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The Design of Wind Tunnel Fans

By

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AERODYNAMIC SYMBOLS

1. GENERAL

m	Mass
t	Time
V	Resultant linear velocity
Ω	Resultant angular velocity
ρ	Density, σ relative density
ν	Kinematic coefficient of viscosity
R	Reynolds number, $R = lV/\nu$ (where l is a suitable linear dimension)

Normal temperature and pressure for aeronautical work are 15° C and 760 mm.

For air under these conditions $\begin{cases} \rho = 0\cdot002378 \text{ slug/cu. ft.} \\ \nu = 1\cdot59 \times 10^{-4} \text{ sq. ft./sec.} \end{cases}$

The slug is taken to be 32·2 lb.-mass.

α	Angle of incidence
ϵ	Angle of downwash
S	Area
b	Span
c	Chord
A	Aspect ratio, $A = b^2/S$
L	Lift, with coefficient $C_L = L/\frac{1}{2}\rho V^2 S$
D	Drag, with coefficient $C_D = D/\frac{1}{2}\rho V^2 S$
γ	Gliding angle, $\tan \gamma = D/L$
L	Rolling moment, with coefficient $C_l = L/\frac{1}{2}\rho V^2 bS$
M	Pitching moment, with coefficient $C_m = M/\frac{1}{2}\rho V^2 cS$
N	Yawing moment, with coefficient $C_n = N/\frac{1}{2}\rho V^2 bS$

2. AIRSCREWS

n	Revolutions per second
D	Diameter
J	V/nD
P	Power
T	Thrust, with coefficient $k_T = T/\rho n^2 D^4$
Q	Torque, with coefficient $k_Q = Q/\rho n^2 D^5$
η	Efficiency, $\eta = TV/P = Jk_T/2\pi k_Q$

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The Design of Wind Tunnel Fans

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Summary.—The present report is one of a series dealing with N.P.L. methods for the design of return flow wind tunnels. Reports already issued deal with the design of the Compressed Air Tunnel[1] and Open Jet Tunnels,[2] with the design of corner cascades[3] and fan straighteners,[4] and with the improvement of velocity distribution by means of windmills[5] and gauzes.[6]

An explanation is now given of the method usually adopted in the design of fans for return flow tunnels; as an example of the method, the design of the fan for the N.P.L. non-turbulent tunnel is considered in detail. It is shown that for a wind tunnel a fan with blades of constant chord is advisable; and a formula is given by means of which a rapid estimate can be made of the variation in rotational speed of the fan corresponding to a variation in power factor at constant power.

1. *Power Factor.*—The power factor λ of a wind tunnel is defined by the equation

$$\tfrac{1}{2}\rho A V^3 \lambda = P, \qquad (1)$$

where P is the power input (ft.lb./sec.), ρ the air density (slug/ft.^{3}), A the area of the working section (ft.^{2}) and V the wind speed in the working section (ft./sec.). The power factor is sometimes defined as the reciprocal of the quantity given by (1); but the form adopted here is in the nature of a drag coefficient, so that a low power factor corresponds to a low tunnel drag and, therefore, implies an efficient tunnel. It is, in fact, possible to obtain an estimate of the power factor and of the variation of power factor with wind speed by direct addition of the drags of the various internal surfaces of a tunnel at different Reynolds numbers; this has been done, for instance, by Wattendorf.[7,8]

For a new wind tunnel of conventional type, the power factor can usually be estimated with sufficient accuracy by comparison of the design with those of other similar tunnels for which the power factors are known. A return flow tunnel with a contraction ratio of about 4 and a closed working section for which A and V are respectively of the order of 100 ft.^{2} and 200 ft./sec. will normally have a power factor in the neighbourhood of 0·25; power factors as low as 0·2 have been achieved in some tunnels which were designed to have the minimum possible internal drag. The provision of such features as a honeycomb and a settling length may raise the power factor to 0·3. If the tunnel has an open working section its power factor will usually lie between 0·35 and 0·45.

When the power factor of a new design has been assessed, equation (1) is used to obtain the wind speed resulting from a given power input, or the power input required to produce a given wind speed. It will be assumed in what follows that the power input P is fixed.

If the area of the fan disc is A_0 and the mean axial speed through the fan is u, then by continuity

$$A_0 u = AV, \qquad (2)$$

whence u may be found. It should be remarked that in the evaluation of A_0, account should be taken of the area of the fan boss; for reasons given in § 4, the boss diameters of the fans of N.P.L. tunnels are usually 0·3 to 0·4 of the fan diameters.

Since uniformity of velocity in the working section is an essential in a wind tunnel, it is obviously advantageous to keep the axial speed across all sections as uniform as possible. In practice, the axial speed through the fan usually varies from point to point; but the differences are not large, and in the absence of information on the distribution (as obtained for instance from experiments on a model of the tunnel) it is assumed to be constant in the design of the fan.

2. *Strip Theory Formulae*.—Corresponding values of u and P for the tunnel are found as in § 1. To proceed to the fan design, the appropriate strip theory formulae are required. These are given in R. & M. 1293,[9] and may be written in the form

$$\frac{dT}{dr} = \frac{1}{2} \rho N c u^2 G \cot \phi, \quad \ldots \quad (3)$$

$$\frac{dQ}{dr} = \frac{1}{2} \rho N c u^2 H r \cot \phi, \quad \ldots \quad (4)$$

$$G = \frac{C_L}{\sin \phi} - \frac{C_D}{\cos \phi}, \quad \ldots \quad (5)$$

$$H = \frac{C_D}{\sin \phi} + \frac{C_L}{\cos \phi}, \quad \ldots \quad (6)$$

$$\tan \phi = \frac{u}{r \Omega (1 - a_2)}, \quad \ldots \quad (7)$$

$$\frac{1}{1 - a_2} = 1 + \frac{Nc}{8 \pi r} H, \quad \ldots \quad (8)$$

$$\phi = \theta - \alpha, \quad \ldots \quad (9)$$

where T = thrust,
Q = torque,
r = local radius,
N = number of blades,
c = local chord,
C_L = local lift coefficient,
C_D = local drag coefficient,
Ω = angular velocity of fan,
a_2 = rotational inflow factor,
θ = local blade angle,
α = local incidence.

The exact application of these formulae will be explained later. To make a preliminary fan design, the equations are simplified by omission of the drag coefficient (which is usually quite small) and of the rotational inflow factor. Moreover, since wind tunnel fans are seldom of high pitch, u is small in comparison with $r\Omega$, and it is, therefore, possible to write

$$\tan \phi = \sin \phi = \phi = \frac{u}{r\Omega}, \quad \ldots \quad (10)$$

with $\cos \phi = 1$. Accordingly (3) and (4) reduce to

$$\frac{dT}{dr} = \frac{1}{2} \rho N c r^2 \Omega^2 C_L, \quad \ldots \quad (11)$$

$$\frac{dQ}{dr} = \frac{1}{2} \rho N c r^2 u \Omega C_L. \quad \ldots \quad (12)$$

It will be seen that, to the degree of approximation adopted, the torque power input ΩQ and the thrust power output uT are equal. The formula (11) can evidently be written down at sight if it is assumed that the velocity of the blade element relative to the wind is the rotational velocity $r \Omega$. Actually, the rotational inflow factor decreases the velocity below $r \Omega$, while the axial velocity component increases it; the corrections are usually of the same order of magnitude, so that (11) gives a sufficiently accurate representation of the thrust grading for preliminary design purposes.

