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A FLIGHT TECHNIQUE FOR THE MEASUREMENT OF THRUST BOUNDARIES AND OF DRAG DUE TO LIFT

By

H. D. Rylands, B.Sc., A.F.R.Ae.S.

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A flight technique for the measurement of thrust boundaries and of drag due to lift

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Summary

None of the existing methods of measuring thrust boundaries is suitable for use with the new high performance aircraft. A new technique is therefore proposed which is acceptable for supersonic aircraft and which will also have many advantages over existing methods for use with subsonic types.

Some data on drag due to lift can also be obtained without the need for thrust measurement.

Preliminary flight trials on a subsonic aircraft proved very satisfactory. No major difficulties are anticipated when the technique is applied to supersonic aircraft.

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1. Introduction

The thrust boundary, which is the limit set by the available thrust to normal acceleration that can be applied in a steady turn at constant height, will remain a most important performance criterion for military aircraft in the supersonic category.

The existing methods of measurement require either a lengthy flying programme or an indirect approach utilising certain assumptions on the variation of drag due to lift.[1,2] Either of these approaches is likely to be unsatisfactory for supersonic aircraft, particularly for interceptors with limited endurance.

The method proposed here effectively gives a direct measurement of the turning performance while avoiding the main drawbacks of the existing methods.

While the technique is thought to be especially applicable for supersonic aircraft it is in many respects preferable to existing methods for subsonic types.

The tests reported here were made on a subsonic fighter in order to obtain practical experience of the technique and to assess the relative accuracy of performance data obtained in such turns compared with levels.

2. Notation

X	nett thrust lb.
D	aircraft drag lb.
C_{D0}	drag coefficient at zero lift.
C_D	total drag coefficient.
W	aircraft weight lb.
C_L	lift coefficient.
S	gross wing area sq.ft.
a_o	speed of sound at sea level ft/sec.
ρ_o	density of air at sea level slugs per cu.ft.
p	relative ambient pressure.
θ	relative ambient absolute temperature.
M	Mach number.
V	true airspeed, ft/sec.
$\frac{dV}{dt}$	longitudinal acceleration ft/sec^2
n	normal acceleration 'g' units.
$\frac{dh}{dt}$	rate of climb ft./sec.
N	engine speed r.p.m.

3. The existing methods

3.1 The direct method. The direct method of obtaining thrust boundaries by actual measurement in turning flight at individual Mach numbers is not practical for modern high performance-aircraft.

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The flying technique for this is difficult as the pilot has to maintain both Mach number and height accurately constant and also control the gradual increase of normal acceleration up to the point at which the altitude can no longer be maintained. In addition, of course, this has to be done under the physical discomforts of high normal acceleration. An alternative technique, that has been used, is to put the aircraft into turns at a given Mach number and at selected normal accelerations and to measure the rate of climb or descent: the thrust boundary at each Mach number then being obtained by interpolating for zero rate of climb. This technique is also difficult to execute.

Apart from these piloting difficulties of the direct method, to cover the required range of speed and height for a high performance aircraft will be most expensive in flying hours and certainly for a supersonic interceptor with its limited endurance it will be prohibitive.

3.2 Accelerated level methods

Method 1

3.2.1 For a turbo-jet engined aircraft, the thrust boundaries can be obtained by making accelerated level runs over a common Mach number range at two heights using the same value of $\frac{N}{\sqrt{\theta}}$. This method, in addition to certain practical limitations requires assumptions which can not be justified for supersonic aircraft. These points will be discussed later.

With the usual assumptions on drag due to lift we may write:-

$$C_D = C_{D_0} + \left(\frac{\partial C_D}{\partial C_L^2}\right) C_L^2 \quad \text{where} \quad \frac{\partial C_D}{\partial C_L^2} \quad \text{is a function only of Mach number.}$$

From which may be obtained

$$\frac{D}{p} = AM^2 + \frac{B}{M^2} \cdot \left(\frac{nW}{p}\right)^2 \quad \text{where } A \propto C_{D_0} \text{ and } B \propto \frac{\partial C_D}{\partial C_L^2} \text{ and both are}$$

constant at a given Mach number.

Since in level flight:

$$X = D + \frac{W}{g}\frac{dV}{dt}$$

$$\text{Then } \frac{X}{p} = AM^2 + \frac{B}{M^2}\left(\frac{nW}{p}\right)^2 + \frac{W}{pg}\frac{dV}{dt}$$

Also $\frac{X}{p} = f\left(\frac{N}{\sqrt{\theta}}, M\right)$ neglecting effects of attitude on intake efficiency and of Reynolds Number.

Hence for the two levels at different heights (i.e. $\frac{W}{p}$ values) but the same $\frac{N}{\sqrt{\theta}}$, at any value of M we have:

$$B'\left(\frac{nW}{p}\right)^2 + \frac{W}{pg}\frac{dV}{dt} = \text{constant, where } B' \propto \frac{1}{M^2} \cdot \left(\frac{\partial C_D}{\partial C_L^2}\right) \text{ and is constant.}$$

Hence at each Mach number $\frac{W}{pg}\frac{dV}{dt}$ is plotted against $\left(\frac{nW}{p}\right)^2$, (n = 1 of course for the levels) and the value of $\frac{nW}{p}$ for which $\frac{dV}{dt} = 0$ obtained by linear extrapolation. These are thus the values of $\frac{nW}{p}$ for steady level flight at the test $\frac{N}{\sqrt{\theta}}$ and from them can be calculated the values of n for any weight and height.

