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Calculated Aerodynamic Forces on a Sweptback Untapered Wing Oscillating in Incompressible Flow

by

D. L. Woodcock, M.A.

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Calculated Aerodynamic forces on a sweptback untapered wing oscillating in incompressible flow

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SUMMARY

The airforces are given for an untapered wing of aspect ratio 2 and sweepback 40° oscillating with symmetric distortion modes in incompressible flow. The results are for frequency parameters of 0(0.6)1.8. These are presented both as influence matrices based on the displacement at a particular set of points and also as overall derivatives for modes of the form $|\eta|^n$ for $n = 0(1)4$ where η is a non-dimensional spanwise co-ordinate. The method used was the vortex lattice method. The standard lattice was modified by making the mesh size (in a chordwise direction) adjacent to the leading edge smaller than that over the main part of the wing. Comparison is made with some values obtained by a variant of the Multhopp method and the agreement is good. An appendix shows how the vortex lattice method is modified to obtain the virtual inertias.

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1 Introduction

Theoretical values of the airforces on a particular oscillating wing were required for comparison with experimental values being measured at the Royal Aircraft Establishment. Details of the wing being used for the experiments are

Aspect ratio	- 1.995
Taper	- none
Sweepback	- 40°

For convenience in the calculations a slightly different wing was considered having aspect ratio 2 and sweepback $\tan^{-1} 5/6 (= 39^\circ 48')$. The effect of the differences should be negligible.

It was decided that the vortex lattice method[1,2,3,4] should be used and opportunity was taken of testing whether any improvement in accuracy over that obtained with a lattice of constant mesh size would be obtained by using a lattice whose mesh size (in a chordwise direction) along the leading edge of the wing was smaller than that over the main part of the wing. It was felt that this arrangement would give a more accurate representation of the doublet distribution* since the rate of change at the leading edge of the strength of the doublet distribution in a direction normal to the leading edge is infinite.

The method[10] being used for the experimental determination of the airforces is one which gives only the differences between the airforces at some airspeed and the airforces when the wing is oscillating with the same frequency in still air. To be able to compare the experimental and theoretical values it is necessary, therefore, to obtain theoretically the airforces for the latter case. These are usually known as the virtual inertia forces. Some modification of the vortex lattice method is required to make it suitable for the determination of these forces. This is described in the Appendix. It is hoped to give numerical results in a later paper.

2 Method

The method used is based on that devised by Jones[1] and applied by Lehrian[3,4]. It is substantially the same as that described in reference 2; the only important differences being the different lattice (Fig.1) and the fact that the downwash velocities due to the semi-infinite doublet sheets were obtained from tables[5,6] rather than from the series expressions given in reference 2. For the case of zero airspeed further slight modifications are necessary and these are described in the Appendix.

The theory will not be repeated here, but we shall only give details of the methods of presentation of the results. The notation is the same as in reference 2. Fig.1 shows the wing planform, the superposed lattice and the position of the collocation points. It will be noted that a mesh size smaller than that used over the main part of the wing has been used a ong those boundaries at which the rate of change of the discontinuity in the velocity potential ($\ell K V e^{i\omega t}$) in a direction normal to the boundary is infinite.

Two methods of presentation are used for the results. One is an influence matrix G based on the displacement at the collocation points. The other is as matrices $[\ell_z]$ etc. of the overall derivatives for a set of modes of the form $|\eta|^i$.

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* The wing is mathematically equivalent to a layer of doublets of strength equal to the discontinuity in the velocity potential across the plane of the wing.

2.1 Influence Matrix G

If $\ell Z_{ij} e^{i\omega t}$ is the downward displacement at the collocation point (ij) then the virtual work done by the resulting pressure distribution in small displacements $\ell \delta Z_{ij} e^{i\omega t}$ is (for one wing) $\rho s^2 c V^2 S e^{2i\omega t}$ where*

$$S = - \delta \bar{Z}' G \bar{Z} \qquad (1)$$

where $\bar{Z}$ is a column matrix of the displacements Z_{ij} i.e.

$$Z = \{Z_i\} \qquad (2)$$

where

$$Z_i = \{Z_{ij}\} \qquad (3)$$

and $\delta\bar{Z}'$ is the transpose of $\delta\bar{Z}$. G is independent of the mode of oscillation of the wing.

In flutter calculations it is usual to write the displacements in terms of the displacements in a set of assumed distortion modes. That is with $\bar{k}$ distortion modes we write

$$\bar{Z} = P \tilde{q} \qquad (4)$$

where

$$\tilde{q} = \{q_k\} \qquad (5)$$

is the column matrix of the coordinates of the degrees of freedom q_k and P is a rectangular matrix whose terms are functions of the modes of the degrees of freedom. We then have

$$S = - \delta \tilde{q}' . P' G P \tilde{q} \quad . \qquad (6)$$

The matrix $P'GP$ is thus a matrix of generalized aerodynamic forces and will give immediately the aerodynamic coefficients for a flutter calculation with these degrees of freedom. If it is desired to obtain the aerodynamic forces with the virtual inertias excluded (e.g. for comparison with experimentally determined values) the matrix G should be replaced by

$$G + \nu^2 G_\infty \qquad (7)$$

where

$$S_\infty = - \delta \bar{Z}' . G_\infty \bar{Z} \quad . \qquad (8)$$

2.2 Overall Derivatives

The matrices of the overall derivatives are defined as follows. If the displacement of any point on the wing is

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* For the special case of zero airspeed we write the virtual work instead as $-\rho s^2 c^3 \omega^2 S_\infty e^{2i\omega t}$

$$z = \sum_{i=0} c|\eta|^i \psi_i - \tfrac{1}{2}c \cos\theta \sum_{r=0} |\eta|^r \chi_r \tag{9}$$

then the virtual work coefficient S is given by

$$\frac{s}{c} S = -[\delta\psi', \delta\chi'] \left\{ \begin{bmatrix} [\ell_z] & [\ell_\alpha] \\ [-m_z] & [-m_\alpha] \end{bmatrix} + i\nu \begin{bmatrix} [\ell_{\dot z}] & [\ell_{\dot\alpha}] \\ [-m_{\dot z}] & [-m_{\dot\alpha}] \end{bmatrix} \right\} \begin{bmatrix} \psi \\ \chi \end{bmatrix} \quad \ldots\ldots (10)$$

where

$$\psi = \{\psi_i\} \tag{11}$$

and

$$\chi = \{\chi_r\} . \tag{12}$$

Thus the overall derivatives are elements of a matrix based this time not on the displacement at a particular set of points but on the displacement in a particular set of modes. For the case of zero airspeed we similarly define

$$\frac{s}{c} S_\infty = -[\delta\psi', \delta\chi'] \begin{bmatrix} [\ell_{\ddot z}] & [\ell_{\ddot\alpha}] \\ [-m_{\ddot z}] & [-m_{\ddot\alpha}] \end{bmatrix} \begin{bmatrix} \psi \\ \chi \end{bmatrix} . \tag{13}$$

A particular element of, for example, $[\ell_\alpha]$ is written as $[\ell_\alpha]_{ij}$ and is the real part of the coefficient of $-\delta\psi_i \cdot \chi_j$ in $\frac{s}{c} S$.

