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Comparison between Experimental
Measurements and a Suggested Formula
for the Variation of Turbulent
Skin-friction in Compressible Flow

By

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LONDON: HIS MAJESTY'S STATIONERY OFFICE

1951

Price 4s. 6d. net.

February, 1950

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Comparison between experimental measurements and a suggested formula for the variation of turbulent skin-friction in compressible flow

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SUMMARY

The formula suggested in ref.1 for the variation of turbulent skin friction with Mach number and heat transfer is

$$C_{F_i} = C_{F_w}$$

when

$$Re_i = Re_w \sqrt{T_1/T_w}$$

where subscript "i" refers to the incompressible values of skin friction coefficient (C_F) and Reynolds number (Re), subscript "w" means that density (ρ) and viscosity (μ) are to be evaluated at the absolute wall (skin) temperature (T_w) and T_1 is the absolute static temperature in the stream outside the boundary layer.

The experimental results in ref.1 gave a check on the worth of the formula only for flow over a flat plate under zero heat transfer conditions at $M_1 = 2.46$.

The present note extends this check by analysis of two sets of American test results^{2,4}.

The first set² covers flow over a flat plate under zero heat transfer conditions for Mach numbers between 1.9 and 2.2, while the second set⁴ covers subsonic flow through a circular pipe for temperature differences up to 684°C. In each case a good correlation is obtained on the basis of a known incompressible flow formula.

In the latter case, the heat transfer results can also be correlated by the same method, substituting k_h for C_F in the above formula.

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1 Introduction

Analysis¹ of measurements of the turbulent boundary on a flat plate, made under zero heat transfer conditions at $M_1 = 2.46$, suggested a formula for the variation of the turbulent skin friction coefficient with Mach number and heat transfer. It is

$$\left. \begin{aligned} C_{F_1} &= C_{T_w} \\ \text{when } Re_1 &= Re_w \frac{T_1}{T_w} \end{aligned} \right\} \dots\dots\dots(1)$$

where

$$C_F = \frac{F}{\frac{1}{2} \rho u_1^2 \ell} \quad (= \text{mean* skin friction coefficient})$$

$$Re = \frac{u_1 \ell}{\nu} \quad (= \text{Reynolds number})$$

subscript "1" refers to the incompressible values, subscript "w" means that density (ρ) and viscosity (ν) are to be evaluated at wall (skin) temperature (T_w) and T_1 and T_w are the static temperatures, in the stream outside the boundary layer and at the wall respectively.

This formula is based on the assumptions

(a) that the general log-law^{**} velocity profile of incompressible flow can serve as an approximation in the compressible flow case if density and viscosity are evaluated at wall temperature. (This was the case under the experimental conditions of ref.1).

and (b) that Reynolds analogy between momentum and heat exchange is valid. (This leads to the assumption of constant total energy in the zero heat transfer case).

It should be noted that Mach number does not appear explicitly in equations 1. Instead, since the static pressure "p" is constant across the boundary layer,

$$C_{F_w} = C_{F_1} \frac{\rho_w}{\rho_1} \dots\dots\dots(2)$$

and $Re_w = Re_1 \frac{\mu_1}{\mu_w} \cdot \frac{T_1}{T_w} \dots\dots\dots(3)$

* Equations 1 should also give the variation of the local skin friction coefficient.

** The various power law velocity profiles are approximations to this law. In particular, the 1/7th power law profile gave good agreement with the experimental results of ref.1.

Hence equations 1 to 3 indicate that the ratio (T_1/T_w) is the only factor in the variation of $C_{F,1}$ with either M_1 or heat transfer, at given Re_1 . (Under zero heat transfer conditions this temperature difference is obtained by aerodynamic heating, the amount of which varies with M_1). Throughout the remainder of this note the subscript "1" will be dropped and C_F and Re will be taken as denoting free stream values of ρ and μ .

The results in ref.1 gave a check on the worth of equations 1 only in the case of flow over a flat plate under zero heat transfer conditions at $M_1 = 2.46$ and for a limited range of Reynolds numbers (based on free stream conditions) between 0.8×10^6 and 3.5×10^6 , when good agreement was found.

The purpose of the present note is to extend the check to other values of M_1 and Re , including cases with heat transfer. To this end, two sets^{2,4} of American test results have been analysed.

The first set² contains measurements of the boundary layer on a flat plate under zero heat transfer conditions, for tunnel Mach numbers between 1.73 and 2.25 and for Reynolds numbers (free stream) between 4×10^6 and 20×10^6 .

The second set⁴ contains measurements of skin friction and heat transfer for subsonic flow through a pipe up to high rates of heat transfer and covers a range of Reynolds numbers (based on pipe diameter and bulk temperature) between 7,000 and 500,000.

In addition to the check on the skin friction variation, the second set⁴ of results affords a means of checking the relation between skin friction coefficient and heat transfer coefficient (k_h) for pipe flows.

2 Skin friction on a flat plate under zero heat transfer conditions at supersonic speeds

2.1 Source of experimental evidence

The experimental data are taken from ref.2 which gives the results of pitot traverses made in the boundary layer of a flat plate at tunnel Mach numbers of 1.73, 2.00 and 2.25. The tests were made in the Ordnance Aerophysics Laboratory (O.A.L.), Daingerfield, wind tunnel by the Defense Research Laboratory (D.R.L.) of the University of Texas. For each tunnel Mach number, the plate was set at two angles of incidence so that results were obtained for mean Mach numbers (in the stream outside the boundary layer) of 1.695, 1.733, 1.897, 2.003, 2.121 and 2.186.

Only a limited amount of data is available at $M_1 = 1.695$ and 1.733 (two traverses at $M_1 = 1.695$ and one at $M_1 = 1.733$) because mechanical defects were found in the apparatus. Most of these defects were corrected before tests were made at the higher Mach numbers, which proved more successful. For that reason the emphasis of the present note is placed on the latter results (at $M_1 = 1.897, 2.003, 2.121$ and 2.186).

2.2 Analysis of results and comparison of skin friction coefficients

2.21 D.R.L. Analysis²

By similar methods to those used in ref.1, the pitot traverses are analysed in ref.2 to give velocity profiles and values of momentum and displacement thicknesses.

With two exceptions, all the velocity profiles are fully turbulent and show reasonable agreement with the $1/7$ th power law profile. Following this it is assumed, by analogy with incompressible flow, that

$$C_F = A Re^{-1/5} \dots\dots\dots(4)$$

where the factor A replaces the constant 0.074 of incompressible flow. The mean skin friction coefficient (C_F) is obtained from the momentum thickness (θ) via the momentum equation,

$$C_F = \frac{2\theta}{x - x_0} \dots\dots\dots(5)$$

where x is distance from leading edge of plate and x_0 is the value of x at the position of the effective start of the turbulent layer.

The distance ($x - x_0$) is used as the characteristic length in both C_F and Re in equation 4. " x_0 " was obtained² by an arithmetical fairing of the test results on the basis of equation 4.

The resulting experimental values of A are shown in fig. 1 as a function of M_1 , and are compared (as in ref. 2) with Von Karman's suggested variation given by

$$C_{F_w} = 0.074 Re_v^{-1/5} \dots\dots\dots(6)$$

from which, by taking (as in ref. 2)

$$T_w = T_0$$

where T_0 is the free stream stagnation temperature

and $\mu \propto T^{0.768}$

we obtain

$$C_F = 0.074 \left[1 + \frac{\gamma - 1}{2} M_1^2 \right]^{-0.6464} Re^{-1/5} \dots\dots\dots(6a)$$

i.e. $A = 0.074 \left[1 + \frac{\gamma - 1}{2} M_1^2 \right]^{-0.6464} \dots\dots\dots(7)$

Fig. 1 shows that equation 7 overestimates the compressibility variation and in ref. 2 an empirical correction is obtained by retaining the form of equation 7 while changing the index. That taken² is

$$A = 0.074 \left[1 + \frac{\gamma - 1}{2} M_1^2 \right]^{-0.348} \dots\dots\dots(8)$$

which gives the uppermost curve in fig. 1. It might be noted that a better mean curve through the experimental results would be obtained by retaining the index (-0.6464) and increasing the incompressible value from 0.074 to 0.0875.

