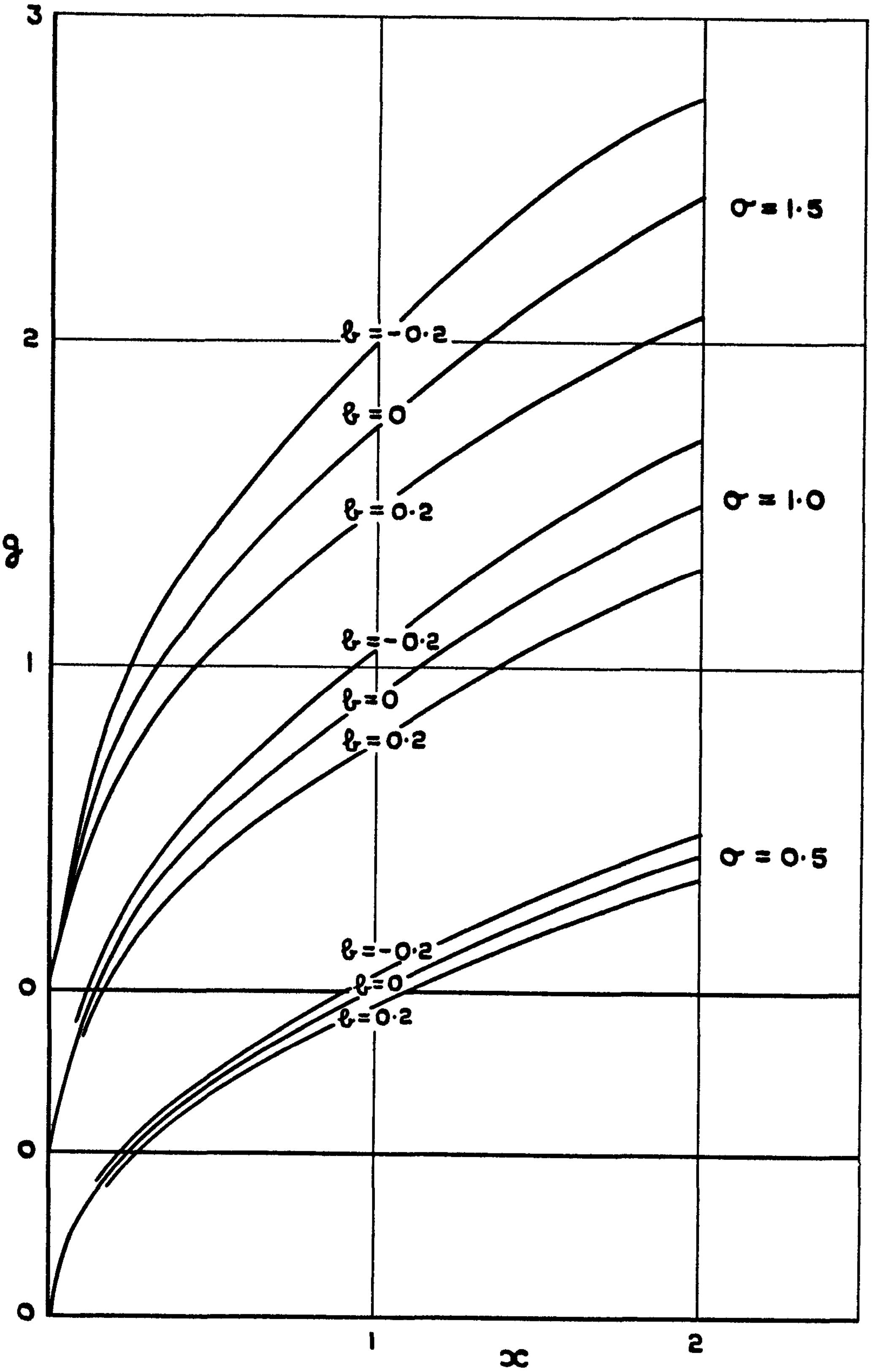


FIG. 8. EFFECT OF b ON FUNCTION g.

