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Summary.

An exact solution is obtained for the upwash interference on wings of finite span in a rectangular wind tunnel with closed side walls and porous-slotted floor and roof.

A résumé of previous results concerning two-dimensional aerofoils and small-span wings is included for completeness.

1. *Introduction.*

The upwash interference on a two-dimensional aerofoil which spans a rectangular wind tunnel, and on a small-span wing, have been obtained by the present author in Ref. 1. Viscous flow through the slotted floor and roof is represented by the homogeneous boundary condition $\partial\phi/\partial x + K\, \partial^2\phi/\partial x\partial n + (1/R)\partial\phi/\partial n = 0$, which was originally derived by Baldwin, Turner and Knechtel. A résumé of these results is given below, and the analysis is extended to provide an exact solution for wings of finite span.

2. *Analysis.*

2.1. *Résumé of Previous Analysis.*

The velocity potential of a two-dimensional aerofoil in free air is taken as

$$\phi_1 = \frac{\Gamma}{2\pi} \tan^{-1}(z/x)$$

and the influence of the tunnel floor and roof is given by the interference velocity potential

$$\phi_2 = -\frac{\Gamma}{2\pi}\int_0^\infty \{A(q)\sin qx + B(q)\cos qx\}\sinh qz\, dq \tag{1}$$

where

$$A(q) = -\frac{e^{-qh}}{q}\left[\frac{(\sinh qh + Kq\cosh qh)(1-Kq) - (1/R)^2\cosh qh}{(\sinh qh + Kq\cosh qh)^2 + \{(1/R)\cosh qh\}^2}\right]$$

and

$$B(q) = \frac{-1/qR}{(\sinh qh + Kq\cosh qh)^2 + \{(1/R)\cosh qh\}^2} \tag{2}$$

* Replaces Hawker Siddeley Aviation Ltd., Wind Tunnel Report No. 107—A.R.C. 25 373.
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For a small-span wing the free-air velocity potential is taken as

$$\phi_1 = \frac{-\Gamma s}{2\pi}\left\{1+\frac{x}{(x^2+y^2+z^2)^{1/2}}\right\}\frac{z}{y^2+z^2}$$

and the boundary condition at the closed side walls is satisfied by the method of images, which, taking into account the free-air contribution yields a velocity potential

$$\phi_P = \frac{-\Gamma s}{2\pi}\sum_{m=0}^{\infty} j\left(\frac{\pi e^{-m\pi z/b}}{b}+\int_0^\infty \frac{2e^{-rz}}{qb}\sin qx\,dq\right)\cos\frac{m\pi y}{b} \tag{3}$$

where

$$r^2 = \left(\frac{m\pi}{b}\right)^2 + q^2 \text{ and } j = \begin{cases}\frac{1}{2}\,(m=0)\\ 1\,(m\neq 0)\end{cases}.$$

Let us write

$$\phi_P = f_P(y,z) + g_P(x,y,z).$$

If we represent the influence of the tunnel floor and roof by the velocity potential

$$\phi_Q = f_Q(y,z) + g_Q(x,y,z),$$

the boundary condition may be written

$$\left.\begin{aligned}&\frac{\partial}{\partial z}(f_P+f_Q) = 0\\ &\left(\frac{\partial}{\partial x}\pm K\frac{\partial^2}{\partial x\partial z}\pm\frac{1}{R}\frac{\partial}{\partial z}\right)(g_P+g_Q) = 0\end{aligned}\right\} z = \pm h. \tag{4}$$

It follows that f_Q represents the interference which would arise in the plane $x = 0$ from a closed floor and roof, and in Ref. 1 it is shown that

$$g_Q = \frac{-\Gamma s}{2\pi b}\sum_{m=0}^{\infty} 2j\left[\int_0^\infty \{A_m(q)\sin qx + B_m(q)\cos qx\}\sinh rz\,dq\right]\cos\frac{m\pi y}{b} \tag{5}$$

where

$$A_m(q) = \frac{-e^{rh}}{q}\frac{(\sinh rh+Kr\cosh rh)(1-Kr)-(r/qR)^2\cosh rh}{(\sinh rh+Kr\cosh rh)^2+\{(r/qR)\cosh rh\}^2}$$

and

$$B_m(q) = \frac{-r/qR}{q[(\sinh rh+Kr\cosh rh)^2+\{(r/qR)\cosh rh\}^2]}. \tag{6}$$

2.2. *Wings of Finite Span.*

A precedure introduced by Polhamus, and outlined in Ref. 4, may be used to extend the analysis to deal with the wings of finite span. A swept or unswept wing of arbitary loading may be represented as illustrated in Fig. 1. The velocity field is represented by a spanwise distribution of vortex doublets whose strength is proportional to the local loading.

