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Generalised Aerodynamic Forces on a T-Tail Oscillating Harmonically in Subsonic Flow

By D. E. DAVIES

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Generalised Aerodynamic Forces on a T-Tail Oscillating Harmonically in Subsonic Flow

By D. E. Davies

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Summary.

The T-tail under investigation consists of a flat horizontal tailplane mounted on top of a flat vertical fin. The chords of the two surfaces at their junction are of the same length and are coincident. The T-tail is assumed to be isolated and to be oscillating harmonically in a subsonic flow whose main stream is parallel to the mean positions of the surfaces of the T-tail. The linearised equations of potential flow are assumed to be valid.

A pair of integral equations relating the normal air velocities on the surfaces of the tailplane and fin with the loading distributions on these surfaces is derived. This pair of simultaneous equations is solved approximately by collocation and the loading functions so determined are used to calculate generalised airforces on the T-tail at any frequency of oscillation. When the T-tail is attached to an aircraft there is some aerodynamic interaction between the aircraft fuselage and the T-tail. It has not been possible to estimate this interaction in general. If the T-tail is attached to an infinite wall with the tailplane parallel to the wall then it is possible to obtain the interaction by the method of images with the infinite wall acting as a reflector. This approaches conditions in a wind tunnel, so a treatment of this case has been given. This case may also be a guide to the more general case of interaction between a fuselage and a T-tail.

The procedures have been programmed for the Ferranti Mercury Computer.

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1. *Introduction.*

The use of T-tail configurations on present day aircraft has initiated the problem of investigating their flutter characteristics. This necessitates the accurate determination of oscillatory airforces acting on the T-tail. There is at present a scarcity of both theoretical and experimental information on these forces, and this paper provides an addition to the theoretical information.

The horse-shoe vortex method of plane wings has been extended to T-tails in a steady subsonic flow by a number of writers[1,2,3]. The process is to replace the surfaces of the T-tail by sets of horse-shoe vortices in the plane of each surface of the T-tail so that the centre of each bound vortex is at a quarter-chord point and the trailing vortices extend downstream to infinity in the direction of the main-stream flow. The boundary condition that the airflow is tangential to a surface is applied at the three-quarter-chord points at the mid-span lines of the horse-shoe vortices. A set of simultaneous equations is then obtained for the strengths of the horse-shoe vortices. These equations are solved and then the airforces on the T-tail are obtained in a straightforward manner using the strengths so obtained.

If a plane wing oscillates in flexible modes or at relatively high frequency parameter the horse-shoe vortex method is not entirely satisfactory. A more complicated horse-shoe vortex method with several horse-shoe vortices at intervals along a chord, or a Multhopp lifting-surface-type method such as that of Acum[4] or Richardson[5] provides better results. It might be expected therefore that a more elaborate method would yield results of wider applicability in the case of the T-tail configuration. This paper extends the method presented by Davies[6] to the oscillating T-tail configuration.

When the T-tail is assumed to be isolated, and the linearised equations of potential flow apply, a pair of integral equations can be derived relating the normal air velocities on the surfaces of the tailplane and fin with the loading distributions on these surfaces. This pair of integral equations is solved approximately by collocation at a number of points on the surfaces of the T-tail, and the loading functions so determined are used to calculate the generalised airforces on the T-tail at any frequency of oscillation.

When the T-tail is attached to an aircraft fuselage there is some aerodynamic interaction between the aircraft fuselage and the T-tail. It has not been possible to estimate this interaction in general. If the T-tail is attached to an infinite wall with the tailplane parallel to the wall then it is possible to obtain the interaction by the method of images with the infinite wall acting as a reflector. This approaches conditions in a wind tunnel, so a treatment of this case is given. This case may also serve as a guide for obtaining the interaction in the more general case of T-tail and fuselage.

A completely different method of solving the problem is to consider the two-dimensional flow in the Trefftz plane behind the T-tail. For steady flow this has been done by Weber and Hawk[9] for a T-tail and fuselage. Many approximations are inherent in the method and an extension to consider oscillatory flow would lead to complications. The method of collocation adopted here does indeed rely on approximations but it would appear to the present writer that the approximations made are more plausible than in the case of the Trefftz plane method.

2. *The Integral Equation Relating the Loadings on the Tailplane and Fin with the Normal Air Velocities on these Surfaces when the T-Tail is Isolated.*

A diagram of the T-tail configuration under consideration is given in Fig. 1. The fin ABCD is attached to the tailplane EFCGHD along CD. The tailplane and fin are assumed to be very thin and nearly plane and the whole T-tail is in a subsonic airstream with the inclination of these surfaces to the main-stream direction being very small. The tailplane and fin oscillate with small amplitude about a mean position in either rigid or flexible modes. Accordingly linearised theory is applicable and the T-tail may be replaced by intersecting flat plates the mean positions of which are parallel to the main-stream direction. It is assumed that the tailplane has no dihedral so that in the mean position EFCGHD may be taken to be a flat plate. The tailplane is taken to be symmetric about the line CD, as usually occurs in practice.

A system of right-handed Cartesian coordinates (x, y, z) is introduced, which is stationary with respect to the mean position of the oscillating surfaces. The origin is taken as some point on the mean position of the line CD. The positive direction of x is that of the main stream and is therefore along DC. The positive axis of z is in the mean plane of the fin along the span of the fin and the axis of y is mutually at right angles to complete a right-handed orthogonal system.

Let the normal displacement at time t in the positive direction of z of a point $(x, y, 0)$ on the undisturbed tailplane be $Z(x, y, t)$ and the normal displacement at time t in the positive direction of y of a point $(x, 0, z)$ on the undisturbed fin be $Y(x, z, t)$. Then in a harmonic oscillation of the T-tail we can write

$$Z(x, y, t) = lf_1(x, y)e^{i\omega t} \tag{1}$$

$$Y(x, z, t) = lf_2(x, z)e^{i\omega t} \tag{2}$$

where l is a typical dimension of the T-tail, and as is usual with using complex functions for harmonic analysis only the real or the imaginary parts represent the pertinent physical quantity.

The boundary conditions that the airflow is tangential to the tailplane and fin surfaces lead to the following linearised equations

$$w_1(x, y) = \left(V\frac{\partial}{\partial x} + i\omega\right) lf_1(x, y) \tag{3}$$

$$w_2(x, z) = \left(V\frac{\partial}{\partial x} + i\omega\right) lf_2(x, z) \tag{4}$$

where V is the main-stream velocity, $w_1(x, y)e^{i\omega t}$ is the component of the air velocity in the z-direction at the surface of the tailplane and $w_2(x, z)e^{i\omega t}$ is the component of the air velocity in the y-direction at the surface of the fin. The functions $w_1(x, y)$ and $w_2(x, z)$ will be called the normal-velocity functions.

Corresponding to the normal-velocity functions given by equations (3) and (4) there is at the point $(x, y, 0)$ on the tailplane surface a pressure force per unit area, or loading $l_1(x, y)e^{i\omega t}$ in the positive direction of z, and at the point $(x, 0, z)$ on the fin surface there is a pressure force per unit area, or loading $l_2(x, z)e^{i\omega t}$ in the positive direction of y.

Reduced normal-velocity functions are introduced by the equations

$$\alpha_1(x, y) = \frac{1}{V} w_1(x, y) \tag{5}$$

$$\alpha_2(x, z) = \frac{1}{V} w_2(x, z) \tag{6}$$

and reduced loading functions are introduced by the equations

$$\lambda_1(x, y) = \frac{1}{\rho_0 V^2} l_1(x, y) \tag{7}$$

$$\lambda_2(x, z) = \frac{1}{\rho_0 V^2} l_2(x, z) \tag{8}$$

where ρ_0 is the density of the air in the undisturbed main stream.

Then the following pair of integral equations (*see* Appendix)

$$\alpha_1(x, y) = \frac{1}{4\pi} \iint\limits_{\text{tailplane}} \lambda_1(x_0, y_0) K_1(x - x_0, y - y_0) dx_0 dy_0 + \\ + \frac{1}{4\pi} \iint\limits_{\text{fin}} \lambda_2(x_0, z_0) K_2(x - x_0, y, z_0) dx_0 dz_0 \tag{9}$$

$$\alpha_2(x, z) = \frac{1}{4\pi} \iint\limits_{\text{tailplane}} \lambda_1(x_0, y_0) K_2(x - x_0, z, y_0) dx_0 dy_0 + \\ + \frac{1}{4\pi} \iint\limits_{\text{fin}} \lambda_2(x_0, z_0) K_1(x - x_0, z - z_0) dx_0 dz_0 \tag{10}$$

are satisfied. The kernel functions K_1 and K_2 are given by

$$K_1(x, y) = e^{-i\omega x/V} \left[\int_{(-x+MR_1)/(1-M^2)}^{\infty} e^{-i\omega u/V} \frac{du}{(u^2+y^2)^{3/2}} + \right. \\ \left. + \frac{M(Mx+R_1)}{R_1(x^2+y^2)} \exp\left\{ -\frac{i\omega}{V} \left(\frac{-x + MR_1}{1 - M^2} \right) \right\} \right] \tag{11}$$

$$K_2(x, y, z) = e^{-i\omega x/V} \left[\int_{(-x+MR)/(1-M^2)}^{\infty} e^{-i\omega u/V} \frac{3yz\, du}{(u^2+y^2+z^2)^{5/2}} + \right. \\ + yz \exp\left\{ -\frac{i\omega}{V} \left(\frac{-x + MR}{1 - M^2} \right) \right\} \left\{ \frac{M(Mx+R)^3}{R(x^2+y^2+z^2)^2} + \frac{M^2(1-M^2)x}{R^3(x^2+y^2+z^2)} + \right. \\ \left. \left. + \frac{2M(Mx+R)}{R(x^2+y^2+z^2)^2} + \frac{i\omega}{V} \frac{M^2(Mx+R)}{R^2(x^2+y^2+z^2)} \right\} \right] \tag{12}$$

where

$$R_1 = \sqrt{\{x^2 + (1-M^2)y^2\}} \tag{13}$$

$$R = \sqrt{\{x^2 + (1-M^2)(y^2+z^2)\}} \tag{14}$$

and M is the Mach number of the main stream.

If now the modified functions

$$\left. \begin{aligned} \hat{\alpha}_1(x, y) &= \alpha_1(x, y) e^{i\omega x/V} \\ \hat{\alpha}_2(x, z) &= \alpha_2(x, z) e^{i\omega x/V} \end{aligned} \right\} \tag{15}$$

$$\left. \begin{aligned} \hat{\lambda}_1(x, y) &= \lambda_1(x, y) e^{i\omega x/V} \\ \hat{\lambda}_2(x, z) &= \lambda_2(x, z) e^{i\omega x/V} \end{aligned} \right\} \tag{16}$$

$$\left. \begin{aligned} \hat{K}_1(x, y) &= K_1(x, y) e^{i\omega x/V} \\ \hat{K}_2(x, y, z) &= K_2(x, y, z) e^{i\omega x/V} \end{aligned} \right\} \tag{17}$$

are introduced into the integral equations (9) amd (10), they become

$$\hat{\alpha}_1(x, y) = \frac{1}{4\pi} \iint\limits_{\text{tailplane}} \hat{\lambda}_1(x_0, y_0)\hat{K}_1(x-x_0, y-y_0)dx_0dy_0 + $$
$$+ \frac{1}{4\pi} \iint\limits_{\text{fin}} \hat{\lambda}_2(x_0, z_0)\hat{K}_2(x-x_0, y, z_0)dx_0dz_0 \tag{18}$$

$$\hat{\alpha}_2(x, z) = \frac{1}{4\pi} \iint\limits_{\text{tailplane}} \hat{\lambda}_1(x_0, y_0)\hat{K}_2(x-x_0, z, y_0)dx_0dy_0 + $$
$$+ \frac{1}{4\pi} \iint\limits_{\text{fin}} \hat{\lambda}_2(x_0, z_0)\hat{K}_1(x-x_0, z-z_0)dx_0dz_0 . \tag{19}$$

Into the integral equations (18) and (19) introduce the new variables

$$\xi = \frac{1}{c_1(y)} [x - x_L^{(1)}(y)] \tag{20}$$

$$\eta = \frac{1}{s_1} y \tag{21}$$

$$\xi_0 = \frac{1}{c_1(y_0)} [x_0 - x_L^{(1)}(y_0)] \tag{22}$$

$$\eta_0 = \frac{1}{s_1} y_0 \tag{23}$$

$$\epsilon = \frac{1}{c_2(z)} [x - x_L^{(2)}(z)] \tag{24}$$

$$\zeta = \frac{1}{s_2} z \tag{25}$$

$$\epsilon_0 = \frac{1}{c_2(z_0)} [x_0 - x_L^{(2)}(z_0)] \tag{26}$$

$$\zeta_0 = \frac{1}{s_2} z_0 \tag{27}$$

where s_1 is the semi-span of the tailplane, $c_1(y)$ is the local chord length and $x_L^{(1)}(y)$ the x-coordinate of the leading edge of the tailplane at the spanwise position y, and s_2 is the span of the fin, $c_2(z)$ is the local chord length and $x_L^{(2)}(z)$ the x-coordinate of the leading edge of the fin at the spanwise position z as shown in Fig. 1.

The pair of integral equations (18) and (19) then become

$$\bar{\alpha}_1(\xi, \eta) = \frac{s_1}{4\pi} \int_{-1}^{+1} c_1(y_0)d\eta_0 \int_0^1 \bar{\lambda}_1(\xi_0, \eta_0)\hat{K}_1(x-x_0, y-y_0)d\xi_0 + $$
$$+ \frac{s_2}{4\pi} \int_0^{+1} c_2(z_0)d\zeta_0 \int_0^1 \bar{\lambda}_2(\epsilon_0, \zeta_0)\hat{K}_2(x-x_0, y, z_0)d\epsilon_0 \tag{28}$$

$$\bar{\alpha}_2(\epsilon, \zeta) = \frac{s_1}{4\pi} \int_{-1}^{+1} c_1(y_0)d\eta_0 \int_0^1 \bar{\lambda}_1(\xi_0, \eta_0)\hat{K}_2(x-x_0, z, y_0)d\xi_0 + $$
$$+ \frac{s_2}{4\pi} \int_0^1 c_2(z_0)d\zeta_0 \int_0^1 \bar{\lambda}_2(\epsilon_0, \zeta_0)\hat{K}_1(x-x_0, z-z_0)d\epsilon_0 \tag{29}$$

where

$$\left.\begin{aligned} \bar{\alpha}_1(\xi, \eta) &= \hat{\alpha}_1(x, y) \\ \bar{\alpha}_2(\epsilon, \zeta) &= \hat{\alpha}_2(x, z) \end{aligned}\right\} \tag{30}$$

$$\left.\begin{aligned} \bar{\lambda}_1(\xi_0, \eta_0) &= \hat{\lambda}_1(x_0, y_0) \\ \bar{\lambda}_2(\epsilon_0, \zeta_0) &= \hat{\lambda}_2(x_0, z_0). \end{aligned}\right\} \tag{31}$$

It is convenient to split the kernel functions $\hat{K}_1$ and $\hat{K}_2$ into

$$\hat{K}_1(x, y) = \hat{K}_1^{(1)}(x, y) + \hat{K}_1^{(2)}(x, y) \tag{32}$$

$$\hat{K}_2(x, y, z) = \hat{K}_2^{(1)}(x, y, z) + \hat{K}_2^{(2)}(x, y, z) \tag{33}$$

where

$$\begin{aligned} \hat{K}_1^{(1)}(x, y) &= \int_{(-x+MR_1)/(1-M^2)}^{\infty} e^{-i\omega u/V} \frac{du}{(u^2+y^2)^{3/2}} \\ &= -\frac{\pi}{y^2}\left(\frac{i\omega|y|}{2V}\right)\left[\mathbf{H}_{-1}\left(\frac{i\omega|y|}{V}\right) + \frac{2i}{\pi} K_1\left(\frac{\omega y}{V}\right) - I_1\left(\frac{\omega y}{V}\right)\right] + \\ &\quad + \int_{(-x+MR_1)/(1-M^2)}^{0} e^{-i\omega u/V} \frac{du}{(u^2+y^2)^{3/2}} \end{aligned} \tag{34}$$

$$\hat{K}_1^{(2)}(x, y) = \frac{M(Mx+R_1)}{R_1(x^2+y^2)} \exp\left\{-\frac{i\omega}{V}\left(\frac{-x+MR_1}{1-M^2}\right)\right\} \tag{35}$$

$$\begin{aligned} \hat{K}_2^{(1)}(x, y, z) &= \int_{(-x+MR)/(1-M^2)}^{\infty} e^{-i\omega u/V} \frac{3yz}{(u^2+y^2+z^2)^{5/2}}\, du \\ &= \frac{2\pi yz}{(y^2+z^2)^2}\left\{\frac{i}{2V}\sqrt{(y^2+z^2)}\right\}^2\left[\mathbf{H}_{-2}\left\{\frac{i\omega}{V}\sqrt{(y^2+z^2)}\right\} - \frac{2}{\pi} K_2\left\{\frac{\omega}{V}\sqrt{(y^2+z^2)}\right\} + \right. \\ &\quad \left. + iI_2\left\{\frac{\omega}{V}\sqrt{(y^2+z^2)}\right\}\right] + \int_{(-x+MR)/(1-M^2)}^{0} e^{-i\omega u/V} \frac{3yz}{(u^2+y^2+z^2)^{5/2}}\, du \end{aligned} \tag{36}$$

and

$$\begin{aligned} \hat{K}_2^{(2)}(x, y, z) &= yz\left[\frac{M(Mx+R)^3}{R(x^2+y^2+z^2)^3} + \frac{M^2(1-M^2)x}{R^3(x^2+y^2+z^2)} + \frac{2M(Mx+R)}{R(x^2+y^2+z^2)^2} + \right. \\ &\quad \left. + \frac{i\omega}{V}\frac{M^2(Mx+R)}{R^2(x^2+y^2+z^2)}\right] \exp\left\{-\frac{i\omega}{V}\left(\frac{-x+MR}{1-M^2}\right)\right\}. \end{aligned} \tag{37}$$

In the above I_1 and I_2 are modified Bessel functions of the first kind, K_1 and K_2 are modified Bessel functions of the second kind and $\mathbf{H}_{-1}$ and $\mathbf{H}_{-2}$ are Struve functions in the usual notation (*see* for example Ref. 7).

If the kernel functions are split up according to equations (32) and (33) and an integration by parts carried out on the integrals involving the first components $\hat{K}_1^{(1)}(x, y)$ and $\hat{K}_2^{(1)}(x, y, z)$ in the integral equations (28) and (29) then there result the pair of integral equations

$$\begin{aligned}\bar{\alpha}_1(\xi, \eta) = {} & \frac{s_1}{4\pi}\int_{-1}^{+1} c_1(y_0)d\eta_0 \int_0^1 \{\bar{\lambda}_1(\xi_0, \eta_0)\hat{K}_1^{(2)}(x-x_0, y-y_0) + \\ & + c_1(y_0)\bar{\lambda}_1^{(1)}(\xi_0, \eta_0)\hat{K}_1^{(3)}(x-x_0, y-y_0)\}d\xi_0 + \\ & + \frac{s_1}{4\pi}\int_{-1}^{+1} c_1(y_0)\bar{\lambda}_1^{(1)}(1, \eta_0)K_1^{(1)}(x-x_T^{(1)}(y_0), y-y_0)d\eta_0 + \\ & + \frac{s_2}{4\pi}\int_0^1 c_2(z_0)d\zeta_0 \int_0^1 \{\bar{\lambda}_2(\epsilon_0, \zeta_0)\hat{K}_2^{(2)}(x-x_0, y, z_0) + \\ & + c_2(z_0)\bar{\lambda}_2^{(1)}(\epsilon_0, \zeta_0)\hat{K}_2^{(3)}(x-x_0, y, z_0)\}d\epsilon_0 + \\ & + \frac{s_2}{4\pi}\int_0^1 c_2(z_0)\bar{\lambda}_2^{(1)}(1, \zeta_0)\hat{K}_2^{(1)}(x-x_T^{(2)}(z_0), y, z_0)d\zeta_0\end{aligned} \tag{38}$$

$$\begin{aligned}\bar{\alpha}_2(\epsilon, \zeta) = {} & \frac{s_1}{4\pi}\int_{-1}^{+1} c_1(y_0)d\eta_0 \int_0^1 \{\bar{\lambda}_1(\xi_0, \eta_0)\hat{K}_2^{(2)}(x-x_0, z, y_0) + \\ & + c_1(y_0)\bar{\lambda}_1^{(1)}(\xi_0, \eta_0)\hat{K}_2^{(3)}(x-x_0, z, y_0)\}d\xi_0 + \\ & + \frac{s_1}{4\pi}\int_{-1}^{+1} c_1(y_0)\bar{\lambda}_1^{(1)}(1, \eta_0)\hat{K}_2^{(1)}(x-x_T^{(1)}(y_0), z, y_0)d\eta_0 + \\ & + \frac{s_2}{4\pi}\int_0^1 c_2(z_0)d\zeta_0 \int_0^1 \{\bar{\lambda}_2(\epsilon_0, \zeta_0)\hat{K}_1^{(2)}(x-x_0, z-z_0) + \\ & + c_2(z_0)\bar{\lambda}_2^{(1)}(\epsilon_0, \zeta_0)\hat{K}_1^{(3)}(x-x_0, z-z_0)\}d\epsilon_0 + \\ & + \frac{s_2}{4\pi}\int_0^1 c_2(z_0)\bar{\lambda}_2^{(1)}(1, \zeta_0)\hat{K}_1^{(1)}(x-x_T^{(2)}(z_0), z-z_0)d\zeta_0\end{aligned} \tag{39}$$

where

$$\bar{\lambda}_1^{(1)}(\xi_0, \eta_0) = \int_0^{\xi_0} \bar{\lambda}_1(u, \eta_0)du \tag{40}$$

$$\bar{\lambda}_2^{(2)}(\epsilon_0, \zeta_0) = \int_0^{\epsilon_0} \bar{\lambda}_1(u, \zeta_0)du \tag{41}$$

$$\hat{K}_1^{(3)}(x, y) = \frac{1}{R_1}\frac{(Mx+R_1)^2}{(x^2+y^2)^2}\exp\left\{-\frac{i\omega}{V}\left(\frac{-x+MR_1}{1-M^2}\right)\right\} \tag{42}$$

$$\hat{K}_2^{(3)}(x, y, z) = \frac{3yz}{R}\frac{(Mx+R)^4}{(x^2+y^2+z^2)^4}\exp\left\{-\frac{i\omega}{V}\left(\frac{-x+MR}{1-M^2}\right)\right\} \tag{43}$$

$x_T^{(1)}(y)$ is the x-coordinate of the trailing edge of the tailplane at spanwise position y and $x_T^{(2)}(z)$ is the x-coordinate of the trailing edge of the fin at the spanwise position z.

The pair of integral equations (38) and (39) are better than the pair (18) and (19) for numerical evaluation since the parts of the kernels involving an infinite integral now occur only in a simple integral.

3. *Approximations to the Loading Functions and Location of Loading and Velocity Points.*

The pair of integral equations (38) and (39) may be solved numerically for values of the loading functions $\bar{\lambda}_1(\xi_0, \eta_0)$ and $\bar{\lambda}_2(\epsilon_0, \zeta_0)$ at only a finite number of points on the tailplane and fin surfaces.

Accordingly a set of points, the loading points, at which the values of the loading are to be determined are chosen at the outset and the values of the loadings at these points are regarded as unknowns. The loading functions are then represented approximately in terms of the values at the loading points by use of interpolation functions which have the same behaviours as the loading functions near the edges of the tailplane and fin.

The normal-velocity distributions on the tailplane and fin, respectively obtained from the integral relations (38) and (39), by using the approximations just described for the loading functions, cannot be made to coincide exactly with the given normal-velocity distributions all over these surfaces. Coincidence at a number of points, the velocity points, equal in number to the number of loading points can be obtained. In this way a set of simultaneous equations for the values of the respective loading functions at the loading points in terms of the values of the normal velocities at the velocity points is set up.

The accuracy with which the loading distributions are determined depends on the number of velocity points chosen and also on the choice of their positions over the tailplane and fin.

Since the harmonic velocity potential of the flow satisfies an elliptic partial differential equation in subsonic flow the loading functions can be expected to be smooth over the tailplane and fin, away from any discontinuities which occur in the normal-velocity functions, such as occur at control-surface edges, and also away from any discontinuities in slope of the edges of the surfaces.

For a T-tail without control surfaces the loading is smooth except in the immediate vicinity of any points of discontinuity of slope of the tailplane or fin edges, so $\bar{\lambda}_1(\xi_0, \eta_0)$ and $\bar{\lambda}_2(\epsilon_0, \zeta_0)$ may be approximated quite well by a few terms of an expansion in terms of elementary orthogonal functions over the whole of the surfaces except in the immediate vicinity of those points of discontinuity of edge slope. The values of total forces on the T-tail obtained by using these approximations should be little different from the actual values.

In the following theory the positions of the leading and trailing edges of the tailplane and fin are specified at only a relatively few stations along the spans and it is assumed that sufficiently good approximations to the leading and trailing edges are obtained by taking the equations of these edges to be polynomials which give the correct values at the specified stations. This leads to small errors in the neighbourhood of discontinuity of slope in the leading and trailing edges but the overall effect on the total forces is expected to be small.

For the present let us confine attention to $\bar{\lambda}_2(\epsilon_0, \zeta_0)$. The loading function $\bar{\lambda}_2(\epsilon_0, \zeta_0)$ has a singular behaviour like $1/\sqrt{\epsilon_0}$ near the leading edge of the fin and tends to zero like $\sqrt{(1-\epsilon_0)}$ near the trailing edge. These are the behaviours near the leading and trailing edges of a two-dimensional wing, which must be followed near the leading and trailing edges of the finite fin of the T-tail.

The selection of velocity points along a chord of the fin will be made on the basis of two-dimensional steady-flow theory. For a particular finite oscillating T-tail there may be better selections but the problem of their choice remains. The selection made on the basis of two-dimensional theory should be better than an arbitrary selection.

An approximation to the loading function $\bar{\lambda}_2(\epsilon_0, \zeta_0)$ along the chord at $\eta = \eta_0$, and which has the correct behaviour at the leading and trailing edges is given by

$$\bar{\lambda}_2^*(\epsilon_0, \zeta_0) = \left\{\sum_{r=0}^{n-1} a_r(\zeta_0)\epsilon_0^{\,r}\right\} \sqrt{\left(\frac{1-\epsilon_0}{\epsilon_0}\right)}. \tag{44}$$

If $\bar{\lambda}_2^*(\epsilon_0, \zeta_0)$, for a particular value of ζ_0, represents the loading on a two-dimensional wing lying between $\epsilon_0 = 0$ and $\epsilon_0 = 1$ in steady subsonic flow, then the corresponding normal velocity at any point ϵ in (0, 1) may be calculated. If this calculated normal velocity is equated to the prescribed normal velocity at each of n points ϵ in (0, 1), then there results a system of n simultaneous linear equations which may be solved for the values $a_r(\zeta_0)$. The values of the $a_r(\zeta_0)$ so obtained will depend on which points have been selected as the n upwash points ξ in (0, 1).

The values of the $a_r(\zeta_0)$ for which $\bar{\lambda}_2{}^*(\epsilon_0, \zeta_0)$ of equation (44) is the best approximation to $\bar{\lambda}_2(\epsilon_0, \zeta_0)$ are deemed to be those for which

$$\int_0^1 [\bar{\lambda}_2(\epsilon_0, \zeta_0) - \bar{\lambda}_2{}^*(\epsilon_0, \zeta_0)]^2 \sqrt{\left(\frac{\epsilon_0}{1-\epsilon_0}\right)}\, d\epsilon_0 \tag{45}$$

is a minimum for a given value of ζ_0. This best set of values of the $a_r(\zeta_0)$ cannot be determined exactly since the function $\bar{\lambda}_2(\epsilon_0, \zeta_0)$ is not known explicitly. However, it is possible to select the n velocity points ϵ in (0, 1) so that the values of the $a_r(\zeta_0)$ calculated in terms of the two-dimensional steady-state normal velocities at these velocity points are, in general, as good approximations to the best set of values of the $a_r(\zeta_0)$ as it is possible to get with only n points. The procedure for doing this involves rewriting equation (44) in terms of orthogonal polynomials.

If the set of polynomials $l_r(\epsilon_0)$ of degree r is defined as an orthogonal set over (0, 1) with respect to $\sqrt{\{(1-\epsilon_0)/\epsilon_0\}}$ as weight function, i.e.

$$\int_0^1 l_r(\epsilon_0) l_s(\epsilon_0) \sqrt{\left(\frac{1-\epsilon_0}{\epsilon_0}\right)}\, d\epsilon_0 = \delta_{r,s} \tag{46}$$

where $\delta_{r,s}$ is Kronecker's delta, and the series (44) is written

$$\bar{\lambda}_2{}^*(\epsilon_0, \zeta_0) = \left\{\sum_{r=0}^{n-1} b_r(\zeta_0) l_r(\epsilon_0)\right\} \sqrt{\left(\frac{1-\epsilon_0}{\epsilon_0}\right)} \tag{47}$$

then the integral (45) is a minimum when

$$b_r(\zeta_0) = \int_0^1 \bar{\lambda}_2(\epsilon_0, \zeta_0) l_r(\epsilon_0) d\epsilon_0, \qquad 0 \leqslant r \leqslant n-1. \tag{48}$$

The $b_r(\zeta_0)$ are the coefficients of the first n terms in the infinite expansion of $\bar{\lambda}_2(\epsilon_0, \zeta_0)$ in terms of the $l_r(\epsilon_0)$:

$$\bar{\lambda}_2(\epsilon_0, \zeta_0) = \left\{\sum_{r=0}^{\infty} b_r(\zeta_0) l_r(\epsilon_0)\right\} \sqrt{\left(\frac{1-\epsilon_0}{\epsilon_0}\right)}. \tag{49}$$

Corresponding to the loading distribution

$$l_n(\epsilon_0) \sqrt{\frac{1-\epsilon_0}{\epsilon_0}} \tag{50}$$

on the two-dimensional wing in steady subsonic flow let there be a normal-velocity distribution $\alpha_n(\epsilon)$. The function $\alpha_n(\epsilon)$ turns out to be a polynomial of degree n in ξ.

Then, corresponding to the loading distribution $\bar{\lambda}_2(\epsilon_0, \zeta_0)$ of equation (49) there is a two-dimensional normal-velocity distribution $u(\epsilon, \zeta_0)$ given by the formula

$$u(\epsilon, \zeta_0) = \sum_{r=0}^{\infty} b_r(\zeta_0) \alpha_r(\epsilon). \tag{51}$$

If equation (51) is written down for n separate points ϵ in (0, 1) a set of equations is obtained which may be solved for the $b_r(\zeta_0)$, $0 \leqslant r \leqslant n-1$, in terms of the values of the two-dimensional normal velocity $u(\epsilon, \zeta_0)$ at these points and of the $b_r(\zeta_0)$, $r \geqslant n$. Approximate values of the $b_r(\zeta_0)$, $0 \leqslant r \leqslant n-1$ are those obtained by neglecting all the $b_r(\zeta_0)$, $r \geqslant n$. If, however, the n separate points ϵ in (0, 1) are chosen to be the n roots

$$\xi_k^{w} \quad k = 1, 2, \ldots, n \tag{52}$$

of the polynomial equation

$$\alpha_n(\epsilon) = 0 \tag{53}$$

then the values of the $b_r(\zeta_0)$, $0 \leqslant r \leqslant n-1$ do not depend on the value of $b_n(\zeta_0)$. The approximations to the $b_r(\zeta_0)$, $0 \leqslant r \leqslant n-1$, will then, in general, be better than those obtainable using the values of $u(\epsilon, \zeta_0)$ at any other selection of n points ϵ in (0, 1). The corresponding values of the $a_r(\zeta_0)$ are then the values which are to be taken as the approximations to the best set of values of the $a_r(\zeta_0)$. It follows that the points (52) are, in general, the best ones to take for the chordwise positions of the velocity points on a two-dimensional wing in steady flow. As mentioned earlier, these points will be taken as the velocity points in the case of the finite fin of the T-tail. The points are numbered in order from the leading edge.

The functions $\alpha_n(\epsilon)$ and $l_n(1-\epsilon)$ are proportional to each other (*see* Ref. 6, Section 3). The n velocity points are therefore given by

$$\xi_k^{(w)} = 1 - \xi_i^{(l)} \qquad k = 1, 2, \ldots, n \tag{54}$$

where

$$i = n - k + 1 \tag{55}$$

and

$$\xi_i^{(l)} \quad i = 1, 2, \ldots, n \tag{56}$$

are the roots, numbered in order of increasing size, of the polynomial equation

$$l_n(\xi) = 0\,. \tag{57}$$

As is shown in Ref. 6, Appendix III, the points $\xi_i^{(l)}$ are given by

$$\xi_i^{(l)} = \tfrac{1}{2} - \tfrac{1}{2}\cos\left(\frac{2i-1}{2n+1}\,\pi\right) \qquad i = 1, 2, \ldots n \tag{58}$$

and they are all in the interval (0, 1).

The approximate values of $\bar{\lambda}_2(\epsilon_0, \zeta_0)$ at n points along a chord may be determined from the approximate formula (44). Reciprocally the approximate formula for $\bar{\lambda}_2(\epsilon_0, \zeta_0)$ may be determined in terms of the approximate values at these n points by the use of interpolation functions having the correct behaviours at the leading and trailing edges. It is very convenient from the point of view of mathematical formulation and numerical computation if these n points are taken to be the n points $\xi_i^{(l)}$ defined above in equation (58). These n points will be called the chordwise loading points.

