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Power Spectrum Analysis of Gust Loads on the Comet Wing and Tailplane

by

D. T. Jones, M.A., B.Sc.

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Power Spectrum Analysis of Gust Loads on the Comet Wing and Tailplane

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D.T. Jones M.A., B.Sc.

SUMMARY

This report describes an analysis of measurements of normal acceleration and stress recorded on Comet aircraft while flying in continuous turbulence at high and low altitude. It is found that the increments of normal acceleration at the centre of gravity and of stress in the wing and tail are affected by resonance, the vibrations of the wing and tail being forced by the turbulence.

Estimates are given of the amplifying effects due to the resonance. It is found that amplifications are proportionately greater in the stresses than in the simultaneously recorded normal accelerations at the centre of gravity.

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1 Introduction

In July 1954 a series of flight tests was made on a Comet aircraft at the Royal Aircraft Establishment. Some of the tests were concerned with the effect of gusts on the aircraft and in these tests measurements of normal acceleration and structural stress were taken while the aircraft was flown straight and level in turbulent air. This report describes one of the methods used to analyse these measurements. The same method was also used to analyse similar measurements obtained from an earlier routine flight made by the De Havilland Aircraft Company. The object of the analysis was to estimate the effects of the natural oscillatory motions of the aircraft on the normal acceleration and stress.

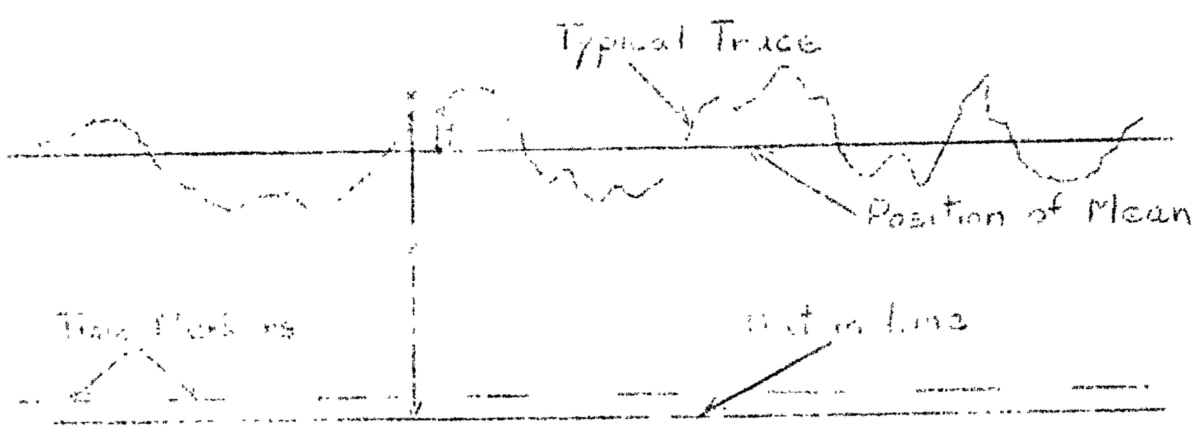
The amplitudes of these motions and the corresponding stresses induced in the structure are increased by resonance. As the increases occur at the natural frequencies of the aircraft an analysis of the average amplitude into its frequency components is made in order to reveal the presence of the increases and to assess their magnitudes. This analysis shows that the contributions to the average amplitude at the structural frequencies are greater than they would be if there were no resonance. The excesses can be estimated and give a measure of the amplification. In this report excesses due to the natural fundamental motions of the wing and tail structures only are considered. The short duration of the records does not allow the effects of rigid body motions which occur at comparatively low frequencies to be included.

The "average" used in the analysis is not the usual arithmetic mean of increments (taken without regard to sign), but the mean of the squares of increments. This quantity is known as power. It is a convenient quantity for, as will be shown, the power of a complex wave form is equal to the sum of the powers of the harmonic components, whatever the phase relationship between the components may be.

2 Method of analysis

2.1 Data

Instruments were installed in Comet aircraft G-ANAV to record normal acceleration at the centre of gravity of the aircraft, normal acceleration at the tail and bending moment (via strain records) at the root of the tailplane while the aircraft was flying in turbulent air. These variables were recorded simultaneously as continuous traces on a roll of paper. The records were taken at low altitude, i.e. between 4,000 and 6,000 ft. The following diagram shows a portion of a typical record. For clarity only one trace is shown.



A record taken on Comet G-ALYT by the De Havilland Aircraft Company while the aircraft was on a routine flight at an altitude of 37,500 ft was also available for analysis. Traces of wing spar stress, tail stress and normal acceleration were recorded in the same manner as on the R.A.E. records.

2.2 Method of reading traces

To find the mean square value (i.e. the power) of a variable and to obtain information required for the analysis of the power into its spectrum, the values assumed by the variable at successive short intervals of time were measured and recorded. These measurements were obtained from traces taken in continuous turbulence for periods of between one and three minutes and the intervals between the measurements were not more than half the period of the highest frequency component likely to be significant. It was thought that structural frequencies above 10 cycles per second would not be significant. Readings were therefore taken at intervals of one-twentieth of a second.

The distance x of a point X on a trace (shown in the above diagram) from a datum line at time t is typical of the measures that were taken. The measures actually required for the analysis were, of course, incremental values, such as y, from the mean but it was found more convenient to read initially from the datum line. This datum was at constant distance from the mean of each trace and a simple adjustment was made in calculations to change the values as read into incremental values from the mean.

As many thousands of these measurements had to be taken a machine* was devised to assist in the reading. The machine embodies a reading head. An operator using the machine brings a cursor which formed part of this head to cover in turn the successive points on the trace (Fig. 1). As each point is covered the measurement (i.e. the distance from the datum line to the cursor) is automatically sensed in the reading head and, when the operator presses a switch is recorded as a set of holes in a Hollerith card. In this way measurements were taken easily and quickly and recorded in a form which was immediately suitable for subsequent calculations on Hollerith machines.

2.3 Power spectrum

A continuous trace of normal acceleration or stress taken when an aircraft is flying through turbulence has a random irregular appearance and it is impossible to discern any single fluctuation in the trace which can be regarded as a typical one. We can, however, take the average of the increments as a measure of the general intensity of the stress or acceleration increments. The simplest average would be the arithmetic mean of the increments (taken without regard to sign) measured at equal intervals of time. This average is easy to compute but is not suitable for our analysis. For this we need the average of the squares of the increments, a quantity known as the mean square, variance or power.

The power of a complex wave form (such as a trace of normal acceleration or stress in turbulence) is simply related to the powers of the simple harmonic wave forms of which the complex form is assumed to be composed.

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* This machine was designed by Mathematical Services Department and Instrumentation Department, R.A.E. in response to a request by Structures Department, R.A.E. It was first used for the analysis of Comet continuous trace records.

