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On the Transient Motion of a Slender Delta Wing with Leading Edge Separation

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On the Transient Motion of a Slender Delta Wing with Leading Edge Separation

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SUMMARY

The linear and non-linear theories of the steady flow past a rigid slender delta wing are reviewed and extended to the two transient motions

(i) entry into a sharp edged gust

(ii) sudden change of incidence.

For slender wings with leading-edge separations it is shown that the solution to the gust problem is trivial; each chordwise section instantaneously jumps from its initial steady state to its final steady state as the gust front passes the section. An approximate method is described for determining the time dependent solution for the problem of the sudden change of incidence.

1. Introduction

The current interest on slender wings is primarily concerned with the steady flow past rigid wings. Theoretically, the application of the linear slender wing theory to these problems, due to Jones[1], leads to a simple result for the loading in terms of the incidence and plan form of the wing. However, the practicability of this result is found to be extremely limited, in particular, to wings with round leading edges at small angles of attack. In general the formation of a pair of leading-edge vortices, due to the separation of the flow from the leading edges and subsequent convection by the free stream, requires a more complex mathematical model for the understanding and prediction of the flow characteristics and the forces involved. Several attempts have been made on this aspect[2,3], the most successful is that due to Mangler and Smith[4]. Together with recent experimental investigations[5,6,7] the main features of the flows, about slender delta wings with zero camber are understood. Some secondary effects on these wings have still to be satisfactorily explained and predicted, for example; the effect of the trailing edge on the loading throughout the subsonic range of speeds, the apparent non-uniformity of different experimental results and the effects of secondary separations. In addition further fundamental work is necessary on the more general types of slender wings with curved leading edges and camber, both chordwise and spanwise.

However it seems opportune at this stage to start discussing some of the transient motions of slender wings in the presence of leading-edge vortices, which will ultimately be needed for the investigations of both

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slender aircraft response and the possibility of flutter. This note is restricted to a discussion of the behaviour of the flow about a rigid slender delta wing as it

(i) passes through a sharp-edged vertical gust

(ii) undergoes a sudden change of incidence.

2. The Linear Slender Wing Theory: Steady Case

The notation for the slender wing model is shown in Fig.1. The apex of the slender wing is taken as the origin of a cartesian system of co-ordinate (x axis is in the direction of the relative free stream V, and the y axis is the spanwise co-ordinate).

It is assumed in this note that the trailing edge is normal to the free stream and that the span is a maximum at the trailing edge. The slender wing approximation (aspect ratio < 1), together with the usual linearisation approximation (perturbation velocities u, v, w are small compared with the free stream velocity V) resolves the problem of finding the perturbation velocity potential $\phi\ (x, y, z)$ (where $u = \partial\phi/\partial x$, $v = \partial\phi/\partial y$, $w = \partial\phi/\partial z$) which satisfies the differential equation

$$\phi_{yy} + \phi_{zz} = 0 \qquad \ldots (1)$$

such that

$$\left(\frac{\partial\phi}{\partial z}\right)_{\substack{z=0 \\ |y|\leqslant S(x) \\ 0\leqslant x\leqslant c_0}} = -\alpha V \qquad \ldots (2)$$

and

$$\phi = 0 \qquad \ldots (3)$$

as $y \to \infty$, $z \to \infty$, $x \to -\infty$. The general solution of equation (1) includes a function of x which depends on the thickness distribution of the wing. In this note we are only concerned with the lifting properties of the wing, so assuming that the thickness distribution does not affect the lifting characteristics, each cross flow plane may be treated independently from its neighbours. The problem in each cross flow plane is essentially two-dimensional, so that the x co-ordinate appears as a dependent variable.

The solution of equation (1) which satisfied equations (2) and (3) gives the discontinuity of the velocity potential across the wing plan form ($|y|\leqslant S(x)$, $0\leqslant x\leqslant c_0$, $z = 0$) as

$$\begin{aligned}\Delta\phi(x,y,0) &= \phi(x,y,-0) - \phi(x,y,+0) \\ &= 2\alpha V\,[S^2(x)-y^2]^{\frac{1}{2}}\end{aligned} \qquad \ldots (4)$$

The pressure coefficient c_p is given by the formula

$$\begin{aligned}c_p &= \frac{P_{\text{lower surface}} - P_{\text{upper surface}}}{\frac{1}{2}\rho V^2} \\ &= \frac{2}{V}\frac{\partial}{\partial x}(\Delta\phi)\end{aligned} \qquad \ldots (5)$$

Substituting equation (4) into equation (5), assuming α is constant,

$$c_p = \frac{4\alpha S'(x)}{[1 - y^2/S^2(x)]^{\frac{1}{2}}} \qquad \ldots (6)$$

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The spanwise lift distribution $L(y)$ is given by

$$L(y) = \tfrac{1}{2}\rho V^2 4\alpha[S^2(c_0)-y^2]^{\frac{1}{2}} \quad \ldots (7)$$

On integration of equation (7) the lift coefficient C_L becomes

$$C_L = \frac{\pi A}{2} \quad \ldots (8)$$

where A is the aspect ratio. These results, presented above, are well known but are included here for the sake of completeness and as a reference for the discussion presented later.

