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The Geometry of Wing Surfaces Generated by straight lines and with a high rate of thickness Taper at the Root

By

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The geometry of wing surfaces generated by straight lines and with a high rate of thickness taper at the root

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D. Peckham, B.Sc.

SUMMARY

This note describes a way in which wings can be designed to have a high rate of thickness taper at the root, while still maintaining a surface shape generated by straight lines.

The method can be most successfully applied to wings of parabolic arc section straight-tapered in planform, in which case there is no change in aerofoil section shape across the span. Other planform shapes, and wing root aerofoil section shapes, result in a variation of aerofoil section shape across the span.

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1 Introduction

Design and construction of the wings of both full size aircraft and wind-tunnel research models would be simplified if they had surfaces generated by straight lines. A simple example is the straight-tapered planform wing of constant thickness/chord ratio whose surface is generated by straight constant-percentage-chord lines. This type of wing is said to have linear "planform taper" and "thickness taper", these terms being defined as:-

Planform taper means that the wing chord varies across the span, it does not imply a corresponding variation of wing thickness.

Thickness taper refers to the spanwise variation of the absolute thickness, and not to the spanwise variation of thickness/chord ratio.

The rate of thickness taper is therefore $\frac{dt(y)}{dy}$.

For the reasons given below, a high rate of thickness taper may be desirable at the root of low aspect ratio wings:-

(a) There may be aerodynamic advantages in having most of the volume close to the root[1].

(b) The resulting high wing thickness at the root gives a better junction shape with a fuselage, or a separate fuselage may not even be necessary.

(c) The high root thickness is structurally desirable in that it allows a high spar depth.

Unfortunately, this high thickness taper at the root generally results in a wing surface which is not generated by straight lines in any way. However, when a sharp leading edge is desirable, it is possible with a straight-tapered planform shape to design a wing surface with straight generators parallel (in plan view) to the leading edge, and to the trailing edge, if a parabolic arc aerofoil section is used. On such a wing, the thickness/chord ratio decreases linearly from the root to zero at the tip, giving a parabolic distribution of maximum thickness across the span. This type of wing is described in section 2 and shown in Fig.3.

As the sharp-edged slender delta wing is a promising shape as a lifting surface because of the one type of flow pattern round it throughout the flight range, the geometry of such a wing with a parabolic arc aerofoil section is discussed in detail in section 3.

Other planform shapes and aerofoil sections are considered in section 4. In these cases straight wing surface generators are obtained only with a variation in aerofoil section shape across the span.

2 The parabolic arc section wing with straight surface generators parallel (in plan view) to the leading edge and to the trailing edge

2.1 Surface shape of wing

The equation of a parabolic arc profile, as shown in Fig.1, can be written as

$$z = \pm k\xi (c - \xi) \tag{1}$$

where c is the chord and ξ is measured from the leading edge. If φ is the leading edge sweep then

$$\xi = x - |y| \tan \varphi .$$

For the straight-tapered planform of any sweep of Fig.2, of root chord c_o and thickness/chord ratio varying from $\left(\frac{t_o}{c_o}\right)$ at the root to zero at the tip,

$$\frac{c}{c_o} = \frac{\left(\frac{t}{c}\right)}{\left(\frac{t_o}{c_o}\right)} = 1 - \eta \tag{2}$$

where η = non-dimensional spanwise ordinate $\frac{|y|}{s}$, the positive right-hand half of the wing only being considered in the following theory.

Substituting in equation (1) values appropriate to the maximum thickness position (mid-chord), we get that at any spanwise position

$$\frac{c_o}{2}\left(\frac{t_o}{c_o}\right)(1-\eta)^2 = k\,\frac{c_o(1-\eta)}{2}\left\{c_o(1-\eta) - \frac{c_o(1-\eta)}{2}\right\}$$

which gives

$$k = \frac{2}{c_o}\left(\frac{t_o}{c_o}\right). \tag{3}$$

As the value of k is independent of η, the aerofoil sections at all spanwise positions are therefore part of one parabolic arc, and the equation of the wing surface is simply

$$z = \pm\frac{2}{c_o}\left(\frac{t_o}{c_o}\right)\xi(c-\xi) \tag{4a}$$

or, with the origin of coordinates at the leading edge apex

$$z = \pm\frac{2}{c_o}\left(\frac{t_o}{c_o}\right)(x - |y| \tan\varphi)\left[c_o - |y|\left(\frac{c_o}{s} - \tan\varphi\right) - x\right]. \tag{4b}$$

In equation (4a) it can be seen that planes having the equations either ξ = constant, or $c-\xi$ = constant, intersect the wing surface in straight lines. This means that the wing surface is generated by two sets of straight lines running from the centre line profile, parallel (in plan view) to the leading edge, and to the trailing edge respectively.

The equation of a series of planes, parallel to the plane containing the z-axis and the maximum thickness line of the half-wing, can be written as

$$\xi = \frac{c}{2} + h = \frac{c_o(1-\eta)}{2} + h$$

where h is the distance of any plane from the plane through the maximum thickness line, measured in the x-direction.

Substituting the above value of ξ in equation (4a) gives

$$z = \frac{c_o}{2}\left(\frac{t_o}{c_o}\right)(1-\eta)^2 - \frac{2}{c_o}\left(\frac{t_o}{c_o}\right)h^2$$

or

$$z = \frac{c_o}{2}\left(\frac{t_o}{c_o}\right)(1-\eta)^2 - kh^2 \; . \qquad (5)$$

Thus the intersection shape with the wing surface of planes parallel to the maximum thickness line, is a series of arcs of the same parabola displaced in the z-direction by kh^2 from the maximum thickness line, and cut off by the leading edge (or trailing edge) at $\eta = 1 - \frac{2h}{c_o}$.

Finally, the intersection shape with the wing surface of any plane, parallel to the z-axis, and swept back at an angle θ is obtained as follows.