3 Results and conclusions

The influence matrices G were determined for frequency parameters $\nu = 0(0.6)1.8$. These are given in Table X. Overall derivatives $[\ell_z]_{ij}$ etc. were obtained from these matrices for modes of the form $|\eta|^n$ with $n = 0(1)4$. These are given in Tables I to VIII as matrices $[\ell_z] = \left[[\ell_z]_{ij}\right]$ etc. In Table IX these values, for modes with $n = 0(2)4$ are compared with values which Minhinnick has obtained by his variant of the Multhopp method*. It will be seen that the agreement is quite good, the differences being nearly always less than 10%.

The generalised aerodynamic forces for any set of modes, which can be expressed with reasonable accuracy in terms of displacements at the nine collocation points, can be obtained from the influence matrix G by means of equation (6). Alternatively these forces can be obtained from the matrices of the overall derivatives by a similar matrix multiplication (see reference 9).

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* Minhinnick's method and results have not yet been issued. The original extension of Multhopp's method to unsteady flow is described in reference 8.

Two further tables are appended. Table XI gives the chordwise loading factors appropriate to the chordwise distribution of doublet layers used in this work; and Table XII gives values of the generalised aerodynamic force for pitch about two spanwise axes. These axes, distant 0.1953 c and 0.9614 c aft of the apex, are to be used in the experimental determination of the derivatives for this wing.

As a check on the calculations a comparison was made with the steady motion values of Falkner[7]. Values of $\frac{dC_L}{d\alpha}$ and the position of the aerodynamic centre were determined from the present results for zero frequency parameter. These are

$$\frac{dC_L}{d\alpha} = 2.352$$

Aerodynamic centre 0.586 c aft of apex

which compares with the following values interpolated from Falkner's[7] results

$$\frac{dC_L}{d\alpha} = 2.357$$

Aerodynamic centre 0.601 c aft of apex .

Falkner's values were obtained, using a 21 × 6 lattice with equal chordwise spacing, the standard solution being modified by an auxiliary solution to reduce the errors caused by the discontinuity at the centre section. The unmodified values are

$$\frac{dC_L}{d\alpha} = 2.411$$

Aerodynamic centre of 0.589 c aft of apex.

From these figures one can conclude that a 21 × 4 lattice with smaller mesh size at the leading edge gives at least as accurate a result as a 21 × 6 lattice with constant chordwise spacing. The amount of work required increases with the size of the lattice and it does therefore appear that the use of a lattice of type used in this report is definitely advantageous at least for desk machine work. For calculations on a digital computer it would be best to retain the lattice of constant mesh size as then simplicity is much more important.

It is hoped to publish values of the virtual inertia derivatives and influence matrix G_∞ at a later date. These will be obtained by the method described in the Appendix.

LIST OF SYMBOLS

b = $\text{Diag}\left\{\text{diag}\left\{\frac{\ell(\eta_i)}{d}\right\}\right\}$ (see reference 2)

c wing chord

d semi-width of doublet strip

G airforce influence matrix (see section 2.1)

G_∞ airforce influence matrix for zero airspeed (see section 2.1)

LIST OF SYMBOLS (Contd)

ℓ	wing semi-chord
$[\ell_z]_{ij}$	etc. overall derivatives (see section 2.2)
$[\ell_z]$	etc. matrices of overall derivatives (see section 2.2)
L_m	$= [L_{mp}]$
L_{mp}	chordwise loading factors
P_i, Q_j	interpolation functions (see reference 2)
s	wing semi-span
S	virtual work coefficient (see section 2.1)
S_∞	virtual work coefficient for zero airspeed (see section 2.1)
T_n	$= \eta^{2n-2}\sqrt{1-\eta^2}$ for symmetric modes, $\eta^{2n-1}\sqrt{1-\eta^2}$ for antisymmetric modes
V	airspeed
η	non-dimensional spanwise coordinate ($s\eta$ = spanwise distance from centre line of wing)
θ	non-dimensional chordwise coordinate ($- \ell \cos \theta$ = distance aft of mid-chord)
ν	$\frac{\omega c}{V}$
ρ	air density
ω	circular frequency

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2	D.L. Woodcock D.E. Williams	Calculation of unsteady airforces for a comprehensive set of planforms and modes using a high speed computing machine. Part I Method R.A.E. Report to be issued
3	D.E. Lehrian	Calculation of stability derivatives for oscillating wings R. & M. 2922. February 1953
4	D.E. Lehrian	Calculation of flutter derivatives for wings of general planform R. & M. 2961. January 1954

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No.	Author	Title, etc.
5	Staff of Math. Div. N.P.L.	Tables of complete downwash due to a rectangular vortex R & M 2461 July 1947
6	Staff of Math. Div. N.P.L.	Downwash tables for the calculation of aerodynamic forces on oscillating wings N.P.L. Aero 229 July 1952
7	V.M. Falkner	Calculated loadings due to incidence of a number of straight and swept back wings R. & M. 2596. June 1948
8	H.C. Garner	Multhopp's subsonic lifting surface theory of wings in slow pitching oscillations R. & M. 2885. July 1952
9	D.L. Woodcock	Aerodynamic derivatives for two cropped delta wings and one arrowhead wing oscillating in distortion modes Current Paper No. 268 April 1956
10	P.R. Guyett D.E.G. Poulter	Measurement of pitching moment derivatives for a series of rectangular wings at low wind speeds Current Paper No. 249 June 1955