There are two important consequences of the slender wing assumption ($A<1$) which are illustrated by the differential equation (3). First, the absence of any terms depending explicitly on the co-ordinate x implies that the results, of the loading distribution, say, are independent of forward speed, thus incidentally eliminating any variation with Mach number. Physically this means that the perturbation velocity $u(=\partial\phi/\partial x)$ in the stream direction is always small compared with the perturbation velocities $v(=\partial\phi/\partial y)$ and $w(=\partial\phi/\partial z)$ in the planes normal to the free stream. Thus the slender wing equation states that

$$u << v, w \quad \ldots (9)$$

together with the initial linearization condition that

$$v, w << V \quad \ldots (10)$$

Therefore to a first approximation (the same order as the linear theory) the velocity components are

$$(V, v, w) \quad \ldots (11)$$

Secondly, the equation for the velocity potential in the cross flow planes is that for a two-dimensional incompressible fluid so that disturbances are propagated infinitely quickly in these planes. This is again a relative effect, since on the linear theory all disturbances are propagated with the speed of sound of the free stream, but the rate of propagation relative to the span is larger than the rate of propagation relative to the chord for a slender wing.

In the discussion above it is stated that the loading is, theoretically, independent of Mach number. This is, of course, optimistic. At supersonic speeds this condition is essentially true because the supersonic trailing edge can sustain a finite load. At subsonic speeds it is expected that the trailing edge will have an increasing effect as the Mach number decreases, since the loading at the trailing edge must be zero. Even so it has been shown experimentally[7] that at low speeds the slender wing type of loading is obtained for approximately 60% of the chord from the leading apex.

3. The Linear Slender Wing Theory: Transient Motion

The transient motion of the slender wing assuming linear aerodynamics has been extensively studied and the results are summarized by Miles[8]. The transient motions which are considered in this paper are

(i) flight through a sharp-edged gust (a constant upwash velocity w_1 exists behind the gust front and is zero in front of it)

(ii) a sudden change of incidence, from α_1 to α_2, say, at time $t = 0$.

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It is instructive in this case to deduct physically the forces during these motions from the ideas presented in the previous paragraph on the steady case.

Consider a sharp-edged gust as it passes over a slender wing at incidence α, with the condition that at $t = 0$ the gust front is at the apex $(x = 0)$. The gust will pass over the wing with the constant free stream velocity V, since, on the basis of the linear approximation, the perturbation velocities in the stream direction are of the second order, and therefore negligible. After time t, the gust front will have reached the chordwise section $x = Vt$. Assuming that the steady case condition that each chordwise section may be considered independently of its neighbours still holds for an unsteady motion, then the section x will have been unaware of the approach of the gust front until time x/V. Then the effective incidence of the section changes instantaneously from α to $\alpha + w_1/V$. In this cross flow plane the fluid may be regarded as incompressible so that pressure disturbances are propagated infinitely quickly away from the wing. Hence the wing instantaneously assumes the loading associated with steady case with the incidence $\alpha + w_1/V$. Thus as the gust front passes over each chordwise section of the wing, it assumes its new steady state.

The main assumption in the above argument is that each chordwise section may be considered separately as in the steady case. Miles[8] has shown that as long as the wing is slender (i.e., aspect ratio $[M^2-1]^{\frac{1}{2}} << 1$) the transient problem may be treated as series of two-dimensional problems in the cross flow planes. However, for the slender wing oscillating at a low frequency equation (3) remains the differential equation, but at high frequencies the cross flow differential equation becomes

$$\phi_{yy} + \phi_{zz} = \frac{1}{a_0^2}\phi_{tt}$$

where a_0 is the speed of sound in the free stream. Physically this means that the incompressibility assumption in the cross flow plane is breaking down because of the high downwash speeds associated with the wing oscillations. This state of affairs cannot be said to apply to the present case under discussion (i.e., sectional sudden change of incidence) where the downwash velocities remain small so that the assumptions are justified.

The other problem, namely, the sudden change of incidence of a slender wing from α_1 to α_2 at time $t = 0$ is simply the sudden change from one steady state loading (associated with α_1) to the second steady state loading (associated with α_2) over the whole wing surface at $t = 0$. This follows by the same arguments as those described above.

4. The Non-Linear Slender Wing Theory: Steady Case

The linear slender wing theory, discussed in Section 2 predicts infinite velocities at the leading edge. This singularity is the usual type associated with the classical thin aerofoil theories but its presence does not invalidate the application of the theories to practical cases as long as the flow follows the contours of wings without separation. For slender wings the free stream separates from the leading edges creating a vortex sheet and this vorticity is convected downstream forming the leading edge vortices. The mathematical model for this type of flow, assuming the existence of the leading edge vortices in inviscid fluid motion, has been most thoroughly investigated by Mangler and Smith[4].