For a uniformly loaded unswept wing, the velocity potential of the wing and the images in the tunnel side walls is easily shown to be

$$\begin{aligned}\phi_P &= \frac{-\Gamma}{4\pi}\int_{-s}^{s}\sum_{m=0}^{\infty} j\left(\frac{\pi e^{-m\pi z/b}}{b}+\int_0^\infty\frac{2e^{-rz}}{qb}\sin qx\,dq\right)\cos\left\{m\pi\frac{(y+b)}{b}\right\}d\epsilon\\ &= \frac{-\Gamma}{2\pi}\sum_{m=0}^{\infty} j\left(\frac{\pi e^{-m\pi z/b}}{b}+\int_0^\infty\frac{2e^{-rz}}{qb}\sin qx\,dq\right)\cos\frac{m\pi y}{b}\left(\sin\frac{m\pi s}{b}\right)\Big/\frac{m\pi s}{b}.\end{aligned} \tag{7}$$

From the similarity of equations (3) and (7), it follows that

$$\phi_Q = f_Q(y, z) + g_Q(x, y, z)$$

where, as before, ϕ_Q represents the influence of a closed floor and roof and

$$g_Q = \frac{-\Gamma s}{2\pi b} \sum_{m=0}^{\infty} 2j \left[\int_0^{\infty} \{A_m(q) \sin qx + B_m(q) \cos qx\} \sinh rz \, dq\right] \cos \frac{m\pi y}{b} \left(\sin \frac{m\pi s}{b}\right) \Big/ \frac{m\pi s}{b} \quad (8)$$

with $A_m(q)$ and $B_m(q)$ as defined in equations (6)

Equations (5) and (8) become identical as $S \to 0$, and it has been shown that a limiting process as $1/R \to 0$ yields results consistent with those of Ref. 5. The interference in a wind tunnel with uniformly porous floor and roof is obtained by putting $K = 0$. For a tunnel with completely open floor and roof $K = 0$ and $1/R \to 0$.

2.3. *Numerical Results.*

The mean incidence correction, obtained by integrating the interference over the span of the wing, is

$$\delta = \frac{\Delta\alpha C}{C_L S} = \frac{-1}{2s} \int_{-s}^{s} \frac{\partial}{\partial z} (\phi_P + \phi_Q - \phi_1) \frac{bh}{\Gamma s} dy \qquad \text{with} \qquad x = z = 0$$

$$= \delta_c - \frac{1}{2\pi} \sum_{m=0}^{\infty} \left[\int_0^{\infty} \frac{2jr}{q} \frac{(rh/qR)\, dq}{(\sinh rh + kr \cosh rh)^2 + \{(r/qR) \cosh rh\}^2}\right] \left(\sin \frac{m\pi s}{b}\right) \Big/ \frac{m\pi s}{b} \quad (9)$$

where δ_c is the corresponding interference factor for a closed wind tunnel.

We may write

$$\delta = \delta_c + \sum_{m=0}^{\infty} \delta_m \left\{\left(\sin \frac{m\pi s}{b}\right) \Big/ \frac{m\pi s}{b}\right\}^2 \quad (10)$$

and δ_m is a function only of K, $1/R$ and tunnel height/width ratio λ. Once δ_c is known and values of δ_m have been calculated, the interference factor may easily be obtained for any wing span and, with a simple extension of the analysis, for any symmetrical wing loading.

The summation is rapidly convergent, and only the first three terms are required for practical purposes. It is easily shown that δ_0 is the interference factor for a two-dimensional aerofoil, and that $\delta_m(\lambda) = \delta_{2m}(\lambda/2)$.

Values of δ_m, $m = 0$, 1 and 2 have been calculated for $\lambda = 1$ and $\lambda = 0{\cdot}5$ (the latter for half-models on the side walls of a square tunnel) for various porosity and slot geometry parameters, *see* Fig. 2. Incidence correction factors have been calculated for span/ tunnel-width ratios $\sigma = 0$ and $0{\cdot}5$, Figs. 3 and 4.

The parameters for zero incidence correction are shown in Fig. 5.

Compressibility effects may be taken into account by dividing the porosity parameter by $\sqrt{(1-M^2)}$.