/The...

The basic assumption of this method that $\frac{\partial C_D}{\partial C_L^2}$ remains constant at a given Mach number for an extrapolation of C_L can not be justified for supersonic aircraft. In addition, with the higher performance capabilities of new aircraft, the effect of changes of Reynolds number on thrust and drag can no longer be neglected and to allow for this effect, extra information is required which may not be readily available.

Some practical limitations of this method are:

(a) with aircraft having high thrust boundaries, in order to minimise the extrapolation on C_L it is necessary that one accelerated level should be measured at an appreciably greater height than the other. Such a height may well be impracticable,

(b) the thrust boundary obtained is limited to the Mach number overlap of the two accelerated levels. If the levels have a large height separation this may be a considerable restriction,

(c) there can be difficulty in achieving the same $\frac{N}{\sqrt{\theta}}$ at the two heights. This can occur due to the difficulty of measuring air temperature accurately with present instrumentation. Also, when the tests are made at or near maximum engine speed at the first height, if the air temperature at the second height is higher relative to standard than at the first; then $\frac{N}{\sqrt{\theta}}$ can not be maintained constant without exceeding the limiting engine speed.

In view of the fact that this method is limited to turbojet engined aircraft only, requires unjustifiable assumptions for aircraft in the supersonic category and has several practical limitations, it can no longer be regarded as suitable for general use.

Method 2

3.2.2 Where thrust measurements are made or drag information is already available the thrust boundaries can be deduced directly from accelerated levels at one height.

From para. 3.2.1 we have for a level acceleration (n = 1):-

$$\frac{X}{p} = AM^2 + \frac{B}{M^2}\left(\frac{W}{p}\right)^2 + \frac{W}{pg}\frac{dV}{dt}$$

and in a steady turn:-

$$\frac{X}{p} = AM^2 + \frac{B}{M^2}\left(\frac{nW}{p}\right)^2$$

Thus at the same M and $\frac{N}{\sqrt{\theta}}$, neglecting effects of attitude on intake efficiency we have

$$\frac{B}{M^2}\left(\frac{W}{p}\right)^2 (n^2 - 1) = \frac{W}{pg}\frac{dV}{dt}$$

$$\text{and } n = \sqrt{\frac{M^2}{B\frac{W}{p}} \cdot \frac{1}{g}\frac{dV}{dt} + 1}$$

Thus if $\frac{\partial C_D}{\partial C_L^2}$ ($\propto$ B) is already known or is obtained from thrust measurements, n can be determined.

/This...

This method is very limited in scope however, the requirement of either knowing $\frac{\partial C_D}{\partial C_L^2}$ or of measuring thrust being undesirable.

Also to obtain the value of $\frac{\partial C_D}{\partial C_L^2}$ from the thrust measurements during the accelerated level it must be assumed that $\frac{\partial C_D}{\partial C_L^2}$ is independent of Mach number. This is known to be unjustified above the critical Mach number.

4. The new method proposed

This consists of measuring an accelerated level over the full available speed range at the required height and engine conditions followed immediately by application of normal acceleration, still maintaining the height such that a fairly steady rate of loss of speed results through the speed range of interest.

The thrust boundary can then be obtained by direct interpolation on lift coefficient at constant Mach number for zero longitudinal acceleration.

Since all the data are obtained at the same height and at the same time this direct interpolation, without any correction process, is possible, involving no assumptions on drag and of effects of attitude on intake efficiency, other than that of linearity over the range of interpolation, or on the effects of Reynolds number on thrust or drag. Neither is there danger for a turbo-jet engined aircraft of air temperature differences arising between different sections of the measurements: any error in temperature measurement will now occur only as an error in the $\frac{N}{\sqrt{\theta}}$ to which the measured result is attributed and will not distort the thrust boundary obtained.

There is, of course, no restriction on the type of engine or mixed power plant for which this method is applicable.

It is convenient to make the interpolation by plotting $\left(\frac{nW}{p}\right)^2$ against $\frac{W}{pg}\frac{dV}{dt}$ at constant M values as in para. 3.2.1, so obtaining the values of $\frac{nW}{p}$ at which $\frac{dV}{dt} = 0$ and hence n for the selected weight and test height. The reason for this is of course that in many conditions this assumption of linearity is virtually correct. In order to minimise errors in the conditions when $\frac{\partial C_D}{\partial C_L^2}$ is not constant with C_L at a given M, the deceleration in the turn should be kept as low as is conveniently possible so that the possible error due to interpolation is small.

5. Flight tests

Some concern was felt initially about the accuracy of measurements that could be achieved in turning flight. To check this, a comparison was made between decelerated levels and decelerated turns under identical non-dimensional conditions with a subsonic fighter, the only aircraft readily available at the time. The decelerated turns, of course, had to be made with constant normal acceleration. Although neither of these manoeuvres correspond precisely with those of the techniques proposed for thrust boundary measurements it was the nearest practical approach in which a fair comparison could be drawn between level and turning flight. From the piloting point of view the turns in this case were probably more difficult as holding the normal acceleration strictly constant is more difficult than just maintaining a fairly steady longitudinal deceleration.