2.22 Analysis on the basis of equations 1

Equations 1 give

when

$$\left. \begin{aligned} C_{F_i} &= C_{F_w} \\ Re_i &= Re_w \frac{T_1}{T_w} \end{aligned} \right\} \dots\dots(1)$$

and if we take

$$C_{F_i} = 0.074 Re_i^{-1/5}$$

then they give

$$C_{F_w} = 0.074 \left(Re_w \frac{T_1}{T_w} \right)^{-1/5} \dots\dots(9)$$

in compressible flow.

At zero heat transfer, experimental evidence indicates that for a turbulent boundary layer

$$\frac{T_w}{T_1} = 1 + 0.88 \frac{\gamma - 1}{2} M_1^2 \dots\dots(10)$$

Also, application of Sutherland's formula for the variation of viscosity with temperature shows that when $M_1 = 2.0$ and $T_0 = 150^\circ\text{F}$ (as in ref. 2), then

$$\frac{\mu_w}{\mu_1} \approx \left(\frac{T_w}{T_1} \right)^{0.82} \dots\dots(11)$$

Combining equations 9, 10 and 11, we obtain

$$C_F = 0.074 \left[1 + 0.88 \frac{\gamma - 1}{2} M_1^2 \right]^{-0.44} Re^{-1/5} \dots\dots(2a)$$

i.e. $A = 0.074 \left[1 + 0.88 \frac{\gamma - 1}{2} M_1^2 \right]^{-0.44} \dots\dots(12)$

The variation of A with M_1 according to equation 12 is shown by the full line in fig. 1, which is lower throughout than the empirical mean curve (equation 8) of ref. 2, and is also lower than the majority of the experimental values from ref. 2. Also shown is the experimental value at $M_1 = 2.46$, obtained from the R.A.E. test results of ref. 1 by a power law analysis.

However, by adjusting the values of x_0 (the assumed starting point of the turbulent boundary layer), the R.A.E. variation for A (equation 12) can give good agreement with the experimental results of ref. 2. This is shown by Figs. 2a to 2d. These are plots of momentum thickness against distance from leading edge and assuming that

$$\partial \alpha R_0^{-1/5} (x-x_0)$$

then

$$\theta^{5/4} \left(\frac{u_1}{v_1} \right)^{1/4}$$

should be a linear function of x . Good agreement is found with experiment and the difference between the R.A.E. (equation 12) and D.R.L. (equation 8) curves is small. (The results for $M_1 = 1.695$ and 1.733 are not included because of their limited number and the possibility of doubts of their accuracy). Thus the apparent discrepancies between the R.A.E. and D.R.L. estimates for the factor "A" in Fig. 1 can arise from small differences in the fairing of the experimental results.

Fig. 3 gives a plot of C_{F_w} against $Re_w^{T_1/T_w}$, where both C_{F_w} and Re_w are based on distance from the leading edge of the plate, i.e. they include the effect of the laminar layer over the forward portion of the plate. Included are the experimental points from the R.A.E. tests¹ and from the D.R.L. tests² at the O.A.L., plus the mean curves resulting from the present analysis. Two conclusions can be drawn. They are

(1) Equations 1 can give good agreement with experiment for the variation of C_F with M_1 over a range of $Re_w^{T_1/T_w}$ from 10^5 to 5×10^6 and for M_1 between 1.9 and 2.46.

and (2) Despite much earlier transition to turbulence, the turbulent boundary layer obtained in the R.A.E. tests¹ is consistent with the order of magnitude of that obtained in the O.A.L. tests².

3 Skin friction and heat transfer in subsonic flow through a circular pipe

3.1 Source of experimental evidence

The experimental data are taken from ref. 4 which covers tests made with air flowing through an electrically heated Inconel tube with a "bell-mouth" entrance, an inside diameter of 0.402 inch and a length of 24 inches. They represent an extension of the data given in ref. 3 and cover average wall temperatures up to 1140°K . At the highest test Reynolds numbers, tube exit Mach numbers of up to 1.0 were obtained.

The tests were made at the N.A.C.A. Lewis Flight Propulsion Laboratory at Cleveland, Ohio.

3.2 Analysis of results

The results are given as curves of Nusselt number against Reynolds number (based on tube diameter) and of mean skin friction coefficient against Reynolds number for different temperature levels. Nusselt number is based on the difference between the mean inside tube wall temperature (T_w in the present notation) and the mean total temperature of the air flow (T_{T_m}). In subsonic flow, the latter is close to the wall temperature for zero heat transfer. The static pressure

drop due to friction (Δp_f) was obtained by subtracting the mean rate of change of momentum from the measured static pressure drop along the tube length. Mean skin friction (F/L) is then given by

$$\frac{F}{L} = \Delta p_f \cdot \frac{D}{4L}$$

where D is the diameter of the tube
and L is its length.

For the present analysis, values of Nusselt number and of mean skin friction coefficient were read from the curves of ref. 4. The former were then transformed into mean heat transfer coefficients (\bar{k}_h) by using the relation

$$k_h = \frac{Nu}{\sigma Re}$$

where σ is the Prandtl number.

Finally, mean skin friction and mean heat transfer coefficients were correlated against Reynolds number on the basis of equations 1, taking the tube diameter as the characteristic length. The results are shown in figs. 4 and 5. Fig. 6 then shows the ratio of mean heat transfer to mean skin friction coefficient. Throughout the analysis the NACA values for the physical properties of air were used, as given in fig. 4 of ref. 3.

3.3 Discussion of results

3.31 Skin friction

On the basis of equations 1, fig. 4 shows a plot of $\frac{1}{2}\gamma_w$ against $Re_w T_m/T_w$ where

$$\gamma_w = \frac{\Delta p_f}{2 \frac{L}{D} \rho_w u_1^2}$$

is the mean skin friction coefficient. (The factor " $\frac{1}{2}$ " is included to aid the comparison with heat transfer coefficient).

" T_m " should be the mean static temperature of the airflow, but this is not given in ref. 4. Consequently the mean total temperature " T_{Hm} " was used in its place in evaluating the factor (T_m/T_w) in $Re_w T_m/T_w$. However since the flow was subsonic, any errors thus introduced are small. (For example, in the extreme cases when the speed of sound was reached at the tube exit, then it might be plausible to assume that the mean Mach number of the flow was of the order of 0.7. This corresponds to

$$\frac{T_{Hm}}{T_m} \approx 1.1$$

so that the maximum error in $Re_w T_m/T_w$ is of the order of 10%. Reference

to fig.4 shows that a shift in $Re_w T_m/T_w$ of this amount would have negligible effect on the correlation).

There is considerable scatter in the experimental points on fig.4, but at each temperature level they show a tendency to approach a common mean curve as the Reynolds number is increased.

For "incompressible" flow, Blasius gave the empirical formula⁵

$$\frac{1}{2} \gamma = 0.0396 \left(\frac{u_{in} D}{\nu} \right)^{-1/4} \dots\dots\dots (13)$$

Adapting this to compressible flow conditions in accordance with equations 1, we obtain

$$\frac{1}{2} \gamma_w = 0.0396 \left(Re_w \frac{T_m}{T_w} \right)^{-1/4} \dots\dots\dots (14)$$

and this curve is in reasonable agreement with the "fully turbulent" experimental values over the whole range of $Re_w T_m/T_w$ (from 1,000 to 400,000).

Thus on the grounds that they bring "fully turbulent" results over a wide range of temperatures on to a common mean curve and that this mean curve agrees with an existing curve for incompressible flow, it can be said that equations 1 give a sufficiently accurate estimate for the variation of skin friction with temperature in subsonic flow through heated pipe.