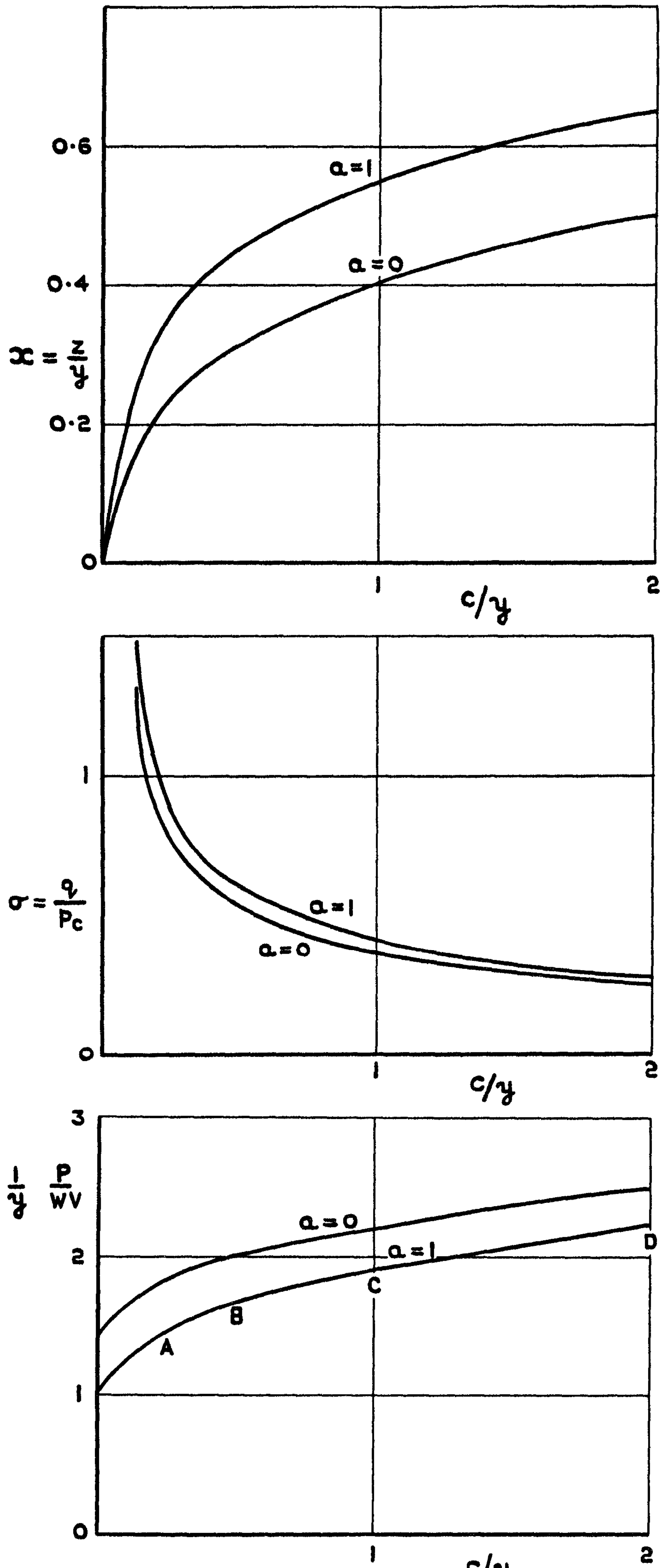


FIG. 9. MINIMA OF $\frac{P}{WV}$ FOR $b = 0$.

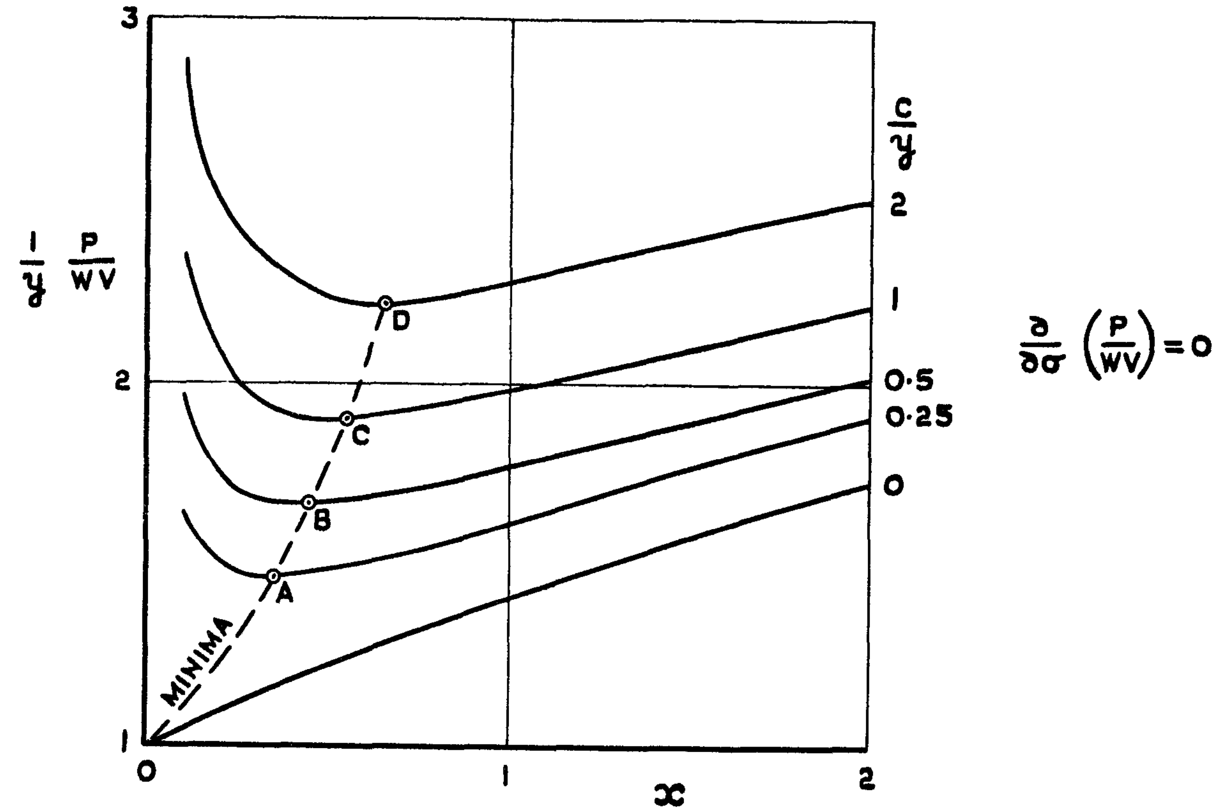


FIG. 10.

