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Summary.

The steady flow incompressible laminar boundary-layer problem in two-dimensions is posed as an integral equation in Crocco variables in the most general case. Numerical investigations are then made of the 'similarity' flows. A transformation is utilised which weakens the singularity at the outer edge of the boundary layer and thereby gives improved numerical results over those obtained from the untransformed equation. An extensive numerical comparison is made with Polhausen's exact analytical solution. Results are also presented in tabular and graphical form illustrating the approach of the boundary-layer characteristics c_f, δ_1, δ_2, δ_3 to their asymptotic values for strong suction for a very wide range of similarity flows. These are in complete agreement with the results obtained by other investigators, and form an extension to the cases previously considered.

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*Replaces A.R.C. 28 116.

1. *Introduction.*

It is well known that Crocco's form of the boundary-layer equation[1] can be converted into an integral equation for constant pressure flow. The integral equation can then be conveniently solved by iteration. This circumstance has been exploited by Crocco[2] himself, and also by Van Driest[3,4] in the calculation of steady compressible flow with variable fluid properties. A similar technique has been employed by von Kármán and Tsien[5] in the solution of the compressible Mises equation.

This method does not appear to have been much used for flow with variable pressure despite the basic simplicity of the notion of solving the boundary-layer equations by a successive approximation procedure. The idea is germane to Weyl's conversion of several of the similarity boundary-layer equations into integral equations, and the construction of an iterative procedure for the solution of the latter (*see* p. 233 Ref. 6). An important property of this and the Crocco algorithm is that the boundary conditions do not require explicit satisfaction, but can be incorporated in the integral equation itself: a well-known feature of integral equation formulations of boundary-value problems. This is particularly useful in numerical work since the general boundary-layer problem has 'two-point' boundary conditions, and the outer edge condition generally has to be satisfied by an interpolation procedure in view of non-linearity of the equation. (It of course happens that the homogeneous Blasius problem can be reduced to an 'initial value' problem.)

It is shown that the general steady incompressible boundary-layer equation in Crocco variables may likewise be converted to an integral equation into which are built the boundary conditions pertinent to an arbitrary pressure distribution and arbitrary suction/blowing velocities at the wall; that is, the most general case. (Other iterative methods have been developed for the general case by Piercy and Preston[7], and Mangler[8]. Both of these methods use a preliminary transformation as in the Crocco method, but they seem on the whole to be somewhat more cumbersome for automatic computation.)

Though we formally obtain a general (implicit) solution of the boundary-layer equation, two practical difficulties arise in any actual numerical work. First, the equations possess a singularity, at worst of the type $r\left[\log\frac{1}{r}\right]^{\frac{1}{2}}$, at the outer edge of the boundary layer. This can be circumvented by using the analytical solution in the neighbourhood of $r \doteq 0$, and joining this to the solution obtained by iteration near $r = 0$. The joining process is not trivial however (*see* Van Driest[3]) and is somewhat unsuited to the systematic refinement procedure used on a digital computer as developed herein.

Second, there is the problem of the *stability* of the iterative procedure used to solve the integral equation. It *happens* that the process is numerically stable for the Blasius flow, but this is not necessarily so for other flows. Furthermore, it may not converge to the correct solution, or it may not converge at all. For the Blasius flow it has been observed that the iterative process 'converges' rapidly if the arithmetic or geometric means of two successive iterates are used as the starting value for the next iteration ('under-relaxation'). The former is a special case of combining the iterates in the ratio $k:1-k$ where we shall call k *the advance factor*. This factor was left as a controllable parameter in the present investigation so as to deal with 'difficult' cases (*see* Section 5), although generally the value 50 per cent was found to be satisfactory. Instability of the numerical process may be associated with instability of the physical flow described by the boundary-layer equation, a situation familiar in iterative computations of the full equations[9].

We propose in this Report to investigate the 'similarity' flows by the Crocco-iterative-method, performing the computations on a high speed digital computer. Evidently the method will be adaptable to non-similar flows if the x-derivative is replaced by a suitable finite difference and an iterative solution made at successive steps Δx from an initial known solution at x_0 (that is, a 'continuation' procedure).

The first part comprises some general investigations of the method in relation to certain well known similarity flows. Initially, the singularity at the outer edge of the boundary layer is simply ignored. Later it is *weakened* by a transformation of the independent variable. The expected improvement in the numerical results is observed. Detailed numerical comparisons are made with Polhausen's analytical solution for the flow between converging planes. The salient characteristics of the Blasius and 'stagnation' flows are also evaluated.

The remainder of the Report consists of a systematic determination of the wall shear stress and boundary-layer thicknesses δ_1, δ_2, δ_3 as functions of the suction parameter for various similarity flows. The results illustrate principally the approach of a wide range of similarity flows to the asymptotic suction flow. Results obtained by other investigators are also included in the graphs.

2. *The Integral Equation Formulation of the Boundary-Layer Problem.*

We start with Crocco's form of the boundary-layer equation for steady flow (*see* pp. 214, 286 Ref. 6).

$$\frac{\partial^2 \tau}{\partial u^2} = \mu\, p'(x) \frac{\partial}{\partial u}\left(\frac{1}{\tau}\right) - \mu\rho \frac{\partial}{\partial x}\left(\frac{u}{\tau}\right) \tag{1}$$

where τ is the shear stress, u the velocity component in the direction of x, $p'(x)$ the pressure gradient, and the other symbols have their usual meanings (*see* List of Symbols). The boundary conditions take the form

$$\frac{\partial \tau}{\partial u} = \frac{\mu\, p'(x)}{\tau} - \rho\, w_s(x) \qquad u = 0 \tag{2}$$

$$\tau = 0 \qquad u = U(x). \tag{3}$$

Now integrate equation (1) with respect to u and incorporate boundary condition (2) so that

$$\frac{\partial \tau}{\partial u} = \frac{\mu\, p'(x)}{\tau} - \mu\rho \int_0^u \frac{\partial}{\partial x}\left(\frac{u}{\tau}\right) du - \rho w_s . \tag{4}$$

Integrating again using (3), we find

$$\tau = \mu \int_{U(x)}^{u} \frac{p'(x)}{\tau}\, du + \mu\rho \int_u^{U(x)} du_2 \int_0^{u_2} \frac{\partial}{\partial x}\left(\frac{u_1}{\tau(x,u_1)}\right) du_1 - \rho\, w_s\{u - U(x)\}. \tag{5}$$

Equation (5) represents the general solution of the incompressible laminar boundary-layer problem for arbitrary mainstream velocity $U(x)$ and arbitrary wall transverse speed $w_s(x)$. The shear stress τ is involved non-linearly and the solution of (5) is thus in general difficult. By means of a well-known identity for repeated integration (*see* the Appendix) we can reduce (5) to a kind of Volterra* integral equation:

$$\tau(x,u) = \int_u^{U(x)} \mu \left[\rho\{U(x)-u\}\frac{\partial}{\partial x}\left(\frac{u}{\tau}\right) - \frac{p'(x)}{\tau}\right] du + \mu\rho\{U(x)-u\}\int_0^u \frac{\partial}{\partial x}\left(\frac{u}{\tau}\right) du$$

$$-\rho\, w_s\{u-U(x)\}\,. \tag{6}$$

We could attempt to solve (6) by means of a power series expansion in u. The coefficients of u, whose complexity increases rapidly with increasing order, are functions of $p'(x)$, $w_s(x)$, and these have been evaluated by Trilling[10] up to u^6. This expansion is of course accurate near the wall only, and would in a precise calculation need to be joined up to an 'outer solution'. The series representation of the solution tends also to obscure the singularity possessed by (1) or (6) at the outer edge of the boundary layer $u = U(x)$.

3. *The Similarity Flows.*

The assumption of similarity involves giving τ the following form

$$\tau = \mu\frac{U(x)}{g(x)}F(u^*) \tag{7}$$

$$u^* = \frac{u}{U(x)}\,. \tag{8}$$

On substitution of these equations into (6) together with the Bernoulli equation

$$p'(x) = -\frac{\rho}{2}\frac{dU^2(x)}{dx} \tag{9}$$

there results the following integral equation for $F(u^*)$

$$F(u^*) = \int_{u^*}^1 \frac{\beta+\alpha u^*-(\alpha+\beta)u^{*2}}{F(u^*)} du^* + \alpha(1-u^*)\int_0^{u^*}\frac{u^*}{F(u^*)}du^* + \gamma(1-u^*) \tag{10}$$

where

$$\alpha = \frac{g}{\nu}\frac{d(Ug)}{dx} \qquad \beta = \frac{g^2}{\nu}\frac{dU}{dx} \qquad \gamma = \frac{w_s g}{\nu} \tag{11}$$

with

$$\begin{aligned}
U(x) &= U_0\,(|x-x_0|/l)^n && \text{(a)}\\
g(x) &= [(2\alpha-\beta)\,(x-x_0)\nu/U(x)]^{\frac{1}{2}} && \text{(b)}\\
w_s(x) &= [U(x)\nu/(2\alpha-\beta)\,(x-x_0)]^{\frac{1}{2}} && \text{(c)}\\
n &= \beta/(2\alpha-\beta) && \text{(d)}\\
(2\alpha-\beta)\,&(x-x_0) > 0 && \text{(e)}
\end{aligned} \tag{12}$$

*Inasmuch as the variable u appears in the limits of the integrals.

and where we have followed exactly the notation of Rosenhead[6] p. 245. Equation (10) is the equivalent of the differential equation (137) (*loc. cit.*) and its associated boundary conditions, but written now as an integral equation in Crocco variables.

3.1. *Behaviour of $F(u^*)$ near $u^* = 1$.*

It may be shown from the differential equation and boundary conditions corresponding to (10) (or *see* p. 247 Ref. 6) that near $u^* = 1$

$$F \sim (1-u^*)\left[\log\frac{1}{1-u^*}\right]^{\frac{1}{2}} \qquad \alpha \neq 0 \qquad \text{(a)}$$

$$F \approx \frac{1}{2}\left[\gamma+\sqrt{\gamma^2+8\beta}\right](1-u^*) \qquad \alpha = 0 \qquad \text{(b)} \tag{13}$$

if the asymptotic approach to $u^* = 1$ is to be of exponential rapidity (*see* also Stewartson[11] p. 32). This is the circumstance which demands special treatment near $u^* = 1$ as exemplified by Crocco[2], Van Driest[3].

3.2. *Weakening the Singularity.*

It is well known (*see*, for example, p. 134 Ref. 12) that singularities of the type (13a) may be weakened by a suitable transformation, with a consequent improvement in numerical work. Such a transformation suggests itself from (13a)

$$F(u^*) \rightarrow \tilde{F}(s) \qquad 1-u^* = s^2. \tag{14}$$

The behaviour of (13a) in the neighbourhood of the singularity $s = 0$ is now

$$\tilde{F}(s) \sim s^2\left[\log\frac{1}{s}\right]^{\frac{1}{2}} \tag{15}$$

which is less singular on account of the fact that the first derivative of $\tilde{F}$ with respect to s tends to zero as $s \rightarrow 0$, a property not shared by dF/du^* as $u^* \rightarrow 1$. It is thus expected that the integral equation written now in terms of $\tilde{F}(s)$

$$\tilde{F}(s) = \int_0^s \frac{2(\alpha+2\beta)s^3-2(\alpha+\beta)s^5}{\tilde{F}(s)}\,ds + s^2\int_s^1 \frac{2\alpha s(1-s^2)}{\tilde{F}(s)}\,ds + \gamma s^2 \tag{16}$$

will yield improved numerical results, and we shall show this to be the case (*see* Section 4.).

The integrand in the second integral still tends to infinity as $s \rightarrow 0$, so it is still necessary to stop short of $s = 0$, say at s_j, in performing the integrations. We can show that for $s_j \leqslant s \leqslant 1$ $s_j > 0$

$$\tilde{F}(s) = \int_{s_j}^s \frac{2(\alpha+2\beta)s^3-2(\alpha+\beta)s^5}{\tilde{F}(s)}\,ds + s^2\int_s^1 \frac{2\alpha s(1-s^2)}{\tilde{F}(s)}\,ds + \gamma s^2 + \tilde{\varepsilon}(s_j) \tag{17}$$

where the 'error' $\tilde{\varepsilon}(s_j)$ is given in the Appendix. Thus by penetrating as close as we please to the singularity we can make the error as small as we please. Although we can in a similar way make the corresponding error $\varepsilon(u_j^*)$ as small as we please when using (10), numerical results obtained from a given integration formula cannot be expected to be as accurate in this case as those obtained from the transformed equation (17) on account of the more singular nature of F and its derivatives near $u^* = 1$: this is confirmed in two examples (*see* Tables 1, 2, 4, 5).

We note in passing that the transformation

$$s = \left[\log \frac{1}{1-u^*}\right]^{\frac{1}{2}} \quad \text{or} \quad 1-u^* = \exp(-s^2) \tag{18}$$

completely removes the singularity and that our transformation (14) is effectively the series truncated after the term in s^2. We are now 'stretching out' the singularity to infinity. The integral equation has exponential kernels and the formulation of the problem is now akin to Weyl's formulation.

3.3. *Boundary-Layer Characteristics.*

In the present formulation of the boundary-layer problem a local coefficient of skin friction c_f is defined as

$$c_f = \frac{\tau_0\, g(x)}{\mu\, U(x)} = F(0) \qquad (\text{or} = \tilde{F}(1)) \tag{19}$$

while the variables z, Z take the forms

$$\frac{z(u^*)}{g(x)} = \int_0^{u^*} \frac{du^*}{F(u^*)} \qquad \frac{Z(u^*)}{g(x)} = \int_0^{u^*} \frac{(1-u^*)}{F(u^*)}\, du^* \tag{20}$$

or in terms of s,

$$\frac{\tilde{z}(s)}{g(x)} = 2\int_s^1 \frac{s}{\tilde{F}(s)}\, ds \qquad \frac{\tilde{Z}(s)}{g(x)} = 2\int_s^1 \frac{s^3}{\tilde{F}(s)}\, ds\,. \tag{21}$$

The variable Z is related to the *displacement thickness* δ_1 by

$$\delta_1 = Z(1) \qquad (\text{or} = \tilde{Z}(0)). \tag{22}$$

Similarly it may be shown that the *momentum* and *energy thickness* δ_2, δ_3 take the forms

$$\frac{\delta_2}{g(x)} = \int_0^1 \frac{u^*(1-u^*)}{F(u^*)}\, du^*\,, \qquad \text{or} = 2\int_0^1 \frac{(1-s^2)s^3}{\tilde{F}(s)}\, ds \tag{23}$$

$$\frac{\delta_3}{g(x)} = \int_0^1 \frac{u^*(1-u^{*2})}{F(u^*)}\, du^*\,, \qquad \text{or} = 2\int_0^1 \frac{(1-s^2)(2s^3-s^5)}{\tilde{F}(s)}\, ds \tag{24}$$

respectively.

