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The Equations of Hydrodynamics in a Very General Form

By

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Summary.—The equations of hydrodynamics are derived in a very general form for a fluid all of whose physical properties are variable. Vector analysis is used.

It is shown that in some circumstances $C_pT + \frac{1}{2}\bar{q}^2 =$ Constant is an integral of the energy equation. The transformation formulae to any co-ordinate system are given.

Introduction.—The equations of hydrodynamics contain symbols which represent the physical properties, density, viscosity and so forth, of the fluid. In the simplest applications these are regarded as constants (including as a particular case zero), and with this assumption the theory has been very thoroughly investigated. The next simplest case is when the fluid is treated as compressible, and here again considerable progress has been made. In recent years, however, it has become increasingly important to have as exact a knowledge as possible of the effect of allowing other quantities, especially viscosity, to vary. Certain particular cases in this category referring to circulation theorems have been worked out[1], but so far as is known no general treatment exists. This report is an attempt to fill this gap, by deriving, once and for all, the general equations when all the physical properties of the fluid vary.

Notation.—In any statement of general principles of this sort, some form of mathematical shorthand is very convenient, otherwise the formulae are apt to become very complicated and the essential statement being made is lost in a cloud of symbols. There are several such systems of which Vector Analysis and Tensor Calculus immediately suggest themselves. The former has been selected on the grounds that it is better known and better represents, in its symbolism, the underlying physical ideas.

There are several competing notations in Vector Analysis; that used by Milne-Thompson[2] in his book " Hydrodynamics " has been selected. A knowledge of vector manipulation will be assumed and frequent reference will be made to this book, hereinafter denoted by " M.T.", and to " Modern Developments in Fluid Dynamics "[3], hereinafter denoted by " F.D." Indeed from one point of view this report is merely an extension of certain parts of these works to the more general case in which the pressure (p), density (ϱ), absolute temperature (T), specific heats (C_p and C_V), thermal conductivity (k) and viscosity (μ) are all scalar point functions. The velocity will be denoted by $\bar{q}$ ($\bar{i}u + \bar{j}v + \bar{k}w$) the dilatation ($\nabla\bar{q}$) by $\triangle$ and the vorticity ($\nabla_\wedge\bar{q}$) by $\bar{\zeta}$. The conversion of any expression obtained to some co-ordinate system will be made via an orthogonal curvilinear system in which $ds^2 = h_1 d\alpha^2 + h_2 d\beta^2 + h_3 d\gamma^2$ with $\bar{i}$, $\bar{j}$, and $\bar{k}$ in the directions of α, β and γ increasing. Throughout $\bar{n}$ denotes the outward drawn normal from the surface S under consideration. Gauss (or the divergence) theorem will be repeatedly used; in its simplest form this states that $\int \bar{n} X dS. = \int \nabla X d\tau$ where X is any scalar or vector point function and in the latter case the product n (or ∇) X may be either scalar or vector. The idea of a " linear vector function" underlies much of the analysis; this states that if $\bar{a}, \bar{b}, \bar{c}$ and $\bar{a}', \bar{b}', \bar{c}'$ are given vectors then $f(\bar{\eta}) \equiv (\bar{a}'\bar{\eta})\bar{a} + (\bar{b}'\bar{\eta})\bar{b} + (\bar{c}'\bar{\eta})\bar{c}$ is a "homogeneous linear function" of the (arbitrary) vector $\bar{\eta}$. If $\bar{\xi}$ be another (arbitrary) vector and h an arbitrary scalar then $f(\bar{\xi} + \bar{\eta}) = f(\bar{\xi}) + f(\bar{\eta})$ and $f(h\bar{\eta}) = hf(\bar{\eta})$. If in addition $\bar{\xi}f(\bar{\eta}) = \bar{\eta}f(\bar{\xi})$ the (linear vector) function is "self conjugate" (M.T. p. 31). The importance of this lies in the fact that the rate of strain of the fluid is given by the self conjugate linear function $(\bar{\eta}\nabla)\bar{q} + \frac{1}{2}\bar{\eta}_\wedge\bar{\xi}$ where $\bar{\eta}$ is the position vector of two adjacent elements of the fluid.