Attached: Appendix
Table I-XII
Figure 1

APPENDIX

Determination of virtual inertias by the vortex lattice method

It can easily be shown that a thin wing oscillating in still air is mathematically equivalent to a layer of doublets of strength $k(= i\ell s\omega K e^{i\omega t})$ where k is the discontinuity in the velocity potential across the plane of the wing. Since K will be zero beyond the wing boundaries we can take

$$K = \sum_{m=1} \sum_{n=1} T_n C_{mn} K_m \quad \text{on the wing}$$

$$= 0 \qquad \text{outside the wing} \tag{A1}$$

where

$$\left.\begin{aligned} K_1 &= \sin\theta \\ K_2 &= \sin\theta + \tfrac{1}{2}\sin 2\theta \\ K_m &= \frac{1}{m}\sin m\,\theta - \frac{1}{m-2}\sin(m-2)\theta \;\; (m>2) \end{aligned}\right\} \tag{A2}$$

and Tn are the usual spanwise loading functions. This is the same form for K as we have used to obtain the forces at zero frequency parameter except that K_1 has been changed from $(\theta + \sin\theta)$ to $\sin\theta$. Thus the procedure to determine the C_{mn} will be similar but with a different set of chordwise loading factors L_{1p}. As before[2] the L_{1p} are given by the formulae

$$\left.\begin{aligned} \sum_{p=1}^{\bar{p}} L_{1p} &= K_1(\pi) \\ \sum_{p=1}^{\bar{p}} \frac{L_{1p}}{2\pi(\cos\theta_p - \cos\vartheta_{p'})} &= U_{1p'}, \quad p' = 1 \ldots (\bar{p}-1) \end{aligned}\right\} \tag{A3}$$

but in this case $K_1(\pi) = 0$ and $U_{1p'}$, the two dimensional downwash due to K_1 is given by

$$U_{1p'} = U_1(\theta_{p'}) = \frac{1}{2\pi}\int_0^\pi \frac{1}{\cos\theta - \cos\phi}\frac{\partial K_1}{\partial\theta}\,d\theta$$

$$= \frac{1}{2\pi}\int_0^\pi \frac{\cos\theta}{\cos\theta - \cos\phi}\,d\theta$$

$$= \frac{1}{2} \,. \tag{A4}$$

The downwash due to the assumed doublet distribution is

$$is\omega e^{i\omega t} \sum_{m=1} \sum_{n=1} C_{mn} W_{mn} \qquad (A5)$$

where W_{mn} is the downwash due to $\ell K_m T_n$. Thus the boundary condition to be satisfied is, with $\ell Z e^{i\omega t}$ the downward displacement,

$$is\omega e^{i\omega t} \sum_{m=1} \sum_{n=1} C_{mn} W_{mn} = \frac{d}{dt} (\ell Z e^{i\omega t}) = i\omega \ell Z e^{i\omega t} \qquad (A6)$$

i.e.

$$\sum_{m=1} \sum_{n=1} C_{mn} W_{mn} = \frac{\ell}{s} Z . \qquad (A7)$$

The pressure distribution will be

$$\rho \frac{\partial k}{\partial t} = - \rho s \ell \omega^2 e^{i\omega t} \sum_{n=1} \sum_{m=1} C_{mn} T_n K_m \qquad (A8)$$

and thus the virtual work in small displacements $\ell . \delta Z e^{i\omega t}$ will be (for one wing) $- \rho c_m^3 s^2 \omega^2 S_\infty e^{2i\omega t}$ where

$$S_\infty = - \left(\frac{s}{c_m}\right)^3 \int_0^1 \int_0^\pi K \left(\frac{\ell}{s}\right)^3 \sin\theta . \delta Z . d\theta \, d\eta . \qquad (A9)$$

Proceeding as in reference 2 we obtain

$$S_\infty = - \delta \bar{Z}' R \bar{W}^{-1} \left(\frac{d}{s}\right) b \bar{Z} \qquad (A10)$$

where

$$R = [I_{im} J] \qquad (A11)$$

$$J = [J_{jn}] \qquad (A12)$$

$$J_{jn} = \left(\frac{s}{c_m}\right)^3 \int_0^1 \left(\frac{\ell}{s}\right)^3 T_n . Q_j(\eta) . d\eta \qquad (A13)$$

and

$$I_{im} = \int_0^\pi K_m \sin\theta . P_i(\theta) \, d\theta . \qquad (A14)$$

The differences compared with the case of non-zero airspeed are

(i) The use of a different K_1 function and hence different factors I_1 .

(ii) Different expressions for the integrals I_{im} and J_{jn}, and hence R different.

(iii) Replacement of the term $(\bar{a}+\alpha b)$ in the expression for the virtual work coefficient by $\frac{d}{s} b$.

TABLE I

Values of $[\ell_z]$

$\nu = 0$ $[\ell_z] = 0$

$\nu = 0.6$ $[\ell_z] =$

-0.1209	-0.06039	-0.03824	-0.02690	-0.01977
-0.05342	-0.03534	-0.02567	-0.01935	-0.01484
-0.03254	-0.02438	-0.01890	-0.01483	-0.01170
-0.02308	-0.01795	-0.01439	-0.01160	-0.009348
-0.01732	-0.01359	-0.01113	-0.009153	-0.007487

$\nu = 1.2$ $[\ell_z] =$

-0.5895	-0.2801	-0.1734	-0.1212	-0.08899
-0.2613	-0.1587	-0.1120	-0.08370	-0.06403
-0.1591	-0.1082	-0.08137	-0.06324	-0.04974
-0.1126	-0.07939	-0.06171	-0.04921	-0.03946
-0.08428	-0.06004	-0.04765	-0.03871	-0.03148

$\nu = 1.8$ $[\ell_z] =$

-1.4814	-0.6857	-0.4198	-0.2928	-0.2152
-0.6583	-0.3825	-0.2661	-0.1983	-0.1517
-0.4009	-0.2594	-0.1921	-0.1487	-0.1168
-0.2833	-0.1902	-0.1454	-0.1153	-0.09229
-0.2118	-0.1438	-0.1122	-0.09058	-0.07344

TABLE II

Values of $[\ell_\alpha]$

$\nu = 0$ $[\ell_\alpha] =$

1.176	0.5014	0.2963	0.2059	0.1524
0.5159	0.2538	0.1652	0.1210	0.09236
0.3096	0.1638	0.1119	0.08420	0.06543
0.2162	0.1174	0.08186	0.06254	0.04911
0.1601	0.08762	0.06182	0.04768	0.03773

$\nu = 0.6$ $[\ell_\alpha] =$

1.133	0.4856	0.2891	0.2022	0.1504
0.4962	0.2466	0.1621	0.1196	0.09189
0.2974	0.1593	0.1099	0.08327	0.06503
0.2075	0.1141	0.08031	0.06171	0.04866
0.1535	0.08515	0.06057	0.04694	0.03724

$\nu = 1.2$ $[\ell_\alpha] =$

1.077	0.4613	0.2768	0.1957	0.1470
0.4701	0.2341	0.1559	0.1166	0.09056
0.2813	0.1516	0.1059	0.08128	0.06410
0.1962	0.1089	0.07759	0.06022	0.04784
0.1453	0.08150	0.05861	0.04577	0.03652

$\nu = 1.8$ $[\ell_\alpha] =$

1.027	0.4423	0.2687	0.1923	0.1459
0.4472	0.2238	0.1511	0.1146	0.09003
0.2671	0.1448	0.1025	0.07968	0.06345
0.1861	0.1041	0.07504	0.05884	0.04711
0.1377	0.07794	0.05663	0.04460	0.03579

