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The Measurement of Sub-Critical Damping on the R.A.E. Flutter Simulator

by

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THE MEASUREMENT OF SUB-CRITICAL DAMPING ON THE R.A.E. FLUTTER SIMULATOR

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SUMMARY

Electronic equipment has been installed in the R.A.E. six-degree-of-freedom flutter simulator which enables the damping of a flutter problem at airspeeds below the critical flutter speed to be obtained more readily than was possible with previous methods. The theory of the method, details of the installation and results obtained with it are given.

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LIST OF SYMBOLS

A	inertia coefficient
B	aerodynamic damping coefficient
C	aerodynamic stiffness coefficient
D	structural damping coefficient
E	structural stiffness coefficient
v	airspeed
v_f	flutter speed
λ	complex root of the flutter solution
λ'	time decay constant
λ''	frequency (radians/sec)
λ_s	root without Damping Control
λ_p	root with Damping Control
Δ	logarithmic decrement
b	damping ratio
V	voltage at amplifier terminals
τ	simulator time constant
τ_f	Damping Control time constant
$^R/S$	amplifier input resistor
C	time constant capacitors
R_f	Damping Control feedback resistor

1 INTRODUCTION

The measurement of the damping of the modes of oscillation of an aircraft flying at airspeeds below the critical flutter speed is of importance to designers because the trend of damping with speed can supply a warning of the approach to flutter. Furthermore, low absolute values of damping are undesirable since prolonged oscillation in particular modes might then occur causing discomfort to the aircrew and passengers and having a deleterious effect on the airframe itself.

A theoretical investigation of the behaviour of an aircraft in flight can be made on an analogue computer, for example, by using the R.A.E. Flutter Simulator[1] but existing methods of obtaining the damping in each mode have been found difficult in operation if a reasonably accurate estimate is required. A superior technique for measuring the damping in particular modes of a flutter problem is described below, and details of the necessary circuit modifications for application of the method with the R.A.E. six-degree-of-freedom simulator are given. Some results obtained with the modified system are presented and discussed.

2 DAMPING IN THE FLUTTER PROBLEM

The equations relating to flutter in n degrees of freedom expressed in the notation used for the R.A.E. Flutter Simulator are:-

$$
\begin{aligned}
A_{11}\ddot{x}_1+(B_{11}v+D_{11})\dot{x}_1+(C_{11}v^2+E_{11})x_1+\dots+A_{1n}\ddot{x}_n+(B_{1n}v+D_{1n})\dot{x}_n+(C_{1n}v^2+E_{1n})x_n &= 0 \\
\dots\dots \qquad\qquad \dots\dots &= 0 \\
A_{n1}\ddot{x}_1+(B_{n1}v+D_{n1})\dot{x}_1+(C_{n1}v^2+E_{n1})x_1+\dots+A_{nn}\ddot{x}_n+(B_{nn}v+D_{nn})\dot{x}_n+(C_{nn}v^2+E_{nn})x_n &= 0 .
\end{aligned}
\qquad \dots\dots (1)
$$

For this system there are n "resonance" frequencies (or latent roots) associated with the n coupled modes of oscillation. If the solutions to these equations are taken to be of the form $x = X_0 e^{\lambda t}$ then the n roots in λ are obtained from the eliminant:-

$$
\begin{vmatrix}
A_{11}\lambda^2+(B_{11}v+D_{11})\lambda+C_{11}v^2+E_{11} & \dots\dots & A_{1n}\lambda^2+(B_{1n}v+D_{1n})\lambda+C_{1n}v^2+E_{1n} \\
\dots\dots & & \dots\dots \\
A_{n1}\lambda^2+(B_{n1}v+D_{n1})\lambda+C_{n1}v^2+E_{n1} & \dots\dots & A_{nn}\lambda^2+(B_{nn}v+D_{nn})\lambda+C_{nn}v^2+E_{nn}
\end{vmatrix} = 0 . \qquad \dots\dots (2)
$$

The damping for each root varies with airspeed and typical curves of damping against airspeed are given in Fig.1. Flutter begins at velocity v_f where the damping in a particular mode becomes zero (mode 1 in this case).

As the term "damping" has different interpretations the definition used here will be outlined. The equation of a damped harmonic oscillation can be given in the form:-

$$x = X_c e^{\lambda t} \tag{3}$$

where x, X_c are the amplitudes at time $t = t$, $t = 0$ and λ is complex, i.e. $(-\lambda' + j\lambda'')$. λ'' is the frequency of oscillation in radians per second and λ' is the "time decay constant" which determines the rate of decay of the oscillation referred to time. A convergent oscillation would have λ' positive.

The damping factor of a decaying oscillation (Fig.1) is defined either by the logarithmic decrement Δ which is the logarithm of the ratio of successive amplitudes giving:-

$$\Delta = 2\pi \frac{\lambda'}{\lambda''} \tag{4}$$

or by the factor b, the ratio of the damping present to the critical damping given by:-

$$b \simeq \frac{\Delta}{2\pi} = \frac{\lambda'}{\lambda''} \tag{5}$$

provided b is small.

It should be noted that $\lambda'' = (1-b^2)^{\frac{1}{2}} \lambda''_n$ where λ''_n is the undamped natural frequency.

The current technique for the measurement of damping on the flutter simulator is by using the sinusoidal driving unit in either of the following methods:-

(i) tune the unit to the mode frequency required, cut it off sharply and count the number of cycles to half amplitude;

(ii) obtain an amplitude-frequency curve in the vicinity of the mode frequency and calculate the damping from the frequency increment between the maximum and half maximum amplitude points.

Both these methods have serious disadvantages. In the first place it is difficult to adjust the excitation so that a 'pure' mode is excited. It is also difficult to cut the driving unit without injecting spurious frequencies and exciting other modes; and to obtain a reliable amplitude frequency curve requires great care in maintaining steady conditions while the measurements are taken. In the case of modes which have nearly coincident natural frequencies, damping measurement by these techniques is virtually impossible. For these reasons a more simple and direct method of measuring damping would be useful.

3 THE PRINCIPLE OF DAMPING CONTROL

The method is based on a technique suggested by Wood and Hansford[2] for use in the solution of flutter problems on analogue computers. It consists of modifying the purely capacitative feedback of all integrating amplifiers by the additions of parallel resistive feedback.

