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Scale Models for Thermo-Aeroelastic Research

by

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SCALE MODELS FOR THERMO-AEROELASTIC RESEARCH

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SUMMARY

An investigation is made of the parameters to be satisfied for thermo-aeroelastic similarity. It is concluded that complete similarity obtains only when aircraft and model are identical in all respects, including size.

By limiting consideration to conduction effects, by assuming the major load carrying parts of the structure are in regions where the flow is either entirely laminar, or entirely turbulent, and by assuming a specific relationship between Reynolds number and Nusselt number, an approach to similarity can be achieved for small scale models. Experimental and analytical work is required to check on the validity of these assumptions.

It appears that existing hot wind tunnels will not be completely adequate for thermo-aeroelastic work, and accordingly a possible layout for the type of tunnel required is described. Automatic programmed control of the tunnel would appear to be necessary.

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1 INTRODUCTION

The problem of simulation of parameters between an aircraft and a scale model, for an adequate representation of aeroelastic effects in the absence of kinetic heating has been considered by various authors[1,2,3]. For the most part, the problems that arise in this case are surmountable, and in consequence a very extensive use is made of scale models for investigating aeroelastic effects, in particular for the investigation of flutter. In the past the speed range to be covered in these investigations has been low enough for effects due to kinetic heating of the structure to be ignored, but it is apparent that for missiles and the future generations of high speed aircraft the effects of kinetic heating will be significant.

In the main, kinetic heating will have its influence on aeroelastic properties of the structure primarily by modifying the structural stiffnesses. There will, of course, be other effects (for example, thermal expansion will result in an increase of wing area), but these will generally be of a secondary nature. The effect on stiffness may be regarded in two phases: namely, an effect of thermal stress in the transient phase when the temperature distribution is in transit from one static state to another, and the effect when the static condition is achieved. There is a greater difficulty in predicting the transient effect than the static one, though there is evidence that the transient condition might often prove the more critical[4]. From such predictions as have been made[5] the thermal effect on stiffness appears to be highly non-linear, so that an attempt to design reduced stiffness, representing thermal effect, into a model to be tested in atmospheric stagnation temperature flow is unlikely to succeed. Furthermore, the prediction of thermal effects requires accurate heat transfer data, which may not be available at the stage where model tests for a prototype aircraft are desirable.

In principle these uncertainties can all be eliminated by testing a scale model of the aircraft in a representative flow. The analogous situation with regard to uncertainties in the aerodynamic derivatives has led to the present extensive use of scale models for aeroelastic investigations, and it is obviously worth considering the extent to which a continuation of this approach will yield reliable results in the kinetic heating field.

There is however little information available on the relationships to be satisfied for the simulation of thermal effects, between aircraft and model structures and such information as is available[6,7] is not directly applicable to the problem of thermo-aeroelastic simulation. Accordingly, the specific problem of thermo-aeroelastic simulation is considered in the present paper.

2 SIMILARITY PARAMETERS FOR THERMO-AEROELASTIC MODELS

The similarity parameters required are developed by dimensional analysis[8]. It is convenient to develop the parameters for purely aeroelastic effects first, and then to determine the additional parameters that arise when thermal effects are introduced. The analysis is confined to the linear range, where local buckling effects are assumed to be absent.

2.1 Aeroelastic similarity

It is assumed that aeroelastic properties are determined by the following physical quantities.

Quantity	Symbol	Dimensions
Linear dimension denoting size*	L	L
Modulus of elasticity	E	$M L^{-1} p^{-2}$
Modulus of elasticity: modulus of rigidity	n	Dimensionless
Material density	σ	$M L^{-3}$
Dynamic viscosity of the fluid	μ	$M L^{-1} p^{-1}$
Density of the fluid	ρ	$M L^{-3}$
Velocity of the fluid	V	$L p^{-1}$
Speed of sound in the fluid	a	$L p^{-1}$
Acceleration due to gravity	g	$L p^{-2}$
Time	p	p

The distinct dimensions that appear in these quantities are those of length L, mass M, and time p. If we select length L, velocity V and air density ρ as the primary physical quantities (which embrace the distinct dimensions) the resulting Π factors of dimensional analysis can be constructed.

For example, considering the modulus of elasticity E. The factor is

$$E L^{a} V^{b} \rho^{c}$$

where a, b, c are the unknown exponents of the selected quantities.

Substituting the dimensions of the physical quantities we have

$$M L^{-1} p^{-2} L^{a} (L p^{-1})^{b} (M L^{-3})^{c}$$

and the conditional equations are then

$$M : \quad 1 + c = 0$$

$$L : \quad -1 + a + b - 3c = 0$$

$$p : \quad -2 - b = 0$$

from which

$$a = 0, \quad b = -2, \quad c = -1.$$

---

*This quantity defines geometrical similarity in the broadest sense, embracing both undistorted and aeroelastically distorted structural components and hence implying similarity in local incidence.

The dimensionless parameter resulting from this Π factor is thus

$$\frac{E}{\rho V^2}\ .$$

In a similar way the exponents for all the Π factors are obtained (see Table 1), and the complete solution yielded by dimensional analysis is then

$$f\left(\frac{E}{\rho V^2},\ n,\ \frac{\sigma}{\rho},\ \frac{\mu}{LV\rho},\ \frac{a}{V},\ \frac{gL}{V^2},\ \frac{pV}{L}\right) = 0. \qquad (1)$$

If we now let the ratio of a physical property for the aircraft to the corresponding physical property for the model be denoted by λ (e.g. λ_L = aircraft size/model size), we obtain from equation (1) the following non-dimensional relationships for aeroelastic similarity between aircraft and model:-

(a) $\lambda_E\,\lambda_\rho^{-1}\,\lambda_V^{-2} = 1$

(b) $\lambda_n = 1$

(c) $\lambda_\sigma\,\lambda_\rho^{-1} = 1$

(d) $\lambda_\mu\,\lambda_L^{-1}\,\lambda_V^{-1}\,\lambda_\rho^{-1} = 1$

(e) $\lambda_a\,\lambda_V^{-1} = 1$

(f) $\lambda_g\,\lambda_L\,\lambda_V^{-2} = 1$

(g) $\lambda_p\,\lambda_V\,\lambda_L^{-1} = 1.$

2.2 Thermal similarity

Many of the quantities that determine aeroelastic similarity are also significant in the thermal case, but since we include the same primary physical quantities in the analysis (i.e. L, V and ρ) no new relationships result. Accordingly the following quantities additional to those of Section 2.1 are assumed to be involved*.

Quantity	Symbols	Dimensions
Specific heat of structural material	c	$L^2\,p^{-2}T^{-1}$
Thermal conductivity of structural material	k	$M\,L\,p^{-3}\,T^{-1}$
Coefficient of expansion of structural material	α	T^{-1}
Absolute temperature	T	T
Specific heat at constant pressure of the fluid	c_1	$L^2\,p^{-2}\,T^{-1}$

---

* Radiation quantities are not included, on the assumption that effects of radiation can be neglected.

Quantity	Symbol	Dimensions
Specific heat at constant pressure: specific heat at constant volume	γ	Dimensionless
Thermal conductivity of the fluid	k_1	$M\,L\,p^{-3}\,T^{-1}$
Heat transfer coefficient	h	$M\,p^{-3}\,T^{-1}$

The distinct dimensions that appear in these quantities are those of length L, mass M, time p and temperature T. Following the procedure of Section 2.1, we will select length L, velocity V, air density ρ and temperature T as the primary physical quantities. The exponents of the resulting Π factors are given in Table 2, from which we obtain the dimensional analysis solution,

$$f\left(\frac{cT}{V^2},\ \frac{kT}{LV^3\rho},\ \alpha T,\ \frac{c_1 T}{V^2},\ \gamma,\ \frac{k_1 T}{LV^3},\ \frac{hT}{V^3\rho}\right) = 0. \qquad (2)$$

From this we obtain further relationships for similarity between aircraft and model, namely

(h) $\lambda_c\,\lambda_T\,\lambda_V^{-2} = 1$

(i) $\lambda_k\,\lambda_T\,\lambda_L^{-1}\,\lambda_V^{-3}\,\lambda_\rho^{-1} = 1$

(j) $\lambda_\alpha\,\lambda_T = 1$

(k) $\lambda_{c_1}\,\lambda_T\,\lambda_V^{-2} = 1$

(l) $\lambda_\gamma = 1$

(m) $\lambda_{k_1}\,\lambda_T\,\lambda_L^{-1}\,\lambda_V^{-3}\,\lambda_\rho^{-1} = 1$

(n) $\lambda_h\,\lambda_T\,\lambda_V^{-3}\,\lambda_\rho^{-1} = 1.$

2.3 Combined relationships for thermo-aeroelastic similarity

Relationships (a) to (n) are now combined and regrouped to provide expressions in terms of more familiar parameters; and we obtain:-

$$\left.\begin{array}{rcll}
\lambda_E\,\lambda_\rho^{-1}\,\lambda_V^{-2} & = & 1 & \text{from (a)} \\
\lambda_n & = & 1 & \text{" (b)} \\
\lambda_V\,\lambda_a^{-1} & = & 1 & \text{" (e)} \\
\lambda_\alpha\,\lambda_T & = & 1 & \text{" (j)} \\
\lambda_\sigma\,\lambda_\rho^{-1} & = & 1 & \text{" (c)} \\
\lambda_g\,\lambda_L\,\lambda_V^{-2} & = & 1 & \text{" (f)}
\end{array}\right\}\text{Aeroelastic parameters}\qquad
\begin{array}{r}(3)\\(4)\\(5)\\(6)\\(7)\\(8)\end{array}$$

$$\left.\begin{array}{rcll}
\lambda_p\,\lambda_V\,\lambda_L^{-1} & = & 1 & \text{from (g)} \\
\lambda_L\,\lambda_V\,\lambda_\rho\,\lambda_\mu^{-1} & = & 1 & \text{" (d)} \\
\lambda_{c_1}\,\lambda_\mu\,\lambda_{k_1}^{-1} & = & 1 & \text{" (d),(k) \& (m)} \\
\lambda_V\,\lambda_a^{-1} & = & 1 & \text{" (e)} \\
\lambda_{c_1}\,\lambda_T\,\lambda_V^{-2} & = & 1 & \text{" (k)} \\
\lambda_\gamma & = & 1 & \text{" (l)} \\
\lambda_h\,\lambda_L\,\lambda_{k_1}^{-1} & = & 1 & \text{" (m) \& (n)} \\
\lambda_g\,\lambda_L\,\lambda_V^{-2} & = & 1 & \text{" (f)}
\end{array}\right\}\begin{array}{l}\text{Fluid thermal}\\ \text{parameters}\end{array}\qquad
\begin{array}{r}(9)\\(10)\\(11)\\(5)\\(12)\\(13)\\(14)\\(8)\end{array}$$

$$\left.\begin{array}{rcll}
\lambda_h\,\lambda_L\,\lambda_k^{-1} & = & 1 & \text{from (i) \& (n)} \\
\lambda_c\,\lambda_\sigma\lambda_L^2\,\lambda_k^{-1}\,\lambda_p^{-1} & = & 1 & \text{" (c),(f),(h) \& (i)}
\end{array}\right\}\begin{array}{l}\text{Thermal parameters}\\ \text{for the structure.}\end{array}\qquad
\begin{array}{r}(15)\\(16)\end{array}$$