3.32 Heat transfer

If Reynolds analogy between momentum and heat exchange were valid throughout the whole of the boundary layer, then we should have⁵

$$\bar{k}_h = \frac{1}{2} \gamma$$

In fact, since there is always a laminar sub-layer present, this could only be the case if $\sigma = 1$. However it suggests that it might be possible to correlate heat transfer results on the basis of equation 1, i. e. by plotting k_{hw} against $Re_w T_m/T_w$.

This is done in fig.5, which shows that the experimental results are correlated on a mean curve with much less scatter than in the case of the skin friction results of fig.4.

For Reynold's numbers greater than 2100, McAdams⁶ gives the formula

$$Nu = 0.023 Re^{0.8} \sigma^{0.4}$$

for flows through pipes, which is equivalent to

$$\bar{k}_h = 0.023 Re^{-0.2} \sigma^{-0.6} \dots\dots\dots (15)$$

However, if the data which support equation 15 were analysed using the physical properties of air given in ref.3, then it is given³, that the constant 0.023 should be altered to 0.0205. Thus

$$\bar{k}_h = 0.0205 \text{ Re}^{-0.2} \sigma^{-0.6} \dots\dots(16)$$

is the comparable "incompressible" equation to the present results⁴. Applying equations 1 (substituting \bar{k}_h for C_F) we get (arbitrarily taking $\sigma = \sigma_w$)

$$\bar{k}_{h_w} = 0.0205 \left(\text{Re}_w \frac{T_m}{T_w} \right)^{-0.2} \sigma_w^{-0.6} \dots\dots(17)$$

(The mean value of $\sigma_w = 0.66$) Fig.5 shows that equation 17 is in good agreement with the experimental points for the higher Reynolds numbers at each temperature level. In general it may be said to give a good fit for $\text{Re}_w \frac{T_m}{T_w} > 4,000$. Thus the application of equations 1 to heat transfer coefficients (k_h) seems justifiable.

The tailing-off of the experimental values at the lower Reynolds numbers would seem to be an entry effect. Subsequent tests by the authors of refs.3 and 4 have shown⁷ that changing the entry shape has considerable influence on the heat transfer results at the lower Reynolds numbers. The illustrations in ref.7 are not however of sufficient size to warrant analysis.

The same entry effect was not noticeable⁷ in the skin friction results.

Consideration of the other curve on fig.5 is deferred till the next section, since it is based on the ratio of heat transfer to skin friction.

3.33 Ratio of heat transfer coefficient to skin friction coefficient

The preceding sections have shown that the correct variations with temperature can be obtained by taking both γ_w and \bar{k}_{h_w} as functions of $\text{Re}_w \frac{T_m}{T_w}$. This suggests that the ratio

$$\bar{k}_{h_w} / \left(\frac{1}{2} \gamma_w \right)$$

might also be plotted against $\text{Re}_w \frac{T_m}{T_w}$.

This is done in fig.6 which shows that despite a large amount of scatter (occasioned by the scatter of the skin friction results, fig.4) the results lie within a definite band. The tailing-off for $\text{Re}_w \frac{T_m}{T_w} < 10^4$ is caused by the corresponding feature in the heat transfer results and must be treated with reserve, since ref.8 shows that this tailing-off varies with the entry shape.

Now for incompressible flow over a flat plate, Von Karman's extension to Reynolds analogy gives

$$\frac{1}{k_h} = \left(\frac{2}{C_f}\right)^{\frac{1}{2}} \left\{ \left(\frac{2}{C_f}\right)^{\frac{1}{2}} + \Sigma(\sigma) \right\} \dots\dots\dots(18)$$

where

$$\Sigma(\sigma) = 5 \left[(\sigma - 1) + \ln \left(\frac{5\sigma + 1}{6} \right) \right]$$

It is easily shown that for the range of Prandtl numbers associated with air (of the order of 0.7), an approximation within $\pm 2\frac{1}{2}\%$ of equation 18, for a range of Reynolds numbers from 10^5 to 10^8 , is given by

$$k_h = \left(\frac{1}{2} C_f\right) \sigma^{-1/3} \dots\dots\dots(19)$$

These expressions are strictly valid only for flows over a flat plate. For pipe flows, Squire⁸ shows that a small correction term δ_m must be included in equation 18, giving

$$\frac{1}{k_h} = \left(\frac{2}{C_f}\right)^{\frac{1}{2}} \left\{ \left(\frac{2}{C_f}\right)^{\frac{1}{2}} + \delta_M + \Sigma(\sigma) \right\} \dots\dots\dots(18a)$$

This correction term δ_M depends on the relation between the velocity and temperature profiles and analysis of some experimental results gives⁸

Re	10^4	10^5	10^6
δ_M	1.08	0.84	0.70

In the present case we shall apply equations 18, 18a and 19 to \bar{k}_h and $\frac{1}{2} \gamma_w$, with σ evaluated at wall temperature. Over the range of test temperatures, σ_w varied from 0.69 to 0.64, according to the values of the physical properties of air given in ref. 3. A mean value of 0.66 has been chosen for the present analysis.

Fig. 6 then shows the comparison between the Karman relation for flat plate flow (eq. 18), Squire's modification⁸ of this relation for pipe flow (eq. 18a), the approximation of equation 19 and the experimental results. (In applying equations 18 and 18a, the variation of $\frac{1}{2} \gamma_w$ with $Re_w T_m/T_w$ according to equation 14 was used).

Both the modified Karman relation (eq. 18a) and the approximate relation (equation 19) are seen to provide a fair mean curve through the experimental values of $\bar{k}_h / \frac{1}{2} \gamma_w$ for $Re_w T_m/T_w > 10^4$. This being so, then combination of equation 14

$$\frac{1}{2} \gamma_w = 0.0396 \left(Re_w \frac{T_m}{T_w} \right)^{-\frac{1}{4}} \dots\dots\dots(14)$$

with equation 19 (with γ_w for C_f)

$$\bar{k}_{h_w} = \left(\frac{1}{2} \gamma_w\right) \sigma^{-1/3} \quad \dots\dots(19)$$

gives

$$\bar{k}_{h_w} = 0.0396 \left(\text{Re}_w \frac{T_m}{T_w} \right)^{-1/4} \sigma^{-1/3} \quad \dots\dots(20)$$

and the variation of \bar{k}_{h_w} with $\left(\text{Re}_w \frac{T_m}{T_w} \right)$ according to equation 20 is included in Fig.5 for comparison with the experimental values of \bar{k}_{h_w} . When $\text{Re}_w \frac{T_m}{T_w} > 10^4$, there is fair agreement, which lends further support to the use of equation 19 as an approximation.

However it should be noted that the agreement obtained with equation 20 is not as good as that already obtained with the modification of McAdam's formula (equation 17).

4 Conclusions

1 Analysis of measurements² of the turbulent boundary layer on a flat plate for Mach numbers between 1.9 and 2.2, under zero heat transfer conditions, shows that the formula

$$\left. \begin{aligned} C_{F_1} &= C_{F_2} \\ \text{when } \text{Re}_1 &= \text{Re}_2 \frac{T_1}{T_2} \end{aligned} \right\} \quad \dots\dots(1)$$

of ref.1, gives good agreement with the experimental skin friction results when applied in conjunction with the incompressible formula

$$C_{F_1} = 0.074 (\text{Re}_1)^{-1/5}$$

2 When considered together with the results of ref.1, this means that the formula has now been checked for flat plates under zero heat transfer conditions over a range of Mach numbers from 1.9 to 2.46 and for free stream Reynolds numbers between 0.8×10^6 and 20×10^6 .

3 Analysis of measurements⁴ of skin friction and heat transfer for subsonic flows through a pipe of circular cross section over a range of Reynolds numbers (based on pipe diameter and bulk temperature) between 7,000 and 500,000 and for temperature differences $(T_w - T_m)$ up to 684°C, shows that application of equation 1 with γ and k_h replacing C_F , gives a good correlation of the experimental results.