3.4. *Integration Formulae.*

In computing the integrals in (10) or (17) it is necessary to decide on an integration formula. The simplest is the trapezium rule which has an error of order h^2 for a fixed range of integration, as is the case in this problem. This allows us to find the new value $F^{(p)}(u_i^*)$ or $\tilde{F}^{(p)}(s_i)$ at each node u_i^* or s_i in the range, using the iterative schemes

$$\begin{aligned} F^{(p)}(u_i^*) &= \phi\{F^{(p-1)}(u_i^*)\} \qquad &\text{(a)} \\ \tilde{F}^{(p)}(s_i) &= \tilde{\phi}\{\tilde{F}^{(p-1)}(s_i)\} \qquad &\text{(b)} \end{aligned} \tag{25}$$

to solve the integral equations (10), (17). The functionals ϕ, $\tilde{\phi}$ represent the right hand sides of equations (10), (17). If we use the more accurate Simpson formula, error $O(h^4)$, we can calculate the iterates only at the even nodes, and it is necessary to use some interpolation formula to calculate the iterates at the odd nodes. The author used quadratic interpolation in the manner suggested by Collatz (p. 516 Ref. 13).

In the present problem we cannot unconditionally expect greater accuracy by using the Simpson formula instead of the trapezium rule. This follows since the derivatives of F (and $\tilde{F}$) become progressively more singular as $u^* \to 1$ (and $s \to 0$), and the error in the Simpson formula involves the fourth derivative whereas only the second derivative is involved in the trapezium rule.

3.5. *Empirical Error Check.*

Suppose the cumulative error is proportional to h^q, that is, if $F(u^*)$ is the correct value and F_1, F_2, F_3 the approximate values of F obtained for step-lengths h, $h/2$, $h/4$, then

$$\begin{aligned} F(u^*)-F_1 &= C\,h^q \\ F(u^*)-F_2 &= C(h/2)^q \\ F(u^*)-F_3 &= C(h/4)^q\,. \end{aligned} \tag{26}$$

Hence

$$\frac{F(u^*)-F_1}{F(u^*)-F_2} = \frac{F(u^*)-F_2}{F(u^*)-F_3} = \frac{1}{2^q} = \frac{1}{t}$$

On solving for $F(u^*)$ we obtain Aitken's extrapolation formula

$$F(u^*) = F_3 - \frac{(F_2-F_3)^2}{F_1-2F_2+F_3}\,. \tag{27}$$

If we introduce the parameter t

$$t = \frac{F_2-F_1}{F_3-F_2} \tag{28}$$

where the 'theoretical' value of t is 2^q, then (27) becomes

$$F(u^*) = F_3 - \frac{1}{1-t}(F_3-F_2). \tag{29}$$

Thus for the trapezium rule t is theoretically $2^2 = 4$ and (29) is the familiar h^2-extrapolation formula. For the Simpson rule t is theoretically 16. In performing computer runs the empirical value of t is determined from (28) and the more closely this relates to the theoretical the more accurate the whole numerical process is regarded.

4. *Preliminary Numerical Investigations.*

The computations in this section were first performed on the University of Strathclyde Sirius computer (at 7–8 significant digits) and some were repeated on the Cambridge University Titan computer (at 13 significant digits) to investigate the effects of 'rounding errors'.

4.1. *No Transformation.*

Using the trapezium rule equation (10) was solved iteratively for the cases (i) $\alpha = 1\ \beta = 0\ \gamma = 0$ (Blasius), (ii) $\alpha = 1\ \beta = 1\ \gamma = 0$ (stagnation flow). The arithmetic mean of successive iterates was used as the next starting value, and generally 16 iterations were performed for a given subdivision of the range of integration. The singularity was ignored, being penetrated to within a distance of $\Delta u^* = 10^{-10}$, thus ensuring a completely negligible error $\varepsilon(u_j^*)$. Virtually any plausible solution can be used to start the process and the actual choice does not materially effect the number of iterations required to bring the solution to within a given difference from its final value. For simplicity the author used $F^{(0)}(u^*) = 1-u^*$ and $\tilde{F}^{(0)}(s) = s$ as starting solutions.

The local coefficient of skin friction c_f and the displacement thickness δ_1 are shown for cases (i) and (ii) above in Tables 1, 4. Three successive extrapolated values are shown, together with the empirical value of t for each (N_f is the final subdivision number, e.g., $N_f = 32$ is used to label the values extrapolated from the results at $h = 1/8, 1/16, 1/32$). In both cases t departs somewhat from the theoretical value 4. This is taken to be the effect of ignoring the singularity in the computation. (For another example of this phenomenon, *see* the article by Noble in Ref. 14.) A similar order of departure from 4 is observed at the intermediate points in the range of integration. The results, nevertheless, are in very satisfactory agreement with the values obtained by other methods. Evidently there is little likelihood of greater accuracy by performing the integrations with the Simpson formula in view of the progressive deterioration of the higher derivatives of F as $u^* \to 1$.

4.2. *Transformation* $s^2 = 1-u^*$.

The above procedure was repeated using equation (17) in cases (i) and (ii) above. The error $\tilde{\varepsilon}(s_j)$ in (17) was similarly rendered negligible by penetrating to within a distance $s_j = 10^{-10}$ of the singularity. The results are displayed in Tables 2, 5. The theoretical value 4 for t was closely attained in all cases, and an overall improvement in accuracy is evident. Bickley's numerical value for the shear stress at the wall as quoted in Ref. 6 is very accurately reproduced. The empirical value of t was generally close to 4 at the intermediate points of the range of integration.

The Simpson formula was next used in case (i) and the results are shown in Table 3 for 8 to 256 subdivisions of the range (no extrapolation). The solution was iterated until the relative difference between successive values of c_f, viz. $2(c_f^{(p)}-c_f^{(p-1)})/(c_f^{(p-1)}+c_f^{(p)})$, was less than 1 in 10^7. As can be seen from Table 3 p was 22 for the Blasius flow, though generally it depends on α, β, γ and on h. The finest subdivision reproduces well the known values. Though it is possible to extrapolate the quoted values, this is not possible for all the intermediate values of the variables since some first increase then decrease (and *vice versa*) as the subdivision becomes finer. As can be seen the Simpson method with a step size of $h = 1/16$ provides an accuracy in the region of 1 in 1000 for both c_f and δ_1. A series of tests was performed for cases in the ranges $\alpha = -1(1)1$, $-1 \leqslant \beta \leqslant 4$, $-1 \leqslant \gamma \leqslant 2$ for which accurate solutions have been obtained by other methods. Providing we are not too near 'separation' conditions a similar order of accuracy is generally obtained. Figs. 1–12, illustrating the variation of the boundary-layer characteristics as functions of the suction parameter γ, were obtained on Sirius using this method.

An extensive comparison of some of the boundary-layer characteristics is shown in Table 6 for (iii) the Polhausen solution $\alpha = 0\ \beta = 1\ \gamma = 0$, using the trapezium rule and extrapolation, and using also the Simpson rule on the mesh $h = 1/128$. Generally the agreement with this known analytical solution (*see* Appendix) is very good.

4.3. *Stability and 'Convergence'.*

On using an integration formula, equation (10) becomes the system of N non-linear algebraic equations (F_N being very near but not actually zero by our 'stopping-short' procedure)

$$F_L = \sum_{M=0}^{N} \frac{a_{LM}}{F_M} + (N-L)h\gamma \qquad \begin{array}{l} L = 0 \ldots N \\ Nh = 1 \end{array} \tag{30}$$

where the a_{LM} are functions of α, β and the step size h.

We start by giving the F_L in (30) (or indeed the $F(u^*)$ in (10)) a small perturbation about their exact values (assuming these exist). For $\gamma = 0$, the modulus of the errors after two successive iterations in which the iterates have been combined in the ratio $k : 1-k$ is then

$$|\xi| = \left| \frac{\delta F_L^{(2)}}{\delta F_L^{(1)}} \right| = |2k-1| \qquad (31)$$

$$\leqslant 1 \text{ for stability.}$$

Thus the perturbation is damped out most rapidly if $k = \frac{1}{2}$. It is this circumstance which permits rapid 'convergence' to a solution* from a given assumed starting solution. The case $k = 1$ (full advance) means that the initial perturbation will not be magnified but this does not in practice lead to convergence. Values of k less than 0·5 cause a change in the sign of the ratio of successive errors and this is useful in aiding the convergence of 'difficult'cases which would not otherwise converge. It may similarly be shown from (10) that the ratio $|\xi|$ will be zero on taking the geometric mean of successive iterates as the next starting value (*see* also Refs. 3, 4). An experiment showed that the number of iterations required to bring the solution to within a specified difference from its final value was substantially the same whether using the arithmetic or the geometric mean. For $0 < k < 0{\cdot}5$ however, the number of iterations required increases as k decreases.

For γ different from zero the present work indicated that positive γ (suction) has a stabilising effect on the numerical procedure with negative γ having the reverse effect, a phenomenon well known in boundary-layer work generally.

The above remarks on stability presuppose that (10) or the system of equations (30) derived from it *have a solution for the given* α, β, γ. While it was never possible to get the numerical process to converge for those α, β, γ for which a solution does not exist, instances were found where a solution is known to exist and in which the numerical process would not converge (*see Case* $\alpha = -1$, next Section). Thus 'convergence' in the present context appears to be a sufficient but not necessary condition for the existence of a solution.

'Direct' solution of the system of non-linear (quadratic) equations (30) would establish the existence or otherwise of solutions for any α, β, γ. Clearly imaginary values for the F_L would be discarded as inadmissable, though negative values might be associated with reversed flow regions. (The inevitable consequence of finite difference or other methods used with non-linear operators, such systems of non-linear algebraic equations are not as yet amenable to 'direct' methods, although Collatz (pp. 146, 212, Ref. 13) solves two pairs of quadratics arising in this way.)

5. *Variation of Boundary Layer Characteristics with Suction Parameter.*

As noted earlier the Simpson rule with a step length of $h = 1/16$ was used in this part of the investigation for the construction of Figs. 1 to 12. For $\alpha = -1(1)1$, γ was varied from -2 (blowing) to $+6$ (suction) with β as a parameter ranging from -4 to $+16$. As it was subsequently recommended that these results be also presented in tabular form, it was decided to carry out very accurate computations on Titan using the Simpson method on the finest mesh $h = 1/256$. The results are displayed in Tables 7 to 33. These correspond to Figs. 1 to 12 which also contain results obtained by other investigators.

Case $\alpha = 1$.

It is known that for $\beta = 0$ the solution does not exist for values of γ below $K_0 = -0{\cdot}87574$. The numerical process failed to converge at or below this value of γ no matter how much the advance factor

*i.e. to the solution of the finite difference scheme approximating the integral equation, as distinct from the convergence of the latter to the solution of the integral equation as $h \to 0$ (*see* for instance, p. 210 Seeger and Temple (eds.), *Research Frontiers in Fluid Dynamics*, 1965).

k was reduced. For positive β solutions were obtained without difficulty with $k = 0{\cdot}5$ and are displayed in Figs. 1 to 4, and in Tables 7 to 17.

For $\beta < 0$ solutions exist only for $\gamma \geqslant \gamma_{01}(\beta)$, the function $\gamma_{01}(\beta)$ being displayed on p. 249 Ref. 6. As the 'separation' profiles are approached the characteristics approach the verticals through the points $\gamma_{01}(\beta)$ on the γ-axis, the numerical procedure giving a solution up to these points but not beyond. It was necessary to reduce the advance factor k and increase the number of iterations as these points were approached. This process was built automatically into the computer programme. If the solution did not converge after a given (large) number of iterations then k was reduced by 0·05 and the computations started afresh. This was repeated successively until the solution did in fact converge to the specified relative accuracy.

Results obtained by Emmons and Leigh[15] (not all of their points are shown), Schlichting and Bussman[16], Brown and Donoughe[17], Thwaites[18] are also displayed in the Figures. Agreement is excellent, in particular Thwaites' analytical solution ($\alpha = 1, \beta = -1$) is reproduced exactly.

It is interesting that the boundary-layer thickness curves for the asymptotic suction solution

$$(\delta_1/g)_{asy} = 1/\gamma \qquad (\delta_2/g)_{asy} = 1/2\gamma \qquad (\delta_3/g)_{asy} = 5/6\gamma \tag{32a,b,c}$$

do not lie intermediate between the curves for $\beta = 0$ and $-0{\cdot}5$ (Figs. 2, 3, 4) as does the skin-friction curve

$$(c_f)_{asy} = \gamma \tag{33}$$

in Fig. 1. This circumstance is connected with the fact that although the wall shear stress for $\beta = -0{\cdot}5$ is less than that of the asymptotic suction solution the shear stress in the outer part of the layer is greater and this implies a lower value of the δ's by equations (20) – (24).

For the flow near separation ($\beta \to -0{\cdot}1988$ from above) the present procedure yielded the Hartree, and not the Stewartson solutions. Generally as separation is approached the tendency to converge to the solution deteriorated and the factor k had to be reduced with consequent increase in the number of iterations.

Case $\alpha = 0$.

With $\beta = -1$ a solution does not exist for $\gamma < \sqrt{8}$ and this is again borne out by the present method. The tendency to converge is even poorer (than for $\alpha = 1$) as this value is approached. As in case $\alpha = 1$ cut-off values $\gamma_{00}(\beta)$ exist for other β but these could not be located with any accuracy by the present method. Broken lines in Figs. 5, 6, 7, 8 show the regions within which the $\gamma_{00}(\beta)$ may lie.