3. *Preliminary Design Considerations.*—In order to maintain across the sections of the tunnel duct the constancy of total head which corresponds to uniformity of velocity in rectilinear flow, it is desirable that the pressure increment through the fan disc shall be constant, i.e., independent of radius. In practice it is sometimes found advantageous to increase the pressure slightly towards the outer radii, more particularly for open jet tunnels, since a considerable part of the energy loss in the tunnel takes place at the boundaries of the stream ; however, the departure from constancy is not great. If the pressure increment is p, then by (11)

$$p = \frac{1}{2\pi r}\frac{dT}{dr} = \frac{1}{4\pi}\ \rho N c r\, \Omega^2 C_L,$$

or

$$\rho\, \Omega^2 N c r C_L = 4\pi p. \qquad (13)$$

If p is to be independent of radius, it follows from (13) that crC_L must be constant along the blades : a condition well-known in tunnel fan design. Also, to the present degree of approximation,

$$P = p A_0 u, \qquad (14)$$

so that (13) becomes

$$\rho A_0 u\, \Omega^2 N c r C_L = 4\pi P. \qquad (15)$$

It remains to choose Ω and the local values of Nc and C_L to satisfy (15), in which the other quantities are known.

Now, for several reasons, it is usual to make c constant along the blades. Thus, if the root chord is much smaller than the tip the stresses in the blade root may be too high ; moreover, a small root chord implies a lift coefficient which may be excessively high. On the other hand, if the root chord is large, blade root interference may appear. There is, however, a more important reason. Suppose the power factor achieved in a wind tunnel differs from the figure used in the fan design (the power factor will in any case vary slightly with wind speed) ; or alternatively, suppose that, by the addition of screens or a honeycomb, the power factor is materially altered from that applicable to the empty tunnel. This will be the case, for instance, in the non-turbulent tunnel at the N.P.L., in which the introduction of certain artificial turbulence screens may increase the power factor from 0·3 to 1·0 or even higher. Then the fan may, and in some cases certainly will, have to work under conditions different from those assumed in the design ; and the local lift coefficients will be different. If, for two conditions of working, the lift coefficients at radius r are C_L and $C_L + \delta C_L$, and if in both cases the pressure increment through the fan is constant, it follows from (13) that both crC_L and $cr\delta C_L$ must be constant along the blades. Now for the linear portion of the lift curve it is possible to write

$$C_L = \frac{\partial C_L}{\partial \alpha}(\mu + \alpha), \qquad (16)$$

where $-\mu$ is the no lift angle, and the slope of the lift curve is for all practical purposes the same at all radii. In view of (9) and (10), (16) becomes

$$C_L = \frac{\partial C_L}{\partial \alpha}\left(\mu + \theta - \frac{u}{r\Omega}\right), \qquad (17)$$

and hence, for the second working condition,

$$\delta C_L = -\ \frac{1}{r}\frac{\partial C_L}{\partial \alpha}\,\delta\left(\frac{u}{\Omega}\right),$$

or

$$cr\delta C_L = -\ c\frac{\partial C_L}{\partial \alpha}\,\delta\left(\frac{u}{\Omega}\right) \ldots \qquad (18)$$

Since $cr\delta C_L$ is to be independent of radius, it follows from (18) that c must be constant along the blades.

Equation (15) may now be used to fix the values of Ω and Nc. In the first place it is usual to choose $C_L = 0 \cdot 6$ at the tip radius $r = r_1$. A lower value of C_L would lead to a fan of less efficiency; and it is an empirical fact (noted for instance in R. & M.1569[2]) that a value in excess of $0 \cdot 7$ is not usually attainable at the tip. In any case, the value chosen leads to sufficiently high values of C_L at the root. Accordingly (13) gives

$$\Omega^2 Nc = \frac{4\pi P}{0 \cdot 6\, r_1 \rho A_0 u}. \qquad (19)$$

Either Ω or Nc may now be arbitrarily fixed, subject to the following limitations. If Ω is chosen too low, Nc will become large and blade root interference may result; moreover, the pitch of the fan will become high, while the motor design may present difficulties. On the other hand, if Ω is chosen to be large, the fan blades will be narrow even when $N = 2$, and the blade strength may be insufficient; in addition, the tip speed may be so high that noise and compressibility effects become serious. On the last account, it is usual to limit the tip speed to be not greater than half the speed of sound; and as a result of the other considerations, the permissible range of Ω is usually not great. When Ω has been fixed, Nc is given by (19). The number of blades N is to some extent arbitrary; it should not be too large, or the blades become unduly weak, while construction of the fan is complicated. For N.P.L. tunnels, N is usually chosen to be 2 or 4.

Since c and crC_L' are constant along the blades, rC_L is constant and equal to the value at the tip. Again, since rC_L is constant, it follows from (17) that $r(\mu + \theta)$ is also constant along the blades.

4. *The Preliminary Design.*—So far, it has been shown how to determine preliminary values of Ω, N, and c for the fan. It has also been shown that rC_L is constant along the blades; hence, unless the boss diameter is fairly large, the blade roots will stall. For N.P.L. tunnels the fan boss diameter is usually chosen to lie between $0 \cdot 3$ and $0 \cdot 4$ of the fan diameter; efficient working of the root sections is thus obtained. Moreover, a boss of this size is generally large enough to allow the fan motor to be placed inside the boss fairing; the power unit is then very compact. The support of the boss fairing and motor on the fan straighteners is described in R. & M.1885.[4]

The blade sections used for N.P.L. fans are usually sections C, D, E and F of the family of airscrews, details of which are given in R. & M.829.[10] To suit the constant chord required for the present purpose, sections C, D and E have been slightly altered, while section F has relatively a much thinner trailing edge. The sections have flat undersurfaces, and are thus easy to construct. Details of the sections are given in Table 6 and Fig. 1. The lift and drag coefficients for infinite aspect ratio are given in Fig. 2 over a sufficiently wide range of incidence.

To complete the preliminary design, therefore, it is usual to take four equally spaced radii, the innermost just outside the boss and the outermost just inside the tip. The sections C, D, E, and F are assigned to these radii in the order of increasing radius. From the local value of C_L, the incidence α of the section is found from Fig. 2; and the blade angle is then given by (9) and (10) as

$$\theta = \alpha + \frac{u}{r\Omega}. \qquad (20)$$

5. *Strip Theory Calculations.*—A more accurate estimate of the performance of the preliminary design of fan found by the methods of § 4 is now obtained by the use of equations (3) to (9). To apply these equations a successive approximation process is used, which in practice converges very rapidly.

The values of u, Ω, N, c and θ obtained in the preliminary design are treated as fixed. The local Reynolds number R at which the blade sections operate is given approximately by $r\Omega c/\nu$, where ν is the kinematic viscosity; it is usually sufficiently accurate to take a mean value of R for the whole blade.

As a first approximation, put $C_D = 0$ and $\cos\phi = 1$ in equation (6), and use the local value of C_L found in the preliminary design. The resulting first approximation $H = C_L$ is then used in (8) to find at each of the reference radii a first approximation to $1/(1 - a_2)$, which in turn is used in (7) to give a first approximation to ϕ. Equation (9) is next used to find α, and thence, from the curves of Fig. 2, values of C_L and C_D at the appropriate Reynolds number are obtained. The values of ϕ, C_L and C_D are then used in (6) to obtain a second approximation to H, which is inserted in (8) to obtain a second approximation to $1/(1 - a_2)$; and thence from (7) a second approximation to ϕ is found. The process of approximation is continued until ϕ repeats. In practice ϕ is computed to the nearest tenth of a degree, and usually it repeats to this order of accuracy after only two steps in the process of approximation.

When final values of ϕ, C_L and C_D have been found, equations (5) and (6) are used to obtain values of H and G, which are inserted in equations (3) and (4) to give curves of thrust and torque grading. These curves are integrated graphically or by Simpson's rule to obtain the thrust and torque* ; and thence the torque power input, thrust power output, and fan efficiency are obtained.