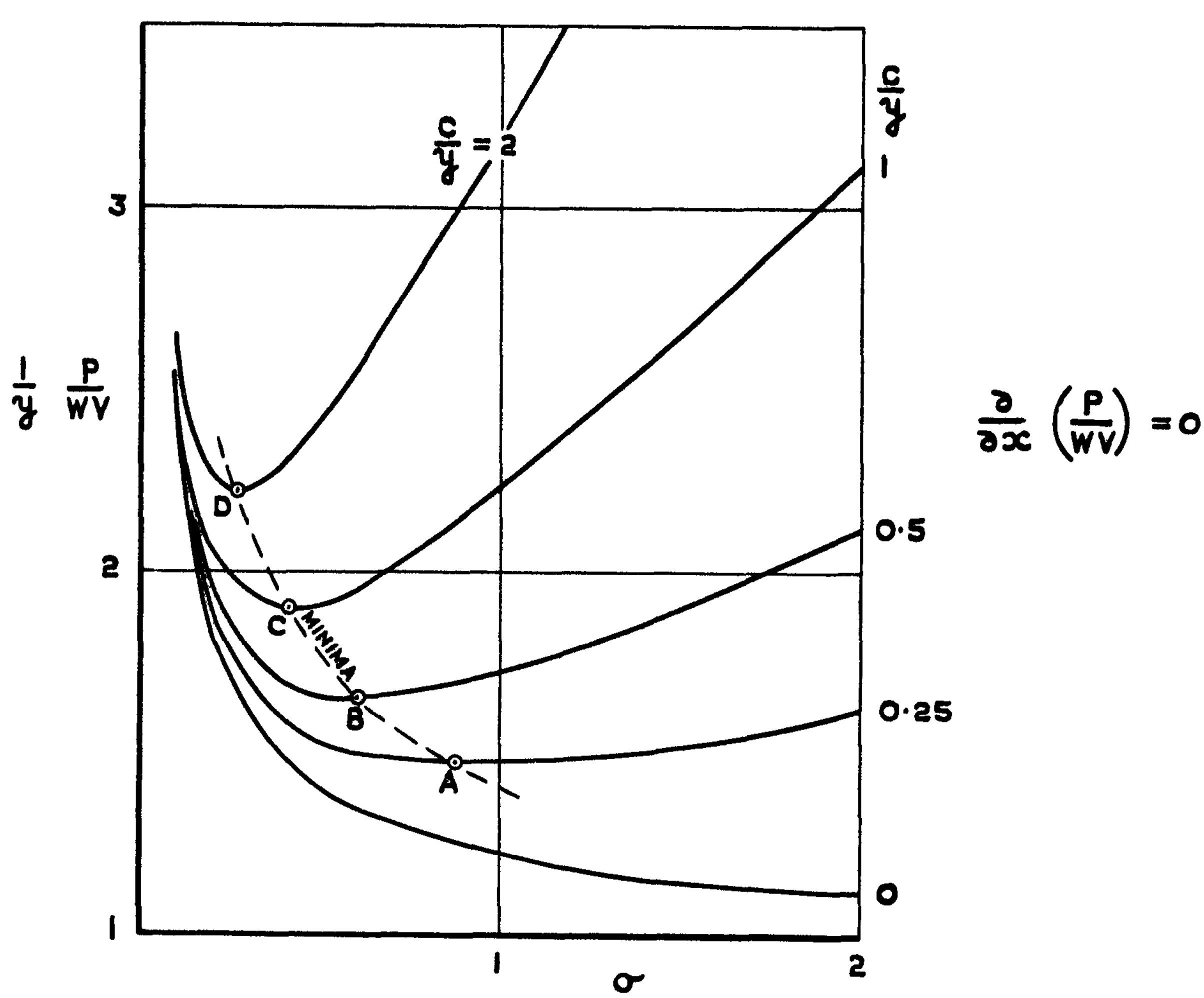


FIG. 11.

FIG. 10 & 11. STRENGTH OF MINIMA OF $\frac{P}{WV}$, $a=1$.

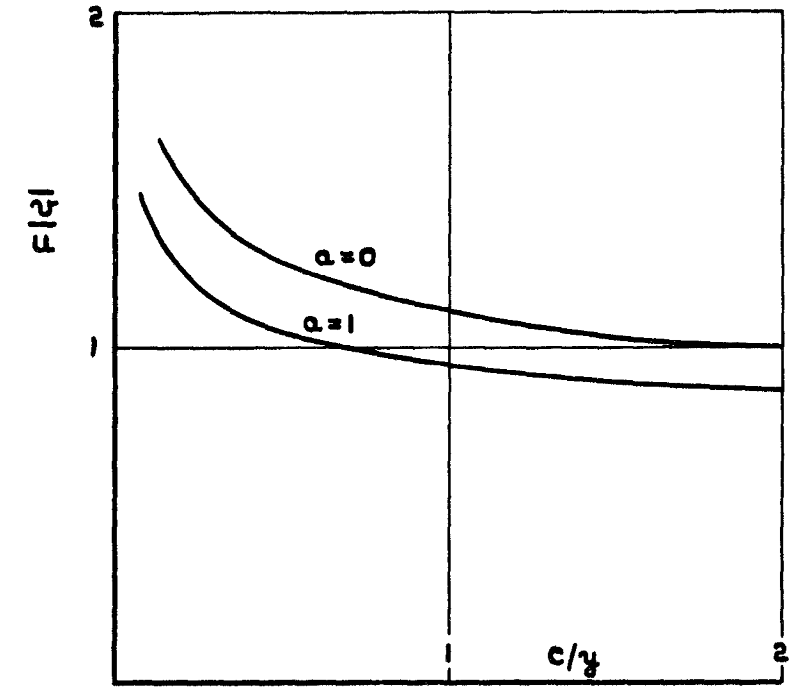


FIG. 12.