It can be shown (see Appendix I) that if a complex form y(t) is the sum of n simple harmonic wave forms and can be written as

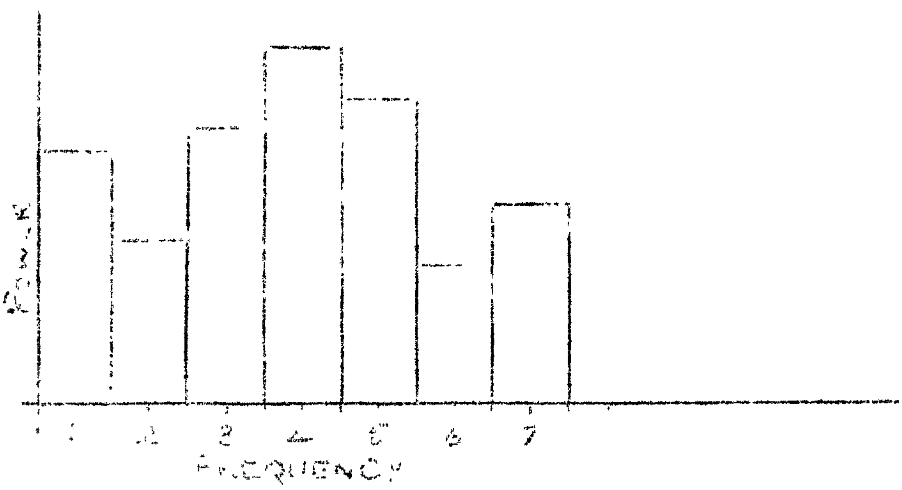
$$y(t) = \sum_{1}^{n} A_k \sin (\omega_k t + \phi_k)$$

where the A's are the amplitudes, the ω's are the frequencies and the ϕ's are the phases of the components, then the power of the k'th component is $\frac{A_k^2}{2}$ and the power of y(t)

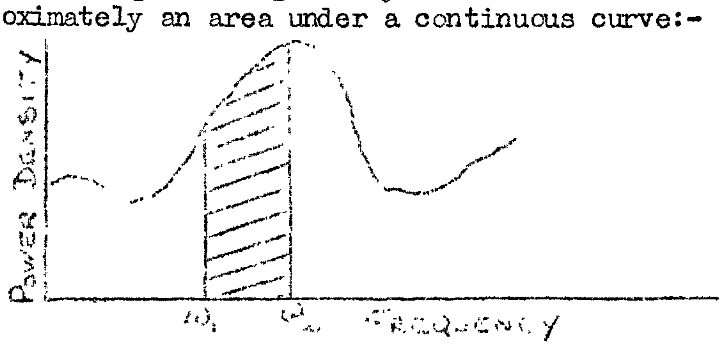
$$\equiv \overline{y^2(t)} = \sum_{1}^{n} \frac{A_k^2}{2}$$

This result is of fundamental importance. It means that if a number of simple harmonic wave forms are combined to produce a complex form, the power of this form is equal to the sum of the powers of the components, whatever the phase relationship between the components may be.

If we have a complex wave form composed of a given finite number of components at frequencies of say 1, 2, ... n cycles per second and the powers of the components are known we can represent the total power by a block diagram such as the following



If the blocks are of unit width the areas of the blocks represent the powers of components having frequencies represented by the mid-points of the block widths, and the total area represents the total power. In a complex wave form such as a continuous trace of stress in turbulence there may be a very large number of components. In a complete analysis of such a trace the block widths would be very small and the heights of the blocks i.e. the ordinates would represent the density of power at frequencies given by the abscissae. The diagram would be approximately an area under a continuous curve:-



The total area again represents the total power and the power due to components at frequencies between any two prescribed frequencies, say ω_1 and ω_2 is given by the shaded area.

The calculation of the power spectrum of a complex wave which may contain a large number of components at unknown frequencies is based on the recently developed theory of generalised harmonic analysis[2]. The calculation is usually made in two parts. The first is to find the autocorrelation function (see Appendix I) of the variable having the complex form, that is the normal acceleration or stress increment. If $y(t)$ represents the value at time t the autocorrelation function $R(\tau)$ is defined as

$$\frac{1}{n} \sum_{n \to \infty} y(t) \cdot y(t + \tau)$$

When this function has been evaluated the next stage is to compute the power spectrum as the Fourier transform of the function. It can be shown (see Appendix I) that if, as before, a complex form $y(t)$ is represented as

$$y(t) = \sum_{1}^{n} A_k \sin (\omega_k t + \phi_k)$$

then

$$R(\tau) = \sum_{1}^{n} \frac{A_k^2}{2} \cdot \cos \omega_k$$

or if we put $p(\omega)$ for an element of power $\left(\text{corresponding to } \frac{A_k^2}{2}\right)$ in the frequency range ω to $\omega + d\omega$, then

$$R(\tau) = \int_0^\infty p(\omega)\, d\omega \cos \omega_k$$

and by a Fourier transform

$$p(\omega) = \frac{2}{\pi} \int_0^\infty R(\tau) \cos \omega\tau \, d\tau$$

Hence the power in a frequency band of width $\omega - \delta\omega$ to $\omega + \delta\omega$ $= 2\int_0^\infty R(\tau) \cos \omega\tau \{(\sin\delta\tau)/\tau\} d\tau$.

When the autocorrelation function has been found this relationship can be used to give estimates of power within any prescribed bands of frequency.

The reliability of autocorrelation functions, and therefore the reliability of power estimates, depends on the length of the records from which they are computed. The reliability also increases with increasing band width. This is reasonable, for suppose that estimates of powers for a number of narrow band widths adjoining one another are obtained. These estimates will err randomly on either side of the true values. If the estimates are combined to give a single estimate for a wider band widtn comprising the narrow ones the result will clearly be more reliable as the component errors will tend to nullify one another. A detailed discussion of this point is given in Reference 3.

3 Results

3.1 Spectra from low altitude flight records

Several records of normal acceleration at the centre of gravity of the aircraft and bending moment at the root of the tailplane were obtained during the R.A.E. test flights at altitudes between 4,000 ft and 6,000 ft. The forms of the spectra obtained did not vary very much between one flight record and another. Fig. 2 and 3 show typical power spectra of normal acceleration and tailplane bending moment obtained from simultaneous recordings of the two variables. When the records were taken the aircraft was flying in continuous turbulence for 160 seconds at an altitude of 6,000 ft and the aircraft speed was 200 knots E.A.S. To obtain the autocorrelation function values of normal acceleration and bending moment were read at intervals of $^1/20$ sec.

The power of normal acceleration decays rapidly as the frequency increases from 0 to 10 cycles per second, but there is a slight peak at between 2 and 3 cycles per second, that is at the fundamental frequency of the wing. There is also a smaller peak at about 5 cycles per second, the fundamental frequency of the tail. The power of normal acceleration therefore appears to be only slightly affected by resonance at the wing and tail frequencies.