Let

$$\xi = x_o + |y| \tan\theta - |y| \tan\varphi$$

be the plane.

Substituting this value of ξ in equation (4a) gives

$$z = \frac{2}{c_o}\left(\frac{t_o}{c_o}\right)(x_o + |y| \tan\theta - |y| \tan\varphi)(c_o - \eta c_o - x_o - |y| \tan\theta + |y| \tan\varphi)$$

which can be re-arranged to

$$z = \frac{2}{c_o}\left(\frac{t_o}{c_o}\right)\left\{x_o(c_o-x_o) + |y|\left[x_o\left(\tan\varphi - \tan\theta - \frac{c_o}{s}\right) + (c_o-x_o)(\tan\theta - \tan\varphi)\right]\right.$$
$$\left. - y^2(\tan\theta - \tan\varphi)\left(\tan\theta - \tan\varphi + \frac{c_o}{s}\right)\right\}, \qquad (6)$$

a quadratic expression in y.

It can be seen in equation (6), that for a plane parallel to the leading edge, when $\tan\theta = \tan\varphi$ (and for a plane parallel to the trailing edge, when $\tan\theta = \tan\varphi - \frac{c_o}{s}$) the equation becomes linear in y. For a plane parallel to the maximum thickness line $\tan\theta = \tan\varphi - \frac{x_o}{s}$, in which case equation (6) reduces to equation (5).

From equation (4a) the equation of the intersection of the plane $z = \text{constant}$ with the wing surface is

$$\xi(c - \xi) = \text{constant} .$$

If a and b are lengths parallel to the leading and trailing edges as defined in Fig. 2, then by similar triangles it follows that

$$\xi(c - \xi) = ab \times \text{constant} .$$

Thus the intersection shape with the wing surface of sections parallel to the $z = 0$ plane, are hyperbolas with the leading and trailing edges as asymptotes.

2.2 Intersection areas of planes with wing surface

The cross-section area of the intersection of the plane $x = x_0 + y \tan\theta$ with both surfaces of half the wing, can be obtained by integrating equation (6) with respect to ζ, the distance along the direction of the plane

$$\int z \, d\zeta = \int z \sqrt{dx^2 + dy^2} = \int z \sqrt{dy^2 \tan^2\theta + dy^2}$$

$$= \sec\theta \int z \, dy .$$

The limits of integration are $y = 0$ and the y-coordinate of the intersection of the plane with the leading edge or trailing edge,

i.e. either $\quad y = \dfrac{c_0 - x_0}{\tan\theta + \tan\psi} \quad$ (intersection with T.E.)

or $\quad y = \dfrac{-x_0}{\tan\theta - \tan\varphi} \quad$ (intersection with L.E.) .

The cross-section area is thus (for half the wing)

$$S(x_0, \theta) = \frac{2}{c_0}\left(\frac{t_0}{c_0}\right) \sec\theta \left\{ x_0(c_0 - x_0)y + \frac{y^2}{2}\left[x_0\left(\tan\varphi - \tan\theta - \frac{c_0}{s}\right) + (c_0 - x_0)(\tan\theta - \tan\varphi)\right] - \frac{y^3}{3}(\tan\theta - \tan\varphi)\left(\tan\theta - \tan\varphi + \frac{c_0}{s}\right)\right\}_0^{\text{edge}} . \tag{7}$$

2.3 Volume enclosed by wing surface

The area of a chordwise section of the wing is

$$2k \int_0^c \xi(c - \xi) \, d\xi = \frac{2}{3}\left(\frac{t_0}{c_0}\right) c_0^2 (1 - \eta)^3 .$$

The volume enclosed by the wing surface, obtained by integrating the above expression across the span, is then found to be

$$V = \frac{1}{3}\left(\frac{t_o}{c_o}\right) c_o^{\,2}\, s \, . \tag{8}$$

Similarly, for a constant thickness/chord ratio wing, the corresponding expressions for area of chordwise section and enclosed volume are

$$2k \int_0^c \xi(c-\xi)\, d\xi = \frac{2}{3}\left(\frac{t}{c}\right) c_o^{\,2}(1-\eta)^2 \, .$$

where in this case

$$k = \frac{2}{c_o}\left(\frac{t}{c}\right)\left(\frac{1}{1-\eta}\right)$$

and

$$V = \frac{4}{9}\left(\frac{t}{c}\right) c_o^{\,2}\, s \, . \tag{9}$$

From equations (8) and (9), it follows that for the two types of wing to have the same volume, the ratio of the root thicknesses is 4 : 3, the wing of constant thickness/chord ratio having the smaller root thickness.

3 The delta planform wing of parabolic arc section

A special case of the type of wing described in section 2 is the sharp-edged, delta planform. This wing has been considered by Newby[1], and is of special interest aerodynamically in that the calculation of the velocity distribution over the surface is particularly simple, and the resulting properties promising both without and with lift. It is now shown that its geometric properties are also simple. A low aspect ratio wing of this type is shown in Fig.3, compared with a similar wing of constant thickness/chord ratio and the same root thickness.

For the delta planform, $\tan \varphi = \frac{c_o}{s}$ and equation (7) becomes

$$S(x_o,\theta) = \frac{2}{c_o}\left(\frac{t_o}{c_o}\right)\left\{x_o(c_o-x_o)y + \frac{y^2}{2}\left[(c_o-x_o)\left(\tan\theta - \frac{c_o}{s}\right) - x_o \tan\theta\right] - \frac{y^3}{3}\tan\theta\left(\tan\theta - \frac{c_o}{s}\right)\right\}_0^{\text{edge}} \, . \tag{10}$$

The cross-section area perpendicular to the wing centre line is, for half the wing

$$S(x_o,0) = 2sc_o\left(\frac{t_o}{c_o}\right)\left(\frac{x_o}{c_o}\right)^2\left(1 - \frac{x_o}{c_o}\right) . \tag{11}$$

The expression corresponding to (11) for a constant thickness/chord ratio delta wing is

$$S(x_o,0) = 4sc_o\left(\frac{t_o}{c_o}\right)\left(1-\frac{y_o}{c_o}\right)\left\{\frac{x_o}{c_o}\left[1-\log\left(1-\frac{x_o}{c_o}\right)\right]+\log\left(1-\frac{x_o}{c_o}\right)\right\}. \tag{12}$$

The area distributions from equations (11) and (12) are plotted in Fig.4 for two wings of the same root thickness/chord ratio. Also plotted, is the area distribution of a constant thickness/chord ratio wing of the same volume as the wing with the linearly decreasing thickness/chord ratio.