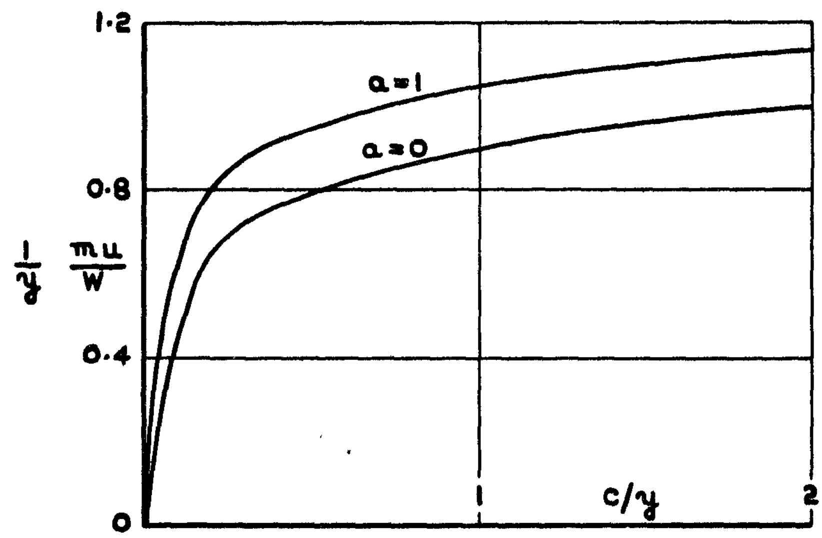


FIG. 13.

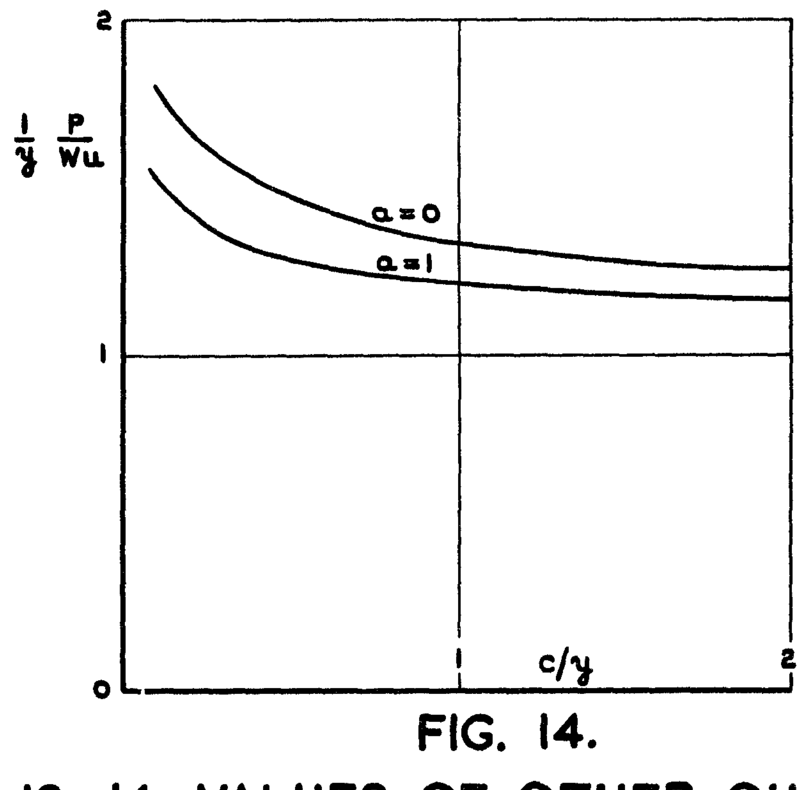


FIG. 14.

FIG. 12–14. VALUES OF OTHER QUANTITIES AT MINIMUM $\frac{P}{WV}$.