For tailplane bending moment there is a slight peak at the wing frequency and a larger one at the tail frequency. Assuming uniform decay in power with increasing frequency when there is no resonance we find that the area above the broken line represents the contribution to total power due to resonance. The ratio of the whole area to the area below the line therefore gives an estimate of the amplification in power due to resonance. For the tailplane bending moment this ratio is about 1.3. As standard deviation is the square root of power, the standard deviation of bending moment increments is amplified by a factor of about 1.14 ($=\sqrt{1.3}$). These estimates depend on the position of the broken line. In Fig. 3, 4, 5 and 6 the lines have been drawn in positions judged to be reasonable as representing the uniform decay in power which would be expected in the absence of resonance peaks. Precise accuracy is not, of course, claimed for the positions adopted.

3.2 Spectra from high altitude flight record

Fig. 4. 5 and 6 show power spectra of normal acceleration, tailplane stress and wing stress obtained from simultaneous recordings of these three variables. The records were taken while the aircraft was flying through continuous turbulence for 200 seconds at an altitude of 37,500 ft. The speed was 200 knots E.A.S.

It can be seen that the spectrum of normal acceleration decays rapidly on the whole as the frequency increases, but that two peaks occur, one at the wing frequency and the other, a considerably smaller one, at the tail frequency. The amplification in power is about 1.20 and the amplification in standard deviation is about 1.10.

The power spectrum of wing stress is in striking contrast to the power spectrum of normal acceleration. There is a very large peak at the wing frequency and the amplification in power is about 2.0. The amplification in standard deviation of wing stress increments is thus about 1.41. Practically the whole of this resonant contribution occurs at the wing frequency.

The power spectrum of tailplane stress has two very large peaks, one at the wing frequency and the other at the tail frequency. The amplification in power is estimated to be about 2.90 and the amplification in standard deviation about 1.70.

4 Discussion

4.1 Main results

The main results of the analysis are as follows:-

(i) Amplifications occur in the power of normal acceleration at the centre of gravity and in the wing and tail stresses. These amplifications are at the natural frequencies of the wing and tail in bending.

(ii) These resonant amplifications are proportionately greater at high altitude than low altitude.

(iii) The resonant amplifications in the power of wing stress and tail stress are greater than the proportional amplification in normal acceleration at the centre of gravity.

(iv) The numerical values obtained for the amplifications suggest the adoption of factors which might be applied to estimates of stress based on measurements of acceleration taken at the centre of gravity. The results have, however, been obtained in turbulence of very low intensity and further work should be done to check their validity at higher intensities of turbulence.

4.2 Effect of altitude

The amplifications in normal acceleration increments and stress increments are much greater at high altitude than low altitude. This striking difference appears to be consistent with the difference in aerodynamic damping. At the high altitude of 37,500 ft the damping is very much less than at 6,000 ft and it is therefore inevitable that there will be a marked difference in resonance effects.

4.3 Comparisons between normal accelerations and stresses

An important point of difference between the spectra is the difference between the amplification of normal acceleration on the one hand and wing and tail stresses on the other. The results show that the amplification of wing and tail stress increments is greater than that of normal acceleration increments. These results may be summarised as follows:-

	Amplification in Standard Deviation of		
	Normal Acc. at c.g.	Wing Stress	Tail Stress
Low altitude flight (R.A.E.)	Very slight	No data	1.14
High altitude flight (De Havilland)	1.10	1.41	1.70

The conclusion is that the effects of resonance on stress increments are only slightly reflected in the normal acceleration at the centre of gravity.

We have seen that in the high altitude flight the standard deviation of normal acceleration near the centre of gravity is slightly affected by resonance at the wing frequency while the standard deviation of wing stress is much more affected. The ratio of "resonant" standard deviation to the "non resonant" standard deviation was found by the power spectrum analysis to be 1.41 for wing stress and 1.10 for normal acceleration at the centre of gravity. It follows that estimates of increment in wing stress derived from increments of normal acceleration at the centre of gravity in the conventional way without regard to resonance should be substantially less than the true values. The ratios given above indicate that for the flight condition in which the measurements were taken a conventional estimate of the standard deviation of stress should be multiplied by $\frac{1.41}{1.10} = 1.29$ to obtain more accurate values.

A direct check on this result has been made. From measurements taken in single manoeuvres in calm air the increment in wing stress corresponding to a given increment of normal acceleration was found. This result was used to compute a wing stress corresponding to the measured standard deviation of normal acceleration. This computed wing stress might be expected to agree with the measured standard deviation of wing stress (if the difference in resonance effects noted above were not present). In fact the ratio of measured standard deviation of wing stress to the computed value was found to be not about 1.34, a value which does not differ greatly from the value of 1.29 estimated above from the power spectrum analysis.

The results show that the ratio of stresses induced in the wing and tail to the normal acceleration at the centre of gravity are greater at high altitude than at low altitude. These ratios were obtained in turbulence of very low intensity and the stresses measured were very low and quite insignificant in relation to the static strength of the wing and tail. There is, however, no evidence that these ratios, or ratios of the same order, would or would not hold good in turbulence of higher intensity. If they were to hold good we might expect the chances of static failure in a patch of severe high altitude turbulence would be greatly increased. For instance let us suppose, for illustrative purposes only, that the aircraft is in a patch of turbulence of a severity such that the standard deviation of tailplane stress increment is calculated (from c.g. acceleration measurements excluding resonance effects) to be a quarter of the design value. The chance of exceeding the design value of stress can easily be computed to be about 0.00003. If, however, the standard deviation of tailplane stress is, due to resonance, actually 1.5 times as great as the calculated value, the chance of exceeding the design value is 0.005 i.e. 150* times as great as when the amplification due to resonance is not included. It thus appears that the consequences of resonance can be important and that records for power spectrum analysis covering a range of altitudes and a variety of turbulence intensities should be obtained.

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* A result such as this must, of course, be considered against a background of operational information on turbulence intensities. Turbulence is known to occur rarely at high altitudes and it might be found that the chance of encountering a turbulent patch of the intensity assumed in this example is negligible. Furthermore it must be emphasised that all the amplification factors in this report are obtained from turbulence of low intensity and that values for high intensity turbulence are not known.

4.4 Resonance and fatigue endurance

The effect of resonance is to amplify loads and therefore to reduce fatigue endurance. Some of the data usually required for estimating fatigue endurance are the numbers of times that peak loads of given values are exceeded. If the power spectrum of a loading process is known these data can be derived from it. It has been shown[4,5] that if $\phi(\omega)$ is the spectral density (i.e. the ordinate of the power spectrum) and N(y) is the number of times that a peak value greater than a given value y (or numerically greater than a value - y) occurs then

$$N(y) \approx \frac{1}{2\pi} \cdot \frac{\sigma_1}{\sigma} \exp\left(-\frac{y^2}{2\sigma^2}\right)$$

where σ is the standard deviation and

$$\sigma_1 = \left[\int_0^\infty \omega^2 \phi(\omega)\, d\omega\right]^{\frac{1}{2}}$$

Applying the formula to the power spectra obtained from the high altitude flight we obtain the result shown in Fig. 7. The continuous line, the broken line and the dotted line give the number of times per second that peaks of wing stress, tailplane stress and normal acceleration exceeding given values of these quantities occur. To facilitate comparison the abscissae are in units of standard deviation of the quantities. It can be seen ~~that~~ in this standard measure peak values greater than a given wing stress (in units of standard deviation) occur about 1.7 times as often as the corresponding value of normal acceleration and that peak values greater than a given tailplane stress occur about 1.4 times as often as the corresponding value of normal acceleration.

