C.P. No. 415
(20,554)
A.R.C. Technical Report

C.P. No. 415
(20,554)
A.R.C. Technical Report

MINISTRY OF SUPPLY

AERONAUTICAL RESEARCH COUNCIL

CURRENT PAPERS

Notes on some Simple Strain Gauge Networks commonly used with Wind Tunnel Balances

by

J. R. Anderson

LONDON: HER MAJESTY'S STATIONERY OFFICE

1959

PRICE 2s. 6d. NET

C.P. No. 415

U.D.C. No. 533.6.071.3 : 531.71.082.73

Technical Note No. Aero 2581

August, 1958

ROYAL AIRCRAFT ESTABLISHMENT

NOTES ON SOME SIMPLE STRAIN GAUGE NETWORKS COMMONLY USED WITH WIND TUNNEL BALANCES

by

J. R. Anderson

SUMMARY

Wheatstone bridge networks of four and of eight resistance strain gauges are considered from the point of view of the errors which may arise from the assumption of a linear relation between applied strain and bridge output. The effects of mismatch of initial resistance and gauge factor are also examined.

LIST OF CONTENTS

LIST OF ILLUSTRATIONS

1 INTRODUCTION

The object of this short Technical Note is to collect together and record a few simple results obtained from the analysis of some Wheatstone bridge networks of resistance strain gauges in common use. The main emphasis is placed on the magnitude of errors which may arise from the assumption that the bridge output is linear with applied strain. The effects of mismatch in resistance and gauge factor between the strain gauges forming a bridge are also examined.

2 BASIC ASSUMPTIONS AND DEFINITIONS

The strain gauge networks considered are of Wheatstone bridge type, although it is immaterial if all the arms are active or not. Supply voltage and out-of-balance indicator are assumed to have no electrical interaction with the state of resistive unbalance resulting from strains applied to the strain gauges, and the networks are considered purely resistive. This enables the voltage unbalance due to strain to be taken as linear with the applied voltage, and to be obtained by the simple consideration of changes in resistance of the gauge elements.

The resistance parameters of interest are defined as follows.

R is the nominal, unstrained resistance of the strain gauge

δ_1 is the initial fractional deviation from nominal of strain gauge No.1, so that its total unstrained resistance is $R(1 + \delta_1)$.

Δ_1 is the fractional resistance change of strain gauge No.1 due to mechanical or electrical strain

so that

$$\begin{aligned} R_1 &= R(1+\delta_1)(1+\Delta_1) \\ R_2 &= R(1+\delta_2)(1+\Delta_2), \text{ etc.} \end{aligned} \qquad (1)$$

We have further to define a gauge factor, k, (assumed negative) such that the change in resistance due to mechanical strain is equal to k times the change in strain, or

$$\begin{aligned} \Delta_1 &= k_1\, e_1 \\ \Delta_2 &= k_2\, e_2 \text{ , etc.,} \end{aligned} \qquad (2)$$

where e_1, e_2, etc. are the mechanical strains applied to strain gauges Nos.1,2, etc.

In normal practice, δ will be limited to about ± 0.005 ohms per ohm, i.e. about $\frac{1}{2}$ per cent, by considerations of linearity in the indicating equipment, which will not be examined here. The value of the mechanical strain, e, using steel balances, is normally limited to about ± 0.001 inches per inch, so that Δ will not exceed about ± 0.002 ohms per ohm, assuming k to have a nominal value of 2, which is roughly appropriate to the strain gauges most commonly used. In later sections, the consideration of possible errors will be restricted to these maximum variations in the main parameters.

3 ANALYSIS OF THE SIMPLE FOUR GAUGE BRIDGE

The network considered is depicted in Fig.1 and consists of four resistances arranged in a conventional Wheatstone bridge: any or all of the resistances may represent strain gauges, although we shall assume here that all do, the most general case. The results for particular cases can be obtained by the simple expedient of equating to zero in the general result the Δ's corresponding to fixed resistances.

A voltage, V, is applied across the corners AC, causing in general a small voltage, dV, to appear across the corners BD, as shown. The solution to the problem is taken to be the variation in the ratio dV/V corresponding to the electrical strains imposed on the strain gauges.