Assuming that the amplifier gain is large so that the input error currents are negligibly small, the relation between input and output voltages for the integrator circuit shown in Fig.2a is given by:-

$$V_{in} = -\frac{CR}{S}\dot{V}_{out}$$

$$\therefore V_{out} = -\frac{S}{CR}\int V_{in}\,dt\,.$$

With an input voltage of the form $V_{in} = V_o\,e^{\lambda_s t}$ and $\tau = \frac{CR}{S}$

$$V_{out} = -\frac{1}{\tau\,\lambda_s}V_{in}\,. \tag{6}$$

The circuit for one-degree-of-freedom oscillation is shown schematically in Fig.3, including the integrating amplifiers 2 and 3. When the feedback paths are quoted in terms of their conductances (which are proportional to the inertia, damping and stiffness coefficients) summation of currents to the input of amplifier 1 gives:-

$$AV_1 - DV_2 + EV_3 + BV_4 - CV_5 = 0$$

also

$$V_4 = -v\,V_2$$

$$V_5 = -v^2\,V_3$$

$$\therefore \quad AV_1 - (D+Bv)V_2 + (E+Cv^2)V_3 = 0\,. \tag{7}$$

Substitution of equation (6) into (7) gives the equation for a one-degree-of-freedom oscillation:-

$$\left[A\tau^2\lambda_s^{\,2} + (D+Bv)\,\tau\,\lambda_s + (E+Cv^2)\right]V_1 = 0 \tag{8}$$

with the solution $V_1 = V_o e^{\lambda_s t}$ where V_o is the amplitude of oscillation at time $t = 0$.

When the integrator is modified to the circuit of Fig.2b the relation between output and input voltage is modified to give:-

$$C\dot{V}_{out} + \frac{V_{out}}{R_f} = -\frac{S}{R}V_{in}\,.$$

With an input voltage of the form $V_{in} = V_o e^{\lambda_p t}$ this gives:-

$$V_{out}\left(\frac{RC}{S}\lambda_p + \frac{R}{SR_f}\right) = -V_{in}.$$

Letting

$$\tau = \frac{RC}{S}$$

and

$$\tau_f = R_f C$$

$$\tau\, V_{out}\left(\lambda_p + \frac{1}{\tau_f}\right) = -V_{in}$$

$$\therefore\ V_{out} = \frac{-V_{in}}{\tau\left(\lambda_p + \frac{1}{\tau_f}\right)}\ . \qquad (9)$$

Substitution of equation (9) into (7) gives the equation for a one-degree-of-freedom oscillation with Damping Control added:-

$$\left(A\,\tau^2\left\{\lambda_p + \frac{1}{\tau_f}\right\}^2 + [D+Bv]\,\tau\left\{\lambda_p + \frac{1}{\tau_f}\right\} + [E+Cv^2]\right)V_1 = 0 \qquad (10)$$

where the solution $V_1 = V_o e^{\lambda_p t}$.

From the identity of equations (8) and (10) it follows that

$$\lambda_p = \lambda_s - \frac{1}{\tau_f}\ . \qquad (11)$$

The solution of equation (10) is therefore:-

$$V_1 = V_o e^{\lambda_p t} = V_o e^{\lambda_s t}\, e^{-(1/\tau_f)t}$$

which is the solution of equation (8) factored by the exponential term $e^{-(1/\tau_f)t}$, <u>i.e. the basic frequency of the solution is unaltered.</u> The factor introduced by Damping Control is by the definition of equation (5):-

$$b_p = \frac{1/\tau_f}{\lambda_s''} = \frac{1}{R_f C \lambda_s''} . \qquad (12)$$

The extension of Damping Control to a multi-degree of freedom problem is straightforward. Fig.4 shows the circuit of a two degree of freedom system, for which the flutter equations are:-

$$\left[A_{11}\,\tau_1^2\left(\lambda + \frac{1}{\tau_f}\right)^2 + \{D_{11} + B_{11}v\}\,\tau_1\left(\lambda + \frac{1}{\tau_f}\right) + \{E_{11} + C_{11}v^2\}\right]V_1 +$$

$$\left[A_{12}\,\tau_2^2\left(\lambda + \frac{1}{\tau_f}\right)^2 + \{D_{12} + B_{12}v\}\,\tau_2\left(\lambda + \frac{1}{\tau_f}\right) + \{E_{12} + C_{12}v^2\}\right]V_2 = 0$$

$$\left[A_{21}\,\tau_1^2\left(\lambda + \frac{1}{\tau_f}\right)^2 + \{D_{21} + B_{21}v\}\,\tau_1\left(\lambda + \frac{1}{\tau_f}\right) + \{E_{21} + C_{21}v^2\}\right]V_1 +$$

$$\left[A_{22}\,\tau_2^2\left(\lambda + \frac{1}{\tau_f}\right)^2 + \{D_{22} + B_{22}v\}\,\tau_2\left(\lambda + \frac{1}{\tau_f}\right) + \{E_{22} + C_{22}v^2\}\right]V_2 = 0$$

..... (13)

with the time constants, τ_1, τ_2, in the respective degrees of freedom and τ_f equal in each. Comparison of equation (13) with equation (2) shows that a root λ_s is replaced by λ_p where

$$\left.\begin{aligned} \lambda_p' &= \lambda_s' + \frac{1}{\tau_f} \\ \lambda_p'' &= \lambda_s'' \\ b_p &= \frac{1}{\tau_f\,\lambda_s''} . \end{aligned}\right\} \qquad (14)$$

The extension to any number of degrees of freedom follows the same procedure.

The Damping Control can be made negative by reversing the sign of R_f. This is achieved by reconnecting R_f to the opposite signed output of the push-pull amplifiers used in the Flutter Simulator.

4 MEASUREMENT OF THE STABILITY OF THE FLUTTER EQUATIONS USING DAMPING CONTROL

A solution to the flutter equations generally consists of a convergent or divergent harmonic oscillation. The rate of convergence or divergence is modified with Damping Control by the factor $e^{-(1/\tau_f)t}$ and it follows that when

$\lambda_s' = 1/\tau_f$ the solution is zero damped, i.e. a steady oscillation. The frequency of the oscillation is then easily measurable and the damping factor is given by

$$b = + \frac{1}{\lambda_s'' \tau_F}$$

where τ_F is the value for zero net damping. Note that for an initially convergent oscillation Damping Control must add negative damping to give a steady oscillation. It was shown in section 2 that the flutter equations in n degrees of freedom have n latent roots corresponding to n resonance modes. Damping Control modifies all the roots and it follows that a zero damped solution is obtained from an initially convergent solution when $-1/\tau_f$ equals λ_s' in the mode with the longest time decay, i.e. the smallest value of λ_s'. If Damping Control is further increased, this mode becomes divergent and no further useful results can be obtained.