For a slender wing at a uniform incidence α, the notation is shown in Fig.2. The wing planform is the same as before, namely $|y| \leq S$, $0 \leq x \leq c_0$, $z = 0$. The vortex sheet is denoted by $S(x, r, \theta) = 0$, where (r, θ) are the polar co-ordinates of the vortex sheet at the chordwise

section/

section x in the cross-flow plane. The equation for the velocity potential is as before

$$\phi_{yy} + \phi_{zz} = 0 \qquad \dots (12)$$

where

$$\left(\frac{\partial\phi}{\partial z}\right)_{\substack{z=0\\ |y|\leqslant S(x)\\ 0\leqslant x\leqslant c_0}} = -\alpha V \qquad \dots (13)$$

is the boundary condition which must still be applied on the wing. The conditions on the vortex sheet, forming the leading edge vortex, are

(i) there is no flow across the sheet

(ii) there is no pressure discontinuity across the sheet.

Condition (i) becomes

$$\left.\frac{\partial\phi}{\partial n}\right|_{S=0} = -V \sin\lambda \left.\frac{\partial r}{\partial x}\right|_{\substack{S=0\\ \theta=\text{const.}}} \qquad \dots (14)$$

where $\partial/\partial n$ denotes the rate of change normal to the sheet in the cross flow plane. Condition (ii) becomes

$$\left(V\Delta\left(\frac{\partial\phi}{\partial x}\right) + \left(\frac{\partial\phi}{\partial\sigma}\right)_m \frac{\partial}{\partial\sigma}(\Delta\phi)\right)_{S=0} = 0 \qquad \dots (15)$$

where $\left(\frac{\partial\phi}{\partial\sigma}\right)_m$ denotes the mean value of $\left(\frac{\partial\phi}{\partial\sigma}\right)$ across the sheet $S = 0$, $\partial/\partial\sigma$ denotes the rate of change tangential to the sheet $S = 0$ in the cross-flow plane, and $\Delta\phi$ signifies the discontinuity of ϕ across the sheet $S = 0$.

Equation (12) is to be solved, satisfying the boundary equations (13), (14) and (15) with the condition that the load is zero at the leading edge.

Mangler and Smith considered the case of a delta wing and assumed conical flow so that

$$\left.\frac{\partial r}{\partial x}\right|_{S=0} = \frac{S'(x)}{S(x)} \qquad \dots (16)$$

where $S'(x) = \tan\gamma$ and γ is the semi-apex angle. Equations (12), (13), (14) and (15) and (16) with the assumption of conical flow, again resolve the problem into a two-dimensional problem in the cross plane, where the shape of the vortex sheet in the cross plane is now one of the features to be determined.

An explicit function $\phi(y, z; x)$ cannot be written down for this problem but it has been computed[4]. The pressure coefficient can then be found in the cross-flow plane from

$$c_p = \frac{2}{V^2}\left(V\frac{\partial}{\partial x}(\Delta\phi) + \left(\frac{\partial\phi}{\partial y}\right)_m \frac{\partial}{\partial y}(\Delta\phi)\right)_{z=0}$$

Thus/

Thus the total spanwise lift distribution can be calculated and the total forces determined. Alternatively the total lift can be obtained by the contour integration of the velocity potential discontinuity over the wing and vortex sheet in the cross flow plane at the trailing edge, so that

$$C_N = \frac{\rho V \int_c \sigma dy}{\frac{1}{2}\rho V^2 S}$$

where C_N is the normal force coefficient and C is a circuit which encloses wing and vortex sheet in the cross-flow plane at the trailing edge of the wing.

It is emphasized that the solution is restricted to the linear and slender wing assumptions so that the orders of magnitude of the velocity perturbation expressed in equations (9), (10) and (11) still apply.

In Fig.3 the slender wing is again shown with its associated leading edge vortices. We are going to consider the vortex sheet at section x_1. Now the vortex sheet is formed by the creation of vorticity due to separation at the leading edge and then convected downstream under the action of the free stream and the cross perturbation velocities. Thus the vorticity formed at point A on the leading edge at position x_A (see Fig.3) at time t_A reaches section x_1 at point A_1 on the vortex sheet at time t_1, where $t_1 - t_A = \frac{x_1 - x_A}{V}$. This relationship is because the perturbation velocities in the streamwise direction are of the second order of magnitude and therefore neglected to a first approximation for a linear theory. Similarly the vorticity formed at the point B at section x_B at time t_B reaches point B_1 at section x_1 at time t_1, where $t_1 - t_B = \frac{x_1 - x_B}{V}$. Therefore the vorticity formed at the apex forms the inner part of the vortex core at section x_1 and takes time x_1/V to pass from the apex to section x_1.