Now clearly, in writing down the equations, we need consider initially only one pair of adjacent arms, such as $R_1\, R_2$, since those for the other pair, $R_3\, R_4$, will follow by changing the subscripts appropriately. Such pairs of gauges connected across the voltage supply are colloquially referred to as 'half-bridges'.

Assuming, then, that each resistance represents an active strain gauge, the value of the voltage at point B, say, referred to the point C is given by

$$\begin{aligned} V_{BC}/V &= R_2/(R_1+R_2) \\ &= R(1+\delta_2)(1+\Delta_2)/\{R(1+\delta_1)(1+\Delta_1)+R(1+\delta_2)(1+\Delta_2)\} \\ &= (1+\delta_2)(1+\Delta_2)/\{(1+\delta_1)(1+\Delta_1)+(1+\delta_2)(1+\Delta_2)\} \end{aligned}$$

in the strained condition of the gauges. The value in the initial, unstrained condition is obtained by putting $\Delta_1 = \Delta_2 = 0$. Thus the change in V_{BC}/V due to strain in the half-bridge is

$$dV_{BC}/V = (1+\delta_2)(1+\Delta_2)/\{(1+\delta_1)(1+\Delta_1)+(1+\delta_2)(1+\Delta_2)\} - (1+\delta_2)/\{(1+\delta_1)+(1+\delta_2)\} \qquad (3)$$

$$= \tfrac{1}{4}\cdot(\Delta_2-\Delta_1)(1+\delta_1+\delta_2+\delta_1\delta_2)\{1+\tfrac{1}{2}\cdot(\delta_1+\delta_2)\}^{-1}\{1+\tfrac{1}{2}\cdot(\delta_1+\delta_2)+\tfrac{1}{2}\cdot\Delta_1(1+\delta_1)+\tfrac{1}{2}\cdot\Delta_2(1+\delta_2)\}^{-1}.$$

Use of the binomial theorem enables this expression to be expanded, and simplified to sufficiently high orders of δ and Δ as

$$dV_{BC}/V = \tfrac{1}{4}\cdot(\Delta_2-\Delta_1)\{1-\tfrac{1}{2}\cdot(\Delta_1+\Delta_2)+\tfrac{1}{4}\cdot(\Delta_1+\Delta_2)^2-\tfrac{1}{4}\cdot(\delta_1-\delta_2)^2+\tfrac{1}{4}\cdot(\Delta_2-\Delta_1)(\delta_1-\delta_2)\} \qquad (4)$$

This result shows among other things that the initial resistive unbalance of the bridge represented by the δ's is restricted to third (and higher) order terms. In fact, in equation (4) above its influence is confined to the terms

$$-\tfrac{1}{4}\cdot(\delta_1-\delta_2)^2+\tfrac{1}{4}\cdot(\Delta_2-\Delta_1)(\delta_1-\delta_2) \;=\; E_1 \text{ (say).}$$

The largest possible value of E_1 under the restricted values of δ and Δ laid down in paragraph 2 above is ± 0.000035 ohms per ohm compared with unity, the approximate value of the terms inside the curly brackets in equation (4). It thus represents a maximum error of ± 0.0035 per cent in the value of dV_{BC}/V, which is negligibly small in all practical applications. The effect of initial resistive unbalance may therefore be ignored, leading to the result, obtained from equation (3)

$$\begin{aligned} dV_{BC}/V &= (1+\Delta_2)/(2+\Delta_1+\Delta_2)-\tfrac{1}{2} \\ &= \tfrac{1}{4}\cdot(\Delta_2-\Delta_1)\{1+\tfrac{1}{2}\cdot(\Delta_1+\Delta_2)\}^{-1} \doteqdot \tfrac{1}{4}\cdot(\Delta_2-\Delta_1)\{1-\tfrac{1}{2}\cdot(\Delta_1+\Delta_2)\}\;. \end{aligned}$$

It is seen that for the classical case where Δ_1 and Δ_2 are equal in magnitude but opposite in sign

$$dV_{BC}/V \;=\; \tfrac{1}{4}\cdot(\Delta_2-\Delta_1) \;=\; \tfrac{1}{2}\cdot\Delta \;.$$

In all other cases, the linear relation

$$dV_{BC}/V \;=\; \tfrac{1}{4}\cdot(\Delta_2-\Delta_1)$$

may be taken as an approximate solution, with an error arising only from ignoring $\tfrac{1}{2}\cdot(\Delta_1+\Delta_2)$ compared with unity. The maximum value of $\tfrac{1}{2}\cdot(\Delta_1+\Delta_2)$ under the restrictions imposed in paragraph 2 is ± 0.002 ohms per ohm, which implies that assumption of the linear relationship for dV_{BC}/V will not introduce errors greater than ± 0.2 per cent in this voltage ratio.