Derivation of the Equations of Motion.—Consider an infinitesimal element of volume $d\tau$ and surface dS. Then from the conservation of mass

$$\frac{\partial}{\partial t}\int \varrho\, d\tau + \int \varrho \bar{q}\, \bar{n}\, dS = 0.$$

$$\therefore \int \left[\frac{\partial \varrho}{\partial t} + \nabla(\varrho\bar{q})\right] d\tau = 0 \text{ from Gauss theorem.}$$

$$\therefore \frac{\partial \varrho}{\partial t} + (\bar{q}\nabla)\varrho + \varrho(\nabla\bar{q}) = 0$$

and we get the Equation of Continuity in the alternative forms

$$\left.\begin{aligned} \frac{d\varrho}{dt} + \varrho\triangle &= 0 \\ \frac{\partial \varrho}{\partial t} + \nabla(\varrho\bar{q}) &= 0 \end{aligned}\right\}.$$

The forces acting on the element are :—

(1) The external force of amount $\int \varrho \bar{F}\, d\tau$.

(2) The normal pressure thrust on the boundary $-\int \bar{n} p\, dS$.

(3) The viscous stresses, these consist of

A term proportional to $\triangle$ $\quad \int \lambda \triangle \bar{n}\, dS$.

A term proportional to the rate of strain $\int [2\mu(\bar{n}\nabla)\bar{q} + \mu\bar{n}_\wedge\bar{\xi}]\, dS$.

By resolving along three mutually perpendicular vectors, it can be shown that $\lambda = -2\mu/3$.

Now apply the second law of motion to the element, we get :—

$$\int \varrho \frac{d\bar{q}}{dt}\, d\tau = \int \varrho \bar{F}\, d\tau - \int p\bar{n}\, dS - \int \frac{2\mu\triangle}{3}\bar{n}\, dS + \int [2\mu(\bar{n}\nabla)\bar{q} + \mu\bar{n}_\wedge\bar{\xi}]\, dS.$$

Using Gauss theorem this becomes

$$\int \varrho \frac{dq}{dt}\, d\tau = \int \left[\varrho\bar{F} - \nabla p - \frac{2}{3}\nabla(\mu\triangle) + 2\nabla\{(\mu\nabla)\bar{q}\} + \nabla_\wedge(\mu\bar{\xi})\right] d\tau$$

or since $d\tau$ is arbitrary

$$\varrho \frac{dq}{dt} = \varrho\bar{F} - \nabla p - \frac{2}{3}\nabla(\mu\triangle) + 2\nabla\{(\mu\nabla)\bar{q}\} + \nabla_\wedge(\mu\bar{\xi})$$

which is the form assumed by the Navier–Stokes equations when the physical properties of the fluid vary. The above is the most compact way of writing them (it is of course equivalent to three

Cartesian equations) ; for further developments it is better to expand the various operands of nabla. Doing this, writing $\bar{\mu}$ for $(\nabla\mu)$ and using the (symbolic) identity $\nabla_\wedge\bar{\xi} = \nabla\triangle - \nabla^2\bar{q}$ we obtain the equations in the alternative forms

$$\varrho\frac{d\bar{q}}{dt} = \rho\bar{F} - \nabla p + \mu\nabla^2\bar{q} + \frac{\mu}{3}\nabla\triangle + 2(\bar{\mu}\nabla)\bar{q} + \bar{\mu}_\wedge\bar{\xi} - \frac{2}{3}\triangle\bar{\mu}$$

$$\varrho\frac{d\bar{q}}{dt} = \varrho\bar{F} - \nabla p - \mu(\nabla_\wedge\bar{\xi}) + \frac{4\mu}{3}\nabla\triangle + 2(\bar{\mu}\nabla)\bar{q} + \bar{\mu}_\wedge\bar{\xi} - \frac{2}{3}\triangle\bar{\mu}$$

in which the increasing complication of the equations as more variations are taken into account is well displayed. The first term on the right-hand side arises from the body forces, the second from the pressures. Together they represent all that is taken into account in the " inviscid, incompressible fluid " of classical hydrodynamics. Allowing for the effect of (constant) viscosity adds the third. Allowing for compressibility adds the fourth. Allowing for the variation of viscosity adds the fifth and sixth. Finally allowing for the variation of both density and viscosity adds the seventh. So far as the first four terms they are well known in Cartesian form (F.D. p. 602). If we neglect body forces and compressibility and assume " slow motion" the equation becomes

$$\nabla p = \mu\nabla^2\bar{q} + 2(\bar{\mu}\nabla)\bar{q} + \bar{\mu}_\wedge\bar{\xi}$$

the Cartesian equivalents of which have been used by Christopherson[4] in connection with lubrication problems. The equivalent cylindrical polar form, with axisymmetric motion, arises when considering the flow through a heated pipe.