The results of these tests for two decelerated levels and two turns are given in Fig. 1 as a plot of $\left(\frac{W}{pg}\frac{dV}{dt} + \frac{W}{pV}\frac{dh}{dt}\right)$ against M. The term $\frac{W}{pV}\frac{dh}{dt}$

/was...

was included to cover variations from the nominal height due to piloting inaccuracies and pressure error changes with speed. The normal acceleration used in the turn was 3'g' and the heights were approximately 35,000 ft. for the levels and 11,000 ft. for the turns.

The Mach number range of the turns is limited in this case because of the speed lost while transferring from level flight at the speed appropriate to the engine conditions to the steady turn at 3 'g'.

From the results it is apparent that good agreement is obtained between the levels and turns and that the degree of scatter is comparable. There is no evidence here of Reynolds' number effects but such effects on the engine are only normally significant at greater altitudes than these.

It is therefore considered that data obtained in the proposed decelerated turn would also be of comparable accuracy from piloting aspects with that from the equivalent level run.

The proposed method of measuring the thrust boundaries was then tried out on the same aircraft. A height of 25,000 ft. was selected for the tests as this gave a reasonably high thrust boundary i.e. 2.6 'g' maximum, which even with the excess 'g' required to decelerate the aircraft, was within the physical capacity of the pilot. Maximum engine speed was used. The results are shown in Figs. 2 to 5 inclusively.

Some difficulty was experienced by the pilots in the initial practice runs in achieving a smooth deceleration in the turns as the thrust boundary varied with the decreasing Mach number; the tendency being to decelerate too quickly. Although it was not tried, it was considered that a longitudinal accelerometer, visible to the pilot with a mark on the dial corresponding to a predetermined desired deceleration would be most helpful.

The results obtained were very satisfactory; the scatter between runs being relatively small. Two pilots did the flying for these tests and in order to demonstrate that approximately the same degree of accuracy was obtained by both, their results are shown separately.

A.11 Hussenot recorders were used for all this work.

6. Drag due to lift

Although thrust measurement is not required, even without it some data on drag due to lift can be calculated. This only involves the assumption that thrust is uneffected by incidence, which is usually justifiable over the incidence changes involved.

These calculations have been made here to demonstrate this point and in Fig. 6 a plot of $\pi A \left(\frac{\partial C_D}{\partial C_L^2}\right)$ v M is shown. It can be seen that up to $M = 0.775$ there is commendably little scatter. For $M = 0.8$ however, owing to the very small accelerations and decelerations the value of $\pi A \left(\frac{\partial C_D}{\partial C_L^2}\right)$ becomes unreliable and is not therefore shown. This unreliability is not also reflected in the analysis of the thrust boundary as the intercept of the $\frac{W}{pg} \cdot \frac{dV}{dt} \cdot v \left(\frac{nW}{p}\right)^2$ line on the abscissa is not particularly sensitive to changes in the value of $\frac{dV}{dt}$.

It may be noted that the calculated drag change is that due to a change of lift coefficient only at constant Mach number and Reynolds number. The data are therefore directly comparable with similar wind tunnel drag measurements obtained with incidence only varying. If the results are interpreted as here, in terms of a drag factor $\pi A \left(\frac{\partial C_D}{\partial C_L^2}\right)$, then this value

/corresponds...

corresponds strictly to the local mean slope of the $C_D - C_L^2$ curve and if this relation is in fact non-linear, caution must be observed in generalising the results.

If desired of course further turns could be made with differing decelerations or with some acceleration to obtain data at intermediate C_L values.

7. Discussion

As the instrumentation required consists only of records of airspeed, altitude and normal acceleration and a knowledge of the aircraft weight, these tests could be made at an early stage of a prototype's flight trials.

For a turbo-jet engined aircraft measurements at one altitude can be generalised to other altitudes very simply[3] ignoring the effect of Reynolds' number variations. It is recommended however that with the high performance aircraft of the future on which such variations can hardly be ignored that it would be advisable to make measurements at two well separated heights above the tropopause and to interpolate between them.

The work shown here has already demonstrated that for an aircraft of this limited performance the technique is satisfactory and certainly preferable to any previously suggested.

No supersonic aircraft has yet been available to try out this method but it is not anticipated that any major problem will be encountered when the opportunity does occur.

It is possible that another limitation may be encountered with high performance aircraft when measuring the thrust boundary e.g. the stall or buffet boundary, lack of longitudinal control to attain the thrust boundary or a physical limitation in normal acceleration of the pilot. If these limits are reached before the thrust boundary is attained then of course the thrust boundary itself is of no practical significance. This technique proposed will demonstrate which limit is attained first and also its magnitude.

With certain aircraft, particularly if a rocket motor is fitted, there may be inadequate fuel to complete the accelerated level and the decelerated turn in one flight. In this case the remedy may be to do the test in two parts; on the first flight decelerating from half way up the Mach range and on the second decelerating from the top speed to half way, also eliminating as much as possible of the low speed end of the accelerated level.