Corresponding to each point $\xi_i^{(l)}$, an interpolation function $h_i^{(n)}(\epsilon_0)$ is formed which is unity at the point $\xi_i^{(l)}$ and zero at the other $(n-1)$ loading points, and which is the product of $\sqrt{\{(1-\epsilon_0)/\epsilon_0\}}$ with a polynomial of degree $(n-1)$ in ϵ_0:

$$h_i^{(n)}(\epsilon_0) = \frac{l_n(\epsilon_0)}{(\epsilon_0 - \xi_i^{(l)})\left[\dfrac{d}{d\epsilon_0}\, l_n(\epsilon_0)\right]} \sqrt{\left(\frac{\xi_i^{(l)}}{1-\xi_i^{(l)}}\right)} \sqrt{\left(\frac{1-\epsilon_0}{\epsilon_0}\right)}\,. \tag{59}$$

The approximation to the loading along a chord of the fin may then be given as the sum

$$\bar{\lambda}_2(\epsilon_0, \zeta_0) = \sum_{i=1}^{n} \bar{\lambda}_2(\xi_i^{(l)}, \zeta_0) h_i^{(n)}(\epsilon_0) \tag{60}$$

where the asterisk has now been dropped from the $\bar{\lambda}_2$, for it is no longer required if one bears in mind that the quantities denoted by $\bar{\lambda}_2$ are henceforth approximations to the actual quantities. Formula (60) is exactly equivalent to formula (44).

Similarly the approximation to the loading on the tailplane may be given as the sum

$$\bar{\lambda}_1(\xi_0, \eta_0) = \sum_{i=1}^{n} \bar{\lambda}_1(\xi_i^{(l)}, \eta_0) h_i^{(n)}(\xi_0). \tag{61}$$

The loading distributions have the behaviour of $\sqrt{(1-\zeta_0)}$ near the tip of the fin, of $\sqrt{(1-\eta_0)}$ near the port tip of the tailplane and of $\sqrt{(1+\eta_0)}$ near the starboard tip of the tailplane. These are the behaviours near the edges of a very slender rectangular wing. Near the junction line DC of the tailplane and fin the spanwise behaviours of the loading are regular except that there may be a finite discontinuity of the tailplane loading function across this junction line. The functions $\bar{\lambda}_2(\xi_i^{(l)}, \zeta_0)$ and $\bar{\lambda}_1(\xi_i^{(l)}, \eta_0)$ must take these behaviours into account.

A suitable approximation to the fin spanwise function $\bar{\lambda}_2(\xi_i^{(l)}, \zeta_0)$ is given by the product of $\sqrt{(1-\zeta_0)}$ with a polynomial of degree $(m-1)$ in ζ_0. Following the procedure of the chordwise variable ϵ_0, we define a set of polynomials $\mu_r(\zeta_0)$ of degree r which are orthogonal over (0, 1) with $\sqrt{(1-\zeta_0)}$ as weight function, i.e.

$$\int_0^1 \mu_r(\zeta_0)\mu_s(\zeta_0)\sqrt{(1-\zeta_0)}d\zeta_0 = \delta_{r,s}. \tag{62}$$

To choose the spanwise locations of the velocity points it is observed that the kernel $K_1(x, z)$ in equation (19) behaves like $1/z^2$ near $z = 0$. The spanwise distribution of normal velocity $w_m(\zeta)$ corresponding to the loading distribution $\mu_m(\zeta_0)\sqrt{(1-\zeta_0)}$ and upon which the choice of spanwise velocity points depends is then taken to be

$$w_m(\zeta) = \int_0^1 \frac{\mu_m(\zeta_0)}{(\zeta-\zeta_0)^2}\sqrt{(1-\zeta_0)}d\zeta_0. \tag{63}$$

The spanwise locations of the velocity are then chosen to be the m real roots

$$\eta_r^{(w)} \quad r = 1, 2, \ldots, m \tag{64}$$

of the equation

$$w_m(\zeta) = 0 \tag{65}$$

for reasons similar to the ones for which the chordwise velocity points were chosen. The spanwise points are numbered in order starting from the function line DC and proceeding towards the tip.

Polynomials which satisfy equation (62) are Jacobi polynomials given by

$$\mu_r(\zeta_0) = G_r\left(\frac{3}{2}, 1, \zeta_0\right) = \sum_{p=0}^{r} \frac{(-1)^p}{2^{2p}}\left(\frac{r\,!}{p\,!}\right)^2 \frac{(2r+2p-1)\,!}{(r-p)\,!\,(r+p)\,!}\frac{\zeta_0^{\,p}}{(2r+1)\,!}. \tag{66}$$

It is again very convenient from the point of view of mathematical formulation if the spanwise loading points are taken to be the m roots

$$\eta_j \quad j = 1, 2, \ldots, m \tag{67}$$

of the m'th degree polynomial

$$\mu_m(\zeta_0) = 0. \tag{68}$$

Corresponding to each point η_j an interpolation function $g_j^{(m)}(\zeta_0)$ is formed which is unity at the point η_j and zero at the other $(m-1)$ spanwise points, and which is the product of $\sqrt{(1-\zeta_0)}$ with a polynomial of degree $(m-1)$ in ζ_0:

$$g_j^{(m)}(\zeta_0) = \frac{\mu_m(\zeta_0)}{(\zeta_0-\eta_j)\left[\dfrac{d}{d\zeta_0}\mu_m(\zeta_0)\right]_{\zeta_0=\eta_j}} \cdot \frac{\sqrt{(1-\zeta_0)}}{\sqrt{(1-\eta_j)}}. \tag{69}$$

The approximation to $\bar{\lambda}_2(\xi_i^{(l)}, \zeta_0)$ is then given by

$$\bar{\lambda}_2(\xi_i^{(l)}, \zeta_0) = \sum_{j=1}^{m} \bar{\lambda}_2(\xi_i^{(l)}, \eta_j) g_j^{(m)}(\zeta_0) \tag{70}$$

so that from equation (60) we get

$$\bar{\lambda}_2(\epsilon_0, \zeta_0) = \sum_{i=1}^{n}\sum_{j=1}^{m} \bar{\lambda}_2(\xi_i^{(l)}, \eta_j) h_i^{(n)}(\epsilon_0) g_j^{(m)}(\zeta_0). \tag{71}$$

To allow for a discontinuity in $\bar{\lambda}_1(\xi_i^{(l)}, \eta_0)$, this function is treated separately for $\eta_0 > 0$ and $\eta_0 < 0$ in a manner similar to the above with ζ_0, for the discontinuities at the ends of the intervals of η_0 correspond with those at the ends of the intervals of ζ_0.

The approximation to $\bar{\lambda}_1(\xi_0, \eta_0)$ is then given by

$$\bar{\lambda}_1(\xi_0, \eta_0) = \begin{cases} \displaystyle\sum_{i=1}^{n}\sum_{j=1}^{m} \bar{\lambda}_1(\xi_i^{(l)}, \eta_j) h_i^{(n)}(\xi_0) g_j^{(m)}(\eta_0) & \text{for} \quad \eta_0 > 0 \\ \displaystyle\sum_{i=1}^{n}\sum_{j=1}^{m} \bar{\lambda}_1(\xi_i^{(l)}, -\eta_j) h_i^{(n)}(\xi_0) g_j^{(m)}(-\eta_0) & \text{for} \quad \eta_0 < 0 \end{cases} \tag{72}$$

It turns out that the positions of the velocity points (64) are very close to the loading points (67), and this is true in particular near the tip of the fin. This introduces complications into the numerical evaluation of some integrals, used later. Since the above process of choosing the spanwise positions of the velocity points can give at most only an indication of the best positions, the points

$$\eta_r \quad r = 1, 2, \ldots, m \tag{73}$$

will be taken to be the velocity points instead of the points (64). The choice of this set has the advantage that the complications in the said numerical evaluation of integrals do not appear and also a certain amount of symmetry is introduced.

To end this section formulae for the loading and velocity points on the T-tail are given. It is assumed that the same number of spanwise and chordwise stations are taken on the fin and each half of the tailplane.

The totality of loading points is therefore:

$$\left.\begin{aligned} x_{2,i,j}^{(l)} &= c_2(z_{2,j})\xi_i^{(l)} + x_L^{(2)}(z_{2,j}) \\ z_{2,j} &= s_2\eta_j \end{aligned}\right\} \begin{aligned} i &= 1, 2, \ldots, n \\ j &= 1, 2, \ldots, m \end{aligned} \tag{74}$$

on the fin;

$$\left.\begin{aligned} x_{1,i,j}^{(l)} &= c_1(y_{1,j}^{+})\xi_i^{(l)} + x_L^{(1)}(y_{1,j}^{+}) \\ y_{1,j}^{+} &= s_1\eta_j \end{aligned}\right\} \begin{aligned} i &= 1, 2, \ldots, n \\ j &= 1, 2, \ldots, m \end{aligned} \tag{75}$$

on the port half-tailplane;

$$\left.\begin{aligned} x_{1,i,j}^{(l)} &= c_1(y_{1,j}^{-})\xi_i^{(l)} + x_L^{(1)}(y_{1,j}^{-}) \\ y_{1,j}^{-} &= -s_1\eta_j \end{aligned}\right\} \begin{aligned} i &= 1, 2, \ldots, n \\ j &= 1, 2, \ldots, m \end{aligned} \tag{76}$$

on the starboard tailplane.

The totality of velocity points is:

$$\left.\begin{aligned} x_{2,k,r}^{(w)} &= c_2(z_{2,r})\xi_k^{(w)} + x_L^{(2)}(z_{2,r}) \\ z_{2,r} &= s_1\eta_r \end{aligned}\right\} \begin{aligned} k &= 1, 2, \ldots, n \\ r &= 1, 2, \ldots, m \end{aligned} \tag{77}$$

on the fin;

$$\left.\begin{aligned} x_{1,k,r}^{(w)} &= c_1(y_{1,r}^{+})\xi_k^{(w)} + x_L^{(1)}(y_{1,r}^{+}) \\ y_{1,r}^{+} &= s_1\eta_r \end{aligned}\right\} \begin{aligned} k &= 1, 2, \ldots, n \\ r &= 1, 2, \ldots, m \end{aligned} \tag{78}$$

on the port half-tailplane;

$$\left.\begin{aligned} x_{1,k,r}^{(w)} &= c_1(y_{1,r}^{-})\xi_k^{(w)} + x_L^{(1)}(y_{1,r}^{-}) \\ y_{1,r}^{-} &= -s_1\eta_r \end{aligned}\right\} \begin{aligned} k &= 1, 2, \ldots, n \\ r &= 1, 2, \ldots, m \end{aligned} \tag{79}$$

on the starboard half-tailplane.

4. *The Integration Procedure.*

Substituting the approximations (60) and (61) for $\bar{\lambda}_2(\epsilon_0, \zeta_0)$ and $\bar{\lambda}_1(\xi_0, \eta_0)$ into the forms (38) and (39) of the integral equations, we obtain

$$\begin{aligned} \bar{\alpha}_1(\xi, \eta) = & \sum_{i=1}^{n} \int_{-1}^{+1} \frac{\bar{\lambda}_1(\xi_i^{(l)}, \eta_0)}{(\eta-\eta_0)^2} I_i^{(n)}(\eta, \eta_0, \xi) d\eta_0 + \\ & + \sum_{i=1}^{n} \int_{0}^{1} \frac{s_1 s_2 \eta \zeta_0}{(s_1^2\eta^2 + s_2^2\zeta_0^2)^2} \bar{\lambda}_2(\xi_i^{(l)}, \zeta_0) J_i^{(n)}(\eta, \zeta_0, \xi) d\zeta_0 \end{aligned} \tag{80}$$

$$\begin{aligned} \bar{\alpha}_2(\epsilon, \zeta) = & \sum_{i=1}^{n} \int_{-1}^{+1} \frac{s_1 s_2 \eta_0 \zeta}{(s_1^2\eta_0^2 + s_2^2\zeta^2)^2} \bar{\lambda}_1(\xi_i^{(l)}, \eta_0) M_i^{(n)}(\eta_0, \zeta, \epsilon) d\eta_0 + \\ & + \sum_{i=1}^{n} \int_{0}^{1} \frac{\bar{\lambda}_2(\xi_i^{(l)}, \zeta_0)}{(\zeta-\zeta_0)^2} N_i^{(n)}(\zeta, \zeta_0, \epsilon) d\zeta_0 \end{aligned} \tag{81}$$

where

$$\begin{aligned} I_i^{(n)}(\eta, \eta_0, \xi) = & \frac{s_1}{4\pi} c_1(y_0)(\eta-\eta_0)^2 \Big[\int_0^1 \{h_i^{(n)}(\xi_0)\hat{K}_1^{(2)}(x-x_0, y-y_0) + \\ & + c_1(y_0) h_i^{(1,n)}(\xi_0)\hat{K}_1^{(3)}(x-x_0, y-y_0)\} d\xi_0 + h_i^{(1,n)}(1)\hat{K}_1^{(1)}(x-x_T^{(1)}(y_0), y-y_0) \Big] \end{aligned} \tag{82}$$

$$\begin{aligned} J_i^{(n)}(\eta, \zeta_0, \xi) = & \frac{1}{4\pi s_1} c_2(z_0) \frac{(s_1^2\eta^2 + s_2^2\zeta_0^2)^2}{\eta\zeta_0} \Big[\int_0^1 \{h_i^{(n)}(\epsilon_0)\hat{K}_2^{(2)}(x-x_0, y, z_0) + \\ & + c_2(z_0) h_i^{(1,n)}(\epsilon_0)\hat{K}_2^{(3)}(x-x_0, y, z_0)\} d\epsilon_0 + h_i^{(1,n)}(1)\hat{K}_2^{(1)}(x-x_T^{(2)}(z_0), y, z_0) \Big] \end{aligned} \tag{83}$$

$$\begin{aligned} M_i^{(n)}(\eta_0, \zeta, \epsilon) = & \frac{1}{4\pi s_2} c_1(y_0) \frac{(s_1^2\eta_0^2 + s_2^2\zeta^2)^2}{\eta_0\zeta} \Big[\int_0^1 \{h_i^{(n)}(\xi_0)\hat{K}_2^{(2)}(x-x_0, z, y_0) + \\ & + c_1(y_0) h_i^{(1,n)}(\xi_0)\hat{K}_2^{(3)}(x-x_0, z, y_0)\} d\xi_0 + h_i^{(1,n)}(1)\hat{K}_2^{(1)}(x-x_T^{(1)}(y_0), y-y_0) \Big] \end{aligned} \tag{84}$$

$$\begin{aligned} N_i^{(n)}(\zeta, \zeta_0, \epsilon) = & \frac{s_2}{4\pi} c_2(z_0)(\zeta-\zeta_0)^2 \Big[\int_0^1 \{h_i^{(n)}(\epsilon_0)\hat{K}_1^{(2)}(x-x_0, z-z_0) + \\ & + c_2(z_0) h_i^{(1,n)}(\epsilon_0)\hat{K}_1^{(3)}(x-x_0, z-z_0)\} d\epsilon_0 + h_i^{(1,n)}(1)\hat{K}_1^{(1)}(x-x_T^{(2)}(z_0), z-z_0) \Big] \end{aligned} \tag{85}$$

and

$$h_i^{(1,n)}(\xi) = \int_0^{\xi} h_i^{(n)}(u) du. \tag{86}$$

The function $I_i^{(n)}(\eta, \eta_0, \xi)$ may be developed into a series of the form

$$I_i^{(n)}(\eta, \eta_0, \xi) = \sum_{s=0}^{\infty} E_{s,i}^{(n)}(\xi, \eta)(\eta-\eta_0)^s + (\eta-\eta_0)^2 \log|\eta-\eta_0| \sum_{s=0}^{\infty} F_{s,i}^{(n)}(\xi, \eta)(\eta-\eta_0)^s \tag{87}$$

in the neighbourhood of $\eta_0 = \eta$, and the function $N_i^{(n)}(\zeta, \zeta_0, \epsilon)$ may be developed into a series of the form

$$N_i^{(n)}(\zeta, \zeta_0, \epsilon) = \sum_{s=0}^{\infty} G_{s,i}^{(n)}(\epsilon, \zeta)(\zeta-\zeta_0)^s + (\zeta-\zeta_0)^2 \log|\zeta-\zeta_0| \sum_{s=0}^{\infty} H_{s,i}^{(n)}(\epsilon, \zeta)(\zeta-\zeta_0)^s \tag{88}$$

in the neighbourhood of $\zeta_0 = \zeta$.

The principal value integrals in equation (81) could be evaluated approximately by using the interpolation formula

$$\bar{\lambda}_2(\xi_i^{(l)}, \zeta_0)N_i^{(n)}(\zeta, \zeta_0, \epsilon) = \sum_{j=1}^{m} \bar{\lambda}_2(\xi_i^{(l)}, \eta_j)N_i^{(n)}(\zeta, \eta_j, \epsilon)g_j^{(m)}(\zeta_0) \tag{89}$$

and integrating each term obtained by putting this series into (81). The accuracy of the value obtained would however be adversely affected by the presence of the logarithmic terms in the expansion (88), and in particular by the lowest-order logarithmic term, especially if the value of m is small. The accuracy can be improved if the lowest-order logarithmic term is removed from $N_i^{(n)}(\zeta, \zeta_0, \epsilon)$ and dealt with separately while the remainder is dealt with by using the interpolation procedure. The procedure is similar to that of Mangler and Spencer[8].

Write the identity

$$\begin{aligned}\bar{\lambda}_2(\xi_i^{(l)}, \zeta_0)N_i^{(n)}(\zeta, \zeta_0, \epsilon) = {} & \bar{\lambda}_2(\xi_i^{(l)}, \zeta)\frac{\sqrt{(1-\zeta_0)}}{\sqrt{(1-\zeta)}} H_{0,i}^{(n)}(\epsilon, \zeta)(\zeta-\zeta_0)^2 \log|\zeta-\zeta_0| + \\ & + \Bigg[\bar{\lambda}_2(\xi_i^{(l)}, \zeta_0)N_i^{(n)}(\zeta, \zeta_0, \epsilon) - \\ & - \bar{\lambda}_2(\xi_i^{(l)}, \zeta)\frac{\sqrt{(1-\zeta_0)}}{\sqrt{(1-\zeta)}} H_{0,i}^{(n)}(\epsilon, \zeta)(\zeta-\zeta_0)^2 \log|\zeta-\zeta_0|\Bigg].\end{aligned} \tag{90}$$

The lowest order logarithmic singularity is missing in the expression in square brackets so the interpolation process is to be applied to that expression. We then obtain approximately

$$\begin{aligned}\bar{\lambda}_2(\xi_i^{(l)}, \zeta_0)N_i^{(n)}(\zeta, \zeta_0, \epsilon) = {} & \bar{\lambda}_2(\xi_i^{(l)}, \zeta)\frac{\sqrt{(1-\zeta_0)}}{\sqrt{(1-\zeta)}} H_{0,i}^{(n)}(\epsilon, \zeta)(\zeta-\zeta_0)^2 \log|\zeta-\zeta_0| + \\ & + \sum_{j=1}^{m}\Bigg[\bar{\lambda}_2(\xi_i^{(l)}, \eta_j)N_i^{n}(\zeta, \eta_j, \epsilon) - \\ & - \bar{\lambda}_2(\xi_i^{(l)}, \zeta)\frac{\sqrt{(1-\eta_j)}}{\sqrt{(1-\zeta)}} H_{0,i}^{(n)}(\epsilon, \zeta)(\zeta-\eta_j)^2 \log|\zeta-\eta_j|\Bigg] g_j^{(m)}(\zeta_0)\end{aligned} \tag{91}$$

and

$$\begin{aligned}\int_0^1 \frac{\bar{\lambda}_2(\xi_i^{(l)}, \zeta_0)}{(\zeta-\zeta_0)^2} N_i^{(n)}(\zeta, \zeta_0, \epsilon)d\zeta = {} & \frac{\bar{\lambda}_2(\xi_i^{(l)}, \zeta)}{\sqrt{(1-\zeta)}} H_{0,i}^{(n)}(\epsilon, \zeta)\Bigg[\int_0^1 \log|\zeta-\zeta_0|\sqrt{(1-\zeta_0)}d\zeta_0 - \\ & - \sum_{j=1}^{m}(\zeta-\eta_j)^2 \log|\zeta-\eta_j|\sqrt{(1-\eta_j)} \int_0^1 \frac{g_j^{(m)}(\zeta_0)}{(\zeta-\zeta_0)^2} d\zeta_0\Bigg] + \\ & + \sum_{j=1}^{m} \bar{\lambda}_2(\xi_i^{(l)}, \eta_j)N_i^{(n)}(\zeta, \eta_j, \epsilon) \int_0^1 \frac{g_j^{(m)}(\zeta_0)}{(\zeta-\zeta_0)^2} d\zeta_0.\end{aligned} \tag{92}$$

The function $H_{0,i}^{(n)}(\epsilon, \zeta)$ can be worked out, as in Ref. 6, Appendix IV. It is

$$H_{0,i}^{(n)}(\epsilon, \zeta) = \frac{1}{4\pi}\frac{s_2}{c_2(z)}\left\{-(1-M^2)h_i^{(n)\prime}(\epsilon) + 2\frac{i\omega}{V}c_2(z)h_i^{(n)}(\epsilon) + \frac{\omega^2}{V^2}c_2^2(z)h_i^{(1,n)}(\epsilon)\right\} \tag{93}$$

Also

$$\int_0^1 \log|\zeta - \zeta_0|\sqrt{(1-\zeta_0)}d\zeta_0 = \frac{2}{3}\log|\zeta| + \frac{2}{3}(1-\zeta)^{3/2}\log\left|\frac{1+\sqrt{(1-\zeta)}}{1-\sqrt{(1-\zeta)}}\right| - \frac{4}{3}|1-\zeta| - \frac{4}{9}. \tag{94}$$

The lowest-order logarithmic term is removed from $I_i^{(n)}(\eta, \eta_0, \epsilon)$ and the remainder dealt with by interpolation procedures in order to evaluate the principal-value integral in equation (80).

For $\eta_0 > 0$ write the identity

$$\begin{aligned}\bar{\lambda}_1(\xi_i^{(l)}, \eta_0)I_i^{(n)}(\eta, \eta_0, \xi) = {} & \bar{\lambda}_1(\xi_i^{(l)}, \eta)\frac{\sqrt{(1-\eta_0)}}{\sqrt{(1-\eta)}}F_{0,i}^{(n)}(\xi, \eta)(\eta-\eta_0)^2\log|\eta-\eta_0| + \\ & + \Big[\bar{\lambda}_1(\xi_i^{(l)}, \eta_0)I_i^{(n)}(\eta, \eta_0, \xi) - \\ & - \bar{\lambda}_1(\xi_i^{(l)}, \eta)\frac{\sqrt{(1-\eta_0)}}{\sqrt{(1-\eta)}}F_{0,i}^{(n)}(\xi, \eta)(\eta-\eta_0)^2\log|\eta-\eta_0|\Big].\end{aligned} \tag{95}$$

For $\eta_0 < 0$ write the identity

$$\begin{aligned}\bar{\lambda}_1(\xi_i^{(l)}, \eta_0)I_i^{(n)}(\eta, \eta_0, \xi) = {} & \bar{\lambda}_1(\xi_i^{(l)}, \eta)\frac{\sqrt{(1+\eta_0)}}{\sqrt{(1+\eta)}}F_{0,i}^{(n)}(\xi, \eta)(\eta-\eta_0)^2\log|\eta-\eta_0| + \\ & + \Big[\bar{\lambda}_1(\xi_i^{(l)}, \eta_0)I_i^{(n)}(\eta, \eta_0, \xi) - \\ & - \bar{\lambda}_1(\xi_i^{(l)}, \eta)\frac{\sqrt{(1+\eta_0)}}{\sqrt{(1+\eta)}}F_{0,i}^{(n)}(\xi, \eta)(\eta-\eta_0)^2\log|\eta-\eta_0|\Big].\end{aligned} \tag{96}$$

The lowest-order logarithmic singularity is missing in the expressions in square brackets in equations (95) and (96), so the interpolation process is to be applied to these expressions. On doing this and integrating, we get

$$\begin{aligned}&\int_{-1}^{+1}\frac{\bar{\lambda}_1(\xi_i^{(l)}, \eta_0)}{(\eta-\eta_0)^2}I_i^{(n)}(\eta, \eta_0, \xi)d\eta_0 \\ &= \int_0^1\frac{\bar{\lambda}_1(\xi_i^{(l)}, \eta_0)}{(\eta-\eta_0)^2}I_i^{(n)}(\eta, \eta_0, \xi)d\eta_0 + \int_{-1}^0\frac{\bar{\lambda}_1(\xi_i^{(l)}, \eta_0)}{(\eta-\eta_0)^2}I_i^{(n)}(\eta, \eta_0, \xi)d\eta_0 \\ &= \bar{\lambda}_1(\xi_i^{(l)}, \eta)F_{0,i}^{(n)}(\xi, \eta)\Bigg[\frac{1}{\sqrt{(1-\eta)}}\left\{\int_0^1\log|\eta-\eta_0|\sqrt{(1-\eta_0)}d\eta_0 - \right. \\ &\quad \left. - \sum_{j=1}^{m}(\eta-\eta_j)^2\log|\eta-\eta_j|\sqrt{(1-\eta_j)}\int_0^1\frac{g_j^{(m)}(\eta_0)}{(\eta-\eta_0)^2}d\eta_0\right\} + \\ &\quad + \frac{1}{\sqrt{(1+\eta)}}\left\{\int_0^1\log|\eta+\eta_0|\sqrt{(1-\eta_0)}d\eta_0 - \right. \\ &\quad \left. - \sum_{j=1}^{m}(\eta+\eta_j)^2\log|\eta+\eta_j|\sqrt{(1+\eta_j)}\int_0^1\frac{g_j^{(m)}(\eta_0)}{(\eta+\eta_0)^2}d\eta_0\right\}\Bigg] + \\ &\quad + \sum_{j=1}^{m}\bar{\lambda}_1(\xi_i^{(l)}, \eta_j)I_i^{(n)}(\eta, \eta_j, \xi)\int_0^1\frac{g_j^{(m)}(\eta_0)}{(\eta-\eta_0)^2}d\eta_0 + \\ &\quad + \sum_{j=1}^{m}\bar{\lambda}_1(\xi_i^{(l)}, -\eta_j)I_i^{(n)}(\eta, -\eta_j, \xi)\int_0^1\frac{g_j^{(m)}(\eta_0)}{(\eta+\eta_0)^2}d\eta_0.\end{aligned} \tag{97}$$

The function $F_{0,i}^{(n)}(\xi, \eta)$ is quite analogous to $H_{0,i}^{(n)}(\epsilon, \zeta)$ of equation (93) and is given by

$$F_{0,i}^{(n)}(\xi, \eta) = \frac{1}{4\pi} \frac{s_1}{c_1(y)} \left\{ -(1-M^2)h_i^{(n)\prime}(\xi) + 2\frac{i\omega}{V} c_1(y) h_i^{(n)}(\xi) + \frac{\omega^2}{V^2} c_1^2(y) h_i^{(1,n)}(\xi) \right\}. \tag{98}$$

The other integrals in equations (80) and (81) have no singularities in the range of integration, though the integrands can be of rapid variation near the origin. This is taken account of by the factors $(s_1 s_2 \eta \zeta_0)/(s_1^2\eta^2 + s_2^2\zeta_0^2)^2$ and $(s_1 s_2 \eta_0 \zeta)/(s_1^2\eta_0^2 + s_2^2\zeta^2)$. These integrals are then evaluated by using the interpolation formulae

$$\bar{\lambda}_2(\xi_i^{(l)}, \zeta_0) J_i^{(n)}(\eta, \zeta_0, \xi) = \sum_{j=1}^{m} \bar{\lambda}_2(\xi_i^{(l)}, \eta_j) J_i^{(n)}(\eta, \eta_j, \xi) g_j^{(m)}(\zeta_0) \tag{99}$$

$$\bar{\lambda}_1(\xi_i^{(l)}, \eta_0) M_i^{(n)}(\eta_0, \zeta, \epsilon) = \begin{cases} \displaystyle\sum_{j=1}^{m} \bar{\lambda}_1(\xi_i^{(l)}, \eta_j) M_i^{(n)}(\eta_j, \zeta, \epsilon) g_j^{(m)}(\eta_0) & \eta_0 > 0 \\ \displaystyle\sum_{j=1}^{m} \bar{\lambda}_1(\xi_i^{(l)}, -\eta_j) M_i^{(n)}(-\eta_j, \zeta, \epsilon) g_j^{(m)}(-\eta_0) & \eta_0 < 0 \end{cases} \tag{100}$$

and integrating term by term.

The equations (80) and (81) may then be replaced by the approximate equations

$$\begin{aligned} \bar{\alpha}_1(\xi, \eta) = \sum_{i=1}^{n} \bar{\lambda}_1(\xi_i^{(l)}, \eta) F_{0,i}^{(n)}(\xi, \eta) \Bigg[\frac{1}{\sqrt{(1-\eta)}} \Bigg\{ \int_0^1 \log|\eta - \eta_0| \sqrt{(1-\eta_0)} d\eta_0 - \\ - \sum_{j=1}^{m} (\eta - \eta_j)^2 \log|\eta - \eta_j| \sqrt{(1-\eta_j)} \int_0^1 \frac{g_j^{(m)}(\eta_0)}{(\eta - \eta_0)^2} d\eta_0 \Bigg\} + \\ + \frac{1}{\sqrt{(1+\eta)}} \Bigg\{ \int_0^1 \log|\eta + \eta_0| \sqrt{(1-\eta_0)} d\eta_0 - \\ - \sum_{j=1}^{m} (\eta + \eta_j)^2 \log|\eta + \eta_j| \sqrt{(1+\eta_j)} \int_0^1 \frac{g_j^{(m)}(\eta_0)}{(\eta + \eta_0)^2} d\eta_0 \Bigg\} \Bigg] + \\ + \sum_{i=1}^{n} \sum_{j=1}^{m} \bar{\lambda}_1(\xi_i^{(l)}, \eta_j) I_i^{(n)}(\eta, \eta_j, \xi) \int_0^1 \frac{g_j^{(m)}(\eta_0)}{(\eta - \eta_0)^2} d\eta_0 + \\ + \sum_{i=1}^{n} \sum_{j=1}^{m} \bar{\lambda}_1(\xi_i^{(l)}, -\eta_j) I_i^{(n)}(\eta, -\eta_j, \xi) \int_0^1 \frac{g_j^{(m)}(\eta_0)}{(\eta + \eta_0)^2} d\eta_0 + \\ + \sum_{i=1}^{n} \sum_{j=1}^{m} \bar{\lambda}_2(\xi_i^{(l)}, \eta_j) J_i^{(n)}(\eta, \eta_j, \xi) \int_0^1 \frac{s_1 s_2 \eta \zeta_0}{(s_1^2\eta^2 + s_2^2\zeta_0^2)^2} g_j^{(m)}(\zeta_0) d\zeta_0 \end{aligned} \tag{101}$$

$$\begin{aligned} \bar{\alpha}_2(\epsilon, \zeta) = \sum_{i=1}^{n} \sum_{j=1}^{m} \bar{\lambda}_1(\xi_i^{(l)}, \eta_j) M_i^{(n)}(\eta_j, \zeta, \epsilon) \int_0^1 \frac{s_1 s_2 \eta_0 \zeta}{(s_1^2\eta_0^2 + s_2^2\zeta^2)^2} g_j^{(m)}(\eta_0) d\eta_0 - \\ - \sum_{i=1}^{n} \sum_{j=1}^{m} \bar{\lambda}_1(\xi_i^{(l)}, -\eta_j) M_i^{(n)}(-\eta_j, \zeta, \epsilon) \int_0^1 \frac{s_1 s_2 \eta_0 \zeta}{(s_1^2\eta_0^2 + s_2^2\zeta^2)^2} g_j^{(m)}(\eta_0) d\eta_0 + \\ + \sum_{i=1}^{n} \bar{\lambda}_2(\xi_i^{(l)}, \zeta) H_{0,i}^{(n)}(\epsilon, \zeta) \Bigg[\frac{1}{\sqrt{(1-\zeta)}} \Bigg\{ \int_0^1 \log|\zeta - \zeta_0| \sqrt{(1-\zeta_0)} d\zeta_0 - \\ - \sum_{j=1}^{m} (\zeta - \eta_j)^2 \log|\zeta - \eta_j| \sqrt{(1-\eta_j)} \int_0^1 \frac{g_j^{(m)}(\zeta_0)}{(\zeta - \zeta_0)^2} d\zeta_0 \Bigg\} \Bigg] + \\ + \sum_{i=1}^{n} \sum_{j=1}^{m} \bar{\lambda}_2(\xi_i^{(l)}, \eta_j) N_i^{(n)}(\zeta, \eta_j, \epsilon) \int_0^1 \frac{g_j^{(m)}(\zeta_0)}{(\zeta - \zeta_0)^2} d\zeta_0. \end{aligned} \tag{102}$$