Note that this error arises from the non-linearity of dV_{BC}/V, which can most easily be seen by putting $\Delta_2 = \Delta$ and $\Delta_1 = a\Delta$, a being a constant in the range $-1 < a < 1$. Then

$$\begin{aligned} dV_{BC}/V &= \tfrac{1}{4}\cdot(\Delta_2-\Delta_1)\{1-\tfrac{1}{2}\cdot(\Delta_1+\Delta_2)\} \\ &= \tfrac{1}{4}\cdot\Delta(1-a)\{1-\tfrac{1}{2}\cdot\Delta(1+a)\}\;. \end{aligned}$$

The error in the voltage ratio incurred by assuming the linear relation is greatest as $a \to 1$ (and the voltage ratio $\to 0$), but the error expressed as a percentage of total output is always less than 0.2 per cent for a finite output, and can be seen to be actually $^1/10 . (1 + a)$ per cent.

The solution for the complete bridge may be written down as

$$
\begin{aligned}
dV/V &= dV_{BC}/V - dV_{DC}/V \\
&= \tfrac{1}{4} . (\Delta_2 - \Delta_1)\{1 + \tfrac{1}{2} . (\Delta_1 + \Delta_2)\}^{-1} - \tfrac{1}{4} . (\Delta_4 - \Delta_3)\{1 + \tfrac{1}{2} . (\Delta_3 + \Delta_4)\}^{-1} \\
&\doteqdot -\tfrac{1}{4} . (\Delta_1 - \Delta_2 - \Delta_3 + \Delta_4)
\end{aligned}
$$

where use of the linear approximation incurs errors of not more than about ±0.2 per cent in dV/V, provided the outputs of the individual half-bridges are of opposite sign, so that their difference is greater than either. Such an arrangement is sometimes called an 'additive' bridge. In the case of an additive bridge with each arm sustaining strains of the same magnitude, note that zero error is incurred.

The alternative arrangement, where the output of the bridge as a whole is less than one or other of its component half-bridges is sometimes called a 'difference' bridge, and appreciable percentage errors may be incurred by the assumption of linearity of output, having regard to the decreased output itself.

These errors arise under conditions where the separate outputs from the half-bridges tend to balance out, whereas the errors involved in the linear assumption tend to accumulate. This can be demonstrated easily to be the case where two strain gauges in opposite arms of the bridge undergo strains which are small and comparable in magnitude, whereas the remaining two gauges suffer large strains, comparable in magnitude but opposite in sign. Such a case is represented by

$$
\begin{aligned}
\Delta_1 &= \Delta \\
\Delta_2 &= -a\Delta \\
\Delta_3 &= -b\Delta \\
\Delta_4 &= -(1 + c)\Delta
\end{aligned}
$$

where a, b and c represent small quantities compared with unity.

The linear output $dV/V = -\tfrac{1}{4} (\Delta_1 - \Delta_2 - \Delta_3 + \Delta_4)$

$$= -\tfrac{1}{4} . \Delta(a + b + c)$$

in the present case, and is small compared with $-\tfrac{1}{4} . \Delta$.

The error in the linear assumption can easily be shown to be

$$-\tfrac{1}{8}(\Delta_1^{\,2}-\Delta_2^{\,2}-\Delta_3^{\,2}+\Delta_4^{\,2}) \; .$$

In our special case this reduces to $-\tfrac{1}{8}.\ \Delta^2(2+2c-a^2-b^2+c^2)$, which may be approximated as $-\tfrac{1}{4}\,.\,\Delta^2$.