It is perhaps worth noting at this stage that, with a suitable choice of the two parameters, it is possible to obtain zero mean upwash interference and zero mean blockage interference at a given subsonic Mach number.

3. *Concluding Remarks.*

It is hoped that the results of this investigation will prove useful for many tunnels, not only the correcting results, but also in obtaining improved testing conditions.

NOTATION

x, y, z	Cartesian co-ordinates with their origin at the centre of the working section: $x + ve$ downstream and $z + ve$ upwards
n	Outward normal at tunnel wall
u	Freestream velocity
Γ	Vortex strength $= \dfrac{USC_L}{4s}$
δ	Mean incidence correction factor $= \dfrac{\Delta\alpha C}{C_L S}$
C_L	Lift$/\frac{1}{2}\rho U^2 S$
s	Wing semi-span
S	Wing area
b	Tunnel semi-width
h	Tunnel semi-height
C	Tunnel cross-sectional area $= 4bh$
K	Slot geometry parameter $= \dfrac{d}{\pi} \log \operatorname{cosec} \dfrac{\pi a}{2d}$ for a rectangular tunnel ($c = K/h$)
a	Slot width
d	Distance between slots
R	Porosity parameter defined by $\Delta p = \dfrac{\rho U}{R} \dfrac{\partial \phi}{\partial n}$
q	Variable of integration
r	$= \sqrt{\left(q^2 + \dfrac{m^2\pi^2}{b^2}\right)}$, where m is an integer

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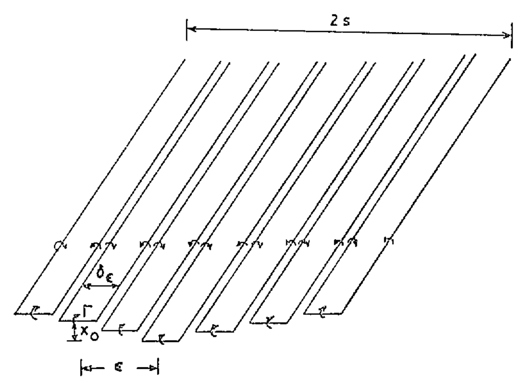

FIG. 1. Representation of a finite-span wing.

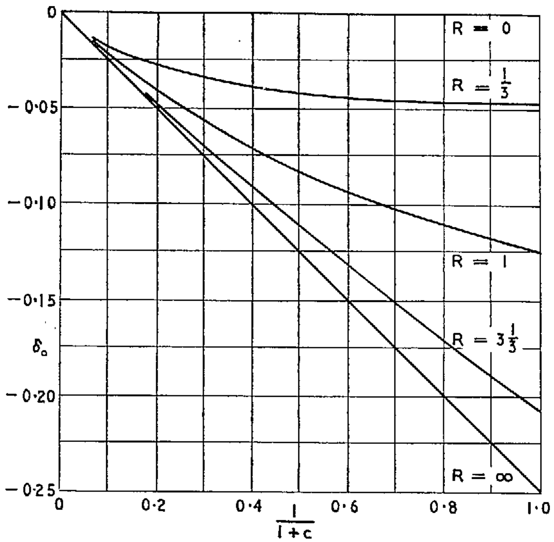

FIG. 2. The incidence correction for a two-dimensional aerofoil for various porosity and slot geometry parameters.

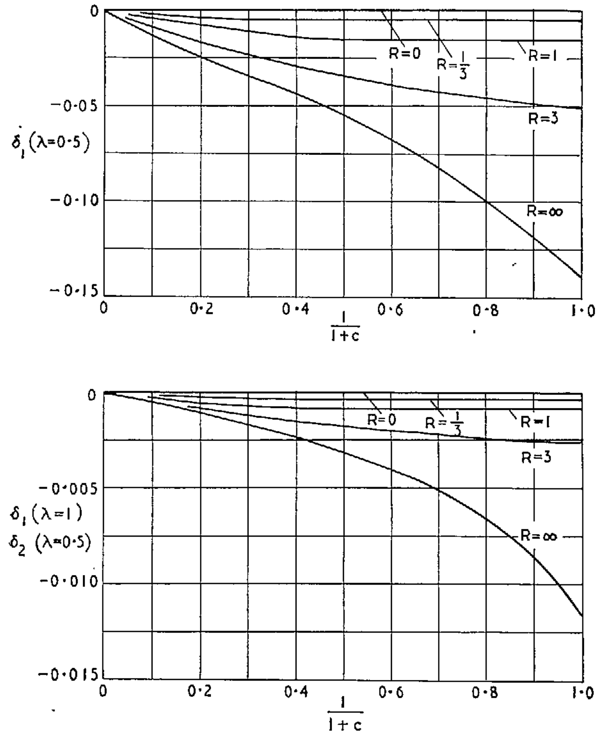

FIG. 2 (*continued*). Values of δ_1 and δ_2 for various porosity and slot geometry parameters.