Numerical results given by Thwaites[18], Holstein[19] are also displayed in the Figures. These are in agreement with the present results, which are given numerically in Tables 18 to 27.

Case $\alpha = -1$.

These results are shown in Figs. 9, 10, 11, 12, and in Tables 28 to 33. The broken lines in the Figures show the regions where it became increasingly difficult and ultimately impossible as γ was reduced to get the numerical process to converge at all. It was sometimes necessary to increase the mesh size to get the solution to converge without requiring an extremely large number of iterations at very low values of k. The curve $\beta = 1$ (Fig. 5) was completed with the aid of Terril's results[20]. Thus we have regions where the present numerical process would not converge and for which solutions are in fact known to exist.

6. *Concluding Discussion.*

With regard to accuracy the transformation $s^2 = 1 - u^*$ used with the trapezium rule and extrapolation gives very satisfactory results in the cases considered. There is a definite improvement over the results obtained with no transformation, evidently the outcome of weakening the singularity. On the basis of Tables 2 and 5 it does not appear worthwhile to go beyond $h = 1/128$ as the final mesh size. The Simpson rule with $s^2 = 1-u^*$ though yielding results which cannot always be extrapolated, gives, nevertheless a comparable accuracy when used only once on the finest mesh. The 'rounding' process in the Sirius computations took place at the 7th–8th significant digit. At the time it was felt that this was somewhat low and possibly affected the accuracy of the 6th significant digit. However, the Titan computations, carried out at 13 significant digits (italics in Tables 2 and 5), indicate that the 6th digit had suffered very little from rounding errors.

As noted in Section 5 Tables 7 to 33 were computed using the Simpson method on the finest mesh $h = 1/256$ and incorporating the device of reducing the value of k until the solution converged to the specified relative accuracy. This gave a simple fully automatic computing procedure producing results of high accuracy. The simultaneous print-out of the number of iterations and the value of k gave a running check on the 'character' of the solution at the appropriate values of α, β, γ – in particular near 'separation' conditions and in the difficult region of small β, γ for $\alpha = -1$. Since analytical and numerical studies to date have strongly suggested the separation point to be a singularity, it was decided in producing these Tables to quote a smaller number of significant figures when near separation conditions; this was done also in the difficult region just referred to. Otherwise the results should be accurate to six significant figures and certainly to five. It may be that in the critical cases the sixth digit (when given) is uncertain to 1 unit. In any event, the present results agree exactly with all the accurate solutions presently known to the author.*

The singularity can be removed altogether by use of the transformation (18), but the resulting integral equation has exponential kernels. The range of integration is now infinite, the main contributions to the integrals coming from the ends of the range. This would seem more fitted to Gaussian integration, for which the present extrapolation methods become inapplicable. It is questionable if increased accuracy could be obtained in this way on a given computer in a given time.

From the viewpoint of stability the method is generally very satisfactory for values of α, β, γ away from 'separation' conditions, namely, very small values of the wall shear stress. It is a rule that as these conditions are approached the tendency to converge deteriorates; the advance factor k then requires progressive reduction, while the number of iterations increases correspondingly. There is also the difficult region of small β, γ for $\alpha = -1$. Evidently similar regions exist for other negative α. As noted previously, 'convergence' in the context of this investigation is a sufficient but not necessary condition for the existence of a solution. Generally suction has a stabilising effect, as it does on the physical flow.

For non-similar flows the method appears feasible as a continuation process from a known solution at x_0. In this connection it should be possible to reformulate the integral equation in terms of Gadd's[21] variables so as to allow x_0 to coincide with a sharp leading edge. No difficulty is anticipated in regions of falling pressure, while the necessity to reduce k as separation conditions are approached may be exploitable as a means of indicating the proximity to the separation point. Further work along these lines would seem to be justified.

In closing, we remark that Mangler's transformation makes possible the application of the technique to the class of axi-symmetric boundary layers.

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*Subsequent to the writing of this Report the author has encountered the Papers by H. L. Evans, ARC CP 857 (1966), and by M. Zamir and A. D. Young, ARC 28 100 (1966). Both of these authors consider the equation of similar profiles in the form $f''' + ff'' + \beta(1-f'^2) = 0$, Evans utilising an alternative representation to deal with our case $\alpha = 0$. Both use Runge-Kutta methods, with an interpolation procedure to meet the outer edge boundary condition. Evans evaluates considerably fewer cases than the present author and does not give the energy thickness δ_3. His results, however, where they overlap, are gratifyingly in agreement with the present results. The Zamir and Young paper restricts its attention to six values of β in the range $-2/9$ to -180 (with our $\alpha = 1$) but gives γ a wider range of variation. In the one overlapping instance, *viz.* $\beta = -0{\cdot}5$, their results are in agreement with the present results. This Report gives accurate values of c_f and accurate velocity profiles $f'(\eta)$, but unfortunately does not give the boundary-layer thicknesses δ_1, δ_2, δ_3.

7. *Acknowledgements.*

The results given in this paper were computed by the author on the University of Strathclyde Sirius computer and on the Cambridge University Titan computer. While performing the Titan computations he was in grateful receipt of an I.C.I. Research Fellowship.

LIST OF SYMBOLS

x, z	Orthogonal curvilinear co-ordinates of the boundary layer
u	x-component of velocity in boundary layer
$U(x)$	Velocity outside boundary layer
$w_s(x)$	Velocity normal to the wall at $z = 0$
τ	Shear stress
$p'(x)$	Pressure gradient
μ, ρ	Coefficient of viscosity and density of fluid
$\nu = \mu/\rho$	Kinematic viscosity of fluid.
$u^* = u/U$	Non-dimensional velocity ratio
$F(u^*)$	Shear-stress function defined in equation (7)
$g(x)$	Length defined in equation (12b)
α, β, γ, n	Parameters characterising the similarity flows (*see* eqs. (11), (12))
U_0, x_0, l	Reference speed, position, length (*see* eq. (12))
s	Transformed independent variable ($s^2 = 1-u^*$)
$\tilde{F}(s)$	Transformed shear-stress function
ε	Stop-short error (*see* eq. (17))
c_f	Local coefficient of skin friction
$Z(u^*), \tilde{Z}(s)$	Variables defined in equations (20), (21)
$\delta_1, \delta_2, \delta_3$	Displacement, momentum, energy thicknesses
$\phi, \tilde{\phi}$	Functionals corresponding to right-hand sides of eqs. (10), (17)
h	Step length in integration formulae ($Nh = 1$)
N	Number of subdivisions of range of integration
t, q	Parameters in extrapolation formula
k	Advance factor in iteration process
a_{LM}	Coefficients depending on α, β and h (*see* eq. (30))
ξ	Error propagation ratio
N.B.	The symbol $\sim$ is used throughout to denote transformed dependent variables. The suffix j refers to the stop-short point. The superscript p in brackets refers to the number of iterations. In Tables 1 to 33 δ_1, δ_2, δ_3 are to be regarded as normalised by the length $g(x)$

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APPENDIX

A.

$$\int_u^{U(x)} du_2 \int_0^{u_2} f(u_1)\, du_1 = (U-u) \int_0^u f(u)\, du + \int_u^U (U-u)\, f(u)\, du$$

B.

The 'error' $\tilde{\varepsilon}(s_j)$ may be shown to be

$$\tilde{\varepsilon}(s_j) = \int_0^{s_j} \frac{2(\alpha+2\beta)s^3 - 2(\alpha+\beta)s^5}{\tilde{F}(s)}\, ds$$

which takes the values (aside from overall multiplicative constants of order unity depending on α, β, γ)

$$(\alpha+2\beta)\,\mathrm{erfc}\left[\sqrt{2}\,(\log\frac{1}{s_j})^{\frac{1}{2}}\right] - \frac{\alpha+\beta}{\sqrt{2}}\,\mathrm{erfc}\left[2\,(\log\frac{1}{s_j})^{\frac{1}{2}}\right] \qquad \alpha \neq 0$$

$$\beta(s_j^{\,2} - s_j^{\,3}) \qquad \alpha = 0$$

on utilising (13a, b) and (14). Analagous results can be obtained for the error $\varepsilon(u_j^*)$.

C.

Polhausen's analytical solution for the flow between converging planes ($\alpha = 0$ $\beta = 1$ $\gamma = 0$) takes the form (*see* p. 236 Ref. 6)

$$u^* = 3\tanh^2\left(\eta_0 + \frac{\eta}{\sqrt{2}}\right) - 2 \qquad \text{(A.1)}$$

where

$$\eta = z/g(x)\,, \qquad \eta_0 = \tanh^{-1}\sqrt{(2/3)} = 1{\cdot}146216 \qquad \text{(A.2)}$$

In the present Report $du^*/d\eta$ corresponds to our F, so that

$$F(\eta) = \frac{6\sinh\left(\eta_0 + \frac{\eta}{\sqrt{2}}\right)}{\sqrt{2}\cosh^3\left(\eta_0 + \frac{\eta}{\sqrt{2}}\right)} \qquad \text{(A.3)}$$

and the relation between η and u^* is obtained by inverting (A.1)

$$\eta(u^*) = \frac{1}{2}\log\frac{\sqrt{3}+\sqrt{(u^*+2)}}{\sqrt{3}-\sqrt{(u^*+2)}} - \sqrt{2}\,\eta_0 \qquad \text{(A.4)}$$

The coefficient of skin friction is then

$$c_f = F(0) = 2/\sqrt{3} = 1{\cdot}154700 \qquad \text{(A.5)}$$

On evaluating the displacement-thickness integral from (A.1) we find our variable Z may be written

$$Z(\eta) = \frac{6}{\sqrt{2}} \tanh\left(\eta_0 + \frac{\eta}{\sqrt{2}} \right) - \tanh \eta_0 \tag{A.6}$$

giving the displacement thickness ($\eta \to \infty$) the value

$$\frac{\delta_1}{g(x)} = \frac{6}{\sqrt{2}} \left[1 - \sqrt{\frac{2}{3}} \right] = 0{\cdot}778539\,. \tag{A.7}$$

TABLE 1

N_f	c_f	t	δ_1 †	t
32	0·469 385 36	2·8	1·217 067 2	3·0
64	0·469 524 06	2·6	1·216 870 8	2·8
128	0·469 574 97	2·5	1·216 810 3	2·6
*	0·469 600	—	1·216 78	—

Blasius ($\alpha = 1$ $\beta = 0$ $\gamma = 0$). Extrapolated c_f, δ_1.
No transformation. Trapezium integration.
Computed on 'Sirius'.
*denotes results quoted on Ref. 6.

TABLE 2

N_f	c_f	t	δ_1	t
32	*603 1* 0·469 597 59	*4·0* 4·0	*024 9* 1·216 853 5	*4·2* 4·2
64	*37 8* 0·469 601 77	*4·0* 4·0	*4 191 4* 1·216 785 7	*4·0* 4·0
128	*1 963 8* 0·469 602 02	*4·0* 4·0	*79 843 4* 1·216 781 0	*4·0* 4·0
256	*0·469 601 969 1*	*4·0*	*1·216 779 577 7*	*4·0*
*	0·469 600	—	1·216 78	—

Blasius ($\alpha = 1$ $\beta = 0$ $\gamma = 0$). Extrapolated c_f, δ_1.
Transformation $s^2 = 1 - u^*$. Trapezium integration.
Italics denote 'Titan' results.
*denotes results quoted in Ref. 6.

†N.B.—For brevity the normalising length $g(x)$, whose value is given by equations (11) and (12), has been suppressed in labelling the boundary-layer thicknesses δ_1, δ_2, δ_3 in Tables 1 to 33.

TABLE 3

N	c_f	δ_1	No. of iterations
8	0·466 954 116 2	1·216 594 644 7	22
16	0·469 232 029 0	1·216 653 194 9	22
32	0·469 548 013 5	1·216 812 127 1	22
64	0·469 591 774 4	1·216 791 898 0	22
128	0·469 598 636 0	1·216 782 823 4	22
256	0·469 599 777 2	1·216 780 979 0	22
*	0·469 600	1·216 78	—

Blasius ($\alpha = 1$ $\beta = 0$ $\gamma = 0$). c_f, δ_1.
Transformation $s^2 = 1 - u^*$. Simpson integration.
Computed on 'Titan'.
*denotes results quoted in Ref. 6.

TABLE 4

N_f	c_f	t	δ_1	t
32	1·232 489 5	3·0	0·647 923 9	2·9
64	1·232 550 9	2·9	0·647 907 2	2·8
128	1·232 574 0	2·8	0·647 902 9	2·7
*	1·232 588	—	0·647 90	—

Stagnation flow ($\alpha = 1$ $\beta = 1$ $\gamma = 0$) Extrapolated c_f, δ_1.
No transformation. Trapezium integration.
Computed on 'Sirius'.
*denotes results quoted in Ref. 6.

TABLE 5

N_f	c_f	t	δ_1	t
32	*89 778 8* 1·232 590 0	*4·0* 4·0	*535 0* 0·647 902 6	*4·0* 4·0
64	*049 5* 1·232 588 4	*4·0* 4·0	*552 3* 0·647 900 6	*4·0* 4·0
128	*890 2* 1·232 587 9	*4·0* 4·0	*437 3* 0·647 900 8	*4·0* 4·0
256	*1·232 587 875 4*	*4·0*	*0·647 900 430 3*	*4·0*
*	1·232 588	—	0·647 90	—

Stagnation flow ($\alpha = 1$ $\beta = 1$ $\gamma = 0$). Extrapolated c_f, δ_1.
Transformation $s^2 = 1-u^*$. Trapezium integration.
Italics denote 'Titan' results.
*denotes results quoted in Ref. 6.