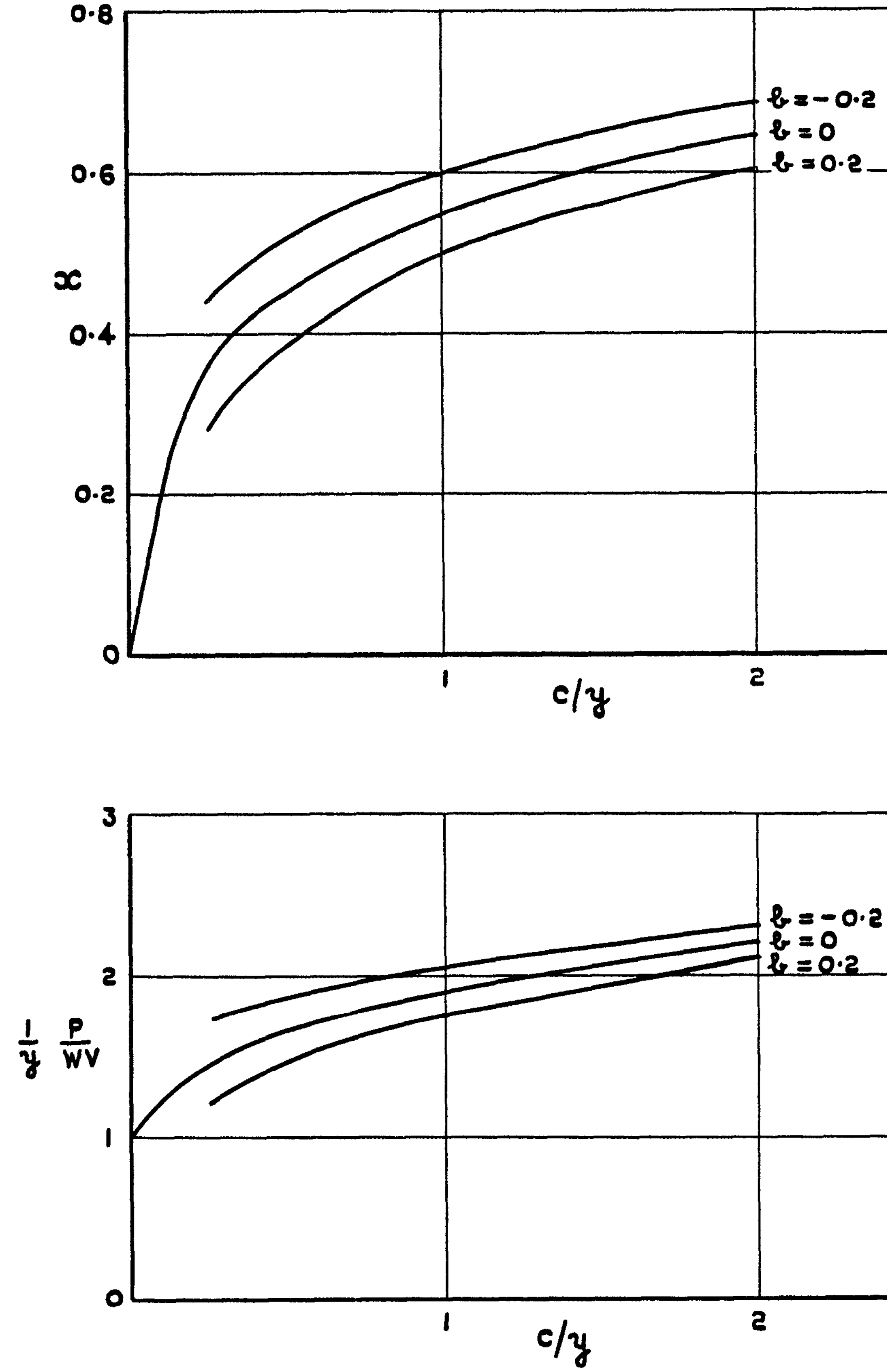
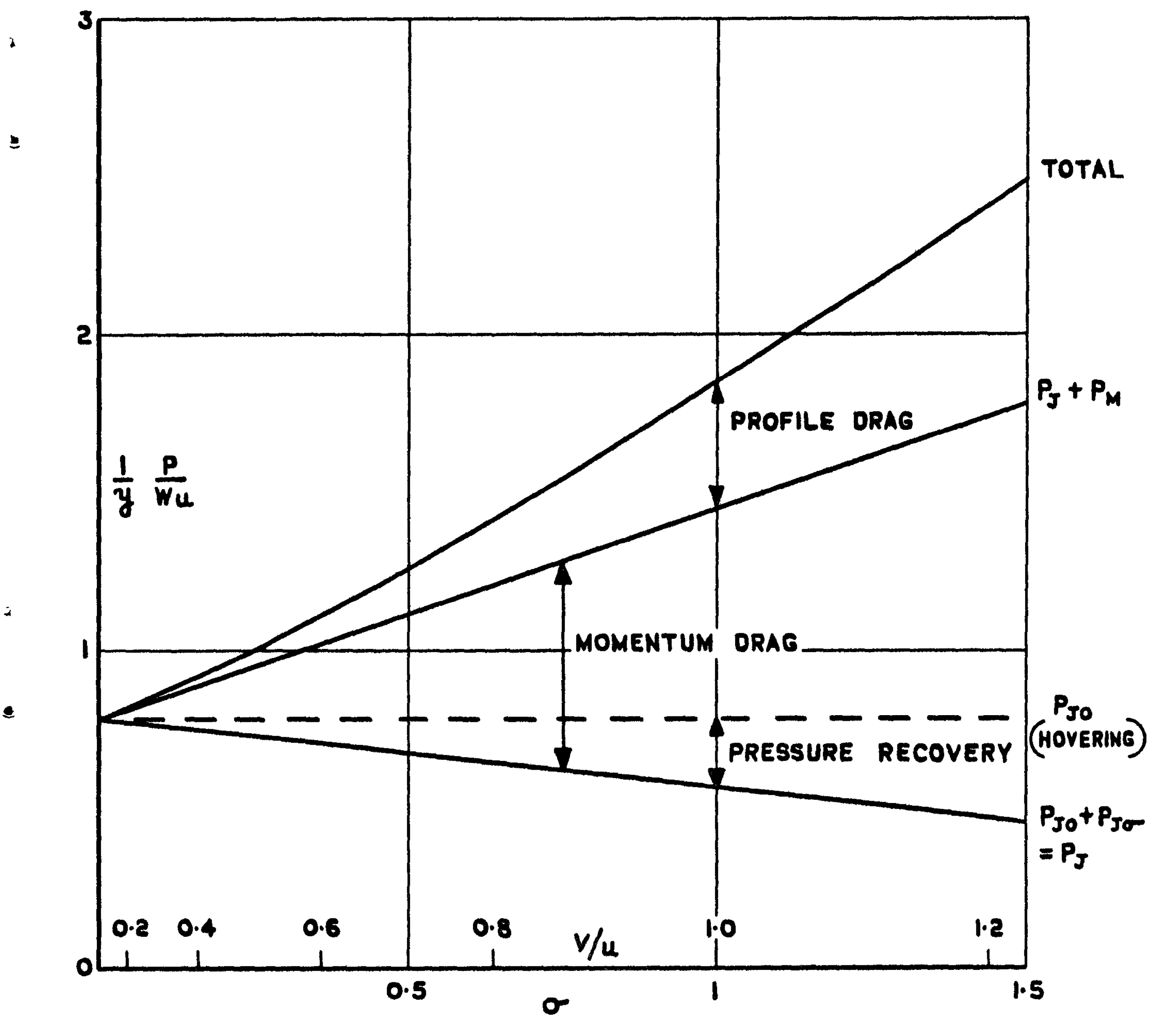


FIG. 15. EFFECT OF b AT POWER MINIMA, a = 1.