4 A mean curve through the skin friction results is given by

$$\gamma_w = 0.0791 \left(\text{Re}_w \frac{T_m}{T_w} \right)^{-1/4}$$

which is the modification according to equations 1 of the Blasius empirical formula⁵ for pipe flows.

$$Y = 0.0791 (Re)^{-1/4}$$

(T_m is the mean temperature of the airflow)

5 A mean curve through the heat transfer results, for $Re_w \frac{T_m}{T_w} > 4,000$, is given by

$$\bar{k}_{h_w} = 0.0205 \left(Re_w \frac{T_m}{T_w} \right)^{-0.2} \sigma_w^{-0.6}$$

which is the modification according to equations 1 and the physical properties of air used in ref. 3, of McAdams⁶ empirical formula

$$Nu = 0.023 Re^{0.8} \sigma^{0.4}$$

where $Nu = \bar{k}_h \cdot \sigma \cdot Re$.

6 For $Re_w \frac{T_m}{T_w} > 10^4$, the results of ref. 4 give

$$\frac{\bar{k}_{h_w}}{\frac{1}{2} \gamma_w} \approx \sigma_w^{-1/3}$$

LIST OF SYMBOLS

(a) Plate Flow

x	distance along plate from leading edge
x_0	value of x at the position of the effective start of the turbulent boundary layer
θ	momentum thickness
T	static temperature (degrees absolute)
ρ	density (mass units)
μ	dynamic viscosity
ν	kinematic viscosity (= μ/ρ)
u	velocity parallel to plate
M	Mach number

subscript "1" denotes free stream conditions

subscript "w" denotes conditions at the surface of the plate

C_F mean skin friction coefficient

$$\left(= \frac{\bar{F}}{\frac{1}{2} \rho_1 u_1^2 x} \text{ if based on distance from L.E.} \right)$$

$$= \frac{\bar{F}}{\frac{1}{2} \rho_1 u_1^2 (x-x_0)} \text{ if based on distance from effective start of turbulent boundary layer;}$$

$$C_{F_w} = \frac{\bar{F}}{\frac{1}{2} \rho_w u_1^2 x}$$

$$Re \quad \text{Reynolds number} \quad \left(= \frac{u_1 x}{\nu_1} \text{ or } \frac{u_1 (x-x_0)}{\nu_1} \right)$$

$$Re_w = \frac{u_1 x}{\nu_w}$$

A factor in relation $C_F = A Re^{-1/5}$

(b) Pipe Flow

L length of pipe

D diameter of pipe

u, ρ , ν , T as for plate flow

subscript "m" denotes mean airflow conditions

γ mean skin friction coefficient

$$= \frac{\Delta p_f}{2 \frac{L}{D} \rho u_m^2}$$

where Δp_f is the pressure drop due to friction

$$Re \quad \text{Reynolds number} = \frac{u_m D}{\nu}$$

$$\sigma \quad \text{Prandtl number} = \frac{C_p \mu}{k}$$

where C_p is the specific heat at constant pressure and

k is the thermal conductivity

\bar{k}_{h_w} mean heat transfer coefficient

$$= \frac{(\dot{Q}/S)}{\rho_w u_m g C_{P_w} (T_w - T_{Hm})}$$

where Q is the overall heat transfer rate
 S is the area of heated surface
 g is the acceleration due to gravity
and T_{Hm} is the mean total temperature of the airflow

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FIG.1

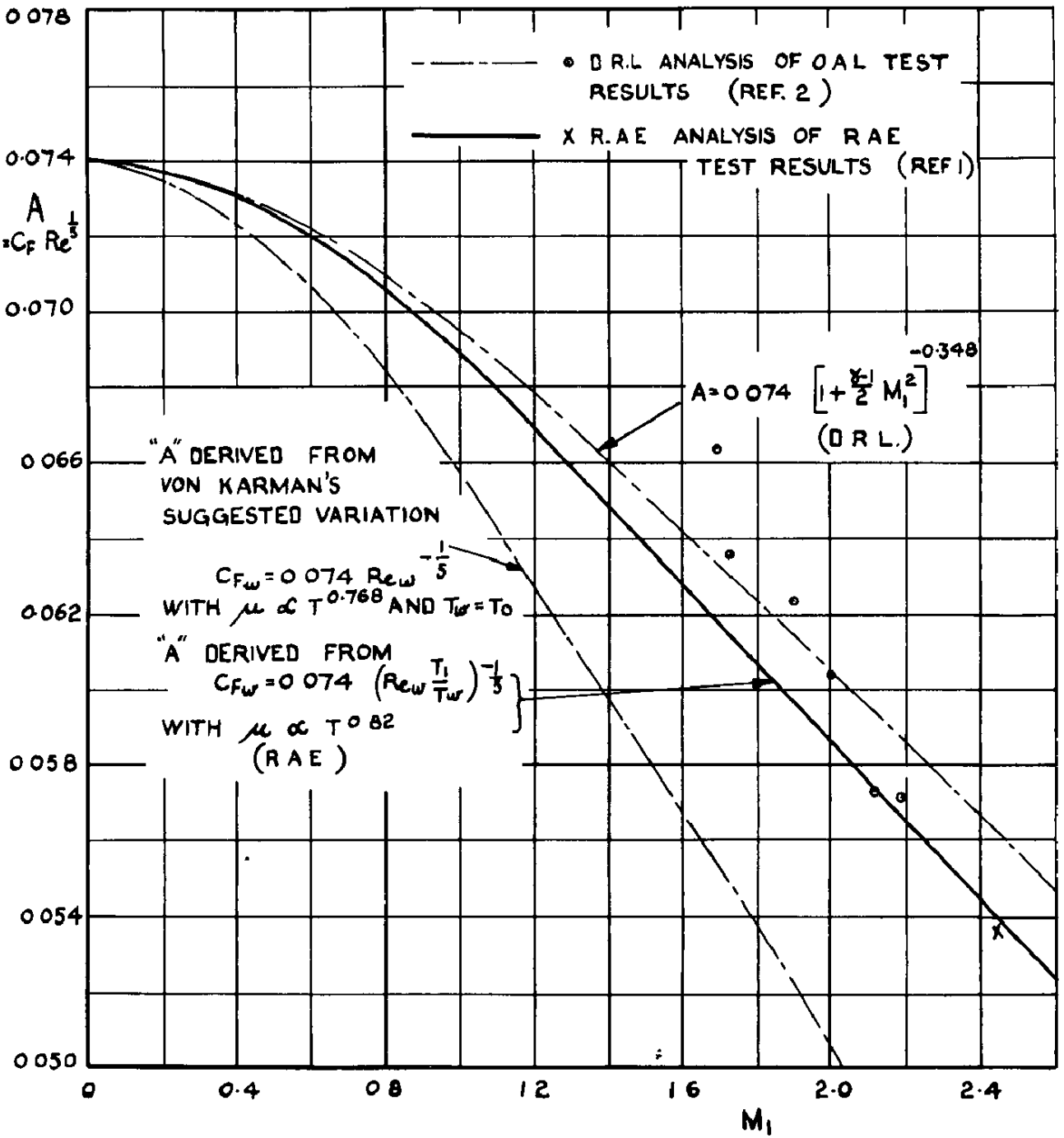


FIG.1 TURBULENT SKIN FRICTION ON A FLAT PLATE. VARIATION WITH MACH NUMBER UNDER ZERO HEAT TRANSFER CONDITIONS.