TABLE 6

s		$u^*(s)$	$u_e^*(s)$	$c_f(s)$	t	$c_{fe}(s)$	$\tilde{Z}(s)/g$	t	$\tilde{Z}_e(s)/g$	$\tilde{z}(s)/g$	t	$\tilde{z}_e(s)/g$
0·000	T'	1·000 000	1·000 000	0·000 000 00	—	0·000 000	0·778 540 2	4·0	0·778 539	∞	—	∞
	T''						0·778 538 5	4·0				
	T'''						0·778 537 2	3·9				
	S						0·778 538 2	—				
0·250	T'	0·937 500	0·937 498	0·087 463 57	3·9	0·087 467	0·734 113 6	4·0	0·734 110	2·089 157 4	3·9	2·089 192
	T''		0·937 500	0·087 461 99	4·6	0·087 463	0·734 111 5	4·0	0·734 112	2·089 193 7	4·0	
	T'''		0·937 500	0·087 462 20	4·1	0·087 463	0·734 109 9	3·9	0·734 112	2·089 188 7	4·0	
	S		0·937 500	0·087 462 47	—	0·087 463	0·734 111 3	—	0·734 112	2·089 191 6	—	
0·500	T'	0·750 000	0·750 000	0·338 500 05	4·0	0·338 502	0·597 918 5	4·0	0·597 918	1·085 929 6	4·0	1·085 929
	T''		0·750 000	0·338 501 56	3·9	0·338 502	0·597 916 2	4·0	0·597 917	1·085 927 7	4·0	
	T'''		0·749 999	0·338 502 88	3·5	0·338 504	0·597 915 2	3·9	0·597 917	1·085 925 4	3·9	
	S		0·750 000	0·338 501 42	—	0·338 502	0·597 916 6	—	0·597 917	1·085 928 2	—	
0·750	T'	0·437 500	0·437 500	0·717 049 5	4·0	0·717 050	0·360 163 24	4·0	0·360 163	0·471 436 88	4·0	0·471 436
	T''		0·437 499	0·717 051 6	3·9	0·717 050	0·360 161 99	4·0	0·360 162	0·471 435 91	4·0	
	T'''		0·437 499	0·717 052 8	3·8	0·717 051	0·360 161 66	3·9	0·360 162	0·471 435 36	3·9	
	S		0·437 499	0·717 050 1	—	0·717 050	0·360 162 40	—	0·360 162	0·471 436 36	—	
1·000	T'	0·000 000	0·000 000	1·154 702 5	4·0	1·154 700	0·000 000	—	0·000 000	0·000 000	—	0·000 000
	T''			1·154 703 8	4·0							
	T'''			1·154 704 4	3·9							
	S			1·154 701 3	—							

Comparison of numerical results with Polhausen's analytical solution ($\alpha = 0$ $\beta = 1$ $\gamma = 0$) using transformation $1-u^* = s^2$. T' denotes first extrapoltation from $h/8$, $h/16$, $h/32$, and so on, using trapezium rule. S denotes Simpson integration at $h = 1/128$. t as defined in equation (28). Suffix e denotes exact analytical solution.

Computed on 'Sirius'.

TABLE 7

γ	c_f	δ_1	δ_2	δ_3
-2.0	3.65608	0.23499	0.11155	0.18397
-1.8	3.74492	0.23027	0.10944	0.18054
-1.6	3.83583	0.22565	0.10738	0.17717
-1.4	3.92881	0.22112	0.10535	0.17386
-1.2	4.02387	0.21668	0.10335	0.17061
-1.0	4.12101	0.21233	0.10140	0.16742
-0.8	4.22021	0.20807	0.09948	0.16429
-0.6	4.32148	0.20390	0.09760	0.16122
-0.4	4.42479	0.19982	0.09575	0.15820
-0.2	4.53015	0.19582	0.09394	0.15525
0.0	4.63753	0.19192	0.09217	0.15235
0.2	4.74692	0.18810	0.09043	0.14951
0.4	4.85831	0.18437	0.08873	0.14673
0.6	4.97166	0.18072	0.08707	0.14400
0.8	5.08697	0.17716	0.08544	0.14133
1.0	5.20420	0.17368	0.08384	0.13872
1.2	5.32333	0.17029	0.08228	0.13617
1.4	5.44435	0.16697	0.08075	0.13367
1.6	5.56721	0.16373	0.07926	0.13122
1.8	5.69190	0.16058	0.07780	0.12883
2.0	5.81837	0.15750	0.07638	0.12649
2.2	5.94661	0.15449	0.07499	0.12420
2.4	6.07658	0.15156	0.07362	0.12197
2.6	6.20825	0.14870	0.07229	0.11979
2.8	6.34159	0.14592	0.07100	0.11765
3.0	6.47656	0.14320	0.06973	0.11557
3.2	6.61313	0.14055	0.06849	0.11353
3.4	6.75127	0.13797	0.06728	0.11155
3.6	6.89094	0.13545	0.06610	0.10960
3.8	7.03212	0.13300	0.06495	0.10771
4.0	7.17475	0.13061	0.06382	0.10586
4.2	7.31883	0.12828	0.06272	0.10405
4.4	7.46429	0.12601	0.06165	0.10229
4.6	7.61113	0.12380	0.06061	0.10056
4.8	7.75929	0.12164	0.05959	0.09888
5.0	7.90876	0.11954	0.05859	0.09724
5.2	8.05949	0.11750	0.05762	0.09564
5.4	8.21145	0.11550	0.05667	0.09408
5.6	8.36462	0.11356	0.05575	0.09255
5.8	8.51895	0.11166	0.05485	0.09106
6.0	8.67443	0.10981	0.05396	0.08961

Boundary Layer Characteristics as Functions of γ.

$\alpha = 1 \quad \beta = 16$

TABLE 8

γ	c_f	δ_1	δ_2	δ_2
-2.0	2.34869	0.35588	0.16685	0.27435
-1.8	2.42994	0.34593	0.16250	0.26730
-1.6	2.51409	0.33623	0.15825	0.26040
-1.4	2.60116	0.32679	0.15409	0.25366
-1.2	2.69116	0.31760	0.15004	0.24709
-1.0	2.78410	0.30866	0.14609	0.24067
-0.8	2.87998	0.29998	0.14224	0.23441
-0.6	2.97878	0.29154	0.13849	0.22832
-0.4	3.08050	0.28336	0.13484	0.22238
-0.2	3.18511	0.27543	0.13130	0.21660
0.0	3.29258	0.26774	0.12785	0.21099
0.2	3.40288	0.26030	0.12450	0.20553
0.4	3.51598	0.25309	0.12125	0.20023
0.6	3.63182	0.24612	0.11809	0.19508
0.8	3.75036	0.23938	0.11503	0.19009
1.0	3.87155	0.23287	0.11207	0.18524
1.2	3.99533	0.22657	0.10920	0.18055
1.4	4.12165	0.22049	0.10641	0.17600
1.6	4.25044	0.21462	0.10372	0.17159
1.8	4.38164	0.20896	0.10111	0.16732
2.0	4.51519	0.20349	0.09859	0.16318
2.2	4.65102	0.19821	0.09615	0.15919
2.4	4.78906	0.19312	0.09379	0.15532
2.6	4.92925	0.18821	0.09150	0.15157
2.8	5.07152	0.18348	0.08930	0.14795
3.0	5.21580	0.17891	0.08717	0.14445
3.2	5.36202	0.17451	0.08510	0.14106
3.4	5.51013	0.17026	0.08311	0.13779
3.6	5.66004	0.16617	0.08119	0.13462
3.8	5.81171	0.16222	0.07933	0.13156
4.0	5.96506	0.15841	0.07753	0.12860
4.2	6.12003	0.15474	0.07579	0.12574
4.4	6.27657	0.15119	0.07411	0.12298
4.6	6.43461	0.14778	0.07249	0.12030
4.8	6.59410	0.14448	0.07092	0.11772
5.0	6.75498	0.14129	0.06940	0.11522
5.2	6.91720	0.13822	0.06794	0.11280
5.4	7.08070	0.13525	0.06652	0.11046
5.6	7.24544	0.13239	0.06515	0.10820
5.8	7.41136	0.12962	0.06382	0.10601
6.0	7.57842	0.12694	0.06254	0.10389

Boundary Layer Characteristics as Functions of γ.

$\alpha = 1 \quad \beta = 8$

TABLE 9

γ	c_f	δ_1	δ_2	δ_3
-2.0	1.45312	0.54885	0.25155	0.41131
-1.8	1.52419	0.52775	0.24264	0.39700
-1.6	1.59921	0.50732	0.23399	0.38309
-1.4	1.67828	0.48758	0.22559	0.36959
-1.2	1.76144	0.46853	0.21746	0.35650
-1.0	1.84875	0.45019	0.20960	0.34383
-0.8	1.94020	0.43255	0.20200	0.33158
-0.6	2.03582	0.41561	0.19468	0.31976
-0.4	2.13557	0.39936	0.18762	0.30836
-0.2	2.23941	0.38381	0.18083	0.29739
0.0	2.34729	0.36893	0.17431	0.28683
0.2	2.45913	0.35472	0.16805	0.27669
0.4	2.57485	0.34115	0.16205	0.26695
0.6	2.69435	0.32822	0.15630	0.25762
0.8	2.81751	0.31589	0.15079	0.24867
1.0	2.94424	0.30415	0.14553	0.24011
1.2	3.07439	0.29298	0.14049	0.23191
1.4	3.20783	0.28236	0.13568	0.22408
1.6	3.34445	0.27225	0.13109	0.21659
1.8	3.48411	0.26264	0.12671	0.20944
2.0	3.62666	0.25351	0.12252	0.20260
2.2	3.77198	0.24483	0.11853	0.19608
2.4	3.91994	0.23658	0.11472	0.18985
2.6	4.07041	0.22874	0.11109	0.18390
2.8	4.22326	0.22128	0.10763	0.17822
3.0	4.37838	0.21419	0.10432	0.17281
3.2	4.53564	0.20745	0.10117	0.16763
3.4	4.69493	0.20103	0.09816	0.16269
3.6	4.85614	0.19493	0.09529	0.15797
3.8	5.01918	0.18911	0.09255	0.15347
4.0	5.18394	0.18357	0.08993	0.14916
4.2	5.35032	0.17830	0.08743	0.14504
4.4	5.51825	0.17326	0.08504	0.14111
4.6	5.68762	0.16846	0.08275	0.13734
4.8	5.85838	0.16388	0.08057	0.13374
5.0	6.03043	0.15951	0.07848	0.13030
5.2	6.20371	0.15533	0.07648	0.12700
5.4	6.37815	0.15134	0.07456	0.12383
5.6	6.55369	0.14752	0.07273	0.12080
5.8	6.73027	0.14386	0.07097	0.11790
6.0	6.90783	0.14036	0.06928	0.11511

Boundary Layer Characteristics as Functions of γ.

$\alpha = 1 \quad \beta = 4$

TABLE 10

γ	c_f	δ_1	δ_2	δ_3
-2.0	0.85606	0.86043	0.37840	0.61241
-1.8	0.91410	0.81595	0.36073	0.58448
-1.6	0.97728	0.77316	0.34366	0.55747
-1.4	1.04587	0.73212	0.32721	0.53142
-1.2	1.12006	0.69292	0.31141	0.50637
-1.0	1.20000	0.65559	0.29627	0.48234
-0.8	1.28578	0.62018	0.28181	0.45935
-0.6	1.37745	0.58667	0.26803	0.43741
-0.4	1.47497	0.55507	0.25494	0.41653
-0.2	1.57827	0.52534	0.24253	0.39669
0.0	1.68722	0.49743	0.23078	0.37790
0.2	1.80165	0.47129	0.21969	0.36011
0.4	1.92136	0.44683	0.20923	0.34332
0.6	2.04612	0.42398	0.19938	0.32747
0.8	2.17569	0.40266	0.19012	0.31255
1.0	2.30982	0.38278	0.18142	0.29850
1.2	2.44823	0.36424	0.17325	0.28529
1.4	2.59067	0.34696	0.16558	0.27286
1.6	2.73688	0.33086	0.15838	0.26119
1.8	2.88660	0.31585	0.15163	0.25023
2.0	3.03961	0.30185	0.14530	0.23993
2.2	3.19566	0.28879	0.13936	0.23025
2.4	3.35453	0.27660	0.13378	0.22115
2.6	3.51603	0.26521	0.12854	0.21260
2.8	3.67996	0.25455	0.12362	0.20455
3.0	3.84614	0.24458	0.11899	0.19698
3.2	4.01441	0.23524	0.11464	0.18986
3.4	4.18460	0.22648	0.11054	0.18314
3.6	4.35658	0.21826	0.10668	0.17681
3.8	4.53021	0.21054	0.10305	0.17083
4.0	4.70537	0.20327	0.09961	0.16519
4.2	4.88196	0.19643	0.09637	0.15985
4.4	5.05986	0.18998	0.09330	0.15481
4.6	5.23898	0.18389	0.09040	0.15003
4.8	5.41924	0.17814	0.08766	0.14551
5.0	5.60056	0.17270	0.08505	0.14121
5.2	5.78286	0.16755	0.08258	0.13714
5.4	5.96607	0.16268	0.08024	0.13327
5.6	6.15013	0.15805	0.07801	0.12959
5.8	6.33499	0.15366	0.07589	0.12609
6.0	6.52058	0.14948	0.07387	0.12276

Boundary Layer Characteristics as Functions of γ.