Equation (5) is the Mach number parameter, equation (8) is the Froude number parameter, equation (10) is the Reynolds number parameter, equation (11) is the Prandtl number parameter, equation (14) is the Nusselt number parameter. Equation (15) determines the flow of heat by conduction, and equation (16) may be recognised as the parameter determining the dissipation of heat within the material, since $\lambda_c\,\lambda_\sigma/\lambda_k$ is the thermal diffusivity parameter. Equations (5) and (8) appear both as aeroelastic and fluid thermal parameters. It is apparent that the Mach number parameter determined by equation (5) will influence the flow pattern for the fluid, and in consequence may be expected to influence both aerodynamic forces and hence aeroelasticity, and the flow of heat in the fluid. The Froude number parameter (equation (8)) determines static deflections for the structure under gravitational load, and though this may generally be expected to have a second order effect on aeroelasticity there are occasions where it has been significant. In association with equation (10), equation (8) may be reframed as the Grashof number, which is a significant parameter for the fluid in relation to free convective heat transfer. Again the effect may be regarded as of second order since it will generally only be significant for the fluid within the structure, forced convection being the over-riding effect for the external fluid. The effect of the Reynolds number parameter (equation (10)) on aeroelasticity is almost invariably ignored[1,2,3], and experience has generally indicated that this procedure is justified. As will be seen later, it is the necessity to satisfy this parameter in relation to fluid thermal effects that is a major factor in limiting the scope of models for thermo-aeroelastic investigations.

3 COMPATIBILITY OF THE SIMILARITY RELATIONSHIPS

It would appear that for there to be any likelihood of an adequate simulation of thermal effects for an aircraft structure the model must be a close replica of the aircraft, at least so far as the major load carrying parts of the structure are concerned. Even when this is achieved the assumption of similarity of conduction properties through joints, etc. must be made. Furthermore, it may occasionally be necessary to simulate parts of the structure that play a minor part in relation to strength, but whose heat capacity may have an appreciable effect on temperature distribution in the major structural members. On these grounds alone it is apparent that a thermo-aeroelastic model will present greater difficulties in design and construction than have been experienced in the past for purely aeroelastic models, the model structure for the latter often differing widely from that for the aircraft while still providing adequate similitude.

In the main, properties of both structural materials and fluids are affected by temperature, and in some cases significant anomalies in thermal properties as functions of temperature are obtained. Obviously, a completely general treatment of the problem is impracticable in these circumstances, and some simplifying assumptions defining variation of properties with temperature must be made. For our purposes it is accordingly assumed that all relevant properties vary in accordance with the law:-

$$\lambda_b = {}_o\lambda_b \, \lambda_T^{S_b} \qquad (17)$$

where ${}_o\lambda_b$ = property ratio at reference temperature

λ_b = property ratio for temperature ratio λ_T

λ_T = temperature ratio

S_b = constant related to particular property b.

Values for the properties of a selection of materials and gases are given in Tables 3 and 4. For these, the values for the exponents of λ_T that give a reasonably close agreement with the measured variation of property values with temperature are as follows:-

$$\left.\begin{aligned} S_n &= S_\sigma = S_k = S_\gamma = 0 && \text{(a)}\\ -S_E &= S_c = S_{c_1} = S_\alpha = 0.2 && \text{(b)}\\ S_a &= 0.5 && \text{(c)}\\ S_\mu &= 0.6 && \text{(d)}\\ S_{k_1} &= 0.8. && \text{(e)} \end{aligned}\right\} \qquad (18)$$

It is worth mentioning here that the condition $S_k = 0$ (equation (18a)) does not imply that there is no variation of material conductivity k with temperature, but rather that the measured variations for this quantity, though generally small in the temperature range of interest, are so inconsistent that they must be ignored.

The remaining quantities L, ρ, V, p, g and h are regarded as independent of temperature and are uniquely determined from the similarity relationships when other properties have been decided upon. The quantities λ_T, λ_L, λ_ρ, λ_V, λ_p, λ_g and λ_h are termed the "derived" ratios, while the λ_T, λ_ρ, λ_V are further termed the "disposable" ratios in the sense that they will generally be controllable to some degree in wind tunnel tests.

On the basis of the foregoing assumptions, the compatibility of the similarity equations is determined by first selecting a set of "leading" equations from equations (3) to (16) that enable the values for the derived ratios to be ascertained, and then substituting these values into the remaining equations to determine how closely the necessary conditions are satisfied.

For our purposes equations (3), (5) and (6) are taken as the leading equations. Past experience shows that equation (3) is of paramount importance for aeroelastic work, equation (5) which determines Mach number similarity is of obvious importance, and equation (6) determines thermal effect on structural stiffness which is the major effect of interest in the present investigation.

From equations (6) and (18b) we obtain

$$\lambda_T = {}_o\lambda_\alpha^{-\frac{1}{1.2}}. \qquad (19)$$

From equations (3) and (18c) we obtain

$$\lambda_V = {}_o\lambda_a \, \lambda_T^{0.5}. \qquad (20)$$

From equations (3), (18b) and (20) we obtain

$$\lambda_\rho = {}_o\lambda_E \, {}_o\lambda_a^{-2} \, \lambda_T^{-1.2} \qquad (21)$$

and from equations (10), (18d) and the above we obtain

$$\lambda_L = {}_o\lambda_E^{-1} \, {}_o\lambda_\mu \, {}_o\lambda_a \, \lambda_T^{1.3} \qquad (22)$$

From equations (14), (18e) and the above we then have

$$\lambda_h = {}_o\lambda_E \, {}_o\lambda_{k_1} \, {}_o\lambda_\mu^{-1} \, {}_o\lambda_a^{-1} \, \lambda_T^{-0.5}. \tag{23}$$

From equations (16), (18b) and the above we have

$$\lambda_p = {}_o\lambda_c \, {}_o\lambda_\sigma \, {}_o\lambda_k^{-1} \, {}_o\lambda_E^{-2} \, {}_o\lambda_\mu^{2} \, {}_o\lambda_a^{2} \, \lambda_T^{2.8} \tag{24}$$

and from equations (8) and the above we obtain

$$\lambda_g = {}_o\lambda_E \, {}_o\lambda_a \, {}_o\lambda_\mu^{-1} \, \lambda_T^{-0.3}. \tag{25}$$

Equations (4), (7), (9), (11), (12), (13) and (15) remain to be dealt with. The left hand sides of these equations are referred to as the "unity parameters", since unit values for these parameters are required if the similarity relationships are to be satisfied. Accordingly, the above derived ratios, together with known property ratios are substituted into the unity parameters, and the results compared with unity.

3.1 Aircraft and model tested in air

The above procedure has been followed in Table 5 for three possible aircraft materials (duralumin, titanium and stainless steel) and for a variety of model materials, in the particular case when both are tested in air.

Unit values for all the unity parameters, implying complete similarity, obtains only for the trivial case in which aircraft and model are identical in <u>all</u> respects (including size). Exact agreement when aircraft and model are of different materials is scarcely to be expected, for the relationship between the separate properties of a particular material is fixed, and to find two different materials with common property ratios would be fortuitous.

The derived ratio λ_g may also be regarded as a unity parameter, since in general a gravity ratio other than unity cannot be obtained in wind tunnel work*. In cases where the derived value of this parameter deviates significantly from unity it follows that representative conditions of deflection under gravitational load, and of free convective heat transfer cannot be obtained, and these effects on thermo-aeroelasticity must accordingly be ignored. Of the unity parameters the values of $\lambda_{c_1} \lambda_\mu \lambda_{k_1}^{-1}$ and λ_γ are unity for tests in air, and for most materials λ_n and $\lambda_{c_1} \lambda_T \lambda_V^{-2}$ are probably close enough to unity to be acceptable. The parameter $\lambda_p \lambda_\sigma^{-1}$ differs considerably from unity, but this parameter can be ignored in static aeroelastic problems (divergence, reversal, etc.), and is generally unimportant for the flutter of structures of high density and finite aspect ratio, provided there is little effect of frequency parameter variation on aerodynamic forces (see Fig.1).

---

* Gravity ratios other than unity in particular directions can of course be obtained using a whirling arm or on rocket models.

Furthermore, in those cases where the parameter has a value less than unity, and where the structure considered is of a hollow shell type, the possibility exists of achieving unit value for the parameter by filling the interior of the model structure with a foamed plastic or other substance thus increasing the effective model density. Providing the stiffness and thermal properties of the filling can be ignored no other parameters would be affected, though of course any simulation of internal convection effects would then be impracticable. The parameter $\lambda_p \lambda_V \lambda_L^{-1}$ also differs widely from unity indicating that certain time dependent aerodynamic effects are not to scale, e.g. the propagation of disturbances in the boundary layer. It seems unlikely that this parameter will have a significant effect for thermo-aeroelastic investigations and accordingly it is assumed that it can be neglected.

The remaining parameter $\lambda_h \lambda_L \lambda_k^{-1}$ is generally of major importance since it determines similarity of heat transfer from fluid to the structure; but unfortunately the parameter differs widely from unity. Even if a deviation from unity of say ±15% is regarded as acceptable for the unity parameters there are still remarkably few materials that could be used for model making. Furthermore, none of the materials considered enables a model smaller than about 0.4 times the size of the aircraft to be made, whereas if effective use is to be made of wind tunnels for thermo-aeroelastic work a range of model scale down to about 0.02 times the aircraft size is desirable.

3.2 Aircraft and model tested in different gases

The possibility exists of satisfying the similarity relationships and also achieving some benefit with regard to model scale by testing the model in a gas other than air.