The cross-sections of the wing perpendicular to the centre line being diamond-shaped, the calculation of the wave drag and supersonic pressure distribution should be simple.

4 Other planform shapes and aerofoil sections

4.1 The parabolic planform wing

The parabolic planform is basically a cropped delta of taper-ratio 0.5 with the leading edge and tip forming a continuous curve, the wing chord varying across the span as

$$\frac{c}{c_o} = \sqrt{1-\eta}.$$

For this planform shape the wing area, aspect ratio and apex angle are, respectively

$$S = \frac{4sc_o}{3}$$

$$A = \frac{3s}{c_o}$$

$$\delta = 2\tan^{-1}\left(\frac{2A}{3}\right).$$

The variation of apex angle with aspect ratio for various delta-type planforms is shown in Fig.6.

With a parabolic arc centre line profile, the surface shape required to give straight-sided transverse sections is

$$z = \frac{2}{c_o}\left(\frac{t_o}{c_o}\right)x_T(c_o-x_T) \times \left\{\frac{s\left(1-\frac{x_T^2}{c_o^2}\right)-\eta s}{s\left(1-\frac{x_T^2}{c_o^2}\right)}\right\}$$

i.e.

$$z = 2x_T\left(\frac{t_o}{c_o}\right)\frac{\left(1-\frac{x_T^2}{c_o^2}-\eta\right)}{\left(1+\frac{x_T}{c_o}\right)} \tag{13}$$

where x_T is measured from the trailing-edge and $x_T = c_o - x$.

Thus the wing section outboard of the centre line is not parabolic, and the maximum thickness position no longer at mid-chord.

By differentiating equation (13) and equating to zero, it is found that the maximum thickness position occurs where

$$1 - 3\left(\frac{x_T}{c}\right)^2 - 2\sqrt{1-\eta}\left(\frac{x_T}{c}\right)^3 = 0 . \tag{14}$$

The spanwise decrease in thickness/chord ratio is nearly linear and is plotted in Fig.7. Fig.8 shows the spanwise variation of the maximum thickness position, and in Fig.9 is plotted a typical chordwise profile with a parabolic arc for comparison.

4.2 The delta wing of non-parabolic section

With an arbitrary section shape at the centre line of a delta wing, the condition that the wing surface is generated by straight lines parallel to the trailing edge is

$$z\,(x_o, \eta s) = z_o\left(\frac{\frac{x_o}{c_o}\,s - \eta s}{\frac{x_o}{c_o}\,s}\right)$$

i.e.

$$z\,(x_o, \eta s) = z_o\left(1 - \frac{\eta c_o}{x_o}\right) \tag{15}$$

where z_o is the ordinate at $(x_o, 0)$.

For a non-parabolic centre line profile, equation (15) can only be satisfied at positions outboard of the centre line by a variation in the section shape across the span. However, for thin sections the circular arc is very nearly the same as the parabolic arc, and the circular arc could be used for the centre line profile, if desired, without causing a large variation in section shape across the span.

As an example of a case where the centre line profile differs greatly from the parabolic shape, the sections at various spanwise positions on a delta wing with a root section of double-wedge shape are shown in Fig.10. The thickness/chord ratio distribution across the span is plotted in Fig.11, and the locus of the maximum thickness position in Fig.12.

4.3 Cambered aerofoil sections

A cambered chordwise profile can be obtained by using different profiles to form the upper and lower surfaces of the wing, each surface still being generated by straight lines. In the simplest case, if two different parabolic arcs are used, a parabolic arc camber line results. Other shape camber lines are obtained by a suitable choice of profile shape for each surface, but the camber line shape will vary across the span if the wing surfaces are still to be generated by straight lines.

The chordwise camber on a wing whose surface is straight-line generated, automatically introduces a camber in the spanwise direction which is a form of dihedral - or it can be termed cross-wise camber. The cross-wise camber line shape is not conical as long as the trailing edge remains straight.

5 Conclusions

It has been shown that a wing of parabolic arc section, straight-tapered in planform, has the following useful properties if the thickness/chord ratio decreases linearly from the root to zero at the tip:-

(a) The wing surface is generated by two sets of straight lines running parallel (in plan view) to the leading edge, and to the trailing edge, respectively.

(b) The aerofoil sections at all spanwise positions are part of one convex parabolic arc.

(c) Sections parallel to the maximum thickness line are part of one concave parabolic arc, cut off at the leading edge and trailing edge.

The above properties should make such a wing simple to manufacture. Furthermore, the spanwise distribution of thickness being parabolic, there is a high rate of thickness taper at the root which is desirable aerodynamically. The high root thickness gives a good junction shape with a fuselage, or may even be sufficient to make a separate fuselage unnecessary, as well as allowing a high spar depth and a therefore economic structure.

With other planform shapes, and root aerofoil sections other than parabolic, a straight line generated wing surface is only obtained at the expense of a variation in section shape across the span.