$\alpha = 1 \quad \beta = 2$

TABLE 11

γ	c_f	δ_1	δ_2	δ_3
-2.0	0.47581	1.36162	0.55709	0.88577
-1.8	0.51856	1.27017	0.52420	0.83518
-1.6	0.56738	1.18235	0.49251	0.78640
-1.4	0.62295	1.09863	0.46216	0.73959
-1.2	0.68585	1.01941	0.43322	0.69489
-1.0	0.75657	0.94498	0.40580	0.65243
-0.8	0.83543	0.87556	0.37993	0.61228
-0.6	0.92256	0.81123	0.35567	0.57450
-0.4	1.01794	0.75195	0.33300	0.53909
-0.2	1.12139	0.69758	0.31190	0.50603
0.0	1.23259	0.64790	0.29234	0.47528
0.2	1.35114	0.60264	0.27425	0.44673
0.4	1.47660	0.56148	0.25756	0.42030
0.6	1.60844	0.52409	0.24218	0.39587
0.8	1.74618	0.49015	0.22802	0.37331
1.0	1.88931	0.45932	0.21500	0.35250
1.2	2.03736	0.43132	0.20302	0.33330
1.4	2.18986	0.40585	0.19200	0.31560
1.6	2.34639	0.38266	0.18187	0.29926
1.8	2.50658	0.36151	0.17253	0.28419
2.0	2.67006	0.34220	0.16393	0.27026
2.2	2.83650	0.32452	0.15599	0.25739
2.4	3.00564	0.30832	0.14866	0.24548
2.6	3.17719	0.29343	0.14188	0.23444
2.8	3.35093	0.27974	0.13559	0.22420
3.0	3.52664	0.26710	0.12977	0.21468
3.2	3.70414	0.25543	0.12435	0.20584
3.4	3.88327	0.24463	0.11932	0.19759
3.6	4.06386	0.23461	0.11463	0.18991
3.8	4.24580	0.22530	0.11025	0.18272
4.0	4.42895	0.21663	0.10616	0.17601
4.2	4.61320	0.20855	0.10233	0.16972
4.4	4.79848	0.20099	0.09874	0.16381
4.6	4.98467	0.19393	0.09537	0.15827
4.8	5.17172	0.18730	0.09221	0.15305
5.0	5.35954	0.18109	0.08923	0.14814
5.2	5.54808	0.17524	0.08642	0.14351
5.4	5.73727	0.16973	0.08377	0.13914
5.6	5.92707	0.16454	0.08127	0.13501
5.8	6.11744	0.15964	0.07890	0.13110
6.0	6.30832	0.15501	0.07665	0.12739

Boundary Layer Characteristics as Functions of γ.

$\alpha = 1 \quad \beta = 1$

TABLE 12

γ	c_f	δ_1	δ_2	δ_3
-2.0	0.24979	2.15036	0.78307	1.21148
-1.8	0.27707	1.97138	0.72759	1.12962
-1.6	0.31033	1.79841	0.67408	1.05060
-1.4	0.35102	1.63329	0.62292	0.97490
-1.2	0.40062	1.47793	0.57444	0.90296
-1.0	0.46043	1.33402	0.52895	0.83518
-0.8	0.53136	1.20273	0.48666	0.77186
-0.6	0.61382	1.08457	0.44769	0.71316
-0.4	0.70773	0.97934	0.41204	0.65911
-0.2	0.81260	0.88634	0.37962	0.60963
0.0	0.92768	0.80455	0.35027	0.56454
0.2	1.05207	0.73279	0.32378	0.52358
0.4	1.18481	0.66985	0.29992	0.48646
0.6	1.32498	0.61462	0.27845	0.45285
0.8	1.47171	0.56604	0.25913	0.42245
1.0	1.62420	0.52321	0.24173	0.39494
1.2	1.78174	0.48533	0.22605	0.37003
1.4	1.94371	0.45172	0.21190	0.34746
1.6	2.10955	0.42179	0.19910	0.32698
1.8	2.27879	0.39505	0.18751	0.30836
2.0	2.45101	0.37106	0.17699	0.29140
2.2	2.62586	0.34948	0.16741	0.27593
2.4	2.80301	0.32999	0.15868	0.26179
2.6	2.98221	0.31233	0.15070	0.24883
2.8	3.16322	0.29628	0.14338	0.23693
3.0	3.34583	0.28165	0.13667	0.22599
3.2	3.52986	0.26827	0.13049	0.21590
3.4	3.71517	0.25600	0.12478	0.20658
3.6	3.90162	0.24471	0.11951	0.19795
3.8	4.08910	0.23431	0.11463	0.18994
4.0	4.27749	0.22469	0.11009	0.18251
4.2	4.46671	0.21578	0.10588	0.17558
4.4	4.65667	0.20751	0.10195	0.16912
4.6	4.84732	0.19980	0.09828	0.16308
4.8	5.03858	0.19262	0.09485	0.15743
5.0	5.23040	0.18591	0.09163	0.15213
5.2	5.42273	0.17963	0.08861	0.14715
5.4	5.61553	0.17374	0.08577	0.14247
5.6	5.80876	0.16821	0.08310	0.13806
5.8	6.00238	0.16300	0.08059	0.13390
6.0	6.19635	0.15809	0.07821	0.12997

Boundary Layer Characteristics as Functions of γ.

$\alpha = 1 \quad \beta = 0{\cdot}5$

TABLE 13

γ	c_f	δ_1	δ_2	δ_3
-0.6	0.0975	2.4834	0.6975	1.0481
-0.4	0.2049	1.8421	0.6049	0.9268
-0.2	0.3305	1.4681	0.5305	0.8248
0.0	0.46960	1.21678	0.46960	0.73848
0.2	0.61904	1.03492	0.41904	0.66497
0.4	0.77661	0.89698	0.37661	0.60202
0.6	0.94068	0.78886	0.34068	0.54784
0.8	1.11001	0.70201	0.31001	0.50097
1.0	1.28363	0.63089	0.28363	0.46021
1.2	1.46080	0.57170	0.26080	0.42460
1.4	1.64091	0.52181	0.24091	0.39334
1.6	1.82348	0.47925	0.22348	0.36576
1.8	2.00812	0.44261	0.20812	0.34131
2.0	2.19451	0.41077	0.19451	0.31956
2.2	2.38239	0.38289	0.18239	0.30010
2.4	2.57156	0.35831	0.17156	0.28264
2.6	2.76183	0.33651	0.16183	0.26691
2.8	2.95305	0.31705	0.15305	0.25268
3.0	3.14510	0.29959	0.14510	0.23976
3.2	3.33788	0.28386	0.13788	0.22800
3.4	3.53129	0.26961	0.13129	0.21725
3.6	3.72526	0.25666	0.12526	0.20740
3.8	3.91973	0.24484	0.11973	0.19834
4.0	4.11463	0.23402	0.11463	0.18999
4.2	4.30993	0.22407	0.10993	0.18228
4.4	4.50558	0.21490	0.10558	0.17513
4.6	4.70155	0.20643	0.10155	0.16849
4.8	4.89780	0.19858	0.09780	0.16232
5.0	5.09430	0.19128	0.09430	0.15655
5.2	5.29103	0.18449	0.09103	0.15117
5.4	5.48798	0.17814	0.08798	0.14613
5.6	5.68511	0.17221	0.08511	0.14140
5.8	5.88242	0.16665	0.08242	0.13695
6.0	6.07989	0.16143	0.07989	0.13277

Boundary Layer Characteristics as Functions of γ.

$\alpha = 1 \quad \beta = 0$

TABLE 14

γ	c_f	δ_1	δ_2	δ_3
0.8	0.4446	1.1354	0.4245	0.6655
1.0	0.7355	0.8961	0.3670	0.5839
1.2	0.9853	0.7534	0.3240	0.5204
1.4	1.21822	0.65361	0.29004	0.46899
1.6	1.44198	0.57845	0.26242	0.42645
1.8	1.66007	0.51932	0.23946	0.39065
2.0	1.87435	0.47136	0.22007	0.36012
2.2	2.08595	0.43159	0.20348	0.33379
2.4	2.29555	0.39802	0.18913	0.31087
2.6	2.50365	0.36930	0.17660	0.29076
2.8	2.71058	0.34442	0.16557	0.27298
3.0	2.91657	0.32266	0.15580	0.25716
3.2	3.12180	0.30347	0.14708	0.24301
3.4	3.32642	0.28642	0.13925	0.23027
3.6	3.53051	0.27116	0.13219	0.21876
3.8	3.73418	0.25743	0.12579	0.20830
4.0	3.93747	0.24502	0.11997	0.19877
4.2	4.14046	0.23373	0.11465	0.19005
4.4	4.34317	0.22344	0.10977	0.18204
4.6	4.54564	0.21400	0.10528	0.17466
4.8	4.74791	0.20532	0.10113	0.16784
5.0	4.94999	0.19731	0.09729	0.16152
5.2	5.15192	0.18990	0.09373	0.15565
5.4	5.35370	0.18302	0.09042	0.15018
5.6	5.55535	0.17662	0.08732	0.14507
5.8	5.75689	0.17065	0.08443	0.14030
6.0	5.95833	0.16506	0.08172	0.13582

Boundary Layer Characteristics as Functions of γ.

$\alpha = 1 \quad \beta = -0{\cdot}5$

TABLE 15

γ	c_f	δ_1	δ_2	δ_3
1.6	0.7483	0.8517	0.3448	0.5476
1.8	1.1136	0.6864	0.2979	0.4792
2.0	1.4142	0.5858	0.2636	0.4274
2.2	1.68523	0.51477	0.23698	0.38628
2.4	1.93907	0.46093	0.21554	0.35269
2.6	2.18174	0.41826	0.19782	0.32463
2.8	2.41661	0.38339	0.18289	0.30081
3.0	2.64575	0.35425	0.17012	0.28030
3.2	2.87054	0.32946	0.15906	0.26245
3.4	3.09193	0.30807	0.14937	0.24676
3.6	3.31059	0.28941	0.14082	0.23286
3.8	3.52704	0.27296	0.13321	0.22045
4.0	3.74166	0.25834	0.12639	0.20931
4.2	3.95474	0.24526	0.12024	0.19925
4.4	4.16653	0.23347	0.11467	0.19011
4.6	4.37721	0.22279	0.10959	0.18178
4.8	4.58694	0.21306	0.10495	0.17416
5.0	4.79583	0.20417	0.10069	0.16715
5.2	5.00400	0.19600	0.09677	0.16068
5.4	5.21153	0.18847	0.09314	0.15470
5.6	5.41849	0.18151	0.08978	0.14915
5.8	5.62494	0.17506	0.08665	0.14398
6.0	5.83095	0.16905	0.08373	0.13916

Boundary Layer Characteristics as Functions of γ.

$\alpha = 1 \quad \beta = -1$

TABLE 16

γ	c_f	δ_1	δ_2	δ_3
2.7	1.0629	0.6760	0.2852	0.4562
2.8	1.3512	0.5937	0.2614	0.4215
3.0	1.7870	0.4931	0.2268	0.3693
3.2	2.13926	0.42936	0.20203	0.33082
3.4	2.44969	0.38364	0.18303	0.30081
3.6	2.73509	0.34854	0.16783	0.27653
3.8	3.00390	0.32040	0.15530	0.25636
4.0	3.26094	0.29716	0.14474	0.23926
4.2	3.50928	0.27752	0.13568	0.22454
4.4	3.75091	0.26064	0.12780	0.21169
4.6	3.98726	0.24593	0.12087	0.20035
4.8	4.21934	0.23297	0.11472	0.19027
5.0	4.44792	0.22143	0.10922	0.18123
5.2	4.67358	0.21108	0.10425	0.17307
5.4	4.89677	0.20174	0.09975	0.16565
5.6	5.11786	0.19325	0.09565	0.15889
5.8	5.33713	0.18549	0.09189	0.15268
6.0	5.55482	0.17837	0.08843	0.14697

Boundary Layer Characteristics as Functions of γ.

$\alpha = 1 \quad \beta = -2$

TABLE 17

γ	c_f	δ_1	δ_2	δ_3
4.4	1.9250	0.4611	0.2102	0.3412
4.6	2.5625	0.3754	0.1787	0.2931
4.8	3.0190	0.3266	0.1583	0.2609
5.0	3.39702	0.29318	0.14342	0.23709
5.2	3.73151	0.26818	0.13192	0.21847
5.4	4.03857	0.24839	0.12263	0.20333
5.6	4.32686	0.23212	0.11488	0.19065
5.8	4.60156	0.21840	0.10828	0.17982
6.0	4.86599	0.20658	0.10256	0.17040

Boundary Layer Characteristics as Functions of γ.

$\alpha = 1 \quad \beta = -4$

TABLE 18

γ	c_f	δ_1	δ_2	δ_3
-2.0	3.64265	0.23860	0.11407	0.18851
-1.8	3.73096	0.23379	0.11190	0.18495
-1.6	3.82133	0.22907	0.10976	0.18146
-1.4	3.91378	0.22445	0.10766	0.17803
-1.2	4.00831	0.21992	0.10560	0.17466
-1.0	4.10491	0.21547	0.10358	0.17135
-0.8	4.20358	0.21113	0.10160	0.16810
-0.6	4.30431	0.20687	0.09965	0.16491
-0.4	4.40710	0.20270	0.09774	0.16178
-0.2	4.51193	0.19862	0.09587	0.15871
0.0	4.61880	0.19463	0.09404	0.15571
0.2	4.72768	0.19074	0.09224	0.15276
0.4	4.83857	0.18692	0.09049	0.14988
0.6	4.95143	0.18320	0.08877	0.14705
0.8	5.06626	0.17956	0.08708	0.14428
1.0	5.18303	0.17601	0.08543	0.14158
1.2	5.30171	0.17254	0.08382	0.13893
1.4	5.42228	0.16915	0.08224	0.13633
1.6	5.54471	0.16584	0.08070	0.13380
1.8	5.66899	0.16262	0.07920	0.13132
2.0	5.79507	0.15947	0.07772	0.12890
2.2	5.92293	0.15640	0.07628	0.12653
2.4	6.05253	0.15340	0.07488	0.12422
2.6	6.18385	0.15049	0.07351	0.12196
2.8	6.31685	0.14764	0.07217	0.11975
3.0	6.45151	0.14486	0.07086	0.11759
3.2	6.58778	0.14216	0.06958	0.11548
3.4	6.72564	0.13952	0.06833	0.11343
3.6	6.86505	0.13695	0.06711	0.11142
3.8	7.00597	0.13445	0.06593	0.10946
4.0	7.14838	0.13201	0.06477	0.10755
4.2	7.29224	0.12963	0.06364	0.10568
4.4	7.43751	0.12731	0.06253	0.10386
4.6	7.58417	0.12505	0.06146	0.10208
4.8	7.73217	0.12286	0.06041	0.10035
5.0	7.88148	0.12071	0.05938	0.09865
5.2	8.03208	0.11862	0.05838	0.09700
5.4	8.18393	0.11659	0.05741	0.09539
5.6	8.33699	0.11461	0.05646	0.09382
5.8	8.49124	0.11267	0.05553	0.09228
6.0	8.64663	0.11079	0.05462	0.09079

Boundary Layer Characteristics as Functions of γ.