From equation (22) it follows that once the materials for aircraft and model are determined the model scale will be reduced as compared with the air: air case if the model can be tested in a gas for which

$${}_o\lambda_\mu \; {}_o\lambda_a > 1. \qquad (26)$$

However, in order that the dynamic temperature difference parameter lies within the supposed allowable tolerance of ±15% we require (from equations (11), (18b) and (20))

$$0.85 < {}_o\lambda_{c_1} \; {}_o\lambda_a^{-2} \; \lambda_T^{0.2} < 1.15. \qquad (27a)$$

For the materials considered in Table 5, $\lambda_T^{0.2}$ lies within the range

$$0.8 < \lambda_T^{0.2} < 1.2$$

and since the materials listed in the table may be regarded as representative of the possible range of materials for model work, it follows that the dynamic temperature requirement is only likely to be satisfied with gases for which

$$0.7 < {}_o\lambda_{c_1} \; {}_o\lambda_a^{-2} < 1.4. \qquad (27b)$$

Finally we also require

$$0.85 < {}_o\lambda_{c_1} \; {}_o\lambda_\mu \; {}_o\lambda_{k_1}^{-1} < 1.15 \qquad (28a)$$

$$0.85 < {}_o\lambda_\gamma < 1.15. \qquad (28b)$$

Air : gas property ratios for various gases are given in Table 4, and it can be seen that conditions (26), (27) and (28) above are satisfied only by methane, air and carbon dioxide (of the gases considered). Of these, carbon dioxide is the most suitable gas in relation to model scale. A minimum model size 0.5 times the aircraft size for tests in air is reduced to 0.3 times the aircraft size for tests in carbon dioxide.

It is apparent that some benefit in reducing the size of the model is obtained by testing in carbon dioxide*, but the minimum model size is still too great for general work.

4 APPROXIMATION TO SIMILARITY

Quite obviously little can be achieved if complete similarity is attempted; which is only to be expected since in the absence of "free" parameters any satisfaction of the similarity conditions is largely fortuitous. A free parameter is one whose value can be varied at will between aircraft and model, and accordingly we will consider possible approximations for thermo-aeroelastic similarity that will provide the necessary free parameters.

It has already been mentioned in Section 1 that the transient effects of kinetic heating may occasionally be more critical from the thermo-aeroelastic viewpoint, than effects when a static condition obtains, due to the rapid variation of stiffness with time that may occur[5].

We will accordingly consider the approximations that may be acceptable in a simulation of the transient heating phase.

Consider an element dx dy dz at the surface of the structure. Co-ordinate x is measured normal to the surface, and co-ordinates y and z lie within the surface (see Fig.2).

The rate of flow of heat into the element through the face dx dy at z is:-

$$Q \quad = \quad - k \, dx \, dy \left(\frac{\partial T}{\partial z}\right)$$

and the rate of flow of heat out of the element through the face dx dy at z + dz is:-

$$Q + \frac{\partial Q}{\partial z} dz \quad = \quad - k \, dx \, dy \left(\frac{\partial T}{\partial z}\right) - k \, dx \, dy \, dz \left(\frac{\partial^2 T}{\partial z^2}\right)$$

so that the rate of gain of heat through the face dx dy is:-

$$k \, dx \, dy \, dz \left(\frac{\partial^2 T}{\partial z^2}\right) . \qquad (a)$$

Similarly the rate of gain of heat through the face dx dz is:-

$$k \, dx \, dy \, dz \left(\frac{\partial^2 T}{\partial y^2}\right) . \qquad (b)$$

---

* Unfortunately, carbon dioxide may be unsuitable for supersonic tunnel work because its temperature of liquefaction (-78°C at standard pressure) is marginal. It is worth noting that investigations that have been made by Chapman[10] on mixtures of less well known gases indicate that wide variations of thermal properties can be achieved. It may be that a mixture could be evolved with better properties than CO_2 for thermo-aeroelastic work.

Due to heat transfer at the surface dy dz that is in contact with the fluid there will be a further rate of flow of heat into the element, given by:-

$$h \, dy \, dz \, (T' - T) \tag{c}$$

where $(T' - T)$ is the difference in temperature between fluid and surface.

The rate of flow of heat out of the element through the face dy dz at dx from the surface will be:-

$$- k \, dy \, dz \left(\frac{\partial T}{\partial x}\right) \tag{d}$$

and the total gain of heat by the element is ultimately:-

$$c \, \sigma \, dx \, dy \, dz \, \frac{\partial T}{\partial p} \, . \tag{e}$$

Hence, from (a), (b), (c), (d) and (e) we can formulate the heat flow equation for the surface of the structure, namely

$$\frac{\partial T}{\partial p} = \frac{k}{c\sigma}\left(\frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right) T + \frac{k}{c\sigma \, dx}\left(\frac{\partial T}{\partial x}\right) + \frac{h}{c\sigma \, dx}\,(T'-T) \, . \tag{29}$$

For the aircraft, equation (29) may be written:-

$$\frac{\partial T_A}{\partial p_A} = \frac{k_A}{c_A \, \sigma_A}\left(\frac{\partial^2}{\partial y_A^2} + \frac{\partial^2}{\partial z_A^2}\right) T_A + \frac{k_A}{c_A \, \sigma_A \, dx_A}\left(\frac{\partial T_A}{\partial x_A}\right) + \frac{h_A}{c_A \, \sigma_A \, dx_A}\,(T'_A - T_A) \tag{30a}$$

and in terms of model parameters this becomes:-

$$\frac{\lambda_T}{\lambda_p}\frac{\partial T_m}{\partial p_m} = \frac{\lambda_k \, \lambda_T}{\lambda_c \, \lambda_\sigma \lambda_L^2}\,\frac{k_m}{c_m \, \sigma_m}\left(\frac{\partial^2}{\partial y_m^2} + \frac{\partial^2}{\partial z_m^2}\right) T_m + \frac{\lambda_k \, \lambda_T}{\lambda_c \, \lambda_\sigma \, \lambda_L^2}\,\frac{k_m}{c_m \, \sigma_m \, dx_m}\left(\frac{\partial T_m}{\partial x_m}\right) + \frac{\lambda_h \, \lambda_T}{\lambda_c \, \lambda_\sigma \, \lambda_L}\,\frac{h_m}{c_m \, \sigma_m \, dx_m}\,(T'_m - T_m) \tag{30b}$$

where $\lambda_{h,c,\sigma,T,p,L}$ are the ratios of aircraft:model properties and $(x_A, y_A, z_A) = \lambda_L(x_m, y_m, z_m)$.

From equation (30) we can derive two equations to be satisfied for similarity of heat flow between aircraft and model, namely:-

$$\frac{\lambda_k \, \lambda_p}{\lambda_c \, \lambda_\sigma \, \lambda_L^2} = 1 \tag{31a}$$

$$\frac{\lambda_h \, \lambda_p}{\lambda_c \, \lambda_\sigma \, \lambda_L} = 1 \tag{31b}$$

and by combining and rearranging these two equations we obtain equations (15) and (16) of Section 2.3, namely:-

$$\lambda_h \lambda_L \lambda_k^{-1} = 1 \tag{15}$$

$$\lambda_c \lambda_\sigma \lambda_L^2 \lambda_k^{-1} \lambda_p^{-1} = 1. \tag{16}$$

4.1 Diffusivity effects ignored

A simple approximation is to ignore diffusivity effects entirely and to regard the structure as a heat sink of uniform temperature in the thickness direction into which heat flows due to heat transfer at the surface*.

On this basis equation (29) reduces to

$$\frac{\partial T}{\partial p} = \frac{h}{c \sigma t} (T'-T) \tag{32}$$

where t is the local thickness of the material. The two similarity relationships (15) and (16) thus reduce to the single relationship

$$\frac{\lambda_h \lambda_p}{\lambda_c \lambda_\sigma \lambda_{t/L} \lambda_L} = 1. \tag{33}$$

4.1.1 Solid structure

For a solid structure t/L determines the wing thickness chord ratio, and since flutter at supersonic speeds is sensitive to this parameter, $\lambda_{t/L}$ must be unity.

In this circumstance the effect of ignoring diffusivity is simply to eliminate the unity parameter $\lambda_h \lambda_L \lambda_k^{-1}$ and to modify the derived ratio λ_p, the remaining derived ratios being unaffected. The expression for λ_p is

$$\lambda_p = {}_o\lambda_c \; {}_o\lambda_\sigma \; {}_o\lambda_E^{-2} \; {}_o\lambda_{k_1}^{-1} \; {}_o\lambda_\mu^2 \; {}_o\lambda_a^2 \; \lambda_T^2 . \tag{34}$$

Minimum model scale is again limited to about 0.4 x full scale (see Table 5).

4.1.2 Thin shell structure

For a thin shell structure without internal webs the possibility exists of treating the skin thickness as a free parameter, in the sense that the scale of skin thickness can differ from the linear model scale. In practice of course some limitation on thickness variation for the model has to be imposed; for our purposes we will suppose the limiting condition to be given by:-

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* This assumption implies that it is only the mid-plane stresses that are important from the aeroelastic viewpoint. While this assumption may be justifiable for a thin wing, it becomes progressively more difficult to justify as the thickness chord ratio is increased.

$$0.4 < \lambda_{t/L} < 2.5 \qquad (35)$$

where t/L is the ratio of skin thickness to size.

For a stressed skin structure, assuming uniform stress across the material thickness, variation of skin thickness can be regarded as an effect on elastic modulus such that the effective modulus ratio becomes $\lambda_E \lambda_{t/L}$. Similarly there is an effect on wing density such that the effective material density becomes $\lambda_\sigma \lambda_{t/L}$. On this basis the expressions for the modified derived ratios become:-

$$\lambda_L = {}_o\lambda_E^{-1} \; {}_o\lambda_\mu \; {}_o\lambda_a \; \lambda_T^{1.3} \; \lambda_{t/L}^{-1} \qquad (36)$$

$$\lambda_h = {}_o\lambda_E \; {}_o\lambda_{k_1} \; {}_o\lambda_\mu^{-1} \; {}_o\lambda_a^{-1} \; \lambda_T^{-0.5} \; \lambda_{t/L} \qquad (37)$$

$$\lambda_p = {}_o\lambda_c \; {}_o\lambda_\sigma \; {}_o\lambda_E^{-2} \; {}_o\lambda_{k_1} \; {}_o\lambda_\mu^{2} \; {}_o\lambda_a^{2} \; \lambda_T^{2} \; \lambda_{t/L}^{-1} \; . \qquad (38)$$

In this case the minimum model scale is limited to about 0.2 x full scale.

For a thin shell wing with internal webs two extreme assumptions that can be made are that:-

(i) the effect of heat flow into the webs can be neglected entirely on the grounds that because of low conduction properties of skin web joints the maximum temperature differences between skin and web exist before the web temperature changes appreciably. This implies that the maximum thermal stress, and the maximum thermal effect on wing stiffness in the transient phase occurs before there is any significant change of temperature for the web,

(ii) heat flow into the webs is governed by the same considerations as heat flow into the skin.