List of symbols

x, y, z	rectangular coordinates, x measured chordwise from leading edge apex, y spanwise
x_T	chordwise distance measured from unswept trailing edge
c	wing chord
h	length measured in x-direction
k	parabolic arc constant
s	wing semi-span
t	wing thickness
δ	apex angle
θ	sweepback of plane intersecting wing surface
φ	leading edge sweepback angle
ψ	trailing edge sweepforward angle
η	non-dimensional spanwise ordinate $\frac{\lvert y \rvert}{s}$
ζ	distance measured along plane intersecting wing surface
ξ	chordwise distance measured from local leading edge
A	aspect ratio
S	wing area

$S(x_o, \theta)$	intersection area of plane with half the wing
T.R.	taper ratio
V	volume enclosed by wing surface

Suffices

o	referring to wing centre line

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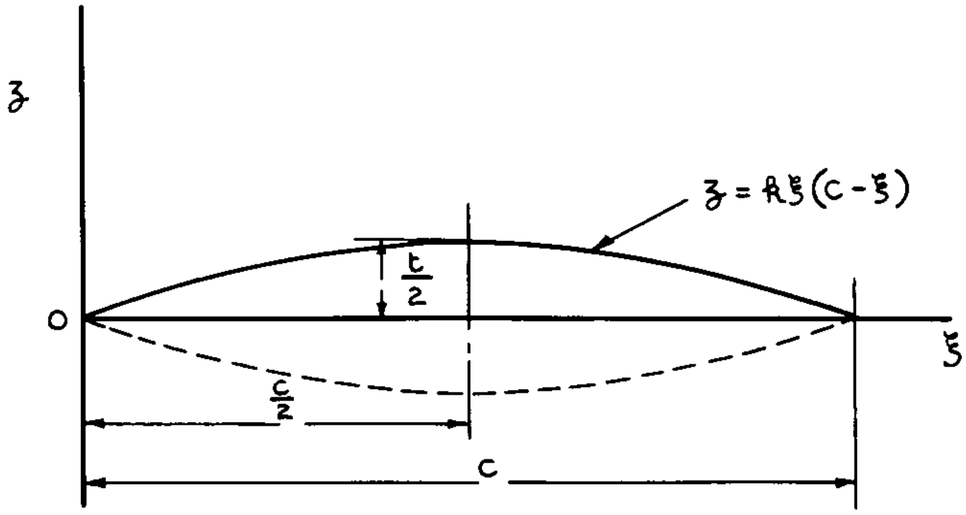


FIG. 1. PARABOLIC ARC AEROFOIL SECTION.