We thus see that the rate of occurrence of peak stresses calculated from normal acceleration data are an underestimate. In addition, as we have already seen, the values of the calculated stresses may themselves be too low.

5 Conclusions

The general level of increments in normal acceleration at the centre of gravity and of stresses in the wing and tail in turbulence are affected by resonance, the vibrations of the wing and tail being forced by the turbulence. These amplifications are greater at high altitude than at low altitude. This is consistent with what may be expected from the comparatively low aerodynamic damping at high altitude.

The proportional amplifications in the standard deviation of wing and tail stress increments are found to be greater than the proportional amplification in standard deviation of normal acceleration increments at the centre of gravity. The results suggest that for high altitude (about 40,000 ft) the amplification factors of the standard deviation of the wing and tail stress increments are about 1.4 and 1.7 respectively. At low altitude, about 4,000 - 6,000 ft the factor has only be obtained for the tail and is about 1.15. These factors are derived from data in turbulence of low intensity and could be used in fatigue calculations.

There is no evidence to show that the suggested factors would be applicable in turbulence of high intensity. They could not be claimed to be valid for considerations of static strength in turbulence. More data and further analysis are required to check the results and to find the magnitudes of resonant effects in various intensities of turbulence.

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APPENDIX I

The Power and Autocorrelation Function of a Complex Wave form

The power of a complex wave form is equal to the sum of the powers of its simple harmonic components[6]. For suppose that one of the components has the form

$$y_k(t) = A_k \sin(\omega_k t + \phi_k)$$

the power of this form is

$$\overline{y_k^2(t)} = \lim_{T\to\infty} \frac{A_k{}^2}{2T} \int_{-T}^{T} \sin^2(\omega_k t + \phi_k)\, dt$$

$$= \lim_{T\to\infty} \frac{A_k^2}{2T} \int_{-T}^{T} \frac{1 - \cos(2\omega_k t + 2\phi_k)}{2}\, dt$$

$$= \lim_{T\to\infty} \frac{A_k^2}{2T} \left[\frac{t}{2} - \frac{1}{2} \left\{ \frac{1}{2\omega_k} \sin(2\omega_k t + 2\phi_k) \right\} \right]_{-T}^{T}$$

$$= \lim_{T\to\infty} \frac{A_k^2}{2T} \left[T - \frac{1}{2\omega_k} (2 \sin 2\, \omega_k t \; \cos 2\, \phi_k) \right]$$

$$= \frac{A_k^2}{2}$$

The power of the component is thus half the square of the amplitude of the wave form. By an extension of the above, if a complex wave form is composed of a number $\underline{n}$ of simple harmonic wave forms, that is

$$\begin{aligned} y(t) &= A_1 \sin(\omega_1 t + \phi_1) \\ &+ A_2 \sin(\omega_2 t + \phi_2) \\ &\cdots\cdots\cdots\cdots\cdots \\ &+ A_k \sin(\omega_k t + \phi_k) \\ &+ \cdots\cdots\cdots\cdots\cdots \\ &+ A_n \sin(\omega_n t + \phi_n) \end{aligned}$$

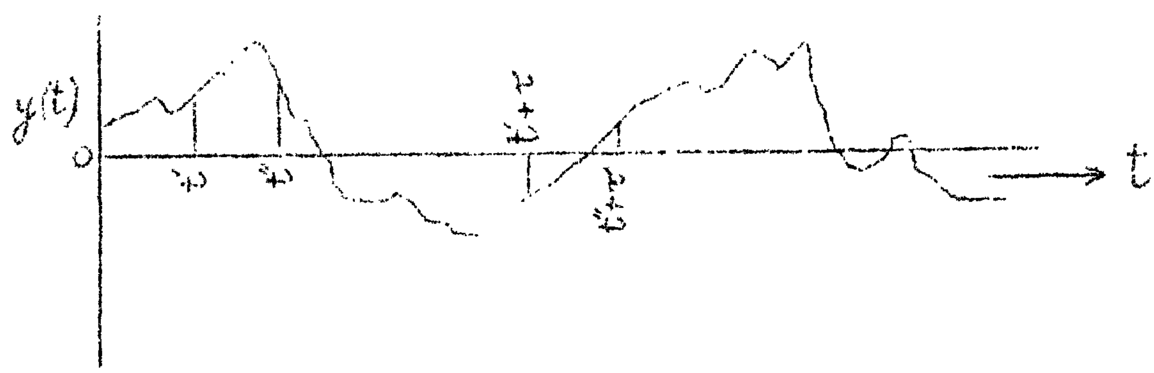
it follows that the power

$$\overline{y^2(t)} = \sum_{1}^{n} \frac{A_k^2}{2}$$

The component powers of $y^2(t)$ cannot easily be calculated directly from a record but they can be derived from a function of the record known as the autocorrelation function. If $y(t)$ represents the value of the variable measured from the mean at time $\underline{t}$ the autocorrelation function $R(\tau)$ is defined as

$$\frac{1}{n} \cdot \sum_{n \to \infty} y(t) \cdot y(t+\tau)$$

The autocorrelation function is thus the mean value of the produce of the value of the variable quantity with the value τ seconds later



For the trace shown in the above diagram the autocorrelation function for any given value of τ would be found by taking the sum of products such as

$$y(t') \cdot y(t' + \tau)$$

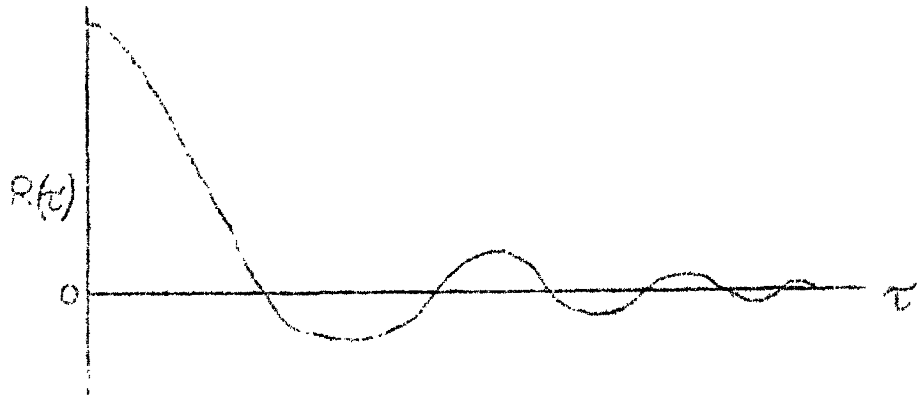
$$y(t'') \cdot y(t'' + \tau) \qquad \text{etc.}$$

for a very large number $\underline{n}$ of values of $\underline{t}$ and dividing the sum by $\underline{n}$. The autocorrelation function for a given value of τ is very similar to the correlation coefficient used in statistics. It gives a measure of the agreement between two sets of values separated by an interval τ, thus when $\tau = 0$ the agreement is perfect, the sum of products has the greatest possible value, and the value $R(0)$ is greater than any other value of $R(\tau)$. $R(0)$ is also the total power, for

$$R(0) = \frac{1}{n} \sum y(t) \cdot y(t)$$

$$= \frac{1}{n} \sum y^2(t)$$

Values of $R(\tau)$ can be plotted against τ and a continuous line drawn through the plotted points is an approximation to the autocorrelation function. The autocorrelation function of a variable such as structural stress in turbulence usually decays at first as τ increases and fluctuates about zero with decreasing amplitude:-



The above diagram shows a typical autocorrelation function.