The effect of assumption (i) is that skin thickness remains a free parameter, which is of some value in enabling the similarity relationships to be satisfied. The assumption is likely to be more nearly satisfied for a riveted structure than one with integral webs. Assumption (ii) on the other hand reduces the considerations for the hollow wing to those for a solid wing, since the depth of the web must be taken into account.

It is apparent that even accepting the above simplifying assumptions the materials for model making are limited, and model scale is too restricted relative to the average size of tunnel working section.

4.2 Diffusivity normal to material thickness ignored

As an alternative to the procedure outlined in Section 4.1, we will consider the case where diffusivity normal to material thickness is ignored but is included in the thickness direction. This procedure would cater in part for the case where the material is so thick that temperature gradient in the thickness direction cannot be ignored.

Equation (29) reduces to:-

$$\frac{\partial T}{\partial p} = \frac{k}{c\,\sigma\,dx}\left(\frac{\partial T}{\partial x}\right) + \frac{h}{c\,\sigma\,dx}\,(T'-T) \qquad (39)$$

from which (since $x_A = \lambda_t \, x_m$) we obtain two similarity equations to replace equations (15) and (16); namely:-

$$\lambda_h \, \lambda_L \, \lambda_k^{-1} \, \lambda_{t/L} = 1 \tag{40}$$

$$\lambda_c \, \lambda_\sigma \, \lambda_L^2 \, \lambda_k^{-1} \, \lambda_p^{-1} \, \lambda_{t/L}^2 = 1 \, . \tag{41}$$

4.2.1 Solid structure

In this case, since $\lambda_{t/L} = 1$, equations (40) and (41) reduce to equations (15) and (16) of Section 2. Accordingly the limitations on model size are those that apply when complete similarity is attempted (Section 3).

4.2.2 Thin shell structure

For the thin shell structure without webs, or where the effect of web heating can be ignored $\lambda_{t/L}$ may be treated as a free parameter (within the limits of equation (35)) in which case equations (36) and (37) hold. It can be seen from Table 5 that $\lambda_{t/L} \geqslant 0.4$ is of some benefit in enabling equation (40) to be satisfied and in affecting some reduction in model size. In fact minimum sizes are comparable with those of Section 4.1.2, but are still too great for general wind tunnel work.

4.3 Diffusivity in the thickness direction ignored

This approximation would apply to circumstances in which large temperature gradients exist in the skin plane; as might occur, for example, if some parts of the structure were insulated while others were not.

Equation (29) reduces to

$$\frac{\partial T}{\partial p} = \frac{k}{c\sigma} \left(\frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \right) T + \frac{h}{c \, \sigma \, dx} \, (T' - T) \tag{42}$$

from which we obtain two similarity relationships to replace equations (15) and (16); namely,

$$\lambda_h \, \lambda_L \, \lambda_k^{-1} \, \lambda_{t/L}^{-1} = 1 \tag{43}$$

$$\lambda_c \, \lambda_\sigma \, \lambda_L^2 \, \lambda_k^{-1} \, \lambda_p^{-1} = 1 \, . \tag{16}$$

4.3.1 Solid structure

For this case the limitations on model size are those that apply when complete similarity is attempted (Section 3).

4.3.2 Thin shell structure

A new expression for λ_p obtains; namely,

$$\lambda_p = {}_o\lambda_c \; {}_o\lambda_\sigma \; {}_o\lambda_k^{-1} \; {}_o\lambda_E^{-2} \; {}_o\lambda_\mu^2 \; {}_o\lambda_a^2 \; \lambda_T^{2.8} \; \lambda_{t/L}^{-1} \, . \tag{44}$$

All other derived ratios are as for Section 4.2.2.

To achieve any benefit in model scale we require $\lambda_{t/L} < 1$, but it can be seen from Table 5 that equation (43) is then satisfied only for a very restricted range of materials, since $\lambda_h \lambda_L \lambda_k^{-1}$ is generally greater than unity.

Model sizes are generally less favourable than for the assumptions of Sections 4.1.2 or 4.2.2.

4.4 Reynolds number similarity ignored

A powerful parameter in limiting model scale is the Reynolds number parameter, and much could be achieved were it feasible for this parameter to be ignored. As has already been mentioned in Section 2.3 it is a common practice to ignore Reynolds number for purely aeroelastic investigations, and Reynolds number ratios of the order 50 : 1 have been accepted on occasions, apparently without any major detrimental effect*. It is obviously worth considering whether latitude in this parameter can be accepted in the thermal regime, and what benefits are likely to result.

The necessity to satisfy the Reynolds number relationship completely derives in part from the necessity to obtain similarity of heat transfer between fluid and model in regions of both laminar and turbulent flow. However, there may be occasions when the main load carrying part of the structure lies <u>wholly</u> within either a laminar or turbulent flow region, in which case it may be adequate to consider specific heat transfer relationships for one type of flow. From a consideration of the theory of heat transfer for flat plates in <u>incompressible</u> flow[9] it appears that a relationship between Nusselt number and Reynolds number can be obtained of the form

$$\lambda_h \lambda_L \lambda_{k_1}^{-1} = \lambda_{Re}^r \tag{45}$$

all other things being equal; where the exponent r has the value 0.5 for laminar flow and 0.8 for turbulent flows.

If we suppose that this same equation also holds for the compressible flow regime and is applicable to aerofoils, then on substituting for λ_{Re} (equation (10)) the above equation becomes:-

$$\lambda_{k_1} \lambda_h^{-1} \lambda_L^{r-1} \lambda_V^r \lambda_\rho^r \lambda_\mu^{-r} = 1 \tag{46}$$

and this now replaces the two basic equations (10) and (14) of Section 2.3.

Proceeding as in Section 3, we have from the leading equation:-

$$\lambda_T = {}_o\lambda_a^{-\frac{1}{1.2}} \tag{19}$$

$$\lambda_V = {}_o\lambda_a \lambda_T^{0.5} \tag{20}$$

$$\lambda_\rho = {}_o\lambda_E \, {}_o\lambda_a^{-2} \lambda_T^{-1.2} \tag{21}$$

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* This remark applies specifically to main surface flutter. Reynolds number is likely to be of greater importance for the flutter of control surfaces and tabs.

From equations (46), (15) and the above we obtain

$$\lambda_L = {}_o\lambda_E^{-1}\ {}_o\lambda_a\ {}_o\lambda_\mu\ \lambda_T^{1.3}\ ({}_o\lambda_h^{-1}\ {}_o\lambda_{k_1}\ \lambda_T^{0.8})^{-\frac{1}{r}} \tag{47}$$

and it then follows that

$$\lambda_h = {}_o\lambda_E\ {}_o\lambda_{k_1}\ {}_o\lambda_a^{-1}\ {}_o\lambda_\mu^{-1}\ \lambda_T^{-0.5}\ ({}_o\lambda_k^{-1}\ {}_o\lambda_{k_1}\ \lambda_T^{0.8})^{(\frac{1}{r}-1)} \tag{48}$$

$$\lambda_p = {}_o\lambda_c\ {}_o\lambda_\sigma\ {}_o\lambda_k^{-1}\ {}_o\lambda_E^{-2}\ {}_o\lambda_a^{2}\ {}_o\lambda_\mu^{2}\ \lambda_T^{2.8}\ ({}_o\lambda_k^{-1}\ {}_o\lambda_{k_1}\ \lambda_T^{0.8})^{-\frac{2}{r}} \tag{49}$$

$$\lambda_g = {}_o\lambda_E\ {}_o\lambda_a\ {}_o\lambda_\mu^{-1}\ \lambda_T^{-0.3}\ ({}_o\lambda_k^{-1}\ {}_o\lambda_{k_1}\ \lambda_T^{0.8})^{\frac{1}{r}}. \tag{50}$$

Equations (47)-(50) may be compared with equations (22)-(25) of Section 3. They differ only by a factor that is a power of the quantity ${}_o\lambda_k^{-1}\ {}_o\lambda_{k_1}\ \lambda_T^{0.8}$.

The value of λ_L in the present case is simply the value of λ_L from Table 5 multiplied by $({}_o\lambda_k^{-1}\ {}_o\lambda_{k_1}\ \lambda_T^{0.8})^{-\frac{1}{r}}$ - and since the main interest is in increasing the value of λ_L, model materials are required with the property:-

$${}_o\lambda_k^{-1}\ {}_o\lambda_{k_1}\ \lambda_T^{0.8} > 1.$$

However, the error in Reynolds number ratio that can be tolerated will not be unlimited, and accordingly we will stipulate an extreme value for this ratio of $\lambda_{Re} < 50$.

Since

$$\lambda_{Re} = ({}_o\lambda_k^{-1}\ {}_o\lambda_{k_1}\ \lambda_T^{0.8})^{-\frac{1}{r}}$$

it follows that model materials are required for which

$$50 > ({}_o\lambda_k^{-1}\ {}_o\lambda_{k_1}\ \lambda_T^{0.8})^{-\frac{1}{r}} > 1. \tag{51}$$

It is readily shown that the quantity ${}_o\lambda_k^{-1}\ {}_o\lambda_{k_1}\ \lambda_T^{0.8}$ is identical with the unity parameter $\lambda_h\ \lambda_L\ \lambda_k^{-1}$ of Table 5.

With the above assumptions a number of materials are acceptable for models of a duralumin aircraft, but for titanium or steel aircraft only the different grades of steel appear to fulfil the requirement. The optimum materials for minimum model size are as follows:-

(a) Major load carrying structure in laminar flow region, $r = 0.5$

Aircraft material	Model material	Model size
Duralumin	Carbon steel	0.026 × aircraft size
Titanium	Nickel steel	0.072 × aircraft size
Chrome steel	Nickel steel	0.15 × aircraft size

(b) Major load carrying structure in turbulent flow region, $r = 0.8$

Aircraft material	Model material	Model size
Duralumin	Chrome steel	0.039 × aircraft size
Titanium	Nickel steel	0.19 × aircraft size
Chrome steel	Nickel steel	0.38 × aircraft size

In both cases the minimum model sizes are well within the useful range for wind tunnel work.

4.4.1 Diffusivity ignored

When the Reynolds number assumption is combined with the assumptions of Section 4.1, the optimum conditions with regard to model scale are realised. Practically all the materials listed in Table 5 could be used for model making, the Reynolds number limitation $\lambda_{Re} < 50$ being the only limiting factor. In particular, models 0.02 × the aircraft size could be made using the same materials for aircraft and model*.

In the same way combining the Reynolds number assumption with the assumptions of Sections 4.2 and 4.3 results in benefits in relation to materials available for model making and in relation to model size, though the benefits are not so great as for the above. In particular smaller scale models can be used if the flow is laminar than if it is turbulent. In this circumstance it is necessary to ensure that the areas of laminar and turbulent flow for model and aircraft correspond. This may necessitate roughening the model surface to ensure transition in appropriate regions, as the Reynolds number for the model will be lower than that for the aircraft.