It is now obvious that when $(a+b+c)$ is comparable in magnitude with Δ, the error is of the same order as the output, and that if $(a+b+c)$ further approaches zero, the linear output will correspondingly decrease further, whereas the error will remain virtually constant. The maximum value of this error is about -1×10^{-6} in our range of interest, and it follows therefore that the standard of ± 0.2 per cent error which applies to more usual strain gauge arrangements can be assumed only when the output of this special type of difference bridge is $\pm 500 \times 10^{-6}$ or greater, i.e. $a+b+c$ is of order plus or minus unity. Fortunately, in practice the arrangement likely to give the largest errors is seldom encountered, since the gauges of any half-bridge are normally strained more or less equally positive and negative.

In conclusion, then, the additive bridge may be used with confidence that the linear approximation will not introduce errors of more than about ± 0.2 per cent in bridge output, whereas the difference bridge must be placed in a different category, and some care exercised in assuming the validity of the linear relation.

4 BRIDGES OF EIGHT STRAIN GAUGES

These Wheatstone bridge arrangements are similar to those for four gauges, but employ the strain gauges connected either in series (Fig.2) or in parallel (Fig.3) in each of the four arms instead of a single gauge. The analysis proceeds in both cases in a similar manner to that employed in the preceding paragraph, namely by expansion of the relation for voltage ratio to sufficiently high orders of δ and Δ, using the binomial theorem. The manipulation of terms, however, is considerably more tedious.

4.1 Series arrangement (Fig.2)

The result for the left-hand half-bridge is

$$dV_{BC}/V = -\tfrac{1}{8}.(\Delta_1-\Delta_2+\Delta_5-\Delta_6)\{1-\tfrac{1}{4}.(\Delta_1+\Delta_2+\Delta_5+\Delta_6)\}$$

$$-\ {}^1\!/_{16}\,.\,(\Delta_1-\Delta_5)(\delta_1-\delta_5)+{}^1\!/_{16}(\Delta_2-\Delta_6)(\delta_2-\delta_6). \qquad (5)$$

The higher order terms in this equation may be considered in two parts. Firstly, the last two terms

$$-\ {}^1\!/_{16}.(\Delta_1-\Delta_5)(\delta_1-\delta_5)+{}^1\!/_{16}.(\Delta_2-\Delta_6)(\delta_2-\delta_6) \qquad (6)$$

which are the only ones involving initial resistance mismatching, may amount in magnitude to as much as $\pm 5 \times 10^{-6}$ under conditions where the output would be expected to be zero, namely, when $\Delta_1+\Delta_5 = 0 = \Delta_2+\Delta_6$. This figure is derived within the limitations on δ and Δ of paragraph 2 above, and compares with a maximum output from the half-bridge of $\pm 1000 \times 10^{-6}$. The error incurred by ignoring these terms is thus significant, and may become very large in some applications. The significance of the terms may be severely reduced, however, by matching the initial resistances of the strain gauges, that is, by making $(\delta_1 - \delta_5)$ and $(\delta_2 - \delta_6)$ sufficiently near zero.

An alternative approach suggests itself, in that the last two terms of equation (5) may be made very small by making $\Delta_1 - \Delta_5 \doteq 0 \doteq \Delta_2 - \Delta_6$. This implies that the pairs of strain gauges in the same arms of the bridge should undergo almost equal electrical strain. This is a little difficult to arrange in practice due to variations between the gauge factors of the individual gauges, and is impossible where the main use of the eight gauge network arises from the desire to eliminate interacting strains, which effectively ensure that this condition cannot be satisfied. (It should also be noted that in situations where the condition can be satisfied for all four arms of the bridge, there would appear to be advantages, and certainly no disadvantages, in using a simple four gauge network). This latter approach would appear, therefore, to have little practical application, and the last two terms in equation (5) may be ignored only if attention is paid to matching the initial resistances of the strain gauges in the same arms of the bridge.