If equation (101) is written down for the 2 mn velocity points on the tailplane and equation (102) is written down for the mn velocity points on the fin, there results a set of 3 mn simultaneous linear equations for the 3 mn unknowns $\bar{\lambda}_1(\xi_i^{(l)}, \eta_j)$, $\bar{\lambda}_1(\xi_i^{(l)}, -\eta_j)$ and $\bar{\lambda}_2(\xi_i^{(l)}, \eta_j)$ for $i = 1, 2, \ldots, n$; $j = 1, 2, \ldots, m$; in terms of the known values of $\bar{\alpha}_1(\xi, \eta)$ or $\bar{\alpha}_2(\epsilon, \eta)$ at the velocity points. These equations are

$$\begin{aligned}
\bar{\alpha}_1(\xi_k^{(w)}, \eta_r) = & \sum_{i=1}^{n} \bar{\lambda}_1(\xi_i^{(l)}, \eta_r) F_{0,i}^{(n)}(\xi_k^{(w)}, \eta_r) \Bigg[\frac{1}{\sqrt{(1-\eta_r)}} \Bigg\{ \int_0^1 \log|\eta_r - \eta_0| \sqrt{(1-\eta_0)}\, d\eta_0 - \\
& - \sum_{j=1}^{m} (\eta_r - \eta_j)^2 \log|\eta_r - \eta_j| \sqrt{(1-\eta_j)} \int_0^1 \frac{g_j^{(m)}(\eta_0)}{(\eta_r - \eta_0)^2}\, d\eta_0 \Bigg\} + \\
& + \frac{1}{\sqrt{(1+\eta_r)}} \Bigg\{ \int_0^1 \log|\eta_r + \eta_0| \sqrt{(1-\eta_0)}\, d\eta_0 - \\
& - \sum_{j=1}^{m} (\eta_r + \eta_j)^2 \log|\eta_r + \eta_j| \sqrt{(1+\eta_j)} \int_0^1 \frac{g_j^{(m)}(\eta_0)}{(\eta_r + \eta_0)^2}\, d\eta_0 \Bigg\} \Bigg] + \\
& + \sum_{i=1}^{n} \sum_{j=1}^{m} \bar{\lambda}_1(\xi_i^{(l)}, \eta_j) I_i^{(n)}(\eta_r, \eta_j, \xi_k^{(w)}) \int_0^1 \frac{g_j^{(m)}(\eta_0)}{(\eta_r - \eta_0)^2}\, d\eta_0 + \\
& + \sum_{i=1}^{n} \sum_{j=1}^{m} \bar{\lambda}_1(\xi_i^{(l)}, -\eta_j) I_i^{(n)}(\eta_r, -\eta_j, \xi_k^{(w)}) \int_0^1 \frac{g_j^{(m)}(\eta_0)}{(\eta_r + \eta_0)^2}\, d\eta_0 + \\
& + \sum_{i=1}^{n} \sum_{j=1}^{m} \bar{\lambda}_2(\xi_i^{(l)}, \eta_j) I_i^{(n)}(\eta_r, \eta_j, \xi_k^{(w)}) \int_0^1 \frac{s_1 s_2 \eta_r \zeta_0}{(s_1^2 \eta_r^2 + s_2^2 \zeta_0^2)^2}\, g_j^{(m)}(\zeta_0)\, d\zeta_0 .
\end{aligned}$$

$$k = 1, 2, \ldots, n$$

$$r = 1, 2, \ldots, m \qquad (103)$$

$$\begin{aligned}
\bar{\alpha}_1(\xi_k^{(w)}, -\eta_r) = & \sum_{i=1}^{n} \bar{\lambda}_1(\xi_i^{(l)}, -\eta_r) F_{0,i}^{(n)}(\xi_k^{(w)}, -\eta_r) \Bigg[\frac{1}{\sqrt{(1-\eta_r)}} \Bigg\{ \int_0^1 \log|\eta_r - \eta_0| \sqrt{(1-\eta_0)}\, d\eta_0 - \\
& - \sum_{j=1}^{m} (\eta_r - \eta_j)^2 \log|\eta_r - \eta_j| \sqrt{(1-\eta_j)} \int_0^1 \frac{g_j^{(m)}(\eta_0)}{(\eta_r - \eta_0)^2}\, d\eta_0 \Bigg\} + \\
& + \frac{1}{\sqrt{(1+\eta_r)}} \Bigg\{ \int_0^1 \log|\eta_r + \eta_0| \sqrt{(1-\eta_0)}\, d\eta_0 - \\
& - \sum_{j=1}^{m} (\eta_r + \eta_j)^2 \log|\eta_r + \eta_j| \sqrt{(1+\eta_j)} \int_0^1 \frac{g_j^{(m)}(\eta_0)}{(\eta_r + \eta_0)^2}\, d\eta_0 \Bigg\} \Bigg] + \\
& + \sum_{i=1}^{n} \sum_{j=1}^{m} \bar{\lambda}_1(\xi_i^{(l)}, \eta_j) I_i^{(n)}(-\eta_r, \eta_j, \xi_k^{(w)}) \int_0^1 \frac{g_j^{(m)}(\eta_0)}{(\eta_r + \eta_0)^2}\, d\eta_0 + \\
& + \sum_{i=1}^{n} \sum_{j=1}^{m} \bar{\lambda}_1(\xi_i^{(l)}, -\eta_j) I_i^{(n)}(-\eta_r, -\eta_j, \xi_k^{(w)}) \int_0^1 \frac{g_j^{(m)}(\eta_0)}{(\eta_r - \eta_0)^2}\, d\eta_0 - \\
& - \sum_{i=1}^{n} \sum_{j=1}^{m} \bar{\lambda}_2(\xi_i^{(l)}, \eta_j) J_i^{(n)}(-\eta_r, \eta_j, \xi_k^{(w)}) \int_0^1 \frac{s_1 s_2 \eta_r \zeta_0}{(s_1^2 \eta^2 + s_2^2 \zeta_0^2)^2}\, g_j^{(m)}(\zeta_0)\, d\zeta_0 .
\end{aligned}$$

$$k = 1, 2, \ldots, n$$

$$r = 1, 2, \ldots, m \qquad (104)$$

$$\bar{\alpha}_2(\xi_k^{(w)}, \eta_r) = \sum_{i=1}^{n}\sum_{j=1}^{m} \bar{\lambda}_1(\xi_i^{(l)}, \eta_j) M_i^{(n)}(\eta_j, \eta_r, \xi_k^{(w)}) \int_0^1 \frac{s_1 s_2 \eta_0 \eta_r}{(s_1^2\eta_0^2 + s_2^2\eta_r^2)^2} g_j^{(m)}(\eta_0) d\eta_0 -$$

$$- \sum_{i=1}^{n}\sum_{j=1}^{m} \bar{\lambda}_1(\xi_i^{(l)}, -\eta_j) M_i^{(n)}(-\eta_j, \eta_r, \xi_k^{(w)}) \int_0^1 \frac{s_1 s_2 \eta_0 \eta_r}{(s_1^2\eta_0^2 + s_2^2\eta_r^2)^2} g_j^{(m)}(\eta_0) d\eta_0 +$$

$$+ \sum_{i=1}^{n} \bar{\lambda}_2(\xi_i^{(l)}, \eta_r) H_{0,i}^{(n)}(\xi_k^{(w)}, \eta_r) \left[\frac{1}{\sqrt{(1-\eta_r)}} \left\{ \int_0^1 \log|\eta_r - \eta_0| \sqrt{(1-\eta_0)} d\eta_0 - \right.\right.$$

$$\left.\left. - \sum_{j=1}^{m} (\eta_r - \eta_j)^2 \log|\eta_r - \eta_j| \sqrt{(1-\eta_j)} \int_0^1 \frac{g_j^{(m)}(\zeta_0)}{(\eta_r - \zeta_0)^2} d\zeta_0 \right\} \right] +$$

$$+ \sum_{i=1}^{n}\sum_{j=1}^{m} \bar{\lambda}_2(\xi_i^{(l)}, \eta_j) N_i^{(n)}(\eta_r, \eta_j, \xi_k^{(w)}) \int_0^1 \frac{g_j^{(m)}(\zeta_0)}{(\eta_r - \zeta_0)^2} d\zeta_0.$$

$$k = 1, 2, \ldots, n$$

$$r = 1, 2, \ldots, m \qquad (105)$$

The simultaneous linear equations (103), (104) and (105) can be put in matrix form and this is done in the next section.

5. *Matrix Formulation of the Equations.*

The set of simultaneous equations (103), (104) and (105) may be written as the matrix equation

$$\begin{bmatrix} \bar{\alpha}_1^+ \\ \bar{\alpha}_2^+ \\ \bar{\alpha}_1^- \end{bmatrix} = \begin{bmatrix} \Lambda_{11}^{++} & \Lambda_{12}^{++} & \Lambda_{11}^{+-} \\ \Lambda_{21}^{++} & \Lambda_{22}^{++} & \Lambda_{21}^{+-} \\ \Lambda_{11}^{-+} & \Lambda_{12}^{-+} & \Lambda_{11}^{--} \end{bmatrix} \begin{bmatrix} \bar{\lambda}_1^+ \\ \bar{\lambda}_2^+ \\ \bar{\lambda}_1^- \end{bmatrix} \qquad (106)$$

where the elements of the submatrices are defined below.

$[\bar{\alpha}_1^+]$ is a column matrix of mn elements with the element

$$\bar{\alpha}_1(\xi_k^{(w)}, \eta_r) \qquad \begin{matrix} k = 1, 2, \ldots, n, \\ r = 1, 2, \ldots, m, \end{matrix} \qquad (107)$$

in the $n(r-1) + k$'th row.

$[\bar{\alpha}_2^+]$ is a column matrix of mn elements with the element

$$\bar{\alpha}_2(\xi_k^{(w)}, \eta_r) \qquad \begin{matrix} k = 1, 2, \ldots, n, \\ r = 1, 2, \ldots, m, \end{matrix} \qquad (108)$$

in the $n(r-1) + k$'th row.

$[\bar{\alpha}_1^-]$ is a column matrix of mn elements with the element

$$\bar{\alpha}_1(\xi_k^{(w)}, -\eta_r) \qquad \begin{matrix} k = 1, 2, \ldots, n, \\ r = 1, 2, \ldots, m, \end{matrix} \qquad (109)$$

in the $n(r-1) + k$'th row.

$[\bar{\lambda}_1^+]$ is a column matrix of mn elements with the element

$$\bar{\lambda}_1(\xi_i^{(l)}, \eta_j) \qquad \begin{matrix} i = 1, 2, \ldots, n, \\ j = 1, 2, \ldots, m, \end{matrix} \qquad (110)$$

in the $n(j-1) + i$'th row.

$[\bar{\lambda}_2^{+}]$ is a column matrix of mn elements with the element

$$\bar{\lambda}_2(\xi_i^{(l)}, \eta_j) \qquad \begin{matrix} i = 1, 2, \ldots, n, \\ j = 1, 2, \ldots, m, \end{matrix} \tag{111}$$

in the $n(j-1)+i$'th row.

$[\bar{\lambda}_1^{-}]$ is a column matrix of mn elements with the element

$$\bar{\lambda}_1(\xi_i^{(l)}, -\eta_j) \qquad \begin{matrix} i = 1, 2, \ldots, n, \\ j = 1, 2, \ldots, m, \end{matrix} \tag{112}$$

in the $n(j-1)+i$'th row.

$[\Lambda_{11}^{++}]$ is a square matrix of order $mn \times mn$ with the element

$$\begin{aligned} & I_i^{(n)}(\eta_r, \eta_j, \xi_k^{(w)}) \int_0^1 \frac{g_j^{(m)}(\eta_0)}{(\eta_r - \eta_0)^2} d\eta_0 + \\ & + \delta_{j,r} F_{0,i}^{(n)}(\xi_k^{(w)}, \eta_r) \left[\frac{1}{\sqrt{(1-\eta_r)}} \left\{ \int_0^1 \log|\eta_r - \eta_0| \sqrt{(1-\eta_0)} d\eta_0 - \right.\right. \\ & \left. - \sum_{j=1}^{m} (\eta_r - \eta_j)^2 \log|\eta_r - \eta_j| \sqrt{(1-\eta_j)} \int_0^1 \frac{g_j^{(m)}(\eta_0)}{(\eta_r - \eta_0)^2} d\eta_0 \right\} + \\ & + \frac{1}{\sqrt{(1-\eta_r)}} \left\{ \int_0^1 \log|\eta_r + \eta_0| \sqrt{(1-\eta_0)} d\eta_0 - \right. \\ & \left.\left. - \sum_{j=1}^{m} (\eta_r + \eta_j)^2 \log|\eta_r + \eta_j| \sqrt{(1+\eta_j)} \int_0^1 \frac{g_j^{(m)}(\eta_0)}{(\eta_r + \eta_0)^2} d\eta_0 \right\} \right] \\ & \qquad i = 1, 2, \ldots, n, \qquad k = 1, 2, \ldots, n, \\ & \qquad j = 1, 2, \ldots, m, \qquad r = 1, 2, \ldots, m, \end{aligned} \tag{113}$$

in the $n(r-1)+k$'th row and $n(j-1)+i$'th column, where $\delta_{j,r}$ is Kronecker's delta.

$[\Lambda_{12}^{++}]$ is a square matrix of order $mn \times mn$ with the element

$$\begin{aligned} & J_i^{(n)}(\eta_r, \eta_j, \xi_k^{(w)}) \int_0^1 \frac{s_1 s_2 \eta_r \zeta_0}{(s_1^2 \eta_r^2 + s_2^2 \zeta_0^2)^2} g_j^{(m)}(\zeta_0) d\zeta_0 \\ & \qquad i = 1, 2, \ldots, n, \qquad k = 1, 2, \ldots, n, \\ & \qquad j = 1, 2, \ldots, m, \qquad r = 1, 2, \ldots, m, \end{aligned} \tag{114}$$

in the $n(r-1)+k$'th row and $n(j-1)+i$'th column.

$[\Lambda_{11}^{+-}]$ is a square matrix of order $mn \times mn$ with the element

$$\begin{aligned} & I_i^{(n)}(\eta_r, -\eta_j, \xi_k^{(w)}) \int_0^1 \frac{g_j^{(m)}(\eta_0)}{(\eta_r + \eta_0)^2} d\eta_0 \\ & \qquad i = 1, 2, \ldots, n, \qquad k = 1, 2, \ldots, n, \\ & \qquad j = 1, 2, \ldots, m, \qquad r = 1, 2, \ldots, m, \end{aligned} \tag{115}$$

in the $n(r-1)+k$'th row and $n(j-1)+i$'th column.

$[\Lambda_{21}{}^{++}]$ is a square matrix of order $mn \times mn$ with the element

$$M_i^{(n)}(\eta_j, \eta_r, \xi_k^{(w)}) \int_0^1 \frac{s_1 s_2 \eta_0 \eta_r}{(s_1^2 \eta_0^2 + s_2^2 \eta_r^2)^2} g_j^{(m)}(\eta_0) d\eta_0$$

$$i = 1, 2, \ldots, n, \qquad k = 1, 2, \ldots, n,$$

$$j = 1, 2, \ldots, m, \qquad m = 1, 2, \ldots, m, \tag{116}$$

in the $n(r-1) + k$'th row and $n(j-1) + i$'th column.

$[\Lambda_{22}{}^{++}]$ is a square matrix of order $mn \times mn$ with the element

$$N_i^{(n)}(\eta_r, \eta_j, \xi_k^{(w)}) \int_0^1 \frac{g_j^{(m)}(\zeta_0)}{(\eta_r - \zeta_0)^2} d\zeta_0 +$$

$$+ \delta_{j,r} H_{0,i}{}^{(n)}(\xi_k^{(w)}, \eta_r) \left[\frac{1}{\sqrt{(1-\eta_r)}} \left\{ \int_0^1 \log |\eta_r - \eta_0| \sqrt{(1-\eta_0)} d\eta_0 - \right.\right.$$

$$\left.\left. - \sum_{j=1}^{m} (\eta_r - \eta_j)^2 \log |\eta_r - \eta_j| \sqrt{(1-\eta_j)} \int_0^1 \frac{g_j^{(m)}(\zeta_0)}{(\eta_r - \zeta_0)^2} d\zeta_0 \right\} \right]$$

$$i = 1, 2, \ldots, n, \qquad k = 1, 2, \ldots, n,$$

$$j = 1, 2, \ldots, m, \qquad m = 1, 2, \ldots, m, \tag{117}$$

in the $n(r-1) + k$'th row and $n(j-1) + i$'th column.

Since the tailplane is symmetric the other submatrices are given by the relations

$$[\Lambda_{21}{}^{+-}] = -[\Lambda_{21}{}^{++}] \tag{118}$$

$$[\Lambda_{11}{}^{-+}] = [\Lambda_{11}{}^{+-}] \tag{119}$$

$$[\Lambda_{12}{}^{-+}] = -[\Lambda_{12}{}^{++}] \tag{120}$$

$$[\Lambda_{11}{}^{--}] = [\Lambda_{11}{}^{++}]. \tag{121}$$

The arrangement of elements in the above matrices corresponds with counting the points along a chord starting with the point nearest the leading edge on the spanwise section nearest the line of junction CD and proceeding outwards towards the tip along each spanwise section in turn for the port half-tailplane, fin, and starboard half-tailplane respectively.

Let the elements in the column matrices in equation (106) be written as the sum of symmetric and antisymmetric components

$$\begin{bmatrix} \bar{\alpha}_1{}^+ \\ \bar{\alpha}_2{}^+ \\ \bar{\alpha}_1{}^- \end{bmatrix} = \begin{bmatrix} \bar{\alpha}_1{}^s \\ 0 \\ \bar{\alpha}_1{}^s \end{bmatrix} + \begin{bmatrix} \bar{\alpha}_1{}^a \\ \bar{\alpha}_2{}^+ \\ -\bar{\alpha}_1{}^a \end{bmatrix}, \quad \begin{bmatrix} \bar{\lambda}_1{}^+ \\ \bar{\lambda}_2{}^+ \\ \bar{\lambda}_1{}^- \end{bmatrix} = \begin{bmatrix} \bar{\lambda}_1{}^s \\ 0 \\ \bar{\lambda}_1{}^s \end{bmatrix} + \begin{bmatrix} \bar{\lambda}_1{}^a \\ \bar{\lambda}_2{}^+ \\ -\bar{\lambda}_1{}^a \end{bmatrix} \tag{122}$$

where

$$[\bar{\alpha}_1{}^s] = \tfrac{1}{2}[\bar{\alpha}_1{}^+ + \bar{\alpha}_1{}^-] \quad [\bar{\lambda}_1{}^s] = \tfrac{1}{2}[\bar{\lambda}_1{}^+ + \bar{\lambda}_1{}^-]$$

$$[\bar{\alpha}_1{}^a] = \tfrac{1}{2}[\bar{\alpha}_1{}^+ - \bar{\alpha}_1{}^-] \quad [\bar{\lambda}_1{}^a] = \tfrac{1}{2}[\bar{\lambda}_1{}^+ - \bar{\lambda}_1{}^-] \tag{123}$$

Then from (106) in virtue of the relations (118) to (121), we have

$$\begin{bmatrix} \bar{\alpha}_1{}^s \\ 0 \\ \bar{\alpha}_1{}^s \end{bmatrix} = \begin{bmatrix} \Lambda_{11}{}^{++} & \Lambda_{12}{}^{++} & \Lambda_{11}{}^{+-} \\ \Lambda_{21}{}^{++} & \Lambda_{22}{}^{++} & \Lambda_{21}{}^{+-} \\ \Lambda_{11}{}^{-+} & \Lambda_{12}{}^{-+} & \Lambda_{11}{}^{--} \end{bmatrix} \begin{bmatrix} \bar{\lambda}_1{}^s \\ 0 \\ \bar{\lambda}_1{}^s \end{bmatrix}, \quad \begin{bmatrix} \bar{\alpha}_1{}^a \\ \bar{\alpha}_2{}^+ \\ -\bar{\alpha}_1{}^a \end{bmatrix} = \begin{bmatrix} \Lambda_{11}{}^{++} & \Lambda_{12}{}^{++} & \Lambda_{11}{}^{+-} \\ \Lambda_{21}{}^{++} & \Lambda_{22}{}^{++} & \Lambda_{21}{}^{+-} \\ \Lambda_{11}{}^{-+} & \Lambda_{12}{}^{-+} & \Lambda_{11}{}^{--} \end{bmatrix} \begin{bmatrix} \bar{\lambda}_1{}^a \\ \bar{\lambda}_2{}^+ \\ \bar{\lambda}_1{}^a \end{bmatrix} \tag{124}$$

which may be replaced by

$$[\bar{\alpha}_1{}^s] = \left[\frac{\Lambda_{11}{}^{++} + \Lambda_{11}{}^{+-}}{2}\right][2\bar{\lambda}_1{}^s], \quad \begin{bmatrix} \bar{\alpha}_1{}^a \\ \bar{\alpha}_2{}^+ \end{bmatrix} = \begin{bmatrix} \dfrac{\Lambda_{11}{}^{++} - \Lambda_{11}{}^{+-}}{2} & \Lambda_{12}{}^{++} \\ \Lambda_{21}{}^{++} & \Lambda_{22}{}^{++} \end{bmatrix} \begin{bmatrix} 2\bar{\lambda}_1{}^a \\ \bar{\lambda}_2{}^+ \end{bmatrix}. \tag{125}$$

6. *Modes of Oscillation and Associated Generalised Forces.*

A number k of independent modes of oscillation of the T-tail will be assumed and these will be numbered from 1 to k. Let

$$Z_p(x, y, t) = lf_1{}^{(p)}(x, y)e^{i\omega t} \tag{126}$$

$$Y_p(x, z, t) = lf_2{}^{(p)}(x, z)e^{i\omega t} \tag{127}$$

be the normal displacements of the tailplane and fin respectively in a harmonic displacement in the p'th mode, $p = 1, 2, \ldots, k$. Let the corresponding loadings be $l_1{}^{(p)}(x, y)e^{i\omega t}$ on the tailplane and $l_2{}^{(p)}(x, z)e^{i\omega t}$ on the fin. Then the generalised airforce $P_{p,q}$ may be defined by

$$P_{p,q} = \iint\limits_{\text{tailplane}} lf_1{}^{(p)}(x_0, y_0)l_1{}^{(q)}(x_0, y_0)dx_0dy_0 + \iint\limits_{\text{fin}} lf_2{}^{(p)}(x_0, z_0)l_2{}^{(q)}(x_0, z_0)dx_0dz_0. \tag{128}$$

If, corresponding to equations (7) and (8) we write

$$l_1{}^{(q)}(x_0, y_0) = \rho_0 V^2\lambda_1{}^{(q)}(x_0, y_0) \tag{129}$$

$$l_2{}^{(q)}(x_0, z_0) = \rho_0 V^2\lambda_2{}^{(q)}(x_0, z_0) \tag{130}$$

then we have

$$P_{p,q} = \rho V^2 l^3 Q_{p,q} \tag{131}$$

where

$$Q_{p,q} = \frac{1}{l^2}\iint\limits_{\text{tailplane}} f_1{}^{(p)}(x_0, y_0)\lambda_1{}^{(q)}(x_0, y_0)dx_0dy_0 + \frac{1}{l^2}\iint\limits_{\text{fin}} f_2{}^{(p)}(x_0, z_0)\lambda_2{}^{(q)}(x_0, z_0)dx_0dz_0. \tag{132}$$

The quantity $Q_{p,q}$ is a generalised aerodynamic force coefficient and is a dimensionless complex number. For similar wings oscillating in similar modes one can see from dimensional considerations that it depends only on the Mach number M of the flow and the frequency parameter ν, where

$$\nu = \frac{\omega l}{V}. \tag{133}$$

By making the transformation of variables from (x_0, y_0) to (ξ_0, η_0) and from (x_0, z_0) to (ϵ_0, ζ_0) as defined in equations (22), (23), (26) and (27) in the integration variables of (132) there results

$$
\begin{aligned}
Q_{p,q} &= \frac{s_1}{l}\int_{-1}^{+1}\frac{c_1(y_0)}{l}\,d\eta_0\int_0^1 f_1^{(p)}(x_0, y_0)\lambda_1^{(q)}(x_0, y_0)d\xi_0 + \\
&+ \frac{s_2}{l}\int_{0}^{+1}\frac{c_2(z_0)}{l}\,d\zeta_0\int_0^1 f_2^{(p)}(x_0, z_0)\lambda_2^{(q)}(x_0, z_0)d\epsilon_0 \\
&= \frac{s_1}{l}\int_{-1}^{+1}\frac{c_1(y_0)}{l}\,d\eta_0\int_0^1 f_1^{(p)}(x_0, y_0)e^{-i\omega x_0/V}\bar{\lambda}_1^{(q)}(\xi_0, \eta_0)d\xi_0 + \\
&+ \frac{s_2}{l}\int_{-1}^{+1}\frac{c_2(z_0)}{l}\,d\zeta_0\int_0^1 f_2^{(p)}(x_0, z_0)e^{-i\omega x_0/V}\bar{\lambda}_2(\epsilon_0, \zeta_0)d\epsilon_0 .
\end{aligned} \tag{134}
$$

Then using the expressions (71) and (72) with the suffix q attached to the λ's this becomes

$$
\begin{aligned}
Q_{p,q} &= \sum_{i=1}^{n}\sum_{j=1}^{m}\frac{s_1}{l}\bar{\lambda}_1^{(q)}(\xi_i^{(l)}, \eta_j)\int_0^1\frac{c_1(y_0)}{l}g_j^{(m)}(\eta_0)d\eta_0 \times \\
&\times \int_0^1 h_i^{(n)}(\xi_0)e^{-i\omega x_0/V}f_1^{(p)}(x_0, y_0)d\xi_0 + \\
&+ \sum_{i=1}^{n}\sum_{j=1}^{m}\frac{s_1}{l}\bar{\lambda}_1^{(q)}(\xi_i^{(l)}, -\eta_j)\int_{-1}^{0}\frac{c_1(y_0)}{l}g_j^{(m)}(-\eta_0)d\eta_0 \times \\
&\times \int_0^1 h_i^{(n)}(\xi_0)e^{-i\omega x_0/V}f_1^{(p)}(x_0, y_0)d\xi_0 + \\
&+ \sum_{i=1}^{n}\sum_{j=1}^{m}\frac{s_2}{l}\bar{\lambda}_2^{(q)}(\xi_i^{(l)}, \eta_j)\int_0^1\frac{c_2(z_0)}{l}g_j^{(m)}(\zeta_0)d\zeta_0 \times \\
&\times \int_0^1 h_i^{(n)}(\epsilon_0)e^{-i\omega x_0/V}f_2^{(p)}(x_0, z_0)d\epsilon_0 .
\end{aligned} \tag{135}
$$

We shall assume that an adequate approximation to

$$c_1(y_0)f_1^{(p)}(x_0, y_0)e^{-i\omega x_0/V} \tag{136}$$

is given by a double polynomial of not greater than the n'th degree in ξ_0 and m'th degree in η_0 over each half of the tailplane, and that an adequate approximation to

$$c_2(z_0)f_2^{(p)}(x_0, z_0)e^{-i\omega x_0/V} \tag{137}$$

is given by a double polynomial of not greater than the n'th degree in ϵ_0 and m'th degree in ζ_0 over the fin.

These approximations may not be so good near points of discontinuity of slope of leading and trailing edges, but this is expected to be only a local effect and is equivalent to modifying the contour of the tailplane and fin surfaces so that there are no such discontinuities. Also the values of $(\omega x_0/V)$ must not be too large anywhere on these surfaces, its greatest permissible magnitude being determined mainly by the number of chordwise points. If large values of $(\omega x_0/V)$ occur then oscillations in the function $e^{-i\omega x_0/V}$ become important and this would need special treatment.

If $a(\xi_0)$ is a polynomial of degree not greater than the n'th in ξ_0 and $b(\eta_0)$ is a polynomial of degree not greater than the m'th in η_0, then by a property of interpolation functions

$$\int_0^1 a(\xi_0)h_i^{(n)}(\xi_0)d\xi_0 = a(\xi_i^{(l)})H_i^{(n)} \tag{138}$$

and

$$\int_0^1 b(\eta_0)g_j^{(m)}(\eta_0)d\eta_0 = b(\eta_j)G_j^{(m)} \tag{139}$$

where

$$H_i^{(n)} = \int_0^1 h_i^{(n)}(\xi_0)d\xi_0 \tag{140}$$

and

$$G_j^{(m)} = \int_0^1 g_j^{(m)}(\eta_0)d\eta_0 . \tag{141}$$

Using this property, we may write instead of (135)

$$\begin{aligned} Q_{p,q} = & \sum_{i=1}^{n}\sum_{j=1}^{m} \frac{s_1}{l} H_i^{(n)}G_j^{(m)} \frac{c_1}{l}(y_{1,j}{}^{+}) \times \\ & \times f_1^{(p)}(x_{1,i,j}{}^{(l)}, y_{1,j}{}^{+}) \exp\left(-\frac{i\omega}{V} x_{1,i,j}\right) \bar{\lambda}_1^{(q)}(\xi_i^{(l)}, \eta_j) + \\ & + \sum_{i=1}^{n}\sum_{j=1}^{m} \frac{s_1}{l} H_i^{(n)}G_j^{(m)} \frac{c_1}{l}(y_{1,j}{}^{-}) \times \\ & \times f_1^{(p)}(x_{1,i,j}{}^{(l)}, y_{1,j}{}^{-}) \exp\left(-\frac{i\omega}{V} x_{1,i,j}\right) \bar{\lambda}_1^{(q)}(\xi_i^{(l)}, -\eta_j) + \\ & + \sum_{i=1}^{n}\sum_{j=1}^{m} \frac{s_2}{l} H_i^{(n)}G_j^{(m)} \frac{c_2}{l}(z_{2,j}) \times \\ & \times f_2^{(p)}(x_{2,i,j}{}^{(l)}, z_{2,j}) \exp\left(-\frac{i\omega}{V} x_{2,i,j}\right) \bar{\lambda}_2^{(q)}(\xi_i^{(l)}, \eta_j) \end{aligned} \tag{142}$$

or, in matrix form

$$Q_{p,q} = [f_{1,p}{}^{+} \quad f_{2,p}{}^{+} \quad f_{1,p}{}^{-}] \begin{bmatrix} B_1{}^{+} & & \\ & B_2{}^{+} & \\ & & B_1{}^{-} \end{bmatrix} \begin{bmatrix} \bar{\lambda}_{1,q}{}^{+} \\ \bar{\lambda}_{2,q}{}^{+} \\ \bar{\lambda}_{1,q}{}^{-} \end{bmatrix} \tag{143}$$

The submatrices appearing as elements in equation (143) are defined below.

$[f_{1,p}{}^{+}]$ is a row matrix of mn elements with the element

$$f_1^{(p)}(x_{1,i,j}{}^{(l)}, y_{1,j}{}^{+}) \qquad \begin{aligned} i &= 1, 2, \ldots, n, \\ j &= 1, 2, \ldots, m, \end{aligned} \tag{144}$$

in the $n(j-1)+i$'th column.

$[f_{2,p}{}^{+}]$ is a row matrix of mn elements with the element

$$f_2^{(p)}(x_{2,i,j}{}^{(l)}, z_{2,j}) \qquad \begin{aligned} i &= 1, 2, \ldots, n, \\ j &= 1, 2, \ldots, m, \end{aligned} \tag{145}$$

in the $n(j-1)+i$'th column.

$[f_{1,p}^-]$ is a row matrix of mn elements with the element

$$f_1^{(p)}(x_{1,i,j}^{(l)}, y_{1,j}^-) \qquad \begin{aligned} i &= 1, 2, \ldots, n, \\ j &= 1, 2, \ldots, m, \end{aligned} \tag{146}$$

in the $n(j-1)+i$'th column.

$[B_1^+]$ is an $mn \times mn$ diagonal matrix with element

$$\frac{s_1}{l^2} c_1(y_{1,j}^+) H_i^{(n)} G_j^{(m)} \exp\left(-\frac{i\omega}{V} x_{1,i,j}^{(l)}\right) \qquad \begin{aligned} i &= 1, 2, \ldots, n, \\ j &= 1, 2, \ldots, m, \end{aligned} \tag{147}$$

in the $n(j-1)+i$'th row and column.

$[B_2^+]$ is an $mn \times mn$ diagonal matrix with element

$$\frac{s_2}{l^2} c_2(z_{2,j}) H_i^{(n)} G_j^{(m)} \exp\left(-\frac{i\omega}{V} x_{2,i,j}^{(l)}\right) \qquad \begin{aligned} i &= 1, 2, \ldots, n, \\ j &= 1, 2, \ldots, m, \end{aligned} \tag{148}$$

in the $n(j-1)+i$'th row and column.

Since the tailplane is symmetric we can write

$$[B_1^-] = [B_1^+]. \tag{149}$$

The column matrices $[\bar{\lambda}_{1,q}^+]$, $[\bar{\lambda}_{2,q}^+]$ and $[\bar{\lambda}_{1,q}^-]$ are defined by (110), (111) and (112) only now with the addition of a suffix q.

The equation (143) may be replaced by

$$Q_{p,q} = [f_{1,p}^s]\,[B_1^+]\,[2\bar{\lambda}_{1,q}^s] + [f_{1,p}^a, f_{2,p}^+] \begin{bmatrix} B_1^+ & \\ & B_2^+ \end{bmatrix} \begin{bmatrix} 2\bar{\lambda}_{1,q}^a \\ \bar{\lambda}_{2,q}^+ \end{bmatrix} \tag{150}$$

where

$$[f_{1,p}^s] = \tfrac{1}{2}[f_{1,p}^+ + f_{1,p}^-], \quad [f_{1,p}^a] = \tfrac{1}{2}[f_{1,p}^+ - f_{1,p}^-] \tag{151}$$

$$[\lambda_{1,q}^s] = \tfrac{1}{2}[\lambda_{1,q}^+ + \lambda_{1,q}^-], \quad [\lambda_{1,q}^a] = \tfrac{1}{2}[\lambda_{1,q}^+ - \lambda_{1,q}^-]. \tag{152}$$

The matrices $[2\bar{\lambda}_{1,q}^s]$, $[2\lambda_{1,q}^a]$ and $(\lambda_{2,q}^+]$ in equation (150) are obtained by solving equations (125) with suffices q added to the $\bar{\lambda}$'s and $\bar{\alpha}$'s, and on using these solutions in (150) there results

$$\begin{aligned} Q_{p,q} = {} & [f_{1,p}^s]\,[B_1^+]\left[\frac{\Lambda_{11}^{++} + \Lambda_{11}^{+-}}{2}\right]^{-1} [\bar{\alpha}_{1,q}^s] + \\ & + [f_{1,p}^a, f_{2,p}^+] \begin{bmatrix} B_1^+ & \\ & B_2^+ \end{bmatrix} \begin{bmatrix} \dfrac{\Lambda_{11}^{++} - \Lambda_{11}^{+-}}{2} & \Lambda_{12}^{++} \\ \Lambda_{21}^{++} & \Lambda_{22}^{++} \end{bmatrix}^{-1} \begin{bmatrix} \bar{\alpha}_{1,q}^a \\ \bar{\alpha}_{2,q}^+ \end{bmatrix} \end{aligned} \tag{153}$$

where $\bar{\alpha}_{1,q}$ and $\bar{\alpha}_{2,q}$ are obtained from equations (3), (4), (5), (6), (15) and (30) on adding suffices q. So

$$\alpha_1^{(q)}(x, y) = l\frac{\partial}{\partial x} f_1^{(q)}(x, y) + \frac{i\omega l}{V} f_1^{(q)}(x, y) \tag{154}$$

$$\alpha_2^{(q)}(x, y) = l\frac{\partial}{\partial x} f_2^{(q)}(x, y) + \frac{i\omega l}{V} f_2^{(q)}(x, y). \tag{155}$$

Now define column matrices $[\alpha_{1,q}^{+}]$, $[\alpha_{1,q}^{-}]$ and $[\alpha_{2,q}^{+}]$ as follows:

$[\alpha_{1,q}^{+}]$ is a column matrix of mn elements with the element

$$\alpha_{1,q}(x_{1,k,r}^{(w)}, y_{1,r}^{+}) \qquad \begin{matrix} k = 1, 2, \ldots, n, \\ r = 1, 2, \ldots, m, \end{matrix} \tag{156}$$

in the $n(r-1)+k$'th row.