The next step is to calculate the dissipation function (Φ) or rate per unit volume at which energy is being dissipated by the viscous stresses. The rate (W_ε) at which the stresses do work on the element is given by

$$W_\varepsilon = \int\bar{q}\varrho\bar{F}d\tau - \int\left[p\,\bar{n}\bar{q} - \bar{q}\left\{\mu 2(\bar{n}\nabla)q - \mu(\bar{n}_\wedge\bar{\xi}) + \frac{2}{3}\mu\bar{n}\triangle\right\}\right]dS.$$

Now

$$2\bar{q}(\bar{n}\nabla)\bar{q} = (\bar{n}\nabla)q^2,$$

so applying Gauss theorem we get

$$W_\varepsilon = \int\left[\bar{q}\bar{F}\varrho - \nabla(p\bar{q}) + \nabla\{(\mu\nabla)\bar{q}^2\} + \nabla\{\mu(\bar{\xi}_\wedge\bar{q})\} - \frac{2}{3}\nabla(\mu\triangle\bar{q})\right]d\tau$$

$$= \int\left[\bar{q}\bar{F}\varrho - \bar{q}\nabla p - p\nabla + \mu\nabla^2\bar{q}^2 + (\bar{\mu}\nabla)\bar{q}^2 + \mu\nabla(\bar{\xi}_\wedge\bar{q}) + \bar{\mu}(\bar{\xi}_\wedge\bar{q}) - \frac{2}{3}\mu\triangle^2 - \frac{2}{3}\mu\bar{q}\nabla\triangle - \frac{2}{3}\mu\bar{q}\triangle\right]d\tau.$$

Now the rate of increase of Kinetic Energy is

$$\frac{d}{dt}\int\left[\tfrac{1}{2}\varrho\bar{q}^2\right]d\tau = \int\left(\varrho q\frac{d\bar{q}}{dt}\right)d\tau$$

$$= \int\left[\bar{q}\bar{F}\varrho - \bar{q}\nabla p + \mu\bar{q}\nabla^2\bar{q} + 2\bar{q}(\mu\nabla)\bar{q} + \bar{q}(\mu_\wedge\xi) + \frac{1}{3}\mu\bar{q}\nabla\triangle - \frac{2}{3}\mu\bar{q}\triangle\right]d\tau.$$

Now

$$2\bar{q}(\bar{\mu}\nabla)\bar{q} = (\bar{\mu}\nabla)\bar{q}^2 \text{ and } \bar{q}\nabla^2\bar{q} = \bar{q}\nabla\triangle - \bar{q}(\nabla_\wedge\bar{\xi}),$$

so we get

$$\frac{dT}{dt} = \int\left[\bar{q}\bar{F}\varrho - \bar{q}\nabla p + (\mu\nabla)\bar{q}^2\mu q(\nabla_\wedge\bar{\xi}) + \mu(\bar{\xi}_\wedge\bar{q}) + \frac{4}{3}\mu\bar{q}\nabla\triangle - \frac{2}{3}\mu\bar{q}\triangle\right]d\tau.$$

Subtracting this from W_ε, the internal work (W_i) done on the element is

$$\int\left[-p\triangle + \mu\nabla^2\bar{q}^2 + \mu\nabla(\bar{\zeta}_\wedge\bar{q}) + \mu\bar{q}(\nabla_\wedge\bar{\zeta}) - 2\mu\bar{q}\,\nabla\triangle - \frac{2\mu}{3}\triangle^2\right]d\tau.$$

The first term is the work done in compressing the fluid, the remainder is the dissipation due to viscosity. Now

$$\bar{q}(\nabla_\wedge\bar{\zeta}) = \bar{\zeta}^2 - \nabla(\bar{q}_\wedge\bar{\zeta})$$

so we get

$$\int\Phi d\tau = \int\mu\left[\nabla^2\bar{q}^2 - 2\nabla(\bar{q}_\wedge\bar{\zeta}) + \bar{\zeta}^2 - 2\bar{q}\nabla\triangle - \frac{2}{3}\triangle^2\right]d\tau$$