In the second place, the inherent non-linearity of the network with strain is contained in the factor

$$\{1 - \tfrac{1}{4} \cdot (\Delta_1 + \Delta_2 + \Delta_5 + \Delta_6)\} \;.$$

Under the restrictions on Δ which we have assumed, this factor departs from the value unity by up to ± 0.002. Thus errors of up to ± 0.2 per cent in dV_{BC}/V may be incurred from this quarter in assuming the linear relation

$$dV_{BC}/V \;=\; -\tfrac{1}{8} \cdot (\Delta_1 - \Delta_2 + \Delta_5 - \Delta_6).$$

We may now write down the approximate solution to the complete network as

$$\begin{aligned} dV/V &= dV_{BC}/V - dV_{DC}/V \\ &= -\tfrac{1}{8} \cdot (\Delta_1 - \Delta_2 + \Delta_5 - \Delta_6 - \Delta_3 + \Delta_4 - \Delta_7 + \Delta_8) \end{aligned}$$

to an accuracy of about ± 0.2 per cent in dV/V, provided the strain gauge resistances in each arm are matched, i.e. $\delta_1 - \delta_5$, $\delta_2 - \delta_6$, $\delta_3 - \delta_7$, $\delta_4 - \delta_8$ are all approximately zero, (note that this is an extra restriction on initial resistive balance compared with the simple arrangement of four gauges), and provided the half-bridges are connected as an additive bridge, i.e. the bridge is arranged such that the total output is greater than either of the individual half-bridges. If this latter condition is not satisfied and a difference bridge arrangement is used, the errors arising from the assumption of linearity may be found to be greater than ± 0.2 per cent in output, the general arguement following the lines of that detailed for the four gauge bridge (Section 3).

4.2 Parallel arrangement (Fig.3)

Once more, the result for the left-hand half-bridge is

$$dV_{BC}/V = -\tfrac{1}{8}\cdot(\Delta_1 - \Delta_2 + \Delta_5 - \Delta_6)\{1 + \tfrac{1}{4}\cdot(\Delta_1 + \Delta_2 + \Delta_5 + \Delta_6)\}$$
$$+ \tfrac{1}{8}\cdot(\Delta_1^2 - \Delta_2^2 + \Delta_5^2 - \Delta_6^2)$$
$$+ \tfrac{1}{16}\cdot(\Delta_1 - \Delta_5)(\delta_1 - \delta_5) - \tfrac{1}{16}\cdot(\Delta_2 - \Delta_6)(\delta_2 - \delta_6). \qquad (6)$$

There is considerable similarity between this result and that for the series arrangement, equation (5). In particular, the last two terms of each are identical except for sign, and the discussion of the contribution of those of equation (6) is the same as for those of equation (5), and need not be repeated here, except for the outcome, which is that the initial resistances of strain gauges in the same arms should be matched to make their contribution to the voltage ratio negligible.

The inherent non-linearity of the network with strain is again obvious in the factor

$$\{1 + \tfrac{1}{4}\cdot(\Delta_1 + \Delta_2 + \Delta_5 + \Delta_6)\}$$

which is the same as for the series arrangement save for a sign change. The pervious arguments therefore apply, and we may assume that errors of up to ±0.2 per cent in dV_{BC}/V may be incurred from this quarter in assuming a linear relationship

$$dV_{BC}/V = -\tfrac{1}{8}\cdot(\Delta_1 - \Delta_2 + \Delta_5 - \Delta_6).$$

There is now left, however, in equation (6), a further term

$$\tfrac{1}{8}\cdot(\Delta_1^2 - \Delta_2^2 + \Delta_5^2 - \Delta_6^2)$$

which expresses a further non-linearity in the network's output. The magnitude of this term is never greater than 1×10^{-6} compared with a maximum output from a complete half-bridge of $\pm 1000 \times 10^{-6}$, and only reaches this value for the case where one resistance in each arm is passive. In the practical case, where the strains in each gauge in the same arm are of the same order, although not necessarily equal in magnitude, the total contribution from this term can be assumed negligible with confidence. The occurrence of this term does, however, suggest that the series arrangement might well be preferred to the parallel, all other considerations apart.

The approximate solution to the complete network of eight gauges may now be written as

$$dV/V = dV_{BC}/V - dV_{DC}/V$$
$$= -\tfrac{1}{8}(\Delta_1 - \Delta_2 + \Delta_5 - \Delta_6 - \Delta_3 + \Delta_4 - \Delta_7 + \Delta_8),$$

which is identical with the result for the series arrangement, to an accuracy of about ±0.2 per cent in dV/V, provided that the strain gauge resistances in each arm are matched i.e. $\delta_1 - \delta_5$, $\delta_2 - \delta_6$, $\delta_3 - \delta_7$, $\delta_4 - \delta_8$ are all approximately zero, and provided that the half-bridges are connected together so as to form an additive bridge. If difference bridge connections are made, under some circumstances the error incurred by the linear assumption may rise considerably above ±0.2 per cent, for similar reasons to those which are discussed in detail in connection with the four gauge bridge (Section 3).