$[\alpha_{1,q}^{-}]$ is a column matrix of mn elements with the element

$$\alpha_{1,q}(x_{1,k,r}^{(w)}, y_{1,r}^{-}) \qquad \begin{matrix} k = 1, 2, \ldots, n, \\ r = 1, 2, \ldots, m, \end{matrix} \tag{157}$$

in the $n(r-1)+k$'th row.

$[\alpha_{2,q}^{+}]$ is a column matrix of mn elements with the element

$$\alpha_{2,q}(x_{2,k,r}^{(w)}, z_{2,r}) \qquad \begin{matrix} k = 1, 2, \ldots, n, \\ r = 1, 2, \ldots, m, \end{matrix} \tag{158}$$

in the $n(r-1)+k$'th row.

As before, define

$$[\alpha_{1,q}^{s}] = \tfrac{1}{2}[\alpha_{1,q}^{+} + \alpha_{1,q}^{-}], \quad [\alpha_{1,q}^{a}] = \tfrac{1}{2}[\alpha_{1,q}^{+} - \alpha_{1,q}^{-}]. \tag{159}$$

Then, with a symmetric tailplane,

$$[\bar{\alpha}_{1,q}^{s}] = [D_1^{+}]\,[\alpha_{1,q}^{s}] \tag{160}$$

$$\begin{bmatrix} \bar{\alpha}_{1,q}^{a} \\ \bar{\alpha}_{2,q}^{+} \end{bmatrix} = \begin{bmatrix} D_1^{+} & \\ & D_2^{+} \end{bmatrix} \begin{bmatrix} \alpha_{1,q}^{a} \\ \alpha_{2,q}^{+} \end{bmatrix} \tag{161}$$

where

$[D_1^{+}]$ is a diagonal matrix of order $mn \times mn$ with the element

$$\exp\left(\frac{i\omega}{V} x_{1,k,r}^{(w)}\right) \qquad \begin{matrix} k = 1, 2, \ldots, n, \\ r = 1, 2, \ldots, m, \end{matrix} \tag{162}$$

in the $n(r-1)+k$'th row and column.

$[D_2^{+}]$ is a diagonal matrix of order $mn \times mn$ with the element

$$\exp\left(\frac{i\omega}{V} x_{2,k,r}^{(w)}\right) \qquad \begin{matrix} k = 1, 2, \ldots, n, \\ r = 1, 2, \ldots, m, \end{matrix} \tag{163}$$

in the $n(r-1)+k$'th row and column.

The expression for $Q_{p,q}$ may now be written

$$Q_{p,q} = [f_{1,p}^{s}]\,[B_1^{+}]\left[\frac{\Lambda_{11}^{++} + \Lambda_{11}^{+-}}{2}\right]^{-1}[D_1^{+}]\,[\alpha_{1,q}^{s}] +$$

$$+ [f_{1,p}^{a}, f_{2,p}^{+}]\begin{bmatrix} B_1^{+} & \\ & B_2^{+} \end{bmatrix}\begin{bmatrix} \dfrac{\Lambda_{11}^{++} - \Lambda_{11}^{+-}}{2} & \Lambda_{12}^{++} \\ \Lambda_{21}^{++} & \Lambda_{22}^{++} \end{bmatrix}^{-1}\begin{bmatrix} D_1^{+} & \\ & D_2^{+} \end{bmatrix}\begin{bmatrix} \alpha_{1,q}^{a} \\ \alpha_{2,q}^{+} \end{bmatrix}. \tag{164}$$

If $f_1^{(p)}(x, y) = f_1^{(p)}(x, -y)$ and $f_2^{(p)}(x, y) = 0$ then the displacements of the T-tail surfaces are symmetric about the mean plane of the fin, whereas if $f_1^{(p)}(x, y) = -f_1^{(p)}(x, y)$ the displacements are antisymmetric. In flutter theory the modes of oscillation usually considered are either symmetric or antisymmetric. If p and q refer to modes which are not both symmetric or not both antisymmetric then

$$Q_{p,q} = 0. \tag{165}$$

If p and q both refer to symmetric modes then

$$Q_{p,q} = [f_{1,p}^{+}]\,[B_1^{+}]\left[\frac{\Lambda_{11}^{++} + \Lambda_{11}^{+-}}{2}\right]^{-1}[D_1^{+}]\,[\alpha_{1,q}^{+}] \tag{166}$$

while if p and q both refer to antisymmetric modes then

$$Q_{p,q} = [f_{1,p}^{+}, f_{2,p}^{+}]\begin{bmatrix} B_1^{+} & \\ & B_2^{+}\end{bmatrix}\begin{bmatrix}\dfrac{\Lambda_{11}^{++} - \Lambda_{11}^{+-}}{2} & \Lambda_{12}^{++} \\ \Lambda_{21}^{++} & \Lambda_{22}^{++}\end{bmatrix}^{-1}\begin{bmatrix} D_1^{+} & \\ & D_2^{+}\end{bmatrix}\begin{bmatrix}\alpha_{1,q}^{+} \\ \alpha_{2,q}^{+}\end{bmatrix} \tag{167}$$

The generalised force coefficient corresponding to T-tail symmetric modes is independent of the presence of the fin, as one would expect. This case can be dealt with by applying plane-wing theory (*see* e.g. Ref. 6) to the tailplane only and so it will not be considered further in this paper.

Equation (166) as it stands determines just one of the possible k^2 generalised airforce coefficients $Q_{p,q}$ if there are k modes of oscillation.

If the rows $[f_{1,p}^{+}, f_{2,p}^{+}]$, $p = 1, 2, \ldots, k$ are arranged consecutively beneath each other to form a rectangular matrix $[f]$ of order $k \times 2mn$ and if the columns

$$\begin{bmatrix}\alpha_{1,q}^{+} \\ \alpha_{2,q}^{+}\end{bmatrix}$$

are arranged consecutively alongside each other to form a rectangular matrix $[\alpha]$ of order $2mn \times k$ then for antisymmetric tailplane modes

$$[Q] = [f]\begin{bmatrix} B_1^{+} & \\ & B_2^{+}\end{bmatrix}\begin{bmatrix}\dfrac{\Lambda_{11}^{++} - \Lambda_{11}^{+-}}{2} & \Lambda_{12}^{++} \\ \Lambda_{21}^{++} & \Lambda_{22}^{++}\end{bmatrix}^{-1}\begin{bmatrix} D_1^{+} & \\ & D_2^{+}\end{bmatrix}[\alpha] \tag{168}$$

where $[Q]$ is a square matrix of order $k \times k$ with the element $Q_{p,q}$ in the p'th row and q'th column.

The matrices $[f]$ and $[\alpha]$ are made up of numbers associated with the displacement and upwash points on the port half-tailplane and on the fin. The matrix obtained from the product

$$\begin{bmatrix} B_1^{+} & \\ & B_2^{+}\end{bmatrix}\begin{bmatrix}\dfrac{\Lambda_{11}^{++} - \Lambda_{11}^{+-}}{2} & \Lambda_{12}^{++} \\ \Lambda_{21}^{++} & \Lambda_{22}^{++}\end{bmatrix}^{-1}\begin{bmatrix} D_1^{+} & \\ & D_2^{+}\end{bmatrix} \tag{169}$$

is called the influence matrix. It depends on the Mach number of the mainstream flow, the frequency parameter of the oscillations and the wing geometry, but it does not depend on the shape of the modes of oscillation.

7. *The T-Tail with Reflector Plate at the Base of the Fin.*

If the T-tail is attached to an infinite wall at the base AB of the fin with the tailplane parallel to the wall, as shown in Fig. 2, then it is possible to obtain the generalised airforces by using the method of images. The generalised airforces on a T-tail in a wind tunnel approach these values if one of the wind-tunnel walls acts as a reflector plate at the base of the fin.

The pair of integral equations corresponding to equations (9) and (10) and appropriate to this case is found from the method of images to be

$$\begin{aligned}\alpha_1(x, y) = & \frac{1}{4\pi}\iint\limits_{\text{tailplane}} \lambda_1(x_0, y_0)K_1(x-x_0, y-y_0)dx_0dy_0 - \\ & -\frac{1}{4\pi}\iint\limits_{\text{tailplane}} \lambda_1(x_0, y_0)K_3(x-x_0, y-y_0, 2s_2)dx_0dy_0 + \\ & +\frac{1}{4\pi}\iint\limits_{\text{fin}} \lambda_2(x_0, z_0)K_2(x-x_0, y, z_0)dx_0dz_0 + \\ & +\frac{1}{4\pi}\iint\limits_{\text{fin}} \lambda_2(x_0, z_0)K_2(x-x_0, y, 2s_2-z_0)dx_0dz_0\end{aligned} \tag{170}$$

$$\begin{aligned}\alpha_2(x, z) = & \frac{1}{4\pi}\iint\limits_{\text{tailplane}} \lambda_1(x_0, y_0)K_2(x-x_0, z, y_0)dx_0dy_0 - \\ & -\frac{1}{4\pi}\iint\limits_{\text{tailplane}} \lambda_1(x_0, y_0)K_2(x-x_0, 2s_2-z, y_0)dx_0dy_0 + \\ & +\frac{1}{4\pi}\iint\limits_{\text{fin}} \lambda_2(x_0, z_0)K_1(x-x_0, z-z_0)dx_0dz_0 + \\ & +\frac{1}{4\pi}\iint\limits_{\text{fin}} \lambda_2(x_0, z_0)K_1(x-x_0, z+z_0-2s_2)dx_0dz_0 .\end{aligned} \tag{171}$$

The kernel functions K_1 and K_2 are given by (11) and (12) as before. The kernel function $K_3(x, y, 2s_2)$ is given by (*see* Appendix)

$$\begin{aligned}K_3(x, y, 2s_2) = & e^{-i\omega x/V}\Bigg[\int_{(-x+MR_3)/(1-M^2)}^{\infty} e^{-i\omega u/V}\frac{u^2+y^2-8s_2^2}{(u^2+y^2+4s_2^2)^{5/2}}du + \\ & + \exp\left\{-\frac{i\omega}{V}\left(\frac{-x+MR_3}{1-M^2}\right)\right\}\left\{\frac{M(Mx+R_3)}{R_3(x^2+y^2+4s_2^2)} - \frac{4s_2^2M(Mx+R_3)^3}{R_3(x^2+y^2+4s_2^2)^3} - \right. \\ & \left. - \frac{4s_2^2M^2(1-M^2)x}{R_3^3(x^2+y^2+4s_2^2)^3} - \frac{8s_2^2M(Mx+R_3)}{R_3(x^2+y^2+4s_2^2)^2} - \frac{i\omega}{V}\frac{4s_2^2M^2(Mx+R_3)}{R_3^2(x^2+y^2+4s_2^2)}\right\}\Bigg]\end{aligned} \tag{172}$$

where

$$R_3 = \sqrt{\{x^2+(1-M^2)(y^2+4s_2^2)\}} . \tag{173}$$

The pair of integral equations (170) and (171) may be reduced following the procedure of Section 2 to

$$
\begin{aligned}
\bar{\alpha}_1(\xi,\eta) = {} & \frac{s_1}{4\pi}\int_{-1}^{+1} c_1(y_0)d\eta_0 \int_0^1 \{\bar{\lambda}_1(\xi_0,\eta_0)\hat{K}_1^{(2)}(x-x_0, y-y_0) + \\
& + c_1(y_0)\bar{\lambda}_1^{(1)}(\xi_0,\eta_0)\hat{K}_1^{(3)}(x-x_0, y-y_0)\}\, d\xi_0 + \\
& + \frac{s_1}{4\pi}\int_{-1}^{+1} c_1(y_0)\bar{\lambda}_1^{(1)}(1,\eta_0)\hat{K}_1^{(1)}(x-x_T^{(1)}(y_0), y-y_0)\, d\eta_0 - \\
& - \frac{s_1}{4\pi}\int_{-1}^{+1} c_1(y_0)d\eta_0 \int_0^1 \{\bar{\lambda}_1(\xi_0,\eta_0)\hat{K}_3^{(2)}(x-x_0, y-y_0, 2s_2) + \\
& + c_1(y_0)\bar{\lambda}_1^{(1)}(\xi_0,\eta_0)\hat{K}_3^{(3)}(x-x_0, y-y_0, 2s_2)\} d\xi_0 - \\
& - \frac{s_1}{4\pi}\int_{-1}^{+1} c_1(y_0)\bar{\lambda}_1^{(1)}(1,\eta_0)\hat{K}_3^{(1)}(x-x_T^{(1)}(y_0), y-y_0, 2s_2)\, d\eta_0 + \\
& + \frac{s_2}{4\pi}\int_0^1 c_2(z_0)d\zeta_0 \int_0^1 \{\bar{\lambda}_2(\epsilon_0,\zeta_0)\hat{K}_2^{(2)}(x-x_0, y, z_0) + \\
& + c_2(z_0)\bar{\lambda}_2^{(1)}(\epsilon_0,\zeta_0)\hat{K}_2^{(3)}(x-x_0, y, z_0)\}\, d\epsilon_0 + \\
& + \frac{s_2}{4\pi}\int_0^1 c_2(z_0)\bar{\lambda}_2^{(1)}(1,\zeta_0)\hat{K}_2^{(1)}(x-x_T^{(2)}(z_0), y, z_0)\, d\zeta_0 + \\
& + \frac{s_2}{4\pi}\int_0^1 c_2(z_0)d\zeta_0 \int_0^1 \{\bar{\lambda}_2(\epsilon_0,\zeta_0)\hat{K}_2^{(2)}(x-x_0, y, 2s_2-z_0) + \\
& + c_2(z_0)\bar{\lambda}_2^{(1)}(\epsilon_0,\zeta_0)\hat{K}_2^{(3)}(x-x_0, y, 2s_2-z_0)d\epsilon_0 + \\
& + \frac{s_2}{4\pi}\int_0^1 c_2(z_0)\bar{\lambda}_2^{(1)}(1,\zeta_0)\hat{K}_2^{(1)}(x-x_T^{(2)}(z_0), y, 2s_2-z_0)d\zeta_0
\end{aligned}
\tag{174}
$$

$$
\begin{aligned}
\bar{\alpha}_2(\epsilon,\zeta) = {} & \frac{s_1}{4\pi}\int_{-1}^{+1} c_1(y_0)d\eta_0 \int_0^1 \{\bar{\lambda}_1(\xi_0,\eta_0)\hat{K}_2^{(2)}(x-x_0, z, y_0) + \\
& + c_1(y_0)\bar{\lambda}_1^{(1)}(\xi_0,\eta_0)\hat{K}_2^{(3)}(x-x_0, z, y_0)\}\, d\xi_0 + \\
& + \frac{s_1}{4\pi}\int_{-1}^{+1} c_1(y_0)\bar{\lambda}_1^{(1)}(1,\eta_0)\hat{K}_2^{(1)}(x-x_T^{(1)}(y_0), z, y_0)d\eta_0 - \\
& - \frac{s_1}{4\pi}\int_{-1}^{+1} c_1(y_0)d\eta_0 \int_0^1 \{\bar{\lambda}_1(\xi_0,\eta_0)\hat{K}_2^{(2)}(x-x_0, 2s_2-z, y_0) + \\
& + c_1(y_0)\bar{\lambda}_1^{(1)}(\xi_0,\eta_0)\hat{K}_2^{(3)}(x-x_0, 2s_2-z, y_0)\}\, d\xi_0 - \\
& - \frac{s_1}{4\pi}\int_{-1}^{+1} c_1(y_0)\bar{\lambda}_1^{(1)}(\xi_0,\eta_0)\hat{K}_2^{(1)}(x-x_0, 2s_2-z, y_0)d\eta_0 + \\
& + \frac{s_2}{4\pi}\int_0^1 c_2(z_0)d\zeta_0 \int_0^1 \{\bar{\lambda}_2(\epsilon_0,\zeta_0)\hat{K}_1^{(2)}(x-x_0, z-z_0) + \\
& + c_2(z_0)\bar{\lambda}_2^{(1)}(\epsilon_0,\zeta_0)\hat{K}_1^{(3)}(x-x_0, z-z_0)\}\, d\epsilon_0 + \\
& + \frac{s_2}{4\pi}\int_0^1 c_2(z_0)\bar{\lambda}_2^{(1)}(1,\zeta_0)\hat{K}_1^{(1)}(x-x_T^{(2)}(z_0), z-z_0)d\zeta_0 + \\
& + \frac{s_2}{4\pi}\int_0^1 c_2(z_0)d\zeta_0 \int_0^1 \{\bar{\lambda}_2(\epsilon_0,\zeta_0)\hat{K}_1^{(2)}(x-x_0, z+z_0-2s_2) + \\
& + c_2(z_0)\bar{\lambda}_2^{(1)}(\epsilon_0,\zeta_0)\hat{K}_1^{(3)}(x-x_0, z+z_0-2s_2)\}\, d\epsilon_0 + \\
& + \frac{s_2}{4\pi}\int_0^1 c_2(z_0)\bar{\lambda}_2^{(1)}(1,\zeta_0)\hat{K}_1^{(1)}(x-x_T^{(2)}(z_0), z+z_0-2s_2)d\zeta_0
\end{aligned}
\tag{175}
$$

where

$$\hat{K}_3^{(1)}(x, y, 2s_2) = \int_{(-x+MR_3)/(1-M^2)}^{\infty} e^{-i\omega u/V} \frac{u^2 + y^2 - 8s_2^2}{(u^2+y^2+4s_2^2)^{5/2}} du$$

$$= -\frac{\pi}{(y^2+4s_2^2)} \left\{\frac{i\omega}{2V} \sqrt{(y^2+4s_2^2)}\right\} \left[\mathbf{H}_{-1}\left\{\frac{i\omega}{V} \sqrt{(y^2+4s_2^2)}\right\} + \right.$$

$$\left. + \frac{2i}{\pi} K_1 \left\{\frac{\omega}{V} \sqrt{(y^2+4s_2^2)}\right\} - I_1 \left\{\frac{\omega}{V} \sqrt{(y^2+4s_2^2)}\right\}\right] -$$

$$- \frac{8\pi s_2^2}{(y^2+4s_2^2)^2} \left\{\frac{i\omega}{2V} \sqrt{(y^2+4s_2^2)}\right\}^2 \left[\mathbf{H}_{-2}\left\{\frac{i\omega}{V} \sqrt{(y^2+4s_2^2)}\right\} - \right.$$

$$\left. - \frac{2}{\pi} K_2 \left\{\frac{\omega}{V} \sqrt{(y^2+4s_2^2)}\right\} + iI_2 \left\{\frac{\omega}{V} \sqrt{(y^2+4s_2^2)}\right\}\right] +$$

$$+ \int_{(-x+MR_3)/(1-M^2)}^{0} e^{-i\omega u/V} \frac{u^2 + y^2 - 8s_2^2}{(u^2+y^2+4s_2^2)^{5/2}} du \, . \tag{176}$$

$$\hat{K}_3^{(2)}(x, y, 2s_2) = \exp\left\{-\frac{i\omega}{V}\left(\frac{-x+MR_3}{1-M^2}\right)\right\} \left\{\frac{M(Mx+R_3)}{R_3(x^2+y^2+4s_2^2)} - \frac{4s_2^2M(Mx+R_3)^3}{R_3(x^2+y^2+4s_2^2)^3} - \right.$$

$$\left. - \frac{4s_2^2M^2(1-M^2)x}{R_3^3(x^2+y^2+4s_2^2)^3} - \frac{8s_2^2M(Mx+R_3)}{R_3(x^2+y^2+4s_2^2)^2} - \frac{i\omega}{V} \frac{4s_2^2M^2(Mx+R_3)}{R_3^2(x^2+y^2+4s_2^2)}\right\} . \tag{177}$$

$$\hat{K}_3^{(3)}(x, y, 2s_2) = \exp\left\{-\frac{i\omega}{V}\left(\frac{-x+MR_3}{1-M^2}\right)\right\} \frac{1}{R_3} \left\{\frac{(Mx+R_3)^2}{(x^2+y^2+4s_2^2)^2} - \frac{12s_2^2(Mx+R_3)^4}{(x^2+y^2+4s_2^2)^4}\right\} . \tag{178}$$

The procedure for solving the pair of integral equations (174) and (175) is similar to the procedure for solving the pair of integral equations (38) and (39). The main difference is that $\bar{\lambda}_2(\epsilon_0, \zeta_0)$ does not tend to zero as $\zeta_0 \to 1$ but remains finite. A suitable approximation to the fin spanwise function $\bar{\lambda}_2(\xi_i^{(l)}, \zeta_0)$ is then given by a polynomial of degree $(m-1)$ in ζ_0. We define a set of polynomials $\bar{\mu}_r(\zeta_0)$ of degree r which are orthogonal over (0, 1) i.e.

$$\int_0^1 \bar{\mu}_r(\zeta_0)\bar{\mu}_s(\zeta_0) d\zeta_0 = \delta_{r,s} \, . \tag{179}$$

These polynomials are related to the Legendre polynomials by the formula

$$\bar{\mu}_r(\zeta_0) = \sqrt{\left(\frac{2r+1}{2}\right)} P_r(\zeta_0) \tag{180}$$

where $P_r(\zeta_0)$ is the Legendre polynomial of order r in the usual notation.

Let

$$\zeta_j \qquad j = 1, 2, \ldots, m, \tag{181}$$

be the m roots of

$$\bar{\mu}_m(\zeta_0) = 0 \, . \tag{182}$$

Then again it is very convenient from the point of view of mathematical formulation if the spanwise loading points on the fin are taken to be the $\zeta_j, j = 1, 2, \ldots, m$.

To avoid complications of numerical evaluation the velocity points on the fin are taken to be the points

$$\zeta_r \qquad r = 1, 2, \ldots, m, \tag{183}$$

instead of the more logical values which are the roots of

$$\overline{w}_m(\zeta) = 0 \tag{184}$$

where

$$\overline{w}_m(\zeta) = \int_0^1 \frac{\bar{\mu}_m(\zeta_0)}{(\zeta-\zeta_0)^2}\, d\zeta_0 . \tag{185}$$

The positions of the loading points and velocity points on the fin are then obtained from equations (74) and (77) by replacing η_j and η_r by ζ_j and ζ_r respectively.

Corresponding to each point ζ_j an interpolation function $\bar{g}_j^{(m)}(\zeta_0)$ is formed which is unity at the point ζ_j and zero at the other $(m-1)$ spanwise points on the fin, and which is a polynomial of degree $(m-1)$ in ζ_0:

$$\bar{g}_j^{(m)}(\zeta_0) = \frac{\bar{\mu}_m(\zeta_0)}{(\zeta_0-\zeta_j)\left[\dfrac{d}{d\zeta_0}\bar{\mu}_m(\zeta_0)\right]_{\zeta_0=\zeta_j}} . \tag{186}$$

The function $g_j^{(m)}(\zeta_0)$ in equation (71) is then to be replaced by $\bar{g}_j^{(m)}(\zeta_0)$ in order to get the approximation to $\bar{\lambda}_2(\epsilon_0, \zeta_0)$ in terms of interpolation functions. The numbers $\bar{G}_j^{(m)}$ are obtained from formula (141) by replacing $g_j^m(\eta_0)d\eta_0$ by $\bar{g}_j^{(m)}(\zeta_0)d\zeta_0$.

By substituting the approximations (60) and (61) for $\bar{\lambda}_2(\epsilon_0, \zeta_0)$ and $\bar{\lambda}_1(\xi_0, \eta_0)$ into the forms (174) and (175) of the integral equations we obtain

$$\begin{aligned}\bar{\alpha}_1(\xi, \eta) = {} & \sum_{i=1}^{n} \int_{-1}^{+1} \frac{\bar{\lambda}_1(\xi_i^{(l)}, \eta_0)}{(\eta-\eta_0)^2} I_i^{(n)}(\eta, \eta_0, \xi) d\eta_0 - \sum_{i=1}^{n} \int_{-1}^{+1} \bar{\lambda}_1(\xi_i^{(l)}, \eta_0) P_i^{(n)}(\eta, \eta_0, \xi) d\eta_0 + \\ & + \sum_{i=1}^{n} \int_0^1 \frac{s_1 s_2 \eta \zeta_0}{(s_1^2\eta^2 + s_2^2\zeta_0^2)^2} \bar{\lambda}_2(\xi_i^{(l)}, \zeta_0) J_i^{(n)}(\eta, \zeta_0, \xi) d\zeta_0 + \\ & + \sum_{i=1}^{n} \int_0^1 \frac{s_1 s_2 \eta (2-\zeta_0)}{[s_1^2\eta^2 + s_2^2(2-\zeta_0)^2]^2} \bar{\lambda}_2(\xi_i^{(l)}, \zeta_0) J_i^{(n)}(\eta, 2-\zeta_0, \xi) d\zeta_0\end{aligned} \tag{187}$$

$$\begin{aligned}\bar{\alpha}_2(\epsilon, \zeta) = {} & \sum_{i=1}^{n} \int_{-1}^{+1} \frac{s_1 s_2 \eta_0 \zeta}{(s_1^2\eta_0^2 + s_2^2\zeta^2)^2} \bar{\lambda}_1(\xi_i^{(l)}, \eta_0) M_i^{(n)}(\eta_0, \zeta, \epsilon) d\eta_0 - \\ & - \sum_{i=1}^{n} \int_{-1}^{+1} \frac{s_1 s_2 \eta_0 (2-\zeta)}{[s_1^2\eta_0^2 + s_2^2(2-\zeta)^2]^2} \bar{\lambda}_1(\xi_i^{(l)}, \eta_0) M_i^{(n)}(\eta_0, 2-\zeta, \epsilon) d\eta_0 + \\ & + \sum_{i=1}^{n} \int_0^1 \frac{\bar{\lambda}_2(\xi_i^{(l)}, \zeta_0)}{(\zeta-\zeta_0)^2} N_i^{(n)}(\zeta, \zeta_0, \epsilon) d\zeta_0 + \\ & + \sum_{i=1}^{n} \int_0^1 \frac{\bar{\lambda}_2(\xi_i^{(l)}, \zeta_0)}{(\zeta+\zeta_0-2)^2} N_i^{(n)}(\zeta, 2-\zeta_0, \epsilon) d\zeta_0\end{aligned} \tag{188}$$

where

$$\begin{aligned}P_i^{(n)}(\eta, \eta_0, \xi) = {} & \frac{s_1}{4\pi} c_1(y_0) \Bigg[\int_0^1 \{h_i^{(n)}(\xi_0)\hat{K}_3^{(2)}(x-x_0, y-y_0, 2s_2) + \\ & + c_1(y_0) h_i^{(1,n)}(\xi_0) \hat{K}_3^{(3)}(x-x_0, y-y_0, 2s_2)\} d\xi_0 \\ & + h_i^{(1,n)}(1) \hat{K}_3^{(1)}(x - x_T^{(1)}(y_0), y-y_0)\Bigg]\end{aligned} \tag{189}$$

and we define

$$c_2(2s_2 - z) = c_2(z) \tag{190}$$

in order to get $J_i^{(n)}(\eta, 2-\zeta_0, \xi)$, $M_i^{(n)}(\eta_0, 2-\zeta, \epsilon)$, $N_i^{(n)}(\zeta, 2-\zeta_0, \epsilon)$ from the definitions (83), (84) and (85).

The evaluation of the principal-value integrals in equation (187) is exactly parallel to the evaluation of the principal-value integrals in equation (80).