There is generally a difference of anything up to about 6 per cent. between the power input P_1 obtained in this manner and the input P assumed in making the preliminary design ; there is therefore (since the axial speed is common to both methods) the same difference in power factor ; an error of this magnitude is usually within the limits of possible error in the estimation of the power factor. To reduce P_1 to P, both u and Ω must be multiplied by $(P/P_1)^{1/3}$, which usually represents a change of only about 2 per cent. and is thus not serious. If, however, the error in the power factor is considered to be serious, the blade angles or chord of the fan may be altered (the approximate extent required is obvious from the calculations) and the strip theory calculations repeated.

Again, from the thrust grading, the distribution of the pressure increment p with radius is found. If this is considered to be unsatisfactory, local alterations in blade angle can be made to improve it. The fan design is thus completed.

As a numerical example, the design of the fan for the N.P.L. non-turbulent tunnel is discussed in detail in § 8.

6. *General Remarks.*—In equation (20), two approximations are involved : the omission of the rotational inflow factor and the approximation $\tan \phi = \phi$. It so happens that for a blade of normal design the errors thus introduced, which are always of opposite sign, are of nearly equal magnitude, so that the value of ϕ given by $u/r\,\Omega$ is very nearly the same, for the whole length of the blade, as that obtained in the strip theory calculations.

It may be argued that the sections of the family of airscrews are not so efficient as other sections developed in recent years. However, the sections are quite good for the ranges of C_L employed, and give tunnel fan efficiencies of the order of 90 per cent. ; since, in addition, the sections have such simple shapes, refinement seems hardly worth while.

A short explanation of the derivation of the curves of Fig. 2 may be given here. The lift and drag coefficients of the sections of the family of airscrews, corresponding to infinite aspect ratio, are tabulated in R. & M. 892[11] and R. & M. 1771[12] ; the first report relates to tests in an atmospheric tunnel, and the second to C.A.T. tests. The range of Reynolds number covered is from about $0{\cdot}1 \times 10^6$ to 6×10^6. The tests show that the scale effect on the lift coefficients in the working range (which for the most part is well below the stall) is not great. Accordingly, for design purposes it is sufficient to use the curves of Fig. 2 for values of R from about $0{\cdot}2 \times 10^6$ to 6×10^6. These curves are mean curves obtained from all the data of the two above reports ; but the incidences for section F have been changed by the addition of a constant factor to give the theoretical no-lift angle for the altered section used in the present report. This no-lift angle was found by means of Glauert's formula,[13] which gives sufficiently good agreement with the experimental no-lift angles of sections C, D and E.

The drag coefficient curves of Fig. 2 are also considered to be of ample accuracy for the purposes of tunnel fan design, though no great absolute accuracy can be attached to them. Examination of the experimental data for sections C, D and E showed that the drag coefficients corresponding to infinite aspect ratio decreased with increasing R in a manner which could be approximately represented by the equation

$$C_D + 0{\cdot}004 \log_{10} R = \text{constant}.$$

* For usual spacings of the sections, the formula given in Appendix II may be used to obtain the integrated values of thrust and torque.

Mean curves of variation of $C_D + 0 \cdot 004 \log_{10} (R \times 10^{-6})$ against α, for the whole range of R covered by the experimental data, were therefore drawn ; these are the curves C, D and E of Fig. 2. The values of minimum drag coefficient thus obtained are only a little higher than those calculated by Squire and Young in R. & M. 1838[14] for modern aerofoil shapes with the transition point at the leading edge ; the rate of variation with $\log_{10}$ R in the neighbourhood of $R = 10^6$ is also in quite good agreement with the results of these writers. The curve shown in Fig. 2 for the modified section F used in the present report was estimated from the curves for the other sections, the difference being based on the results of R. & M. 1838.[14]

The stresses and deflections in the blades due to centrifugal force, thrust, and twisting moment are briefly discussed in Appendix I. A representative blade is examined ; and since the ratio of boss radius to tip radius, and the disposition of the sections along the blades, do not vary much in N.P.L. fans, the results for this representative blade can be used to obtain a rough estimate of the stresses and deflections to be expected in any other blade. If γ is the density of the material of the blade and E is Young's modulus, it is shown that the maximum tensile stress, which occurs near the root, is given approximately by

$$\sigma_c + \sigma_b = 0 \cdot 25\, \gamma r_1^2 \Omega^2 + 12 \cdot 5\, \rho r_1^2 \Omega^2 (r_1/c)^2, \qquad (21)$$

and that the extension due to centrifugal force and deflection due to thrust, measured at the tip, are respectively

$$e_c = 0 \cdot 11\, \gamma \Omega^2 r_1^3/E, \qquad (22)$$

$$e_b = 26\, \rho \Omega^2 r_1^3 (r_1/c)^3/E. \qquad (23)$$

A rough estimate of the critical flutter speed of the blades of a new fan can be made on the basis of formulae given in R. & M. 1518,[15] which discusses the flutter of solid cylindrical blades in a uniform airstream.

7. *Variation of Rotational Speed with Power Factor.*—If the power input P is fixed, equations (1) and (2) show that λV^3 and λu^3 are constant. The variations in axial speed corresponding to small variations in λ are thus not great. The speed obtained for a given power in a new wind tunnel is for this reason usually quite close to the anticipated figure. The rotational speed of the fan is usually also very close to the design figure, though the reason for this is not so immediately obvious.

The calculation of the variation of Ω with λ by the exact strip theory equations is best effected indirectly. Any reasonable relative values of u and Ω are assumed, and by the methods of § 5, the corresponding power input P_1 is calculated. The values of u and Ω are then multiplied by $(P/P_1)^{1/3}$ to bring the power input to the fixed value P ; corresponding values of Ω and λ are thus obtained. The calculations are repeated for other values of u and Ω, and a curve of variation of Ω with λ can then be drawn.

For most practical purposes, however, it is sufficiently accurate, besides being simpler and more instructive, to use a formula derived from the approximate equations of § 3. Substitution from (17) in (15) for C_L gives

$$\rho A_0 u \Omega^2 N c \frac{\partial C_L}{\partial \alpha} \left\{ r(\mu + \theta) - \frac{u}{\Omega} \right\} = 4\pi P,$$

and substitution from (1) and (2) for u in terms of P then yields a quadratic for Ω of which the solution is

$$\Omega = K_1 \lambda^{-1/3} \left\{ 1 + \sqrt{1 + K_2 \lambda} \right\}, \qquad (24)$$

where

$$K_1 = \frac{1}{2} \left(\frac{2P}{\rho A} \right)^{1/3} \frac{A}{A_0 r(\mu + \theta)},$$

$$K_2 = \frac{8\pi}{Nc} \frac{\partial \alpha}{\partial C_L} \left(\frac{A_0}{A} \right)^2 r(\mu + \theta).$$

The quantities K_1 and K_2 are constants since, as shown above, c and $r(\mu + \theta)$ are constant along the blades. As λ increases, the first factor in the expression (24) for Ω decreases while the second increases ; for the practical range of λ the resulting variation in Ω is usually quite small.

By differentiation of (24), a very simple expression results which can be used directly to obtain the small variation $\delta\Omega$ corresponding to a small variation $\delta\lambda$. The equation obtained, after a little reduction, is

$$\frac{d\Omega}{d\lambda} = \frac{\Omega}{3\lambda}\left[\frac{\Omega r(\mu + \theta) - 2u}{2\Omega r(\mu + \theta) - u}\right]. \qquad (25)$$

Numerical examples of equations (24) and (25) are given in § 8.