5 THE EFFECT OF UNMATCHED GAUGE FACTORS

The electrical strain, Δ, is related to the mechanical strain, e, by the equation

$$\Delta = k \cdot e$$

where k is the gauge factor, assumed negative in this case. Now k depends not only on the strain sensitivity and mechanical condition of the resistance wire used in the strain gauge but also on temperature and, in an imperfectly understood manner, on the materials of the gauge backing, the adhesive employed and even upon the method of application of the strain gauge to the host specimen. It is therefore appropriate that the effect of any mismatch of gauge factors between the gauges in a bridge arrangement should be examined briefly. Let us take the case of a simple four gauge network.

The linear relation is

$$dV/V = -\tfrac{1}{4} \cdot (\Delta_1 - \Delta_2 - \Delta_3 + \Delta_4)$$

$$= -\tfrac{1}{4} \cdot (k_1 e_1 - k_2 e_2 - k_3 e_3 + k_4 e_4) \, .$$

Now let $k_1 = k(1 + a_1)$ etc., so that the a's are a measure of gauge factor mismatching, then

$$dV/V = -\tfrac{1}{4} \cdot \{k(1+a_1) e_1 - k(1+a_2) e_2 - k(1+a_3) e_3 + k(1+a_4) e_4\}$$

$$= -\tfrac{1}{4} \cdot k(e_1 - e_2 - e_3 + e_4)\{1 + (a_1 e_1 - a_2 e_2 - a_3 e_3 + a_4 e_4)/(e_1 - e_2 - e_3 + e_4)\} \, .$$

This demonstrates that even when mechanical strains are applied perfectly, gauge factor variations may produce, in the limit, a markedly non-linear calibration.

In the case of an 'additive' bridge with equally strained arms, where $e_1 = -e_2 = -e_3 = e_4$, the effect is zero, since

$$dV/V = -\tfrac{1}{4} \cdot k(4e_1)\{1 + e_1(a_1 + a_2 + a_3 + a_4)/4e_1\}$$

$$= -k \cdot e_1 \quad \text{if k is now defined as the mean.}$$

Non-linear effects may obviously be minimised by the approximate matching of gauge factors. This, however, is a process which would appear to have little practical possibility. It is evident, then, that in designing a strain gauge balance, it is useless to aim for performance or interactions of better than about ±1 per cent, which is the tolerance on gauge factor usually quoted by the manufacturer of the resistance strain gauges.

6 NON-CANCELLATION OF INTERFERING STRAINS DUE TO UNMATCHED GAUGE FACTORS

It is frequently found in the design of strain gauge balances that strains due to the designed structural load occur only in the presence of strains arising from other loads which are not required to contribute to the output of the bridge. It is convenient to arrange the strain gauges on the balance structure and in the Wheatstone bridge in such a manner that the interfering strains are effectively cancelled, and the required strains effectively summed, so far as the bridge output is concerned. Taking the simple four gauge bridge as an example,

$$dV/V = -\tfrac{1}{4}\cdot(\Delta_1 - \Delta_2 - \Delta_3 + \Delta_4)$$

$$= -\tfrac{1}{4}\cdot(k_1\, e_1 - k_2\, e_2 - k_3\, e_3 + k_4\, e_4)$$

where the symbols are as defined in paragraph 5.

Now suppose that the strains e_1, etc., are compounded of a strain e_m required to contribute to the measurement and an interfering strain e_c required to be cancelled. Then if

$$e_1 = p_1\, e_m + q_1\, e_c$$
$$e_2 = p_2\, e_m + q_2\, e_c$$
$$e_3 = p_3\, e_m + q_3\, e_c$$
$$e_4 = p_4\, e_m + q_4\, e_c$$

where the p's and q's are constants which may have any values within the range ±1, we have

$$dV/V = -\tfrac{1}{4}\cdot\{e_m(k_1\, p_1 - k_2 p_2 - k_3\, p_3 + k_4\, p_4) + e_c(k_1\, q_1 - k_2\, q_2 - k_3\, q_3 + k_4\, q_4)\}\,.$$