$\alpha = 0 \quad \beta = 16$

TABLE 19

γ	c_f	δ_1	δ_2	δ_3
-2.0	2.33259	0.36696	0.17461	0.28830
-1.8	2.41282	0.35663	0.16997	0.28073
-1.6	2.49593	0.34655	0.16544	0.27332
-1.4	2.58195	0.33674	0.16101	0.26608
-1.2	2.67089	0.32718	0.15668	0.25901
-1.0	2.76277	0.31788	0.15247	0.25211
-0.8	2.85758	0.30884	0.14836	0.24538
-0.6	2.95532	0.30006	0.14436	0.23883
-0.4	3.05598	0.29153	0.14046	0.23245
-0.2	3.15954	0.28327	0.13667	0.22624
0.0	3.26599	0.27526	0.13299	0.22020
0.2	3.37528	0.26749	0.12942	0.21434
0.4	3.48740	0.25998	0.12595	0.20864
0.6	3.60230	0.25271	0.12258	0.20311
0.8	3.71993	0.24567	0.11932	0.19775
1.0	3.84024	0.23888	0.11615	0.19255
1.2	3.96319	0.23231	0.11309	0.18751
1.4	4.08871	0.22597	0.11012	0.18263
1.6	4.21676	0.21984	0.10725	0.17791
1.8	4.34726	0.21393	0.10448	0.17333
2.0	4.48015	0.20823	0.10179	0.16891
2.2	4.61538	0.20273	0.09920	0.16463
2.4	4.75286	0.19742	0.09669	0.16050
2.6	4.89254	0.19231	0.09426	0.15650
2.8	5.03435	0.18737	0.09192	0.15263
3.0	5.17821	0.18262	0.08966	0.14890
3.2	5.32407	0.17803	0.08747	0.14529
3.4	5.47185	0.17362	0.08537	0.14181
3.6	5.62148	0.16936	0.08333	0.13844
3.8	5.77291	0.16525	0.08136	0.13519
4.0	5.92606	0.16129	0.07946	0.13206
4.2	6.08088	0.15748	0.07763	0.12902
4.4	6.23729	0.15380	0.07586	0.12610
4.6	6.39525	0.15025	0.07415	0.12327
4.8	6.55468	0.14683	0.07250	0.12054
5.0	6.71554	0.14354	0.07091	0.11790
5.2	6.87777	0.14035	0.06937	0.11535
5.4	7.04130	0.13728	0.06788	0.11289
5.6	7.20610	0.13432	0.06644	0.11051
5.8	7.37211	0.13146	0.06505	0.10820
6.0	7.53928	0.12870	0.06371	0.10598

Boundary Layer Characteristics as Functions of γ.

$\alpha = 0 \quad \beta = 8$

TABLE 20

γ	c_f	δ_1	δ_2	δ_3
-2.0	1.43557	0.58327	0.27563	0.45453
-1.8	1.50481	0.56064	0.26556	0.43812
-1.6	1.57792	0.53871	0.25577	0.42215
-1.4	1.65500	0.51748	0.24627	0.40663
-1.2	1.73612	0.49697	0.23706	0.39159
-1.0	1.82133	0.47719	0.22814	0.37701
-0.8	1.91067	0.45814	0.21952	0.36292
-0.6	2.00415	0.43983	0.21121	0.34931
-0.4	2.10179	0.42225	0.20320	0.33619
-0.2	2.20355	0.40540	0.19549	0.32356
0.0	2.30940	0.38927	0.18808	0.31142
0.2	2.41928	0.37385	0.18097	0.29975
0.4	2.53313	0.35912	0.17416	0.28857
0.6	2.65084	0.34507	0.16764	0.27785
0.8	2.77236	0.33169	0.16140	0.26760
1.0	2.89753	0.31894	0.15545	0.25780
1.2	3.02626	0.30681	0.14976	0.24843
1.4	3.15843	0.29528	0.14433	0.23949
1.6	3.29389	0.28432	0.13916	0.23097
1.8	3.43252	0.27390	0.13423	0.22284
2.0	3.57419	0.26401	0.12953	0.21509
2.2	3.71876	0.25462	0.12506	0.20772
2.4	3.86608	0.24571	0.12081	0.20069
2.6	4.01604	0.23725	0.11676	0.19400
2.8	4.16849	0.22921	0.11291	0.18763
3.0	4.32332	0.22158	0.10924	0.18157
3.2	4.48038	0.21434	0.10576	0.17580
3.4	4.63958	0.20746	0.10244	0.17031
3.6	4.80078	0.20092	0.09928	0.16507
3.8	4.96387	0.19470	0.09627	0.16009
4.0	5.12876	0.18879	0.09340	0.15534
4.2	5.29535	0.18317	0.09067	0.15082
4.4	5.46353	0.17782	0.08807	0.14650
4.6	5.63321	0.17272	0.08558	0.14238
4.8	5.80432	0.16786	0.08322	0.13846
5.0	5.97676	0.16323	0.08096	0.13471
5.2	6.15047	0.15882	0.07880	0.13113
5.4	6.32536	0.15461	0.07674	0.12771
5.6	6.50138	0.15058	0.07476	0.12443
5.8	6.67846	0.14674	0.07288	0.12130
6.0	6.85654	0.14306	0.07108	0.11831

Boundary Layer Characteristics as Functions of γ.

$\alpha = 0 \quad \beta = 4$

TABLE 21

γ	c_f	δ_1	δ_2	δ_3
-2.0	0.83967	0.96717	0.45267	0.74520
-1.8	0.89482	0.91690	0.43053	0.70917
-1.6	0.95481	0.86834	0.40907	0.67422
-1.4	1.01990	0.82160	0.38835	0.64044
-1.2	1.09032	0.77678	0.36838	0.60789
-1.0	1.16630	0.73393	0.34922	0.57661
-0.8	1.24797	0.69311	0.33087	0.54664
-0.6	1.33545	0.65435	0.31337	0.51802
-0.4	1.42879	0.61768	0.29672	0.49077
-0.2	1.52799	0.58307	0.28092	0.46490
0.0	1.63299	0.55051	0.26599	0.44041
0.2	1.74370	0.51996	0.25189	0.41728
0.4	1.85996	0.49135	0.23863	0.39550
0.6	1.98159	0.46462	0.22618	0.37502
0.8	2.10838	0.43968	0.21451	0.35581
1.0	2.24008	0.41646	0.20358	0.33782
1.2	2.37643	0.39484	0.19337	0.32099
1.4	2.51717	0.37475	0.18384	0.30527
1.6	2.66203	0.35607	0.17495	0.29059
1.8	2.81074	0.33871	0.16666	0.27689
2.0	2.96303	0.32259	0.15893	0.26411
2.2	3.11865	0.30760	0.15172	0.25219
2.4	3.27734	0.29367	0.14500	0.24107
2.6	3.43888	0.28071	0.13873	0.23070
2.8	3.60305	0.26864	0.13289	0.22101
3.0	3.76964	0.25739	0.12743	0.21196
3.2	3.93846	0.24691	0.12232	0.20350
3.4	4.10933	0.23712	0.11755	0.19558
3.6	4.28209	0.22797	0.11308	0.18817
3.8	4.45659	0.21940	0.10889	0.18122
4.0	4.63269	0.21138	0.10496	0.17470
4.2	4.81027	0.20386	0.10127	0.16857
4.4	4.98920	0.19680	0.09780	0.16281
4.6	5.16939	0.19016	0.09454	0.15739
4.8	5.35074	0.18391	0.09146	0.15228
5.0	5.53316	0.17802	0.08856	0.14746
5.2	5.71656	0.17246	0.08582	0.14290
5.4	5.90088	0.16721	0.08323	0.13860
5.6	6.08605	0.16224	0.08078	0.13452
5.8	6.27200	0.15754	0.07846	0.13066
6.0	6.45869	0.15309	0.07626	0.12700

Boundary Layer Characteristics as Functions of γ.

$\alpha = 0 \quad \beta = 2$

TABLE 22

γ	c_f	δ_1	δ_2	δ_3
-2.0	0.46385	1.68489	0.77895	1.27954
-1.8	0.50299	1.57292	0.73007	1.20009
-1.6	0.54733	1.46464	0.68269	1.12305
-1.4	0.59751	1.36053	0.63698	1.04868
-1.2	0.65413	1.26102	0.59311	0.97727
-1.0	0.71778	1.16653	0.55125	0.90906
-0.8	0.78896	1.07742	0.51154	0.84430
-0.6	0.86806	0.99395	0.47411	0.78317
-0.4	0.95533	0.91629	0.43904	0.72584
-0.2	1.05089	0.84450	0.40639	0.67239
0.0	1.15470	0.77854	0.37616	0.62283
0.2	1.26657	0.71824	0.34832	0.57714
0.4	1.38618	0.66337	0.32281	0.53520
0.6	1.51313	0.61362	0.29951	0.49686
0.8	1.64695	0.56863	0.27831	0.46194
1.0	1.78710	0.52803	0.25907	0.43019
1.2	1.93304	0.49142	0.24162	0.40138
1.4	2.08425	0.45843	0.22582	0.37527
1.6	2.24019	0.42868	0.21151	0.35160
1.8	2.40039	0.40184	0.19855	0.33015
2.0	2.56438	0.37758	0.18680	0.31068
2.2	2.73176	0.35563	0.17613	0.29300
2.4	2.90216	0.33573	0.16643	0.27692
2.6	3.07523	0.31764	0.15759	0.26226
2.8	3.25069	0.30116	0.14953	0.24887
3.0	3.42827	0.28612	0.14215	0.23662
3.2	3.60774	0.27235	0.13539	0.22539
3.4	3.78890	0.25973	0.12917	0.21506
3.6	3.97157	0.24811	0.12345	0.20556
3.8	4.15558	0.23741	0.11817	0.19678
4.0	4.34081	0.22752	0.11329	0.18866
4.2	4.52713	0.21836	0.10877	0.18114
4.4	4.71443	0.20987	0.10456	0.17415
4.6	4.90262	0.20196	0.10065	0.16764
4.8	5.09161	0.19460	0.09700	0.16158
5.0	5.28133	0.18773	0.09360	0.15591
5.2	5.47171	0.18130	0.09041	0.15060
5.4	5.66269	0.17527	0.08742	0.14563
5.6	5.85423	0.16962	0.08461	0.14095
5.8	6.04627	0.16430	0.08197	0.13656
6.0	6.23877	0.15929	0.07948	0.13242

Boundary Layer Characteristics as Functions of γ.

$\alpha = 0 \quad \beta = 1$

TABLE 23

γ	c_f	δ_1	δ_2	δ_3
-1.8	0.268	2.837	1.299	2.131
-1.6	0.2965	2.598	1.195	1.962
-1.4	0.3306	2.3667	1.0946	1.7981
-1.2	0.3713	2.1449	0.9977	1.6407
-1.0	0.41983	1.9343	0.9053	1.4904
-0.8	0.47741	1.73667	0.81814	1.34844
-0.6	0.54516	1.55356	0.73677	1.21577
-0.4	0.62398	1.38622	0.66175	1.09328
-0.2	0.71439	1.23535	0.59344	0.98154
0.0	0.81650	1.10102	0.53197	0.88082
0.2	0.92998	0.98270	0.47727	0.79099
0.4	1.05419	0.87937	0.42901	0.71162
0.6	1.18822	0.78968	0.38675	0.64198
0.8	1.33102	0.71214	0.34990	0.58117
1.0	1.48152	0.64518	0.31785	0.52822
1.2	1.63867	0.58734	0.29000	0.48215
1.4	1.80153	0.53728	0.26577	0.44202
1.6	1.96923	0.49382	0.24464	0.40700
1.8	2.14105	0.45593	0.22616	0.37634
2.0	2.31635	0.42277	0.20993	0.34940
2.2	2.49460	0.39360	0.19561	0.32563
2.4	2.67537	0.36781	0.18293	0.30456
2.6	2.85828	0.34492	0.17165	0.28581
2.8	3.04302	0.32449	0.16156	0.26905
3.0	3.22934	0.30618	0.15251	0.25400
3.2	3.41702	0.28969	0.14436	0.24043
3.4	3.60588	0.27479	0.13698	0.22816
3.6	3.79577	0.26127	0.13027	0.21700
3.8	3.98656	0.24895	0.12416	0.20683
4.0	4.17813	0.23769	0.11857	0.19753
4.2	4.37040	0.22737	0.11344	0.18899
4.4	4.56329	0.21787	0.10872	0.18113
4.6	4.75673	0.20910	0.10436	0.17388
4.8	4.95066	0.20099	0.10033	0.16716
5.0	5.14503	0.19347	0.09659	0.16093
5.2	5.33979	0.18647	0.09310	0.15513
5.4	5.53490	0.17995	0.08986	0.14972
5.6	5.73034	0.17386	0.08682	0.14467
5.8	5.92607	0.16816	0.08398	0.13994
6.0	6.12206	0.16281	0.08132	0.13550

Boundary Layer Characteristics as Functions of γ.

$\alpha = 0 \quad \beta = 0{\cdot}5$

TABLE 24

γ	c_f	δ_1	δ_2	δ_3
2.2	1.743	0.605	0.308	0.516
2.4	2.0113	0.5161	0.2613	0.4366
2.6	2.2571	0.4558	0.2301	0.3842
2.8	2.49130	0.41056	0.20684	0.34527
3.0	2.71834	0.37477	0.18856	0.31466
3.2	2.94046	0.34544	0.17363	0.28969
3.4	3.15902	0.32082	0.16113	0.26880
3.6	3.37488	0.29978	0.15047	0.25099
3.8	3.58862	0.28152	0.14124	0.23557
4.0	3.80067	0.26551	0.13316	0.22206
4.2	4.01134	0.25132	0.12600	0.21011
4.4	4.22087	0.23865	0.11962	0.19946
4.6	4.42943	0.22725	0.11388	0.18988
4.8	4.63718	0.21694	0.10869	0.18122
5.0	4.84423	0.20756	0.10397	0.17335
5.2	5.05067	0.19899	0.09966	0.16616
5.4	5.25659	0.19112	0.09571	0.15956
5.6	5.46203	0.18387	0.09207	0.15349
5.8	5.66707	0.17716	0.08870	0.14787
6.0	5.87174	0.17094	0.08557	0.14266

Boundary Layer Characteristics as Functions of γ.

