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NUMERICAL ASPECTS OF UNSTEADY LIFTING-SURFACE THEORY AT SUPERSONIC SPEEDS

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1. Introduction

The following brief remarks are concerned with the general problems of computing the unsteady aerodynamic loading on finite wings in a supersonic stream, when the planform, Mach number, mode and frequency of oscillation are given. In the corresponding subsonic problem the absence of exact solutions made imperative the early development of general collocation methods and much attention is now being given to the establishment of an optimum routine and its limits of accuracy. By contrast, the hyperbolic nature of the differential equations of motion in supersonic flow has led to analytical linearized solutions for planforms without subsonic edges and exact solutions in other special cases, which usually involve rather difficult numerical evaluation. Algebraic solutions in powers of the frequency parameter exist for particular classes of planform over restricted ranges of Mach number, but these introduce heavy direct computation unless the frequency is fairly small. Neither approach is ideally suited to mechanized computation. It is comparatively recently that collocation methods have been proposed to deal with wings of arbitrary planform in a supersonic stream; steps are being taken to programme such methods for electronic machines. There has also arisen the prospect of a unified numerical procedure for oscillating wings in subsonic or supersonic flow. It is important to decide how this broad field of computational research should be explored.

2. Method of Approach

A brief formal discussion of the supersonic problem is included in Ref.1 (Garner and Acum, 1956). In its simplest form the differential equation for periodic linearized supersonic flow is

$$(M^2 - 1)\frac{\partial^2\psi}{\partial x^2} - \frac{\partial^2\psi}{\partial y^2} - \frac{\partial^2\psi}{\partial z^2} + \frac{M^2\omega^2\psi}{U^2(M^2-1)} = 0, \qquad \ldots(1)$$

the perturbation velocity potential being

$$\Phi = \psi(x, y, z)\exp\left[-\frac{iM^2\omega x}{U(M^2-1)}\right]\exp(i\omega t). \qquad \ldots(2)$$

Many workers in this field have obtained exact solutions with restrictions on planform, Mach number and frequency. For example, Stewartson[2] (1950) has found analytical expressions for the pressure distribution on a semi-infinite wing, slender body of revolution and a swept-back wing with a supersonic leading edge; Miles[3] (1951) has considered rectangular wings of aspect ratio greater than $(M^2-1)^{-\frac{1}{2}}$. In each case fairly simple formulae for the lift and pitching moment are derived. Stewartson[4] (1952)

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has also given a general treatment of the differential equation and has formulated a practical method for wings with supersonic leading and trailing edges. His analysis for wings with a subsonic leading edge is very complicated and would probably defy computation.

In current work it is more usual to consider instead the equivalent integral equation, whereby $\alpha(\xi, \eta)$, the specified flow-direction at the wing, may be expressed in the following three ways:-

$$\alpha(\xi, \eta) = \iint \ell(x, y) K_1(x - \xi, y - \eta, \omega/U, M)\, dx\, dy, \quad \ldots(3)$$

$$\alpha(\xi, \eta) = \iint \Phi(x, y) K_2(x - \xi, y - \eta, \omega/U, M)\, dx\, dy, \quad \ldots(4)$$

$$\Phi(x, y) = \iint \alpha(\xi, \eta) K_3(x - \xi, y - \eta, \omega/U, M)\, d\xi\, d\eta, \quad \ldots(5)$$

where the area of integration is the part of the plane $z = 0$ intercepted by the forward Mach cone. Equation (3) gives the required non-dimensional wing loading, ℓ, instead of the velocity potential and has been fully discussed by Watkins and Berman[5] (1955); the kernel function K_1 is an integral which needs careful numerical evaluation. On the other hand, the kernels K_2 and K_3 contain no integrals and can be evaluated easily, but the solution gives Φ, which has to be differentiated to give the wing loading.

Equations (3) and (4) suffer from the disadvantage that a matrix inversion is always required to obtain a solution in terms of the known α. When all the edges of the planform are supersonic, equation (5) is clearly superior to (3) or (4), since the answer is given directly and is equivalent to that of Stewartson[4]. When subsonic edges exist, the best choice of integral equation is uncertain. The solution of (5) is complicated, since $\alpha(\xi, \eta)$ is non-zero in a region forward of the leading edge and is no longer defined over the whole area of integration. The analytical treatment due to Evvard[6] (1950) by use of steady source distributions leads to integral expressions valid for low frequencies. In general, a matrix solution has to be obtained in the form

$$\begin{bmatrix} \Phi_1 \\ \Phi_2 \end{bmatrix} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \begin{bmatrix} \alpha_1 \\ \alpha_2 \end{bmatrix}, \quad \ldots(6)$$

where suffix 1 denotes values on the wing surface and suffix 2 those ahead of the wing and behind the Mach cone trailing from the leading apex. Then $\Phi_2 = 0$ and α_1 is known. By elimination of the unknown α_2, the required solution is

$$\Phi_1 = [A_{11} - A_{12} A_{22}^{-1} A_{21}]\, \alpha_1 \quad \ldots(7)$$

and involves the inversion of the matrix A_{22}.

Thus, for wings with subsonic leading edges, equation (5) has little advantage over (4), while the relative merit of obtaining ℓ instead of Φ in (3) is largely offset by the difficult numerical integration to evaluate the kernel K_1. As will be seen, equation (5) has special applications, but for the general problem equation (3) is preferred.