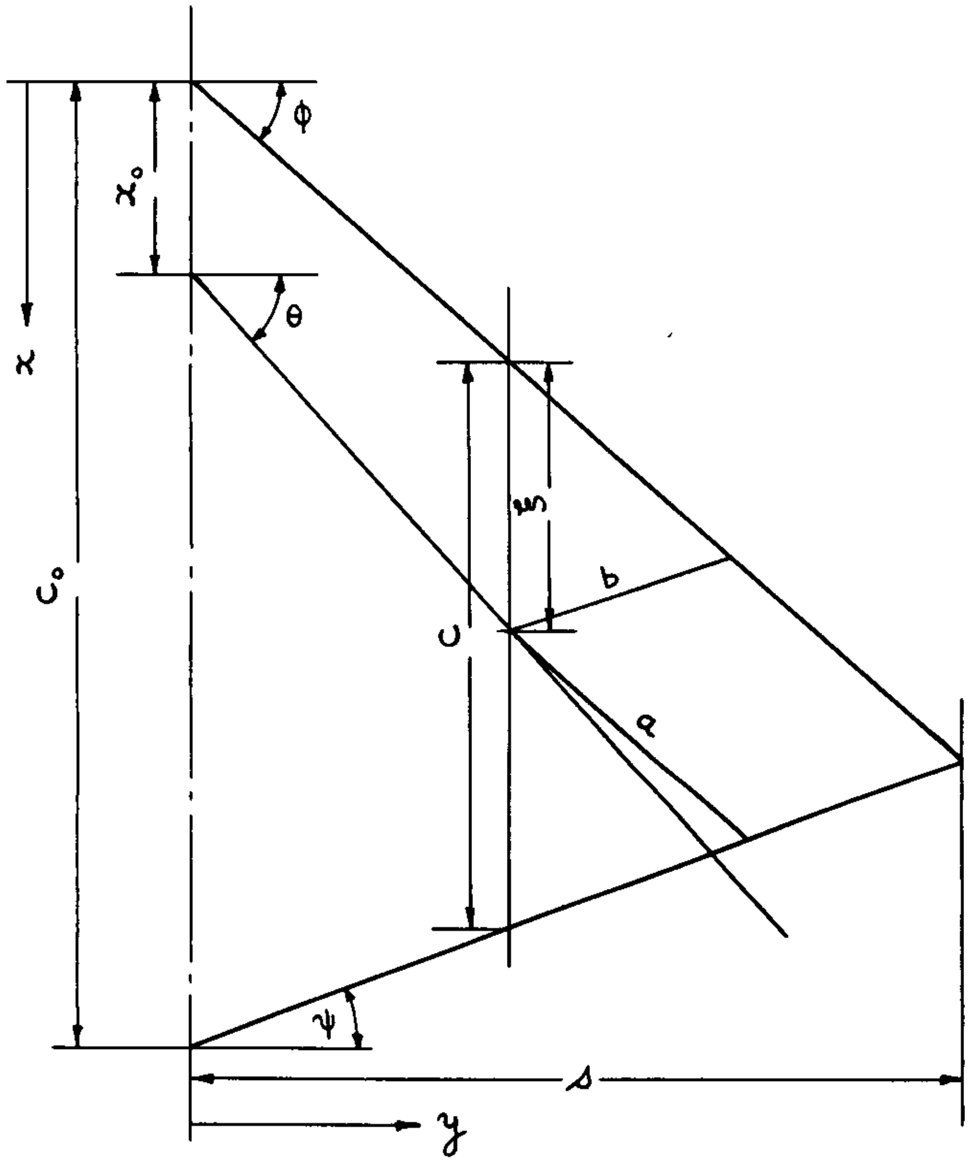
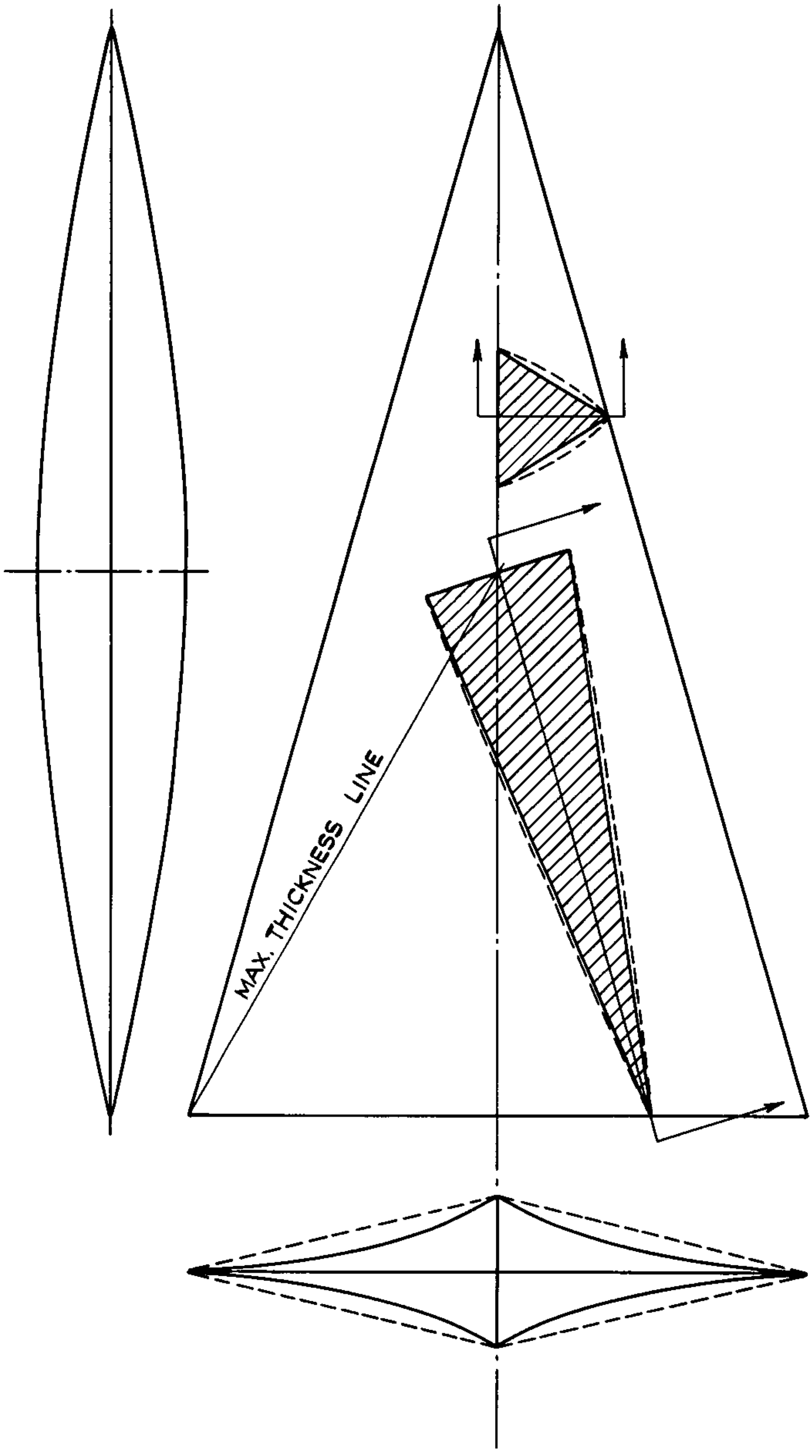


FIG. 2. STRAIGHT - TAPERED PLANFORM. NOTATION.



---- CONSTANT THICKNESS / CHORD RATIO WING.

FIG. 3. PARABOLIC ARC AEROFOIL SECTION DELTA WING WITH THICKNESS / CHORD RATIO DECREASING FROM ROOT TO ZERO AT TIP.