The mode with the shortest decay time or highest value of λ' can, however, be determined by changing the sign of all the damping terms in the original set of equations thus making the problem initially divergent, and then finding the amount of Damping Control required to give a zero damped oscillation. For this process the sign of the Damping Control will also require changing. The sign of the damping terms can be reversed by reversing the sign of the feedbacks from amplifiers 2 and 4 to amplifier 1 in each degree of freedom.

It is possible therefore to measure the damping in the modes with the longest and shortest time decays. It does not necessarily mean however, that these have the smallest and largest damping factors. Equation (5) shows that a small value of λ' for a low frequency mode may well correspond with a higher value of b than that for a greater value of λ' in a high frequency mode. Close to the flutter speed, the lowest λ' usually corresponds with the lowest b but remote from the flutter speed this is not necessarily so. However, for most flutter work, damping in the neighbourhood of the critical condition is required and in general Damping Control will provide this result adequately.

5 PRACTICAL CIRCUIT INSTALLED IN THE R.A.E. FLUTTER SIMULATOR

Equation (9) gives the relation between input and output voltages of the circuit of Fig.2b with the time constant τ, τ_F as defined.

The time constant τ is changed in the R.A.E. simulator by changing both the feedback capacitors C and the input resistors R/S. The feedback capacitors are changed simultaneously in all degrees of freedom by one control giving overall frequency ranges of "times 1", "times 10" and "times 100". The input resistors R/S in each degree of freedom are also variable in steps and provide frequency scaling factors of $\frac{1}{4}$, $\frac{1}{2}$, 1, 2 or 4. These scaling factors are used to make the numerical settings for the inertia and stiffness terms as large as possible. For instance, if the stiffness term in a degree of freedom were numerically equal to one quarter of the inertia term, the practice would be to quadruple the stiffness term and use a frequency scaling factor of $\frac{1}{2}$ to maintain the original frequency of solution. In these circumstances C λ_s'' in a given problem will have the same value whatever the settings for overall frequency range or time constant in each degree of freedom. The definition of the factor introduced by Damping Control (equation (12)) shows therefore, that b_p is inversely proportional to R_f. When $b_p = 0$, R_f is infinite, so that R_f has to be variable from a high fixed value to infinity. This is most conveniently achieved by using a fixed high value resistor R_f' tapping into a variable potentiometer R_v on the output of the integrator amplifier as shown in Fig.5. With this arrangement $R_f = \frac{R_f'}{S_f}$ where S_f is the potentiometer setting, so that the damping control factor $b_p = \frac{S_f}{R_f' C \lambda_s''}$ and b_p is directly proportional to $\frac{S_f}{\lambda_s''}$ since $R_f' C$ is constant in a given problem (provided $R_f' \gg R_v$).

The value of R_f' determines the range of damping factor b_p which can be added. When the machine is operating at a frequency such that $C \lambda_s'' = 1$ (i.e. the optimum design condition) b_p maximum is 10% and 100% respectively for values of R_f' of 10 megohms and 1 megohm. These ranges should be adequate both for the investigation of low damped modes and for high damped modes in the problem inverted as described in section 4.

Since push-pull amplifiers are used in this simulator, twenty-four ganged potentiometers are required for simultaneous equal settings in all six degrees of freedom. These are ten-turn helical potentiometers of value 50 kilohms geared together in a central unit with provision for manual adjustment of the setting S_f. Also situated in the central unit is the range switch for changing R_f' from 10 megohms to 1 megohm and a switch which operates a relay in each of the six degrees of freedom to invert the sense of the feedback lines R_f and the sign of all damping terms in the problem. The circuit is shown in Fig.6. An indicator light is provided which is illuminated only when the potentiometers R_v are set at zero and the sign-changing relays are inoperative. This is the condition for operation of the simulator without Damping Control (i.e. "normal" operation).

6 TESTS ON DAMPING CONTROL

6.1 Tests on individual degrees of freedom

The inertia and stiffness coefficients on all degrees of freedom were set to their maximum values (unity), and the overall time constant set to provide a circular frequency of 10 rads/sec. Each degree of freedom was then taken in turn and the damping coefficient varied in steps. The required amount of opposing Damping Control to maintain the oscillation at constant amplitude was then measured for each step. A typical example of the six curves obtained is shown in Fig.7. These tests were sufficient to show that the circuits were behaving as intended, and demonstrated the existence of inherent damping factor error of approximately 0.5% of critical damping in each degree of freedom.

This inherent error is due to losses in the circuit components. It is variable with the frequency of the solution to a flutter problem. As the setting of the Damping Control is also dependent on frequency it is not possible to compensate for this error by installing a simple correction circuit. Though small in itself the error can be significant since damping factor values around 1% of critical commonly occur in flutter solutions.

6.2 Application to binary flutter problem

A sample binary flutter problem originally considered by Broadbent and Hartley[3] was investigated and the damping in each degree of freedom obtained at a range of settings of the velocity control. The results are compared in Fig. 8 with the analytical curves of Ref. 3, the ordinates of which were computed using "DEUCE". Reasonable agreement in damping values for both roots is obtained.

6.3 Application to a quaternary problem

A four-degree-of-freedom problem for which dampings at sub-critical speeds had already been computed on "DEUCE" was then tested on the Flutter Simulator. In this case only dampings for the modes with longest and shortest time decays can be obtained. In examining the mode with the shortest time decay in this problem the damping range became restricted because of the high frequency of the root (see section 5) and it was found necessary to re-scale the problem. It may often happen, therefore, that a wide separation of frequencies between modes with longest and shortest time decays will necessitate different scaling for each condition. The results are given in Fig. 9.

7 CONCLUSIONS

In a problem having many degrees of freedom the sub-critical damping for the two modes with the shortest and longest time decays may be obtained by this method of damping control. It is essentially a process whereby the damping in these modes is reduced to zero, resulting in oscillations of constant amplitude. The errors observed with the system installed in the R.A.E. Flutter Simulator are considered to be acceptable for general investigations of sub-critical damping for flutter problems. The control equipment is convenient in use and the measurements are presented as a scale reading of a potentiometer setting which is easily converted to percentage critical damping.