8. *The Design of the Fan for the N.P.L. Non-Turbulent Tunnel.*—As an illustration of the methods described above, details will now be given of the fan design for the N.P.L. non-turbulent tunnel. The tunnel has a working section in the shape of a regular 16-sided polygon, the diameter of the inscribed circle being 7 ft. Accordingly,

$$A = 39{\cdot}0 \text{ ft.}^2 \qquad (26)$$

The tunnel is unusually long ; the working section has a length of 45 ft. There follows a short expansion leading to the fan section ; there is then a long expanding cone, the diameter reached being 20 ft. A fine mesh honeycomb and a settling chamber about 30 ft. long are provided. The contraction ratio is about 8. Since the tunnel has so many unusual features, the estimate of power factor is comparatively rough. For the empty tunnel it may be as low as 0·3. When artificial turbulence screens are introduced, however, it is anticipated that the power factor may rise to 1·0 or even higher ; the fan will therefore have to work under widely varying conditions. It must evidently be designed to be unstalled at the highest power factor, and will then be further from the stall for the less resistant conditions.

The fan diameter is 8·5 ft., and the boss diameter 3·0 ft. ; a motor of 100 H.P. is installed in the boss fairing. Accordingly

$$A_0 = 49{\cdot}65 \text{ ft.}^2 \qquad (27)$$

$$P = 5{\cdot}5 \times 10^4 \text{ ft. lb./sec.} \qquad (28)$$

For a power factor of $\lambda = 1$, therefore, equations (1) and (2) give, with $\rho = 23{\cdot}8 \times 10^{-4}$ slug/ft.^{3},

$$V = 105{\cdot}8 \text{ ft./sec.} \qquad (29)$$

$$u = 83{\cdot}1 \text{ ft./sec.} \qquad (30)$$

Hence from (19), since $r_1 = 4{\cdot}25$ ft.,

$$Nc\Omega^2 = 2{\cdot}76 \times 10^4 \text{ ft./sec.}^2 \qquad (31)$$

It was decided that a rotational speed of about 1,000 r.p.m., giving a value of Ω just in excess of 100 rad./sec. and a fan tip speed of about 450 ft./sec., would be suitable. Hence the following figure was chosen

$$Nc = 2{\cdot}5 \text{ ft.,} \qquad (32)$$

and therefore

$$\Omega = 105{\cdot}1 \text{ rad./sec.} \qquad (33)$$

The reference radii were chosen to be 1·75, 2·5, 3·25, and 4·0 ft., and the sections C, D, E and F were assigned to these radii, respectively. The lift coefficient at radius r is given by

$$rC_L = 2{\cdot}55. \qquad (34)$$

From the lift coefficient the local incidence α is found from the curves of Fig. 2 ; the blade angle is then given by (20), in which u and Ω have the values given by (30) and (33). The results are summarised in Table 1.

TABLE 1

r (ft.)	Section	C_L	α (deg.)	$\frac{u}{r\Omega}$ (rad.)	$\frac{u}{r\Omega}$ (deg.)	θ (deg.)
1·75	C	1·458	10·0	0·452	25·9	35·9
2·5	D	1·020	4·8	0·316	18·1	22·9
3·25	E	0·785	3·2	0·243	13·9	17·1
4·0	F	0·638	2·6	0·198	11·3	13·9

The above is an illustration of the usual method for the preliminary design of a fan. In the case of the non-turbulent tunnel, however, the blade angles given by Table 1 were not finally adopted, since in view of the very long and narrow working section it was thought that the total head losses near the walls would be considerable. Accordingly, instead of keeping the pressure increment p constant, it was decided to increase it linearly with radius. For constant chord, (13) shows that rC_L must then increase linearly with radius also. The relation adopted was

$$rC_L = 1\cdot64 + 0\cdot28\,r, \qquad (35)$$

so that at section F, p is roughly 30 per cent. greater than at section C. With the expression (35) for rC_L and the values of u, Ω, and Nc given by (30), (33), and (32), the integral of the thrust power output found from (11) is 98·6 H.P., which is sufficiently accurate for the preliminary design.

The blade angles were, therefore, determined as in Table 2.

TABLE 2

r (ft.)	Section	C_L	α (deg.)	ϕ (deg.)	θ (deg.)
1·75	C	1·215	6·2	25·9	32·1
2·5	D	0·935	3·9	18·1	22·0
3·25	E	0·785	3·2	13·9	17·1
4·0	F	0·690	3·1	11·3	14·4

The preliminary design was then chosen to be a 2-bladed fan, of constant chord 15 in., and having the sections and blade angles given by Table 2.

A computation of the performance of this fan by the exact strip theory equations will now be carried out exactly as in § 3. A mean Reynolds number for the blades is $R = 3 \times 10^6$ approximately; accordingly the curves of Fig. 2 represent $C_D + 0\cdot0019$. Details of the process of successive approximation to ϕ are given in Table 3. It will be seen that, except for the innermost radius, the second approximation to ϕ repeats the first; and only one step further is required for the radius $r = 1\cdot75$ ft. It will be noticed that the final values of ϕ obtained in Table 3 are practically identical with those given in Table 2, which are found from the approximate equation (10) (see remarks in § 6).

The final values of ϕ, C_L and C_D are used in Table 4 to obtain values of thrust and torque grading from equations (3) to (6). The pressure increment through the fan is

$$p = \frac{1}{4\pi}\,\rho\,Ncu^2\left(\frac{G\cot\phi}{r}\right), \qquad (36)$$

and from the last column of Table 4 it will be seen that p increases steadily towards the outer radii. Integration of (3) and (4) using the values of $Hr\cot\phi$ and $G\cot\phi$ contained in Table 4 gives the torque power input and thrust power output respectively as

$$\Omega Q = 56{,}600 \text{ ft. lb./sec.} \qquad (37)$$

$$= 102\cdot9 \text{ H.P.}$$

$$uT = 51{,}700 \text{ ft. lb./sec.} \qquad (38)$$

The ratio of (38) to (37) gives the fan efficiency as

$$\eta = 0{\cdot}914. \quad \ldots \quad (39)$$

TABLE 3

r	Section	θ	$\frac{Nc}{8\pi r}$	$\frac{u}{r\Omega}$	C_L	$\frac{1}{1-a_2}$	tan ϕ	ϕ
1·75	C	32·1	0·0569	0·452	1·215	1·069	0·483	25·8
2·5	D	22·0	0·0398	0·316	0·935	1·037	0·328	18·2
3·25	E	17·1	0·0306	0·243	0·785	1·024	0·249	14·0
4·0	F	14·4	0·0249	0·198	0·690	1·017	0·201	11·4

sin ϕ	cos ϕ	α	C_L	C_D	H	$\frac{1}{1-a_2}$	tan ϕ	ϕ
0·435	0·900	6·3	1·218	0·0196	1·398	1·079	0·487	26·0
0·312	0·950	3·8	0·925	0·0130	1·015	1·040	0·328	18·2
0·242	0·970	3·1	0·771	0·0108	0·840	1·026	0·249	14·0
0·198	0·980	3·0	0·682	0·0098	0·746	1·019	0·202	11·4

sin ϕ	cos ϕ	α	C_L	C_D	H	$\frac{1}{1-a_2}$	tan ϕ	ϕ
0·438	0·899	6·1	1·202	0·0192	1·381	1·079	0·487	26·0
—	—	—	—	—	—	—	—	—
—	—	—	—	—	—	—	—	—
—	—	—	—	—	—	—	—	—

TABLE 4

r	H	G	Hr cot ϕ	G cot ϕ	$\frac{\text{G cot }\phi}{r}$
1·75	1·381	2·725	4·97	5·59	3·20
2·5	1·015	2·950	7·73	8·99	3·60
3·25	0·840	3·175	10·97	12·78	3·94
4·0	0·746	3·435	14·78	17·02	4·26

The value of V given by (29) was calculated on the assumption that $\lambda = 1$ and P = 100 H.P. Since the power is 102·9 H.P. it follows that $\lambda = 1{\cdot}029$. To reduce the power to 100 H.P., the speeds must all be divided by $\sqrt[3]{1{\cdot}029}$. The resulting value of Ω is

$$\Omega = 104{\cdot}1 \text{ rad./sec.}, \quad \ldots \quad (40)$$

the rotational speed being 995 r.p.m.