Again

$$2(\bar{q}\nabla)\,\bar{q} = \nabla\bar{q}^2 - 2(\bar{q}_\wedge\bar{\zeta})$$ so that we get finally

$$\Phi = \left|\begin{array}{l}\mu\left[2\nabla\{(\bar{q}\nabla)\bar{q}\} + \bar{\zeta}^2 - 2\bar{q}\nabla\triangle - \frac{2}{3}\triangle^2\right]\\ \mu\left[\nabla^2\bar{q}^2 - \bar{\zeta}^2 - 2\bar{q}\,\nabla^2\bar{q} - \frac{2}{3}\triangle^2\right]\end{array}\right.$$

The first form is vectorially more satisfying as it puts the effect of compressibility in evidence; the second is probably better for expressing results in terms of co-ordinate systems. Millikan[5] gives the second form, without the $\triangle^2$ term, claiming that compressibility has been taken into account, the error probably arising from neglecting the $\triangle$ term in the stress equations. Notice that the μ terms have disappeared, thereby proving that the form of the function is not affected by variation of viscosity: that this would be so can be inferred from a careful inspection of the method of obtaining Φ in Cartesians. The Cartesian form is given in F.D. p. 603, being there derived for constant viscosity.

As a digression it may be noticed that one method of deriving the "Strain-Energy Function" of Elasticity is formally precisely the same. It follows that this function, also is not affected if the "elastic constants" are functions of position.

The next, and last, step in the general analysis is to derive the Energy equation. We define the total energy (ε) per unit volume as $\frac{1}{2}\bar{q}^2 + C_V T$ and assume that the perfect gas law $p = (C_p - C_v)\varrho T$ is obeyed.

We have, as in F.D. pp. 604–607,

Time rate of increase of total energy + Rate of working against external forces
= Rate of working on element by surface stresses + Rate of heat conduction through boundaries + Rate of convection of total energy through boundaries. In symbols

$$\frac{\partial t}{\partial t}\int\varrho\varepsilon d\tau - \int\varrho\,\bar{n}\,\bar{q}\,\bar{F}\,dS = \left[-\int\bar{n}\,\bar{q}\,\bar{F}\,dS - \int p\triangle d\tau + \int\tfrac{1}{2}\varrho\frac{d\bar{q}^2}{dt} + \int\Phi\,d\tau\right]$$

$$+\int k(\bar{n}\nabla)T dS - \int\varrho\varepsilon\,\bar{n}\,\bar{q}\,dS.$$

$$\therefore \frac{\partial \varrho \varepsilon}{\partial t} = - p\triangle + \tfrac{1}{2}\varrho \frac{d\bar{q}^2}{dt} + \Phi + \nabla\{(k\nabla)\mathrm{T}\} - \nabla(\varrho\varepsilon\bar{q}).$$

Now $\dfrac{d}{dt} = \dfrac{\partial}{\partial t} + (\bar{q}\nabla)$ and $\dfrac{d\varrho}{dt} = - \varrho\triangle$ so that

$$\frac{d\varrho\varepsilon}{dt} = - p\triangle + \tfrac{1}{2}\varrho\frac{d\bar{q}^2}{dt} + \varepsilon\frac{d\varrho}{dt} + \Phi + \nabla\{(k\nabla)\mathrm{T}\}$$

or since $\varepsilon \equiv \tfrac{1}{2}\bar{q}^2 + \mathrm{C_v T}$

$$\varrho\frac{d(\mathrm{C_v T})}{dt} + p\triangle = \Phi + \nabla\{(k\nabla)\,\mathrm{T}\}.$$

An alternative form can be obtained by substituting $\left(\mathrm{C_p T} - \dfrac{p}{\varrho}\right)$ for $\mathrm{C_v T}$ yielding

$$\varrho\frac{d(\mathrm{C_p T})}{dt} - \frac{dp}{dt} = \Phi + \nabla\{(k\nabla)\mathrm{T}\}.$$

If we assume steady motion, allow for compressibility but otherwise neglect variation of the physical properties of the fluid (or substitute mean values) we get writing out Φ in full

$$\varrho(\bar{q}\nabla)\,(\mathrm{C_p T}) - (\bar{q}\nabla)p = \frac{k}{\mathrm{C_p}}\nabla^2\,(\mathrm{C_p T}) + 2\mu\nabla\{(\bar{q}\nabla)\bar{q}\} - 2\mu\bar{q}\nabla\triangle + \mu\bar{\zeta}^2 - \frac{2\mu}{3}\triangle^2.$$