If, as before, one of the simple harmonic components has the form:-

$$y_k(t) = A_k \sin(\omega_k t + \phi_k)$$

its autocorrelation function is

$$R_k(\tau) = \lim_{T\to\infty} \frac{1}{2T} \int_0^\infty A_k \sin(\omega_k t + \phi_k) \cdot A_k \sin(\omega_k t + \phi_k + \omega_k \tau)\, dt$$

$$= \lim_{T\to\infty} \frac{A_k^2}{2T} \int_0^\infty \left\{ \tfrac{1}{2} \cos \omega_k t - \tfrac{1}{2} \cos(2\omega_k t + \omega_k \tau + 2\phi_k) \right\} dt$$

$$= \lim_{T\to\infty} \frac{A_k^2}{2T} \left[T \cos \omega_k \tau - \tfrac{1}{2} \left(\tfrac{1}{2} \sin(2\omega_k t + \omega_k \tau + 2\phi_k)\right) \right]_{-T}^{T}$$

$$= \lim_{T\to\infty} \frac{A_k^2}{2T} \left\{ T \cos \omega\tau - \tfrac{1}{2} \frac{\sin(2\omega t + \omega\tau + 2\phi)}{2\omega} - \tfrac{1}{2} \frac{\sin(2\omega t - \omega\tau - 2\phi)}{2\omega} \right\}$$

$$= \frac{A^2}{2} \cos \omega\tau$$

By an extension of the above the autocorrelation function of the complex form

$$y(t) = R(\tau) = \sum \frac{A_k^2}{2} \cos \omega_k \tau$$

$\frac{A_k^2}{2}$ is the power due to one of the components. If we assume that we have an infinitely large number of components we may write $p(\omega)$ for the power density at frequency ω and $p(\omega)\, d\omega$ for the element of power due to components in the range ω to $\omega + d\omega$. $R(\tau)$ can then be expressed as

$$R(\tau) = \int_0^\infty p(\omega)\, d\omega \cos \omega\tau$$

By a Fourier transform

$$p(\omega) = \frac{2}{\pi} \int_0^\infty R(\tau) \cos \omega\tau \, d\tau$$

It can be deduced[6] that the power in a frequency bands of width $\omega - \delta\omega$ to $\omega + \delta\omega$

$$= 2 \int_0^\infty R(\tau) \cos \omega\tau \, \{(\sin \delta\tau)/\tau\} \, d\tau$$

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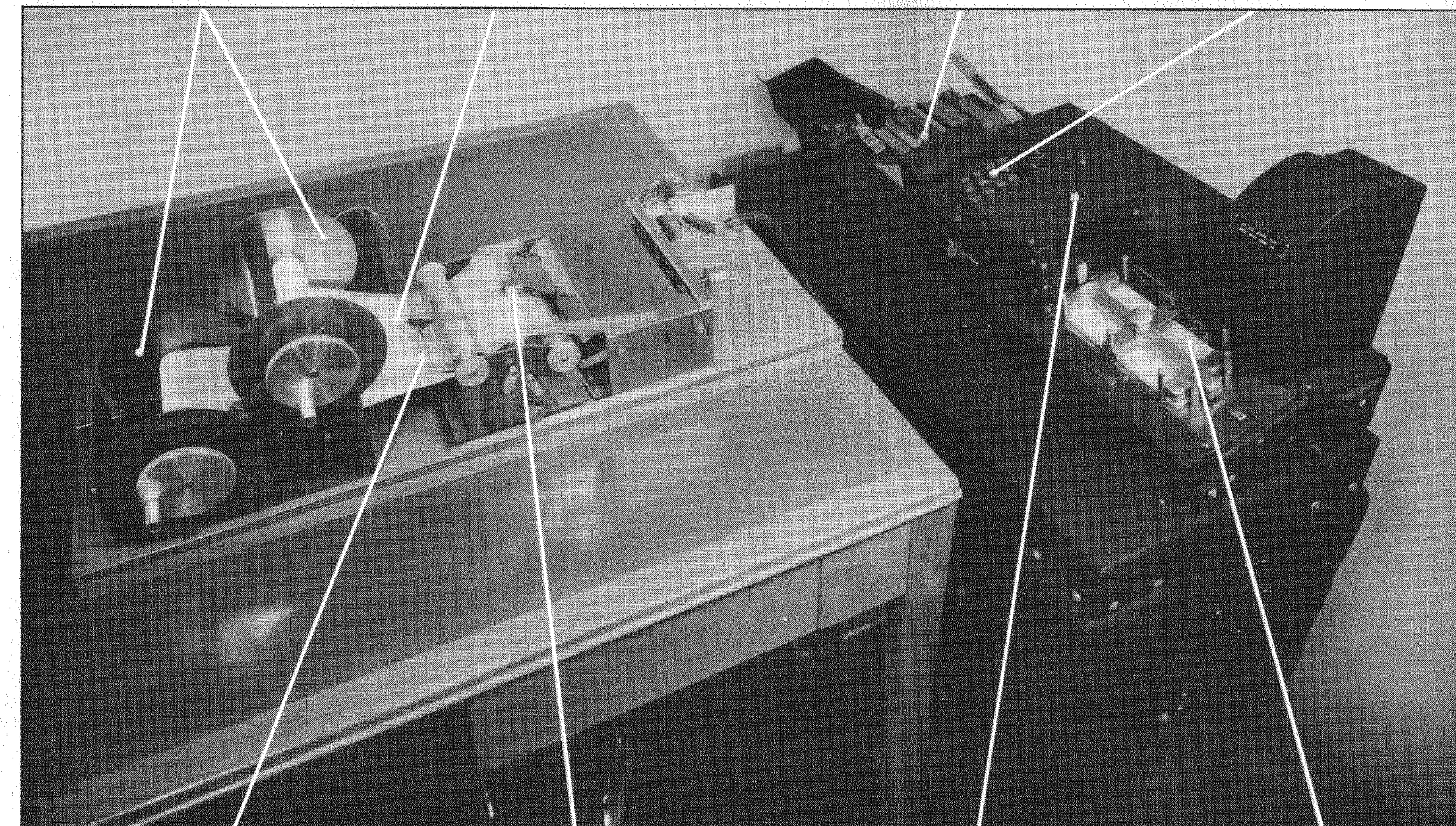