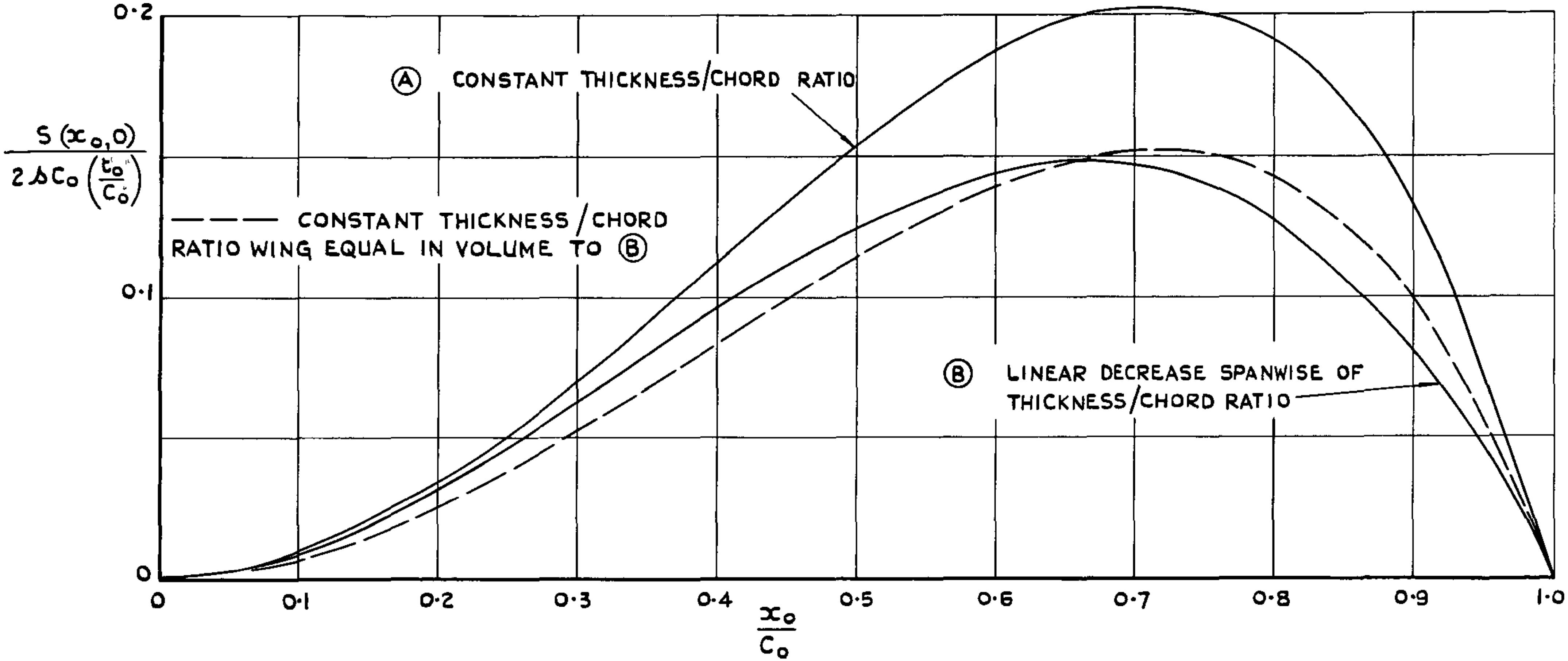


FIG. 4. TRANSVERSE CROSS-SECTION AREAS OF DELTA WINGS WITH PARABOLIC ARC AEROFOIL SECTION.

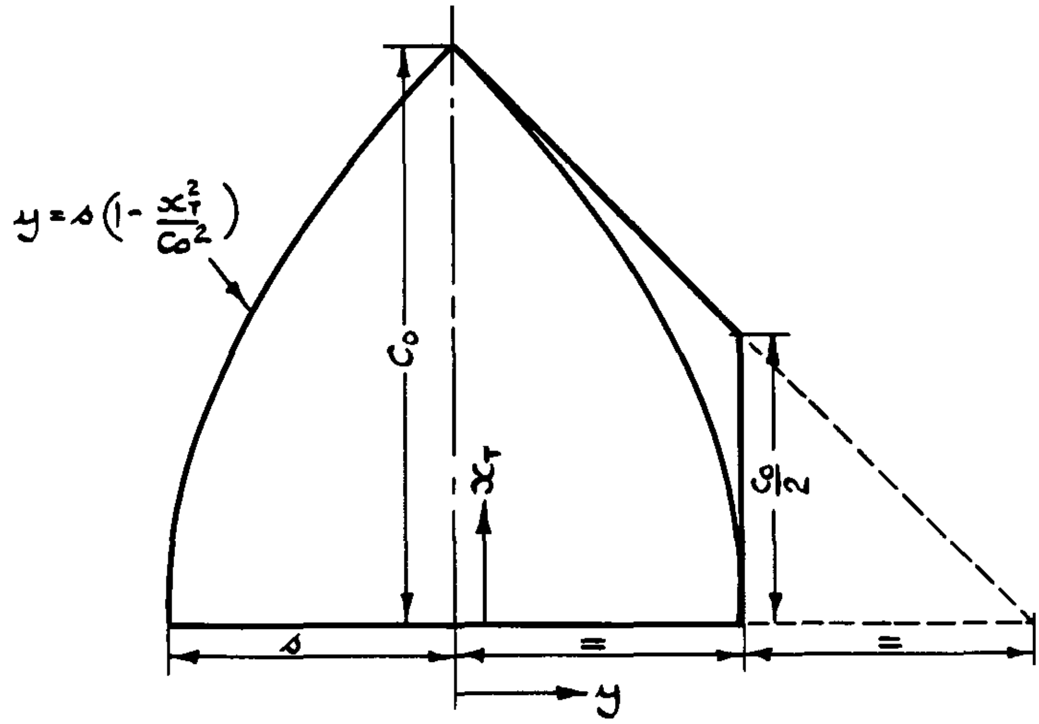


FIG.5 CROPPED DELTA (T.R. 0·5) AND PARABOLIC PLANFORMS.