$x = 0.4, \quad a = \frac{1}{2}, \quad b = 0, \quad \frac{c}{y} = 0.4$

FIG. 16. ANALYSIS OF POWER REQUIRED FOR GIVEN DESIGN.

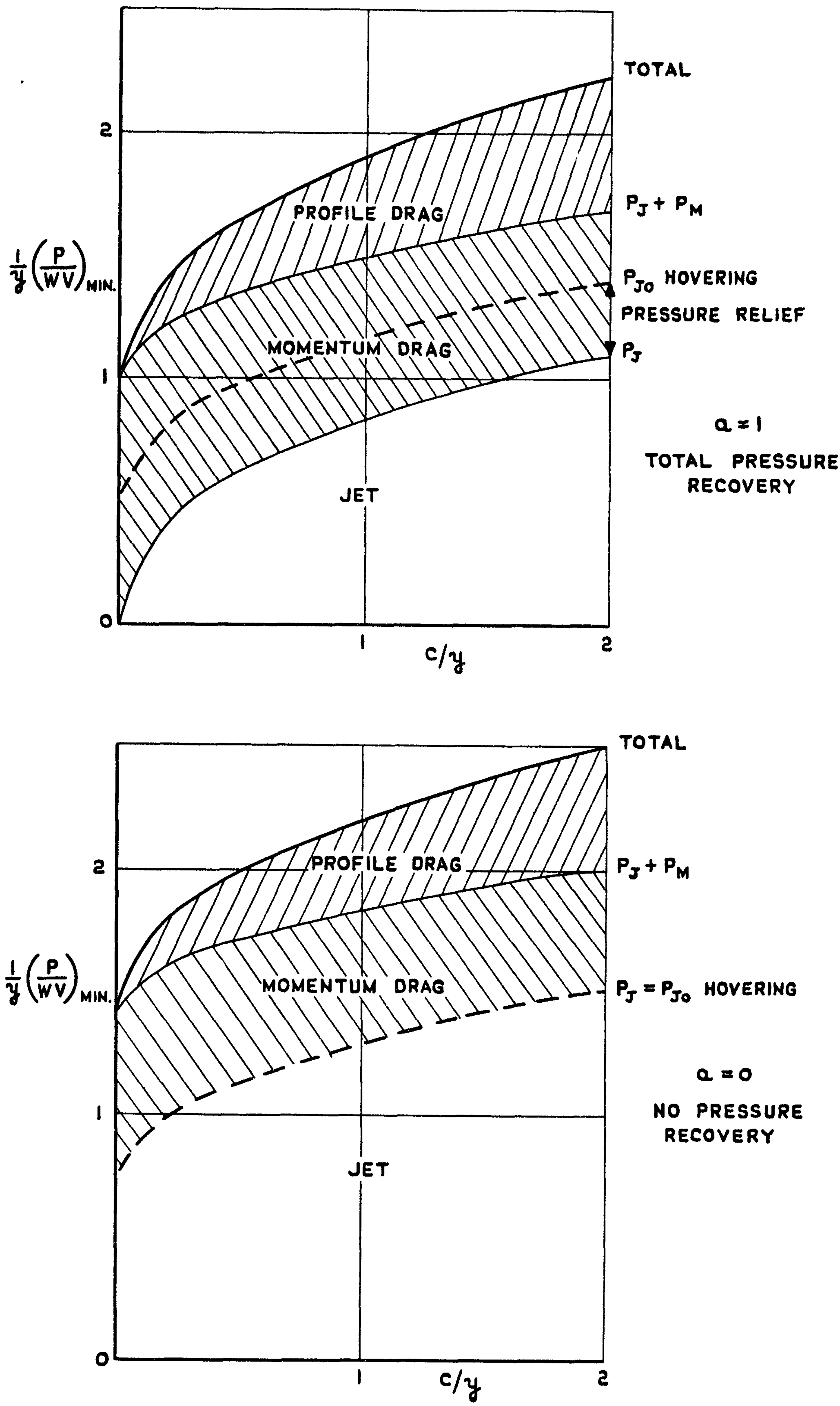


FIG. 17. ANALYSIS OF MINIMUM POWER, $b = 0$.