Putting as before $k_1 = k(1 + a_1)$ etc., where k is the mean gauge factor (i.e. $a_1 + a_2 + a_3 + a_4 = 0$).

$$dV/V = -\tfrac{1}{4}\, k\{e_m(p_1 - p_2 - p_3 + p_4) + e_m(p_1\, a_1 - p_2\, a_2 - p_3\, a_3 + p_4\, a_4) + e_c(q_1 - q_2 - q_3 + q_4) + e_c(q_1\, a_1 - q_2\, a_2 - q_3\, a_3 + q_4\, a_4)\}$$

The condition that the interfering strains should cancel to the first order, is now seen to be

$$q_1 - q_2 - q_3 + q_4 = 0$$

and the interaction then remaining can be expressed as a percentage of bridge output,

$$I\% = 100 \times (e_c/e_m)\,(q_1\,a_1 - q_2\,a_2 - q_3\,a_3 + q_4\,a_4)/\{(p_1 - p_2 - p_3 + p_4) + (p_1\,a_1 - p_2\,a_2 - p_3\,a_3 + p_4\,a_4)\}.$$

Maximum signal (dV/V) and minimum interaction are obtained from the network if $p_1 = -p_2 = -p_3 = p_4 = 1$, thus defining an 'additive' bridge with equally strained arms. The interaction may then be written

$$I\% = 100 \times (e_c/e_m)\,.\,\tfrac{1}{4}\,.\,(q_1\,a_1 - q_2\,a_2 - q_3\,a_3 + q_4\,a_4).$$

In the worst case, where gauge factors may differ by ±1 per cent, the interaction becomes

$$I\% = \pm\tfrac{1}{4}\,.\,(e_c/e_m)(q_1 + q_2 + q_3 + q_4).$$

The factors q are required to satisfy two conditions; that shown above for first order cancellation of interference, and the restriction of value to ±1. In the majority of practical cases, these conditions are satisfied by

$$q_1 = q_2 = q_3 = q_4 = 1,$$

and then the interference may be expressed as

$$I\% = \pm\, e_c/e_m\,.$$

Good strain gauge balance designs aim at keeping the ratio e_c/e_m to a maximum of unity, but cases do arise in practice where this is difficult, and the value may be as high as 10 or 15. It must be borne in mind, then, that unfavourable combinations of gauge factor variations may penalise the balance design with interactions of equivalent percentage.

WT.2078.C.P.415.K3 - Printed in Great Britain

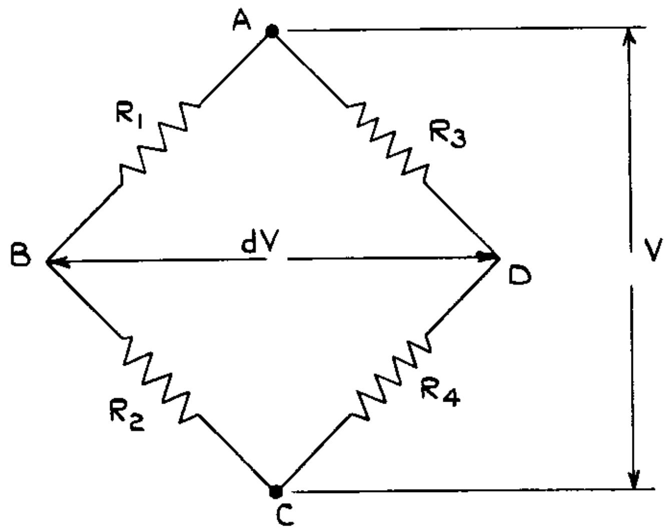

FIG. 1. FOUR-GAUGE WHEATSTONE BRIDGE.