(ASSUMING THAT $C_F = A Re^{-1/5}$)

$$C_F = \frac{F}{\frac{1}{2} \rho_1 u_1^2 (X-X_0)} \quad Re = \frac{u_1 (X-X_0)}{\nu_1}$$

FIG. 2(a&b)

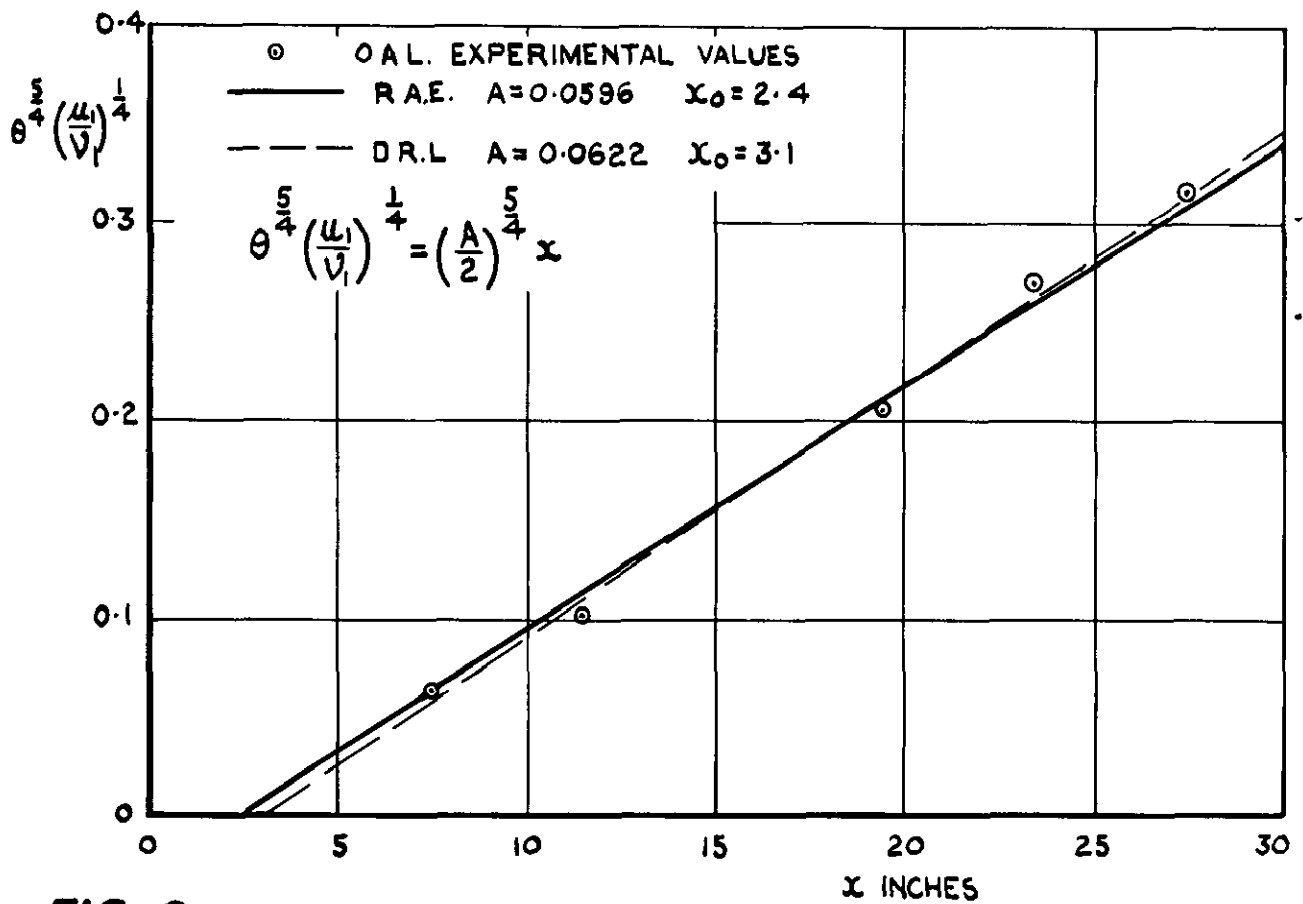


FIG. 2(a) $M_1=1.097$

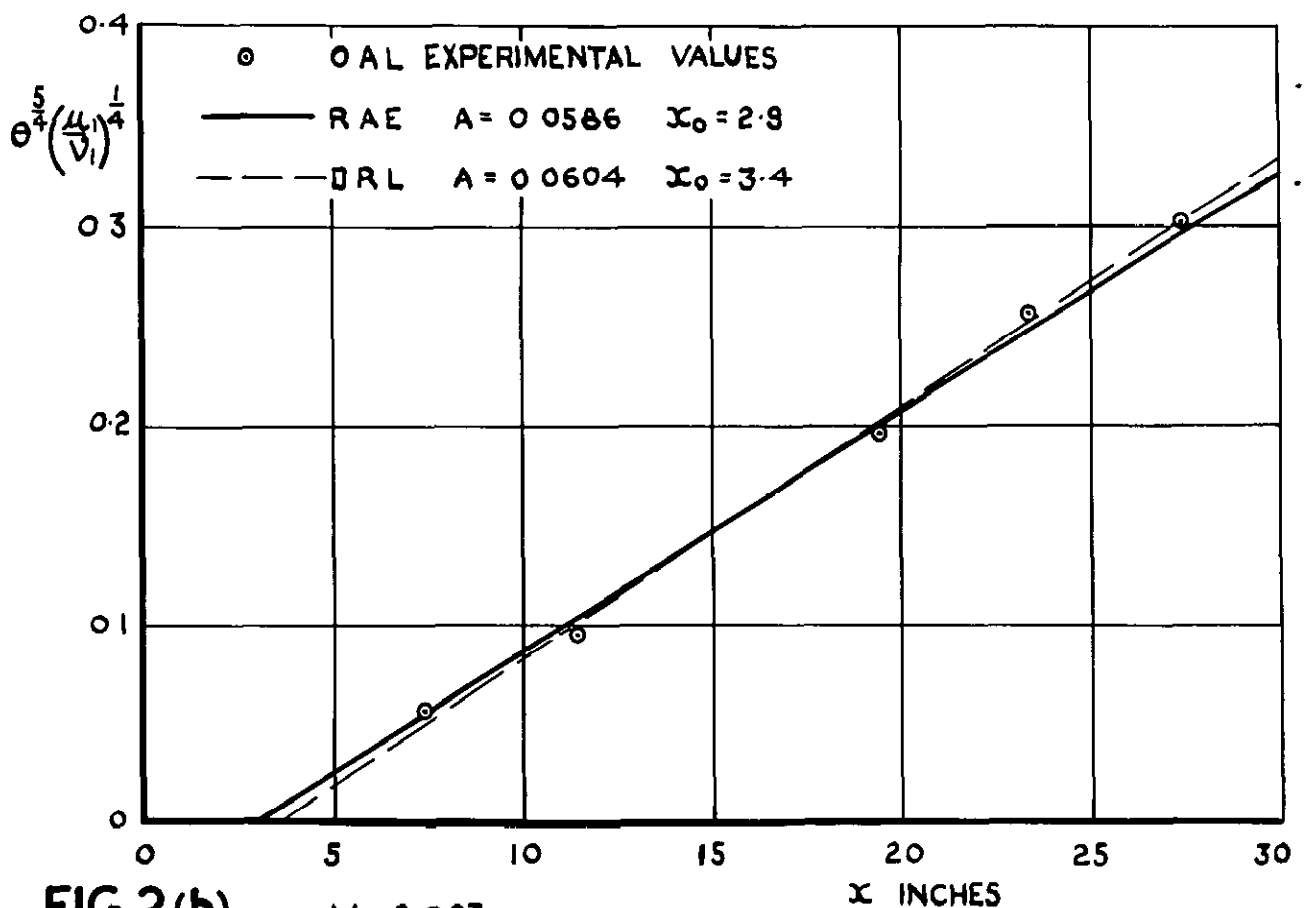


FIG.2(b) $M_1=2.003$

FIG.2(a&b) COMPARISON WITH EXPERIMENT OF R.A.E. (REF.1) AND D.R.L. (REF.2) ESTIMATES FOR MOMENTUM THICKNESS. (FACTOR "A" FROM FIG.1.)