If for any reason particular doubt is felt about the interpolation on a linear basis between the points obtained from accelerated level or decelerated turn, then an intermediate point could be obtained from an accelerated turn which will help to indicate more clearly the nature of the relationship. Such a situation could occur if the decelerated levels were made too rapidly so that a large interpolation was required.

8. Conclusions

A new technique for measuring thrust boundaries is proposed. It requires a relatively small amount of flying time to carry out and involves no major assumptions in the analysis.

Some data on drag due to lift can also be obtained without the need for thrust measurement.

Preliminary flight trials on a subsonic aircraft proved very satisfactory. No serious difficulties are anticipated when the technique is applied to supersonic aircraft.

/References....

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2.	A.& A.E.E. Discussion Memo. No.11.	R.T. Shields	Techniques employed for the measurement of the turning performance limits of aircraft. Unpublished M.O.S. Report.
3.	AGARD Flight Test Manual, Volume 1.		Section 4.15.

FIG. 1.

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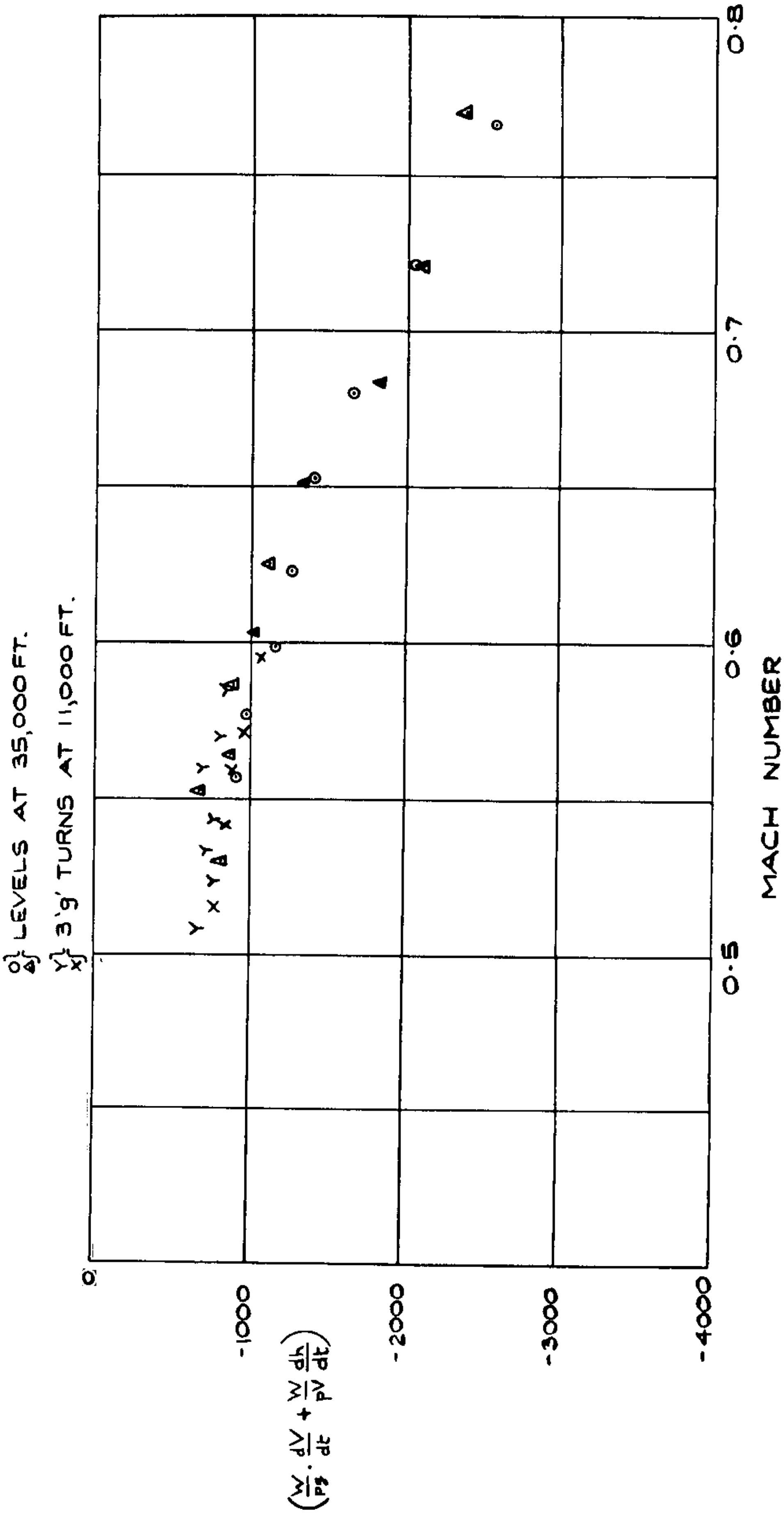

DECELERATED LEVELS AND TURNS AT CONSTANT $\frac{nW}{p}$ AND $\frac{N}{\sqrt{\theta}}$

$\left(\frac{W}{pg}\cdot\frac{dV}{dt}+\frac{W}{pV}\cdot\frac{dh}{dt}\right)$ v M.