$\alpha = 0 \quad \beta = -0{\cdot}5$

TABLE 25

γ	c_f	δ_1	δ_2	δ_3
3.2	2.589	0.405	0.206	0.344
3.4	2.8519	0.3639	0.1842	0.3078
3.6	3.0994	0.3326	0.1680	0.2806
3.8	3.33764	0.30736	0.15501	0.25879
4.0	3.56939	0.28637	0.14424	0.24076
4.2	3.79641	0.26849	0.13511	0.22547
4.4	4.01980	0.25299	0.12721	0.21226
4.6	4.24030	0.23939	0.12030	0.20071
4.8	4.45849	0.22733	0.11418	0.19047
5.0	4.67476	0.21653	0.10871	0.18133
5.2	4.88942	0.20680	0.10378	0.17310
5.4	5.10272	0.19796	0.09932	0.16564
5.6	5.31485	0.18991	0.09525	0.15885
5.8	5.52596	0.18252	0.09152	0.15262
6.0	5.73619	0.17572	0.08809	0.14690

Boundary Layer Characteristics as Functions of γ.

$\alpha = 0 \quad \beta = -1$

TABLE 26

γ	c_f	δ_1	δ_2	δ_3
4.2	3.179	0.337	0.173	0.290
4.4	3.4861	0.3027	0.1542	0.2580
4.6	3.7626	0.2777	0.1410	0.2357
4.8	4.02263	0.25803	0.13066	0.21832
5.0	4.27214	0.24174	0.12219	0.20411
5.2	4.51417	0.22788	0.11503	0.19210
5.4	4.75066	0.21584	0.10884	0.18171
5.6	4.98260	0.20528	0.10342	0.17264
5.8	5.21110	0.19585	0.09860	0.16456
6.0	5.43668	0.18738	0.09428	0.15733

Boundary Layer Characteristics as Functions of γ.

$\alpha = 0 \quad \beta = -2$

TABLE 27

γ	c_f	δ_1	δ_2	δ_2
5.6	3.742	0.305	0.160	0.268
5.8	4.2462	0.2564	0.1321	0.2215
6.0	4.5943	0.2325	0.1189	0.1991

Boundary Layer Characteristics as Functions of γ.

$\alpha = 0 \quad \beta = -4$

TABLE 28

γ	c_f	δ_1	δ_2	δ_3
-2.0	3.62941	0.24238	0.11675	0.19335
-1.8	3.71719	0.23747	0.11451	0.18966
-1.6	3.80703	0.23266	0.11230	0.18604
-1.4	3.89894	0.22794	0.11013	0.18247
-1.2	3.99293	0.22331	0.10800	0.17897
-1.0	4.08899	0.21878	0.10590	0.17553
-0.8	4.18712	0.21433	0.10385	0.17216
-0.6	4.28732	0.20999	0.10184	0.16885
-0.4	4.38958	0.20573	0.09986	0.16560
-0.2	4.49389	0.20156	0.09793	0.16241
0.0	4.60023	0.19749	0.09603	0.15929
0.2	4.70860	0.19350	0.09417	0.15623
0.4	4.81897	0.18961	0.09235	0.15324
0.6	4.93134	0.18580	0.09057	0.15030
0.8	5.04568	0.18208	0.08883	0.14743
1.0	5.16197	0.17844	0.08712	0.14462
1.2	5.28018	0.17490	0.08545	0.14187
1.4	5.40030	0.17143	0.08382	0.13918
1.6	5.52230	0.16805	0.08223	0.13655
1.8	5.64614	0.16476	0.08067	0.13397
2.0	5.77181	0.16154	0.07915	0.13146
2.2	5.89928	0.15840	0.07766	0.12900
2.4	6.02850	0.15534	0.07621	0.12660
2.6	6.15946	0.15235	0.07479	0.12426
2.8	6.29212	0.14944	0.07340	0.12197
3.0	6.42644	0.14660	0.07205	0.11973
3.2	6.56241	0.14384	0.07073	0.11755
3.4	6.69997	0.14114	0.06944	0.11542
3.6	6.83910	0.13852	0.06819	0.11334
3.8	6.97977	0.13596	0.06696	0.11132
4.0	7.12193	0.13347	0.06576	0.10934
4.2	7.26557	0.13104	0.06460	0.10740
4.4	7.41063	0.12867	0.06346	0.10552
4.6	7.55709	0.12637	0.06235	0.10368
4.8	7.70492	0.12412	0.06127	0.10189
5.0	7.85408	0.12193	0.06021	0.10014
5.2	8.00453	0.11980	0.05918	0.09844
5.4	8.15625	0.11772	0.05818	0.09677
5.6	8.30920	0.11570	0.05720	0.09515
5.8	8.46335	0.11373	0.05624	0.09357
6.0	8.61867	0.11181	0.05531	0.09202

Boundary Layer Characteristics as Functions of γ.

$\alpha = -1 \quad \beta = 16$

TABLE 29

γ	c_f	δ_1	δ_2	δ_3
-2.0	2.31700	0.37916	0.18339	0.30421
-1.8	2.39623	0.36841	0.17842	0.29604
-1.6	2.47831	0.35792	0.17357	0.28804
-1.4	2.56329	0.34769	0.16883	0.28023
-1.2	2.65117	0.33772	0.16420	0.27260
-1.0	2.74197	0.32803	0.15968	0.26515
-0.8	2.83570	0.31860	0.15528	0.25789
-0.6	2.93236	0.30943	0.15099	0.25081
-0.4	3.03195	0.30053	0.14681	0.24392
-0.2	3.13445	0.29190	0.14275	0.23721
0.0	3.23984	0.28353	0.13880	0.23069
0.2	3.34810	0.27541	0.13497	0.22436
0.4	3.45921	0.26756	0.13125	0.21820
0.6	3.57312	0.25995	0.12764	0.21224
0.8	3.68979	0.25260	0.12414	0.20645
1.0	3.80918	0.24549	0.12075	0.20084
1.2	3.93125	0.23862	0.11747	0.19540
1.4	4.05593	0.23198	0.11430	0.19014
1.6	4.18318	0.22558	0.11122	0.18505
1.8	4.31292	0.21939	0.10825	0.18013
2.0	4.44511	0.21343	0.10538	0.17537
2.2	4.57967	0.20768	0.10261	0.17077
2.4	4.71655	0.20213	0.09993	0.16632
2.6	4.85567	0.19678	0.09734	0.16203
2.8	4.99696	0.19163	0.09485	0.15789
3.0	5.14036	0.18666	0.09244	0.15389
3.2	5.28581	0.18188	0.09011	0.15003
3.4	5.43322	0.17727	0.08787	0.14630
3.6	5.58254	0.17283	0.08570	0.14270
3.8	5.73369	0.16855	0.08361	0.13924
4.0	5.88661	0.16443	0.08160	0.13589
4.2	6.04124	0.16045	0.07966	0.13266
4.4	6.19750	0.15663	0.07779	0.12955
4.6	6.35535	0.15294	0.07598	0.12654
4.8	6.51471	0.14938	0.07424	0.12365
5.0	6.67553	0.14596	0.07255	0.12085
5.2	6.83774	0.14265	0.07093	0.11815
5.4	7.00130	0.13947	0.06936	0.11555
5.6	7.16615	0.13640	0.06785	0.11303
5.8	7.33224	0.13343	0.06639	0.11060
6.0	7.49951	0.13058	0.06499	0.10826

Boundary Layer Characteristics as Functions of γ.

$\alpha = -1 \quad \beta = 8$

TABLE 30

γ	c_f	δ_1	δ_2	δ_3
-2.0	1.419	0.6250	0.3064	0.5107
-1.8	1.487	0.6005	0.2949	0.4915
-1.6	1.5580	0.57677	0.28365	0.47290
-1.4	1.6332	0.55374	0.27273	0.45479
-1.2	1.7123	0.53147	0.26214	0.43720
-1.0	1.7954	0.50995	0.25188	0.42015
-0.8	1.8826	0.48920	0.24195	0.40365
-0.6	1.9740	0.46923	0.23236	0.38771
-0.4	2.0695	0.45003	0.22312	0.37233
-0.2	2.1691	0.43160	0.21422	0.35752
0.0	2.2728	0.41394	0.20567	0.34328
0.2	2.3806	0.39705	0.19746	0.32961
0.4	2.49240	0.38090	0.18960	0.31650
0.6	2.60817	0.36549	0.18207	0.30395
0.8	2.72782	0.35080	0.17487	0.29195
1.0	2.85125	0.33681	0.16801	0.28049
1.2	2.97836	0.32350	0.16146	0.26956
1.4	3.10902	0.31084	0.15522	0.25915
1.6	3.24312	0.29882	0.14928	0.24923
1.8	3.38053	0.28741	0.14363	0.23980
2.0	3.52111	0.27658	0.13827	0.23083
2.2	3.66474	0.26631	0.13317	0.22232
2.4	3.81126	0.25657	0.12833	0.21423
2.6	3.96055	0.24734	0.12374	0.20655
2.8	4.11247	0.23858	0.11938	0.19927
3.0	4.26688	0.23029	0.11524	0.19236
3.2	4.42364	0.22242	0.11132	0.18580
3.4	4.58265	0.21497	0.10759	0.17957
3.6	4.74376	0.20790	0.10406	0.17367
3.8	4.90686	0.20119	0.10071	0.16806
4.0	5.07185	0.19482	0.09752	0.16274
4.2	5.23860	0.18878	0.09450	0.15769
4.4	5.40702	0.18303	0.09163	0.15289
4.6	5.57700	0.17758	0.08890	0.14833
4.8	5.74846	0.17239	0.08630	0.14399
5.0	5.92130	0.16746	0.08383	0.13986
5.2	6.09545	0.16276	0.08147	0.13593
5.4	6.27083	0.15828	0.07923	0.13218
5.6	6.44736	0.15402	0.07710	0.12861
5.8	6.62497	0.14995	0.07506	0.12521
6.0	6.80361	0.14607	0.07311	0.12196

Boundary Layer Characteristics as Functions of γ.

$\alpha = -1 \quad \beta = 4$

TABLE 31

γ	c_f	δ_1	δ_2	δ_3
0.0	1.58	0.629	0.324	0.546
0.2	1.689	0.5920	0.3047	0.5137
0.4	1.801	0.5572	0.2866	0.4831
0.6	1.919	0.5245	0.2697	0.4543
0.8	2.0419	0.4941	0.2538	0.4274
1.0	2.1704	0.4657	0.2390	0.4023
1.2	2.3039	0.43930	0.22526	0.37899
1.4	2.44212	0.41482	0.21248	0.35730
1.6	2.58491	0.39214	0.20063	0.33720
1.8	2.73194	0.37114	0.18966	0.31860
2.0	2.88294	0.35172	0.17951	0.30141
2.2	3.03761	0.33374	0.17011	0.28548
2.4	3.19577	0.31715	0.16148	0.27088
2.6	3.35706	0.30179	0.15348	0.25735
2.8	3.52126	0.28758	0.14610	0.24487
3.0	3.68812	0.27442	0.13927	0.23333
3.2	3.85743	0.26224	0.13296	0.22267
3.4	4.02897	0.25093	0.12711	0.21281
3.6	4.20255	0.24043	0.12169	0.20367
3.8	4.37800	0.23067	0.11666	0.19519
4.0	4.55515	0.22159	0.11198	0.18731
4.2	4.73386	0.21312	0.10762	0.17999
4.4	4.91398	0.20521	0.10356	0.17316
4.6	5.09540	0.19781	0.09977	0.16679
4.8	5.27800	0.19089	0.09623	0.16084
5.0	5.46170	0.18439	0.09291	0.15526
5.2	5.64639	0.17829	0.08980	0.15004
5.4	5.83199	0.17256	0.08687	0.14513
5.6	6.01844	0.16716	0.08412	0.14051
5.8	6.20566	0.16207	0.08153	0.13617
6.0	6.39360	0.15726	0.07908	0.13207

Boundary Layer Characteristics as Functions of γ.

$\alpha = -1 \quad \beta = 2$

TABLE 32

γ	c_f	δ_1	δ_2	δ_3
1.6	2.12	0.524	0.283	0.484
1.8	2.282	0.4824	0.2587	0.4410
2.0	2.446	0.4455	0.2371	0.4030
2.2	2.613	0.4131	0.2183	0.3703
2.4	2.784	0.3844	0.2019	0.3417
2.6	2.959	0.3591	0.1875	0.3168
2.8	3.137	0.3365	0.1749	0.2950
3.0	3.3165	0.31646	0.16373	0.27583
3.2	3.49849	0.29849	0.15386	0.25889
3.4	3.68235	0.28235	0.14507	0.24384
3.6	3.86780	0.26780	0.13722	0.23045
3.8	4.05462	0.25462	0.13016	0.21842
4.0	4.24264	0.24264	0.12378	0.20757
4.2	4.43170	0.23170	0.11799	0.19775
4.4	4.62169	0.22169	0.11271	0.18881
4.6	4.81248	0.21248	0.10788	0.18065
4.8	5.00400	0.20400	0.10345	0.17316
5.0	5.19615	0.19615	0.09936	0.16626
5.2	5.38888	0.18888	0.09558	0.15989
5.4	5.58211	0.18211	0.09208	0.15399
5.6	5.77581	0.17581	0.08883	0.14851
5.8	5.96992	0.16993	0.08579	0.14341
6.0	6.16441	0.16441	0.08296	0.13865

Boundary Layer Characteristics as Functions of γ.

$\alpha = -1 \quad \beta = 1$

TABLE 33

γ	c_f	δ_1	δ_2	δ_3
3.0	3.08	0.36	0.19	0.33
3.2	3.278	0.335	0.179	0.304
3.4	3.474	0.3043	0.1571	0.2641
3.6	3.6695	0.2897	0.1507	0.2543
3.8	3.86583	0.27235	0.14069	0.23681
4.0	4.06253	0.25769	0.13263	0.22299
4.2	4.25955	0.24421	0.12512	0.21004
4.4	4.45684	0.23244	0.11876	0.19920
4.6	4.65436	0.22204	0.11331	0.19001
4.8	4.85210	0.21233	0.10814	0.18122
5.0	5.05001	0.20347	0.10345	0.17328
5.2	5.24808	0.19535	0.09918	0.16606
5.4	5.44630	0.18787	0.09527	0.15944
5.6	5.64465	0.18096	0.09166	0.15335
5.8	5.84311	0.17454	0.08833	0.14773
6.0	6.04167	0.16858	0.08523	0.14252

Boundary Layer Characteristics as Functions of γ.