TABLE III

Values of $[-m_z]$

$\nu = 0 \quad [-m_z] = 0$

$\nu = 0.6 \quad [-m_z] =$

-0.01568	-0.006542	-0.003658	-0.002387	-0.001666
-0.007271	-0.002962	-0.001659	-0.001102	-0.0007865
-0.004505	-0.001743	-0.0009675	-0.0006557	-0.0004812
-0.003187	-0.001166	-0.0006380	-0.0004406	-0.0003319
-0.002374	-0.0008255	-0.0004450	-0.0003124	-0.0002410

$\nu = 1.2 \quad [-m_z] =$

-0.03728	-0.01640	-0.009482	-0.006284	-0.004422
-0.01626	-0.006931	-0.004067	-0.002800	-0.002055
-0.009697	-0.003822	-0.002232	-0.001597	-0.001226
-0.006738	-0.002424	-0.001392	-0.001032	-0.0008253
-0.004972	-0.001639	-0.0009207	-0.0007050	-0.0005856

$\nu = 1.8 \quad [-m_z] =$

-0.05658	-0.02644	-0.01606	-0.01104	-0.007993
-0.02263	-0.01029	-0.006548	-0.004831	-0.003741
-0.01269	-0.005239	-0.003403	-0.002682	-0.002208
-0.008540	-0.003101	-0.002011	-0.001680	-0.001457
-0.006190	-0.001968	-0.001259	-0.001113	-0.001013

TABLE IV

Values of $[-m_\alpha]$

$\nu = 0 \quad [-m_\alpha] =$

-0.3291	-0.1354	-0.07595	-0.05018	-0.03556
-0.1544	-0.07581	-0.04793	-0.03389	-0.02508
-0.09682	-0.05195	-0.03504	-0.02588	-0.01976
-0.06913	-0.03851	-0.02694	-0.02047	-0.01598
-0.05184	-0.02941	-0.02107	-0.01634	-0.01295

$\nu = 0.6 \quad [-m_\alpha] =$

-0.3226	-0.1338	-0.07506	-0.04940	-0.03484
-0.1512	-0.07540	-0.04788	-0.03382	-0.02496
-0.09471	-0.05178	-0.03516	-0.02599	-0.01983
-0.06759	-0.03839	-0.02707	-0.02062	-0.01611
-0.05067	-0.02931	-0.02119	-0.01649	-0.01310

$\nu = 1.2 \quad [-m_\alpha] =$

-0.3297	-0.1388	-0.07804	-0.05105	-0.03569
-0.1539	-0.07883	-0.05041	-0.03551	-0.02608
-0.09624	-0.05425	-0.03719	-0.02752	-0.02097
-0.06869	-0.04021	-0.02869	-0.02194	-0.01716
-0.05153	-0.03068	-0.02248	-0.01760	-0.01402

$\nu = 1.8 \quad [-m_\alpha] =$

-0.3596	-0.1539	-0.08723	-0.05713	-0.03989
-0.1672	-0.08799	-0.05688	-0.04019	-0.02952
-0.1045	-0.06065	-0.04208	-0.031267	-0.02387
-0.07458	-0.04494	-0.03248	-0.02497	-0.01958
-0.05598	-0.03426	-0.02544	-0.02004	-0.01602

TABLE V

Values of $[\ell_z]$

$\nu = 0$ $[\ell_z] =$

1.176	0.5014	0.2963	0.2059	0.1524
0.5159	0.2538	0.1652	0.1210	0.09236
0.3096	0.1638	0.1119	0.08420	0.06543
0.2162	0.1174	0.08186	0.06254	0.04911
0.1601	0.08762	0.06182	0.04769	0.03773

$\nu = 0.6$ $[\ell_z] =$

1.1170	0.4801	0.2856	0.1993	0.1480
0.4892	0.2442	0.1605	0.1181	0.09049
0.2932	0.1579	0.1089	0.08237	0.06421
0.2045	0.1131	0.07967	0.06117	0.04817
0.1514	0.08441	0.06014	0.04660	0.03696

$\nu = 1.2$ $[\ell_z] =$

1.042	0.4486	0.2679	0.1877	0.1398
0.4549	0.2288	0.1516	0.1123	0.08640
0.2723	0.1485	0.1034	0.07877	0.06167
0.1900	0.1068	0.07604	0.05873	0.04644
0.1408	0.07997	0.05759	0.04489	0.03572

$\nu = 1.8$ $[\ell_z]$

0.9919	0.4273	0.2561	0.1803	0.1348
0.4323	0.2183	0.1455	0.1083	0.08375
0.2586	0.1420	0.09954	0.07619	0.05988
0.1804	0.1023	0.07331	0.05688	0.04511
0.1336	0.07672	0.05561	0.04350	0.03470

TABLE VI

Values of $[\ell_\alpha]$

$\nu = 0$ $[\ell_\alpha] =$

0.5480	0.2534	0.1483	0.09739	0.06740
0.2438	0.1491	0.1018	0.07291	0.05364
0.1491	0.1032	0.07604	0.05750	0.04413
0.1058	0.07621	0.05857	0.04594	0.03631
0.07937	0.05779	0.04569	0.03680	0.02971

$\nu = 0.6$ $[\ell_\alpha] =$

0.6402	0.2915	0.1713	0.1139	0.08012
0.2858	0.1664	0.1123	0.08060	0.05961
0.1747	0.1138	0.08244	0.06216	0.04773
0.1237	0.08366	0.06303	0.04914	0.03876
0.09265	0.06331	0.04897	0.03912	0.03146

$\nu = 1.2$ $[\ell_\alpha] =$

0.6942	0.3109	0.1827	0.1226	0.08721
0.3100	0.1749	0.1174	0.08479	0.06319
0.1894	0.1190	0.08563	0.06471	0.04988
0.1340	0.08743	0.06528	0.05086	0.04016
0.1002	0.06621	0.05065	0.04034	0.03241

$\nu = 1.8$ $[\ell_\alpha] =$

0.7289	0.3231	0.1903	0.1290	0.09271
0.3258	0.1802	0.1211	0.08805	0.06616
0.1990	0.1224	0.08785	0.06667	0.05165
0.1407	0.08985	0.06682	0.05213	0.04125
0.1051	0.06806	0.05177	0.04120	0.03311

TABLE VII

Values of $[-m_z]$

$\nu = 0$ $[-m_z] =$

-0.3291	-0.1354	-0.07595	-0.05018	-0.03556
-0.1544	-0.07581	-0.04793	-0.03389	-0.02508
-0.09682	-0.05196	-0.03505	-0.02588	-0.01977
-0.06913	-0.03851	-0.02694	-0.02047	-0.01598
-0.05184	-0.02941	-0.02107	-0.01634	-0.01295