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2	Wood, K.E. Hansford, I.V.	Flutter Simulator in two degrees of freedom. (Vickers Armstrong Technical Report No. 104). A Flutter Computer with low gain amplifiers. (Electronic Engineering, November 1955).
3	Broadbent, E.G. Hartley, E.V.	Vectorial analysis of flight flutter test results. A.R.C. R. & M. No. 3125. February 1958.

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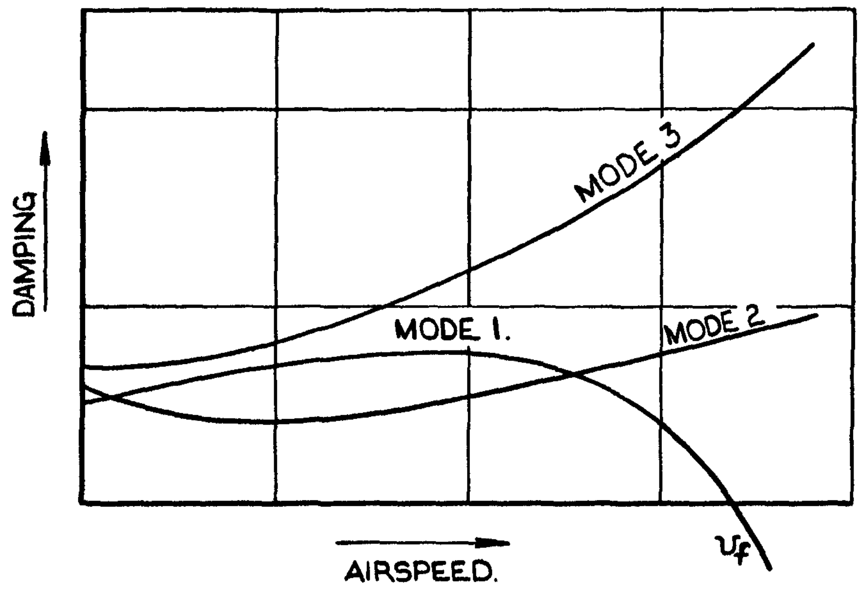


FIG. 1. TYPICAL DAMPING/AIRSPEED CURVES.

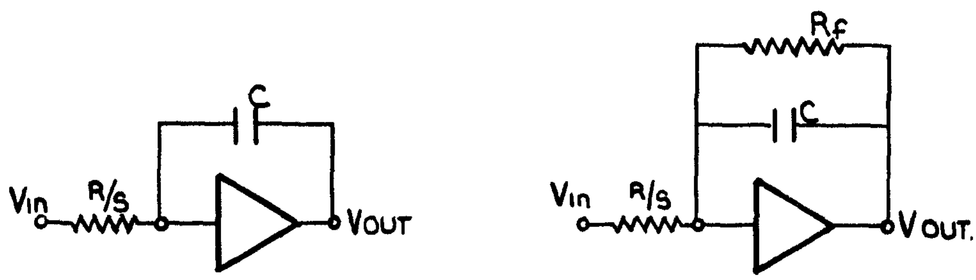


FIG. 2. (a) INTEGRATING CIRCUIT. FIG. 2. (b) MODIFIED INTEGRATING CIRCUIT.

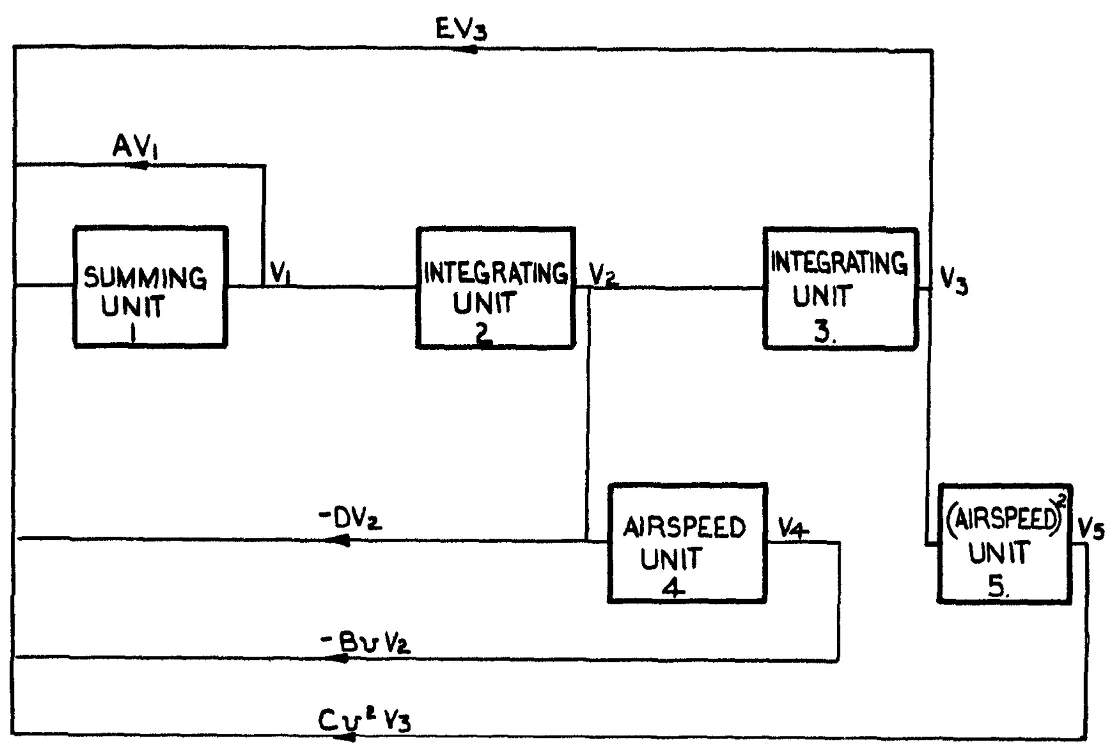


FIG. 3. CIRCUIT FOR ONE DEGREE OF FREEDOM OSCILLATION.