3. Numerical Solutions

Among the methods of approach towards a numerical solution of the integral equation are the following:-

(a) restrictions on planform, Mach number or frequency to permit an exact solution or an analytical one by iteration or successive approximation,

(b)/

(b) expansion of the kernel function in powers of frequency,

(c) continuous lifting-surface theory in which the flow direction is satisfied by collocation at a limited number of points on the wing,

(d) use of a simplified model, such as a lattice of uniformly loaded elements to represent the wing.

Recent examples of (a), based on equation (5), use Evvard's[6] concepts for a subsonic loading edge. Watson[7,8] (1955, 1956) has given approximate expressions for the derivatives of a slowly oscillating combination of cropped delta wing and constant-chord control surface. There is also an unpublished iterative method due to the late I. T. Minhinnick. It seems that such approximate solutions lead to lengthy algebraic expressions, which are not suitable for mechanized computation. The same is true of solutions based on superposed conical fields and of (b) generally; this is illustrated by some results for a triangular wing with subsonic leading edge and quadratic harmonic deformation. Watkins and Berman[9] (1953) give lengthy formulae up to the third power in frequency; some unpublished work of D. E. Davies of R.A.E., shows that higher-order terms in either the deformation or the frequency become progressively harder to calculate. Similar results for the theoretical lift and pitching moment on rigid pointed arrowhead wings with supersonic trailing edges have been evaluated[10] (Cunningham, 1955). Further developments in this field are not likely to be fruitful from the numerical standpoint.

The outstanding example of (c) is Richardson's[11] (1955) theory, in which he formulates a collocation method for arbitrary planform incorporating principles and techniques analogous to those of Multhopp's subsonic theory (Ref.12). Distinct basic loadings and collocation positions in the chordwise direction are derived in the four cases of subsonic or supersonic, leading and trailing edges. This two-dimensional concept introduces spanwise discontinuities in loading wherever the leading or trailing edge becomes sonic. The treatment of the spanwise integrations is precisely that of Ref.12. Richardson[13] (1956) has applied his method to the steady flow of Mach number 1.25 past a slender triangular wing. With only eight collocation points, good accuracy is obtained over most of the span; however, comparison with exact theory shows that the treatment of the kink at the centre section is a source of error. As suggested by Richardson, it would seem sensible to try an even number of spanwise stations in order to avoid a station at the central kink, though some modification to Multhopp's spanwise integration formula would then be necessary. Difficulties involving discontinuities of this kind can only be resolved by means of lengthy calculations with systematically increasing numbers of chordwise loadings and collocation stations. It may not always be best to use the same number of terms in the chordwise loading at all collocation stations. The maximum number would be needed near the centre section and the tip where the load distributions are much distorted from that of an infinite sheared wing; however, it might be practicable to avoid superfluous collocation points at some intermediate stations.

The approach (d) is regarded as being without restriction on planform, Mach number or frequency. A simplified representation of the wing may lead to a rapid but crude method of uncertain accuracy or alternatively to an element theory which reduces to a very large number of simple operations. The former, however imperfect, may have a place in semi-empirical work in which the yard-stick is experimental. The latter may require so many small elements that it is impracticable on a desk machine but becomes powerful and accurate with the aid of an automatic computer. In the box-grid method[14] (Ta Li, 1956), the wing area is subdivided into a number of rectangles with their diagonals parallel to the Mach lines. Ta Li has obtained highly satisfactory

results/

results for two-dimensional flutter coefficients at Mach numbers down to 1.1. He has also formulated a general treatment of subsonic leading edges. The numerical work involves the evaluation of coefficients representing the influence of one rectangular box on another, and these seem suitable for programming on a high-speed computer and can be used directly in flutter calculations for arbitrary planforms. Stewartson's[4] theory for wings with supersonic leading and trailing edges has been developed by Hunt[15] (1955) into a method of calculation, in which the double integrals are replaced by double summations at lattice points identical to those used in Ref.14.

Programming of General Methods

Although methods (a) and (b) lead to precise formulae which can feasibly be calculated on a desk machine, the results are limited and require extremely careful calculation; it seems that the formulae would be just as laborious to programme for an electronic computer. In this respect it is best to avoid any method which involves integration of the downwash over a subsonic edge. On the other hand, desk calculations of methods (c) and (d) would become prohibitive in many problems, since the kernel function K_1 would need to be evaluated an excessive number of times. It is therefore essential to programme the calculation of K_1, given in equation (23) of Ref.1. If the awkward singularities in the integrand at the limits of integration can be so handled, then it should be possible to obtain results in all cases by Richardson's[11] theory.

In the practical problem of calculating flutter characteristics of a thin wing, the frequency and modes of oscillation are unknown. The modes of structural deformation are roughly determined by the elastic properties of the wing, but the frequency is usually obtained by trial and error, separate calculations being made for selected values of the frequency parameter. Hunn[16] (1955) has formulated such a treatment for wings with straight supersonic leading edges, but computation, if attempted, would be very heavy. He also gives a method valid for small frequencies, by which a complete flutter calculation has been performed. A similar method without restriction on planform would be worth-while and should be sought as a limited objective. It would, however, seem inexpedient to attempt to extend the range of frequency by expansion in power series. The necessary automatic programming is rarely easier when the method is valid for a limited range of frequency.

The mechanization of Hunt's[15] method for wings with supersonic edges has been taken up by Sadler[17] (1956) and Wicks[18] (1956), so that a standard computational form is now available for modal deflections of arbitrary frequency. It would seem useful to attempt to modify the programme of Ref.18 to deal with wings of arbitrary planform oscillating in elastic modes. The lattice points should be extended forward of a subsonic leading edge, as suggested in Ref.14; additional linear equations would be introduced to satisfy the conditions of zero pressure difference ahead of the wing.

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