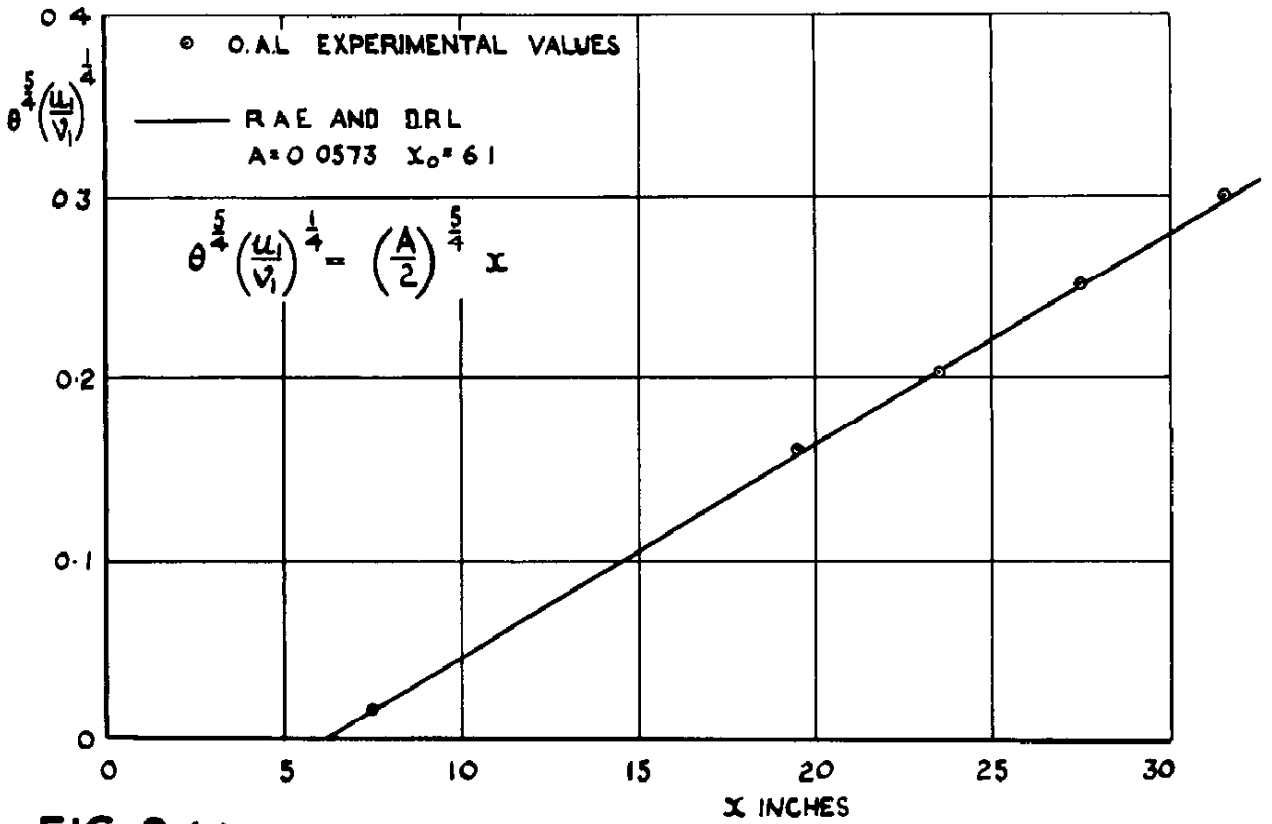


FIG.2 (c) $M_1 = 2.121$

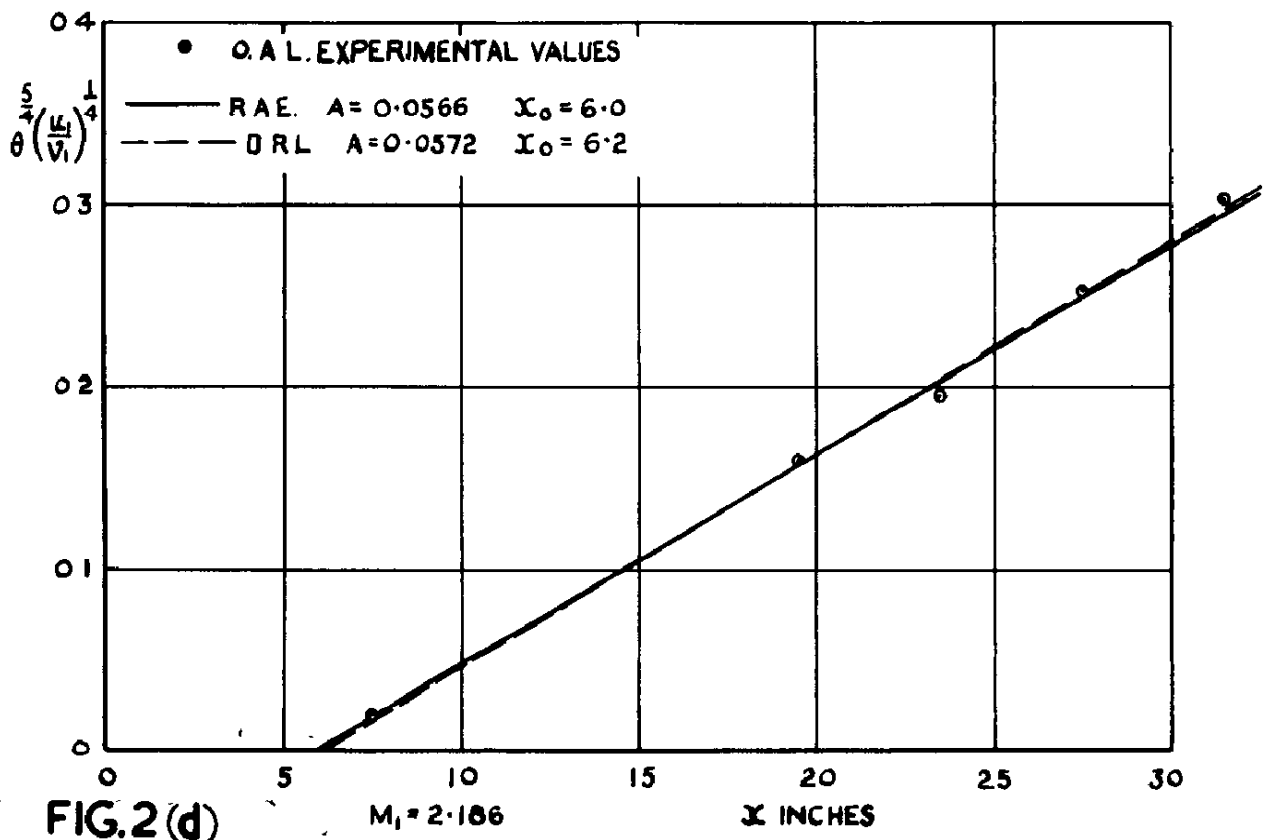


FIG.2 (d) $M_1 = 2.186$

FIG.2 (c&d) - COMPARISON WITH EXPERIMENT OF RAE (REF.1) AND D.R.L. (REF.2) ESTIMATES FOR MOMENTUM THICKNESS. (FACTOR "A" FROM FIG.1.)