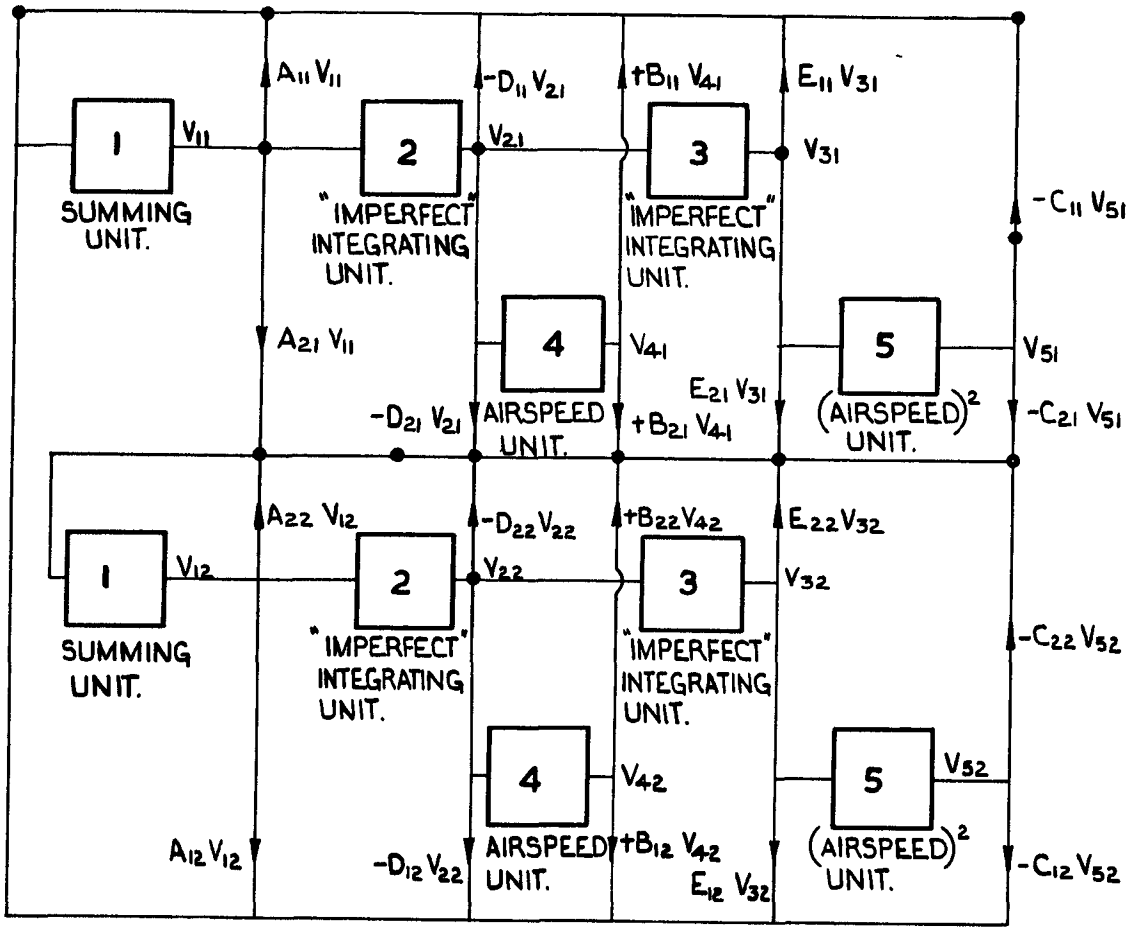


FIG. 4. CIRCUIT FOR TWO DEGREE OF FREEDOM OSCILLATION.

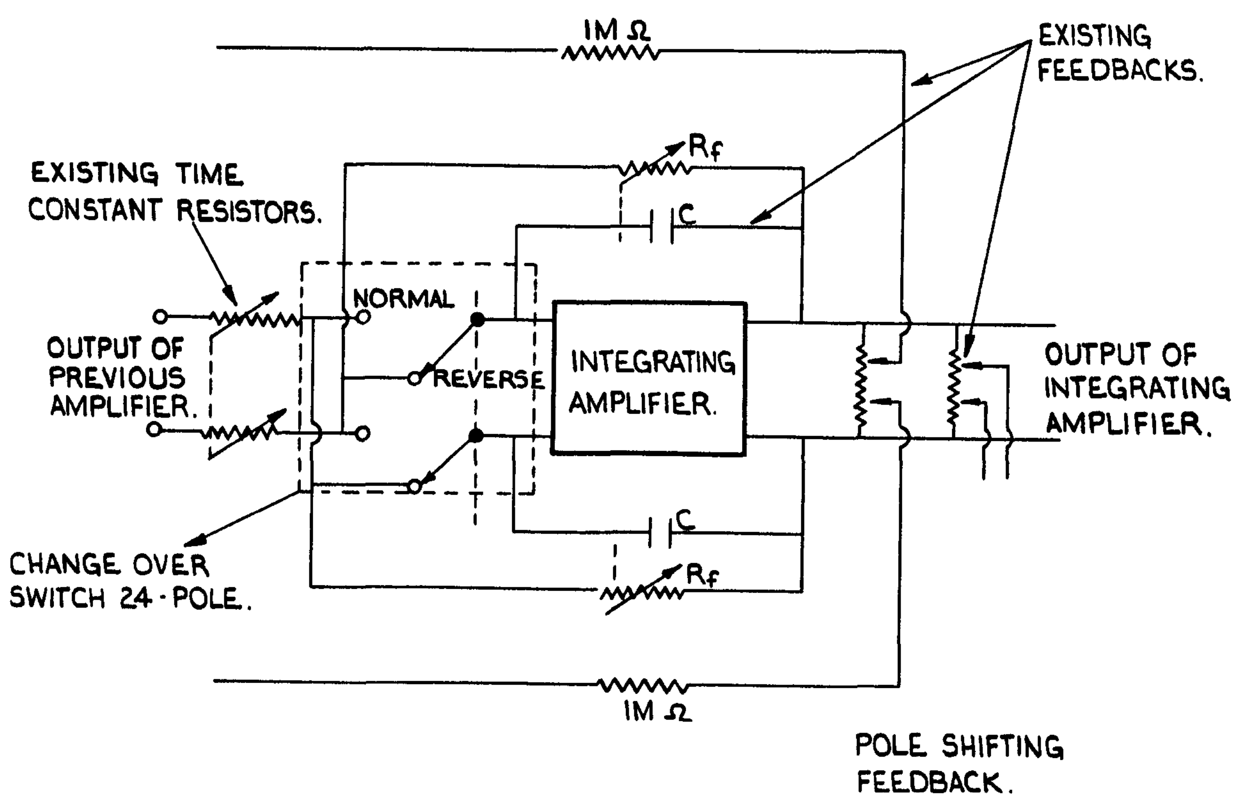


FIG. 5. CIRCUIT GIVING A COMBINED SIGN CHANGE OF DAMPING AND POLE SHIFTING.