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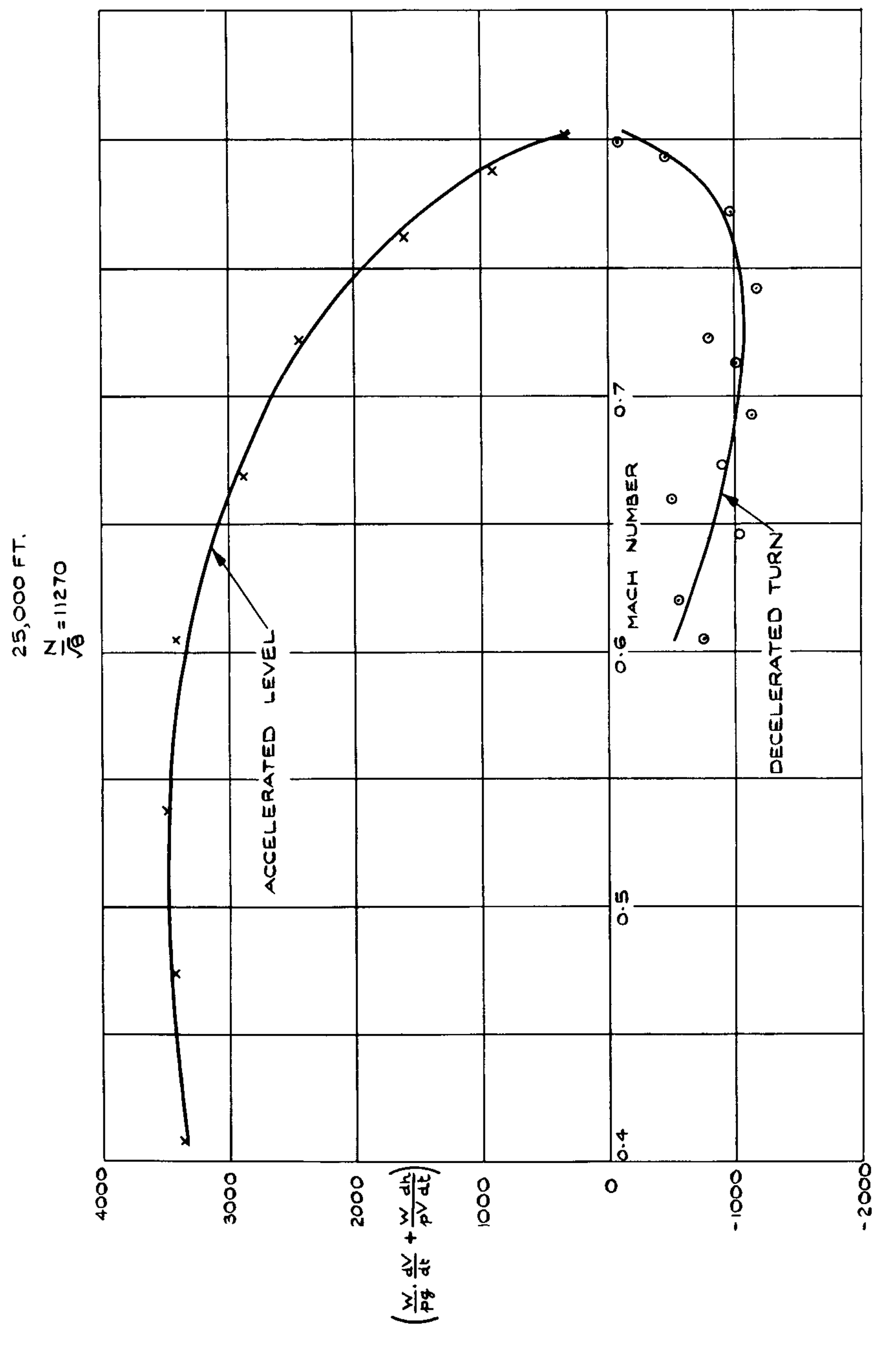
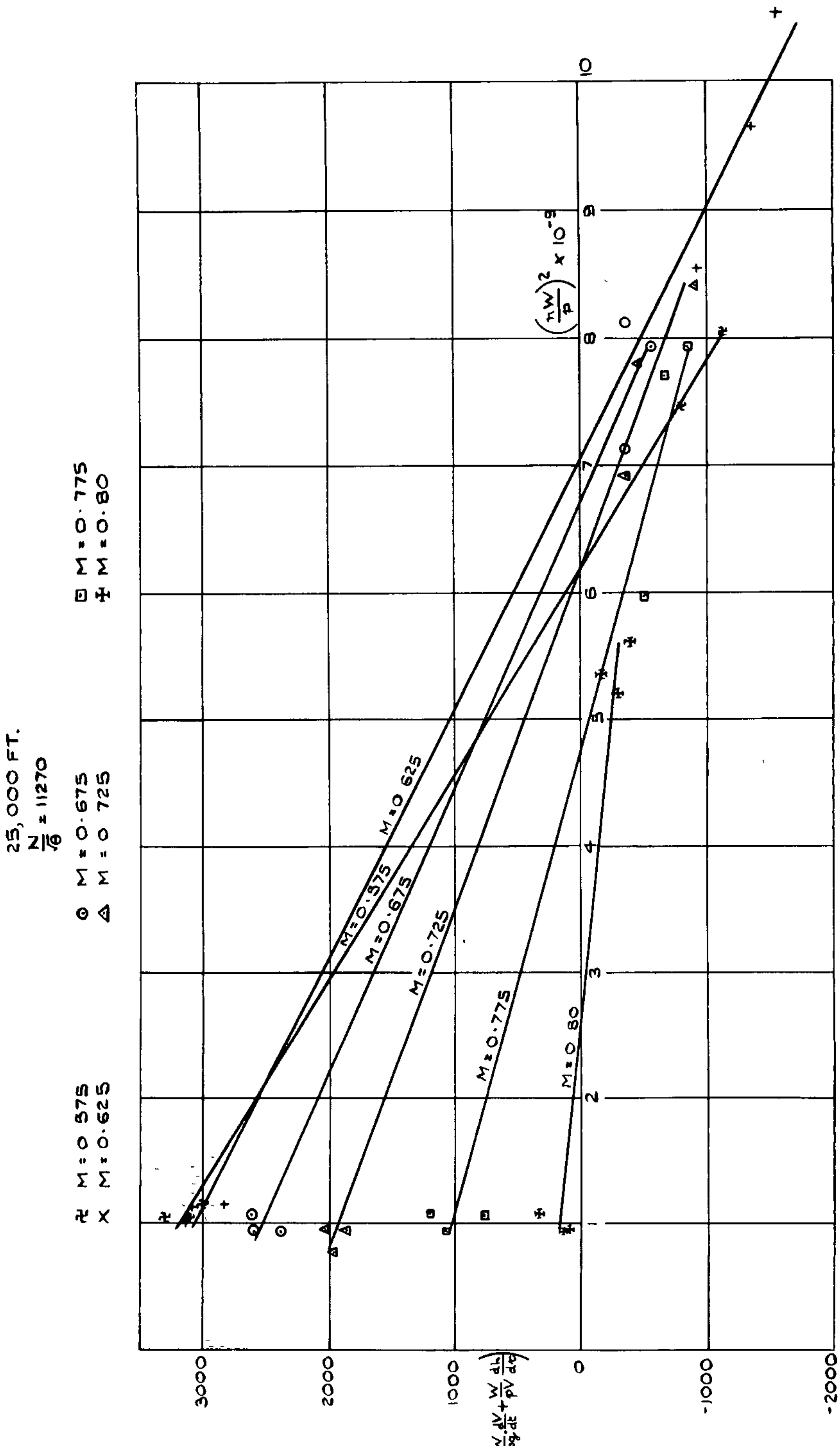
FIG. 2.