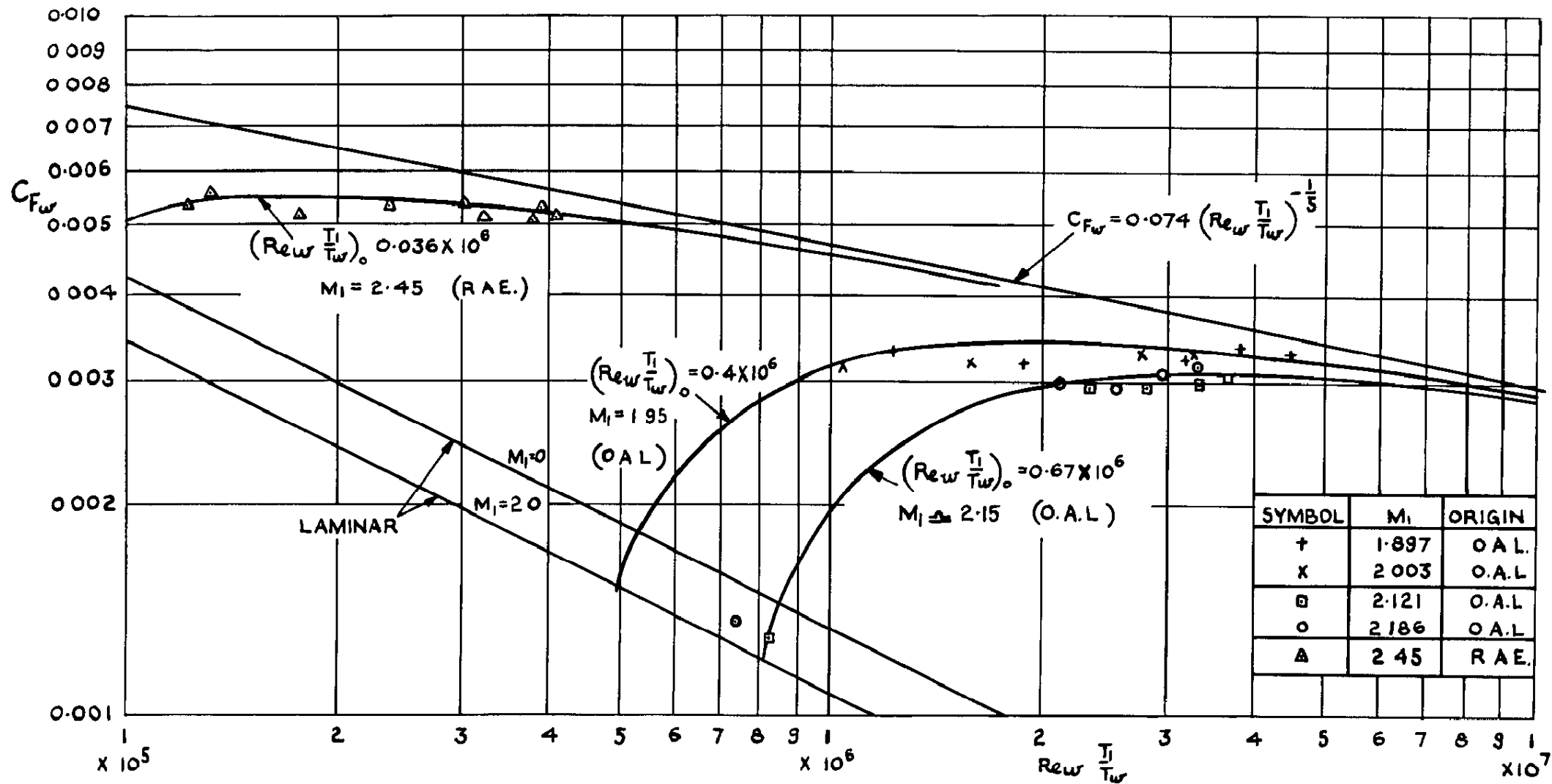


FIG. 3 CORRELATION OF SKIN FRICTION RESULTS FOR FLAT PLATES UNDER ZERO HEAT TRANSFER CONDITIONS.

$$C_{F_w} = \frac{F}{\frac{1}{2} \rho_w u_1^2 x}$$

$$Re_w = \frac{u_1 x}{\nu_w}$$

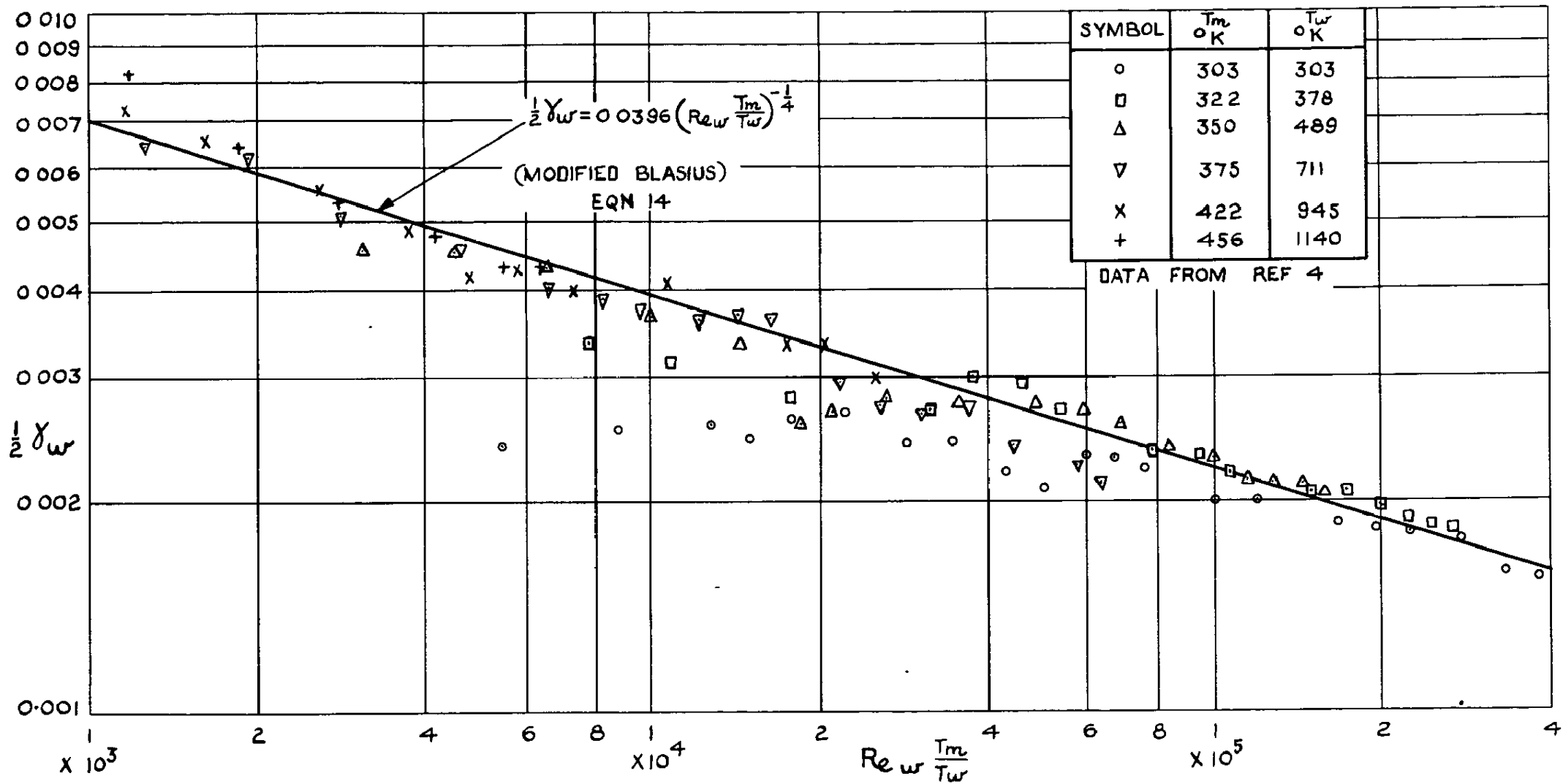


FIG.4 CORRELATION OF SKIN FRICTION RESULTS FOR PIPES WITH HEAT TRANSFER.