Calculations have also been made for this fan for conditions giving power factors of $\lambda = 0{\cdot}70$ and $\lambda = 0{\cdot}33$. The fan was considered to give a satisfactory performance throughout the probable range of operation ; accordingly, no alteration was made in the preliminary design.

The values of Ω corresponding to an input of 100 H.P. at the three power factors 1·03, 0·70 and 0·33 are plotted in Fig. 3 as isolated points. It will be seen that the variation in Ω is quite small. In Fig. 3 the approximate curve of Ω against λ given by equation (24) is also drawn. To determine this curve, values of the slope of the lift curve and of $r\,(\mu + \theta)$ were required. In view of the altered distribution of p, $r\,(\mu + \theta)$ is not now independent of radius ; the variation is, however, slight, and an arithmetical mean

$$r\,(\mu + \theta) = 1{\cdot}213 \quad \ldots \quad (41)$$

was obtained from Table 5, which also gives values of the slope of the lift curve, estimated from the incidence range 0 – 5°. The mean value was taken to be

$$\frac{\partial C_L}{\partial \alpha} = 5{\cdot}60. \quad \ldots \quad (42)$$

Equation (24) now becomes

$$\Omega = 34 \cdot 3 \lambda^{-1/3} \left\{ 1 + \sqrt{1 + 3 \cdot 52 \lambda} \right\}, \qquad (43)$$

which is the curve of Fig. 3. The agreement between this curve and the three isolated points found from the exact strip theory equations is quite good.

TABLE 5

r	Section	μ (deg.)	θ (deg.)	$\mu + \theta$ (rad.)	$r(\mu + \theta)$	$\partial C_L/\partial \alpha$
1·75	C	6·7	32·1	0·677	1·184	5·16
2·5	D	5·5	22·0	0·480	1·200	5·56
3·25	E	4·5	17·1	0·377	1·225	5·73
4·0	F	3·7	14·4	0·316	1·265	5·96

For comparison, Fig. 3 gives also the curve of variation of u with λ. It is

$$u = 83 \cdot 1 \lambda^{-1/3}, \qquad (44)$$

and it will be seen that the variation in u is much greater than the variation in Ω.

Through each of the three isolated points in Fig. 3 is drawn a short straight line having a slope given by (25). It is apparent that (25) gives a very good estimate of the rate of variation of Ω with λ. Fig. 3 shows that, for the extreme conditions of working of the non-turbulent tunnel, the variation in rotational speed of the fan for a power input of 100 H.P. is only from about 975 r.p.m. to 1,125 r.p.m.

The stresses in the fan may be estimated on the basis of the representative fan discussed in Appendix I, for which the stresses and deflections are given by equations (21), (22) and (23). For the fan discussed in the present paragraph

$$r_1/c = 3 \cdot 40, \qquad (45)$$

while the maximum value of Ω will be about 120 rad./sec. (see Fig. 3) so that

$$\Omega r_1 = 510 \text{ ft./sec.} \qquad (46)$$

It was decided that the fan should be made of spruce, for which the density γ and Young's modulus E are approximately

$$\gamma = 1 \cdot 0 \text{ slug/ft.}^3, \qquad (47)$$

$$E = 2 \cdot 4 \times 10^8 \text{ lb./ft.}^2 \qquad (48)$$

Accordingly, equation (21) gives the maximum tensile stress as

$$\sigma_c + \sigma_b = 6 \cdot 5 \times 10^4 + 8 \cdot 9 \times 10^4$$
$$= 15 \cdot 4 \times 10^4 \text{ lb./ft.}^2, \qquad (49)$$

which is about one-fifth of the elastic limit of spruce.

The extension due to the centrifugal force becomes

$$e_c = 5 \cdot 1 \times 10^{-4} \text{ ft.}$$
$$= 0 \cdot 006 \text{ in.}, \qquad (50)$$

approximately. Similarly, the tip deflection due to thrust is roughly

$$e_b = 112 \times 10^{-4} \text{ ft.}$$
$$= 0 \cdot 13 \text{ in.} \qquad (51)$$

APPENDIX I

9. *Stresses and Deflections in the Blades of a Representative Fan.*—The stresses and strains occurring in an airscrew are discussed in some detail in R. & M.1274.[16] The main stresses may be classified under the following heads :—

(*a*) Tensile stress due to centrifugal force.
(*b*) Tensile and compressive stress resulting from the bending moment due to thrust.
(*c*) Shear stress due to thrust.
(*d*) Shear stress resulting from the untwisting effect of the centrifugal pull.
(*e*) Shear stress due to the twisting moment of the centrifugal forces.
(*f*) Shear stress due to aerodynamical twisting moment.

If the blades have any " hang-forward " or " hang-back " there will be a contribution to the bending moment from the centrifugal forces ; in the present discussion it will be assumed that the blade sections are so disposed that there is no such contribution.

The forces occurring in the blades of a fan obviously increase toward the root ; the thickness of the fan blades discussed in the present paper has been chosen to increase toward the root to minimise the resultant stresses. Consider a representative fan blade and regard it as being encastré at the root radius

$$r0 = 0 \cdot 4 r_1, \qquad (52)$$

where r_1 is the tip radius ; and write

$$r = x r_1, \qquad (53)$$

where r is the current radius. If in the representative fan, sections C, D, E and F are assigned to the points $x = 0 \cdot 475$, $0 \cdot 625$, $0 \cdot 775$ and $0 \cdot 925$ respectively, then to a close degree of approximation

$$\frac{t}{c} = \frac{0 \cdot 08}{x}, \qquad (54)$$

where t is the maximum thickness of the section. Moreover, the sections are derivable from each other by orthogonal projection ; and the area of cross-section and moment of inertia about the neutral axis parallel to the chord are respectively

$$a = 0 \cdot 745 ct,$$
$$I = 0 \cdot 05 ct^3,$$

which in view of (54) become

$$a = 59 \cdot 6 \times 10^{-3} c^2 x^{-1}, \qquad (55)$$

$$I = 25 \cdot 6 \times 10^{-6} c^4 x^{-3}. \qquad (56)$$

The material of the blade will be assumed to be isotropic and of density γ. A caution should be added here that all densities must be expressed in the proper units ; if the unit of force is one pound weight and the unit of length one foot, then the density must be expressed in slugs/ft.^{3} ; if, however, the inch is chosen as the unit of length, the unit of mass becomes 12 slugs, and the density must be expressed as the number of these units per cubic foot.

The centrifugal force on a slice of the blade of thickness $r_1 dx$ is $\gamma a r_1^2 \Omega^2 x dx$; hence, in view of (55), the total tension at any radius x is

$$\gamma r_1^2 \Omega^2 \int_x^1 a x dx = 59 \cdot 6 \times 10^{-3} c^2 \gamma r_1^2 \Omega^2 (1 - x), \qquad (57)$$

and the stress, i.e., the tension per unit area, is therefore

$$\sigma_c = \gamma r_1^2 \Omega^2 x (1 - x). \qquad (58)$$

The maximum value of this stress occurs when $x = 0 \cdot 5$ and is of magnitude

$$\sigma_c = 0 \cdot 25 \gamma r_1^2 \Omega^2. \qquad (59)$$

The variation in the stress over the inner third of the blade is very slight; for the range $x = 0 \cdot 4$ to $x = 0 \cdot 6$ it can be assumed that the stress is given by (59).