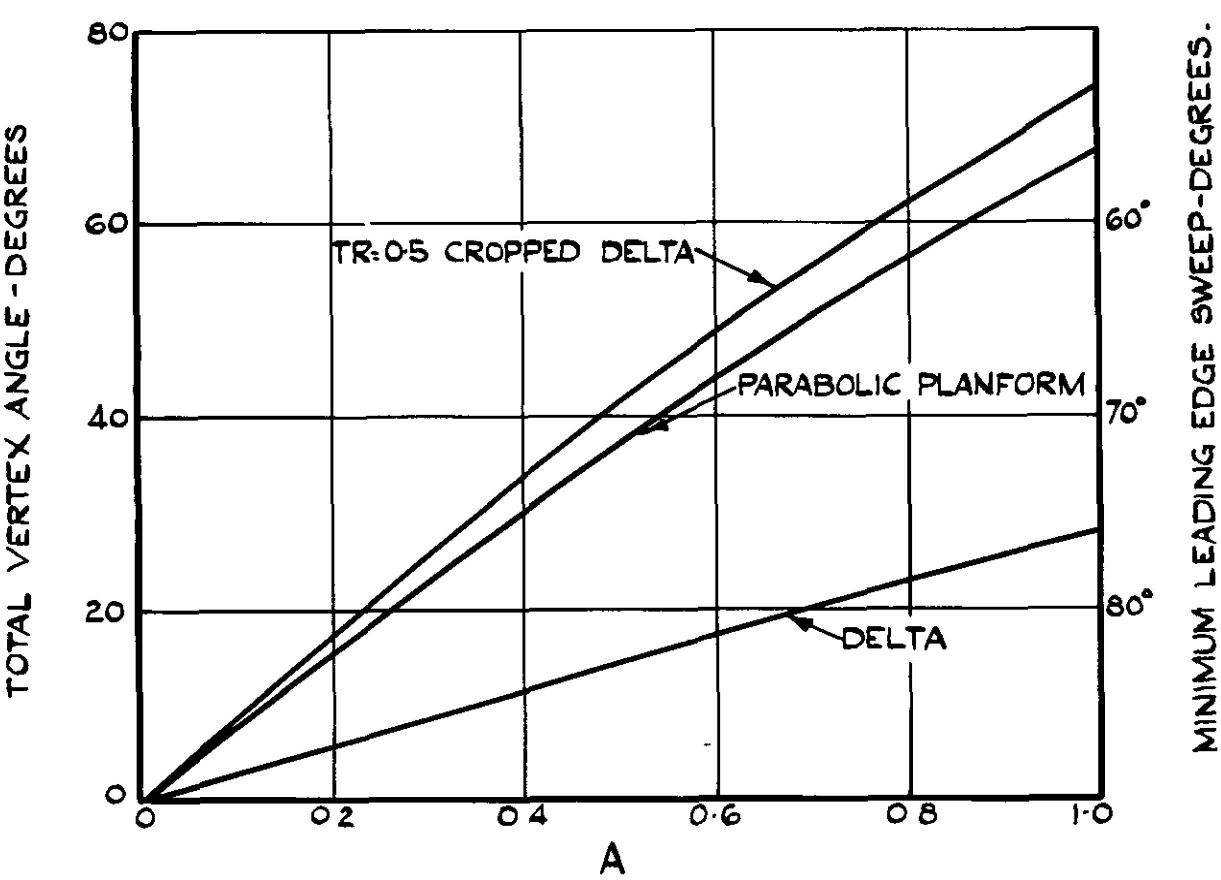


FIG.6. VARIATION OF VERTEX ANGLE WITH ASPECT RATIO FOR VARIOUS PLANFORM SHAPES.

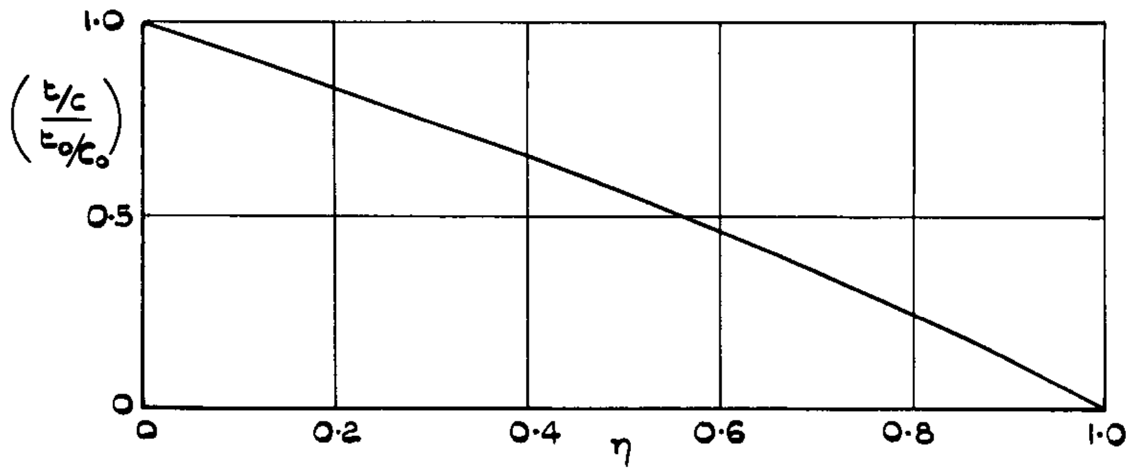


FIG.7. SPANWISE VARIATION OF THICKNESS/ CHORD RATIO ON PARABOLIC PLANFORM WING.

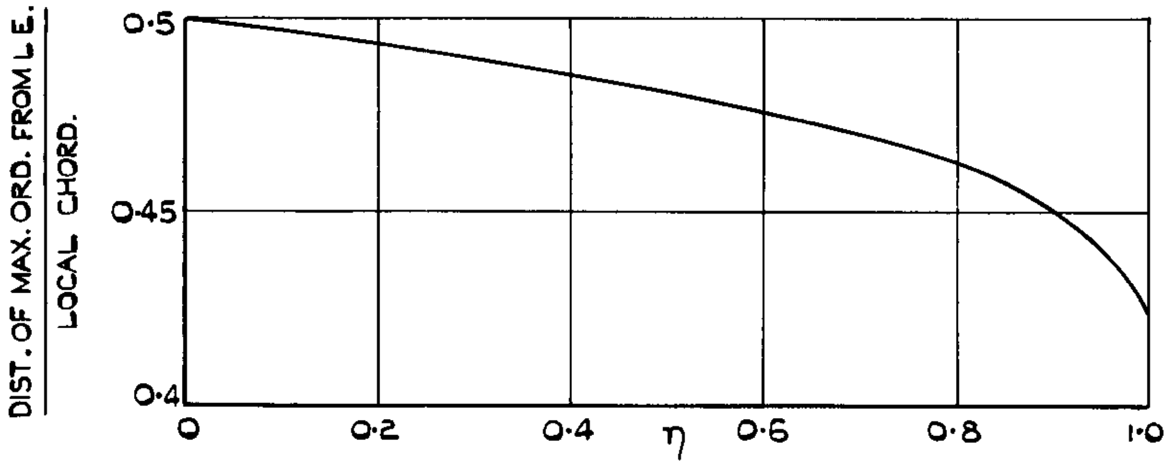


FIG.8. SPANWISE VARIATION OF LOCAL MAX. THICKNESS POSITION ON PARABOLIC PLANFORM WING.