$\nu = 0.6$ $[-m_z] =$

-0.3121	-0.1294	-0.07282	-0.04812	-0.03407
-0.1462	-0.07292	-0.04643	-0.03291	-0.02436
-0.09152	-0.05008	-0.03408	-0.02525	-0.01931
-0.06528	-0.03713	-0.02623	-0.02001	-0.01565
-0.04894	-0.02835	-0.02052	-0.01599	-0.01270

$\nu = 1.2$ $[-m_z] =$

-0.2942	-0.1236	-0.06990	-0.04615	-0.03258
-0.1371	-0.07004	-0.04501	-0.03195	-0.02363
-0.08566	-0.04818	-0.03314	-0.02463	-0.01885
-0.06108	-0.03571	-0.02552	-0.01955	-0.01532
-0.04579	-0.02725	-0.01997	-0.01564	-0.01246

$\nu = 1.8$ $[-m_z] =$

-0.2866	-0.01220	-0.06939	-0.04584	-0.03232
-0.1331	-0.06934	-0.04490	-0.03191	-0.02360
-0.08299	-0.04769	-0.03307	-0.02464	-0.01886
-0.05916	-0.03531	-0.02545	-0.01956	-0.01535
-0.04435	-0.02691	-0.01990	-0.01564	-0.01249

TABLE VIII

Values of $[-m_\alpha]$

$\nu = 0$ $[-m_\alpha] =$

0.1197	0.05662	0.03383	0.02258	0.01586
0.05425	0.02947	0.01938	0.01382	0.01022
0.03345	0.01917	0.01345	0.01016	0.007871
0.02378	0.01372	0.01004	0.007913	0.006351
0.01784	0.01021	0.007697	0.006269	0.005160

$\nu = 0.6$ $[-m_\alpha] =$

0.09565	0.04654	0.02807	0.01871	0.01308
0.04261	0.02458	0.01661	0.01198	0.008904
0.02603	0.01608	0.01168	0.008970	0.007017
0.01845	0.01151	0.008766	0.007046	0.005716
0.01383	0.008560	0.006739	0.005605	0.004667

$\nu = 1.2$ $[-m_\alpha] =$

0.08283	0.04115	0.02533	0.01720	0.01223
0.03617	0.02185	0.01525	0.01127	0.008533
0.02183	0.01434	0.01080	0.008480	0.006736
0.01540	0.01029	0.008122	0.006657	0.005466
0.01152	0.007657	0.006251	0.005288	0.004445

$\nu = 1.8$ $[-m_\alpha] =$

0.07647	0.03825	0.02401	0.01669	0.01213
0.03294	0.02033	0.01457	0.01102	0.008510
0.01971	0.01337	0.01033	0.008267	0.006657
0.01385	0.009610	0.007769	0.006456	0.005348
0.01033	0.007168	0.005975	0.005105	0.004314

TABLE IX

Comparative values of derivatives

Comparison of values by vortex-lattice and Multhopp-Minhinnick method. These results are for modes 1, η^2, η^4 and the values in brackets are the Multhopp-Minhinnick ones.

(i) $\nu = 0$

$$[\ell_z] = 0$$

$$[\ell_\alpha] = \begin{bmatrix} 1.176\ (1.161) & 0.2963\ (0.2820) & 0.1524\ (0.1398) \\ 0.3096\ (0.3064) & 0.1119\ (0.1072) & 0.06543\ (0.06140) \\ 0.1601\ (0.1559) & 0.06182\ (0.06245) & 0.03773\ (0.03838) \end{bmatrix}$$

$$[-m_z] = 0$$

$$[-m_\alpha] = \begin{bmatrix} -0.3291\ (-0.3091) & -0.07595\ (-0.07391) & -0.03556\ (-0.03657) \\ -0.09682\ (-0.09970) & -0.03504\ (-0.03570) & -0.01976\ (-0.02054) \\ -0.05184\ (-0.05367) & -0.02107\ (-0.02228) & -0.01295\ (-0.01384) \end{bmatrix}$$

$$[\ell_{\dot z}] = [\ell_\alpha]$$

$$[\ell_{\dot\alpha}] = \begin{bmatrix} 0.5480\ (0.6152) & 0.1483\ (0.1620) & 0.06740\ (0.08448) \\ 0.1491\ (0.1656) & 0.07604\ (0.08117) & 0.04413\ (0.05076) \\ 0.07937\ (0.08621) & 0.04569\ (0.05136) & 0.02971\ (0.03484) \end{bmatrix}$$

$$[-m_{\dot z}] = [-m_\alpha]$$

$$[-m_{\dot\alpha}] = \begin{bmatrix} 0.1197\ (0.1224) & 0.03383\ (0.03369) & 0.01586\ (0.01771) \\ 0.03345\ (0.03210) & 0.01345\ (0.01354) & 0.007871\ (0.008364) \\ 0.01784\ (0.01689) & 0.007697\ (0.007974) & 0.005160\ (0.005324) \end{bmatrix}$$

(ii) $\nu = 0.6$

$$[\ell_z] = \begin{bmatrix} -0.1209\ (-0.1353) & -0.03824\ (-0.04008) & -0.01977\ (-0.02144) \\ -0.03254\ (-0.03517) & -0.01890\ (-0.01936) & -0.01170\ (-0.01233) \\ -0.01732\ (-0.01807) & -0.01113\ (-0.01211) & -0.007487\ (-0.008358) \end{bmatrix}$$

$$[\ell_\alpha] = \begin{bmatrix} 1.133\ (1.116) & 0.2891\ (0.2731) & 0.1504\ (0.1357) \\ 0.2974\ (0.2936) & 0.1099\ (0.1044) & 0.06503\ (0.06008) \\ 0.1535\ (0.1492) & 0.06057\ (0.06087) & 0.03724\ (0.03761) \end{bmatrix}$$

$$[-m_z] = \begin{bmatrix} -0.01568\ (-0.01504) & -0.003658\ (-0.003155) & -0.001666\ (-0.001561) \\ -0.004505\ (-0.004210) & -0.0009675\ (-0.0009335) & -0.0004812\ (-0.0004860) \\ -0.002374\ (-0.002289) & -0.0004450\ (-0.000468) & -0.0002410\ (-0.0002430) \end{bmatrix}$$

$$[-m_\alpha] = \begin{bmatrix} -0.3226\ (-0.3068) & -0.07506\ (-0.07421) & -0.03484\ (-0.03690) \\ -0.09471\ (-0.09801) & -0.03516\ (-0.03600) & -0.01983\ (-0.02088) \\ -0.05067\ (-0.05268) & -0.02119\ (-0.02252) & -0.01310\ (-0.01412) \end{bmatrix}$$