For the evaluation of the principal-value integrals in equation (188) write the identities

$$\bar{\lambda}_2(\xi_i^{(l)}, \zeta_0)N_i^{(n)}(\zeta, \zeta_0, \epsilon) = \bar{\lambda}_2(\xi_i^{(l)}, \zeta)H_{0,i}^{(n)}(\epsilon, \zeta)\,(\zeta-\zeta_0)^2 \log|\zeta-\zeta_0| + \\ + [\bar{\lambda}_2(\xi_i^{(l)}, \zeta_0)N_i^{(n)}(\zeta, \zeta_0, \epsilon) - \bar{\lambda}_2(\xi_i^{(l)}, \zeta)H_{0,i}^{(n)}(\epsilon, \zeta)\,(\zeta-\zeta_0)^2 \log|\zeta-\zeta_0|] \tag{191}$$

and

$$\bar{\lambda}_2(\xi_i^{(l)}, \zeta_0)N_i^{(n)}(\zeta, 2-\zeta_0, \epsilon) = \bar{\lambda}_2(\xi_i^{(l)}, \zeta)H_{0,i}^{(n)}(\epsilon, \zeta)\,(\zeta+\zeta_0-2)^2 \log|\zeta+\zeta_0-2| + \\ + [\bar{\lambda}_2(\xi_i^{(l)}, \zeta_0)N_i^{(n)}(\zeta, 2-\zeta_0, \epsilon) - \bar{\lambda}_2(\xi_i^{(l)}, \zeta)H_{0,i}^{(n)}(\epsilon, \zeta)\,(\zeta+\zeta_0-2)^2 \log|\zeta+\zeta_0-2|]. \tag{192}$$

In virtue of the expansion (88), the lowest-order logarithmic singularity is missing in the expressions in square brackets in identities (191) and (192). In identity (192) the definition (190) has to be used. We therefore expand these expressions in terms of interpolation functions $\bar{g}_j^{(m)}(\zeta_0)$. On doing this and integrating, we get

$$\int_0^1 \frac{\bar{\lambda}_2(\xi_i^{(l)}, \zeta_0)}{(\zeta-\zeta_0)^2} N_i^{(n)}(\zeta, \zeta_0, \epsilon)d\zeta_0 \\ = \bar{\lambda}_2(\xi_i^{(l)}, \zeta)H_{0,i}^{(n)}(\epsilon, \zeta)\left[\int_0^1 \log|\zeta-\zeta_0|d\zeta_0 - \sum_{j=1}^{m} (\zeta-\zeta_j)^2 \log|\zeta-\zeta_j| \int_0^1 \frac{\bar{g}_j^{(m)}(\zeta_0)}{(\zeta-\zeta_0)^2} d\zeta_0\right] + \\ + \sum_{j=1}^{m} \bar{\lambda}_2(\xi_i^{(l)}, \zeta_j)N_i^{(n)}(\zeta, \zeta_j, \epsilon) \int_0^1 \frac{\bar{g}_j^{(m)}(\zeta_0)}{(\zeta-\zeta_0)^2} d\zeta_0 \tag{193}$$

and

$$\int_0^1 \frac{\bar{\lambda}_2(\xi_i^{(l)}, \zeta_0)}{(\zeta+\zeta_0-2)^2} N_i^{(n)}(\zeta, 2-\zeta_0, \epsilon)d\zeta_0 \\ = \bar{\lambda}_2(\xi_i^{(l)}, \zeta)H_{0,i}^{(n)}(\epsilon, \zeta)\left[\int_0^1 \log|\zeta+\zeta_0-2|d\zeta_0 - \right. \\ - \sum_{j=1}^{m} (\zeta+\zeta_j-2)^2 \log|\zeta+\zeta_j-2| \int_0^1 \frac{\bar{g}_j^{(m)}(\zeta_0)}{(\zeta+\zeta_0-2)^2} d\zeta_0 + \\ \left. + \sum_{j=1}^{m} \bar{\lambda}_2(\xi_i^{(l)}, \zeta_j)N_i^{(n)}(\zeta, 2-\zeta_j, \epsilon) \int_0^1 \frac{g_j^{(m)}(\zeta_0)}{(\zeta+\zeta_0-2)^2} d\zeta_0\right]. \tag{194}$$

To evaluate the other integrals in equations (187) and (188) we use the interpolation formulae

$$\bar{\lambda}_1(\xi_i^{(l)}, \eta_0)P_i^{(n)}(\eta, \eta_0, \xi) = \begin{cases} \sum_{j=1}^{m} \bar{\lambda}_1(\xi_i^{(l)}, \eta_j)P_i^{(n)}(\eta, \eta_j, \xi)g_j^{(m)}(\eta_0) & \eta_0 > 0 \\ \sum_{j=1}^{m} \bar{\lambda}_1(\xi_i^{(l)}, -\eta_j)P_i^{(n)}(\eta, -\eta_j, \xi)g_j^{(m)}(-\eta_0) & \eta_0 < 0 \end{cases} \tag{195}$$

$$\bar{\lambda}_2(\xi_i^{(l)}, \zeta_0)J_i^{(n)}(\eta, \zeta_0, \xi) = \sum_{j=1}^{m} \bar{\lambda}_2(\xi_i^{(l)}, \zeta_j)J_i^{(n)}(\eta, \zeta_j, \xi)\bar{g}_j^{(m)}(\zeta_0) \tag{196}$$

$$\bar{\lambda}_2(\xi_i^{(l)}, \zeta_0)J_i^{(n)}(\eta, 2-\zeta_0, \xi) = \sum_{j=1}^{m} \bar{\lambda}_2(\xi_i^{(l)}, \zeta_j)J_i^{(n)}(\eta, 2-\zeta_j, \xi)\bar{g}_j^{(m)}(\zeta_0) \tag{197}$$

$$\bar{\lambda}_1(\xi_i^{(l)}, \eta_0)M_i^{(n)}(\eta_0, 2-\zeta, \epsilon) = \begin{cases} \sum_{j=1}^{m} \bar{\lambda}_1(\xi_i^{(l)}, \eta_j)M_i^{(n)}(\eta_j, 2-\zeta, \epsilon)g_j^{(m)}(\eta_0) & \eta_0 > 0 \\ \sum_{j=1}^{m} \bar{\lambda}_1(\xi_i^{(l)}, -\eta_j)M_i^{(n)}(-\eta_j, 2-\zeta, \epsilon)g_j^{(m)}(-\eta_0) & \eta_0 < 0 \end{cases} \tag{198}$$

and the formula (97). The equations (187) and (188) may then be replaced by

$$\bar{\alpha}_1(\xi, \eta) = \sum_{i=1}^{n} \bar{\lambda}_1(\xi_i^{(l)}, \eta)F_{0,i}^{(n)}(\xi, \eta) \left[\frac{1}{\sqrt{(1-\eta)}} \left\{\int_0^1 \log|\eta - \eta_0|\sqrt{(1-\eta_0)}d\eta_0 - \right.\right.$$

$$\left. - \sum_{j=1}^{m} (\eta-\eta_j)^2 \log|\eta - \eta_j|\sqrt{(1-\eta_j)} \int_0^1 \frac{g_j^{(m)}(\eta_0)}{(\eta-\eta_0)^2} d\eta_0 \right\} +$$

$$+ \frac{1}{\sqrt{(1+\eta)}} \left\{\int_0^1 \log|\eta + \eta_0|\sqrt{(1-\eta_0)}d\eta_0 - \right.$$

$$\left.\left. - \sum_{j=1}^{m} (\eta+\eta_j)^2 \log|\eta + \eta_j|\sqrt{(1+\eta_j)} \int_0^1 \frac{g_j^{(m)}(\eta_0)}{(\eta+\eta_0)^2} d\eta_0 \right\}\right] +$$

$$+ \sum_{i=1}^{n}\sum_{j=1}^{m} \bar{\lambda}_1(\xi_i^{(l)}, \eta_j)I_i^{(n)}(\eta, \eta_j, \xi) \int_0^1 \frac{g_j^{(m)}(\eta_0)}{(\eta-\eta_0)^2} d\eta_0 +$$

$$+ \sum_{i=1}^{n}\sum_{j=1}^{m} \bar{\lambda}_1(\xi_i^{(l)}, -\eta_j)I_i^{(n)}(\eta, -\eta_j, \xi) \int_0^1 \frac{g_j^{(m)}(\eta_0)}{(\eta+\eta_0)^2} d\eta_0 -$$

$$- \sum_{i=1}^{n}\sum_{j=1}^{m} \bar{\lambda}_1(\xi_i^{(l)}, \eta_j)P_i^{(n)}(\eta, \eta_j, \xi)G_j^{(m)} -$$

$$- \sum_{i=1}^{n}\sum_{j=1}^{m} \bar{\lambda}_1(\xi_i^{(l)}, -\eta_j)P_i^{(n)}(\eta, -\eta_j, \xi)G_j^{(m)} +$$

$$+ \sum_{i=1}^{n}\sum_{j=1}^{m} \bar{\lambda}_2(\xi_i^{(l)}, \zeta_j)J_i^{(n)}(\eta, \zeta_j, \xi) \int_0^1 \frac{s_1 s_2 \eta \zeta_0}{(s_1^2\eta^2 + s_2^2\zeta_0^2)^2} \bar{g}_j^{(m)}(\zeta_0)d\zeta_0 +$$

$$+ \sum_{i=1}^{n}\sum_{j=1}^{m} \bar{\lambda}_2(\xi_i^{(l)}, \zeta_j)J_i^{(n)}(\eta, 2-\zeta_j, \xi) \int_0^1 \frac{s_1 s_2 \eta(2-\zeta_0)}{[s_1^2\eta^2 + s_2^2(2-\zeta_0)^2]^2} \bar{g}_j^{(m)}(\zeta_0)d\zeta_0 \qquad (199)$$

$$\bar{\alpha}_2(\epsilon, \zeta) = \sum_{i=1}^{n}\sum_{j=1}^{m} \bar{\lambda}_1(\xi_i^{(l)}, \eta_j)M_i^{(n)}(\eta_j, \zeta, \epsilon) \int_0^1 \frac{s_1 s_2 \eta_0 \zeta}{(s_1^2\eta_0^2 + s_2^2\zeta^2)^2} g_j^{(m)}(\eta_0)d\eta_0 -$$

$$- \sum_{i=1}^{n}\sum_{j=1}^{m} \bar{\lambda}_1(\xi_i^{(l)}, -\eta_j)M_i^{(n)}(-\eta_j, \zeta, \epsilon) \int_0^1 \frac{s_1 s_2 \eta_0 \zeta}{(s_1^2\eta_0^2 + s_2^2\zeta^2)^2} g_j^{(m)}(\eta_0)d\eta_0 -$$

$$- \sum_{i=1}^{n}\sum_{j=1}^{m} \bar{\lambda}_1(\xi_i^{(l)}, \eta_j)M_i^{(n)}(\eta_j, 2-\zeta, \epsilon) \int_0^1 \frac{s_1 s_2 \eta_0(2-\zeta)}{[s_1^2\eta_0^2 + s_2^2(2-\zeta)^2]^2} g_j^{(m)}(\eta_0)d\eta_0 +$$

$$+ \sum_{i=1}^{n}\sum_{j=1}^{m} \bar{\lambda}_1(\xi_i^{(l)}, -\eta_j)M_i^{(n)}(-\eta_j, 2-\zeta, \epsilon) \int_0^1 \frac{s_1 s_2 \eta_0(2-\zeta)}{[s_1^2\eta_0^2 + s_2^2(2-\zeta)^2]^2} g_j^{(m)}(\eta_0)d\eta_0 +$$

$$+ \sum_{i=1}^{n}\sum_{j=1}^{m} \bar{\lambda}_2(\xi_i^{(l)}, \zeta_j)N_i^{(n)}(\zeta, \zeta_j, \epsilon) \int_0^1 \frac{\bar{g}_j^{(m)}(\zeta_0)}{(\zeta-\zeta_0)^2} d\zeta_0 +$$

$$+ \sum_{i=1}^{n}\sum_{j=1}^{m} \bar{\lambda}_2(\xi_i^{(l)}, \zeta_j)N_i^{(n)}(\zeta, 2-\zeta_j, \epsilon) \int_0^1 \frac{\bar{g}_j^{(m)}(\zeta_0)}{(\zeta+\zeta_0-2)^2} +$$

$$+ \sum_{i=1}^{n} \bar{\lambda}_2(\xi_i^{(l)}, \zeta)H_{0,i}^{(n)}(\epsilon, \zeta) \left[\int_0^2 \log|\zeta - \zeta_0|d\zeta_0 - \right.$$

$$- \sum_{j=1}^{m} (\zeta-\zeta_j)^2 \log|\zeta - \zeta_j| \int_0^1 \frac{\bar{g}_j^{(m)}(\zeta_0)}{(\zeta-\zeta_0)^2} d\zeta_0 -$$

$$\left. - \sum_{j=1}^{m} (\zeta+\zeta_j-2)^2 \log|\zeta + \zeta_j - 2| \int_0^1 \frac{\bar{g}_j^{(m)}(\zeta_0)}{(\zeta+\zeta_0-2)^2} d\zeta_0 \right]. \qquad (200)$$

If the equation (199) is written down for the $2mn$ velocity points on the tailplane and equation (200) is written down for the mn velocity points on the fin, there results a set of $3mn$ simultaneous equations for the $3mn$ unknowns $\bar{\lambda}_1(\xi_i^{(l)}, \eta_j)$, $\bar{\lambda}_1(\xi_i^{(l)}, -\eta_j)$ and $\bar{\lambda}_2(\xi_i^{(l)}, \eta_j)$ for $i = 1, 2, \ldots, n$, $j = 1, 2, \ldots, m$; in terms of the known values of $\bar{\alpha}_1(\xi, \eta)$ or $\bar{\alpha}_2(\epsilon, \eta)$ at the velocity points. These simultaneous equations can be written as the matrix equation

$$\begin{bmatrix} \bar{\alpha}_1^{+} \\ \bar{\alpha}_2^{+} \\ \bar{\alpha}_1^{-} \end{bmatrix} = \begin{bmatrix} \Lambda_{11}^{++} - \Lambda_{13}^{++}, & \dagger\Lambda_{12}^{++} + \dagger\Lambda_{12}^{+-}, & \Lambda_{11}^{+-} - \Lambda_{13}^{+-} \\ \dagger\Lambda_{21}^{++} - \dagger\Lambda_{23}^{++}, & \dagger\Lambda_{2,2}^{++} + \dagger\Lambda_{2,2}^{+-}, & \dagger\Lambda_{21}^{+-} - \dagger\Lambda_{23}^{+-} \\ \Lambda_{11}^{-+} - \Lambda_{13}^{-+}, & \dagger\Lambda_{12}^{-+} + \dagger\Lambda_{12}^{--}, & \Lambda_{11}^{--} - \Lambda_{13}^{--} \end{bmatrix} \begin{bmatrix} \bar{\lambda}_1^{+} \\ \bar{\lambda}_2^{+} \\ \bar{\lambda}_1^{-} \end{bmatrix}. \qquad (201)$$

The submatrices that are different from the ones appearing in equation (106) are defined below.

Λ_{13}^{++} is a square matrix of order $mn \times mn$ with the element

$$P_i^{(n)}(\eta_r, \eta_j, \xi_k^{(w)})G_j^{(m)}$$
$$i = 1, 2, \ldots, n, \qquad k = 1, 2, \ldots, n,$$
$$j = 1, 2, \ldots, m, \qquad r = 1, 2, \ldots, m, \qquad (202)$$

in the $n(r-1) + k$'th row and $n(j-1) + i$'th column.

$\dagger\Lambda_{12}^{++}$ is a square matrix of order $mn \times mn$ with the element

$$J_i^{(n)}(\eta_r, \zeta_j, \xi_k^{(w)}) \int_0^1 \frac{s_1 s_2 \eta_r \zeta_0}{(s_1^2\eta_r^2 + s_2^2\zeta_0^2)^2} \bar{g}_j^{(m)}(\zeta_0)d\zeta_0$$
$$i = 1, 2, \ldots, n, \qquad k = 1, 2, \ldots, n,$$
$$j = 1, 2, \ldots, m, \qquad r = 1, 2, \ldots, m, \qquad (203)$$

in the $n(r-1) + k$'th row and $n(j-1) + i$'th column.

$\dagger\Lambda_{12}^{+-}$ is a square matrix of order $mn \times mn$ with the element

$$J_i^{(n)}(\eta_r, 2-\zeta_j, \xi_k^{(w)}) \int_0^1 \frac{s_1 s_2 \eta_r (2-\zeta_0)}{[s_1^2\eta_r^2 + s_2^2(2-\zeta_0)^2]^2} \bar{g}_j^{(m)}(\zeta_0)d\zeta_0$$
$$i = 1, 2, \ldots, n, \qquad k = 1, 2, \ldots, n,$$
$$j = 1, 2, \ldots, m, \qquad r = 1, 2, \ldots, m, \qquad (204)$$

in the $n(r-1) + k$'th row and $n(j-1) + i$'th column.

Λ_{13}^{+-} is a square matrix of order $mn \times mn$ with the element

$$P_i^{(n)}(\eta_r, -\eta_j, \xi_k^{(w)})G_j^{(m)}$$
$$i = 1, 2, \ldots, n, \qquad k = 1, 2, \ldots, n,$$
$$j = 1, 2, \ldots, m, \qquad r = 1, 2, \ldots, m, \qquad (205)$$

in the $n(r-1) + k$'th row and $n(j-1) + i$'th column.

$\dagger\Lambda_{21}^{++}$ is a square matrix of order $mn \times mn$ with the element

$$M_i^{(n)}(\eta_j, \zeta_r, \xi_k^{(w)}) \int_0^1 \frac{s_1 s_2 \eta_0 \zeta_r}{(s_1^2\eta_0^2 + s_2^2\zeta_r^2)^2} g_j^{(m)}(\eta_0)d\eta_0$$
$$i = 1, 2, \ldots, n, \qquad k = 1, 2, \ldots, n,$$
$$j = 1, 2, \ldots, m, \qquad r = 1, 2, \ldots, m, \qquad (206)$$

in the $n(r-1) + k$'th row and $n(j-1) + i$'th column.

$\dagger\Lambda_{2,3}^{++}$ is a square matrix of order $mn \times mn$ with the element

$$M_i^{(n)}(\eta_j, 2-\zeta_r, \xi_k^{(w)}) \int_0^1 \frac{s_1 s_2 \eta_0 (2-\zeta_r)}{[s_1^2\eta_0^2 + s_2^2(2-\zeta_r)^2]^2} g_j^{(m)}(\eta_0) d\eta_0$$

$$\begin{aligned} i &= 1, 2, \ldots, n, & k &= 1, 2, \ldots, n, \\ j &= 1, 2, \ldots, m, & r &= 1, 2, \ldots, m, \end{aligned} \tag{207}$$

in the $n(r-1)+k$'th row and $n(j-1)+i$'th column.

$\dagger\Lambda_{2,2}^{++}$ is a square matrix of order $mn \times mn$ with the element

$$N_i^{(n)}(\zeta_r, \zeta_j, \xi_k^{(w)}) \int_0^1 \frac{\bar{g}_j^{(m)}(\zeta_0)}{(\zeta_r - \zeta_0)^2} d\zeta_0 + \delta_{j,r} H_{0,i}^{(n)}(\xi_k^{(w)}, \zeta_r) \left[\int_0^1 \log|\zeta_r - \zeta_0| d\zeta_0 - \right.$$

$$\left. - \sum_{j=1}^{m} (\zeta_r - \zeta_j)^2 \log|\zeta_r - \zeta_j| \int_0^1 \frac{\bar{g}_j^{(m)}(\zeta_0)}{(\zeta - \zeta_0)^2} d\zeta_0 \right]$$

$$\begin{aligned} i &= 1, 2, \ldots, n, & k &= 1, 2, \ldots, n, \\ j &= 1, 2, \ldots, m, & r &= 1, 2, \ldots, m, \end{aligned} \tag{208}$$

in the $n(r-1)+k$'th row and $n(j-1)+i$'th column.

$\dagger\Lambda_{2,2}^{+-}$ is a square matrix of order $mn \times mn$ with the element

$$N_i^{(n)}(\zeta_r, 2-\zeta_j, \xi_k^{(w)}) \int_0^1 \frac{\bar{g}_j^{(m)}(\zeta_0)}{(\zeta_r + \zeta_0 - 2)^2} d\zeta_0 + \delta_{j,r} H_{0,i}^{(n)}(\xi_k^{(w)}, \zeta_r) \left[\int_0^1 \log|\zeta + \zeta_0 - 2| d\zeta_0 - \right.$$

$$\left. - \sum_{j=1}^{m} (\zeta_r + \zeta_j - 2)^2 \log|\zeta_r + \zeta_j - 2| \int_0^1 \frac{\bar{g}_j^{(m)}(\zeta_0)}{(\zeta_r + \zeta_0 - 2)^2} d\zeta_0 \right]$$

$$\begin{aligned} i &= 1, 2, \ldots, n, & k &= 1, 2, \ldots, n, \\ j &= 1, 2, \ldots, m, & r &= 1, 2, \ldots, m, \end{aligned} \tag{209}$$

in the $n(r-1)+k$'th row and $n(j-1)+i$'th column.

Since the tailplane is symmetric the other submatrices are given by the relations

$$[\dagger\Lambda_{21}^{+-}] = -[\dagger\Lambda_{21}^{++}] \tag{210}$$

$$[\dagger\Lambda_{23}^{+-}] = -[\dagger\Lambda_{23}^{++}] \tag{211}$$

$$[\Lambda_{13}^{-+}] = [\Lambda_{13}^{+-}] \tag{212}$$

$$[\dagger\Lambda_{12}^{-+}] = -[\dagger\Lambda_{12}^{++}] \tag{213}$$

$$[\dagger\Lambda_{12}^{--}] = -[\dagger\Lambda_{12}^{+-}] \tag{214}$$

$$[\Lambda_{13}^{--}] = [\Lambda_{13}^{++}] \tag{215}$$

If we assume only antisymmetric T-tail oscillations then

$$[\bar{\alpha}_1^{+}] = -[\bar{\alpha}_1^{-}], \tag{216}$$

and in virtue of relations (210) to (215) we must have

$$[\bar{\lambda}_1^{+}] = -[\bar{\lambda}_1^{-}] \tag{217}$$

and the matrix equation (201) may be replaced by

$$\begin{bmatrix} \bar{\alpha}_1{}^+ \\ \\ \bar{\alpha}_2{}^+ \end{bmatrix} = \begin{bmatrix} \dfrac{\Lambda_{11}{}^{++} - \Lambda_{11}{}^{--} - \Lambda_{13}{}^{++} + \Lambda_{13}{}^{--}}{2} & , \dagger\Lambda_{12}{}^{++} + \dagger\Lambda_{12}{}^{+-} \\ \\ \dagger\Lambda_{21}{}^{++} - \dagger\Lambda_{23}{}^{++} & , \dagger\Lambda_{22}{}^{++} + \dagger\Lambda_{22}{}^{+-} \end{bmatrix} \begin{bmatrix} 2\bar{\lambda}_1{}^+ \\ \\ \bar{\lambda}_2{}^+ \end{bmatrix} \tag{218}$$

Equation (218) may be compared with the second equation (125). It is seen that these equations are similar, but that equation (218) contains extra elements arising from the reflection effect of the wall. There are also the minor differences connected with the different location of the velocity and loading points on the fin.

By following the argument of Section 6 it is easily shown that the matrix $[\dagger \underset{\sim}{Q}]$ of the generalised airforces for antisymmetric oscillations of the T-tail is given by

$$[\dagger \underset{\sim}{Q}] = [\dagger f] \begin{bmatrix} B_1{}^+ & \\ & \dagger B_2{}^+ \end{bmatrix} \begin{bmatrix} \dfrac{\Lambda_{11}{}^{++} - \Lambda_{11}{}^{--} - \Lambda_{13}{}^{++} + \Lambda_{13}{}^{--}}{2} & , \dagger\Lambda_{12}{}^{++} + \dagger\Lambda_{12}{}^{+-} \\ \\ \dagger\Lambda_{21}{}^{++} - \dagger\Lambda_{23}{}^{++} & , \dagger\Lambda_{22}{}^{++} + \dagger\Lambda_{22}{}^{+-} \end{bmatrix}^{-1} \times$$

$$\times \begin{bmatrix} D_1{}^+ & \\ & \dagger D_2{}^+ \end{bmatrix} [\dagger\alpha] \tag{219}$$

where $[\dagger f]$ and $[\dagger\alpha]$ correspond with the $[f]$ and $[\alpha]$ of equation (168) only that the values of the elements of $[\dagger f]$ and $[\dagger\alpha]$ are for the new loading and velocity points on the fin. The definition of $[\dagger B_2{}^+]$ is similar to $[B_2{}^+]$ given in equation (148) only that $G_j{}^{(m)}$ has to be replaced by $\bar{G}_j{}^{(m)}$ and $z_{2,j}$ and $x_{2,i,j}{}^{(l)}$ replaced by the values corresponding to the new loading and velocity points on the fin. The definition of $[\dagger D_2{}^+]$ is similar to the definition of $[D_2{}^+]$ given by equation (163) only that $x_{2,k,r}{}^{(w)}$ has to be replaced by the value corresponding to the new loading and velocity points on the fin.

8. *The Treatment of Control Surfaces.*

There may be control surfaces on the T-tail, such as elevators on the tailplane and a rudder on the fin. The displacement functions $f_1{}^{(p)}(x, y)$ and $f_2{}^{(p)}(x, z)$ of equations (126) and (127) will not be smooth across all the inboard edges of the control surfaces, when the mode p is a mode of oscillation involving relative motions of the T-tail and these control surfaces. Also the reduced normal-velocity functions $\alpha_1{}^{(q)}(x, y)$ and $\alpha_2{}^{(q)}(x, z)$ of equations (154) and (155) are not smooth across the inboard edges of the control surfaces, when the mode q is a mode of oscillation involving relative motions of the T-tail and these control surfaces. When a mode of oscillation does involve relative motion of the T-tail and control surfaces we shall say that it is a control-surface mode.

The generalised airforce coefficients $\underset{\sim}{Q}_{p,q}$ may be determined when both p and q do not both refer to control-surface modes at the same time. The functions $f_1{}^{(p)}(x, y)$ and $f_2{}^{(p)}(x, z)$ of the control-surface modes are replaced by equivalent smooth functions and the functions $\alpha_1{}^{(q)}(x, y)$ and $\alpha_2{}^{(q)}(x, z)$ of the control-surface modes are replaced by equivalent smooth functions. The procedure for doing this is described in Ref. 6.

The values of these equivalent functions at the loading and velocity points are then taken instead of the values of the actual function $f_1{}^{(p)}(x, y)$, $f_2{}^{(p)}(x, z)$, $\alpha_1{}^{(q)}(x, y)$ and $\alpha_2{}^{(q)}(x, z)$ to form the matrices $[f]$ and $[\alpha]$ of equation (168) and the matrices $[\dagger f]$ and $[\dagger\alpha]$ of equations (219). It is necessary only to write down the values of these functions at the loading and velocity points since their derivation is exactly analogous to the derivation given in Ref. 6.

If $f_1^{(e,p)}(x, y)$, $f_2^{(e,p)}(x, z)$, $\alpha_1^{(e,q)}(x, y)$ and $\alpha_2^{(e,q)}(x, z)$ denote the functions equivalent to the functions $f_1^{(p)}(x, y)$, $f_2^{(p)}(x, z)$, $\alpha_1^{(q)}(x, y)$, $\alpha_2^{(q)}(x, z)$ then for the isolated T-tail

$$f_1^{(e,p)}(x_{1,i,j}^{(l)}, y_{1,j}^{+}) = \frac{1}{H_i^{(n)}G_j^{(m)}}\frac{1}{c_1(y_{1,j}^{+})}\int_0^1 c_1(y_0)g_j^{(m)}(\eta_0)d\eta_0 \int_0^1 f_1^{(p)}(x_0, y_0)h_i^{(n)}(\xi_0)d\xi_0 \tag{220}$$

$$f_2^{(e,p)}(x_{2,i,j}^{(l)}, z_{2,j}) = \frac{1}{H_i^{(n)}G_j^{(m)}}\frac{1}{c_2(z_{2,j})}\int_0^1 c_2(z_0)g_j^{(m)}(\zeta_0)d\zeta_0 \int_0^1 f_2^{(p)}(x_0, z_0)h_i^{(n)}(\xi_0)d\xi_0 \tag{221}$$

$$\alpha_1^{(e,q)}(x_{1,k,r}^{(w)}, y_{1,r}^{+}) = \frac{1}{H_{n-k+1}^{(n)}G_r^{(m)}}\frac{1}{c_1(y_{1,r}^{+})}\int_0^1 c_1(y_0)g_r^{(m)}(\eta_0)d\eta_0 \int_0^1 \alpha_1^{(q)}(x_0, y_0)h_{n-k+1}^{(n)}(1-\xi_0)d\xi_0 \tag{222}$$

$$\alpha_2^{(e,q)}(x_{2,k,r}^{(w)}, z_{2,r}) = \frac{1}{H_{n-k+1}^{(n)}G_r^{(m)}}\frac{1}{c_2(z_{2,r})}\int_0^1 c_2(z_0)g_r^{(m)}(\zeta_0)d\zeta_0 \int_0^1 \alpha_2^{(q)}(x_0, z_0)h_{n-k+1}^{(n)}(1-\xi_0)d\xi_0 . \tag{223}$$

When there is a reflector plate at the base of the fin, the equivalent values of the displacement and reduced normal-velocity functions on the tailplane are still given by equations (220) and (222). On the fin, however, we now have

$$f_2^{(e,p)}(x_{2,i,j}^{(l)}) = \frac{1}{H_i^{(n)}\bar{G}_j^{(m)}}\frac{1}{c_2(z_{2,j})}\int_0^1 c_2(z_0)\bar{g}_j^{(m)}(\zeta_0)d\zeta_0 \int_0^1 f_2^{(p)}(x_0, z_0)h_i^{(n)}(\xi_0)d\xi_0 \tag{224}$$

$$\alpha_2^{(e,q)}(x_{2,k,r}^{(w)}, z_{2,r}) = \frac{1}{H_{n-k+1}^{(n)}\bar{G}_r^{(m)}}\frac{1}{c_2(z_{2,r})}\int_0^1 c_2(z_0)\bar{g}_r^{(m)}(\zeta_0)d\zeta_0 \int_0^1 \alpha_2^{(q)}(x_0, z_0)h_{n-k+1}^{(n)}(1-\xi_0)d\xi_0 \tag{225}$$

where now $x_{2,i,j}^{(l)}$, $z_{2,j}$, $x_{2,k,r}^{(w)}$, $z_{2,r}$ refer to loading and velocity points on the fin, modified as a result of introducing the reflecting plane and obtained from (74) and (77) by replacing η_j and η_r by ζ_j and ζ_r respectively.

The matrix of generalised airforce coefficients is then obtained either from equation (168) or equation (219) according as to whether the T-tail is isolated or has a reflector plate at the base of the fin. Values for $Q_{p,q}$ when p and q both refer to control-surface modes at the same time are obtained in these matrices, but it must be remembered that the procedure for obtaining these is not valid. Nevertheless these values may be used as estimates of the correct values until a proper procedure for obtaining them becomes available.

9. *The Numerical Procedure.*

The values of the displacement functions and of the reduced normal-velocity functions are to be given at the loading points and at the velocity points respectively on the T-tail. The loading points and velocity points are given in equations (74) to (79) in terms of $\xi_i^{(l)}$, $\xi_k^{(w)}$ and η_j. If there is a reflector at the base of the fin then ζ_j is also required.

The $\xi_i^{(l)}$ are obtained by solving equation (57) and then the $\xi_k^{(w)}$ are obtained from (54) and (55). Values of $\xi_i^{(l)}$ and $\xi_k^{(w)}$ are given in Table 1, corresponding to the first few values of n.

The η_j are obtained by solving equation (68) and the ζ_j are obtained by solving equation (182). Values of η_j and ζ_j are given in Table 2, corresponding to the first few values of m.

The numerical values of the elements in the matrices

$$\begin{bmatrix} \dfrac{\Lambda_{11}{}^{++} - \Lambda_{11}{}^{+-}}{2}, & \Lambda_{12}{}^{++} \\ \Lambda_{21}{}^{++}, & \Lambda_{22}{}^{++} \end{bmatrix} \tag{226}$$

and

$$\begin{bmatrix} \dfrac{\Lambda_{11}{}^{++} - \Lambda_{11}{}^{--} - \Lambda_{13}{}^{++} + \Lambda_{13}{}^{--}}{2}, & \dagger\Lambda_{12}{}^{++} + \dagger\Lambda_{12}{}^{+-} \\ \dagger\Lambda_{21}{}^{++} - \dagger\Lambda_{23}{}^{++}, & \dagger\Lambda_{22}{}^{++} + \dagger\Lambda_{22}{}^{+-} \end{bmatrix} \tag{227}$$

of equations (125) and (218) have to be determined. The elements are determined by use of expressions (113) to (117) and (202) to (209). These expressions involve the integrals

$$P_{j,r}{}^{(m)} = \int_0^1 \frac{g_j{}^{(m)}(\eta_0)}{(\eta_r - \eta_0)^2}\, d\eta_0 \tag{228}$$

$$Q_{j,r}{}^{(m)} = \int_0^1 \frac{g_j{}^{(m)}(\eta_0)}{(\eta_r + \eta_0)^2}\, d\eta_0 \tag{229}$$

$$\bar{P}_{j,r}{}^{(m)} = \int_0^1 \frac{\bar{g}_j{}^{(m)}(\zeta_0)}{(\zeta_r - \zeta_0)^2}\, d\zeta_0 \tag{230}$$

$$\bar{Q}_{j,r}{}^{(m)} = \int_0^1 \frac{\bar{g}_j{}^{(m)}(\zeta_0)}{(\zeta_r + \zeta_0)^2}\, d\zeta_0 \tag{231}$$

which are independent of the shapes of the tailplane and fin. These can be worked out and their values corresponding to the first few values of m are given in Table 3.

The expressions (113) to (117) and (202) to (209) involve the integrals

$$\begin{aligned} &\int_0^1 \frac{s_1 s_2 \eta_r \zeta_0}{(s_1{}^2\eta_r{}^2 + s_2{}^2\zeta_0{}^2)^2}\, g_j{}^{(m)}(\zeta_0) d\zeta_0, && \int_0^1 \frac{s_1 s_2 \eta_0 \eta_r}{(s_1{}^2\eta_0{}^2 + s_2{}^2\eta_r{}^2)^2}\, g_j{}^{(m)}(\eta_0) d\eta_0, \\ &\int_0^1 \frac{s_1 s_2 \eta_0 \zeta_0}{(s_1{}^2\eta_r{}^2 + s_2{}^2\zeta_0{}^2)^2}\, \bar{g}_j{}^{(m)}(\zeta_0) d\zeta_0, && \int_0^1 \frac{s_1 s_2 \eta_0 \zeta_r}{(s_1{}^2\eta_0{}^2 + s_2{}^2\zeta_r{}^2)^2}\, g_j{}^{(m)}(\eta_0) d\eta_0, \\ &\int_0^1 \frac{s_1 s_2 \eta_r (2-\zeta_0)}{[s_1{}^2\eta_r{}^2 + s_2{}^2(2-\zeta_0)^2]^2}\, \bar{g}_j{}^{(m)}(\zeta_0) d\zeta_0, && \int_0^1 \frac{s_1 s_2 \eta_r (2-\zeta_r)}{[s_1{}^2\eta_0{}^2 + s_2{}^2(2-\zeta_r)^2]^2}\, g_j{}^{(m)}(\eta_0) d\eta_0 \end{aligned} \tag{232}$$

which are dependent on s_1 and s_2. These integrals can be worked out only when s_1 and s_2 are known so their values cannot be given here. In order to work out their values expressions for the functions $g_j{}^{(m)}(\eta_0)$ and $\bar{g}_j{}^{(m)}(\zeta_0)$ must be known. These are given in Table 4. Values of $G_j{}^{(m)}$ and $\bar{G}_j{}^{(m)}$, which are also required, are given in Table 2.

The expressions (113) to (117) and (202) to (209) involve $F_{0,i}{}^{(n)}(\xi_k{}^{(w)}, \eta_r)$, $H_{0,i}{}^{(n)}(\xi_k{}^{(w)}, \eta_r)$ and $H_{0,i}{}^{(n)}(\xi_k{}^{(w)}, \zeta_r)$ and these depend on the values of $h_i{}^{(n)\prime}(\xi_k{}^{(w)})$, $h_i{}^{(n)}(\xi_k{}^{(w)})$ and $h_i{}^{(1,n)}(\xi_k{}^{(w)})$, which are given in Table 5 for the first few values of n.

The expressions (113) to (117) and (202) to (209) also involve $I_i^{(n)}(\eta_r, \eta_j, \xi_k^{(w)})$, $I_i^{(n)}(\eta_r, -\eta_j, \xi_k^{(w)})$, $J_i^{(n)}(\eta_r, \eta_j, \xi_k^{(w)})$, $M_i^{(n)}(\eta_j, \eta_r, \xi_k^{(w)})$, $N_i^{(n)}(\eta_r, \eta_j, \xi_k^{(w)})$, $P_i^{(n)}(\eta_r, \eta_j, \xi_k^{(w)})$, $P_i^{(n)}(\eta_r, -\eta_j, \xi_k^{(w)})$, $J_i^{(n)}(\eta_r, \zeta_j, \xi_k^{(w)})$, $J_i^{(n)}(\eta_r, 2-\zeta_j, \xi_k^{(w)})$, $M_i^{(n)}(\eta_j, \zeta_r, \xi_k^{(w)})$, $M_i^{(n)}(\eta_j, 2-\zeta_r, \xi_k^{(w)})$, $N_i^{(n)}(\zeta_r, \zeta_j, \xi_k^{(w)})$, $N_i^{(n)}(\zeta_r, 2-\zeta_j, \xi_k^{(w)})$.

If $r = j$, then $I_i^{(n)}(\eta_r, \eta_j, \xi_k^{(w)})$, $N_i^{(n)}(\eta_r, \eta_j, \xi_k^{(w)})$, $N_i^{(n)}(\zeta_r, \zeta_j, \xi_k^{(w)})$ cannot be worked out using numerical integration for the integrands in the definitions (82) and (85) are singular. The values are obtained from the formulae

$$I_i^{(n)}(\eta, \eta, \xi) = \frac{1}{2\pi} \frac{c_1(y)}{s_1} h_i^{(1, n)}(\xi) \tag{233}$$

$$I_i^{(n)}(\zeta, \zeta, \xi) = \frac{1}{2\pi} \frac{c_2(z)}{s_2} h_i^{(1, n)}(\xi) \tag{234}$$

which are obtained from (82) and (85) by proceeding to the limits $\eta_0 = \eta$ and $\zeta_0 = \zeta$. The derivation of these formulae is given in Ref. 6, Appendix V.