$\alpha = -1 \quad \beta = 0{\cdot}5$

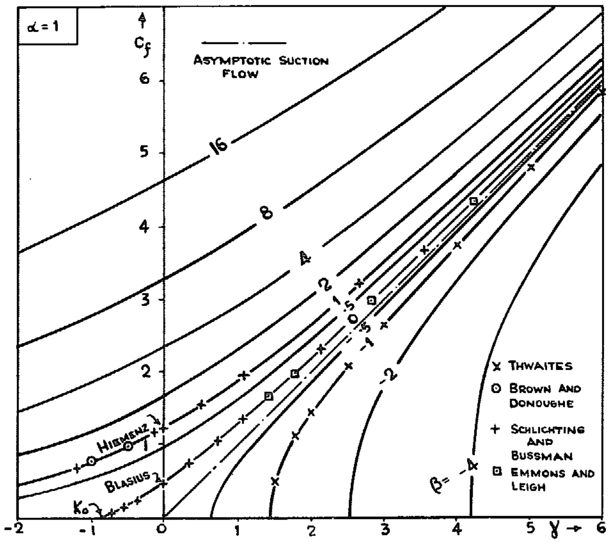

FIG. 1. Skin friction coefficient c_f.

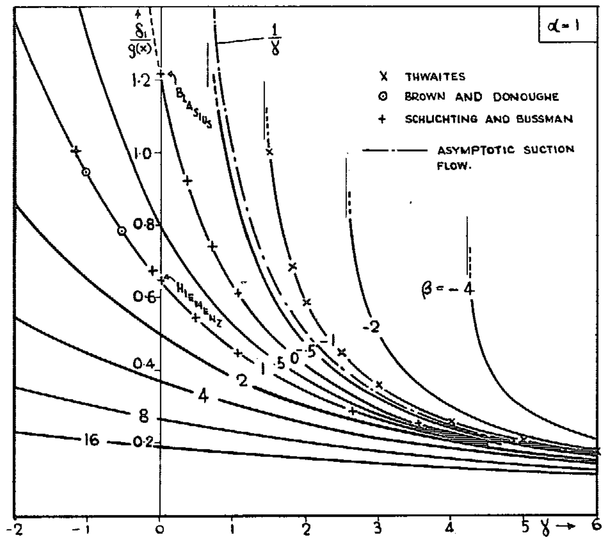

FIG. 2. Displacement thickness δ_1.

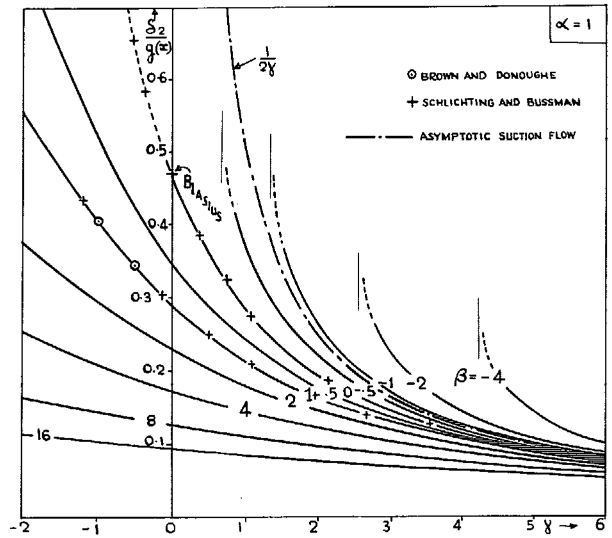

FIG. 3. Momentum thickness δ_2.

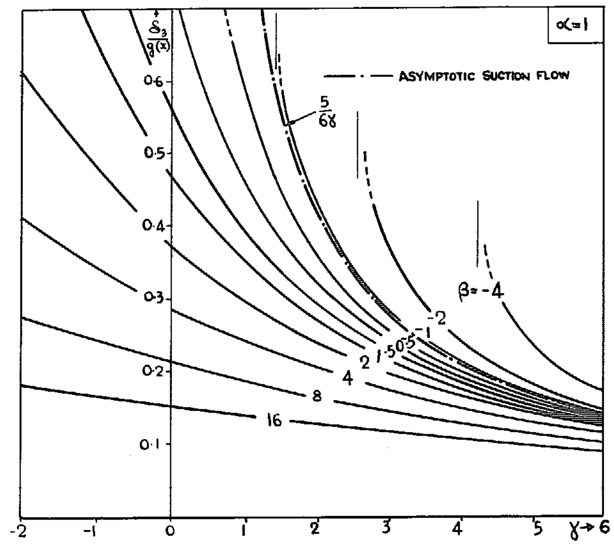

FIG. 4. Energy thickness δ_3.

Boundary-layer characteristics as functions of the suction parameter γ for α = 1.

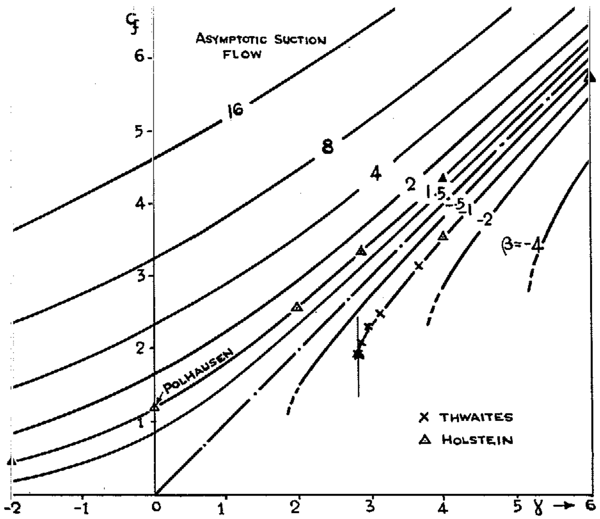

FIG. 5. Skin friction coefficient c_f.

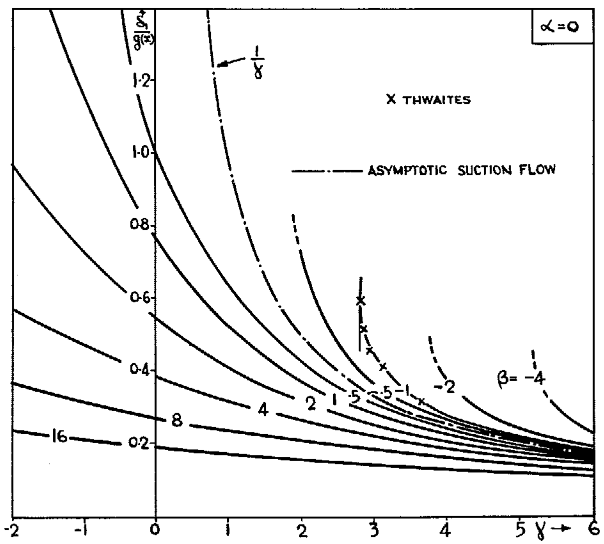

FIG. 6. Displacement thickness δ_1.

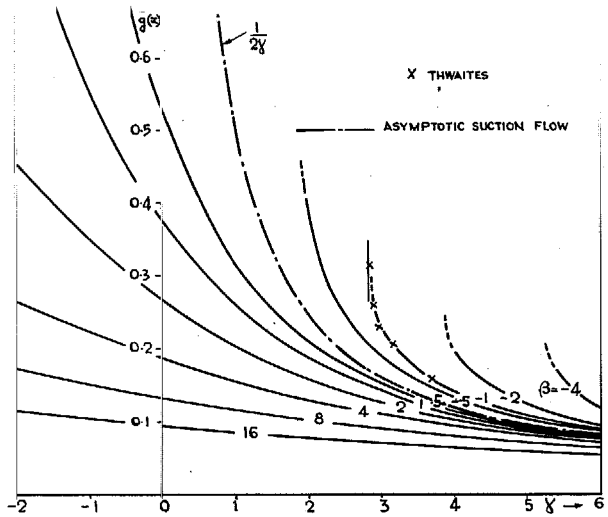

FIG. 7. Momentum thickness δ_2.

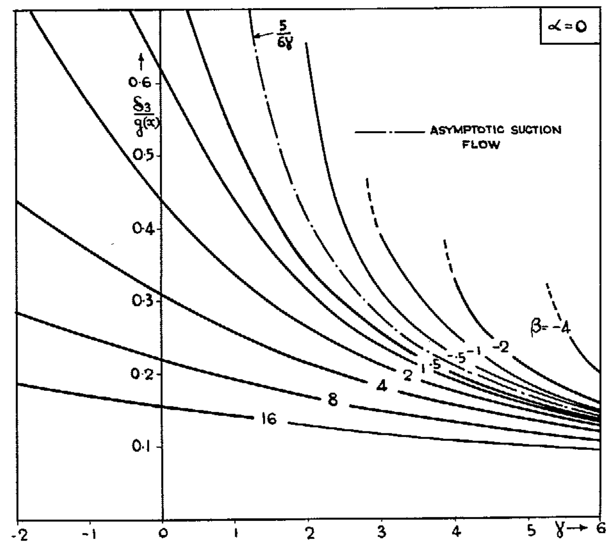

FIG. 8. Energy thickness δ_3.

Boundary-layer characteristics as functions of the suction parameter γ for $\alpha \doteq 0$.

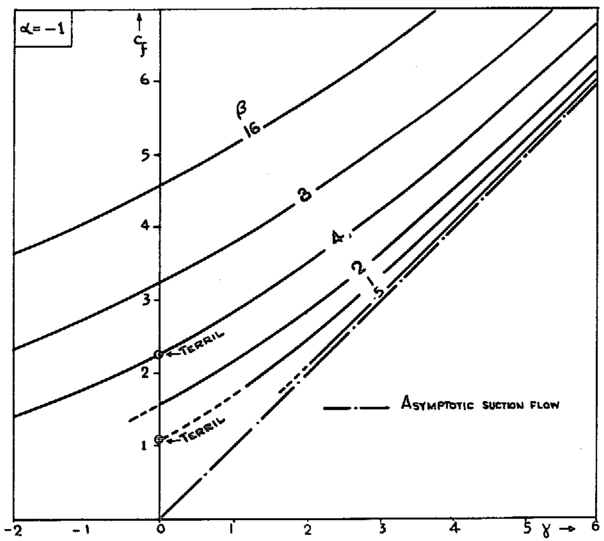

FIG. 9. Skin friction coefficient c_f.

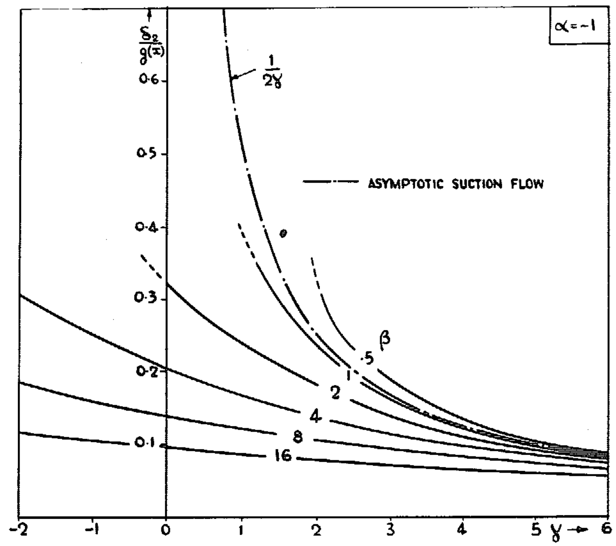

FIG. 11. Momentum thickness δ_2.

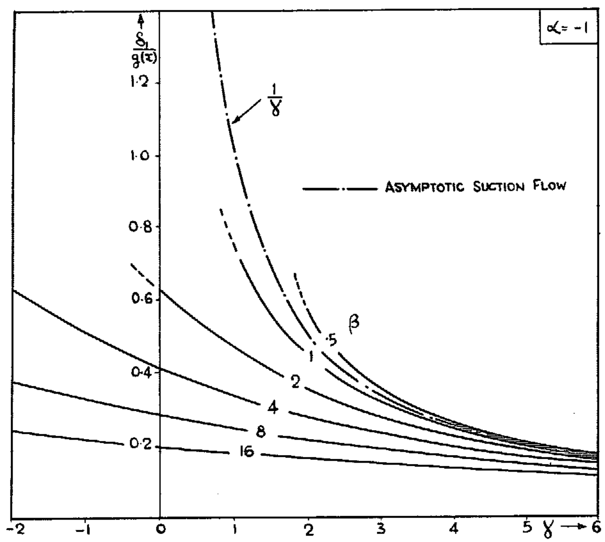

FIG. 10. Displacement thickness δ_1.

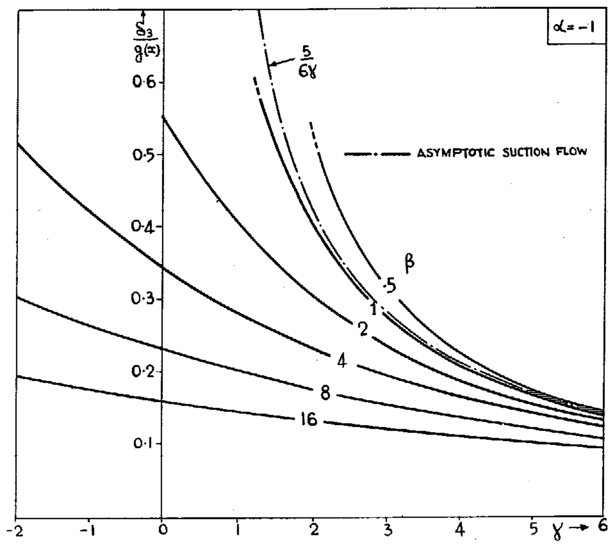

FIG. 12. Energy thickness δ_3.

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