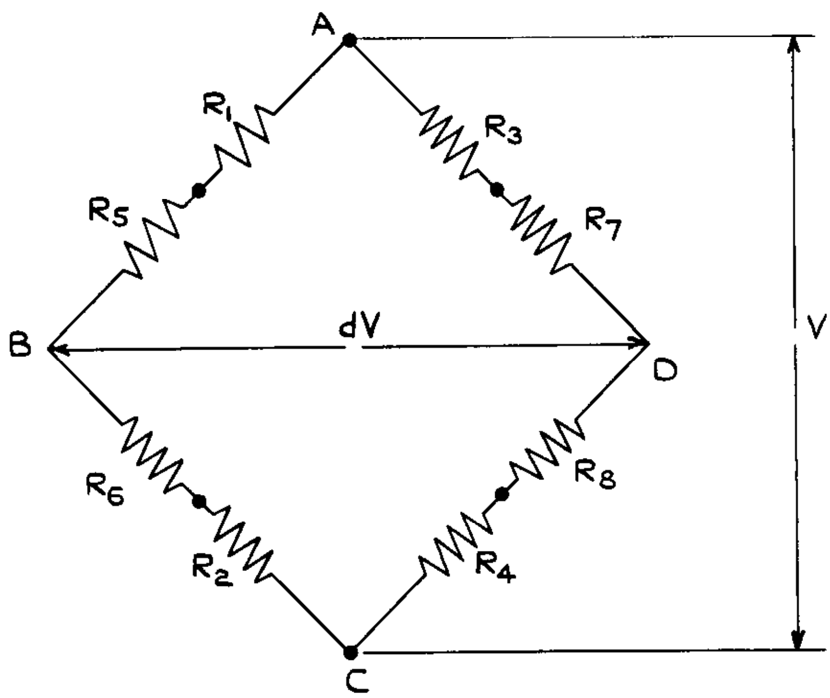

FIG. 2. EIGHT-GAUGE BRIDGE
(SERIES ARRANGEMENT)

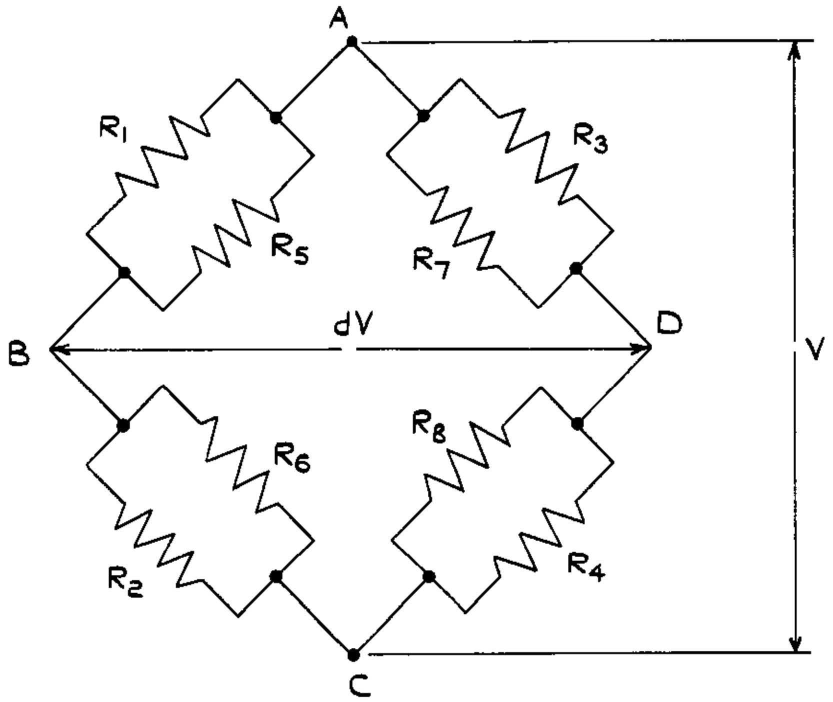

FIG. 3. EIGHT - GAUGE BRIDGE
(PARALLEL ARRANGEMENT)

C.P. No. 415
(20,554)
A.R.C. Technical Report

Published by
HER MAJESTY'S STATIONERY OFFICE

To be purchased from
York House, Kingsway, London w.c 2
423 Oxford Street, London w 1
13A Castle Street, Edinburgh 2
109 St. Mary Street, Cardiff
39 King Street, Manchester 2
Tower Lane, Bristol 1
2 Edmund Street, Birmingham 3
80 Chichester Street, Belfast
or through any bookseller

Printed in Great Britain

S.O. Code No. 23-9011-15

C.P. No. 415