All the other quantities listed can be worked out by numerical integration. In order to do this, expressions for the functions $h_i^{(n)}(\xi)$ and $h_i^{(1, n)}(\xi)$ are required and these are given in Table 6 for the first few values of n.

The numerical values of the elements in the matrices (226) and (227) can then be determined and the matrices inverted for use in equations (168) and (219). Equations (168) and (219) also require the numerical values of the elements of the matrices $[B_1^+]$, $[B_2^+]$, $[\dagger B_2^+]$, $[D_1^+]$, $[D_2^+]$, $[\dagger D_2^+]$, and these are easily determined. The values of $G_j^{(m)}$, $\bar{G}_j^{(m)}$ and $H_i^{(n)}$ required are given in Tables 1 and 2.

10. *Examples.*

As a first example we shall consider the tailplane to be rectangular and of aspect ratio 2 and the fin to be rectangular and of aspect ratio 1. The chords of the tailplane and the fin are of equal length and this length is taken to be the typical length l of the T-tail. The T-tail is immersed in a subsonic flow with free stream Mach number $M = 0{\cdot}866$ and is assumed to be oscillating with a frequency parameter $\nu = 0{\cdot}3$ in one of the six modes of oscillation defined by

$$f_1^{(1)}(x, y) = 0 \qquad f_2^{(1)}(x, z) = 1 \tag{235}$$

$$f_1^{(2)}(x, y) = 0 \qquad f_2^{(2)}(x, z) = x/l \tag{236}$$

$$f_1^{(3)}(x, y) = y/l \qquad f_2^{(3)}(x, z) = \frac{1}{l}(s_2 - z) \tag{237}$$

$$f_1^{(4)}(x, y) = xy/l^2 \qquad f_2^{(4)}(x, z) = \frac{1}{l^2} x(s_2 - z) \tag{238}$$

$$f_1^{(5)}(x, y) = 2s_2 y/l^2 \qquad f_2^{(5)}(x, z) = \frac{1}{l^2}(s_2 - z)^2 \tag{239}$$

$$f_1^{(6)}(x, y) = s_2 xy/l^3 \qquad f_2^{(6)}(x, z) = \frac{1}{l^2} x(s_2 - z)^2 \tag{240}$$

where the origin has been taken at the leading point of the chord of junction.

Calculations were made on an electronic digital computer to obtain the matrix $[Q]$ of the generalised airforce coefficients using different numbers m and n of spanwise and chordwise points. The results for the elements $Q_{p,q}$ of the matrix $[Q]$ are given below for the different combinations of m and n used. $Q_{p,q}$ is a complex number so it is separated into real and imaginary parts by

$$Q_{p,q} = Q_{p,q}' + iQ_{p,q}''. \tag{241}$$

The results are:

m	n	Q_{11}'	Q_{12}'	Q_{13}'	Q_{14}'	Q_{15}'	Q_{16}'
3	2	0·0556	– 1·3849	0·0381	– 1·1334	0·0348	– 0·9000
3	3	0·0550	– 1·3836	0·0372	– 1·1304	0·0337	– 0·8971
3	4	0·0538	– 1·3828	0·0357	– 1·1297	0·0319	– 0·8966
4	2	0·0561	– 1·3839	0·0383	– 1·1257	0·0353	– 0·8861
4	3	0·0564	– 1·3838	0·0382	– 1·1240	0·0351	– 0·8844
4	4	0·0560	– 1·3837	0·0376	– 1·1242	0·0343	– 0·8845
5	2	0·0556	– 1·3839	0·0377	– 1·1251	0·0345	– 0·8843
5	3	0·0560	– 1·3841	0·0378	– 1·1238	0·0344	– 0·8830
5	4	0·0560	– 1·3843	0·0375	– 1·1241	0·0339	– 0·8833

(242)

m	n	Q_{11}''	Q_{12}''	Q_{13}''	Q_{14}''	Q_{15}''	Q_{16}''
3	2	– 0·4165	– 0·5300	– 0·3390	– 0·4004	– 0·3670	– 0·3106
3	3	– 0·4162	– 0·5275	– 0·3380	– 0·3957	– 0·3654	– 0·3063
3	4	– 0·4158	– 0·5213	– 0·3378	– 0·3887	– 0·3651	– 0·3003
4	2	– 0·4166	– 0·5329	– 0·3370	– 0·4008	– 0·3623	– 0·3090
4	3	– 0·4166	– 0·5351	– 0·3365	– 0·4007	– 0·3612	– 0·3085
4	4	– 0·4166	– 0·5328	– 0·3365	– 0·3975	– 0·3613	– 0·3057
5	2	– 0·4166	– 0·5310	– 0·3368	– 0·3984	– 0·3617	– 0·3060
5	3	– 0·4167	– 0·5341	– 0·3364	– 0·3992	– 0·3608	– 0·3063
5	4	– 0·4167	– 0·5335	– 0·3364	– 0·3975	– 0·3609	– 0·3048

(243)

m	n	Q_{21}'	Q_{22}'	Q_{23}'	Q_{24}'	Q_{25}'	Q_{26}'
3	2	0·0495	– 0·2123	0·0391	– 0·1953	0·0414	– 0·1598
3	3	0·0499	– 0·2058	0·0397	– 0·1935	0·0423	– 0·1592
3	4	0·0494	– 0·2096	0·0390	– 0·1986	0·0413	– 0·1637
4	2	0·0500	– 0·2118	0·0393	– 0·1928	0·0413	– 0·1559
4	3	0·0508	– 0·2037	0·0402	– 0·1893	0·0426	– 0·1539
4	4	0·0506	– 0·2032	0·0399	– 0·1903	0·0421	– 0·1549
5	2	0·0499	– 0·2129	0·0392	– 0·1938	0·0411	– 0·1567
5	3	0·0508	– 0·2059	0·0402	– 0·1913	0·0425	– 0·1555
5	4	0·0508	– 0·2046	0·0400	– 0·1915	0·0422	– 0·1558

(244)

m	n	Q_{21}''	Q_{22}''	Q_{23}''	Q_{24}''	Q_{25}''	Q_{26}''
3	2	– 0·0707	– 0·3116	– 0·0639	– 0·2445	– 0·0746	– 0·1918
3	3	– 0·0690	– 0·3153	– 0·0635	– 0·2473	– 0·0747	– 0·1940
3	4	– 0·0700	– 0·3118	– 0·0648	– 0·2433	– 0·0764	– 0·1906
4	2	– 0·0706	– 0·3139	– 0·0632	– 0·2454	– 0·0730	– 0·1911
4	3	– 0·0685	– 0·3200	– 0·0624	– 0·2504	– 0·0727	– 0·1950
4	4	– 0·0684	– 0·3185	– 0·0626	– 0·2482	– 0·0731	– 0·1931
5	2	– 0·0709	– 0·3133	– 0·0635	– 0·2447	– 0·0732	– 0·1902
5	3	– 0·0692	– 0·3202	– 0·0630	– 0·2503	– 0·0733	– 0·1946
5	4	– 0·0688	– 0·3198	– 0·0630	– 0·2491	– 0·0735	– 0·1935

(245)

m	n	Q_{31}'	Q_{32}'	Q_{33}'	Q_{34}'	Q_{35}'	Q_{36}'	
3	2	0·0374	− 1·1234	0·0584	− 1·3713	0·0854	− 1·1869	
3	3	0·0365	− 1·1208	0·0576	− 1·3694	0·0845	− 1·1853	
3	4	0·0351	− 1·1201	0·0562	− 1·3686	0·0830	− 1·1847	
4	2	0·0381	− 1·1223	0·0585	− 1·3637	0·0855	− 1·1731	
4	3	0·0381	− 1·1209	0·0585	− 1·3630	0·0856	− 1·1726	
4	4	0·0375	− 1·1210	0·0580	− 1·3630	0·0851	− 1·1726	
5	2	0·0377	− 1·1223	0·0578	− 1·3633	0·0845	− 1·1716	
5	3	0·0378	− 1·1213	0·0580	− 1·3628	0·0848	− 1·1713	
5	4	0·0375	− 1·1216	0·0578	− 1·3630	0·0846	− 1·1714	(246)

m	n	Q_{31}''	Q_{32}''	Q_{33}''	Q_{34}''	Q_{35}''	Q_{36}''	
3	2	− 0·3359	− 0·3956	− 0·4133	− 0·5365	− 0·5617	− 0·4721	
3	3	− 0·3350	− 0·3912	− 0·4127	− 0·5327	− 0·5613	− 0·4690	
3	4	− 0·3348	− 0·3844	− 0·4124	− 0·5258	− 0·5608	− 0·4631	
4	2	− 0·3360	− 0·3992	− 0·4113	− 0·5361	− 0·5570	− 0·4694	
4	3	− 0·3355	− 0·3993	− 0·4111	− 0·5371	− 0·5570	− 0·4704	
4	4	− 0·3355	− 0·3961	− 0·4110	− 0·5341	− 0·5569	− 0·4679	
5	2	− 0·3360	− 0·3975	− 0·4111	− 0·5333	− 0·5565	− 0·4661	
5	3	− 0·3356	− 0·3984	− 0·4110	− 0·5352	− 0·5566	− 0·4678	
5	4	− 0·3356	− 0·3968	− 0·4110	− 0·5338	− 0·5566	− 0·4667	(247)

m	n	Q_{41}'	Q_{42}'	Q_{43}'	Q_{44}'	Q_{45}'	Q_{46}'	
3	2	0·0386	− 0·1935	0·0495	− 0·2100	0·0683	− 0·1779	
3	3	0·0392	− 0·1919	0·0499	− 0·2052	0·0687	− 0·1734	
3	4	0·0384	− 0·1968	0·0492	− 0·2097	0·0680	− 0·1773	
4	2	0·0391	− 0·1923	0·0497	− 0·2083	0·0683	− 0·1750	
4	3	0·0399	− 0·1890	0·0504	− 0·2018	0·0691	− 0·1691	
4	4	0·0396	− 0·1899	0·0501	− 0·2021	0·0689	− 0·1693	
5	2	0·0390	− 0·1932	0·0495	− 0·2096	0·0681	− 0·1762	
5	3	0·0400	− 0·1910	0·0504	− 0·2043	0·0691	− 0·1712	
5	4	0·0397	− 0·1911	0·0503	− 0·2038	0·0690	− 0·1708	(248)

m	n	Q_{41}''	Q_{42}''	Q_{43}''	Q_{44}''	Q_{45}''	Q_{46}''	
3	2	− 0·0633	− 0·2417	− 0·0701	− 0·3095	− 0·0914	− 0·2699	
3	3	− 0·0629	− 0·2445	− 0·0689	− 0·3128	− 0·0893	− 0·2727	
3	4	− 0·0642	− 0·2404	− 0·0701	− 0·3089	− 0·0905	− 0·2695	
4	2	− 0·0630	− 0·2442	− 0·0696	− 0·3103	− 0·0904	− 0·2691	
4	3	− 0·0622	− 0·2490	− 0·0680	− 0·3159	− 0·0879	− 0·2738	
4	4	− 0·0624	− 0·2468	− 0·0681	− 0·3139	− 0·0877	− 0·2723	
5	2	− 0·0633	− 0·2436	− 0·0700	− 0·3094	− 0·0909	− 0·2680	
5	3	− 0·0628	− 0·2492	− 0·0688	− 0·3158	− 0·0888	− 0·2734	
5	4	− 0·0628	− 0·2479	− 0·0686	− 0·3148	− 0·0884	− 0·2727	(249)

m	n	Q_{51}'	Q_{52}'	Q_{53}'	Q_{54}'	Q_{55}'	Q_{56}'	
3	2	0·0340	− 1·2090	0·0853	− 1·8470	0·1407	− 1·6517	
3	3	0·0329	− 1·2051	0·0844	− 1·8454	0·1400	− 1·6507	
3	4	0·0312	− 1·2043	0·0829	− 1·8444	0·1383	− 1·6499	
4	2	0·0349	− 1·2076	0·0853	− 1·8383	0·1406	− 1·6358	
4	3	0·0347	− 1·2052	0·0854	− 1·8379	0·1410	− 1·6359	
4	4	0·0339	− 1·2053	0·0849	− 1·8378	0·1407	− 1·6358	
5	2	0·0344	− 1·2076	0·0845	− 1·8379	0·1393	− 1·6341	
5	3	0·0344	− 1·2056	0·0847	− 1·8378	0·1399	− 1·6345	
5	4	0·0339	− 1·2060	0·0846	− 1·8380	0·1400	− 1·6345	(250)

m	n	Q_{51}''	Q_{52}''	Q_{53}''	Q_{54}''	Q_{55}''	Q_{56}''	
3	2	− 0·3600	− 0·3979	− 0·5583	− 0·7506	− 0·8295	− 0·6878	
3	3	− 0·3586	− 0·3919	− 0·5579	− 0·7471	− 0·8298	− 0·6852	
3	4	− 0·3582	− 0·3837	− 0·5575	− 0·7390	− 0·8292	− 0·6783	
4	2	− 0·3600	− 0·4025	− 0·5560	− 0·7496	− 0·8241	− 0·6841	
4	3	− 0·3591	− 0·4015	− 0·5560	− 0·7517	− 0·8248	− 0·6863	
4	4	− 0·3591	− 0·3973	− 0·5559	− 0·7484	− 0·8246	− 0·6837	
5	2	− 0·3600	− 0·4005	− 0·5558	− 0·7462	− 0·8235	− 0·6800	
5	3	− 0·3592	− 0·4004	− 0·5559	− 0·7492	− 0·8243	− 0·6831	
5	4	− 0·3593	− 0·3979	− 0·5559	− 0·7478	− 0·8243	− 0·6821	(251)

m	n	Q_{61}'	Q_{62}'	Q_{63}'	Q_{64}'	Q_{65}'	Q_{66}'	
3	2	0·0300	− 0·1565	0·0428	− 0·1762	0·0614	− 0·1504	
3	3	0·0304	− 0·1561	0·0431	− 0·1717	0·0616	− 0·1459	
3	4	0·0298	− 0·1604	0·0425	− 0·1754	0·0610	− 0·1491	
4	2	0·0303	− 0·1553	0·0430	− 0·1748	0·0613	− 0·1480	
4	3	0·0311	− 0·1534	0·0435	− 0·1690	0·0620	− 0·1424	
4	4	0·0307	− 0·1544	0·0433	− 0·1691	0·0618	− 0·1424	
5	2	0·0302	− 0·1561	0·0428	− 0·1761	0·0611	− 0·1492	
5	3	0·0311	− 0·1551	0·0435	− 0·1711	0·0619	− 0·1444	
5	4	0·0309	− 0·1554	0·0435	− 0·1707	0·0619	− 0·1438	(252)

m	n	Q_{61}''	Q_{62}''	Q_{63}''	Q_{64}''	Q_{65}''	Q_{66}''	
3	2	− 0·0509	− 0·1873	− 0·0591	− 0·2678	− 0·0787	− 0·2379	
3	3	− 0·0508	− 0·1894	− 0·0579	− 0·2706	− 0·0764	− 0·2403	
3	4	− 0·0520	− 0·1859	− 0·0589	− 0·2674	− 0·0774	− 0·2377	
4	2	− 0·0507	− 0·1894	− 0·0587	− 0·2684	− 0·0778	− 0·2371	
4	3	− 0·0502	− 0·1932	− 0·0572	− 0·2731	− 0·0753	− 0·2411	
4	4	− 0·0505	− 0·1912	− 0·0572	− 0·2716	− 0·0751	− 0·2399	
5	2	− 0·0509	− 0·1890	− 0·0591	− 0·2677	− 0·0783	− 0·2361	
5	3	− 0·0507	− 0·1933	− 0·0579	− 0·2730	− 0·0762	− 0·2407	
5	4	− 0·0508	− 0·1921	− 0·0577	− 0·2723	− 0·0758	− 0·2402	(253)

The results show that the calculated generalised airforce coefficients do not change very much with the different combinations of m and n, especially those relevant to rigid oscillations. The greatest change seems to occur on increasing m from 3 to 4 and on increasing n from 2 to 3. For the remaining examples we shall take $m = 4$ and $n = 3$ and assume that results are obtained with good accuracy for this choice of m and n.

As a second example we shall consider the tailplane to be rectangular and of aspect ratio 1 and the fin to be rectangular and of aspect ratio 1. The typical length l is again taken to be the length of the chord of either tailplane or fin. The T-tail is immersed in a subsonic flow of free-stream Mach number zero, and is assumed to be oscillating in one of the four modes of oscillation defined by

$$f_1^{(1)}(x, y) = 0 \qquad f_2^{(1)}(x, z) = 1 \tag{254}$$

$$f_1^{(2)}(x, y) = 0 \qquad f_2^{(2)}(x, z) = x/l - \tfrac{1}{2} \tag{255}$$

$$f_1^{(3)}(x, y) = y/l \qquad f_2^{(3)}(x, z) = \frac{1}{l}\left(\frac{3}{2}s_2 - z\right) \tag{256}$$

$$f_1^{(4)}(x, y) = y/l \qquad f_2^{(4)}(x, z) = 0 \tag{257}$$

where the origin has been taken at the leading edge at the chord of junction. Calculation of $[Q]$ was made on an electronic digital computer for a selection of values of the frequency parameter ν. Some of the elements of $[Q]$ obtained are given below.

	$\nu = 0$	$\nu = 0\cdot1$	$\nu = 0\cdot2$	$\nu = 0\cdot5$	$\nu = 0\cdot7$	$\nu = 1\cdot0$
$Q_{12}' =$	$-1\cdot0865$	$-1\cdot0854$	$-1\cdot0832$	$-1\cdot0748$	$-1\cdot0696$	$-1\cdot0640$
$Q_{22}' =$	$+0\cdot3282$	$+0\cdot3280$	$+0\cdot3280$	$+0\cdot3300$	$+0\cdot3331$	$+0\cdot3418$
$Q_{32}' =$	$-1\cdot2306$	$-1\cdot2293$	$-1\cdot2268$	$-1\cdot2172$	$-1\cdot2112$	$-1\cdot2047$
$Q_{42}' =$	$-0\cdot0717$	$-0\cdot0716$	$-0\cdot0715$	$-0\cdot0708$	$-0\cdot0704$	$-0\cdot0698$
$Q_{12}'' =$	$0\cdot0000$	$-0\cdot0774$	$-0\cdot1560$	$-0\cdot3972$	$-0\cdot5615$	$-0\cdot8116$
$Q_{22}'' =$	$0\cdot0000$	$-0\cdot0104$	$-0\cdot0204$	$-0\cdot0490$	$-0\cdot0672$	$-0\cdot0936$
$Q_{32}'' =$	$0\cdot0000$	$-0\cdot0849$	$-0\cdot1710$	$-0\cdot4358$	$-0\cdot6163$	$-0\cdot8913$
$Q_{42}'' =$	$0\cdot0000$	$-0\cdot0036$	$-0\cdot0073$	$-0\cdot0187$	$-0\cdot0265$	$-0\cdot0386$. (258)

Using the theory of Fin-Body-Tailplane arrangements given in Ref. 9 and taking the radius of the body zero we calculate

$$\left.\begin{aligned} Q_{12}' &= -1\cdot0587 \\ Q_{42}' &= -0\cdot0705 \end{aligned}\right\} \tag{259}$$

when $\nu = 0$, and these are in good agreement with the relevant results in (258).

Experimental values have been obtained for an oscillating T-tail with rectangular tailplane and fin surfaces and are reported in Ref. 10. As the experimental values were obtained in a wind tunnel we should use the theoretical model of the T-tail with a reflector plate at the base of the fin. Calculations were performed on an electronic digital machine for $\nu = 0\cdot5$ and the following results were obtained.

$$\begin{aligned} Q_{12} &= -1\cdot7816 - i\,0\cdot3878 = -1\cdot8233 \exp(i\,12°17') \\ Q_{22} &= +0\cdot4805 - i\,0\cdot0857 = +0\cdot4881 \exp(-i\,10°7') \\ Q_{32} &= -1\cdot8714 - i\,0\cdot4014 = -1\cdot9140 \exp(i\,12°6') \\ Q_{42} &= -0\cdot1006 - i\,0\cdot0140 = +0\cdot1392 \exp(i\,7°55'). \end{aligned}$$

The corresponding experimental values, in our notation, are

$$Q_{12} = -2\cdot 06 \exp(i\,9°)$$
$$Q_{22} = +0\cdot 54 \exp(-i\,20°)$$
$$Q_{32} = -1\cdot 81 \exp(i\,3°)$$
$$Q_{42} = +0\cdot 16 \exp(-i\,3°).$$

The moduli of the generalised airforces are thus seen to be of the same order of magnitude in both the experimental and theoretical cases. The phases are not in good agreement.

As a final example we consider the case of swept back fin and tailplane, as shown in Fig. 3. The typical length l of the T-tail is taken to be the length of the chord of junction and the origin is taken as the leading point of the chord of junction. The T-tail is immersed in a subsonic flow with free-stream Mach number $M = 0\cdot 8$ and is assumed to be oscillating with a frequency parameter $\nu = 0\cdot 5$ in one of the six modes of oscillation defined by

$$f_1^{(1)}(x, y) = 0 \qquad f_2^{(1)}(x, z) = 1 \tag{260}$$

$$f_1^{(2)}(x, y) = 0 \qquad f_2^{(2)}(x, z) = x/l \tag{261}$$

$$f_1^{(3)}(x, y) = y/l \qquad f_2^{(3)}(x, z) = \frac{1}{l}(s_2 - z) \tag{262}$$

$$f_1^{(4)}(x, y) = 0 \qquad f_2^{(4)}(x, z) = \frac{1}{l^2}x(s_2 - z) \tag{263}$$

$$f_1^{(5)}(x, y) = 2s_2 y/l^2 \qquad f_2^{(5)}(x, z) = \frac{1}{l^2}(s_2 - z)^2 \tag{264}$$

$$f_1^{(6)}(x, y) = y/l \qquad f_2^{(6)}(x, z) = 0. \tag{265}$$

Calculations were made on an electronic digital computer with $m = 4$ and $n = 3$ for the isolated T-tail and for the T-tail with reflector plate at the base of the fin. For the isolated T-tail the matrix $[Q]$ of the generalised airforce coefficients is

$$[Q] = \begin{bmatrix} +0\cdot 2164 & -2\cdot 0977 & 0\cdot 1589 & -1\cdot 7000 & 0\cdot 1529 & -0\cdot 0010 \\ -0\cdot 0129 & +0\cdot 6847 & 0\cdot 0261 & +0\cdot 3520 & 0\cdot 0529 & +0\cdot 0047 \\ +0\cdot 0859 & -2\cdot 2202 & 0\cdot 1759 & -2\cdot 0574 & 0\cdot 3505 & +0\cdot 0764 \\ +0\cdot 0201 & +0\cdot 2922 & 0\cdot 0414 & +0\cdot 1658 & 0\cdot 0661 & +0\cdot 0047 \\ -0\cdot 0451 & -3\cdot 1942 & 0\cdot 2505 & -3\cdot 0975 & 0\cdot 7556 & +0\cdot 2304 \\ -0\cdot 0563 & -0\cdot 4522 & 0\cdot 0334 & -0\cdot 4372 & 0\cdot 1889 & +0\cdot 0762 \end{bmatrix}$$
$$+ i \begin{bmatrix} -1\cdot 0246 & -0\cdot 7804 & -0\cdot 9654 & -0\cdot 6946 & -1\cdot 2249 & -0\cdot 1191 \\ +0\cdot 3185 & -0\cdot 1498 & +0\cdot 1489 & -0\cdot 1612 & +0\cdot 0703 & -0\cdot 0122 \\ -1\cdot 0942 & -0\cdot 6104 & -1\cdot 6627 & -0\cdot 7025 & -3\cdot 0422 & -0\cdot 6391 \\ +0\cdot 1324 & -0\cdot 1723 & +0\cdot 0477 & -0\cdot 1987 & -0\cdot 0271 & -0\cdot 0222 \\ -1\cdot 5684 & -0\cdot 5691 & -3\cdot 2206 & -0\cdot 8174 & -6\cdot 8032 & -1\cdot 6910 \\ -0\cdot 2163 & +0\cdot 0264 & -0\cdot 7215 & -0\cdot 0163 & -1\cdot 7757 & -0\cdot 5126 \end{bmatrix}. \tag{266}$$

For the T-tail with a reflector plate attached to the fin the matrix $[Q]$ of the generalised airforce coefficients is

$$[Q] = \begin{bmatrix} -0\cdot1633 & -4\cdot0155 & -0\cdot2935 & -2\cdot7973 & -0\cdot5674 & -0\cdot1266 \\ +0\cdot1813 & +1\cdot2695 & +0\cdot2143 & +0\cdot4973 & +0\cdot2890 & +0\cdot0262 \\ -0\cdot3858 & -3\cdot8891 & -0\cdot2594 & -3\cdot1439 & -0\cdot2869 & -0\cdot0252 \\ +0\cdot1201 & +0\cdot4187 & +0\cdot1356 & +0\cdot1743 & +0\cdot1826 & +0\cdot0129 \\ -0\cdot8453 & -5\cdot6930 & -0\cdot3958 & -4\cdot8000 & -0\cdot1032 & +0\cdot1166 \\ -0\cdot2158 & -0\cdot9024 & -0\cdot0763 & -0\cdot7532 & +0\cdot0702 & +0\cdot0696 \end{bmatrix}$$

$$+ i \begin{bmatrix} -1\cdot8334 & -0\cdot0574 & -1\cdot4569 & -0\cdot1158 & -1\cdot7800 & -0\cdot1712 \\ +0\cdot5678 & -0\cdot7720 & +0\cdot1472 & -0\cdot6228 & -0\cdot0736 & -0\cdot0611 \\ -1\cdot8023 & +0\cdot2201 & -2\cdot2442 & -0\cdot1877 & -3\cdot8973 & -0\cdot7819 \\ +0\cdot1798 & -0\cdot4995 & -0\cdot0039 & -0\cdot4681 & -0\cdot1844 & -0\cdot0659 \\ -2\cdot6355 & +0\cdot7964 & -4\cdot2391 & -0\cdot0469 & -8\cdot4671 & -2\cdot0150 \\ -0\cdot4099 & +0\cdot2922 & -0\cdot9365 & +0\cdot1285 & -2\cdot1611 & -0\cdot5973 \end{bmatrix} . \qquad (267)$$

It is seen that the presence of the reflector surface at the base of the fin modifies the isolated T-tail generalised aerodynamic force coefficients considerably. No comparisons with other theoretical work or with experimental results to check the calculated effect of the reflector surface have been possible.

11. *Conclusions.*

A theory, based on ordinary plane-wing lifting-surface theory, for determining generalised airforces on a T-tail oscillating in subsonic flow has been described. The calculations are long but straightforward and are best carried out on an electronic digital computer. For this purpose programmes RAE264A and RAE265A have been constructed for use with the Ferranti Mercury Computer. Programme RAE264A obtains generalised airforces on an isolated T-tail and programme RAE265A obtains generalised airforces on a T-tail with a reflector plate attached to the base of the fin.

Some examples have been given. The first example illustrates the effect of taking different numbers of spanwise and chordwise points. Some of the results in the second example may be compared with theoretical results in steady flow and also with experimental results in oscillatory flow. The third example gives results for a T-tail which has swept-back tailplane and fin. The comparison of the theoretical results is good, but the comparison of the experimental results, though it gives tolerable agreement in the absolute magnitude, shows discrepancies in the phases of the generalised airforce coefficients.

SYMBOLS

a	Speed of sound in undisturbed main stream
$a_r(\zeta_0)$	Coefficients appearing in equation (44)
$b_r(\zeta_0)$	Coefficients appearing in equation (47)
$[B_1^+]$, $[B_2^+]$, $[B_1^-]$	Diagonal submatrices, the elements of which are defined in equations (147), (148) and (149)
$c_1(y)$	Chord of tailplane at spanwise station y
$c_2(z)$	Chord of fin at spanwise station z
$[D_1^+]$, $[D_2^+]$	Diagonal submatrices, the elements of which are defined in equations (162) and (163)
$E_{s,i}^{(n)}(\xi, \eta)$, $F_{s,i}^{(n)}(\xi, \eta)$	Coefficients appearing in expansion (87)
$f_1(x, y)$	Shape of tailplane surface, *see* equation (1)
$f_2(x, z)$	Shape of fin surface, *see* equation (2)
$f_1^{(p)}(x, y)$	Shape of tailplane surface in the p'th mode of oscillation, *see* equation (126)
$f_2^{(p)}(x, z)$	Shape of fin surface in the p'th mode of oscillation, *see* equation (127)
$[f_{1,p}^+]$, $[f_{2,p}^+]$, $[f_{1,p}^-]$	Row matrices, the elements of which are defined in equations (144), (145) and (146)
$[f]$	Matrix whose rows are the row matrices $[f_{1,p}^+, f_{2,p}^+]$
$f_1^{(e,p)}(x_{1,i,j}^{(l)}, y_{1,j}^+)$	Equivalent values of tailplane displacement defined in equation (220)
$f_2^{(e,p)}(x_{2,i,j}^{(l)}, z_{2,j})$	Equivalent values of fin displacement defined in equation (221) for an isolated tailplane and in equation (224) for a tailplane with a reflector surface
$[f_{1,p}^s]$, $[f_{1,p}^a]$	Defined in equations (151)
$g_j^{(m)}(\zeta_0)$	Spanwise interpolation functions, defined in equation (69)
$\bar{g}_j^{(m)}(\zeta_0)$	Spanwise interpolation functions, defined in equation (186)
$G_j^{(m)}$	Defined in equation (141)
$\bar{G}_j^{(m)}$	Defined in equation (141) with $g_j^{(m)}(\eta_0)d\eta_0$ replaced by $\bar{g}_j^{(m)}(\zeta_0)d\zeta_0$
$G_r\left(\frac{3}{2}, 1, \zeta_0\right)$	A Jacobi polynomial, *see* equation (66)
$G_{s,i}^{(n)}(\epsilon, \zeta)$, $H_{s,i}^{(n)}(\epsilon, \zeta)$	Coefficients appearing in expansion (88)
$h_i^{(n)}(\xi_0)$	Chordwise interpolation functions, defined in equation (59)
$h_i^{(1,n)}(\xi)$	Defined in equation (86)
$H_i^{(n)}$	Defined in equation (140)

$I_i^{(n)}(\eta, \eta_0, \xi)$		Defined in equation (82)
$J_i^{(n)}(\eta, \zeta_0, \xi)$		Defined in equation (83)
$K_1(x, y)$		Kernel function defined in equation (11)
$K_2(x, y, z)$		Kernel function defined in equation (12)
$K_3(x, y, z)$		Kernel function defined in equation (302)
$\hat{K}_1(x, y)$, $\hat{K}_2(x, y, z)$		Modified kernel functions defined in equations (17)
$\hat{K}_1^{(1)}(x, y)$, $\hat{K}_1^{(2)}(x, y)$		Constituents of $\hat{K}_1(x, y)$ defined in equations (34) and (35)
$\hat{K}_2^{(1)}(x, y, z)$, $\hat{K}_2^{(2)}(x, y, z)$		Constituents of $\hat{K}_2(x, y, z)$ defined in equations (36) and (37)
$\hat{K}_1^{(3)}(x, y)$, $\hat{K}_2^{(3)}(x, y, z)$		Defined in equations (42) and (43)
$\hat{K}_3^{(1)}(x, y, 2s_2)$, $\hat{K}_3^{(2)}(x, y, 2s_2)$, $\hat{K}_3^{(3)}(x, y, 2s_2)$		Defined in equations (176), (177) and (178)
l		Typical dimension of the T-tail
$l_1(x, y)$		Loading function on the tailplane
$l_2(x, z)$		Loading function on the fin
$l_1^{(p)}(x, y)$		Loading function on the tailplane in the p'th mode of oscillation
$l_2^{(p)}(x, z)$		Loading function on the fin in the p'th mode of oscillation
$l_1(x, y, t)$		Loading at time t at a point x, y on the tailplane
$L_1(X, Y, t)$		Loading at time t at a point X, Y on the tailplane
m		Number of spanwise points on the half-tailplane and on the fin
M	=	V/a, Mach number of the main stream
$M_i^{(n)}(\eta_0, \zeta, \epsilon)$		Defined in equation (84)
n		Number of chordwise points on the half-tailplane and on the fin
$N_i^{(n)}(\zeta, \zeta_0, \epsilon)$		Defined in equation (85)
p		Ambient pressure
p_0		Free-stream pressure
$P_{p,q}$		Generalised aerodynamic force, defined in equation (128)
$P_{j,r}^{(m)}$, $\bar{P}_{j,r}^{(m)}$		Defined in equations (228) and (230)
$P_i^{(n)}(\eta, \eta_0, \xi)$		Defined in equation (189)
q		Velocity of a fluid particle relative to X, Y, Z coordinate axes
Q		Matrix with elements $Q_{p,q}$
$Q_{p,q}$		Generalised aerodynamic force coefficient, defined in equation (132)