FIG.1. MECHANICAL READER WITH PROGRAMME BOARD PUNCH

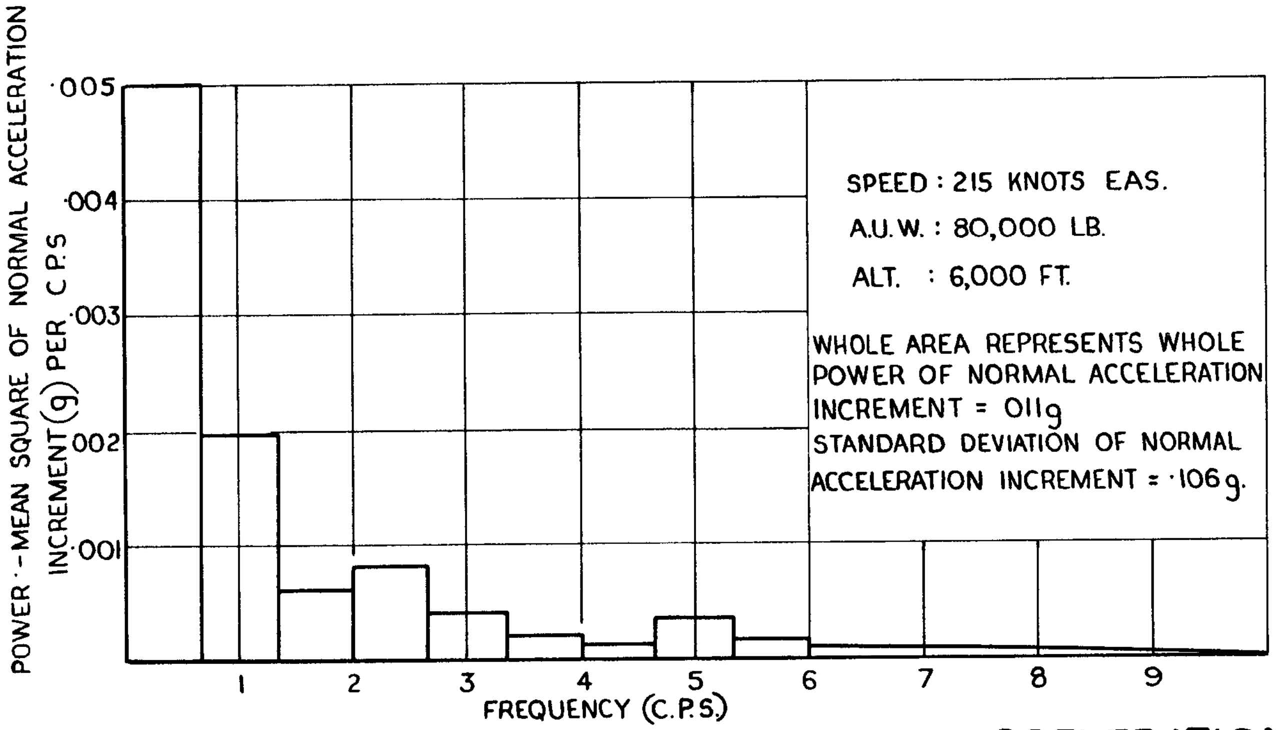

FIG. 2. POWER SPECTRUM OF NORMAL ACCELERATION NEAR AIRCRAFT C.G. (LOW ALTITUDE.)

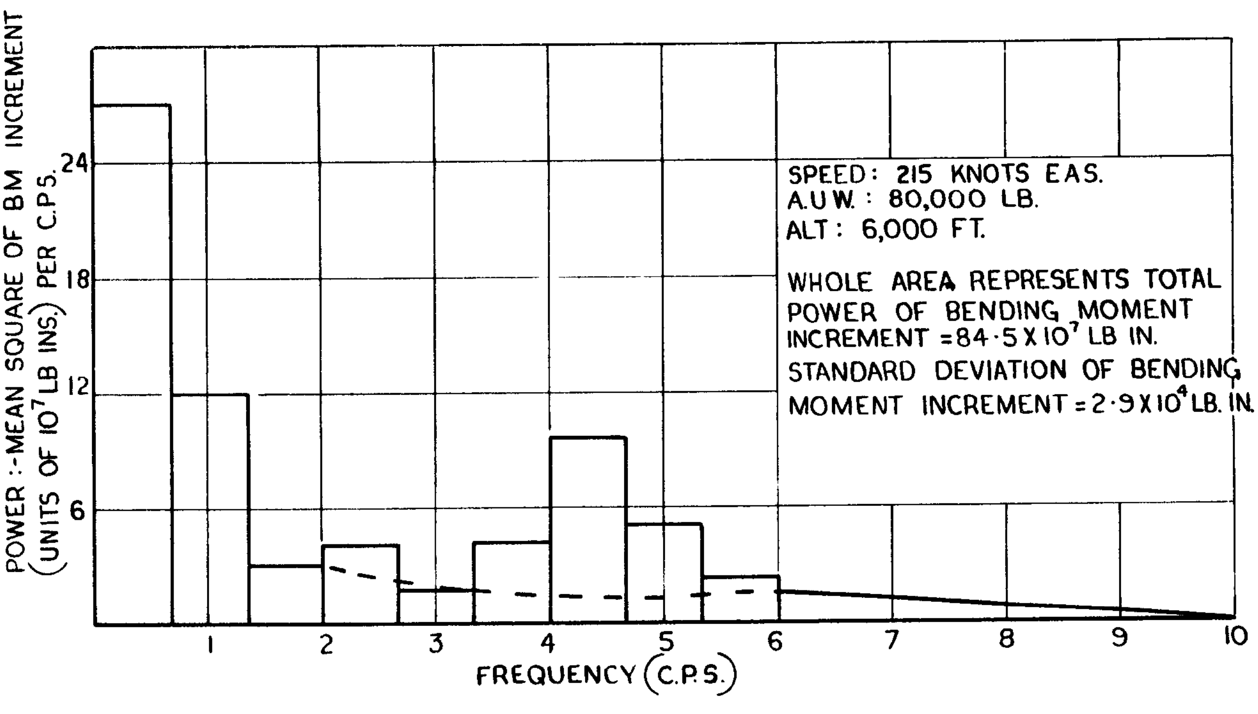

FIG. 3. POWER SPECTRUM OF BENDING MOMENT AT TAILPLANE ROOT (LOW ALTITUDE.)