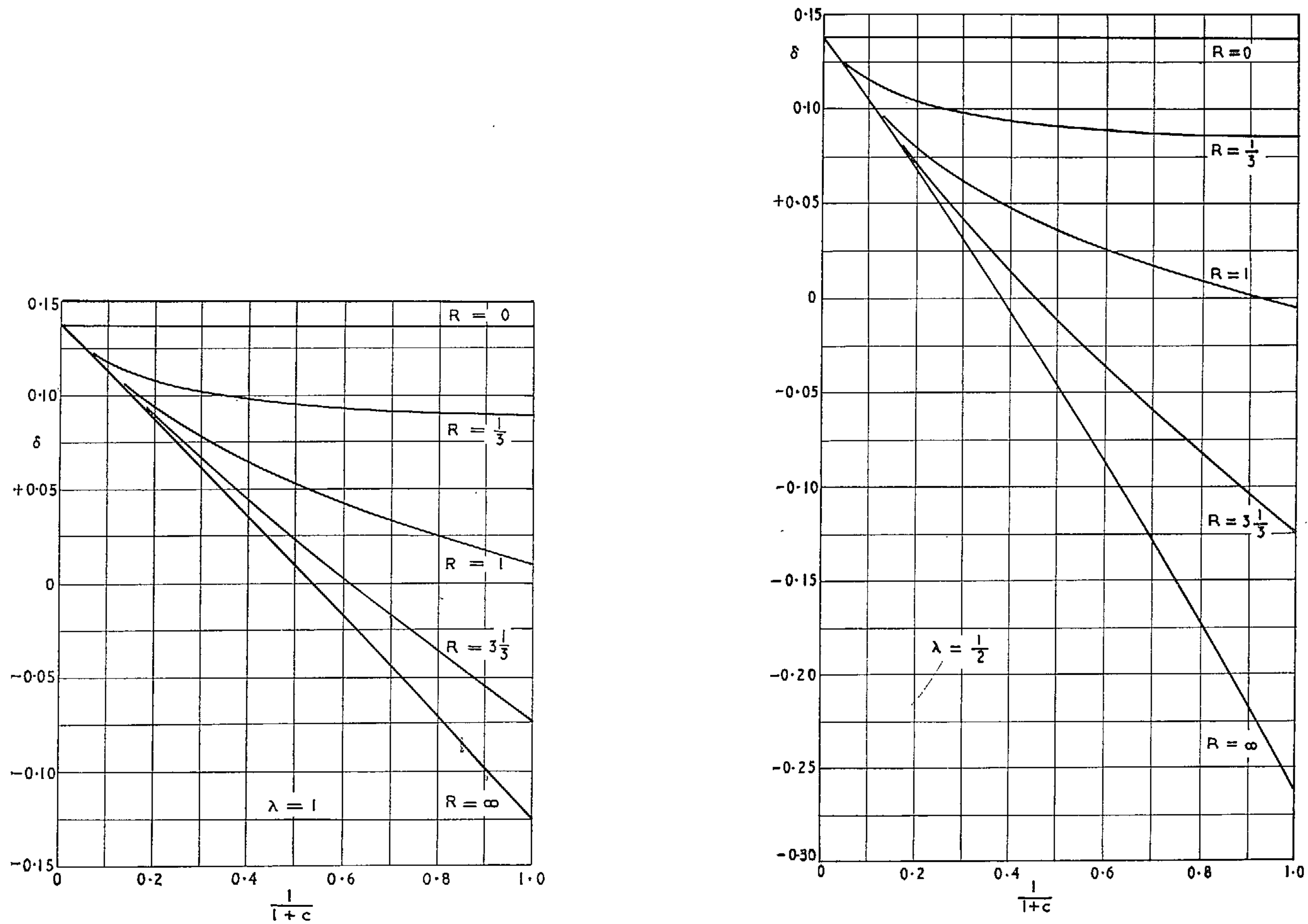

FIG. 3. The incidence correction factor for a small-span wing, for various porosity and slot geometry parameters.