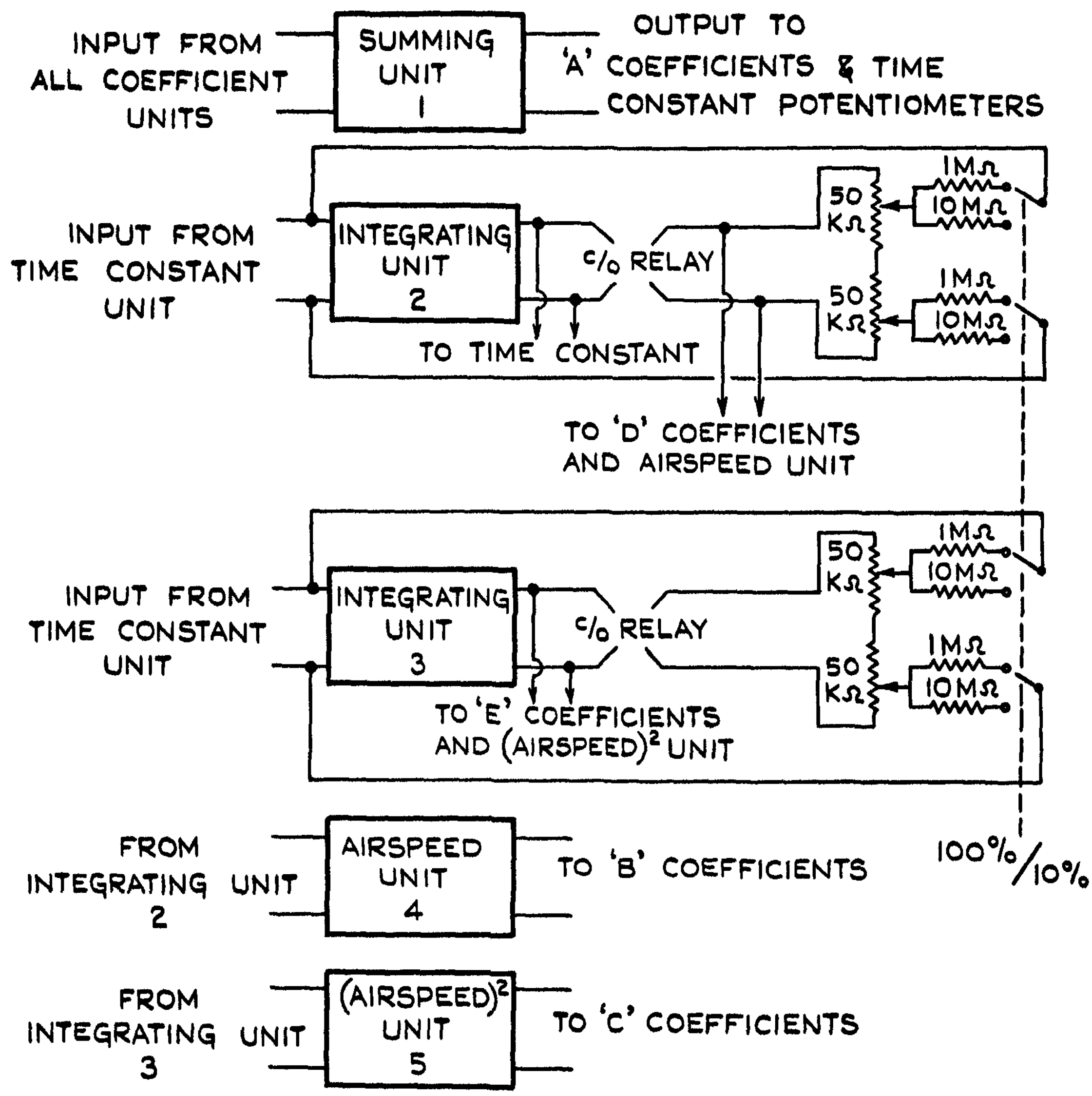


FIG. 6. MANUAL DAMPING CONTROL CIRCUIT FOR ONE DEGREE OF FREEDOM.

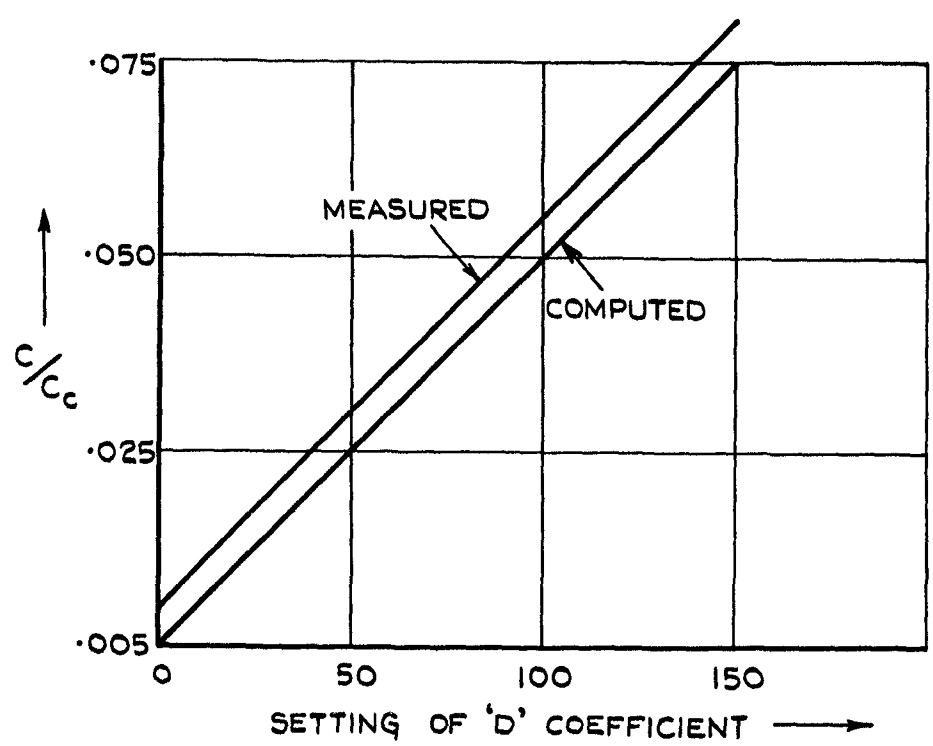


FIG. 7. ERROR CURVE IN A SINGLE DEGREE OF FREEDOM.