If we denote $\Gamma(x)$ as the circulation around one of the leading-edge vortices at section x, say the one for $y > 0$, then

$$\Gamma(x) = \int_c \Delta\phi \, d\sigma \qquad \ldots (19)$$

where the circuit C (which lies in the cross-flow plane at section x) encloses the whole of the leading edge vorticity, for $y > 0$, but <u>not</u> the wing plan form ($|y| \leq S(x)$, $z = 0$). Then the rate at which circulation is shed from the leading edge is

$$\frac{d\Gamma}{dt} = V \frac{\partial \Gamma}{\partial x} = V \frac{\partial}{\partial x}\left(\int_c \Delta\phi \, d\sigma\right) \qquad \ldots (20)$$

and therefore we can write

$$\Gamma = \int_{t - \frac{x}{V}}^{t} \left(\frac{d\Gamma}{dt}\right)_{t-\tau} d\tau \qquad \ldots (21)$$

where Γ is the circulation at section x at time t and $\left(\frac{d\Gamma}{dt}\right)_{t-\tau} d\tau$

is/

is the circulation which is shed in time $\delta\tau$ upstream at section $x - V\tau$ at time $t - \tau$. The substitution of (20) into equation (21) gives an identity for the steady case, but the ideas presented here are fundamental for the discussion of the unsteady motions.

5. The Non-Linear Slender Wing Theory: Sharp-Edged Gust

We now consider the case of a slender wing entering a sharp-edged vertical gust which has a constant vertical velocity w_1 behind the gust front.

First we shall consider the wing at zero incidence initially before the gust passes over the wing so that there is no initial vortex system or loading. Since the slender wing approximations are still assumed the gust front will not accelerate as it passes over the body and therefore proceeds past the wing at the free stream velocity. As the gust passes each chordwise section it is assumed that rate of vorticity associated with effective w_1/V is instantly generated and then convected downstream.

Thus as the gust proceeds along the chord the vorticity which is created at each chordwise section is then convected with the gust front. When the gust reaches section x, say, the section is instantaneously at an effective incidence w_1/V, together with the full complement of leading edge vorticity associated with the steady case of the section at the effective incidence of w_1/V. Since the fluid motion in the cross-flow plane is essentially incompressible there are no time lags for the propagation of the pressure field at the wing to infinity. Therefore the problem in the cross-flow plane becomes instantaneously the steady flow problem, since there is no additional leading-edge vorticity after the gust front has passed the section. Assuming that the solution to the steady problem is unique the section instantaneously takes up the steady state loading associated with the effective incidence w_1/V as the gust front passes the section. This statement implies that the vortex rolls up as it is convected and is fully rolled up behind the gust front.

In the preceding paragraph it is stated initially that the results are based on the assumption that each section creates the vorticity associated with the steady case at incidence w_1/V instantaneously as the gust front passes that section, and then at the same rate subsequently. An alternative physical approach is to argue that if the steady state condition is instantaneously attained at one section as the gust passes that section then all subsequent sections will instantaneously attain their steady state conditions as the gust front passes them. But the initial section can be taken as the apex, where the leading-edge vorticity is zero. Therefore the solution follows.

If the wing is at an initial incidence α the above remarks still apply. Mathematically each section instantaneously takes up the steady loading condition associated with the effective incidence $\alpha + w_1/V$. Physically this is more obscure than the case discussed above since in this case this result means that the leading edge vortex associated with the initial incidence α instantaneously increases its strength and changes the position of its core. However, the approximation of the slender wing theory that the rates of change in the cross-flow planes are greater than rate of change in the streamwise direction imply that the rate of change of the vortex strength and its position is greater than the rate at which the gust front passes over the wing.

It is claimed that the solution presented for this problem is the correct mathematical one on the basis of the linear slender wing approximation, and not a crude approximation.

In real fluids two viscous effects will modify these analytical solutions. The first will be the viscous lag which will delay the formation of the leading edge of vorticity (this is analagous to the lag of the formation of trailing edge vorticity for a two-dimensional oscillating wing in incompressible flow). There will also be a lag in the formation of the secondary separations on the upper wing surface near the leading edge.

6. The Non-Linear Slender Wing Theory: Sudden Change of Incidence

If a slender wing at zero incidence in a stream of velocity V suddenly changes to a finite incidence α at time $t = 0$, then initially at $t = +0$, before any leading edge vortices are formed the velocity potential is given by the linear theory (presented in Section 2). In this case vorticity is created along the whole of the leading edges and then convected downstream. Thus, at a reference chordwise section x aft of the apex the strength of the leading edge vorticity builds up from zero at time $t = 0$ to the full strength associated with the steady case at incidence α after a certain time (probably infinite).

Mathematically, the potential problem to be solved, by extending the equations presented in Section 4 to this unsteady case, is the solution of the differential equation,

$$\phi_{yy} + \phi_{zz} = 0 \qquad \ldots (22)$$

with the boundary conditions

(i)
$$\left(\frac{\partial\phi}{\partial z}\right)_{\substack{z=0\\ |y|\leqslant S(x)\\ t>0}} = -\alpha V \qquad \ldots (23)$$

(ii) there is no flow across the leading-edge vortex sheet, so

$$\left.\frac{\partial\phi}{\partial n}\right|_{S=0} = -\sin\lambda\left(\frac{\partial r}{\partial t} + V\frac{\partial r}{\partial x}\right)\Bigg|_{\substack{S=0\\ \theta=\text{const.}}} \qquad \ldots (24)$$

where
$$S(x,\ r,\ \theta,\ t) = 0$$

is the equation of the leading-edge vortex.