5 COMPOSITE STRUCTURES

In practice it may prove convenient to build the structure with more than one material. For example a steel skin may be used on an inner structure of light alloy, or an insulating cover may be applied to the outer surface of the structure. This obviously results in additional complication as regards thermo-aeroelastic simulation.

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* This assumes the condition $\lambda_p \lambda_V \lambda_L^{-1} = 1$ can be neglected. This condition could, however, be introduced as a leading equation, when we have

$$\lambda_L = {}_o\lambda_E^{-1} \; {}_o\lambda_a \; {}_o\lambda_\mu \; {}_o\lambda_T^{1.3} \; \lambda_{t/L}^{-1} \left({}_o\lambda_E^{-1} \; {}_o\lambda_c \; {}_o\lambda_\sigma \; {}_o\lambda_{k_1}^{-1} \; {}_o\lambda_a^2 \; {}_o\lambda_\mu \; \lambda_T^{1.2} \right)^{\frac{1}{r-1}} .$$

The range of model scale would then be more restricted.

5.1 More than one structural material

If both structural materials for the aircraft are exposed to the airflow and are major stress carriers then all the similarity relationships must be satisfied by both. Furthermore, for compatibility between the multiple materials of aircraft and model identical values for the derived ratios and unity parameters for each material must obtain. For example, if the aircraft is made of two materials of conductivities k_1, and k_2, and the model is made of two materials of conductivities k_3 and k_4, then we require:-

$$\frac{k_1}{k_2} = \frac{k_3}{k_4} \qquad (52)$$

and ideally a similar relationship must apply to all other material properties. Such conditions will obviously be practically impossible to satisfy without a great deal of approximation.

5.2 Insulated structures

The principal effect of the application of an insulating layer to the outer surface of a structure is to reduce the rate of heat flow into the structure. If the insulation also has appreciable thermal capacity then a rapid rise of temperature on the outer surface will be attenuated by the insulation and in consequence the temperature rise on the inner surface will be more gradual.

The effect of reduced rate of heat flow is to expand the time scale for the transient phase in which a reduction of stiffness may occur, and consequently diffusion of heat in the structure will have a relatively larger effect thus reducing the severity of the fall in stiffness. By the same token a rapid rise in temperature (thermal shock) is the most severe condition for loss of stiffness.

The difficulties in simulating the insulation will depend to some extent on whether the insulation is stress carrying or has an appreciable heat capacity. In cases where the stress and heat capacity of the insulation can be neglected the quantity that determines the flow of heat through the insulation is its conductivity, and for similarity between aircraft and model we accordingly require the insulation to satisfy equation (15) only. If the insulation has appreciable mass, then equation (7) would also be affected, but in most of the foregoing work it has been assumed that the latter equation is relatively unimportant.

If we consider thickness of the insulation as a free parameter then the equation to be satisfied is

$$\lambda_h \lambda_L \lambda_{k_i}^{-1} \lambda_{t/L} = 1 \qquad (53)$$

where λ_{k_i} is the ratio of insulation conductivities for aircraft and model.

6 STABLE THERMAL CONDITIONS

In the absence of any device for cooling the structure, a condition will ultimately be reached in which all points of the structure are at a temperature corresponding to the recovery temperature of the flow. Thermal stresses then arise only if the structure is of the composite type; otherwise the effect is simply an effect of temperature on material properties. Accordingly, it is sufficient in this case to satisfy only the aeroelastic equations (3)-(9). Since all the relevant leading equations are the same as those that have been used for the foregoing analysis it follows that any of the models designed to satisfy the assumptions of the preceding sections would also provide a representative solution when a stable thermal condition obtains.

7 SUMMARY

The results of the foregoing investigations are best summarised in tabular form.

Similarity approximations	Gas flow conditions	Thermo-aeroelastic similarity
1. Complete similarity	Unrestricted	Complete similarity impossible for small scale models. By ignoring density effects, and accepting deviations from ideal values of ±15% in other parameters, models of different materials than the aircraft material may be acceptable. However model sizes less than 0.4 × aircraft size are impracticable.
2. Structural diffusivity ignored	Unrestricted	Many of the important similarity parameters can be closely satisfied. Models of shell type structures can be made using materials different than the aircraft material, but conditions are less favourable for solid structures. Model scale too restricted for general wind tunnel work.
3. Diffusivity normal to material thickness ignored	Unrestricted	As for (2) above. Model scale parameters somewhat less favourable, (i.e. larger models required).
4. Diffusivity in thickness direction ignored	Unrestricted	As for (2) above. Model scale parameters less favourable than for (3) above.
5. Reynolds number similarity ignored	(a) Laminar flow over major load carrying part of the structure.	Quite small scale models can be made using materials different than the aircraft material, though the choice of materials is very limited.
	(b) Turbulent flow over major load carrying part of the structure.	As above.
6. Reynolds number similarity and diffusivity ignored	(a) Laminar flow over major load carrying part of the structure. (b) Turbulent flow over major load carrying part of the structure.	A wide range of materials is available for model making including the same materials as the aircraft, and for all practical purposes there is no limitation on model size.

Similarity approximations	Gas flow conditions	Thermo-aeroelastic similarity
7. Reynolds number similarity and structure diffusivity normal to material thickness ignored.	(a) Laminar flow over major load carrying part of the structure.	Model scale more restricted than 6(a) above, but better than (3) above.
	(b) Turbulent flow over major load carrying structure.	Model scale more restricted than 6(b) above, but better than (3) above.
8. Reynolds number similarity and structural diffusivity normal to material thickness ignored.	(a) Laminar flow over major load carrying part of the structure.	Model scale more restricted than 7(a) above, but better than (4) above.
	(b) Turbulent flow over major load carrying structure.	Model scale more restricted than 7(b) above, but better than (4) above.

8 DISCUSSION

It is quite obvious that thermo-aeroelastic similarity in the widest sense is impossible to achieve for small scale models. By limiting consideration to conduction effects, by assuming the major load carrying parts of the structure are in regions where the flow is entirely laminar or entirely turbulent, and by assuming a specific relationship between Reynolds number and Nusselt number, an approach to similarity for small scale models seems feasible, but further data are required to check these assumptions.

At the same time, it is worth keeping in mind that thermal effects on aeroelasticity will not of necessity be large. For a solid wing under zero initial load the transient thermal effect can undoubtedly lead to a significant reduction in wing stiffness *for a small range of wing amplitudes*[4], but the effect is likely to be far less pronounced for the built up sections that are typical of aircraft construction, and will in any case be attenuated by the initial deflections of the aircraft structure under the aerodynamic loads of normal cruising conditions. In these circumstances it may well be that the difference in the thermal effect on stiffness between an exactly similar and an approximately similar model is of the second order.

Furthermore, even for models designed for tests without thermal effects, exact similarity between model and full scale structures is never achieved, though a close approximation to similarity is generally obtained. Consequently, experimental data for the model are rarely accepted as providing information directly applicable to the full scale aircraft. Instead, theoretical investigations are made to compare with the experimental results from the model, and the theoretical approach that provides correlation with the model experiments is then applied to the full scale structure. An extension of this approach to include thermal effects seems feasible, though many more measurements on the model will be required.

8.1 Model construction

There is, of course, little point in demonstrating that an approximation to thermo-aeroelastic similarity can be achieved for small scale models if it should then prove that the difficulties of model construction are unsurmountable. If the tunnel size is large enough for a model to be constructed using conventional riveting, welding and shaping procedures, the problem is simplified, but unfortunately this will rarely be the case.

It has already been mentioned that the construction must be a closer replica of that for the aircraft than is usually the case for purely aeroelastic models, and the construction of the latter is formidable enough. Furthermore, it would appear that few of the established techniques for the construction of small scale aeroelastic models[2,11] are likely to be applicable in the thermo-aeroelastic case.

Of the established techniques for construction of aeroelastic models the technique developed at Cornell University[12] of wrapping and gluing a skin to a prepared "former" is the one likely to have greatest application for thermo-aeroelastic work. Stress and thermal paths for the skin are represented, though the thermal effects of the "former" may not be negligible and temperature resistant glues are required. Difficulties arise when a composite structure is to be represented, and there is the possibility that the desired materials for model making are not available in sheet form. Also, the representation of internal webs is inconvenient.

To overcome some of these difficulties an alternative technique is in course of development by Guyett at R.A.E. It so happens that in many cases the materials that are desirable for small scale thermo-aeroelastic models are also materials for which established electro-plating procedures exist. Accordingly, it would seem straightforward enough to electro-plate a model skin of the desired thickness onto a prepared former. This procedure promises obvious advantages; for example, a skin with double curvature presents no problems, as it would by the wrapping procedure. Needless to say, in practice there are many difficulties to be surmounted; accurate control of the plating process is required to avoid built-in stresses for the model and it is difficult to ensure a deposit of the required thickness and uniformity. However, progress is being made and the technique promises well for the future.

In Fig.3, a nickel wing torsion box formed by electro-plating is shown. More complex structures, including internal webs, can be constructed without great difficulty; for example, leading and trailing edges can readily be plated onto the torsion box shown.

8.2 Wind tunnel facilities

In general existing wind tunnels for kinetic heating work do not provide for simultaneous control of Mach number, stagnation pressure and stagnation temperature during a tunnel run. Available tunnels are generally of the intermittent type running from compressed air storage through a pre-heated exchanger into a fixed Mach number working section, the only variable being stagnation pressure. Tests in these circumstances simulate thermal shock conditions in which the model is instantaneously accelerated to a particular Mach number.

In general these tests will be more severe than anything the aircraft itself will experience. Although it follows that aircraft will have an adequate margin of safety when cleared on the basis of such model tests, it may prove that to satisfy this margin an unacceptable structure weight penalty is imposed. Tunnels more capable of approximating the aircraft flight plan may thus be required.

The range of operating conditions for aircraft is very wide, particularly in terms of air density and Mach number. The range of static air temperature will be less extensive, and in many cases the operational role of the aircraft may be such that the static temperature can be regarded as constant (e.g. high Mach number flight restricted to heights greater than 37,000 ft static temperature -56.5°C).

It follows that controlled variation of density and Mach number is required for the wind tunnel, with related heat control to maintain the appropriate static temperature conditions in the working section as the Mach number is varied. At the same time continuous variation of Mach number without shut-down of the tunnel is by no means easy, requiring variation in the geometry of the liners. Methods have however been evolved for this; for example programmed deformation of the liner plates using hydraulic jacks, or a sliding block nozzle[13].