On the above assumptions the equation of motion becomes

$$\varrho(\bar{q}\nabla)\bar{q} = - \nabla p + \frac{4\mu}{3}\nabla\triangle - \mu(\nabla\,\bar{\zeta}).$$

$$\therefore\ \varrho\bar{q}(\bar{q}\nabla)\bar{q} + \bar{q}\nabla p = \frac{4\mu}{3}\bar{q}\,\nabla\triangle - \bar{q}\mu(\nabla_{\wedge}\bar{\zeta}).$$

$$\therefore\ \tfrac{1}{2}\varrho\,(\bar{q}\nabla)\bar{q}^2 + \bar{q}\nabla p = \frac{4\mu}{3}\bar{q}\nabla\triangle + \mu\nabla(\bar{q}_{\wedge}\bar{\zeta}) - \mu\bar{\zeta}^2.$$

Eliminating $\bar{q}\nabla p$ by adding this to the energy equation we get

$$p\,(\bar{q}\nabla)\left[\mathrm{C_p T} + \tfrac{1}{2}\bar{q}^2\right] = \frac{k}{\mathrm{C_p}}\nabla^2\,[\mathrm{C_p T}] + 2\mu\nabla[(\bar{q}\nabla)\bar{q}] + \mu\nabla(\bar{q}_{\wedge}\bar{\zeta}) - \frac{2}{3}\mu(\bar{q}\nabla\triangle + \triangle^2).$$

$$\therefore\ (\bar{q}\nabla)\left[\mathrm{C_p T} + \tfrac{1}{2}\bar{q}^2\right] = \frac{k}{\mathrm{C_p}\varrho}\nabla^2\left[\mathrm{C_p T} + \tfrac{1}{2}\sigma\bar{q}^2\right] + \nu\left[\nabla(\bar{q}\nabla)\bar{q} - \frac{2}{3}\nabla(\bar{q}\triangle)\right].$$

So that if $\sigma = 1$ and the circumstances of the motion are such that $\nu\left[\nabla\left\{(\bar{q}\nabla)\bar{q} - \dfrac{2}{3}\bar{q}(\nabla\bar{q})\right\}\right]$ can be neglected $\mathrm{C_p T} + \tfrac{1}{2}\bar{q}^2 = \text{Constant}$ is an integral of the energy equation. The practical value of this will of course be decided by the particular application under discussion at the time. The first condition restricts it to gases.

The derivation of the energy equation completes the analysis in its general form and it seems likely that short of the introduction of a " second coefficient of viscosity ", justified by arguments from the Kinetic Theory of Gases, it is *the* most general form possible. On the practical side it should be adequate for a long time to come.

In the course of the preceding discussion some of the cases in which the variation of the physical properties of the fluid is of importance have been noticed in passing. The compressible flow of a gas at high speed [6, 7, 8, & 9,] is another important example. It is not proposed in this report to go into details of these or any other possible applications. For convenience of reference, however, the curvilinear orthogonal equivalents of many of the expressions which will be required in any such application are collated below.

$$\triangle = \nabla\bar{q} = \frac{1}{h_1h_2h_3}\left\{\Sigma \frac{\partial}{\partial\alpha}(h_2h_3u)\right\}$$

$$\bar{\zeta} \equiv \nabla_\wedge\bar{q} = \frac{1}{h_1h_2h_3}\begin{vmatrix} h_1\bar{i} & h_2\bar{j} & h_3\bar{k} \\ \frac{\partial}{\partial\alpha} & \frac{\partial}{\partial\beta} & \frac{\partial}{\partial\gamma} \\ h_1u & h_2v & h_3w \end{vmatrix}$$

$$\nabla\varphi = \Sigma \frac{\bar{i}}{h_1}\frac{\partial\varphi}{\partial\alpha},$$

$$\nabla\bar{i} = \frac{1}{h_1h_2h_3}\frac{\partial}{\partial\alpha}(h_2h_3)$$

$$\nabla_\wedge\bar{i} = \frac{1}{h_1}\left\{\frac{\bar{j}}{h_3}\frac{\partial h_1}{\partial\gamma} - \frac{\bar{k}}{h_2}\frac{\partial h_1}{\partial\beta}\right\}$$

With similar expressions for $\nabla\bar{j}$, $\nabla\bar{k}$, $\nabla_\wedge\bar{j}$, $\nabla_\wedge\bar{k}$.