$$[\ell_{\dot z}] = \begin{bmatrix} 1.1170\ (1.1064) & 0.2856\ (0.2711) & -0.1480\ (0.1347) \\ 0.2932\ (0.2906) & 0.1089\ (0.1040) & 0.06421\ (0.05990) \\ 0.1514\ (0.1476) & 0.06014\ (0.06074) & 0.03696\ (0.03757) \end{bmatrix}$$

$$[\ell_{\dot\alpha}] = \begin{bmatrix} 0.6402\ (0.6937) & 0.1713\ (0.1797) & 0.08012\ (0.09311) \\ 0.1747\ (0.1877) & 0.08244\ (0.08613) & 0.04773\ (0.05316) \\ 0.09265\ (0.09761) & 0.04897\ (0.05392) & 0.03146\ (0.03607) \end{bmatrix}$$

$$[-m_{\dot z}] = \begin{bmatrix} -0.3121\ (-0.2968) & -0.07282\ (-0.07179) & -0.03407\ (-0.03564) \\ -0.09152\ (-0.09491) & -0.03408\ (-0.03486) & -0.01931\ (-0.02018) \\ -0.04894\ (-0.05100) & -0.02052\ (-0.02182) & -0.01270\ (-0.01366) \end{bmatrix}$$

$$[-m_{\dot\alpha}] = \begin{bmatrix} 0.09565\ (0.09602) & 0.02807\ (0.02811) & 0.01308\ (0.01515) \\ 0.02603\ (0.02454) & 0.01168\ (0.01191) & 0.007017\ (0.007589) \\ 0.01383\ (0.01284) & 0.006739\ (0.007084) & 0.004667\ (0.004901) \end{bmatrix}$$

TABLE IX (Contd)

(iii) $\nu = 1.2$

$$[\ell_z] = \begin{bmatrix} -0.5895\ (-0.6384) & -0.1734\ (-0.1808) & -0.08899\ (-0.09545) \\ -0.1591\ (-0.1709) & -0.08137\ (-0.08409) & -0.04974\ (-0.05254) \\ -0.08428\ (-0.08808) & -0.04765\ (-0.05198) & -0.03148\ (-0.03516) \end{bmatrix}$$

$$[\ell_\alpha] = \begin{bmatrix} 1.077\ (1.035) & 0.2768\ (0.2600) & 0.1470\ (0.1303) \\ 0.2813\ (0.2727) & 0.1059\ (0.1001) & 0.06410\ (0.05813) \\ 0.1453\ (0.1386) & 0.05861\ (0.05836) & 0.03652\ (0.03638) \end{bmatrix}$$

$$[-m_z] = \begin{bmatrix} -0.03728\ (-0.04623) & -0.009482\ (-0.01022) & -0.004422\ (-0.005254) \\ -0.009697\ (-0.009484) & -0.002232\ (-0.002492) & -0.001226\ (-0.001472) \\ -0.004972\ (-0.004768) & -0.0009207\ (-0.001128) & -0.0005856\ (-0.0008905) \end{bmatrix}$$

$$[-m_\alpha] = \begin{bmatrix} -0.3297\ (-0.3155) & -0.07804\ (-0.07762) & -0.03569\ (-0.03918) \\ -0.09624\ (-0.1001) & -0.03719\ (-0.03848) & -0.02097\ (-0.02272) \\ -0.05153\ (-0.05350) & -0.02248\ (-0.02415) & -0.01402\ (-0.01543) \end{bmatrix}$$

$$[\ell_{\dot{z}}] = \begin{bmatrix} 1.042\ (1.023) & 0.2679\ (0.2563) & 0.1398\ (0.1285) \\ 0.2723\ (0.2691) & 0.1034\ (0.1001) & 0.06167\ (0.05820) \\ 0.1408\ (0.1370) & 0.05759\ (0.05869) & 0.03572\ (0.03664) \end{bmatrix}$$

$$[\ell_{\dot{\alpha}}] = \begin{bmatrix} 0.6942\ (0.7384) & 0.1827\ (0.1911) & 0.08721\ (0.09889) \\ 0.1894\ (0.2028) & 0.08563\ (0.08999) & 0.04988\ (0.05510) \\ 0.1002\ (0.1056) & 0.05065\ (0.05595) & 0.03241\ (0.03709) \end{bmatrix}$$

$$[-m_{\dot{z}}] = \begin{bmatrix} -0.2942\ (-0.2793) & -0.06990\ (-0.06837) & -0.03258\ (-0.03428) \\ -0.08566\ (-0.08964) & -0.03314\ (-0.03412) & -0.01885\ (-0.01997) \\ -0.04579\ (-0.04798) & -0.01997\ (-0.02146) & -0.01246\ (-0.01359) \end{bmatrix}$$

$$[-m_{\dot{\alpha}}] = \begin{bmatrix} 0.08283\ (0.09120) & 0.02533\ (0.02711) & 0.01223\ (0.01465) \\ 0.02183\ (0.02083) & 0.01080\ (0.01112) & 0.006736\ (0.007236) \\ 0.01152\ (0.01059) & 0.006251\ (0.006580) & 0.004445\ (0.004665) \end{bmatrix}$$

TABLE X

Values of Influence Matrix G

Collocation points at $\eta = 0.2,\ 0.6,\ 0.8$

$\cos\vartheta = \frac{1}{2},\ 0,\ -\frac{1}{2}$

$[G]_{\nu=0} =$

−0.50628	−0.013616	−0.053260	0.23490	−0.064972	0.023878	0.27138	0.078588	0.029382
0.049890	−0.25654	0.021264	−0.15545	0.11248	0.048299	0.10556	0.14407	−0.069563
−0.035116	0.15248	−0.47015	−0.18252	−0.21369	0.50158	0.21763	0.061211	−0.031430
0.70826	−0.043288	0.066529	−0.86493	0.10872	−0.076718	0.15667	−0.065437	0.010188
0.11774	−0.0031265	0.15384	−0.18652	0.25025	−0.34519	0.068780	−0.24712	0.19135
−0.025558	0.11223	0.40971	0.22312	−0.24155	−0.51569	−0.19756	0.12932	0.10598
0.15343	0.015404	0.022143	−0.54653	−0.069711	−0.075865	0.39310	0.054307	0.053722
−0.30890	0.87258	−0.52696	0.57353	−1.73176	0.96812	−0.26463	0.85918	−0.44117
0.18453	−0.40296	0.29800	−0.43310	0.77172	−0.67291	0.24857	−0.36876	0.37491