$$\gamma_w = \frac{\Delta P_f}{2 \frac{L}{D} \rho_w \mu_m^2}$$

$$Re_w = \frac{\mu_m D}{\nu_w}$$

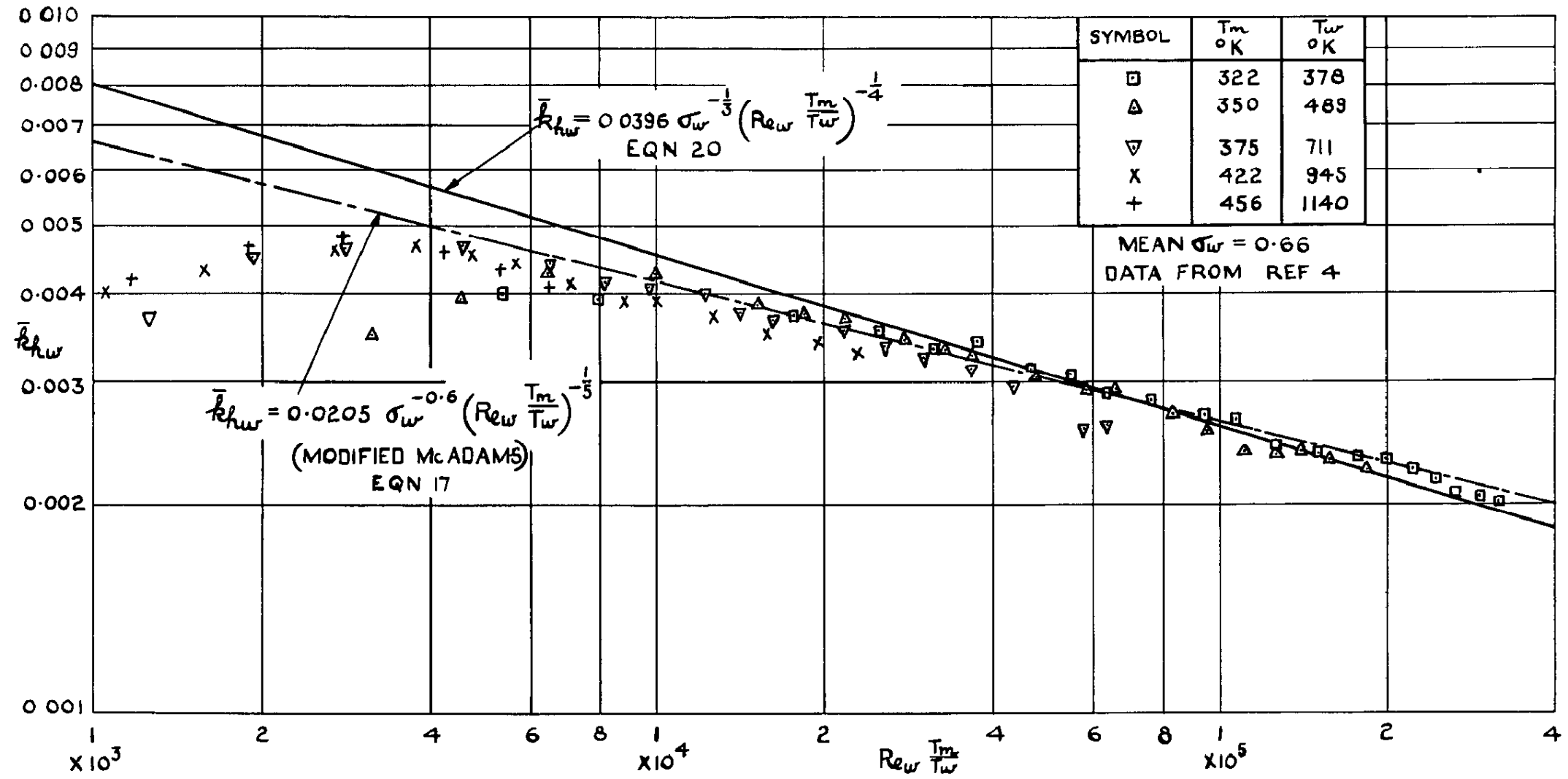


FIG.5 CORRELATION OF HEAT TRANSFER RESULTS FOR PIPES.

$$\bar{h}_{hw} = \frac{(Q/S)}{e_w u_m g C_{pw} (T_w - T_m)} \quad Re_w = \frac{u_m D}{\nu_w}$$

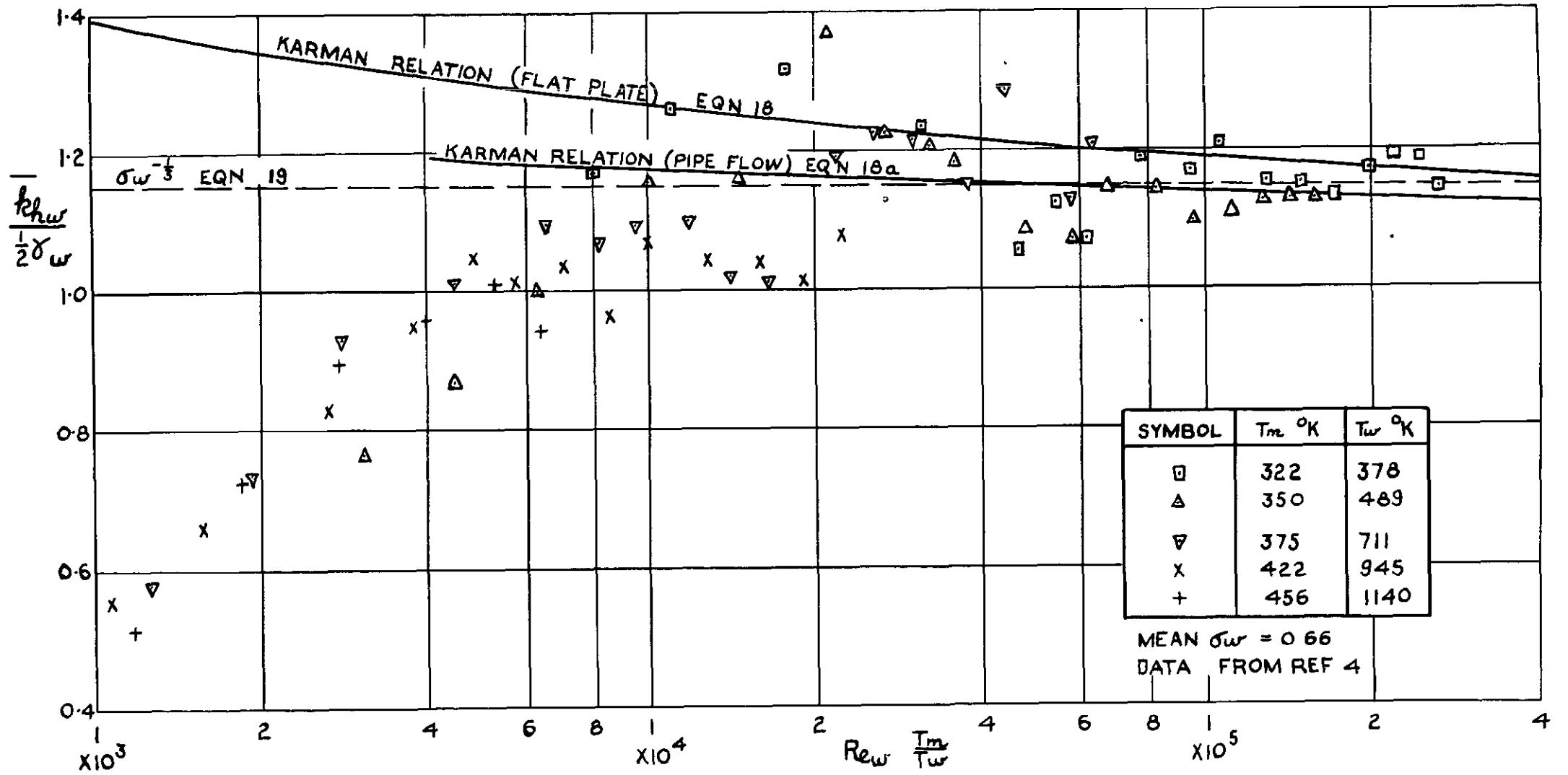


FIG. 6 RELATION BETWEEN HEAT TRANSFER AND SKIN FRICTION COEFFICIENTS FOR PIPE FLOWS OF REF. 4.

FIG. 6

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