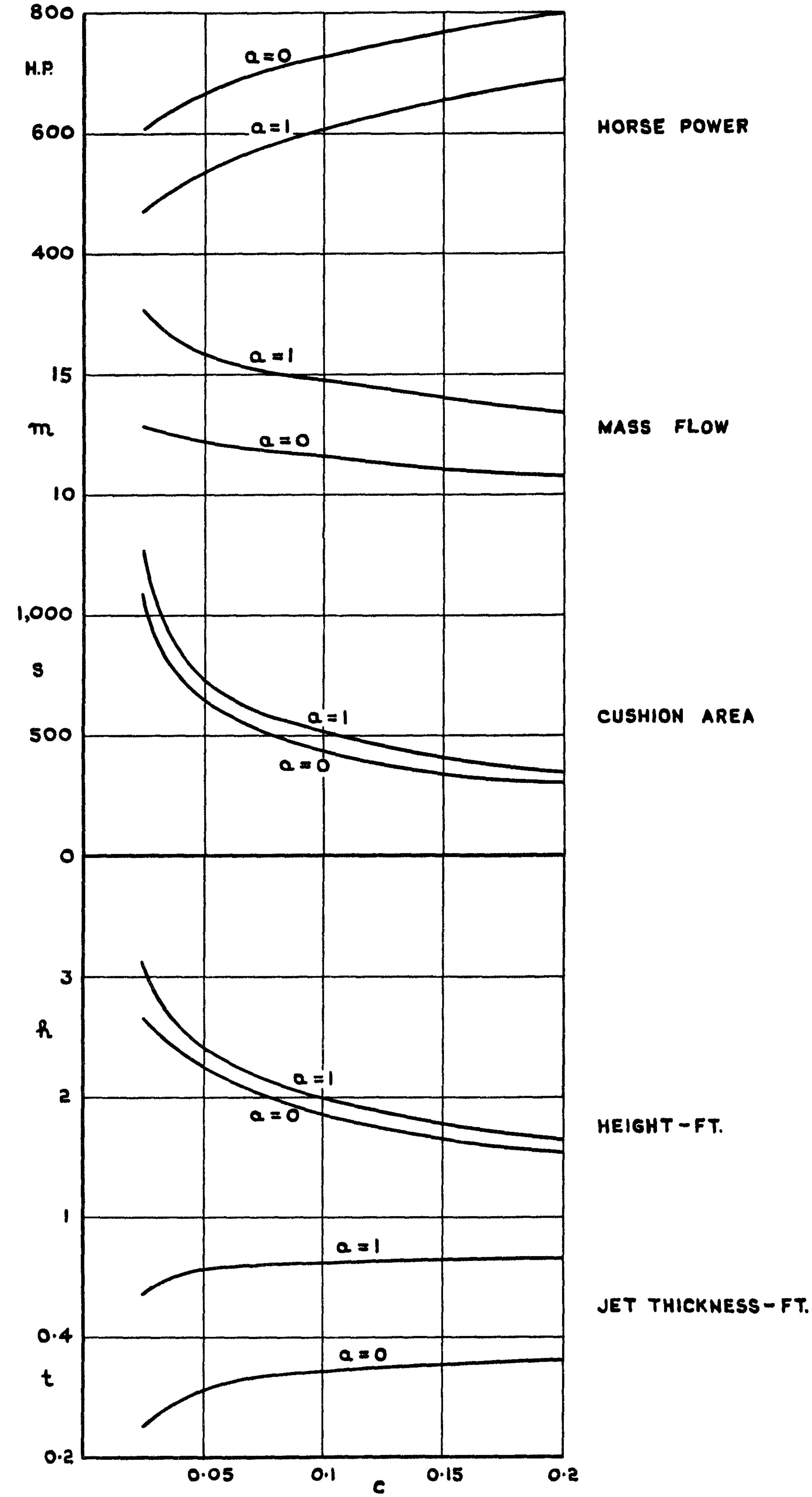
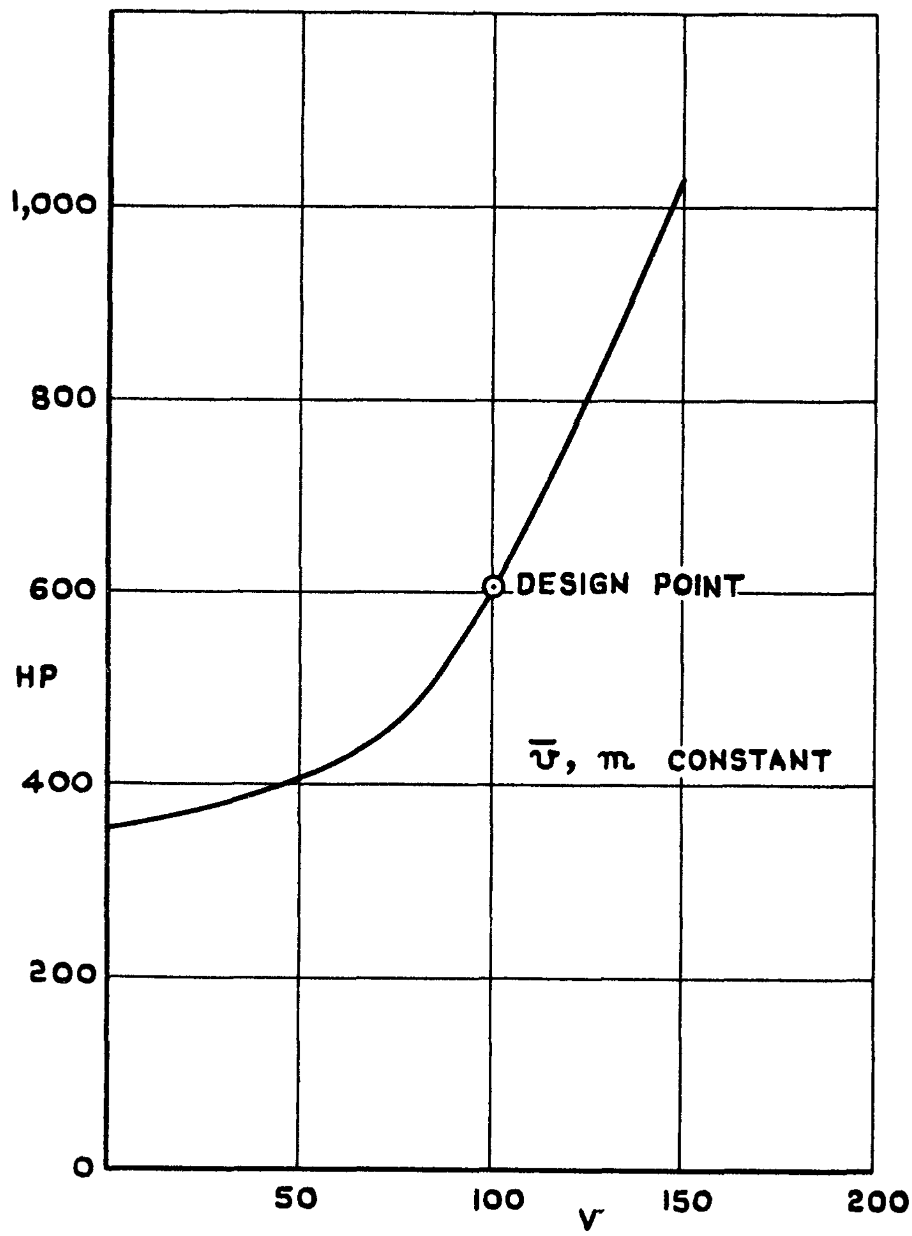