TYPICAL EXAMPLES OF ACCELERATED LEVEL AND DECELERATED TURN $\left(\frac{W}{\rho g}\cdot\frac{dV}{dt}+\frac{W}{\rho V}\cdot\frac{dh}{dt}\right)$ V M.

FIG. 3.



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ACCELERATED LEVELS AND DECELERATED TURNS $\left(\frac{W}{pg}\cdot\frac{dV}{dt}+\frac{W}{pV}\frac{dh}{dt}\right) v \left(\frac{nW}{p}\right)^2$

PILOT I

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ACCELERATED LEVELS AND DECELERATED TURNS $\left(\frac{W}{pg}\frac{dV}{dt}+\frac{W}{pV}\cdot\frac{dh}{dt}\right)V \left(\frac{nW}{p}\right)^2$

PILOT 2

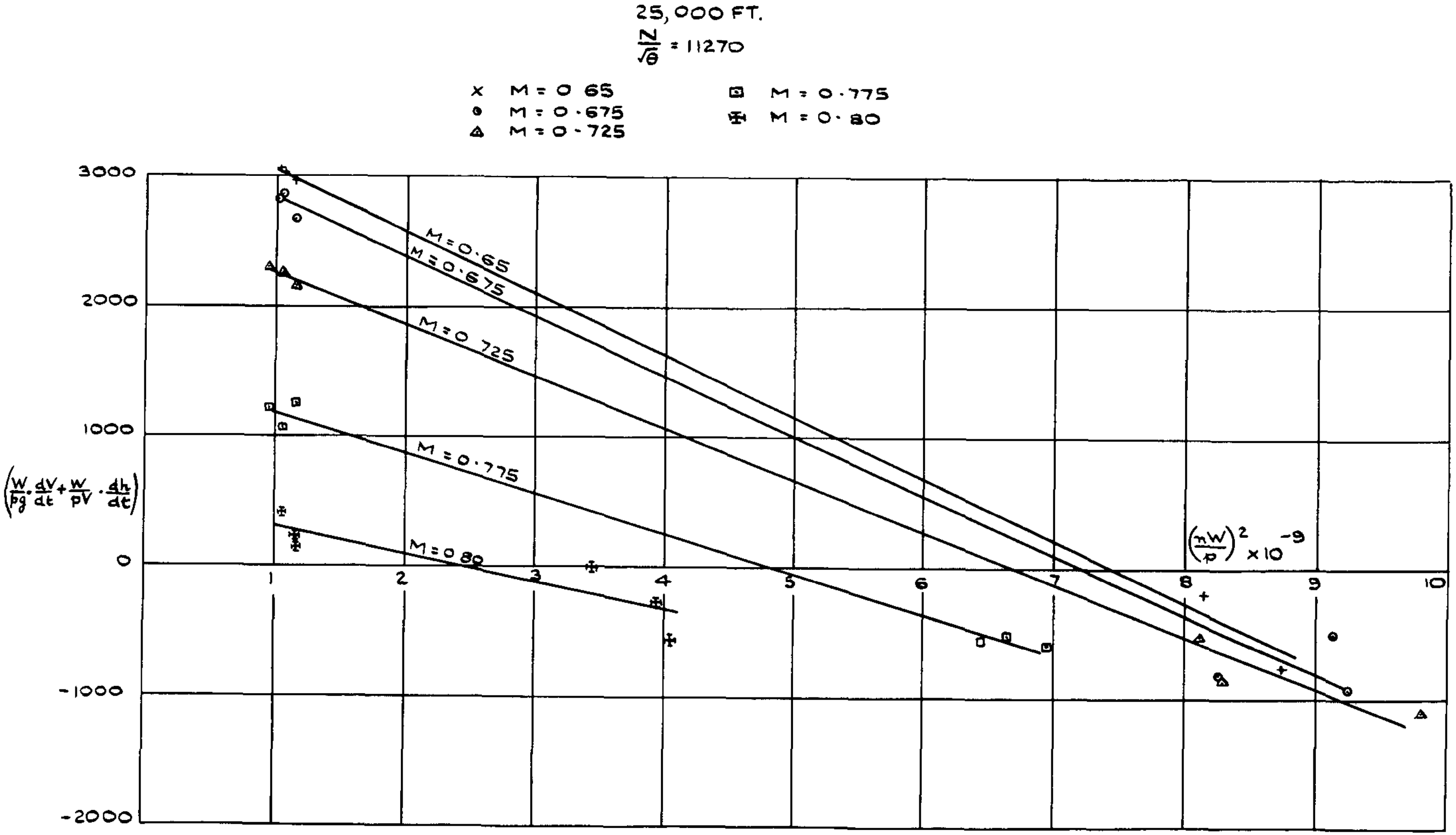
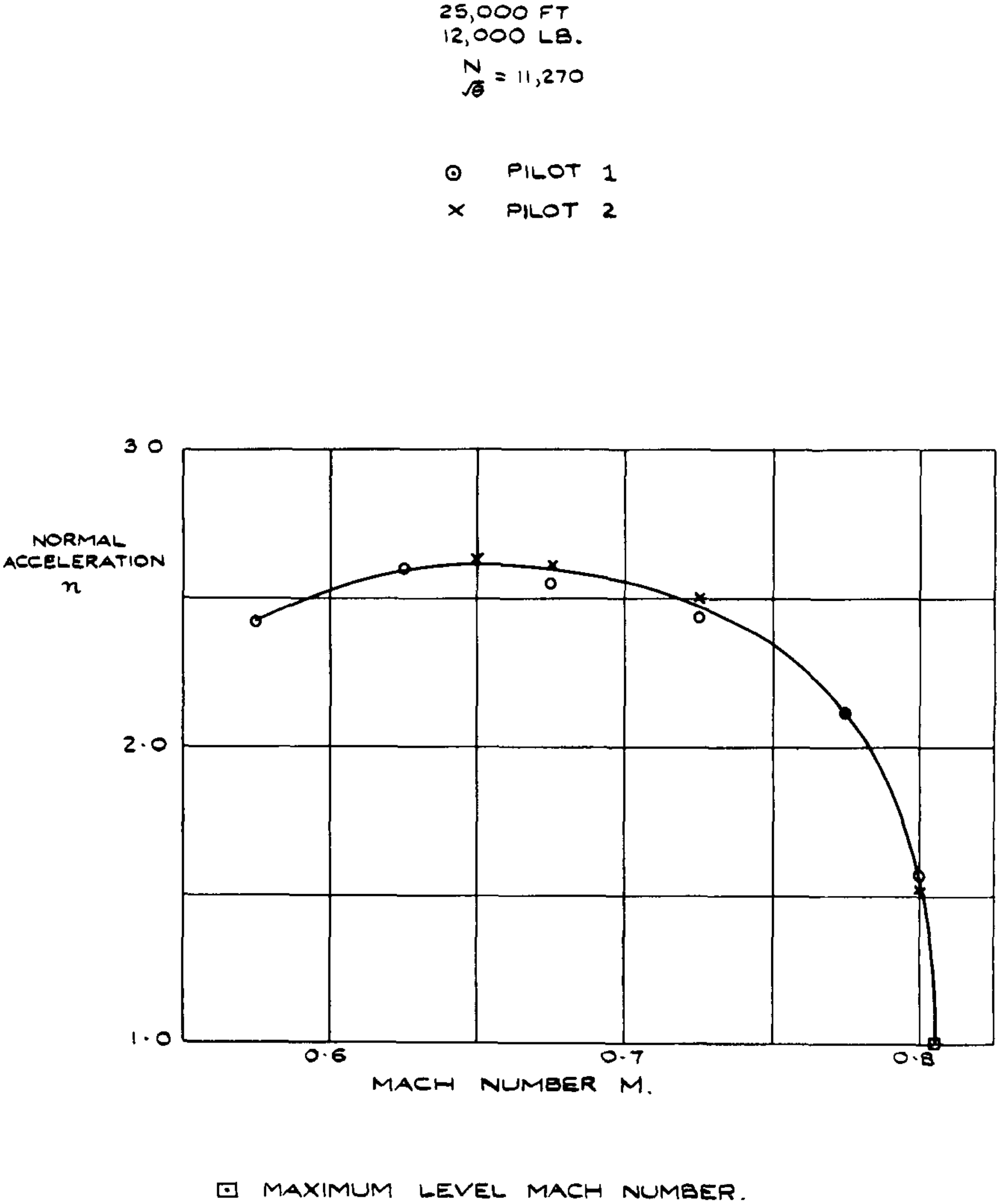