$[G]_{\nu=0.6} =$

−0.53113 −0.0056194i	−0.023190 −0.013204i	−0.051051 +0.0071868i	0.29832 +0.12673i	−0.038942 +0.045577i	0.027141 +0.0035333i	0.22990 −0.0090382i	0.063562 −0.019866i	0.024594 +0.00054187i
0.054123 −0.038842i	−0.29473 +0.078546i	0.042915 −0.049692i	−0.15788 +0.10678i	0.20051 −0.13116i	0.0015493 +0.10652i	0.10879 −0.061052i	0.087145 +0.11113i	−0.041394 −0.070610i
−0.054041 +0.0070310i	0.16561 −0.066013i	−0.48771 +0.049254i	−0.12146 +0.024692i	−0.24186 +0.13209i	0.53819 −0.033639i	0.17747 +0.0024928i	0.081045 −0.080118i	−0.056450 +0.049076i
0.72210 −0.096399i	−0.036449 +0.0054139i	0.064093 −0.011802i	−0.91165 +0.014140i	0.088649 −0.017009i	−0.077627 +0.0035391i	0.18496 +0.0025540i	−0.053759 +0.0091927i	0.012555 +0.00056551i
0.10099 +0.083300i	0.061429 −0.24542i	0.11464 +0.13566i	−0.14301 −0.21950i	0.074324 +0.48117i	−0.24892 −0.292228i	0.036834 +0.12905i	−0.13308 −0.26977i	0.13316 +0.16246i
−0.0031551 −0.048910i	0.079544 +0.15389i	0.43661 −0.17089i	0.14965 +0.071815i	−0.15247 −0.34111i	−0.59053 +0.25284i	−0.14868 −0.045314i	0.069436 +0.18917i	0.15375 −0.12632i
0.14270 +0.065692i	0.017438 −0.030594i	0.018789 +0.015567i	−0.51743 −0.18512i	−0.076984 +0.056690i	−0.064365 −0.034827i	0.36736 +0.15380i	0.058878 −0.020472i	0.045112 +0.023514i
−0.28826 −0.060596i	0.80421 +0.19286i	−0.48297 −0.12001i	0.51843 +0.20474i	−1.54997 −0.57974i	0.85558 +0.33892i	−0.22458 −0.13698i	0.73492 +0.38232i	−0.36953 −0.20544i
0.17011 +0.028475i	−0.37444 −0.081086i	0.27528 +0.097636i	−0.39111 −0.095108i	0.69765 +0.26060i	−0.61555 −0.28038i	0.21846 +0.074711i	−0.31724 −0.17357i	0.33289 +0.19352i

TABLE X (Contd)

$[G]_{\nu=1.2} =$

−0.57683 +0.0072541i	−0.029140 −0.012857i	−0.052473 +0.0090933i	0.37316 +0.17931i	−0.022125 +0.052274i	0.034959 +0.0073776i	0.18033 +0.027896i	0.053432 −0.016938i	0.017461 +0.0038923i
0.071543 −0.055209i	−0.37755 +0.12610i	0.89480 −0.076981i	−0.19290 +0.13898i	0.37739 −0.17824i	−0.99769 +0.14736i	0.13433 −0.071861i	−0.027192 +0.16373i	0.020342 −0.097478i
−0.084814 +0.018963i	0.20378 −0.099122i	−0.53351 +0.079726i	−0.035501 +0.019205i	−0.32352 +0.17411i	0.61369 −0.028818i	0.12060 +0.021237i	0.13472 −0.10266i	−0.10572 +0.075926i
0.74501 −0.20507i	−0.034967 +0.0022513i	0.064294 −0.020269i	−0.97564 +0.077172i	0.078038 −0.0091768i	−0.083157 +0.0069799i	0.21987 −0.025292i	−0.046993 +0.0037511i	0.016731 −0.00075916i
0.057125 +0.13072i	0.20020 −0.41568i	0.023155 +0.22001i	−0.020022 −0.32814i	−0.30159 +0.75544i	−0.010314 −0.44317i	−0.050327 +0.17993i	0.10766 −0.39761i	−0.014422 +0.23100i
0.037201 −0.089938i	0.0027693 +0.25194i	0.49792 −0.30712i	0.023248 +0.13270i	0.057081 −0.52796i	−0.75149 +0.42177i	−0.064822 −0.077997i	−0.068455 +0.27593i	0.25346 −0.20018i
0.11889 +0.13692i	0.025354 −0.049482i	0.010624 +0.024484i	−0.46396 −0.39443i	−0.10308 +0.079799i	−0.042422 −0.056946i	0.31101 +0.32188i	0.073103 −0.019278i	0.029133 +0.039958i
−0.23565 −0.088665i	0.65289 +0.30943i	−0.38647 −0.18842i	0.37251 +0.31032i	−1.15316 −0.94508i	0.60754 +0.53531i	−0.12009 −0.20174i	0.46186 +0.61619i	−0.21123 −0.31700i
0.14129 +0.048566i	−0.31348 −0.12661i	0.22691 +0.17316i	−0.31107 −0.16998i	0.53973 +0.42158i	−0.49878 −0.50448i	0.15860 +0.13252i	−0.20539 −0.27950i	0.24222 +0.35073i

TABLE X (Contd)

$[G]_{\nu=1.8} =$

-0.63587 +0.027803i	-0.032346 -0.0098023i	-0.056484 +0.0097398i	0.43016 +0.20303i	-0.015809 +0.048799i	0.041220 +0.011385i	0.13795 +0.082413i	0.048489 -0.0073718i	0.011982 +0.0077092i
0.092518 -0.052817i	-0.46981 +0.13417i	0.13836 -0.077383i	-0.23636 +0.11493i	0.54440 -0.12157i	-0.19750 +0.11737i	0.16175 -0.044521i	-0.13209 +0.14752i	0.077952 -0.079019i
-0.11411 +0.025795i	0.24990 -0.10334i	-0.59574 +0.092304i	0.029066 +0.022101i	-0.41094 +0.13928i	0.69140 +0.017971i	0.077296 +0.032480i	0.18768 -0.075705i	-0.15463 +0.077562i
0.77176 -0.31915i	-0.035547 -0.0020923i	0.065074 -0.028274i	-1.0378 +0.16025i	0.071934 +0.0035685i	-0.087163 +0.011364i	0.25196 -0.065595i	-0.042295 -0.0048139i	0.019348 -0.0030884i
0.011907 +0.14209i	0.34182 -0.49698i	-0.067212 +0.24439i	0.10814 -0.33183i	-0.67599 +0.79050i	0.21810 -0.43644i	-0.13578 +0.15701i	0.33621 -0.36538i	-0.14838 +0.19908i
0.074460 -0.11358i	-0.079503 +0.29224i	0.56838 -0.40597i	-0.087516 +0.14814i	0.27734 -0.55858i	-0.92535 +0.49596i	0.0058965 -0.076914i	-0.20653 +0.25884i	0.35613 -0.21417i
0.084556 +0.21021i	0.033754 -0.059518i	0.00076116 +0.027606i	-0.39674 -0.61435i	-0.13349 +0.076335i	-0.020002 -0.065622i	0.22827 +0.49460i	0.085822 -0.00032007i	0.012314 +0.048016i
-0.17921 -0.081927i	0.48607 +0.33818i	-0.28204 -0.19728i	0.22010 +0.31140i	-0.73320 -1.0646i	0.34655 +0.56893i	-0.015333 -0.19038i	0.17365 +0.68065i	-0.048425 -0.32178i
0.10991 +0.056437i	-0.24351 -0.13300i	0.16528 +0.22378i	-0.23186 -0.21134i	0.36687 +0.47380i	-0.35903 -0.66386i	0.096849 +0.16511i	-0.083015 -0.31092i	0.12695 +0.46474i