FIG. 18. OPTIMA FOR W = 10,000, V = 100.



W = 10,000, a = 1, b = 0, c = 0·1, y = 0·2

⊙ MINIMUM POWER AT V = 100

FIG. 19. OFF DESIGN PERFORMANCE, VARIATION WITH SPEED.

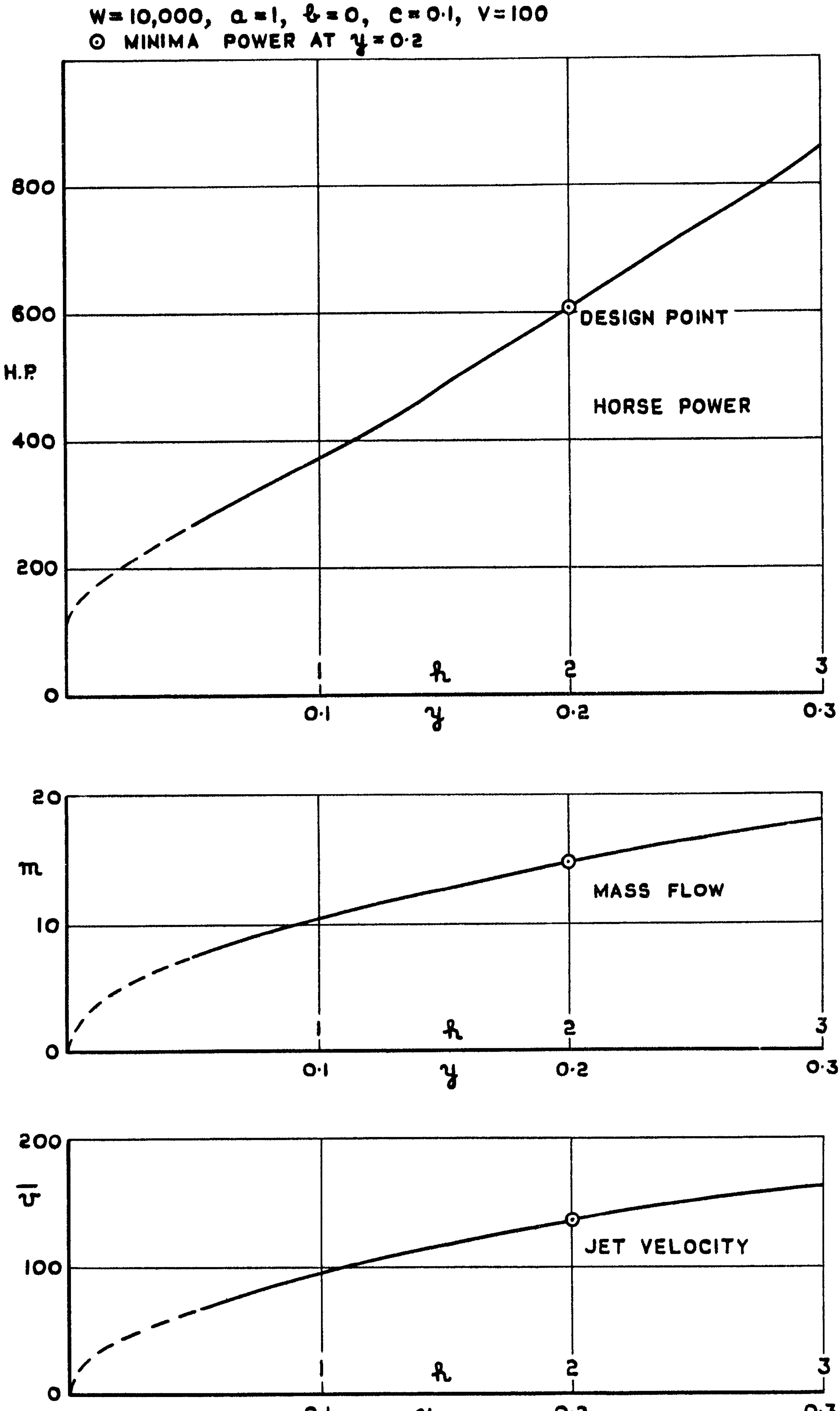


FIG. 20. OFF DESIGN PERFORMANCE – VARIATION WITH HEIGHT.

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A CRUDE THEORY OF HOVERCRAFT PERFORMANCE AT ZERO TILT.
Gates, S.B. November, 1961.

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(1) to give a rather clearer view of the assumptions and parameters involved in the crude theory,

(2) to encourage a stricter comparison between prediction and ad hoc test results as they become available,

(3) as a point of departure in planning basic experiments that would lead most economically to a better understanding of the matter.

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