If e_c is the extension of the blade due to the centrifugal forces, then

$$\frac{1}{r_1} \frac{de_c}{dx} = \frac{\sigma_c}{E},$$

where E is Young's modulus for the material of the blade, and σ_c is given by (58). The integral of this equation over the whole of the blade gives approximately

$$e_c = 0 \cdot 11 \, \gamma r_1^3 \Omega^2 / E. \qquad (60)$$

In the determination of the bending stresses due to the thrust the assumption will be made that the blade angles are small and that the thrust is normal to the chord. The stresses are then probably not smaller than those occurring in an actual twisted blade.

Equation (11) gives the approximate value of the element of thrust at $x = \xi$ on each blade as

$$dT = \tfrac{1}{2} \rho c r_1^3 \Omega^2 C_L \xi^2 d\xi, \qquad (61)$$

and the total bending moment at the point x, due to the elements of thrust external to x, is therefore

$$M = \tfrac{1}{2} \rho c r_1^4 \Omega^2 \int_x^1 C_L \xi^2 (\xi - x) d\xi. \qquad (62)$$

Now since rC_L is constant along the blade, and $C_L = 0 \cdot 6$ at the tip, $C_L \xi = 0 \cdot 6$; hence (62) reduces to

$$M = 0 \cdot 05 \, \rho c r_1^4 \, \Omega^2 \, (1 - x)^2 \, (2 + x). \qquad (63)$$

Moreover, the neutral axis of the section is very approximately at $0 \cdot 5 \, t$ from the chord; hence the maximum stress at the section is

$$\sigma_b = 0 \cdot 5 \, tM/I,$$

which on substitution from (54), (56) and (63) gives

$$\sigma_b = 78 \cdot 1 \, \rho \, r_1^2 \Omega^2 \left(\frac{r_1}{c}\right)^2 x^2 \, (1 - x)^2 \, (2 + x). \qquad (64)$$

This is itself a maximum for $x = 0 \cdot 525$; as for the case of the centrifugal stress, however, its value is practically constant over the inner third of the blade, and is of magnitude, approximately

$$\sigma_b = 12 \cdot 5 \, \rho r_1^2 \Omega^2 \, (r_1/c)^2. \qquad (65)$$

If e_b is the total deflection of the blade tip due to bending, then

$$\frac{1}{r_1} \frac{de_b}{dx} = \frac{M r_1 \, (1 - x)}{EI},$$

which on substitution from (56) and (63) gives approximately

$$e_b = 26 \, \frac{\rho \, r_1^2 \Omega^2}{E} \left(\frac{r_1}{c}\right)^2. \qquad (66)$$

The stress given by (65) is tensile on the flat face of the blade and compressive on the curved face. Actually, the maximum compressive stress is probably a little greater than the maximum tensile stress, since the neutral axis is, in fact, rather nearer the flat face than is assumed above. However, the centrifugal tensile stress has to be added to the bending stress ; since the maximum stresses due to both causes occur over the inner third of the blade, it is sufficient in practice to say that the maximum stress is tensile and of magnitude

$$\sigma_c + \sigma_b = 0\cdot25\,\gamma r_1{}^2\Omega^2 + 12\cdot5\,\rho r_1{}^2\Omega^2\,(r_1/c)^2. \quad \ldots \quad (67)$$

In a similar manner it is readily shown that the shear stress due to the thrust is

$$\sigma_s = 0\cdot97\,\rho r_1{}^2\Omega^2\,(r_1/c), \quad \ldots \quad (68)$$

and the deflection due to shear is

$$e_s = 0\cdot45\,\frac{\rho r_1{}^3\Omega^2}{\mu}\left(\frac{r_1}{c}\right), \quad \ldots \quad (69)$$

where μ is the modulus of rigidity. As is usual for a loaded cantilever of slender form, however, these shearing actions are small in comparison with the bending actions.

As regards the torsional actions classified above under the headings (*d*) and (*e*), these involve the blade angle and rate of twist, and cannot profitably be discussed for the representative blade. They can be evaluated by the methods described in R. & M. 1274.[16] It may be remarked, however, that the twisting moments due to these two causes are of opposite sense : the direct centrifugal pull tends to untwist the blade, i.e., to increase the blade angles, while for normal blade angles the moment of the centrifugal forces tends to throw the blade sections into the plane of rotation, and thus to decrease the blade angles. For fans with relatively low tip speeds, such as are discussed in the present paper, the resultant moment is probably not great.

Finally, the aerodynamical twisting moment is in practice negligibly small. Values of the aerodynamical pitching moment coefficient for the sections of the family of airscrews, measured about the quarter-chord position, are given in R. & M. 1771.[12] For each section, the moment is almost independent of the value of C_L ; it follows that the lift force acts at the quarter-chord point. These pitching moment coefficients are, as might be expected, very nearly proportional to the thickness-chord ratio of the section, and for the representative blade, they can in view of (54) be taken to be given by $-\,0\cdot05x^{-1}$. Now the centre of twist of the sections, estimated from the formulae of R. & M. 1444,[17] is at $0\cdot34\,c$ from the leading edge. Hence the total pitching moment coefficient, measured about this axis of twist, is for the section at radius r_1x

$$C_m = C_L\,(0\cdot34 - 0\cdot25) - 0\cdot05x^{-1}.$$

But xC_L has the constant value $0\cdot6$ along the blade ; hence

$$\begin{aligned} C_m &= 0\cdot054x^{-1} - 0\cdot05x^{-1} \\ &= 0\cdot004x^{-1}. \quad \ldots \quad (70) \end{aligned}$$

No accuracy can be attached to the numerical coefficient in (70) ; it can only be said that it is of this order. It is, however, so small that the resultant torsional moments are negligibly small. On the basis of equation (70), a blade of chord $c = 1$ ft., tip radius $r_1 = 5$ ft., and having a tip speed of 500 ft./sec., will be subjected to a total aerodynamical twisting moment, measured at the root, of only about $2\cdot5$ lb. ft.

To sum up, it may be said that in practice equations (66) and (67) provide useful criteria for the strength of a fan blade of the type discussed in the present paper. If the deflection given by (66) is unimportant and the stress given by (67) is well within the elastic limit of the material, then in practice the blade is of adequate strength.

APPENDIX II

10. *A Formula for the Integration of the Thrust and Torque Grading Curves.*—The thrust and torque grading are each defined by four points, one at each reference section. In drawing a smooth curve through four typical points, and integrating graphically or by Simpson's rule from root to tip, an error of the order of $\pm 0 \cdot 5$ per cent. may be introduced; thus the error in efficiency may be ± 1 per cent. This error may be reduced by determining, for both thrust and torque grading, the cubic curve which passes through the four given points, and by analytical integration of this curve. When, as is nearly always the case, the distance from the root to section C is the same as the distance from section F to the tip, and when there is equal spacing s between the four reference sections, the integral of the cubic curve is given by equation (71), which is an extension of Simpson's three-eighths rule. If the given ordinates are y_1, y_2, y_3 and y_4, and if the blade length $r_1 - r_0$ is l, then

$$\int_{r_0}^{r_1} y\,dr = \frac{l}{48}\left\{\left(\frac{l^2}{s^2} - 3\right)(y_1 + y_4) + \left(27 - \frac{l^2}{s^2}\right)(y_2 + y_3)\right\} \quad \ldots \quad (71)$$

TABLE 6

Details of Airscrew Blade Sections

η = fraction of chord from leading edge.
R_l = leading edge radius.
R_t = trailing edge radius.