An unfortunate feature that arises in the simulation of aircraft conditions to model scale is that the stagnation density required for the model is higher than in the aircraft case. The smaller the model scale the higher the stagnation density required. This is very inconvenient for wind tunnel work since it implies that the tunnel shell must withstand a high degree of pressurisation, and the power requirements both for the tunnel drive and heat exchanger are increased. However, provided the expansion coefficient for the model material exceeds that for the aircraft the temperature scale for the model will be less than that for the aircraft, which is beneficial from the heat exchanger viewpoint.

A further factor that leads to economy in tunnel power requirements is the achievement of a reduced time scale for the model as compared with the aircraft, and this may have an important bearing on the choice of the model material. Of the materials considered in Table 5 silver is outstanding in this respect. However, it is obvious that a compromise may be required so far as reduced time scale is concerned, as the speed at which tunnel parameters can be varied and measurements recorded will be limited. It seems likely that because of the many variables involved and the need for rapid adjustments during a tunnel run, complete automation of tunnel control will be necessary. Furthermore, because of the difficulties in rapid variation of tunnel parameters in continuous as compared with intermittent tunnels, it seems almost certain that tunnels of the latter type will be used.

In Fig.4 a block diagram of the tunnel layout envisaged is given.

8.3 Rocket tests

In some circumstances free flight rockets or rocket sleds can be used for thermo-aeroelastic work. In general the reference datum temperature for such tests will be higher than in the aircraft case ($0.7 < \lambda_T < 1$) and hence model materials are required having lower coefficients of expansion than the aircraft material. It can be seen from Table 5 that although the choice of materials is limited the possibility nevertheless exists of constructing a thermo-aeroelastic model for rocket tests to be representative of conditions for the aircraft at a higher altitude.

9 AERODYNAMIC EFFECTS ABSENT

A less ideal approach to the problem, but one which may make less demands on facilities in the way of high stagnation density wind tunnels with heat control, is to build a structural model for investigating purely thermal effects on structural modes and stiffnesses, with heat supplied electrically. A separate model can then be constructed to represent the most adverse condition (from the aeroelastic viewpoint) obtained from the heating investigations, and this is tested in a wind tunnel with heating effects absent.

Disadvantages of this approach are that the effects on stiffness of thermal stress are generally highly non-linear with amplitude so that the stiffness to be represented must be defined in relation to a particular amplitude of displacement of the structure. As we are also concerned principally with stiffnesses in the transient phase, the time available for stiffness or frequency measurement may be so short as to require special measuring techniques to be developed. One technique in use at present is to oscillate the structure at a fixed amplitude, driving the system at a resonance frequency through a self tuning feedback network. However, all such systems require a finite time interval to stabilise the oscillation and for rapidly changing phenomena this may lead to errors in the measurements. Furthermore, although it may be feasible to construct a wind tunnel model having the stiffnesses determined for the thermal model, the stiffnesses will then be linear and in consequence the aeroelastic behaviour may differ from what would actually occur with heating present. Finally, this approach requires the prior determination and simulation of local heat transfer coefficients which themselves may have to be determined from wind tunnel or free flight tests on models.

However, despite its drawbacks, since this approach utilises existing facilities it is one that is likely to be followed to some extent.

10 CONCLUSIONS

The principal conclusion is that complete similarity of thermo-aeroelastic effects for an aircraft and model can only be obtained in the trivial case where the two have a 1 : 1 correspondence throughout.

A degree of similarity between the aircraft and a small scale model is practicable only if certain major simplifying assumptions can be justified. The main assumption is with regard to the importance of Reynolds number, in affecting heat transfer at the surface. Optimum conditions in determining model scale are achieved when a particular law of variation of heat transfer with Reynolds number for laminar and turbulent flows is assumed. The law assumed is determined for incompressible flow over a flat plate.

The requirements for wind tunnels for thermo-aeroelastic testing are considered, and it appears that existing wind tunnels are unlikely to be completely suitable. Accordingly a possible layout for a suitable tunnel is described. In view of the number of tunnel parameters that need to be varied to simulate the aircraft flight case, and the rapidity with which these variations must be made because of the reduced time scale for the model, it is envisaged that a completely automatic programmed control of the tunnel may be necessary.

It would appear that a limited use can be made of rocket test facilities, a necessary requirement in this case being a lower coefficient of expansion for the model material than for the aircraft.

The possibilities of investigating transient thermal effects on a purely structural model with aerodynamic effects absent are considered briefly and it seems likely that despite its difficulties this approach is likely to be followed to some extent, since facilities for the work already exist.

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TABLE 1

Exponents for aeroelastic quantities

Quantity with unit exponent	Exponents of selected quantities		
	L	V	ρ
E	0	-2	-1
n	0	0	0
σ	0	0	-1
μ	-1	-1	-1
a	0	-1	0
g	1	-2	0
p	-1	1	0

From the above table the solution determining aeroelastic effects is:-

$$f\left(\frac{E}{\rho V^2},\ n,\ \frac{\sigma}{\rho},\ \frac{\mu}{LV\rho},\ \frac{a}{V},\ \frac{gL}{V^2},\ \frac{pV}{L}\right) = 0.$$

TABLE 2

Exponents for thermal quantities

Quantity with unit exponent	Exponents of selected quantities			
	L	V	ρ	T
c	0	-2	0	1
k	-1	-3	-1	1
α	0	0	0	1
c_1	0	-2	0	1
γ	0	0	0	0
k_1	-1	-3	-1	1
h	0	-3	-1	1

From the above table, the solution determining thermal effects is:-

$$f\left(\frac{cT}{V^2},\ \frac{kT}{LV^3\rho},\ \alpha T,\ \frac{c_1 T}{V^2},\ \gamma,\ \frac{k_1 T}{LV^3\rho},\ \frac{hT}{V^3\rho}\right) = 0.$$

TABLE 3

Average material properties at 68°F

Material	Coefficient of expansion per °F x 10^6 α	Modulus of elasticity (lb/in^2) x 10^6 E	Modulus ratio n	Conductivity $\frac{B.T.U.}{hr\ ft\ ^{o}F}$ k	Specific heat $\frac{B.T.U.}{lb\ ^{o}F}$ c	Density lb/ft^3 σ
Duralumin	12.5	10.0	0.38	95	0.21	174
Nickel steel	3	29.5	0.39	6	0.11	490
Chrome steel	5.5	"	"	15	"	"
Carbon steel	7	"	"	30	"	"
Titanium	5	16.0	0.38	15	0.13	280
Magnesium alloy	16	6.5	0.37	40/87	0.24	112
Copper alloy	9.3	15.0	"	15/100	0.09	540
Glass	4.7	8.0	0.40	0.44	0.20	169
Bakelite	0.5	1.3	"	0.02	0.35	81
Cadmium	16.5	5.0	0.38	53	0.055	536
Chromium	3.7	30.0	-	40	0.062	440
Nickel	7.1	28.5	0.38	52	0.11	550
Gold	7.8	11.3	0.35	180	0.03	1200
Silver	10.6	11.1	0.37	242	0.056	650
Tin	11.7	7.7	0.375	37	0.054	453
Platinum	4.9	16.8	0.36	40	0.032	1340
Zinc	16.5	8.7	0.436	65	0.092	440

TABLE 4

Average gas properties at 32^{o}F and standard pressure

Gas	Density lb/ft^3 ρ	Specific heat at constant pressure BTU/lb ^{o}F c_1	Specific heat ratio γ	Dynamic viscosity lb sec/ft^2 μ	Conductivity $\frac{\text{BTU}}{\text{hr ft }^o\text{F}}$ k_1	Speed of sound ft/sec a	${}_o\lambda_\rho$	${}_o\lambda_{c_1}$	${}_o\lambda_\gamma$	${}_o\lambda_\mu$	${}_o\lambda_{k_1}$	${}_o\lambda_a$	${}_o\lambda_\mu\,{}_o\lambda_a$	${}_o\lambda_{c_1}\,{}_o\lambda_a^{-2}$	${}_o\lambda_{c_1}\,{}_o\lambda_\mu\,{}_o\lambda_{k_1}^{-1}$
Hydrogen	0.0054	3.41	1.41	1.73×10^{-7}	0.0950	4500	14.5	0.07	0.99	2.07	0.15	0.24	0.50	1.21	0.97
Helium	0.0108	1.26	1.63	3.80×10^{-7}	0.080	3200	7.2	0.19	0.86	0.95	0.18	0.34	0.32	1.65	1.0
Methane	0.0435	0.59	1.32	2.17×10^{-7}	0.0175	1410	1.79	0.41	1.06	1.66	0.80	0.77	1.28	0.69	0.85
Ammonia	0.048	0.51	1.31	1.95×10^{-7}	0.0123	1360	1.63	0.47	1.07	1.85	1.14	0.80	1.48	0.72	0.76
Air	0.078	0.24	1.40	3.6×10^{-7}	0.0140	1090	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
Carbon dioxide	0.120	0.20	1.30	2.9×10^{-7}	0.0082	850	0.65	1.20	1.08	1.24	1.69	1.28	1.59	0.73	0.88
Sulphur dioxide	0.176	0.15	1.28	2.40×10^{-7}	0.0044	690	0.44	1.60	1.09	1.50	3.18	1.58	2.37	0.64	0.76
Freon	0.310	0.16	1.16	2.44×10^{-7}	0.0048	470	0.25	1.50	1.21	1.48	2.92	2.32	2.43	0.28	0.76