(iii) there is no pressure loading across the vortex sheet, hence

$$\left(\frac{\partial(\Delta\phi)}{\partial t} + V\Delta\left(\frac{\partial\phi}{\partial x}\right) + \left(\frac{\partial\phi}{\partial\sigma}\right)_m \frac{\partial}{\partial\sigma}(\Delta\phi)\right)\Bigg|_{S=0} = 0 \qquad \ldots (25)$$

Unfortunately this set of equations cannot be resolved for a slender delta wing by the assumption of conical flow to the solution in only one cross flow plane, as performed by Mangler and Smith[4] in the steady case. The concept of conical flow must break down in this problem on a dimensional basis, because the time t cannot be non-dimensionalised by a combination of x, y and V without upsetting the condition that the non-dimensional variables remain a function of y/x only at each instant of time.

The following technique is suggested for this problem with a slender delta wing for its approximate solution. The aim is to attempt to obtain a time dependent solution from the solution which is available for the steady case of the slender delta wing.

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The velocity potential discontinuity across the planform of a delta wing in steady motion is denoted by

$$\Delta \Phi = \Delta \Phi (x, y, \alpha) \qquad \ldots (26)$$

This solution is available although it is not given directly in Ref.4.

For the case of the sudden change of incidence, in a particular cross-flow plane, at section x say, the leading-edge vortex will be zero at time $t = 0$, and then build up in strength with a movement of the vortex core with increasing time, until the final steady values are reached. Qualitatively the distributions of $\Delta\phi$ across the wing with increasing time will resemble the distributions in the steady case as the incidence increases from zero to α. Therefore it is suggested that a simple representation of this may be written

$$\Delta \phi(x, y, t) = \frac{\Delta \Phi(x, y, \alpha[x, t])}{\alpha[x, t]} \alpha \qquad \ldots (27)$$

where $\Delta \phi(x, y, t)$ represents the time dependent solution across the wing for the section x for a sudden change of incidence from zero to α at $t = 0$; $\dfrac{\Delta \Phi(x, y, \alpha[x, t])}{\alpha[x, t]}$ represents the <u>distribution</u> of load expected with increasing time, and the term α must be included since the wing is at the constant incidence α for $t > 0$. Therefore

$$\alpha (x, 0) = 0 \qquad \ldots (28)$$

and

$$\alpha (x, t) \to \alpha$$

after a sufficient time.

It is assumed from equations (27) and (28) that

$$[\Delta\phi(x,y,t)]_{t\to 0} = \left[\Delta\Phi(x,y,\alpha[x,t]) \frac{\alpha}{\alpha[x,t]}\right]_{t\to 0} - 2V(S^2(x)-y^2)^{\frac{1}{2}}\alpha \quad \ldots (29)$$

which is the linear solution. The convergence of the steady solution as α decreases to the linear solution is shown by Mangler and Smith although the convergence is non-uniform at $\alpha = 0$. This is due to the different singularities of $\Delta\Phi$ for $\alpha > 0$ and $\alpha = 0$.

The next point which needs clarifying is the function $\alpha(x, t)$, introduced in equation (27). For our unsteady problem it is now assumed that the leading-edge vorticity which is shed during the transient motion at any particular section is the same as that in the steady case, when this section is at the same effective incidence. Thus the rate of creation of leading edge vorticity at section x, for $t > 0$, is

$$\left[V \frac{\partial \Gamma}{\partial x} (x, \alpha)\right]_{\text{steady case}} \qquad \ldots (30)$$

This is the same as equation (20) and has therefore been calculated.[4] Vorticity is convected with the free stream velocity, so that the strength of the leading-edge vorticity at section x after time t is

$$\Gamma(x,t) = \int_{x-Vt}^{x} \frac{\partial\Gamma(x,\alpha)}{\partial x}\, dx = [\Gamma(x,\alpha) - \Gamma(x-Vt,\alpha)]_{\text{steady case}} \qquad \ldots (31)$$

This equation states that $\Gamma(x,0)$ is zero at time $t = 0$ and builds up to its steady value in time $t = x/V$ (which will vary with chordwise section position x).

It is stated for equation (30) that $\Gamma(x,\alpha)$ in the steady case has been calculated, so by inversion, the function $\alpha(x,\Gamma)$ is known for the steady case. Hence we now assume that

$$\alpha(x,t) = \alpha(x, \Gamma[x,t]) \qquad \ldots (32)$$

where $\Gamma[x,t]$ is given by equation (31).