η	Ordinate, as fraction of chord			
	C	D	E	F
0	0·0212	0·0163	0·0130	0·0108..
0·05	0·1000	0·0768	0·0613	0·0512
0·1	0·1326	0·1018	0·0813	0·0679
0·2	0·1611	0·1237	0·0988	0·0825
0·3	0·1680	0·1290	0·1030	0·0860
0·4	0·1660	0·1275	0·1018	0·0850
0·5	0·1591	0·1222	0·0975	0·0814
0·6	0·1460	0·1121	0·0895	0·0747
0·7	0·1253	0·0962	0·0768	0·0642
0·8	0·0968	0·0743	0·0593	0·0495
0·9	0·0627	0·0481	0·0384	0·0321
1·0	0·0124	0·0095	0·0076	0·0064
R_l	0·0212	0·0163	0·0130	0·0108
R_t	0·0124	0·0095	0·0076	0·0064

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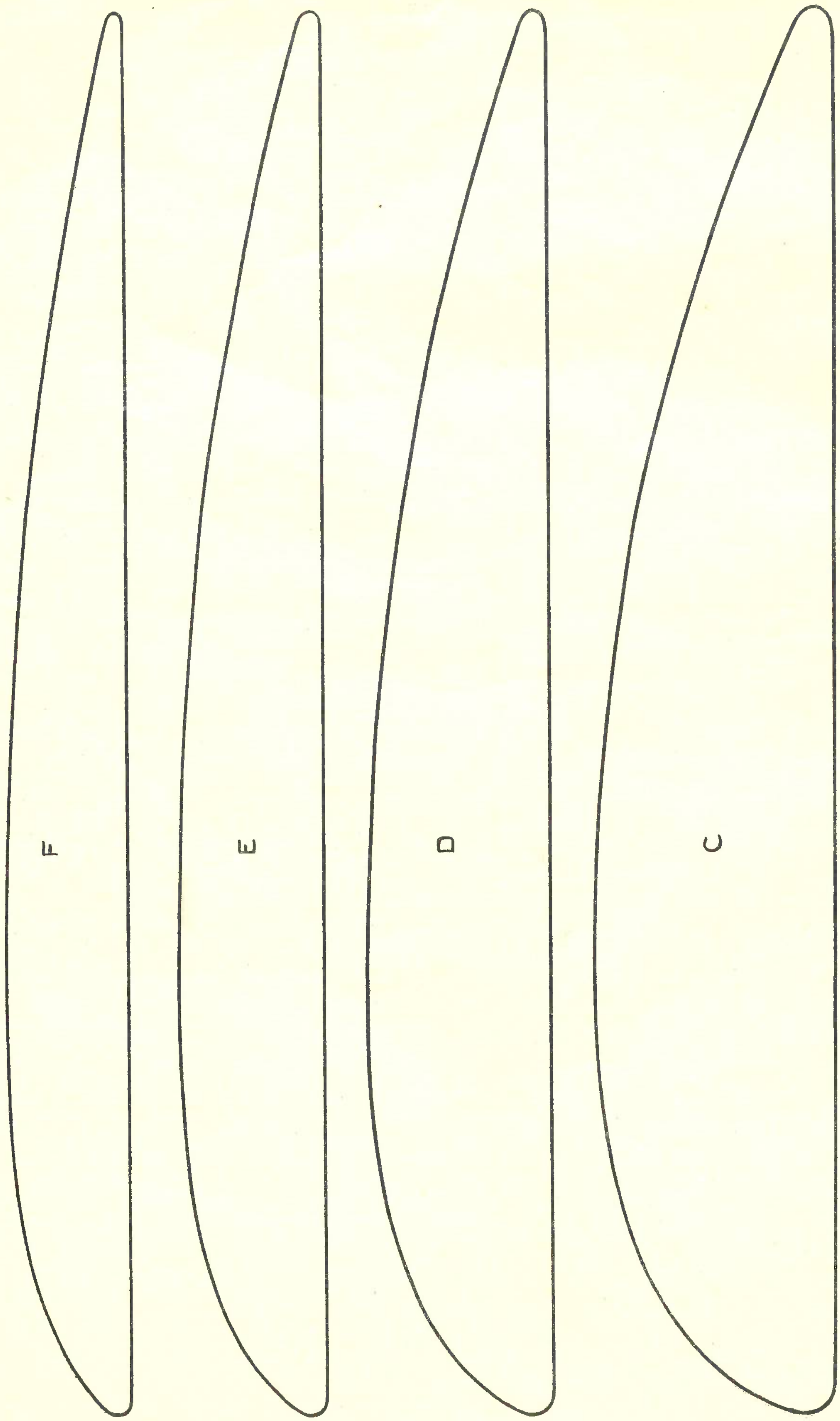
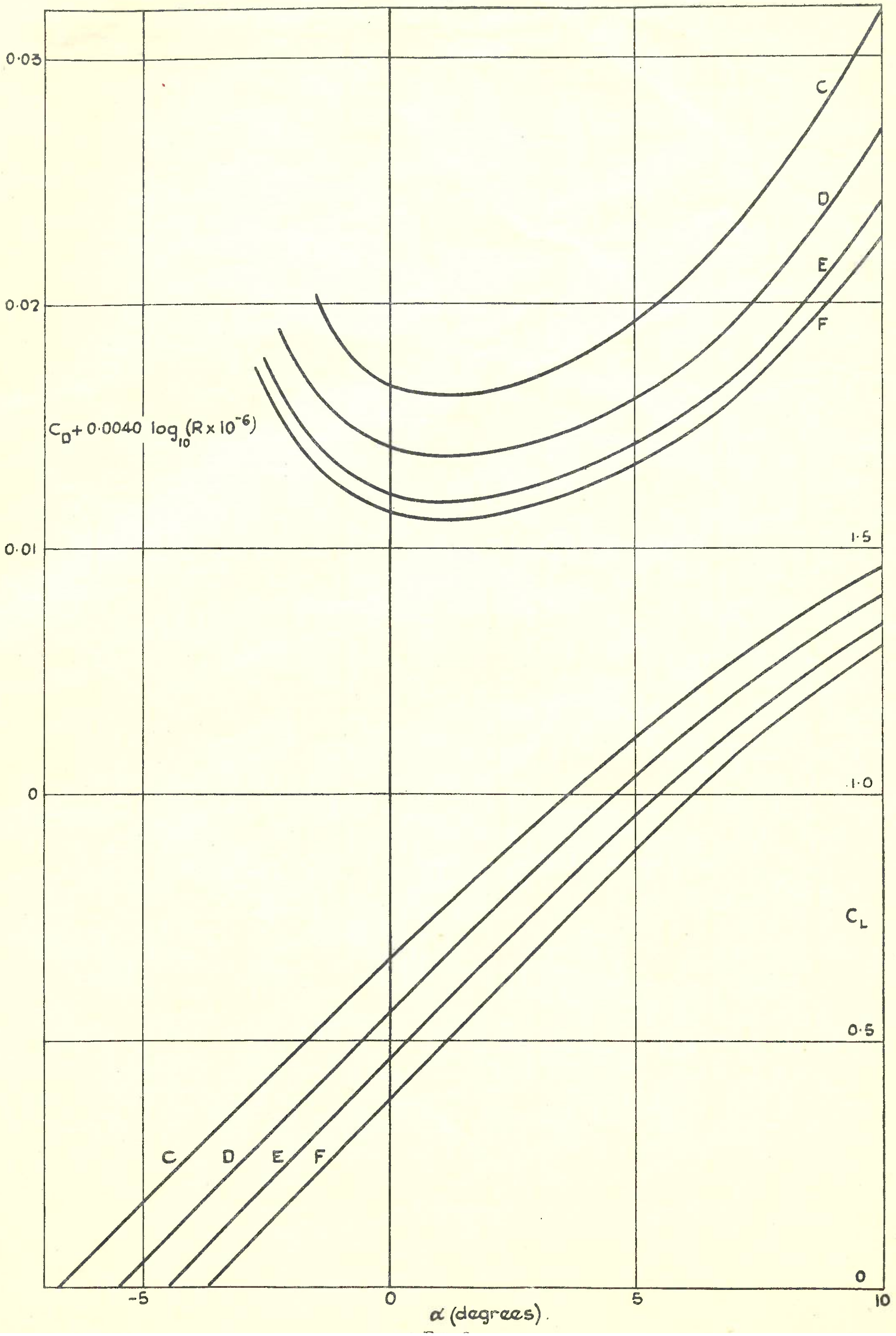