$Q_{p,q}'$, $Q_{p,q}''$	Real and imaginary parts of $Q_{p,q}$
$Q_{j,r}^{(m)}$, $\bar{Q}_{j,r}^{(m)}$	Defined in equations (229) and (231)
r	Defined in equation (277)
$[r]$	Defined in equation (294)
R	Defined in equation (14)
R_1	Defined in equation (13)
R_3	Defined in equation (173)
s_1	Semi-span of tailplane, *see* Fig. 1
s_2	Span of fin, *see* Fig. 1
t	Time
$u(\epsilon, \zeta_0)$	Defined in equation (51)
V	Main-stream speed
$w_m(\zeta)$	Defined in equation (63)
$\bar{w}_m(\zeta)$	Defined in equation (185)
$w_1(x; y)$	Tailplane velocity function defined in equation (3)
$w_2(x, z)$	Fin velocity function defined in equation (4)
$w_1(x, y, t)$	Defined in equation (273)
$w_2(x, z, t)$	Defined in equation (274)
$w_{12}(x, y, z, t)$	Defined in equation (304)
x, y, z	Rectangular Cartesian coordinates, stationary with respect to the mean position of the oscillating wing
X, Y, Z	Rectangular Cartesian coordinates, stationary with respect to the main-stream flow
$x = x_L^{(1)}(y)$	equation of leading edge of tailplane
$x = x_L^{(2)}(z)$	equation of leading edge of fin
$x = x_T^{(1)}(y)$	equation of trailing edge of tailplane
$x = x_T^{(2)}(z)$	equation of trailing edge of fin
$x_{2,i,j}^{(l)}$, $z_{2,j}$	Loading points on fin, defined in equations (74)
$x_{1,i,j}^{(l)}$, $y_{1,j}^{+}$	Loading points on port half-tailplane, defined in equations (75)
$x_{1,i,j}^{(l)}$, $y_{1,j}^{-}$	Loading points on starboard half-tailplane, defined in equations (76)
$x_{2,k,r}^{(w)}$, $z_{2,r}$	Velocity points on fin, defined in equations (77)
$x_{1,k,r}^{(w)}$, $y_{1,r}^{+}$	Velocity points on port half-tailplane, defined in equations (78)
$x_{1,k,r}^{(w)}$, $y_{1,r}^{-}$	Velocity points on starboard half-tailplane, defined in equations (79)

$Y(x, z, t)$	Normal displacement of a point x, z on the surface of the fin at time t
$Z(x, y, t)$	Normal displacement of a point x, y, on the surface of the tailplane at time t
$Y_p(x, z, t)$	Normal displacement in the p'th mode of oscillation of a point x, z on the surface of the fin at time t
$Z_p(x, y, t)$	Normal displacement in the p'th mode of oscillation of a point x, y on the surface of the tailplane at time t
$\alpha_1(x, y)$	Reduced tailplane velocity function, defined in equation (5)
$\alpha_2(x, z)$	Reduced fin velocity function, defined in equation (6)
$\hat{\alpha}_1(x, y)$, $\hat{\alpha}_2(x, z)$	Defined in equations (15)
$\bar{\alpha}_1(\xi, \eta)$, $\bar{\alpha}_2(\xi, \zeta)$	Defined in equations (30)
$\bar{\alpha}_{1, q}(\xi, \eta)$, $\bar{\alpha}_{2, q}(\xi, \zeta)$	Functions $\bar{\alpha}_1(\xi, \eta)$, $\bar{\alpha}_2(\xi, \zeta)$ appropriate to the q'th mode of oscillation
$\alpha_1^{(e, q)}(x_{1, k, r}^{(w)}, y_{1, r}^{+})$	Equivalent values of tailplane reduced normal-velocity function, defined in equation (222)
$\alpha_2^{(e, q)}(x_{2, k, r}^{(w)}, z_{2, r})$	Equivalent values of fin reduced normal-velocity function, defined in equations (223) or (225)
$\alpha_n(\epsilon)$	Normal velocity on a two-dimensional wing, corresponding to a loading distribution $l_r(\xi_0)\sqrt{\{(1-\epsilon_0)/\epsilon_0\}}$
$[\bar{\alpha}_1^{+}]$, $[\bar{\alpha}_2^{+}]$, $[\bar{\alpha}_1^{-}]$	Matrices, the elements of which are defined in equations (107), (108) and (109)
$[\bar{\alpha}_1^{s}]$, $[\bar{\alpha}_1^{a}]$	Matrices defined in equations (123)
$[\alpha_{1, q}^{+}]$, $[\alpha_{2, q}^{+}]$, $[\alpha_{1, q}^{-}]$	Matrices, the elements of which are defined in equations (156), (157) and (158)
$[\alpha_{1, q}^{s}]$, $[\alpha_{1, q}^{a}]$	Matrices defined in equations (159)
$[\alpha]$	Matrix whose columns are the column matrices $\begin{bmatrix}\alpha_{1, q}^{+} \\ \alpha_{2, q}^{+}\end{bmatrix}$
ϵ	Defined in equation (24)
ϵ_0	Defined in equation (26)
ζ	Defined in equation (25)
ζ_0	Defined in equation (27)
ζ_j	The roots of equation (182)
η	Defined in equation (21)
η_0	Defined in equation (23)
η_j	The roots of equation (68)
$\eta_r^{(w)}$	The real roots of equation (65)

$\lambda_1(x, y)$, $\lambda_2(x, z)$	Reduced loading functions defined in equations (7) and (8)
$\hat{\lambda}_1(x, y)$, $\hat{\lambda}_2(x, z)$	Defined in equations (16)
$\bar{\lambda}_1(\xi_0, \eta_0)$, $\bar{\lambda}_2(x, z)$	Defined in equations (31)
$\lambda_1^{(q)}(x_0, y_0)$, $\lambda_2^{(q)}(x_0, z_0)$	Defined in equations (129) and (130)
$\bar{\lambda}_2^*(\epsilon_0, \zeta_0)$	Approximation to $\bar{\lambda}_2(\epsilon_0, \zeta_0)$, defined in equation (44)
$[\bar{\lambda}_1^+]$, $[\bar{\lambda}_2^+]$, $[\bar{\lambda}_1^-]$	Matrices, the elements of which are defined in equations (110), (111) and (112)
$[\bar{\lambda}_1^s]$, $[\bar{\lambda}_1^a]$	Matrices defined in equations (123)
$[\bar{\lambda}_{1,q}^+]$, $[\bar{\lambda}_{2,q}^+]$, $[\bar{\lambda}_{1,q}^-]$	Matrices $[\bar{\lambda}_1^+]$, $[\bar{\lambda}_2^+]$, $[\bar{\lambda}_1^-]$ appropriate to the q'th mode
$[\lambda_{1,q}^s]$, $[\lambda_{1,q}^a]$	Matrices defined in equations (152)
$[\Lambda_{1,1}^{++}]$, $[\Lambda_{1,2}^{++}]$, $[\Lambda_{1,1}^{--}]$	Submatrices, the elements of which are defined in equations (113) to (121)
$[\Lambda_{2,1}^{++}]$, $[\Lambda_{2,2}^{++}]$, $[\Lambda_{21}^{+-}]$	
$[\Lambda_{11}^{-+}]$, $[\Lambda_{12}^{-+}]$, $[\Lambda_{11}^{--}]$	
$[\dagger\Lambda_{12}^{++}]$, $[\dagger\Lambda_{12}^{+-}]$, $[\Lambda_{13}^{+-}]$	Submatrices, the elements of which are defined in equations (202) to (215)
$[\dagger\Lambda_{2,1}^{++}]$, $[\dagger\Lambda_{2,3}^{++}]$, $[\dagger\Lambda_{2,2}^{++}]$	
$[\dagger\Lambda_{2,2}^{++}]$, $[\dagger\Lambda_{2,2}^{+-}]$, $[\dagger\Lambda_{2,1}^{+-}]$	
$[\dagger\Lambda_{2,3}^{+-}]$, $[\Lambda_{13}^{-+}]$, $[\Lambda_{12}^{-+}]$	
$[\dagger\Lambda_{12}^{--}]$, $[\Lambda_{13}^{--}]$	
$\mu_r(\zeta_0)$	The polynomial of degree r in ζ_0 satisfying equation (62)
$\bar{\mu}_r(\zeta_0)$	The polynomial of degree r in ζ_0 satisfying equation (179)
$\mu_1(t)$, $\mu_2(t)$	Doublet strengths, appearing in equations (275) and (276)
$\mu_1(X, Y, t)$, $\mu_2(X, Z, t)$	Doublet strengths, defined in equations (279) and (280)
ν	$= \dfrac{\omega l}{V}$, frequency parameter
ξ	Defined in equation (20)
ξ_0	Defined in equation (22)
$\xi_i^{(l)}$	Chordwise loading points, defined in equation (58)
$\xi_k^{(w)}$	Chordwise velocity points, defined in equations (54) and (55)
ρ_0	Free-stream velocity
ϕ	Velocity potential
χ_0	Defined in equation (288)
ω	Circular frequency

The sign † placed before symbol indicates that the symbol is appropriate to the case of a reflector plate attached to the base of the fin.

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APPENDIX

Derivation of the Integral Equations

Besides the system (x, y, z) of right-handed Cartesian coordinates in Section 2 of the main text, another system (X, Y, Z) of right-handed Cartesian coordinates is introduced which is stationary with respect to the main-stream flow and which coincides with the system (x, y, z) at time $t = 0$. Then at time t the following relationships exist between the coordinates of a point in the two systems

$$\left.\begin{aligned} x &= X + Vt \\ y &= Y \\ z &= Z. \end{aligned}\right\} \tag{268}$$

If the flow of air about the wing is assumed to be irrotational then a velocity potential ϕ exists such that the velocity $\mathbf{q}$ of a fluid particle relative to the (X, Y, Z) coordinate system is given by

$$\mathbf{q} = \mathbf{i}\frac{\partial \phi}{\partial X} + \mathbf{j}\frac{\partial \phi}{\partial Y} + \mathbf{k}\frac{\partial \phi}{\partial Z} \tag{269}$$

where $\mathbf{i}$, $\mathbf{j}$ and $\mathbf{k}$ are unit vectors directed along the X, Y and Z axes respectively.

With the usual assumptions of linearised theory it is found from Euler's equation of motion of inviscid flow, the continuity equation, and the adiabatic equation of state that ϕ satisfies the wave equation

$$\left(\frac{\partial^2}{\partial X^2} + \frac{\partial^2}{\partial Y^2} + \frac{\partial^2}{\partial Z^2}\right)\phi = \frac{1}{a^2}\frac{\partial^2 \phi}{\partial t^2}. \tag{270}$$

The airflow must be tangential to the surfaces of the tailplane and fin and this leads to boundary conditions. Within the accuracy of linearised theory it is permissible to apply the boundary conditions at the mean position of the tailplane in the plane $Z = 0$ and at the mean position of the fin in the plane $Y = 0$ rather than on the surfaces. The conditions may then be written

$$\left(\frac{\partial \phi}{\partial Z}\right)_{Z=0} = w_1(x, y, t) \tag{271}$$

$$\left(\frac{\partial \phi}{\partial Y}\right)_{Y=0} = w_2(x, z, t) \tag{272}$$

over the mean positions of the tailplane and fin respectively, where

$$w_1(x, y, t) = \left(V\frac{\partial}{\partial x} + \frac{\partial}{\partial t}\right) Z(x, y, t) \tag{273}$$

$$w_2(x, z, t) = \left(V\frac{\partial}{\partial x} + \frac{\partial}{\partial t}\right) Y(x, z, t) \tag{274}$$

and $Z(x, y, t)$ and $Y(x, z, t)$ are respectively the normal displacements of a point (x, y) on the surface of the tailplane and of a point (x, z) on the surface of the fin at time t.

The functions

$$\phi_1(X, Y, Z, t) = -\frac{1}{4\pi}\frac{\partial}{\partial Z}\left\{\frac{\mu_1(t - r/a)}{r}\right\} \tag{275}$$

$$\phi_2(X, Y, Z, t) = -\frac{1}{4\pi}\frac{\partial}{\partial Y}\left\{\frac{\mu_2(t - r/a)}{r}\right\} \tag{276}$$

where

$$r = \sqrt{\{(X - X_0)^2 + (Y - Y_0)^2 + (Z - Z_0)^2\}} \tag{277}$$

and $\mu_1(t)$ and $\mu_2(t)$ are arbitrary differentiable functions, satisfy the wave equation (270) and correspond to potentials about doublets of strengths $\mu_1(t)$ and $\mu_2(t)$ at time t placed at the point (X_0, Y_0, Z_0) and pointing in the positive directions of Z and Y respectively.

If there are doublet layers on the planes $Z = Z_1$ and $Y = Y_1$ then the potential of the flow about them, by the principle of superposition, is

$$\phi(X, Y, Z, t) = -\frac{1}{4\pi}\iint_{\substack{X_0, Y_0 \\ \text{plane}}} \left[\frac{\partial}{\partial Z}\left\{\frac{\mu_1(X_0, Y_0, t-r/a)}{r}\right\}\right]_{Z_0=Z_1} dX_0 dY_0 - $$
$$-\frac{1}{4\pi}\iint_{\substack{X_0, Z_0 \\ \text{plane}}} \left[\frac{\partial}{\partial Y}\left\{\frac{\mu_1(X_0, Z_0, t-r/a)}{r}\right\}\right]_{Y_0=Y_1} dX_0 dZ_0 \tag{278}$$

As is usual with doublet layers, there is a discontinuity of potential across the layer. It may be shown, as in Appendix II of Ref. 6, that the discontinuity in potential across a layer at any point on it is of amount equal to the strength of the layer at that point, so that

$$\phi(X_0, Y_0, Z_1+0, t) - \phi(X_0, Y_0, Z_1-0, t) = \mu_1(X_0, Y_0, t) \tag{279}$$
$$\phi(X_0, Y_1+0, Z_0, t) - \phi(X_0, Y_1-0, Z_0, t) = \mu_2(X_0, Z_0, t). \tag{280}$$

The linearised Bernoulli equation is

$$\frac{\partial \phi}{\partial t} = -\frac{(p-p_0)}{\rho_0} \tag{281}$$

where p is the pressure at a point in the flow, p_0 is the free-stream pressure and ρ_0 is the free-stream density.

In linearised theory the wakes shed from the trailing edges are plane and parallel to the main-stream flow. The T-tail and the wakes will be replaced by doublet sheets and the strengths of the sheets will be adjusted so that the boundary conditions (271) and (272) are satisfied on the T-tail surfaces and so that no loading is sustained by the wakes. If there is a reflector surface at the base of the fin then the flow about the T-tail is the same as if the T-tail and the wall were replaced by the T-tail and its image in the wall. The wakes on the T-tail will also have corresponding image wakes.

Let us investigate the velocity field about a surface which sustains a given load distribution. We shall take the tailplane to be this surface. We replace the tailplane and its wake in the plane $Z = 0$ by a doublet sheet of strength $\mu_1(X_0, Y_0, t)$ at the point (X_0, Y_0) at time t.

Let

$$\begin{aligned} L_1(X_0, Y_0, t) &= p(X_0, Y_0, -0, t) - p(X_0, Y_0, +0, t) \\ &= \rho_0 \frac{\partial}{\partial t}[\phi(X_0, Y_0, +0, t) - \phi(X_0, Y_0, -0, t)] \\ &= \rho_0 \frac{\partial}{\partial t}\mu_1(X_0, Y_0, t) \end{aligned} \tag{282}$$

be the pressure force per unit area, or loading, in the positive direction of z at the point (X_0, Y_0) on the tailplane or its wake at time t.

The equations of the leading and trailing edges of the tailplane are respectively

$$x = x_L^{(1)}(y), \qquad z = 0 \tag{283}$$
$$x = x_T^{(1)}(y), \qquad z = 0. \tag{284}$$

Since the two systems of axes coincide at time $t = 0$, the point $(X_0, Y_0, 0)$ is on the leading edge of the tailplane at time

$$t_0^{(1)} = \frac{x_L^{(1)}(Y_0) - X_0}{V} \tag{285}$$

provided

$$|Y_0| < s_1 . \tag{286}$$

Before time $t_0^{(1)}$ the strength of the doublet layer at the point $(X_0, Y_0, 0)$ is zero for the tailplane has not yet reached it. By integrating equation (282) and making use of this last observation we get

$$\mu_1(X_0, Y_0, t) = \frac{1}{\rho_0} \int_{\{x_L^{(1)}(Y_0) - X_0\}/V}^{t} L_1(X_0, Y_0, u) du \tag{287}$$

and then on making the change of variables

$$\chi_0 = X_0 + Vu \tag{288}$$

in the integral in equation (287) we get

$$\mu_1(X_0, Y_0, t) = \frac{1}{\rho_0 V} \int_{x_L^{(1)}(Y_0)}^{X_0 + Vt} L_1 \left\{ X_0, Y_0, \frac{\chi_0 - X_0}{V} \right\} d\chi_0 . \tag{289}$$

The contribution to the velocity potential from the doublet layer is then

$$\phi_1 = -\frac{1}{4\pi\rho_0 V} \frac{\partial}{\partial Z} \iint_{\substack{\text{tailplane} \\ \text{and wake}}} dX_0 dY_0 \left[\frac{1}{r} \int_{x_L^{(1)}(Y_0)}^{X_0 + V(t - r/a)} L_1 \left\{ X_0, Y_0, \frac{\chi_0 - X_0}{V} \right\} d\chi_0 \right]_{Z_0 = 0} . \tag{290}$$

If we write

$$l_1(x_0, y_0, t) = L_1(X_0, Y_0, t) \tag{291}$$

where (x_0, y_0) in the (x, y, z) coordinate system corresponds to (X_0, Y_0) in the (X, Y, Z) coordinate system, then $l_1(x, y_0, t)$ is the loading distribution on the tailplane and its wake as a function of the coordinates fixed relative to the mean position of the tailplane, and it is non zero only on the tailplane.

Since

$$\begin{aligned} L_1 \left\{ X_0, Y_0, \frac{\chi_0 - X_0}{V} \right\} &= l_1 \left\{ X_0 + V \left(\frac{\chi_0 - X_0}{V} \right), Y_0, \frac{\chi_0 - X_0}{V} \right\} \\ &= l_1 \left\{ \chi_0, Y_0, \frac{\chi_0 - x_0}{V} + t \right\} \end{aligned} \tag{292}$$

the expression (290) for ϕ becomes

$$\phi_1(x, y, z, t) = -\frac{1}{4\pi\rho_0 V} \frac{\partial}{\partial z} \int_{-s_1}^{+s_1} dy_0 \int_{x_L^{(1)}(y_0)}^{\infty} \frac{dx_0}{[r]} \int_{x_L^{(1)}(y_0)}^{x_0 - M[r]} l_1 \left\{ \chi_0, y_0, \frac{\chi_0 - x_0}{V} + t \right\} d\chi_0 \tag{293}$$

where

$$[r] = \sqrt{\{(x - x_0)^2 + (y - y_0)^2 + z^2\}} \tag{294}$$

$$M = \frac{V}{a} . \tag{295}$$

The order of integration of the inner two integrals in (293) is to be changed. The expression for ϕ then becomes

$$\phi_1(x, y, z, t) = -\frac{1}{4\pi\rho_0 V}\frac{\partial}{\partial z}\int_{-s_1}^{+s_1} dy_0 \int_{x_2(y_0)}^{\infty} d\chi_0 \times$$

$$\times \int_{x+\{1/(1-M^2)\}\{(\chi_0-x)+M\sqrt{[(\chi_0-x)^2+(1-M^2)\{(y-y_0)^2+z^2\}]}\}}^{\infty} l_1\left\{\chi_0, y_0, \frac{\chi_0 - x_0}{V} + t\right\}\frac{dx_0}{[r]}$$

$$= -\frac{1}{4\pi\rho_0 V}\frac{\partial}{\partial z}\int_{-s_1}^{+s_1} dy_0 \int_{x_L(y_0)}^{\infty} d\chi_0 \times$$

$$\times \int_{\{M/(1-M^2)\}\{M(\chi_0-x)+\sqrt{[(\chi_0-x)^2+(1-M^2)\{(y-y_0)^2+z^2\}]}\}}^{\infty} l_1\left\{\chi_0, y_0, t - \frac{\sigma}{V}\right\}\frac{d\sigma}{\sqrt{\{(\sigma-x+\chi_0)^2 + (y-y_0)^2 + z^2\}}} \quad \ldots (296)$$

If the tailplane is oscillating harmonically, then we may write

$$l_1(x, y, t) = l_1(x, y)e^{i\omega t} \quad (297)$$

where only the real or imaginary part of a complex function represents the pertinent physical quantity. So, using the fact that

$$l_1(x, y, t) = 0 \quad (298)$$

for

$$x > x_T^{(1)}(y) \quad (299)$$

i.e. beyond the trailing edge, the expression for the potential becomes

$$\phi_1(x, y, z, t) = -\frac{e^{i\omega t}}{4\pi\rho_0 V}\frac{\partial}{\partial z}\int_{-s_1}^{+s_1} dy_0 \int_{x_L^{(1)}(y_0)}^{x_T^{(1)}(y_0)} l_1(\chi_0, y_0)d\chi_0 \times$$

$$\times \int_{\{M/(1-M^2)\}\{M(\chi_0-x)+\sqrt{[(\chi_0-x)^2+(1-M^2)\{(y-y_0)^2+z^2\}]}\}}^{\infty} e^{-i\omega\sigma/V}\frac{d\sigma}{\sqrt{\{(\sigma-x+\chi_0)^2 + (y-y_0)^2 + z^2\}}} \quad (300)$$

The contribution $w_{11}(x, y, z, t)$ from the tailplane and its wake to the component of velocity in the positive direction of z is then

$$w_{11}(x, y, z, t) = \left(\frac{\partial\phi_1}{\partial z}\right)$$

$$= -\frac{e^{i\omega t}}{4\pi\rho_0 V}\int_{-s_1}^{+s_1} dy_0 \int_{x_L^{(1)}(y_0)}^{x_T^{(1)}(y_0)} l_1(\chi_0, y_0)d\chi_0 \times$$

$$\times \frac{\partial^2}{\partial z^2}\int_{\{M/(1-M^2)\}\{M(\chi_0-x)+\sqrt{[(\chi_0-x)^2+(1-M^2)\{(y-y_0)^2+z^2\}]}\}}^{\infty} e^{-i\omega\sigma/V}\frac{d\sigma}{\sqrt{\{(\sigma-x+\chi_0)^2 + (y-y_0)^2 + z^2\}}}$$

$$= \frac{e^{i\omega t}}{4\pi\rho_0 V}\iint_{\text{tailplane}} l_1(x_0, y_0)K_3(x-x_0, y-y_0, z)dx_0 dy_0 \quad (301)$$

where

$$K_3(x, y, z) = -\frac{\partial^2}{\partial z^2}\int_{\{M/(1-M^2)\}\{-Mx+R\}}^{\infty} e^{-i\omega\sigma/V}\frac{d\sigma}{\sqrt{\{(\sigma-x)^2+y^2+z^2\}}}$$

$$= -e^{-i\omega x/V}\frac{\partial^2}{\partial z^2}\int_{(-x+MR)/(1-M^2)}^{\infty} e^{-i\omega u/V}\frac{du}{\sqrt{(u^2+y^2+z^2)}}$$

$$= -e^{-i\omega x/V}\frac{\partial}{\partial z}\left[-z\int_{(-x+MR)/(1-M^2)}^{\infty} e^{-i\omega u/V}\frac{du}{(u^2+y^2+z^2)^{3/2}} - \frac{Mz}{R}\frac{\exp\left\{-\frac{i\omega}{V}\left(\frac{-x+MR}{1-M^2}\right)\right\}}{\sqrt{\left\{\left(\frac{-x+MR}{1-M^2}\right)^2+y^2+z^2\right\}}}\right]$$

$$= -e^{-i\omega x/V}\frac{\partial}{\partial z}\left[-z\int_{(-x+MR)/(1-M^2)}^{\infty} e^{-i\omega u/V}\frac{du}{(u^2+y^2+z^2)^{3/2}} - \frac{Mz(Mx+R)}{R(x^2+y^2+z^2)}\exp\left\{-\frac{i\omega}{V}\left(\frac{-x+MR}{1-M^2}\right)\right\}\right]$$

$$= e^{-i\omega x/V}\left[\int_{(-x+MR)/(1-M^2)}^{\infty} e^{-i\omega u/V}\frac{u^2+y^2-2z^2}{(u^2+y^2+z^2)^{5/2}} + \right.$$
$$+\exp\left\{-\frac{i\omega}{V}\left(\frac{-x+MR}{1-M^2}\right)\right\}\left\{\frac{M(Mx+R)}{R(x^2+y^2+z^2)} - \frac{Mz^2(Mx+R)^3}{R(x^2+y^2+z^2)^3} - \right.$$
$$\left.\left. -\frac{M^2(1-M^2)z^2x}{R^3(x^2+y^2+z^2)} - \frac{2Mz^2(Mx+R)}{R(x^2+y^2+z^2)^2} - \frac{i\omega}{V}\frac{M^2z^2(Mx+R)}{R^2(x^2+y^2+z^2)}\right\}\right]. \tag{302}$$

If we take $z = 0$ in equation (302) we get

$$K_1(x, y) = K_3(x, y, 0)$$
$$= e^{-i\omega x/V}\left[\int_{(-x+MR_1)/(1-M^2)}^{\infty} e^{-i\omega u/V}\frac{du}{(u^2+y^2)^{3/2}} + \right.$$
$$\left. + \frac{M(Mx+R_1)}{R_1(x^2+y^2)}\exp\left\{-\frac{i\omega}{V}\left(\frac{-x+MR}{1-M^2}\right)\right\}\right] \tag{303}$$

$K_1(x, y)$ has a non-integrable singularity at $x = 0$, $y = 0$ and the resulting integral in (301) has to be dealt with by Hadamard's 'Finite Part' method of integration when $z = 0$.

The contribution $w_{12}(x, y, z, t)$ from the tailplane and its wake to the component of velocity in the positive direction of y is then

$$w_{12}(x, y, z, t) = \left(\frac{\partial\phi_1}{\partial y}\right)$$
$$= \frac{-e^{i\omega t}}{4\pi\rho_0 V}\int_{-s_1}^{+s_1} dy_0 \int_{x_L^{(1)}(y_0)}^{x_T^{(1)}(y_0)} l_1(\chi_0, y_0)d\chi_0 \times$$
$$\times\frac{\partial^2}{\partial y\partial z}\int_{\{M/(1-M^2)\}\{M(\chi_0-x)+\sqrt{[(\chi_0-x)^2+(1-M^2)\{(y-y_0)^2+z^2\}]}\}}^{\infty} e^{-i\omega\sigma/V}\frac{d\sigma}{\sqrt{\{(\sigma-x+\chi_0)^2+(y-y_0)^2+z^2\}}}$$
$$= \frac{e^{i\omega t}}{4\pi\rho_0 V}\iint_{\text{tailplane}} l_1(x_0, y_0)K_2(x-x_0, z, y_0-y)dx_0dy_0 \tag{304}$$

where

$$K_2(x, z, y) = \frac{\partial^2}{\partial y \partial z} \int_{\{M/(1-M^2)\}\{-Mx+R\}} e^{-i\omega\sigma/V} \frac{d\sigma}{\sqrt{\{(\sigma-x)^2 + y^2 + z^2\}}}$$

$$= e^{-i\omega x/V} \frac{\partial^2}{\partial y \partial z} \int_{(-x+MR)/(1-M^2)} e^{-i\omega u/V} \frac{du}{\sqrt{(u^2+y^2+z^2)}}$$

$$= e^{-i\omega x/V} \frac{\partial}{\partial y} \left[- \int_{(-x+MR)/(1-M^2)}^{\infty} e^{-i\omega u/V} \frac{z\, du}{(u^2+y^2+z^2)^{3/2}} - \right.$$

$$\left. - \frac{Mz(Mx+R)}{R(x^2+y^2+z^2)} \exp\left\{ -\frac{i\omega}{V}\left(\frac{-x+MR}{1-M^2}\right)\right\}\right]$$

$$= e^{-i\omega x/V} \left[\int_{(-x+MR)/(1-M^2)}^{\infty} e^{-i\omega u/V} \frac{3yz\, du}{(u^2+y^2+z^2)^{5/2}} + \right.$$

$$+ yz \exp\left\{-\frac{i\omega}{V}\left(\frac{-x+MR}{1-M^2}\right)\right\} \left\{ \frac{M(Mx+R)^3}{R(x^2+y^2+z^2)^3} + \frac{M^2(1-M^2)x}{R^3(x^2+y^2+z^2)} + \right.$$

$$\left.\left. + \frac{2M(Mx+R)}{R(x^2+y^2+z^2)^2} + \frac{i\omega}{V} \frac{M^2(Mx+R)}{R^2(x^2+y^2+z^2)} \right\}\right]. \tag{305}$$

We have found above the contributions to the velocities in the directions of y and z from a surface in the $Z = 0$ plane which sustains a given load distribution. By similar arguments we can obtain the contributions to the velocities in the directions of y and z from a surface in the $Z = Z_1$ plane which sustains a given load distribution and from a surface in the plane $Y = 0$ which sustains a given load distribution.

If there are several load sustaining surfaces then by the principle of superposition the velocities in the directions of y and z at any point is obtained as the sum of the separate contributions from each of the load sustaining surfaces. Thus the velocity in the z direction at the surface of the isolated tailplane is

$$w_1(x, y, t) = w_1(x, y)e^{i\omega t} \tag{306}$$

where

$$w_1(x, y) = \frac{1}{4\pi\rho_0 V} \iint_{\text{tailplane}} l_1(x_0, y_0)K_1(x-x_0, y-y_0)dx_0dy_0 +$$

$$+ \frac{1}{4\pi\rho_0 V} \iint_{\text{fin}} l_2(x_0, z_0)K_2(x-x_0, y, z_0)dx_0dz_0. \tag{307}$$

The velocity in the y direction at the surface of the fin is

$$w_2(x, z, t) = w_2(x, z)e^{i\omega t} \tag{308}$$

where

$$w_2(x, z) = \frac{1}{4\pi\rho_0 V} \iint_{\text{tailplane}} l_1(x_0, y_0)K_2(x-x_0, z, y_0)dx_0dy_0 +$$

$$+ \frac{1}{4\pi\rho_0 V} \iint_{\text{fin}} l_2(x_0, z_0)K_1(x-x_0, z-z_0)dx_0dz_0. \tag{309}$$

If equations (5), (6), (7) and (8) of the main text are used in the integral equations (307) and (309), then the pair of integral equations (9) and (10) of the main text are obtained.

When there is a reflector plate at the base of the fin, then, as has already been mentioned we replace the T-tail and reflector plate by the T-tail and its image in the plane wall.

If $l_1(x, y, t)$ is the loading at the point $(x, y, 0)$ on the tailplane, then the loading at the point $(x, y, 2s_2)$ on the image tailplane is $-l_1(x, y, t)$. If $l_2(x, z, t)$ is the loading at the point $(x, z, 0)$ on the fin, then $l_2(x, 2s_2-z, t)$ is the loading at the point $(x, 2s_2-z, 0)$ on the fin. The velocity in the direction of z on the surface of the tailplane and the velocity in the direction of y on the surface of the fin are then obtained by summing the contributions from each of the load sustaining surfaces on the T-tail and its image. This leads to

$$
\begin{aligned}
w_1(x, y) = \frac{1}{4\pi\rho_0 V} & \iint\limits_{\text{tailplane}} l_1(x_0, y_0)K_1(x-x_0, y-y_0)dx_0 dy_0 - \\
& - \frac{1}{4\pi\rho_0 V} \iint\limits_{\text{tailplane}} l_1(x_0, y_0)K_3(x-x_0, y-y_0, 2s_2)dx_0 dy_0 + \\
& + \frac{1}{4\pi\rho_0 V} \iint\limits_{\text{fin}} l_2(x_0, z_0)K_2(x-x_0, y, z_0)dx_0 dz_0 + \\
& + \frac{1}{4\pi\rho_0 V} \iint\limits_{\text{fin}} l_2(x_0, z_0)K_2(x-x_0, y, 2s_2-z_0)dx_0 dz_0
\end{aligned} \tag{310}
$$

and

$$
\begin{aligned}
w_2(x, z) = \frac{1}{4\pi\rho_0 V} & \iint\limits_{\text{tailplane}} \lambda_1(x_0, y_0)K_2(x-x_0, z, y_0)dx_0 dy_0 - \\
& - \frac{1}{4\pi\rho_0 V} \iint\limits_{\text{tailplane}} \lambda_1(x_0, y_0)K_2(x-x_0, 2s_2-z, y_0)dx_0 dy_0 + \\
& + \frac{1}{4\pi\rho_0 V} \iint\limits_{\text{fin}} \lambda_2(x_0, z_0)K_1(x-x_0, z-z_0)dx_0 dz_0 + \\
& + \frac{1}{4\pi\rho_0 V} \iint\limits_{\text{fin}} \lambda_2(x_0, z_0)K_1(x-x_0, z+z_0-2s_2)dx_0 dz_0 .
\end{aligned} \tag{311}
$$

If equations (5), (6), (7) and (8) of the main text are used in the integral equations (310) and (311), then the pair of integral equations (170) and (171) of the main text are obtained.