Thus equations (32), (31) and (27) given an approximate time dependent solution based on our knowledge of the steady state characteristics. The pressure coefficient can be determined from equation (27) by the formula

$$c_p = \frac{2}{V^2}\left[\frac{\partial(\Delta\phi)}{\partial t} + V\,\Delta\left(\frac{\partial\phi}{\partial x}\right) + \left(\frac{\partial\phi}{\partial y}\right)_m \frac{\partial(\Delta\phi)}{\partial y}\right]_{\substack{|y|\leqslant S(x)\\ z=0}} \qquad \ldots (33)$$

The application of equation (33) introduces the following points

(i) at time $t = 0$, c_p will <u>not</u> be identical to the linear solution because the term $\partial(\Delta\phi)/\partial t$, which involves $\partial\alpha(x,t)/\partial t$, is not zero. However c_p should remain finite since

$$\frac{\partial\alpha(x,t)}{\partial t} = \frac{\partial\alpha}{\partial\Gamma}\cdot\frac{\partial\Gamma(x,t)}{\partial t}$$

where $\partial\Gamma/\partial t = O(1)$ at $t = 0$, and $\partial\alpha/\partial\Gamma = O(1)$ at time $t = 0$. This latter condition is based on the observation that at very small incidences the effect of the leading-edge separation is to transfer the singularity from the wing to the leading-edge vortex, so the strength of the vortex should then be proportional to α. This point requires further investigation.

(ii) at time $t = x/V$, c_p will be discontinuous because of the discontinuity in $\partial\alpha/\partial t$. However, this is a sectional discontinuity in the respect that it occurs at different sections at different times. But the overall force and moment coefficients will be continuous functions of time. It could be argued that the magnitude of this sectional discontinuity will indicate the usefulness and correctness of this approximate method.

Unfortunately it is difficult to assess whether or not the results from the above approximate theory, which will involve some computation, are satisfactory. An exact solution of equation (22) with boundary equations (23), (24) and (25), will involve a considerable amount of numerical work. Experimentally, this problem of sudden change of incidence is virtually impossible to perform. However, this particular problem is an academic one, rather than one which idealizes some particular motion of an aircraft during a transient condition.

It is suggested that it would be profitable to extend the ideas presented here to more complex transient motions (rigid wing harmonic oscillations, for example) which can be experimentally tested in a wind tunnel.

Notation

x, y, z	cartesian co-ordinates, origin at the wing apex
u, v, w	perturbation velocities
V	free stream velocity, parallel to x direction
ϕ	velocity potential
c_o	maximum chord of slender wing
$\|y\| = S(x)$	equation of leading edges
$\Delta\phi$	discontinuity of velocity potential
α	incidence
w_1	uniform upwash velocity behind the gust front
$S(x,r,\theta) = 0$	equation of leading-edge vortex in the steady case
(r,θ)	polar co-ordinates in cross-flow plane, origin at $y = z = 0$
λ	additional polar angle (see Fig.2)
σ	tangential co-ordinate along a surface
$\Gamma(x)$	strength of leading-edge vorticity in steady case
$\Delta\Phi(x,y,\alpha)$	discontinuity of velocity potential across the wing in the steady case
$\Delta\phi(x,y,t)$	discontinuity of velocity potential across the wing in the unsteady case
$\alpha(x,t)$	empirical incidence function
$\Gamma(x,t)$	strength of leading vorticity in unsteady case.