$$\nabla^2\varphi = \frac{1}{h_1h_2h_3}\left[\Sigma\frac{\partial}{\partial\alpha}\frac{h_2h_3}{h_1}\frac{\partial\varphi}{\partial\alpha}\right]$$

$$\nabla^2\bar{q} = \Sigma\bar{i}\left[\frac{1}{h_1}\frac{\partial\triangle}{\partial\alpha} + \frac{1}{h_2h_3}\left\{\frac{\partial}{\partial\gamma}(h_2\bar{\zeta}_2) - \frac{\partial}{\partial\beta}(h_3\bar{\zeta}_3)\right\}\right]$$

$$(\bar{q}\nabla)\varphi = \Sigma\frac{u}{h_1}\frac{\partial\varphi}{\partial\alpha}$$

$$(\bar{q}\nabla)\bar{q} = \Sigma\left[\bar{i}\left\{\frac{1}{2h_1}\frac{\partial\bar{q}^2}{\partial\alpha} - (v\bar{\zeta}_3 - w\bar{\zeta}_2)\right\}\right].$$

BIBLIOGRAPHY

1. Cartesian Tensors.—H. Jeffreys.
2. Theoretical Hydrodynamics.—L. M. Milne-Thompson.
3. Modern Developments in Fluid Dynamics.—Edited by S. Goldstein. Especially Chapters III, IV, Vol. 1 and XIV, Vol. II.
4. A New Mathematical Method for the Solution of Film Lubrication Problems.—D. G. Christopherson. Proceedings I.Mech.E. 146 (1941).
5. On the Steady Motion of Viscous, Incompressible Fluids.—C. B. Millikan. Phil. Mag., Vol. 7, 1929.
6. Motion of a Compressible Fluid in the Laminar Boundary Layer of a Flat Plate.—A. Busemann. Z.A.M.M., Vol. 15, 1935.
7. Boundary Layer in Compressible Fluids.—Th. von Kármán and H. S. Tsien. Journal of the Aeronautical Sciences, Vol. 5, 1937–38.
8. Effect of Variable Viscosity on Boundary Layers, with a Discussion on Drag Measurements. J. G. Brainerd and H. W. Emmons. A.S.M.E. Preprint 1941.
9. Temperature Effects in a Laminar Compressible-Fluid Boundary Layer along a Flat Plate.—J. G. Brainerd and H. W. Emmons. Journal of Applied Mechanics. Vol. 8, 1941.

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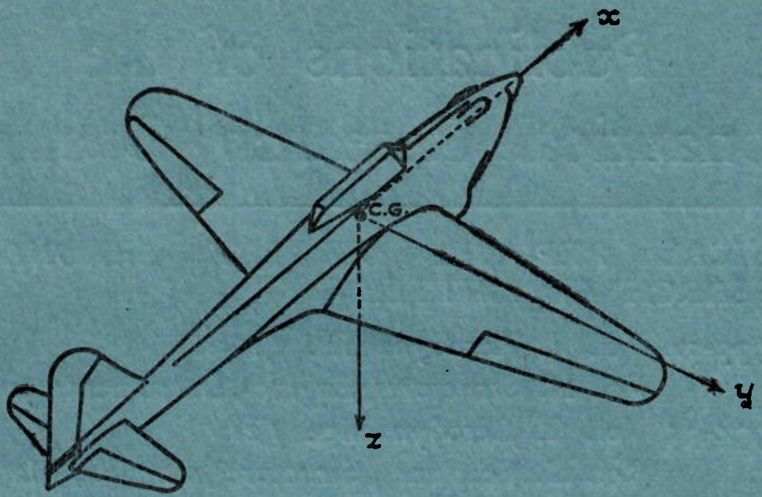
SYSTEM OF AXES



Axes	Symbol Designation Positive direction	x longitudinal forward	y lateral starboard	z normal downward
Force	Symbol	X	Y	Z
Moment	Symbol Designation	L rolling	M pitching	N yawing
Angle of Rotation	Symbol	ϕ	θ	ψ
Velocity	Linear Angular	u p	v q	w r
Moment of Inertia		A	B	C

Components of linear velocity and force are positive in the positive direction of the corresponding axis.

Components of angular velocity and moment are positive in the cyclic order y to z about the axis of x, z to x about the axis of y, and x to y about the axis of z.

The angular movement of a control surface (elevator or rudder) is governed by the same convention, the elevator angle being positive downwards and the rudder angle positive to port. The aileron angle is positive when the starboard aileron is down and the port aileron is up. A positive control angle normally gives rise to a negative moment about the corresponding axis.

The symbols for the control angles are :—

ξ aileron angle
η elevator angle
η_T tail setting angle
ζ rudder angle

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