TABLE 1

Values of $\xi_k^{(l)}$, $\xi_k^{(w)}$ and $H_k^{(n)}$

$n = 2$

k =	1	2
$\xi_k^{(l)}$ =	0·095492	0·654508
$\xi_k^{(w)}$ =	0·345492	0·904509
$H_k^{(n)}$ =	0·369316	0·597566

$n = 3$

k =	1	2	3
$\xi_k^{(l)}$ =	0·049516	0·388740	0·811745
$\xi_k^{(w)}$ =	0·188255	0·611260	0·950484
$H_k^{(n)}$ =	0·194727	0·437547	0·350885

$n = 4$

k =	1	2	3	4
$\xi_k^{(l)}$ =	0·030154	0·250000	0·586824	0·883022
$\xi_k^{(w)}$ =	0·116978	0·413176	0·750000	0·969846
$H_k^{(n)}$ =	0·119388	0·302300	0·343763	0·224375

TABLE 2

Values of η_j, $G_j^{(m)}$, ζ_j and $\bar{G}_j^{(m)}$

$m = 2$

j =	1	2
η_j =	0·178838	0·710051
$G_j^{(m)}$ =	0·429397	0·515454
ζ_j =	0·211325	0·788675
$\bar{G}_j^{(m)}$ =	0·500000	0·500000

$m = 3$

j =	1	2	3
η_j =	0·099194	0·450132	0·835290
$G_j^{(m)}$ =	0·245790	0·414821	0·309928
ζ_j =	0·112702	0·500000	0·887298
$\bar{G}_j^{(m)}$ =	0·277778	0·444444	0·277778

$m = 4$

j =	1	2	3	4
η_j =	0·062666	0·301052	0·623775	0·894860
$G_j^{(m)}$ =	0·157373	0·302055	0·319702	0·202559
ζ_j =	0·069432	0·330009	0·669991	0·930568
$\bar{G}_j^{(m)}$ =	0·173927	0·326073	0·326073	0·173927

TABLE 2—*continued*

$m = 5$

j =	1	2	3	4	5
η_j =	0·043069	0·213120	0·466878	0·730539	0·927346
$G_j^{(m)}$ =	0·108913	0·222795	0·272040	0·242100	0·141674
ζ_j =	0·046910	0·230765	0·500000	0·769235	0·953090
$\bar{G}_j^{(m)}$ =	0·118463	0·239314	0·284444	0·239314	0·118463

$m = 6$

j =	1	2	3	4	5	6
η_j =	0·031384	0·158013	0·357473	0·587387	0·798854	0·946889
$G_j^{(m)}$ =	0·079687	0·169061	0·222636	0·228864	0·186408	0·104298
ζ_j =	0·033765	0·169395	0·380690	0·619310	0·830605	0·966235
$\bar{G}_j^{(m)}$ =	0·085662	0·180381	0·233957	0·233957	0·180381	0·085662

TABLE 3

Values of $P_{j,r}^{(m)}$, $Q_{j,r}^{(m)}$, $\bar{P}_{j,r}^{(m)}$ *and* $\bar{Q}_{j,r}^{(m)}$

$m = 2$

$P_{j,r}^{(m)}$

r	$j = 1$	2
1	−9·518531	2·548367
2	2·807973	−9·417170

$Q_{j,r}^{(m)}$

r	$j = 1$	2
1	4·567635	−0·031926
2	0·563100	0·237099

TABLE 3—*continued*

$\bar{P}_{j,r}^{(m)}$

r	$j = 1$	2
1	−8·281038	2·281038
2	2·281038	−8·281038

$\bar{Q}_{j,r}^{(m)}$

r	$j = 1$	2
1	0·190525	0·518351
2	0·164525	3·741983

$m = 3$

$P_{j,r}^{(m)}$

r	$j = 1$	2	3
1	−16·398798	4·070202	1·341310
2	3·638673	−11·581868	4·008660
3	0·126345	5·469741	−15·853708

$Q_{j,r}^{(m)}$

r	$j = 1$	2	3
1	8·430222	−0·223962	1·133921
2	0·833395	0·486518	0·202195
3	0·282898	0·248674	0·112578

TABLE 3—*continued*

$\bar{P}_{j,r}^{(m)}$

r	$j = 1$	2	3
1	−14·658324	3·988877	0·669448
2	3·333333	−10·666667	3·333333
3	0·669448	3·988877	−14·658324

$\bar{Q}_{j,r}^{(m)}$

r	$j = 1$	2	3
1	0·088982	0·228858	0·279318
2	0·150672	0·425942	0·756719
3	0·607306	0·056694	7·310269

$m = 4$

$P_{j,r}^{(m)}$

r	$j = 1$	2	3	4
1	−25·451941	5·932427	3·104863	−0·956458
2	5·022717	−15·825107	5·770737	0·189552
3	0·152620	5·631828	−15·308302	5·445832
4	0·372449	0·095233	8·627186	−24·322979

$Q_{j,r}^{(m)}$

r	$j = 1$	2	3	4
1	13·475880	−0·493871	2·447914	−0·642971
2	1·211599	0·799357	0·400114	0·127882
3	0·334841	0·351460	0·206935	0·086985
4	0·171784	0·210896	0·138913	0·063072

TABLE 3—*continued*

$\bar{P}_{j,r}^{(m)}$

r	$j = 1$	2	3	4
1	−23·188159	5·848081	2·043919	−0·181066
2	4·644974	−14·652359	5·277936	0·206675
3	0·206675	5·277936	−14·652359	4·644974
4	−0·181066	2·043919	5·848081	−23·188159

$\bar{Q}_{j,r}^{(m)}$

r	$j = 1$	2	3	4
1	0·050135	0·127515	0·204893	0·174086
2	0·067554	0·182648	0·324604	0·318946
3	0·104272	0·343017	0·721327	1·109725
4	−0·152388	1·732847	−0·172940	12·060018

$m = 5$

$P_{j,r}^{(m)}$

r	$j = 1$	2	3	4	5
1	−36·654032	8·204778	5·284580	−2·441495	1·718142
2	6·812513	−21·395018	7·842744	0·415356	0·345390
3	0·193307	6·692443	−17·946446	6·828673	0·070062
4	0·371843	0·111502	7·728426	−20·312798	7·278869
5	0·058549	0·797841	0·067790	12·451713	−34·806374

TABLE 3—*continued*

$Q_{j,r}^{(m)}$

r	$j = 1$	2	3	4	5
1	19·706913	−0·832375	4·028973	−1·613904	1·092911
2	1·686414	1·181859	0·627216	0·244812	0·122642
3	0·419532	0·480300	0·313584	0·167662	0·073508
4	0·182049	0·250043	0·189902	0·113274	0·051617
5	0·115671	0·171255	0·139994	0·088044	0·041206

$\overline{P}_{j,r}^{(m)}$

r	$j = 1$	2	3	4	5
1	−33·861386	8·062735	3·965818	−1·174384	0·640618
2	6·349251	−19·949937	7·332571	0·333492	0·301223
3	0·198913	6·334420	−17·166667	6·334420	0·198913
4	0·301227	0·333492	7·332571	−19·949937	6·349251
5	0·640618	−1·174384	3·965818	8·062735	−33·861386

$\overline{Q}_{j,r}^{(m)}$

r	$j = 1$	2	3	4	5
1	0·032612	0·080647	0·134758	0·170711	0·118482
2	0·039962	0·101021	0·176710	0·239171	0·177913
3	0·056323	0·147837	0·285653	0·446802	0·396718
4	0·089206	0·223424	0·561607	1·085863	1·560804
5	0·442992	−0·775040	3·183321	−0·483796	17·994713

TABLE 3—*continued*

$m = 6$

$P_{j,r}^{(m)}$

r	$j = 1$	2	3	4	5	6
1	−49·997973	10·900261	7·879665	−4·284168	3·785728	−1·629086
2	8·974522	−28·147670	10·305251	0·680211	0·449913	0·229954
3	0·244704	8·200450	−21·895089	8·444675	0·147159	0·525785
4	0·412270	0·135013	8·296396	−21·458991	8·265228	0·036664
5	0·059968	0·741709	0·078680	10·170993	−26·427640	9·495349
6	0·145874	0·061559	1·221566	0·049932	16·977256	−47·296467

$Q_{j,r}^{(m)}$

r	$j = 1$	2	3	4	5	6
1	27·124013	−1·237307	5·895373	−2·570997	2·328977	−0·906849
2	2·254827	1·635617	0·890982	0·370479	0·231522	0·071421
3	0·527679	0·634688	0·437271	0·254895	0·140445	0·060773
4	0·208168	0·304171	0·249505	0·165717	0·097088	0·044262
5	0·115613	0·184630	0·166528	0·119077	0·073039	0·034215
6	0·083268	0·138478	0·130865	0·097216	0·061170	0·029079

TABLE 3—*continued*

$\bar{P}_{j,r}^{(m)}$

r	$j = 1$	2	3	4	5	6
1	−46·674227	10·678482	6·366758	−2·701571	2·117525	−0·438162
2	8·424154	−26·429205	9·732129	0·529855	0·507723	0·128051
3	0·236004	7·783803	−20·809157	7·971187	0·195836	0·380819
4	0·380819	0·195836	7·971187	−20·809157	7·783803	0·236004
5	0·128051	0·507723	0·529854	9·732129	−26·429205	8·424154
6	−0·438162	2·117525	−2·701571	6·366758	10·678482	−46·674227

$\bar{Q}_{j,r}^{(m)}$

r	$j = 1$	2	3	4	5	6
1	0·022937	0·055873	0·093057	0·128966	0·139861	0·085665
2	0·026529	0·065373	0·111273	0·159473	0·180365	0·114661
3	0·034060	0·085852	0·152412	0·234064	0·289880	0·200886
4	0·047050	0·123506	0·232928	0·404973	0·594749	0·499325
5	0·062006	0·195600	0·347214	0·815892	1·521543	2·105954
6	−0·237792	1·373232	−1·679025	4·943319	−0·866137	25·115319

TABLE 4

Expressions for $g_j^{(m)}(\eta)$ and $\bar{g}_j^{(m)}(\zeta)$

$m = 2$

$$g_1^{(2)}(\eta) = (1\cdot475049 - 2\cdot077385\eta)\sqrt{(1-\eta)}$$
$$g_2^{(2)}(\eta) = (-0\cdot625217 + 3\cdot495993\eta)\sqrt{(1-\eta)}$$
$$\bar{g}_1^{(2)}(\zeta) = (1\cdot366025 - 1\cdot732051\zeta)$$
$$\bar{g}_2^{(2)}(\zeta) = (-0\cdot366025 + 1\cdot732051\zeta)$$

$m = 3$

$$g_1^{(3)}(\eta) = (1\cdot533547 - 5\cdot242835\eta + 4\cdot078690\eta^2)\sqrt{(1-\eta)}$$
$$g_2^{(3)}(\eta) = (-0\cdot826657 + 9\cdot323386\eta - 9\cdot977043\eta^2)\sqrt{(1-\eta)}$$
$$g_3^{(3)}(\eta) = (0\cdot388054 - 4\cdot774151\eta + 8\cdot690931\eta^2)\sqrt{(1-\eta)}$$
$$\bar{g}_1^{(3)}(\zeta) = (1\cdot478831 - 4\cdot624328\zeta + 3\cdot333333\zeta^2)$$
$$\bar{g}_2^{(3)}(\zeta) = (-0\cdot666667 + 6\cdot666667\zeta - 6\cdot666667\zeta^2)$$
$$\bar{g}_3^{(3)}(\zeta) = (0\cdot187836 - 2\cdot042339\zeta + 3\cdot333333\zeta^2)$$

$m = 4$

$$g_1^{(4)}(\eta) = (1\cdot559280 - 9\cdot421666\eta + 16\cdot884801\eta^2 - 9\cdot278958\eta^3)\sqrt{(1-\eta)}$$
$$g_2^{(4)}(\eta) = (-0\cdot915869 + 17\cdot106893\eta - 41\cdot403255\eta^2 + 26\cdot183033\eta^3)\sqrt{(1-\eta)}$$
$$g_3^{(4)}(\eta) = (0\cdot560687 - 11\cdot436254\eta + 41\cdot799757\eta^2 - 33\cdot211906\eta^3)\sqrt{(1-\eta)}$$
$$g_4^{(4)}(\eta) = (-0\cdot270920 + 5\cdot657485\eta - 22\cdot733949\eta^2 + 23\cdot021879\eta^3)\sqrt{(1-\eta)}$$
$$\bar{g}_1^{(4)}(\zeta) = (1\cdot526788 - 8\cdot546023\zeta + 14\cdot325858\zeta^2 - 7\cdot420540\zeta^3)$$
$$\bar{g}_2^{(4)}(\zeta) = (-0\cdot813632 + 13\cdot807167\zeta - 31\cdot388222\zeta^2 + 18\cdot795449\zeta^3)$$
$$\bar{g}_3^{(4)}(\zeta) = (0\cdot400762 - 7\cdot417070\zeta + 24\cdot998126\zeta^2 - 18\cdot795449\zeta^3)$$
$$\bar{g}_4^{(4)}(\zeta) = (-0\cdot113917 + 2\cdot155927\zeta - 7\cdot935762\zeta^2 + 7\cdot420540\zeta^3)$$

$m = 5$

$$g_1^{(5)}(\eta) = (1\cdot572822 - 14\cdot597797\eta + 44\cdot433144\eta^2 - 54\cdot549321\eta^3 + 23\cdot332778\eta^4)\sqrt{(1-\eta)}$$
$$g_2^{(5)}(\eta) = (-0\cdot962982 + 26\cdot778413\eta - 109\cdot077332\eta^2 + 153\cdot247397\eta^3 - 70\cdot691533\eta^4)\sqrt{(1-\eta)}$$
$$g_3^{(5)}(\eta) = (0\cdot652261 - 19\cdot801420\eta + 116\cdot576273\eta^2 - 200\cdot774720\eta^3 + 104\cdot893899\eta^4)\sqrt{(1-\eta)}$$
$$g_4^{(5)}(\eta) = (-0\cdot414763 + 12\cdot912033\eta - 83\cdot423700\eta^2 + 172\cdot250664\eta^3 - 104\cdot368207\eta^4)\sqrt{(1-\eta)}$$
$$g_5^{(5)}(\eta) = (0\cdot202926 - 6\cdot376286\eta + 42\cdot587490\eta^2 - 94\cdot222069\eta^3 + 64\cdot819543\eta^4)\sqrt{(1-\eta)}$$
$$\bar{g}_1^{(5)}(\zeta) = (1\cdot551408 - 13\cdot470285\zeta + 38\cdot644499\zeta^2 - 44\cdot988985\zeta^3 + 18\cdot339721\zeta^4)$$
$$\bar{g}_2^{(5)}(\zeta) = (-0\cdot893158 + 22\cdot924334\zeta - 88\cdot222811\zeta^2 + 117\cdot863415\zeta^3 - 51\cdot939721\zeta^4)$$
$$\bar{g}_3^{(5)}(\zeta) = (0\cdot533333 - 14\cdot933333\zeta + 82\cdot133333\zeta^2 - 134\cdot400000\zeta^3 + 67\cdot200000\zeta^4)$$
$$\bar{g}_4^{(5)}(\zeta) = (-0\cdot267942 + 7\cdot689927\zeta - 46\cdot270892\zeta^2 + 89\cdot895469\zeta^3 - 51\cdot939721\zeta^4)$$
$$\bar{g}_5^{(5)}(\zeta) = (0\cdot076359 - 2\cdot210643\zeta + 13\cdot715871\zeta^2 - 28\cdot369899\zeta^3 + 18\cdot339721\zeta^4)$$

TABLE 4—*continued*

$m = 6$

$$g_1^{(6)}(\eta) = (1\cdot580813 - 20\cdot766095\eta + 94\cdot142330\eta^2 - 191\cdot546669\eta^3 + 179\cdot426751\eta^4 - 62\cdot987336\eta^5)\sqrt{(1-\eta)}$$

$$g_2^{(6)}(\eta) = (-0\cdot990835 + 38\cdot317004\eta - 231\cdot250693\eta^2 + 536\cdot758939\eta^3 - 541\cdot064783\eta^4 + 198\cdot775681\eta^5)\sqrt{(1-\eta)}$$

$$g_3^{(6)}(\eta) = (0\cdot706560 - 29\cdot818666\eta + 254\cdot408849\eta^2 - 714\cdot680690\eta^3 + 808\cdot902634\eta^4 - 320\cdot671491\eta^5)\sqrt{(1-\eta)}$$

$$g_4^{(6)}(\eta) = (-0\cdot500279 + 21\cdot660840\eta - 202\cdot319164\eta^2 + 665\cdot498511\eta^3 - 855\cdot333967\eta^4 + 373\cdot082630\eta^5)\sqrt{(1-\eta)}$$

$$g_5^{(6)}(\eta) = (0\cdot323501 - 14\cdot152607\eta + 136\cdot957984\eta^2 - 481\cdot625472\eta^3 + 682\cdot833293\eta^4 - 328\cdot104504\eta^5)\sqrt{(1-\eta)}$$

$$g_6^{(6)}(\eta) = (-0\cdot159325 + 7\cdot001364\eta - 68\cdot783206\eta^2 + 248\cdot996133\eta^3 - 370\cdot261279\eta^4 + 191\cdot536478\eta^5)\sqrt{(1-\eta)}$$

$$\bar{g}_1^{(6)}(\zeta) = (1\cdot565673 - 19\cdot388900\zeta + 83\cdot356172\zeta^2 - 161\cdot633449\zeta^3 + 144\cdot893361\zeta^4 - 48\cdot847570\zeta^5)$$

$$\bar{g}_2^{(6)}(\zeta) = (-0\cdot940463 + 33\cdot947557\zeta - 194\cdot590041\zeta^2 + 431\cdot244211\zeta^3 - 416\cdot671896\zeta^4 + 147\cdot202432\zeta^5)$$

$$\bar{g}_3^{(6)}(\zeta) = (0\cdot616930 - 24\cdot290507\zeta + 195\cdot304165\zeta^2 - 523\cdot416261\zeta^3 + 568\cdot416487\zeta^4 - 217\cdot010043\zeta^5)$$

$$\bar{g}_4^{(6)}(\zeta) = (-0\cdot379228 + 15\cdot315224\zeta - 134\cdot546123\zeta^2 + 419\cdot850741\zeta^3 - 516\cdot633728\zeta^4 + 217\cdot010043\zeta^5)$$

$$\bar{g}_5^{(6)}(\zeta) = (0\cdot191800 - 7\cdot824684\zeta + 71\cdot135538\zeta^2 - 236\cdot580950\zeta^3 + 319\cdot340266\zeta^4 - 147\cdot202432\zeta^5)$$

$$\bar{g}_6^{(6)}(\zeta) = (-0\cdot054713 + 2\cdot241310\zeta - 20\cdot659712\zeta^2 + 70\cdot535708\zeta^3 - 99\cdot344492\zeta^4 + 48\cdot847570\zeta^5)$$

TABLE 5

Values of $h_i^{(n)\prime}(\xi_k^{(w)})$, $h_i^{(n)}(\xi_k^{(w)})$ and $h_i^{(1,n)}(\xi_k^{(w)})$

$n = 2$

$h_1^{(2)\prime}(\xi_1^{(w)}) = -1{\cdot}346625$	$h_1^{(2)}(\xi_1^{(w)}) = 0{\cdot}247214$	$h_1^{(1,2)}(\xi_1^{(w)}) = 0{\cdot}349981$
$h_2^{(2)\prime}(\xi_1^{(w)}) = 1{\cdot}515542$	$h_2^{(2)}(\xi_1^{(w)}) = 0{\cdot}847214$	$h_2^{(1,2)}(\xi_1^{(w)}) = 0{\cdot}036772$
$h_1^{(2)\prime}(\xi_2^{(w)}) = 0{\cdot}084458$	$h_1^{(2)}(\xi_2^{(w)}) = -0{\cdot}047214$	$h_1^{(1,2)}(\xi_2^{(w)}) = 0{\cdot}372707$
$h_2^{(2)\prime}(\xi_2^{(w)}) = -2{\cdot}946625$	$h_2^{(2)}(\xi_2^{(w)}) = 0{\cdot}647214$	$h_2^{(1,2)}(\xi_2^{(w)}) = 0{\cdot}555307$

$n = 3$

$h_1^{(3)\prime}(\xi_1^{(w)}) = -2{\cdot}260026$	$h_1^{(2)}(\xi_1^{(w)}) = 0{\cdot}229125$	$h_1^{(1,3)}(\xi_1^{(w)}) = 0{\cdot}184626$
$h_2^{(3)\prime}(\xi_1^{(w)}) = 2{\cdot}327923$	$h_2^{(2)}(\xi_1^{(w)}) = 0{\cdot}998274$	$h_2^{(1,3)}(\xi_1^{(w)}) = 0{\cdot}026019$
$h_3^{(3)\prime}(\xi_1^{(w)}) = 0{\cdot}391361$	$h_3^{(2)}(\xi_1^{(w)}) = -0{\cdot}371982$	$h_3^{(1,3)}(\xi_1^{(w)}) = -0{\cdot}003760$
$h_1^{(3)\prime}(\xi_2^{(w)}) = 0{\cdot}081594$	$h_1^{(2)}(\xi_2^{(w)}) = -0{\cdot}031405$	$h_1^{(1,3)}(\xi_2^{(w)}) = 0{\cdot}196205$
$h_2^{(3)\prime}(\xi_2^{(w)}) = -2{\cdot}651387$	$h_2^{(2)}(\xi_2^{(w)}) = 0{\cdot}499137$	$h_2^{(1,3)}(\xi_2^{(w)}) = 0{\cdot}410162$
$h_3^{(3)\prime}(\xi_2^{(w)}) = 2{\cdot}677075$	$h_3^{(2)}(\xi_2^{(w)}) = 0{\cdot}641994$	$h_3^{(1,3)}(\xi_2^{(w)}) = 0{\cdot}023056$
$h_1^{(3)\prime}(\xi_3^{(w)}) = -0{\cdot}025688$	$h_1^{(2)}(\xi_3^{(w)}) = 0{\cdot}015702$	$h_1^{(1,3)}(\xi_3^{(w)}) = 0{\cdot}194119$
$h_2^{(3)\prime}(\xi_3^{(w)}) = 0{\cdot}365672$	$h_2^{(2)}(\xi_3^{(w)}) = -0{\cdot}158559$	$h_2^{(1,3)}(\xi_3^{(w)}) = 0{\cdot}443600$
$h_3^{(3)\prime}(\xi_3^{(w)}) = -5{\cdot}753671$	$h_3^{(2)}(\xi_3^{(w)}) = 0{\cdot}743964$	$h_3^{(1,3)}(\xi_3^{(w)}) = 0{\cdot}325165$

$n = 4$

$h_1^{(4)\prime}(\xi_1^{(w)}) = -3{\cdot}509302$	$h_1^{(4)}(\xi_1^{(w)}) = 0{\cdot}222222$	$h_1^{(1,4)}(\xi_1^{(w)}) = 0{\cdot}113207$
$h_2^{(4)\prime}(\xi_1^{(w)}) = 3{\cdot}429725$	$h_2^{(4)}(\xi_1^{(w)}) = 1{\cdot}057505$	$h_2^{(1,4)}(\xi_1^{(w)}) = 0{\cdot}017865$
$h_3^{(4)\prime}(\xi_1^{(w)}) = 1{\cdot}119379$	$h_3^{(4)}(\xi_1^{(w)}) = -0{\cdot}521621$	$h_3^{(1,4)}(\xi_1^{(w)}) = -0{\cdot}003375$
$h_4^{(4)\prime}(\xi_1^{(w)}) = -0{\cdot}760505$	$h_4^{(4)}(\xi_1^{(w)}) = 0{\cdot}256156$	$h_4^{(1,4)}(\xi_1^{(w)}) = 0{\cdot}000879$
$h_1^{(4)\prime}(\xi_2^{(w)}) = 0{\cdot}102414$	$h_1^{(4)}(\xi_2^{(w)}) = -0{\cdot}026803$	$h_1^{(1,4)}(\xi_2^{(w)}) = 0{\cdot}120262$
$h_2^{(4)\prime}(\xi_2^{(w)}) = -3{\cdot}366212$	$h_2^{(4)}(\xi_2^{(w)}) = 0{\cdot}458706$	$h_2^{(1,4)}(\xi_2^{(w)}) = 0{\cdot}283682$
$h_3^{(4)\prime}(\xi_2^{(w)}) = 3{\cdot}415907$	$h_3^{(4)}(\xi_2^{(w)}) = 0{\cdot}750974$	$h_3^{(1,4)}(\xi_2^{(w)}) = 0{\cdot}021663$
$h_4^{(4)\prime}(\xi_2^{(w)}) = -0{\cdot}204048$	$h_4^{(4)}(\xi_2^{(w)}) = -0{\cdot}222222$	$h_4^{(1,4)}(\xi_2^{(w)}) = -0{\cdot}002844$
$h_1^{(4)\prime}(\xi_3^{(w)}) = -0{\cdot}021762$	$h_1^{(4)}(\xi_3^{(w)}) = 0{\cdot}010585$	$h_1^{(1,4)}(\xi_3^{(w)}) = 0{\cdot}119090$
$h_2^{(4)\prime}(\xi_3^{(w)}) = 0{\cdot}296296$	$h_2^{(4)}(\xi_3^{(w)}) = -0{\cdot}111111$	$h_2^{(1,4)}(\xi_3^{(w)}) = 0{\cdot}305700$
$h_3^{(4)\prime}(\xi_3^{(w)}) = -4{\cdot}030576$	$h_3^{(4)}(\xi_3^{(w)}) = 0{\cdot}593166$	$h_3^{(1,4)}(\xi_3^{(w)}) = 0{\cdot}321609$
$h_4^{(4)\prime}(\xi_3^{(w)}) = 3{\cdot}991186$	$h_4^{(4)}(\xi_3^{(w)}) = 0{\cdot}582580$	$h_4^{(1,4)}(\xi_3^{(w)}) = 0{\cdot}015019$
$h_1^{(4)\prime}(\xi_4^{(w)}) = 0{\cdot}011265$	$h_1^{(4)}(\xi_4^{(w)}) = -0{\cdot}007131$	$h_1^{(1,4)}(\xi_4^{(w)}) = 0{\cdot}119558$
$h_2^{(4)\prime}(\xi_4^{(w)}) = -0{\cdot}129271$	$h_2^{(4)}(\xi_4^{(w)}) = 0{\cdot}067868$	$h_2^{(1,4)}(\xi_4^{(w)}) = 0{\cdot}300683$
$h_3^{(4)\prime}(\xi_4^{(w)}) = 0{\cdot}694747$	$h_3^{(4)}(\xi_4^{(w)}) = -0{\cdot}222222$	$h_3^{(1,4)}(\xi_4^{(w)}) = 0{\cdot}348976$
$h_4^{(4)\prime}(\xi_4^{(w)}) = -9{\cdot}444874$	$h_4^{(4)}(\xi_4^{(w)}) = 0{\cdot}784909$	$h_4^{(1,4)}(\xi_4^{(w)}) = 0{\cdot}207712$

TABLE 6

Expressions for $h_i^{(n)}(\xi)$ and $h_i^{(1,n)}(\xi)$

$n = 2$

$$h_1^{(2)}(\xi) = (0\cdot380423 - 0\cdot581234\xi)\sqrt{\left(\frac{1-\xi}{\xi}\right)}$$

$$h_2^{(2)}(\xi) = (-0\cdot235114 + 2\cdot462147\xi)\sqrt{\left(\frac{1-\xi}{\xi}\right)}$$

$$h_1^{(1,2)}(\xi) = 0\cdot235114 \sin^{-1}\sqrt{\xi} + (0\cdot525731 - 0\cdot290617\xi)\sqrt{\{\xi(1-\xi)\}}$$

$$h_2^{(1,2)}(\xi) = 0\cdot380423 \sin^{-1}\sqrt{\xi} + (-0\cdot850651 + 1\cdot231073\xi)\sqrt{\{\xi(1-\xi)\}}$$

$n = 3$

$$h_1^{(3)}(\xi) = (0\cdot278551 - 1\cdot059699\xi + 0\cdot882727\xi^2)\sqrt{\left(\frac{1-\xi}{\xi}\right)}$$

$$h_2^{(3)}(\xi) = (-0\cdot223380 + 4\cdot786503\xi - 5\cdot557555\xi^2)\sqrt{\left(\frac{1-\xi}{\xi}\right)}$$

$$h_3^{(3)}(\xi) = (0\cdot123967 - 2\cdot282249\xi + 6\cdot440282\xi^2)\sqrt{\left(\frac{1-\xi}{\xi}\right)}$$

$$h_1^{(1,3)}(\xi) = 0\cdot123967 \sin^{-1}\sqrt{\xi} + (0\cdot433135 - 0\cdot603410\xi + 0\cdot294242\xi^2)\sqrt{\{\xi(1-\xi)\}}$$

$$h_2^{(1,3)}(\xi) = 0\cdot278551 \sin^{-1}\sqrt{\xi} + (-0\cdot725313 + 2\cdot856381\xi - 1\cdot852518\xi^2)\sqrt{\{\xi(1-\xi)\}}$$

$$h_3^{(1,3)}(\xi) = 0\cdot223380 \sin^{-1}\sqrt{\xi} + (0\cdot024553 - 1\cdot947933\xi + 2\cdot146761\xi^2)\sqrt{\{\xi(1-\xi)\}}$$

$n = 4$

$$h_1^{(4)}(\xi) = (0\cdot218846 - 1\cdot496156\xi + 2\cdot905421\xi^2 - 1\cdot689349\xi^3)\sqrt{\left(\frac{1-\xi}{\xi}\right)}$$

$$h_2^{(4)}(\xi) = (-0\cdot192450 + 6\cdot928203\xi - 18\cdot475209\xi^2 + 12\cdot316806\xi^3)\sqrt{\left(\frac{1-\xi}{\xi}\right)}$$

$$h_3^{(4)}(\xi) = (0\cdot142842 - 5\cdot470253\xi + 24\cdot960213\xi^2 - 21\cdot458674\xi^3)\sqrt{\left(\frac{1-\xi}{\xi}\right)}$$

$$h_4^{(4)}(\xi) = (-0\cdot076004 + 2\cdot954106\xi - 14\cdot895625\xi^2 + 17\cdot181092\xi^3)\sqrt{\left(\frac{1-\xi}{\xi}\right)}$$

$$h_1^{(1,4)}(\xi) = 0\cdot076004 \sin^{-1}\sqrt{\xi} + (0\cdot361688 - 0\cdot902209\xi + 1\cdot038863\xi^2 - 0\cdot422337\xi^3)\sqrt{\{\xi(1-\xi)\}}$$

$$h_2^{(1,4)}(\xi) = 0\cdot192450 \sin^{-1}\sqrt{\xi} + (-0\cdot577350 + 4\cdot362202\xi - 6\cdot671603\xi^2 + 3\cdot079201\xi^3)\sqrt{\{\xi(1-\xi)\}}$$

$$h_3^{(1,4)}(\xi) = 0\cdot218846 \sin^{-1}\sqrt{\xi} + (0\cdot066837 - 3\cdot697505\xi + 9\cdot214182\xi^2 - 5\cdot364668\xi^3)\sqrt{\{\xi(1-\xi)\}}$$

$$h_4^{(1,4)}(\xi) = 0\cdot142842 \sin^{-1}\sqrt{\xi} + (-0\cdot294851 + 1\cdot823507\xi - 5\cdot681087\xi^2 + 4\cdot295273\xi^3)\sqrt{\{\xi(1-\xi)\}}$$

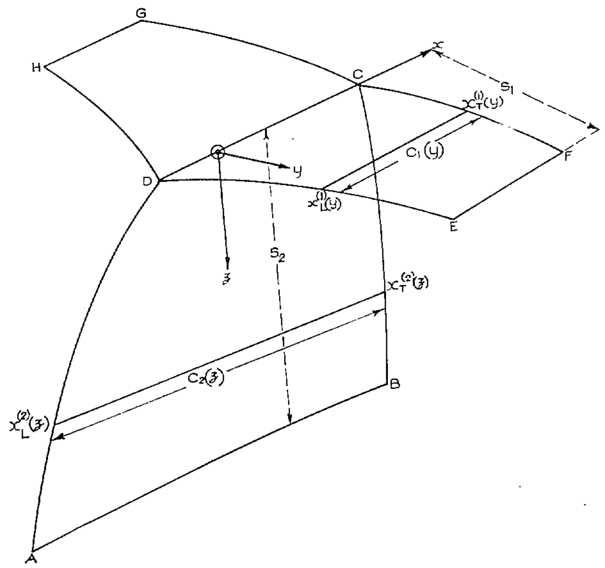

FIG. 1. Diagram of the isolated T-tail.

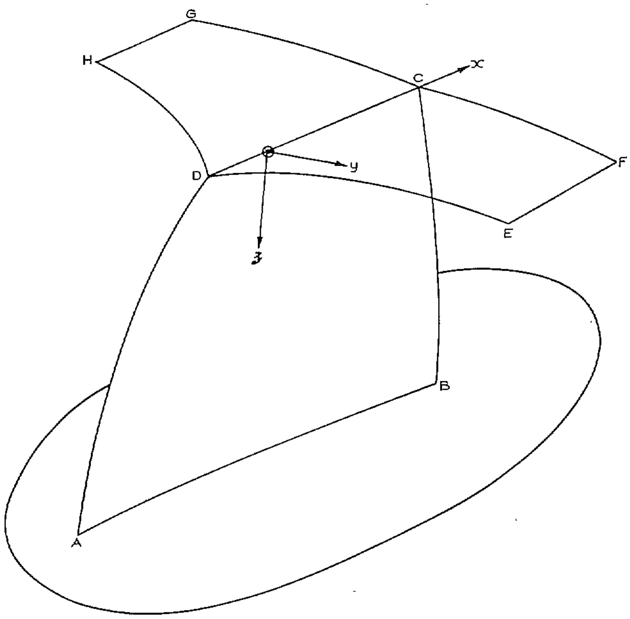

FIG. 2. Diagram of T-tail attached to a reflector plate at the base AB of the fin.

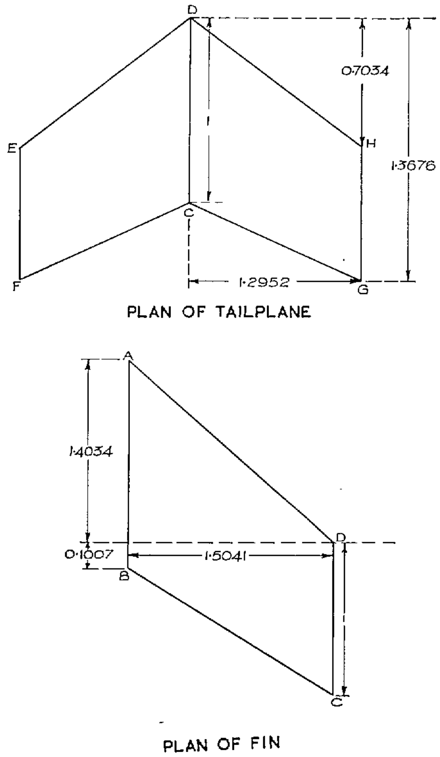

FIG. 3. Planforms of a tailplane and fin of a swept-back T-tail.

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