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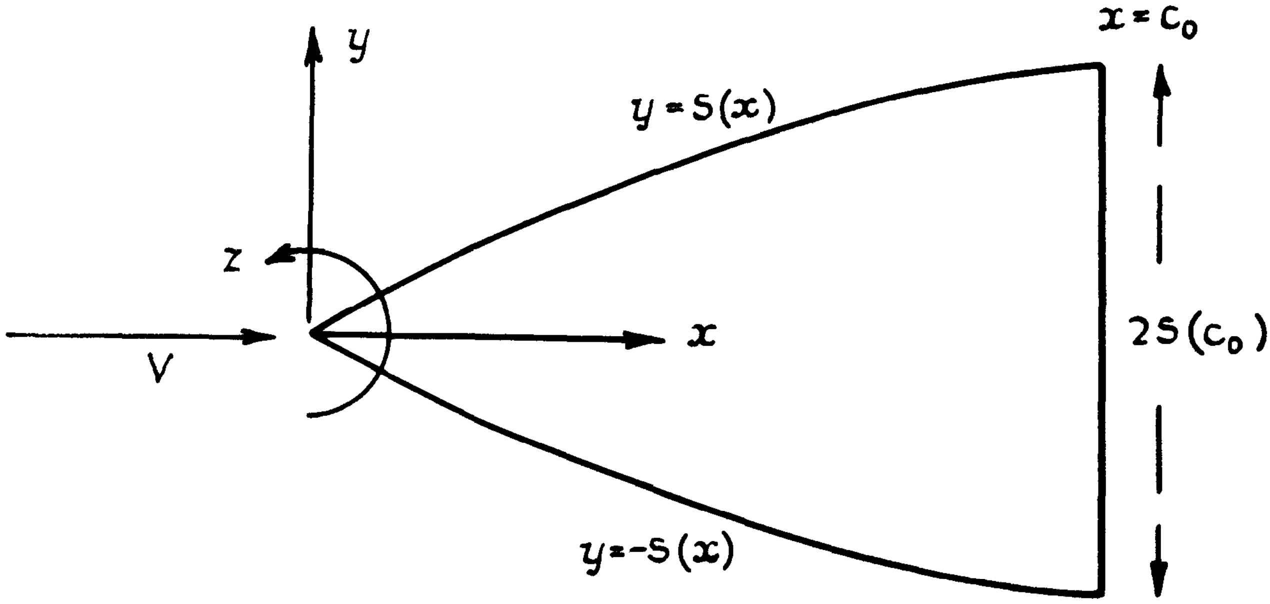
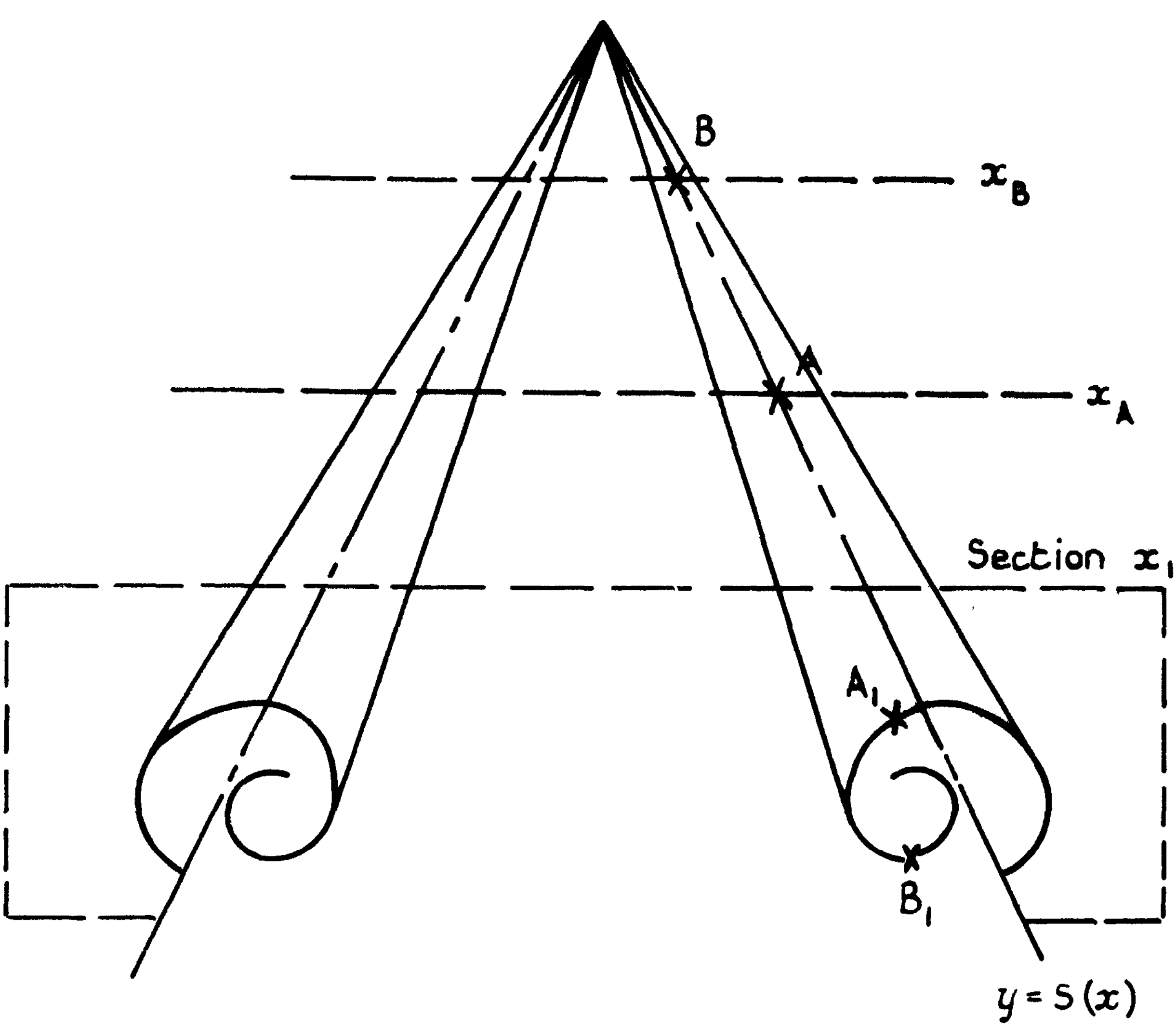
FIGS. 1 & 3.

FIG. 1.