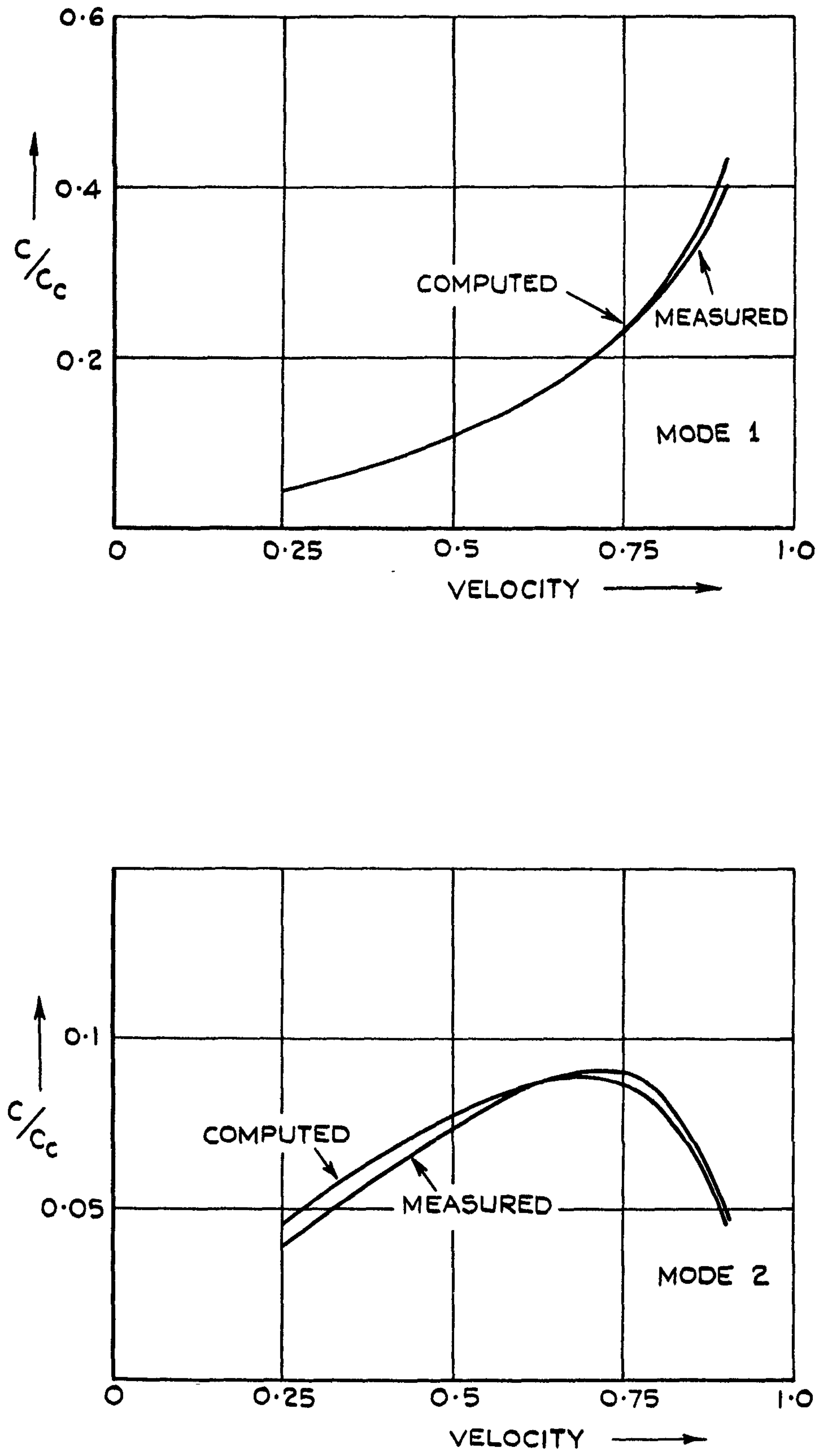
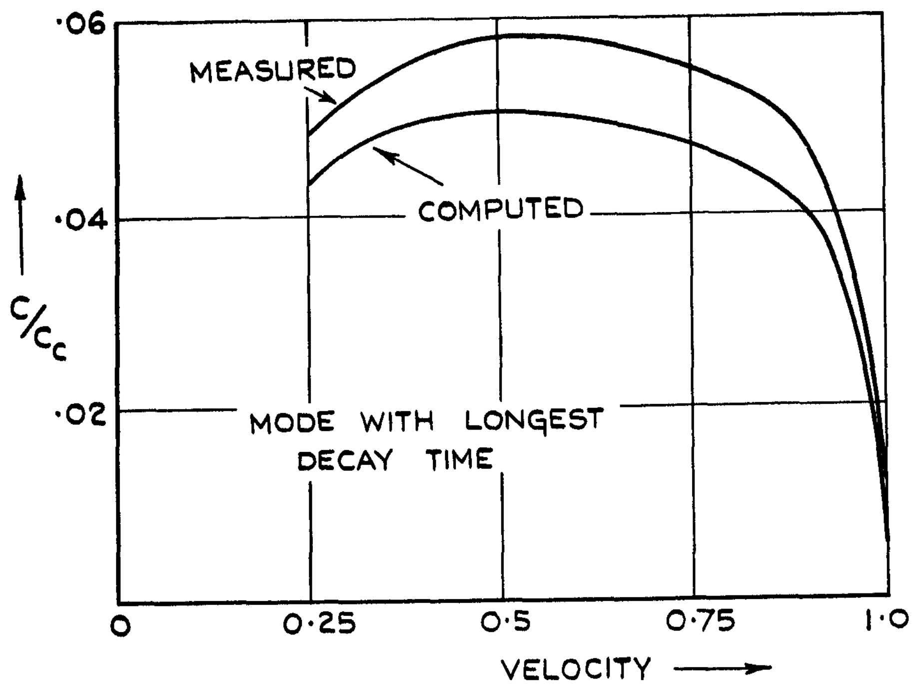


FIG. 8. SUB - CRITICAL DAMPING IN BINARY FLUTTER PROBLEM.