Fig. 1



0·03
0·02
0·01
0
C
D
E
F
$C_D + 0{\cdot}0040\ \log_{10}(R \times 10^{-6})$
1·5
1·0
C_L
0·5
0
C D E F
-5
0
5
10
α (degrees).

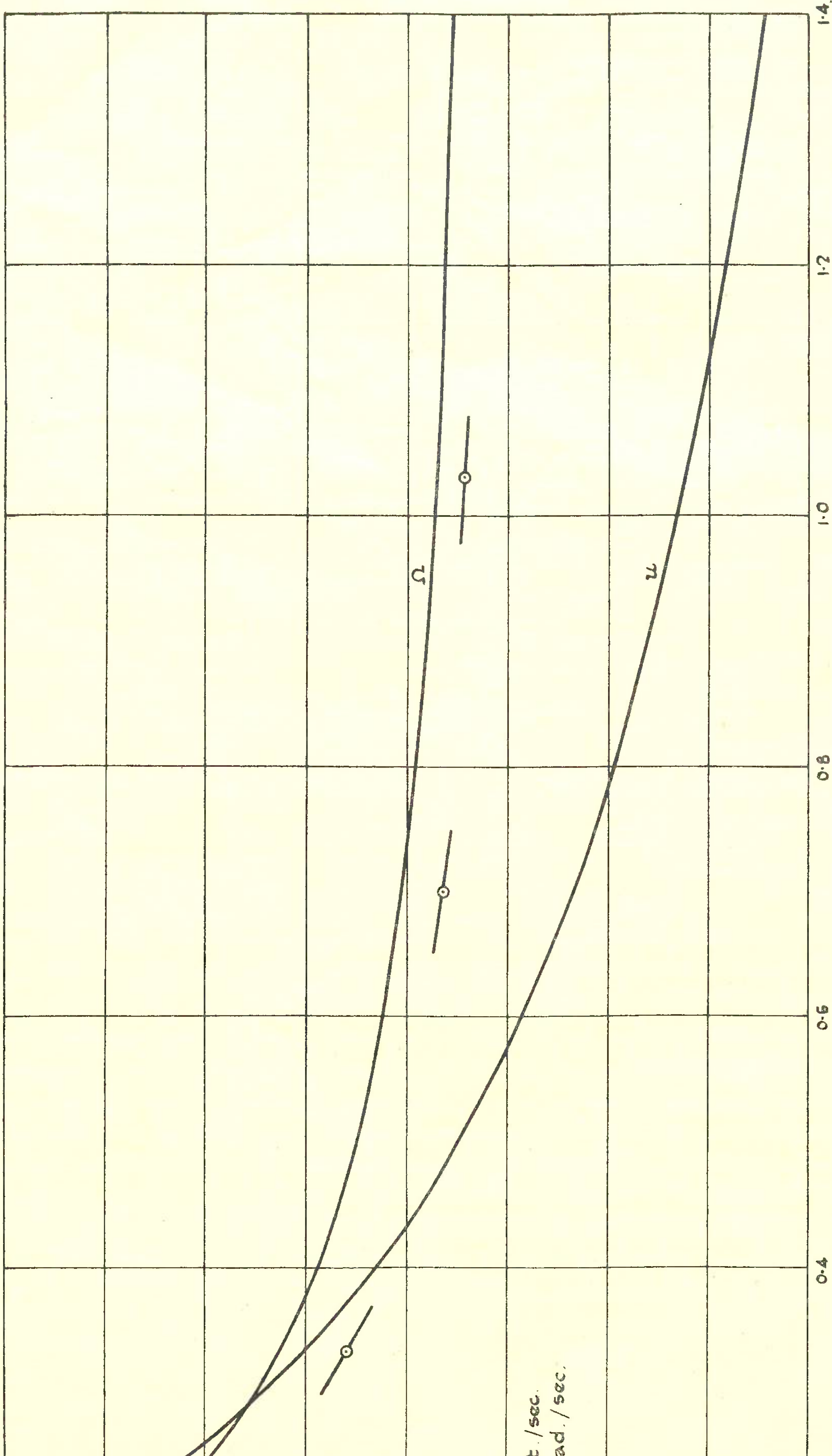
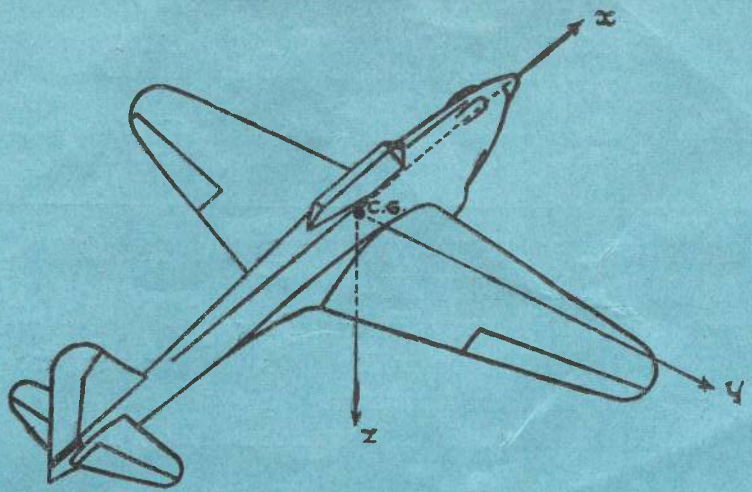


FIG. 3

(53487) Wt. 4 500 12/40 Hw. G.377/1

SYSTEM OF AXES



Axes	Symbol Designation Positive direction	x longitudinal forward	y lateral starboard	z normal downward
Force	Symbol	X	Y	Z
Moment	Symbol Designation	L rolling	M pitching	N yawing
Angle of Rotation	Symbol	ϕ	θ	ψ
Velocity	Linear Angular	u p	v q	w r
Moment of Inertia		A	B	C

Components of linear velocity and force are positive in the positive direction of the corresponding axis.

Components of angular velocity and moment are positive in the cyclic order y to z about the axis of x, z to x about the axis of y, and x to y about the axis of z.

The angular movement of a control surface (elevator or rudder) is governed by the same convention, the elevator angle being positive downwards and the rudder angle positive to port. The aileron angle is positive when the starboard aileron is down and the port aileron is up. A positive control angle normally gives rise to a negative moment about the corresponding axis.

The symbols for the control angles are :—

ξ aileron angle
η elevator angle
η_T tail setting angle
ζ rudder angle

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