TABLE XI

Chordwise loading factors

$\nu = 0 \quad L_1' = [1.2021 \quad 0.8181 \quad 0.6013 \quad 0.3646 \quad 0.1554]$

$\nu = 0.6 \quad L_1' = [1.1985-0.0754i, \ 0.8093-0.1395i, \ 0.5482-0.3288i, \ 0.2550-0.3975i, \ -0.0146-0.4116i]$

$\nu = 1.2 \quad L_1' = [1.1884-0.1491i, \ 0.7835-0.2763i, \ 0.3933-0.6278i, \ -0.0517-0.6959i, \ -0.4574-0.6121i]$

$\nu = 1.8 \quad L_1' = [1.1731-0.2202i, \ 0.7420-0.4083i, \ 0.1488-0.8698i, \ -0.4926-0.8148i, \ -0.9934-0.4668i]$

Zero airspeed $L_1 = [0.5464, \ 0.3272, \ 0.0859, \ -0.1823, \ -0.7772]$

$L_2 = [0.9653, \ 0.3818, \ -0.3293, \ -0.5774, \ -0.4404]$

$L_3 = [-0.3946, \ -0.6250, \ -0.4343, \ 0.5039, \ 0.9499]$

TABLE XII

Generalised force for pitch about two spanwise axes

(i) Pitch about axis 0.1953c aft of apex

ν		
0	0.4592	
0.6	0.3611	+0.5072i
1.2	0.06265	+1.0142i
1.8	-0.4578	+1.5175i

(ii) Pitch about axis 0.9614c aft of apex

ν		
0	-0.4417	
0.6	-0.4364	+0.06555i
1.2	-0.4587	+0.1045i
1.8	-0.5209	+0.1342i

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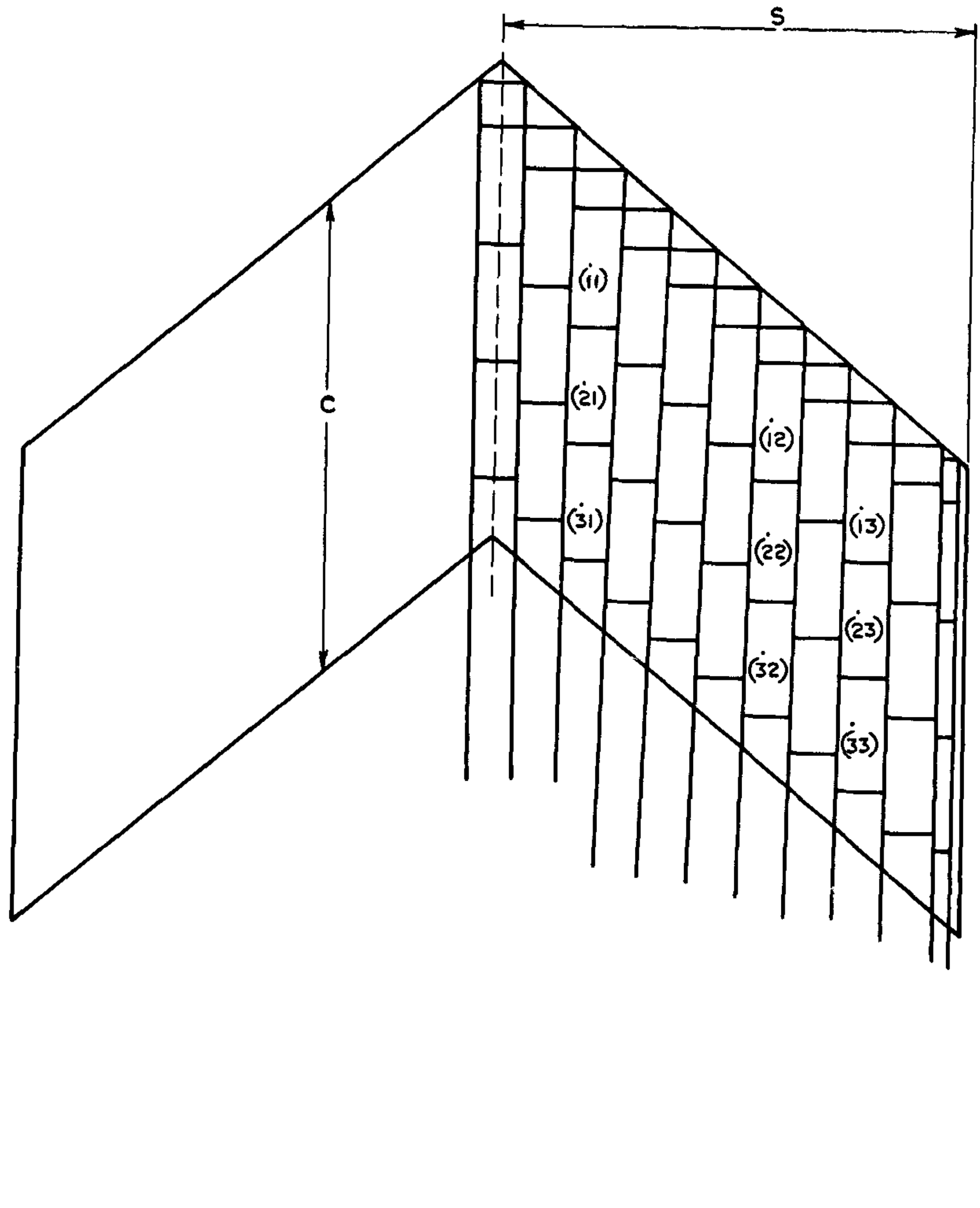

FIG. 1. WING PLAN SHOWING LATTICE AND COLLOCATION POINTS.

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