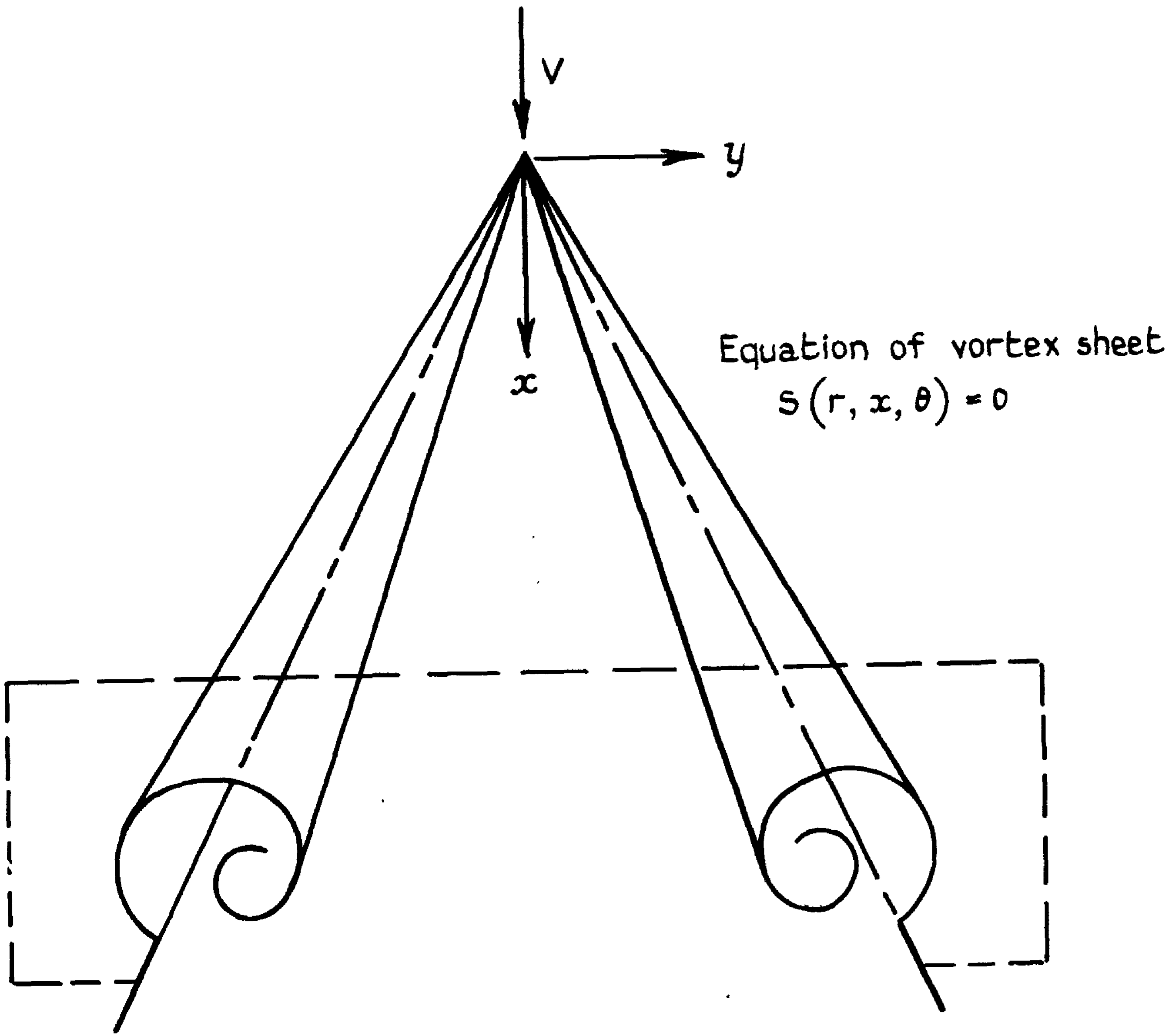
Notation

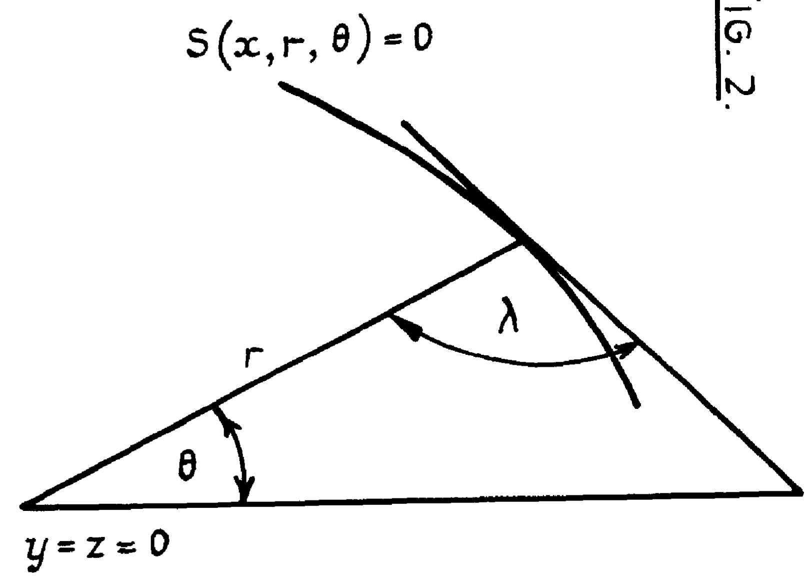
FIG. 3.



Convection of vorticity

FIG. 2.





Slender wing notation

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