FIG. 4.

FIG. 5.



THRUST BOUNDARY n v M.

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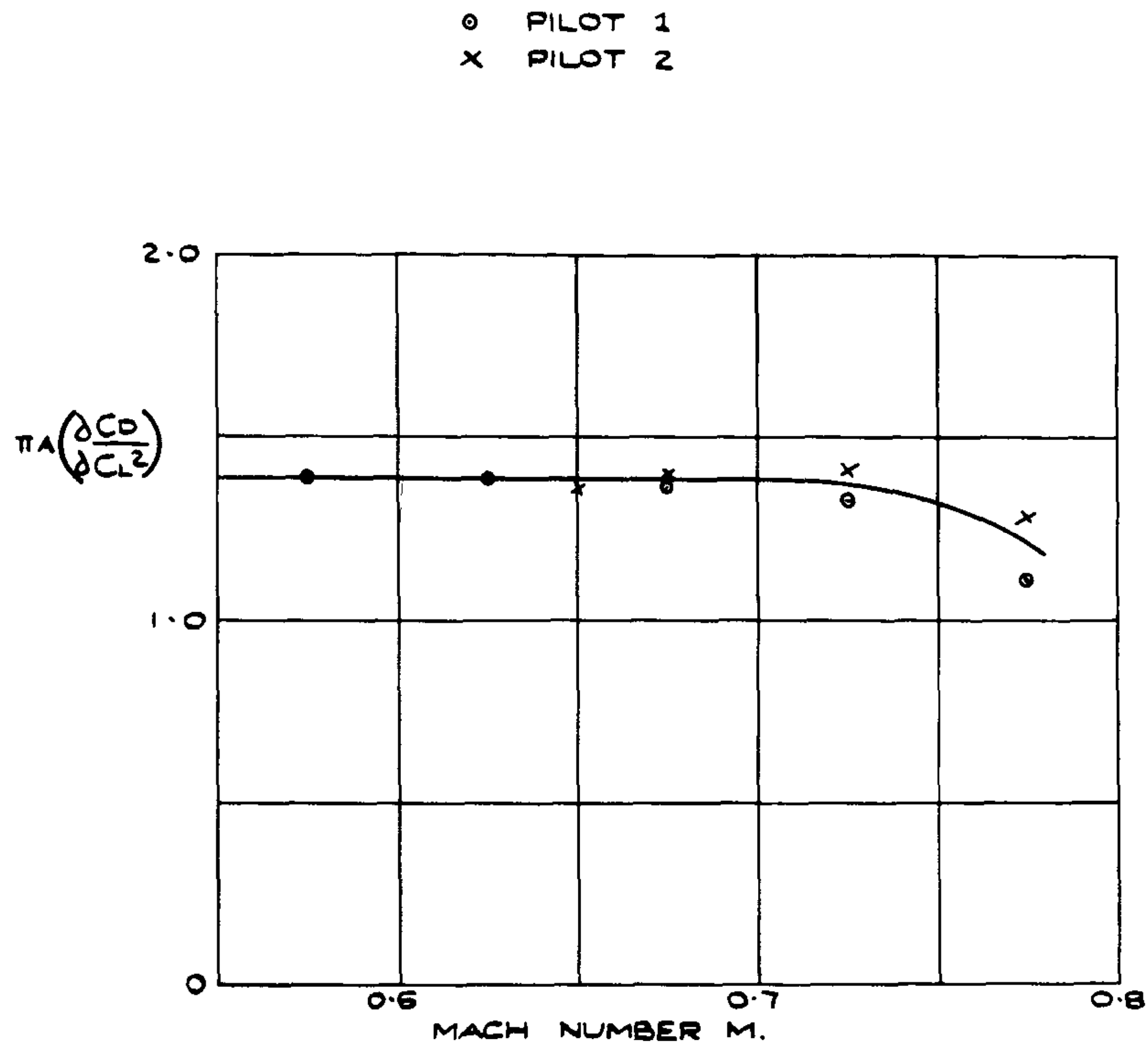
FIG. 6.



DRAG DUE TO LIFT FACTOR $\pi A\left(\frac{\partial C_D}{\partial C_L^2}\right)$ v M.

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