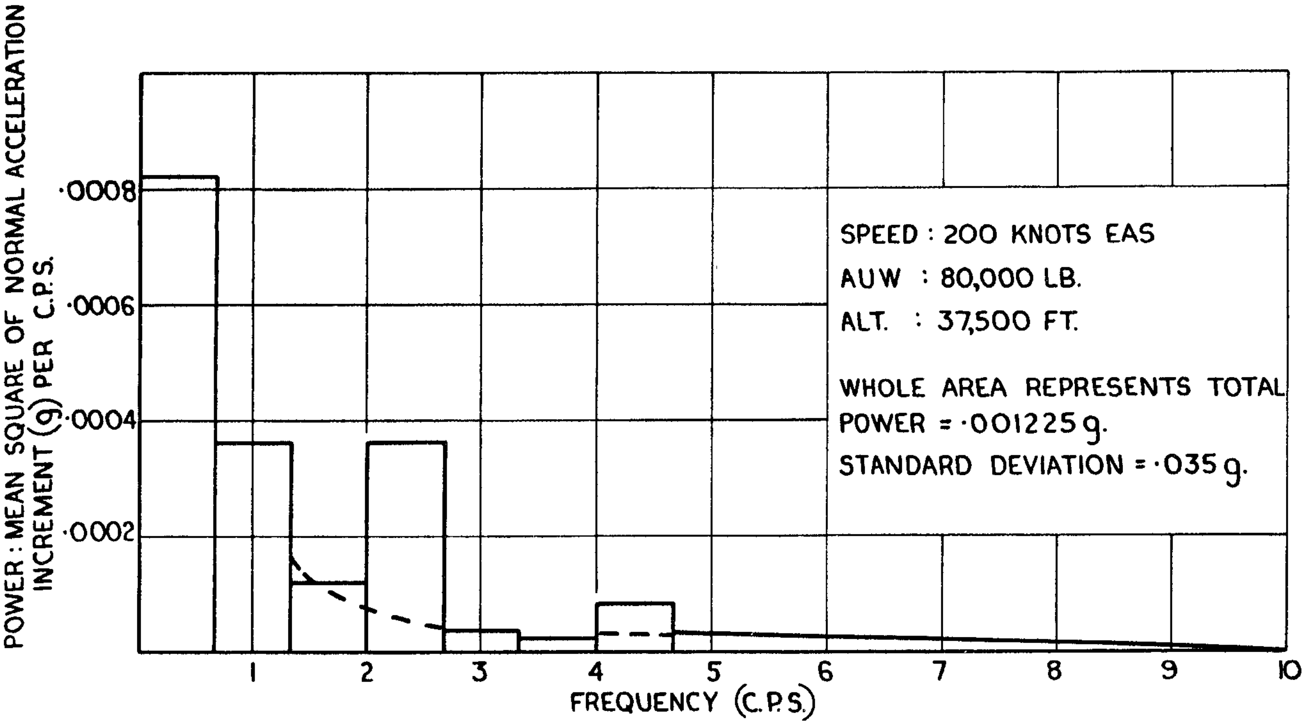

FIG. 4. POWER SPECTRUM OF NORMAL ACCELERATION IN FRONT FUSELAGE (HIGH ALTITUDE)

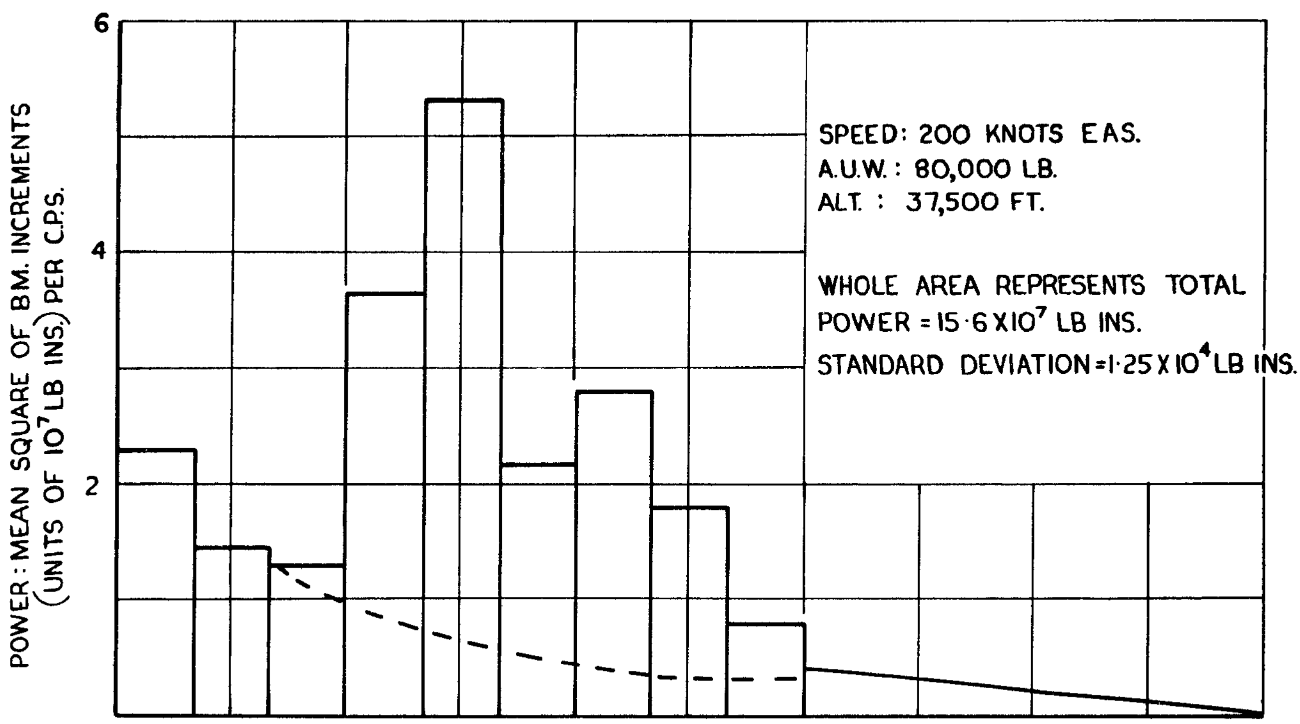

FIG. 5. POWER SPECTRUM OF BENDING MOMENT AT TAILPLANE ROOT (HIGH ALTITUDE)