Similarity relationships for aircraft and model tested in air

Aircraft material	Model material	Basic property ratios						Complete similarity													
								Derived property ratios for tests in air							Unity parameters for tests in air						
		$_0\lambda_\alpha$	λ_σ	$_0\lambda_E$	λ_n	λ_k	$_0\lambda_c$	λ_T	λ_V	λ_ρ	λ_L	λ_h	λ_p	λ_g	λ_n	$\frac{\lambda_\rho}{\lambda_\sigma}$	$\frac{\lambda_{c_1}\lambda_T}{\lambda_V^2}$	$\frac{\lambda_h\lambda_L}{\lambda_k}$	$\frac{\lambda_p\lambda_V}{\lambda_L}$	$\frac{\lambda_{c_1}\lambda_\mu}{\lambda_{k_1}}$	λ_γ
Duralumin	Duralumin	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0
	Nickel steel	4.2				15.8		0.30	0.55	1.34	0.65	0.58	0.014	0.46		3.83	0.79	0.024	0.012	"	"
	Chrome steel	2.3	0.35	0.32	0.98	6.3	1.91	0.50	0.71	0.73	1.28	0.45	0.15	0.39	0.98	2.08	0.87	0.091	0.083	"	"
	Carbon steel	1.8				3.2		0.61	0.78	0.57	1.69	0.41	0.54	0.36		1.63	0.91	0.21	0.25	"	"
	Titanium	2.5	0.62	0.62	1.0	6.3	1.61	0.47	0.69	1.55	1.41	0.90	0.28	0.33	1.0	2.50	0.86	0.09	0.14	"	"
	Magnesium	0.78	1.55	1.55	1.03	2.4 1.09	0.88	1.22	1.10	1.21	0.82	1.41	0.18 0.85	1.49	1.03	0.78	1.04	0.50 1.09	0.24 1.14	"	"
	Copper	1.35	0.32	0.68	1.03	6.30 0.95	2.33	0.83	0.91	0.92	1.18	0.75	0.07 1.19	0.7	1.03	2.9	0.96	0.13 0.89	0.05 0.92	"	"
	Glass	2.70	1.04	1.35	0.95	2.16	1.05	0.44	0.66	3.65	0.26	2.05	0.003	1.69	0.95	3.4	0.85	0.0024	0.008	"	"
	Cadmium	0.76	0.33	2.0	0.98	1.79	3.82	1.23	1.11	1.52	0.65	1.8	0.30	1.9	0.98	4.6	1.04	0.66	0.51	"	"
	Chromium	3.40	0.39	0.33	-	2.37	3.4	0.36	0.60	1.13	0.87	0.55	0.38	0.41	-	3.4	0.81	0.19	0.26	"	"
	Nickel	1.76	0.32	0.35	0.98	1.83	1.91	0.63	0.79	0.56	1.58	0.44	0.75	0.40	0.98	1.75	0.91	0.38	0.38	"	"
	Gold	1.60	0.15	0.89	1.09	0.53	7.0	0.68	0.82	1.43	0.70	1.09	0.88	0.97	1.09	9.5	0.92	1.35	1.03	"	"
	Silver	1.18	0.27	0.90	1.03	0.39	3.86	0.87	0.93	1.06	0.93	0.97	2.17	0.94	1.03	3.9	0.97	2.27	2.17	"	"
	Tin	1.07	0.38	1.30	0.99	2.57	3.89	0.94	0.97	1.39	0.72	1.34	0.30	1.31	0.99	3.65	0.99	0.37	0.41	"	"
	Platinum	2.55	0.13	0.60	1.06	2.37	6.6	0.46	0.68	1.53	0.61	0.88	0.12	0.75	1.06	1.17	0.86	0.31	0.13	"	"
	Zinc	0.76	0.40	1.15	0.87	1.46	2.3	1.23	1.11	0.88	1.11	1.04	0.80	1.11	0.87	2.2	1.04	0.83	0.80	"	"
Titanium	Duralumin	0.40	1.61	1.61	1.0	0.16	0.62	2.13	1.46	0.64	1.63	1.10	19.1	1.31	1.0	0.40	1.16	11.1	17.3	"	"
	Nickel steel	1.67				2.5		0.65	0.81	0.90	1.08	0.67	0.29	0.61		1.6	0.92	0.28	0.22		
	Chrome steel	0.9	0.57	0.54	0.98	1.0	1.18	1.02	1.05	0.49	2.13	0.52	3.1	0.52	0.98	0.87	1.01	1.00	1.52	"	"
	Carbon steel	0.71				0.5		1.35	1.16	0.38	2.7	0.47	10.2	0.50		0.67	1.06	2.56	4.37		
	Titanium	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	"	"
	Magnesium	0.31	2.5	2.5	1.03	0.38 0.17	0.54	2.63	1.62	0.77	1.42	1.54	8.9 19.2	1.85	1.03	0.31	1.21	5.56 12.5	10.1 21.8	"	"
	Copper	0.54	0.52	1.07	1.03	1.0 0.15	1.44	1.67	1.29	0.58	1.82	0.83	2.7 17.9	0.92	1.03	1.12	1.11	1.51 10.0	1.98 12.7	"	"
	Glass	1.06	1.66	2.0	0.95	34.0	0.65	0.95	0.97	2.12	0.48	2.06	0.007	1.98	0.95	1.27	0.99	0.028	0.014	"	"
	Cadmium	0.30	0.52	3.2	0.98	0.28	2.37	2.71	1.65	0.96	1.15	1.94	7.0	2.35	0.98	1.85	1.22	7.69	10.0	"	"
	Chromium	1.35	0.64	0.53	-	0.38	2.10	0.83	0.91	0.72	1.46	0.58	7.3	0.57	-	1.12	0.96	2.27	4.5	"	"
	Nickel	0.70	0.51	0.56	0.98	0.29	1.18	1.33	1.15	0.39	2.66	0.49	15.5	0.50	0.98	0.76	1.06	4.35	6.7	"	"
	Gold	0.64	0.23	1.42	1.09	0.08	4.33	1.43	1.20	0.91	1.12	1.18	16.0	1.28	1.09	4.0	1.07	16.7	17.2	"	"
	Silver	0.47	0.43	1.44	1.03	0.06	2.33	1.89	1.37	0.68	1.60	1.05	47.0	1.18	1.03	1.58	1.14	25.0	40.0	"	"
	Tin	0.43	0.62	2.08	0.99	0.41	2.41	2.0	1.41	0.90	1.20	1.48	5.9	1.67	0.99	1.45	1.15	4.17	6.9	"	"
	Platinum	1.02	0.21	0.95	1.06	0.38	4.07	0.90	0.99	0.99	1.03	0.96	2.34	0.88	1.06	4.7	1.00	2.63	2.25	"	"
	Zinc	0.30	0.64	1.84	0.87	0.23	1.42	2.71	1.65	0.55	1.97	1.11	18.6	1.38	0.87	0.86	1.22	10.0	15.5	"	"
Chrome steel	Duralumin	0.44	2.82	2.95	1.03	0.16	0.53	1.96	1.40	1.30	0.82	2.10	7.3	2.40	1.03	0.46	1.14	11.1	12.5	"	"
	Nickel steee	1.85				2.5		0.60	0.77	1.85	0.53	1.30	0.10	1.13		1.85	0.90	0.28	0.14		
	Chrome steel	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	1.0	"	"
	Carbon steel	0.79				0.5		1.22	1.10	0.79	1.36	0.91	3.84	0.93		0.79	1.04	2.56	3.2		
	Titanium	1.10	1.75	1.85	1.03	1.0	0.85	0.93	0.96	2.03	0.50	1.93	0.38	1.86	1.03	1.17	0.99	1.0	0.73	"	"
	Magnesium	0.34	4.40	4.45	1.05	0.38 0.17	0.46	2.44	1.56	1.54	0.71	2.91	3.3 7.3	3.44	1.05	0.35	1.20	5.56 12.5	7.3 16.0	"	"
	Copper	0.55	0.91	1.97	1.05	1.0 0.15	1.22	1.54	1.24	1.08	0.90	1.59	1.01 6.7	1.71	1.05	1.19	1.09	1.51 10.0	1.39 8.4	"	"
	Glass	1.17	2.90	3.7	0.98	34.0	0.55	0.87	0.93	4.3	0.22	3.97	0.0023	3.95	0.98	1.48	0.97	0.028	0.01	"	"
	Cadmium	0.33	0.92	5.9	1.03	0.28	2.00	2.50	1.58	1.95	0.56	3.74	2.5	4.47	1.03	2.12	1.20	7.69	7.0	"	"
	Chromium	1.50	1.11	0.98	-	0.38	1.77	0.71	0.84	1.47	0.65	1.17	2.16	1.09	-	1.32	0.93	2.27	2.8	"	"
	Nickel	0.77	0.89	1.03	1.03	0.29	1.0	1.22	1.11	0.79	1.26	0.93	5.25	0.97	1.03	0.89	1.04	4.35	4.60	"	"
	Gold	0.71	0.41	2.62	1.12	0.08	3.66	1.32	1.15	1.86	0.55	2.28	5.9	2.40	1.12	4.5	1.06	16.7	12.3	"	"
	Silver	0.52	0.75	2.66	1.05	0.06	1.97	1.72	1.31	1.38	0.75	2.03	15.4	2.30	1.05	1.84	1.11	25.0	27.0	"	"
	Tin	0.47	1.08	3.84	1.04	0.41	2.04	1.89	1.37	1.80	0.61	2.80	2.4	3.10	1.04	1.67	1.14	4.17	5.4	"	"
	Platinum	1.12	0.37	1.76	1.08	0.38	3.43	0.92	0.96	1.97	0.51	1.83	0.89	1.81	1.08	5.3	0.98	2.63	1.68	"	"
	Zinc	0.33	1.11	3.4	0.90	0.23	1.20	2.50	1.58	1.12	0.97	2.15	6.7	2.58	0.90	1.0	1.20	10.0	4.20	"	"

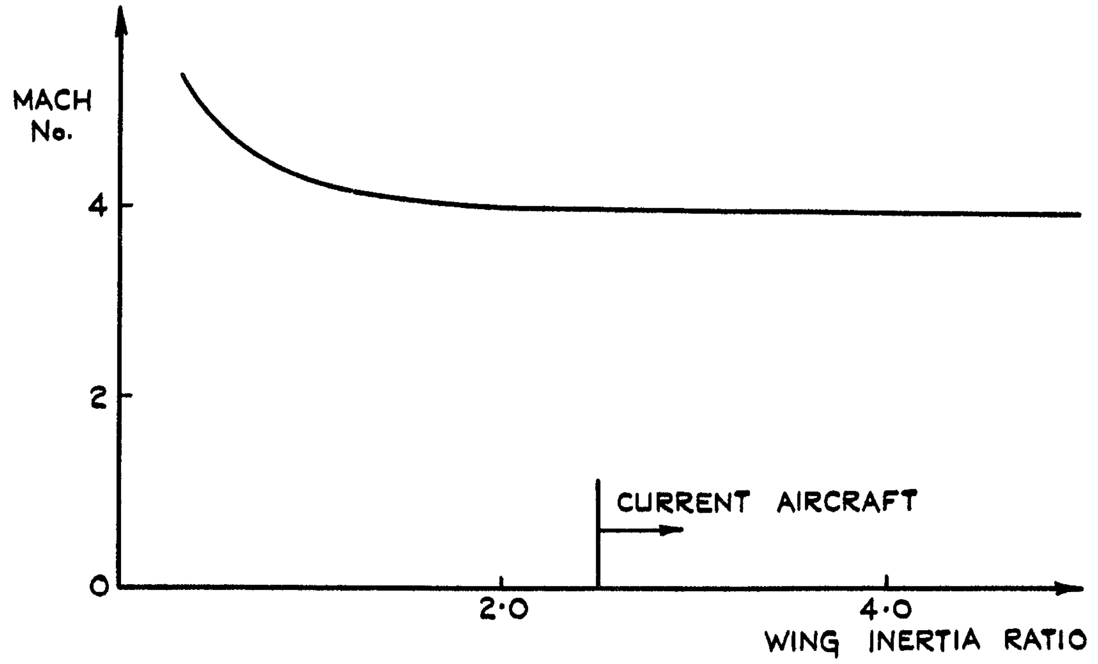

FIG. 1. EFFECT OF WING DENSITY ON WING FLUTTER.