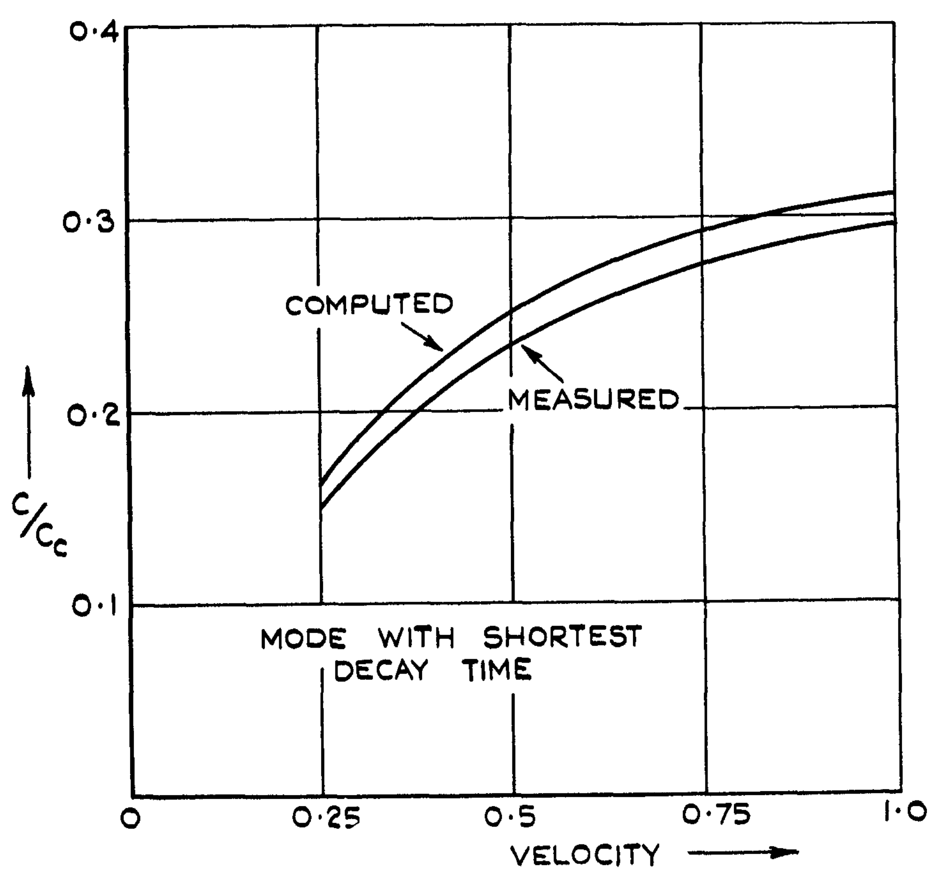


FIG.9. SUB-CRITICAL DAMPING IN QUATERNARY FLUTTER PROBLEM.

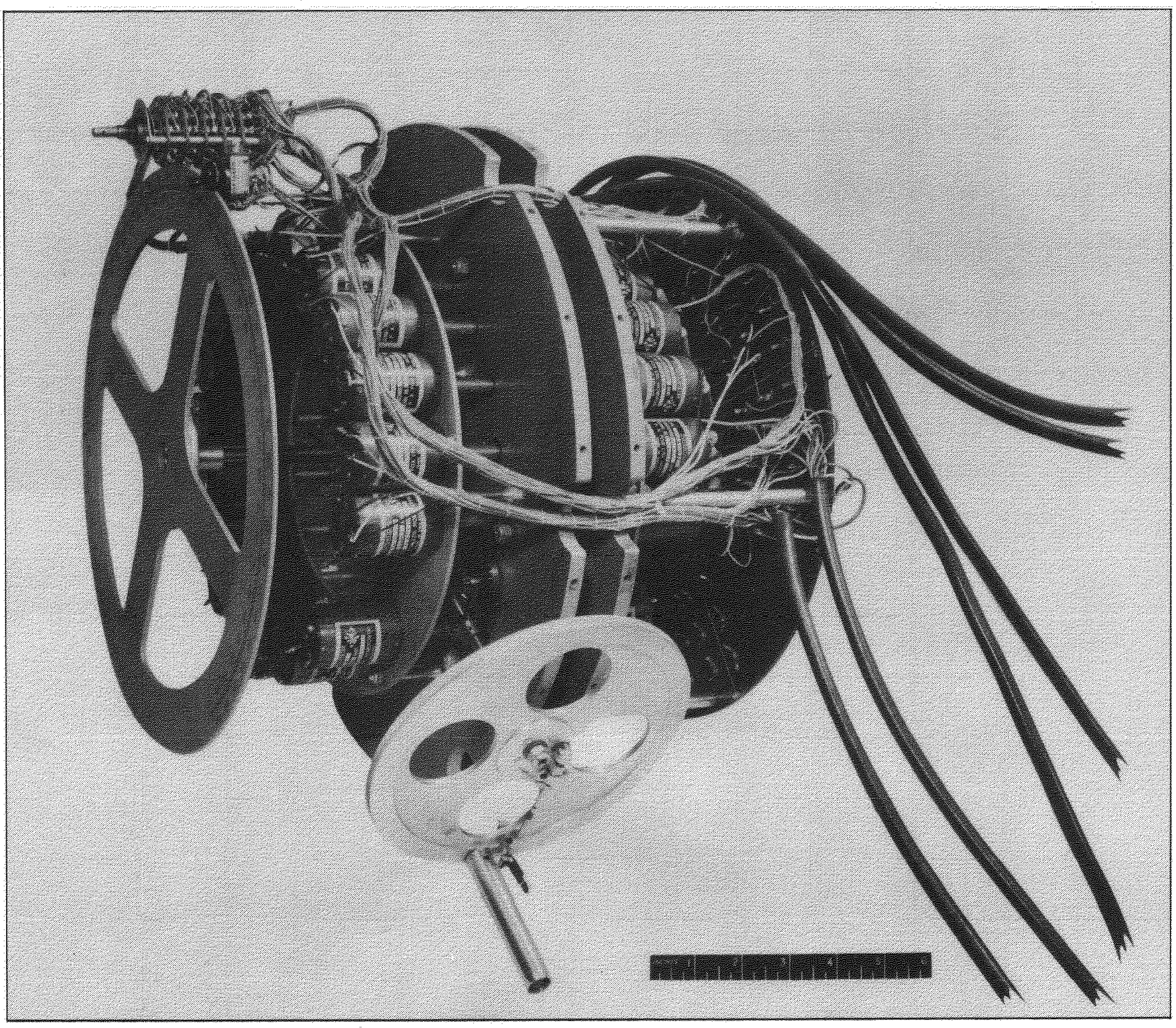


FIG. 10. DAMPING CONTROL DRUM

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