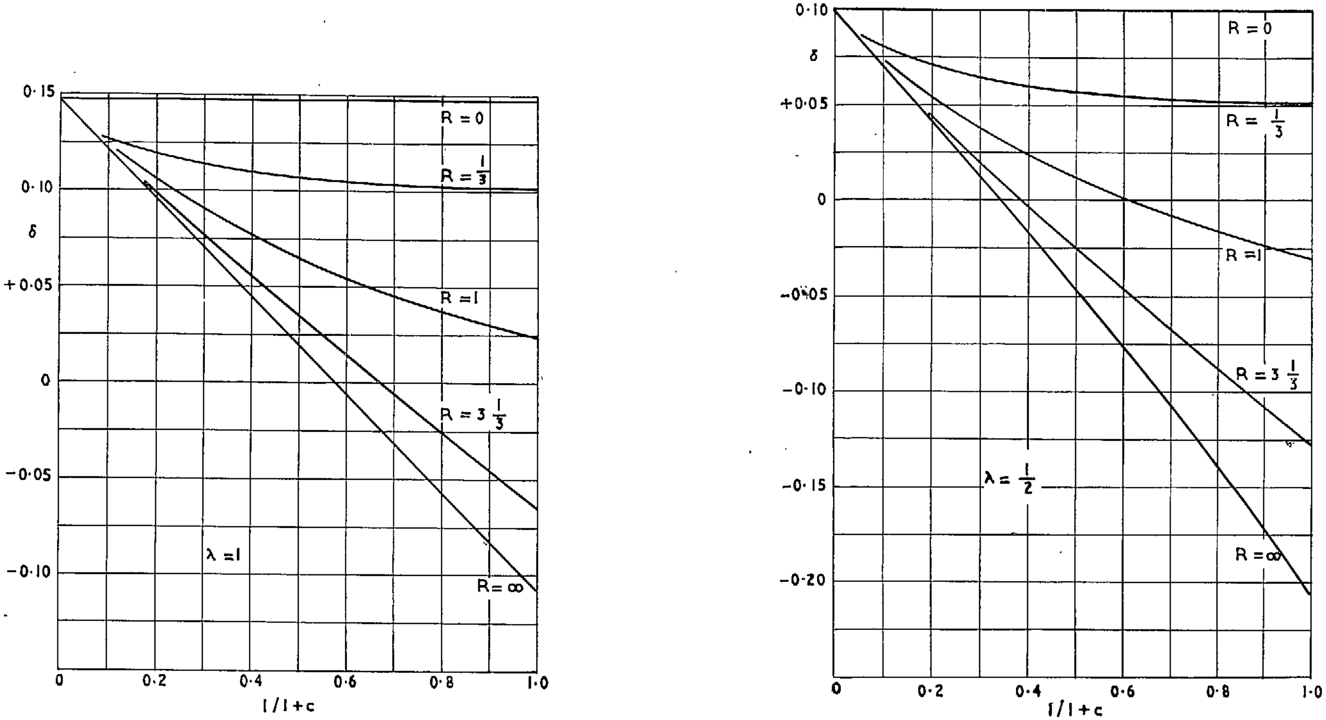

FIG. 4. The incidence correction factor for a finite-span wing ($\sigma = 0 \cdot 5$) for various porosity and slot geometry parameters.

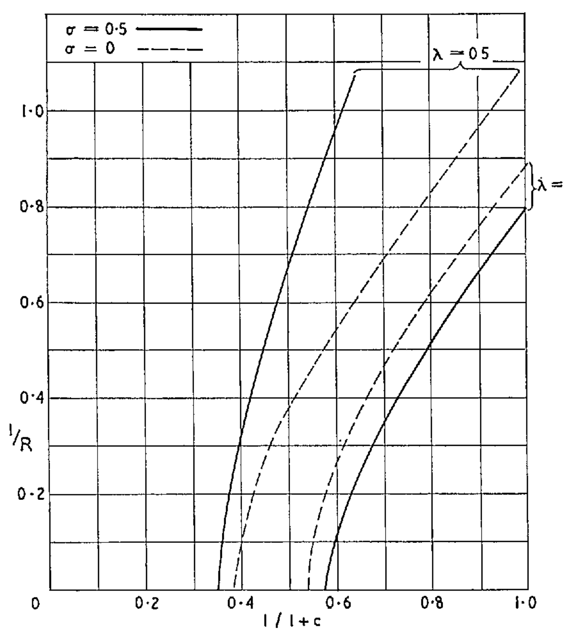

FIG. 5. Values of the porosity and slot geometry parameters for zero incidence corrections.

(90848) Wt- 66/2301 K5 2/65 Hw.

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