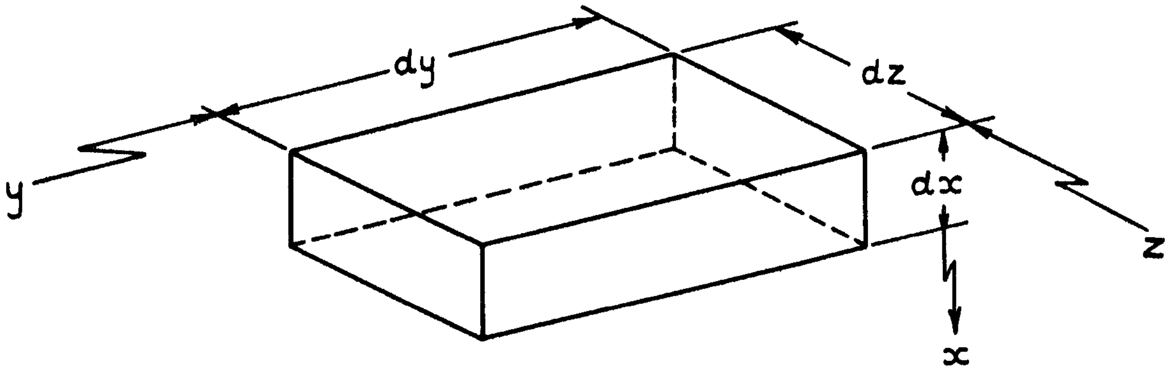

FIG. 2. ELEMENT OF WING SURFACE.

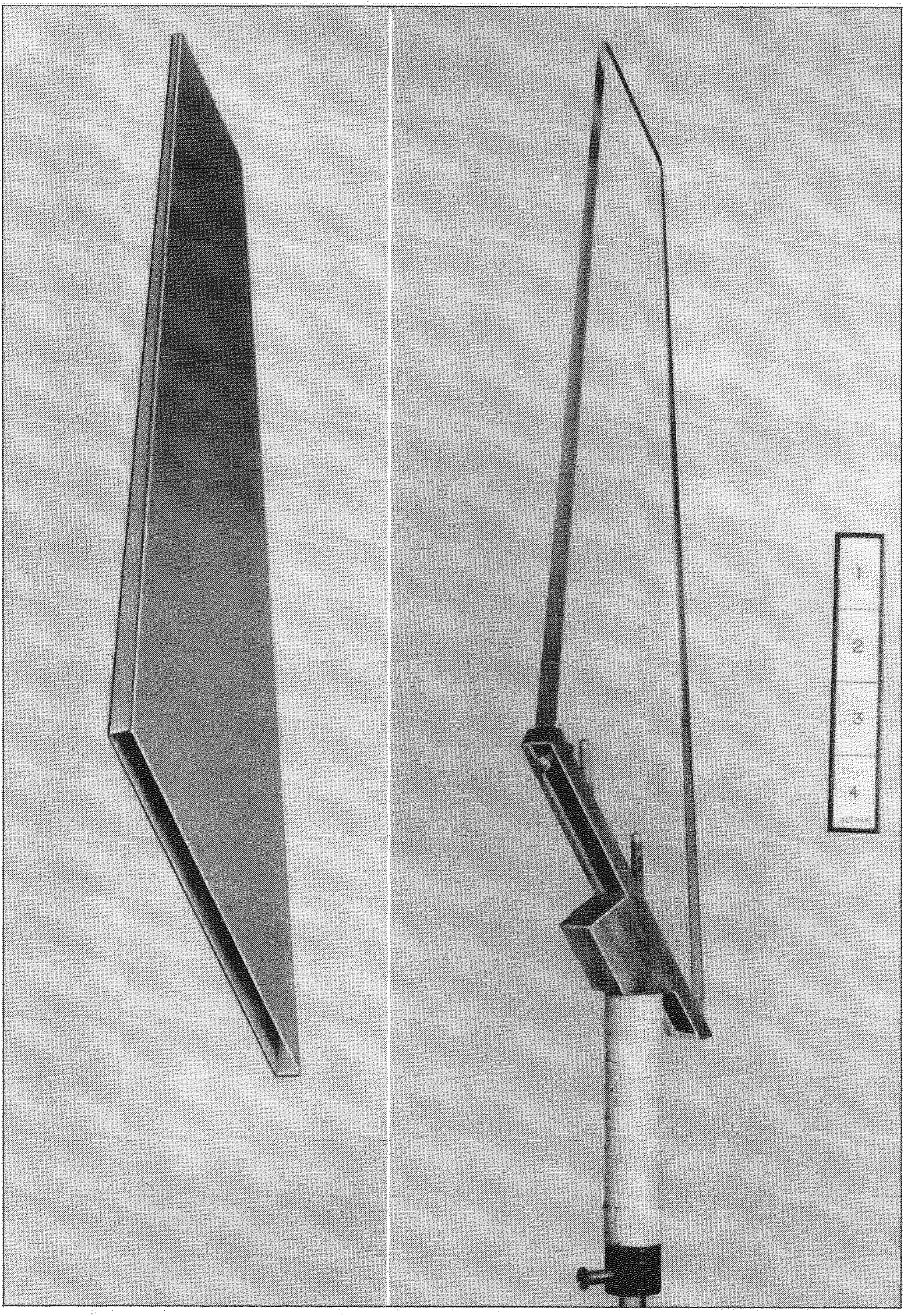

FIG.3. NICKEL PLATE TORSION BOX AND FORMER

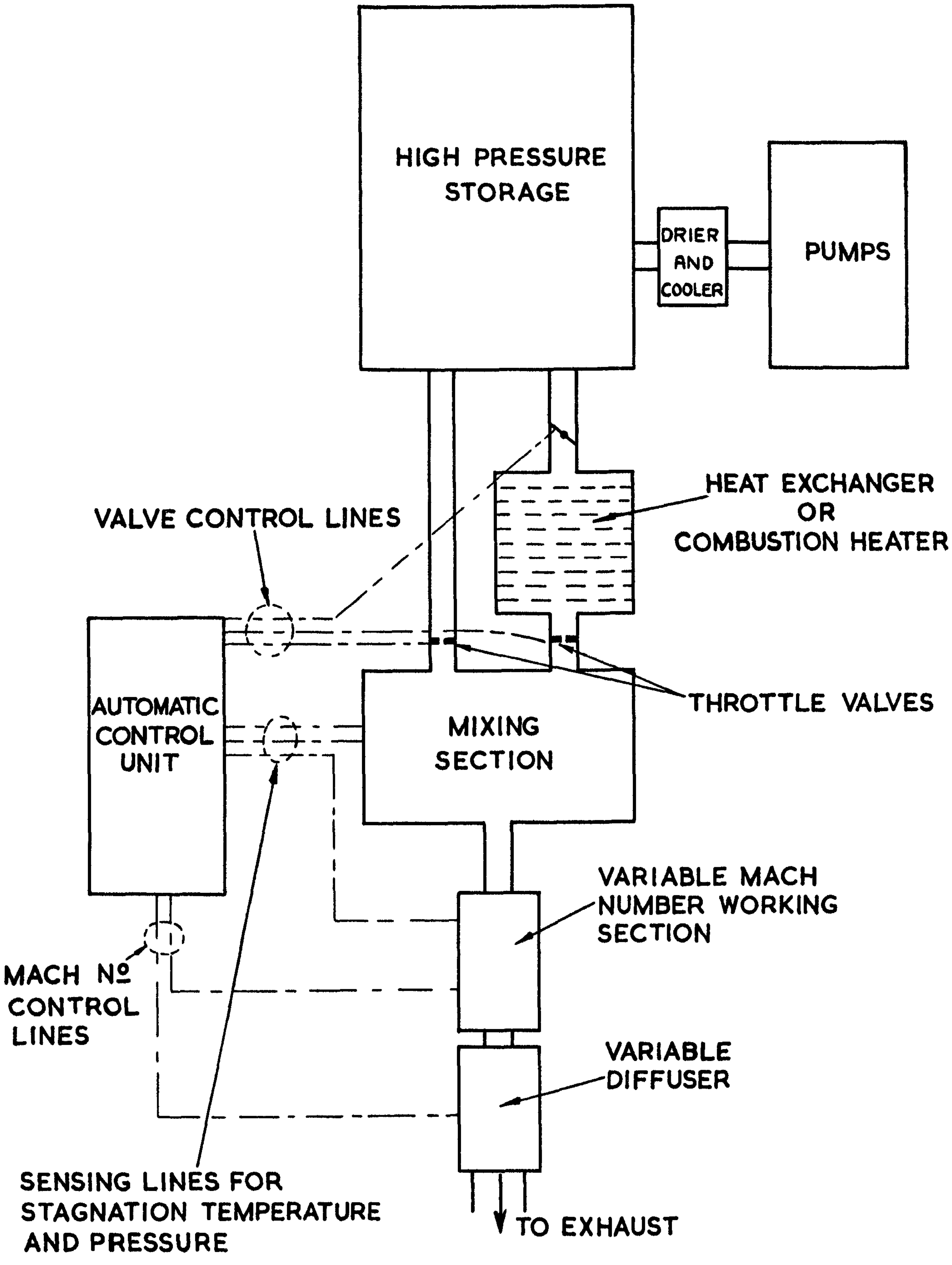

FIG. 4. POSSIBLE TUNNEL LAYOUT

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SCALE MODELS FOR THERMO-AEROELASTIC RESEARCH. Molyneux, W.G. March 1961.

An investigation is made of the parameters to be satisfied for thermo-aeroelastic similarity. It is concluded that complete similarity obtains only when aircraft and model are identical in all respects, including size.

By limiting consideration to conduction effects, by assuming the major load carrying parts of the structure are in regions where the flow is either entirely laminar, or entirely turbulent, and by assuming a specific relationship between Reynolds number and Nusselt number, an approach to similarity can be achieved for small scale models. Experimental and analytical work is required to check on the validity of these assumptions.

It appears that existing hot wind tunnels will not be completely adequate for thermo-aeroelastic work, and accordingly a possible layout for the type of tunnel required is described. Automatic programmed control of the tunnel would appear to be necessary.

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