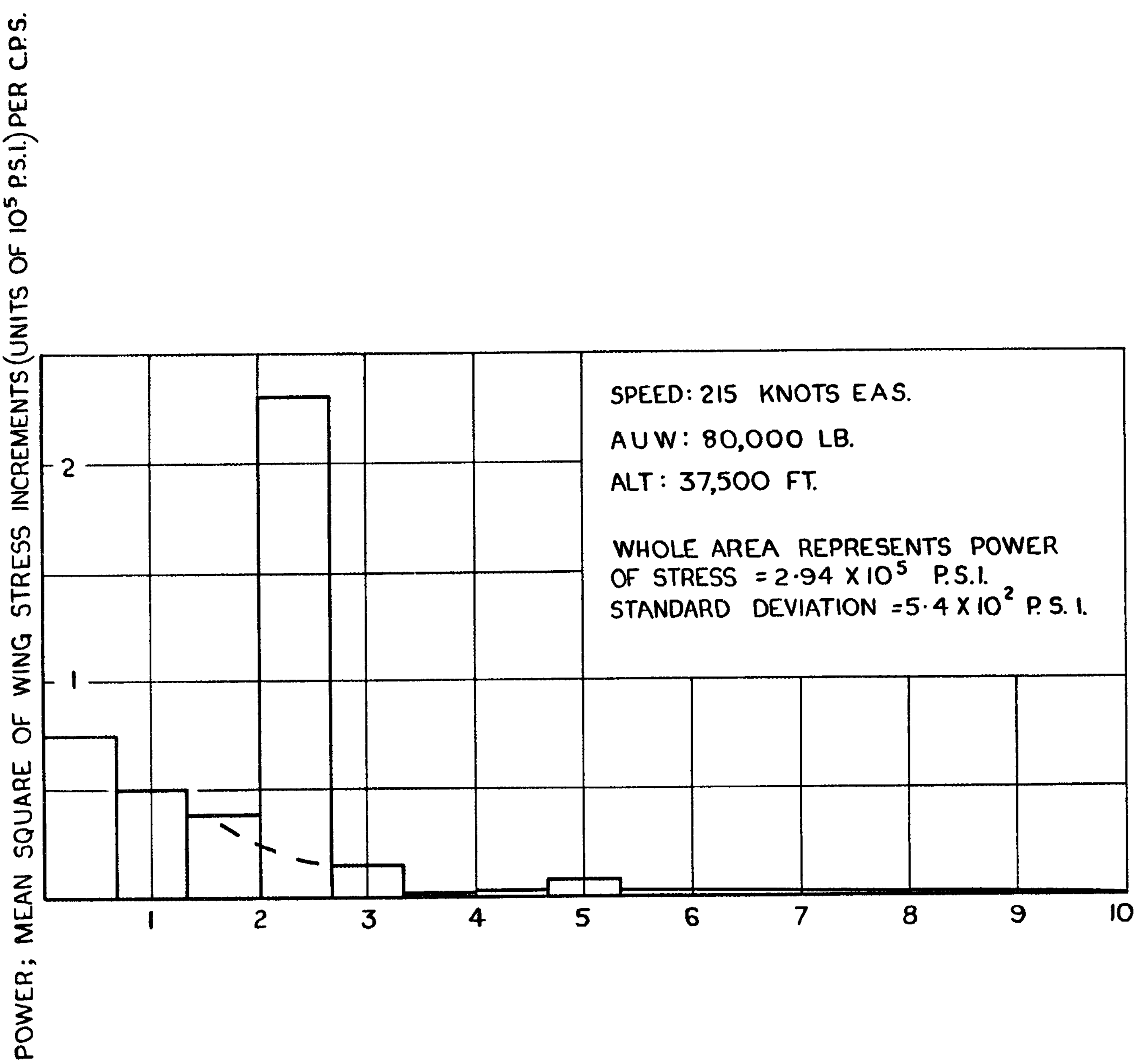

FIG. 6. POWER SPECTRUM OF WING SPAR STRESS (HIGH ALTITUDE.)

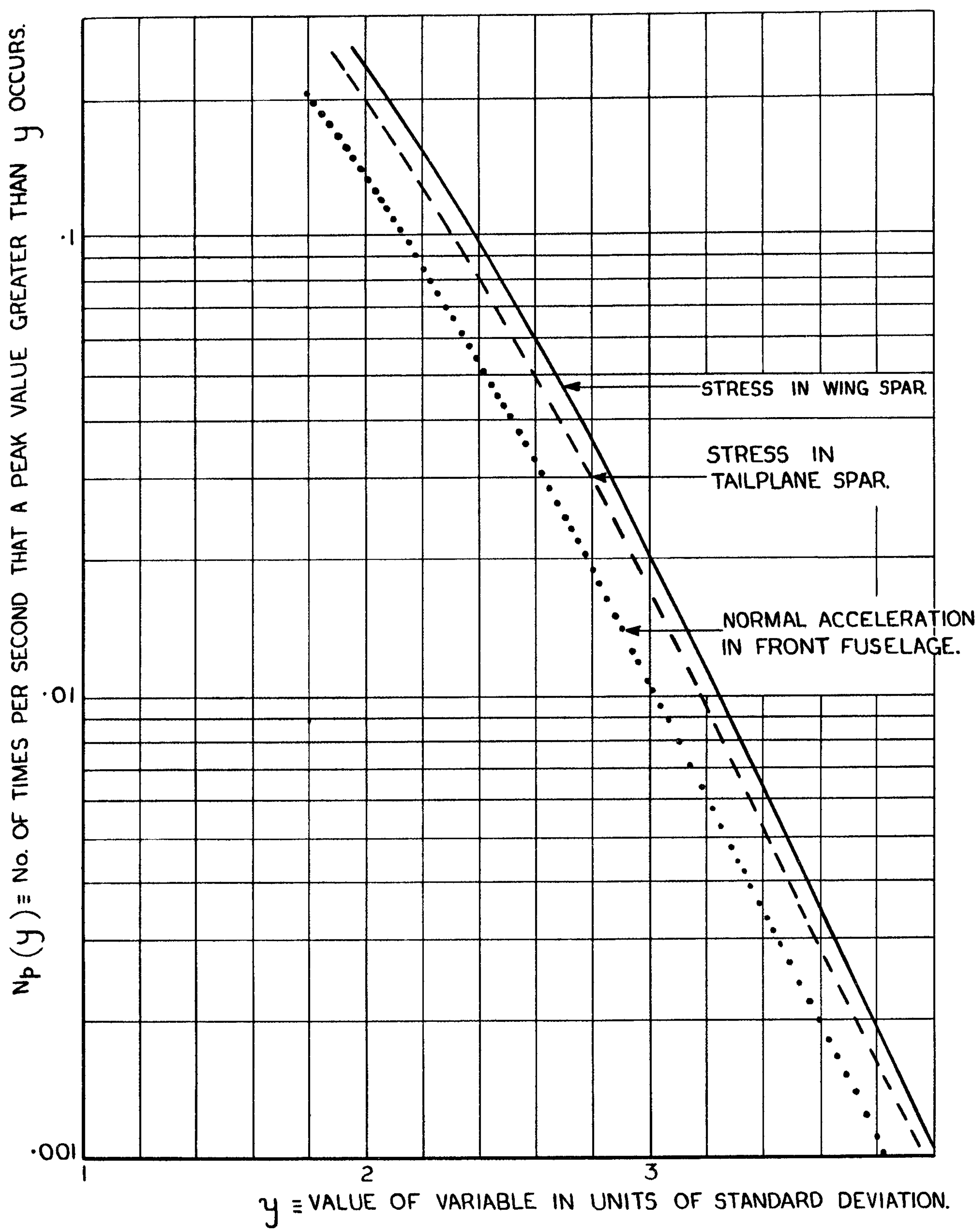

FIG. 7. RATE OF OCCURRENCE OF PEAK VALUES.

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