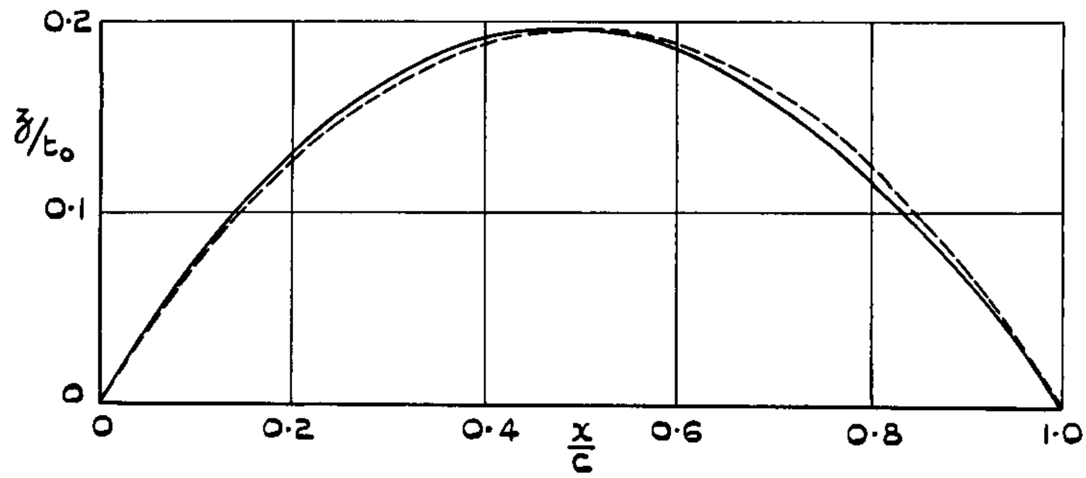


FIG.9. TYPICAL CHORDWISE PROFILE ON PARABOLIC PLANFORM WING.

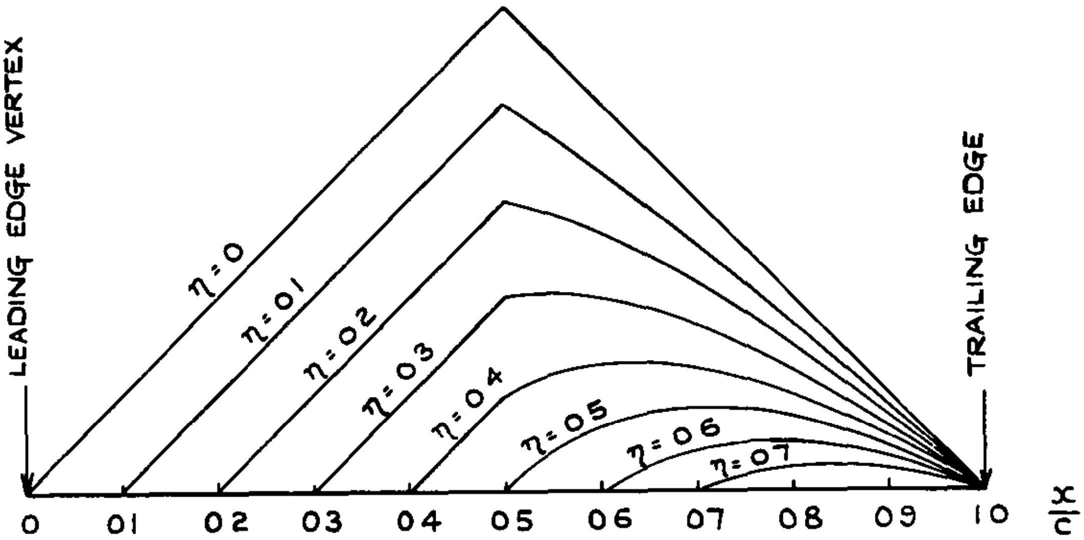


FIG. 10. CHORDWISE PROFILES ON DELTA WING WITH WEDGE ROOT PROFILE AND STRAIGHT SURFACE GENERATORS PARALLEL IN PLAN VIEW TO TRAILING EDGE.

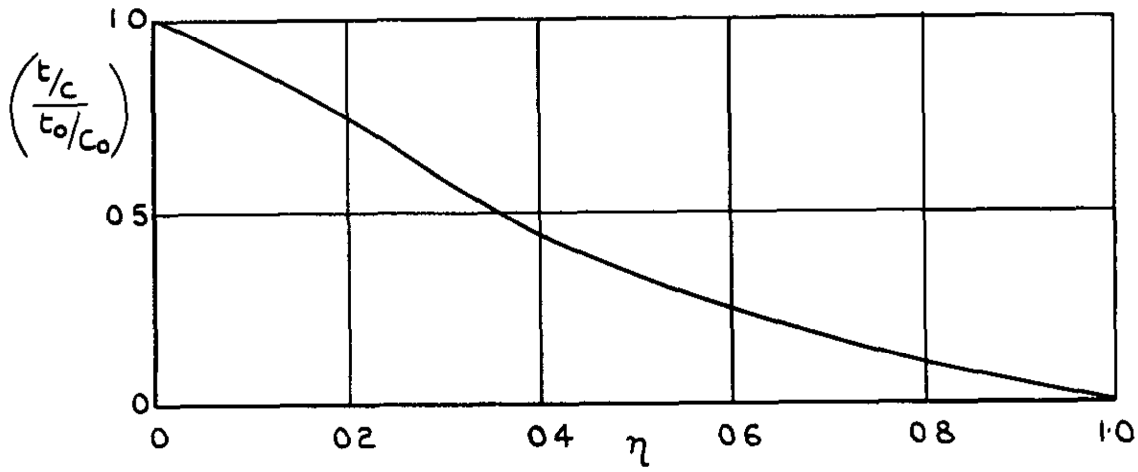


FIG. 11. THICKNESS / CHORD RATIO DISTRIBUTION ON WING OF FIG. 10.

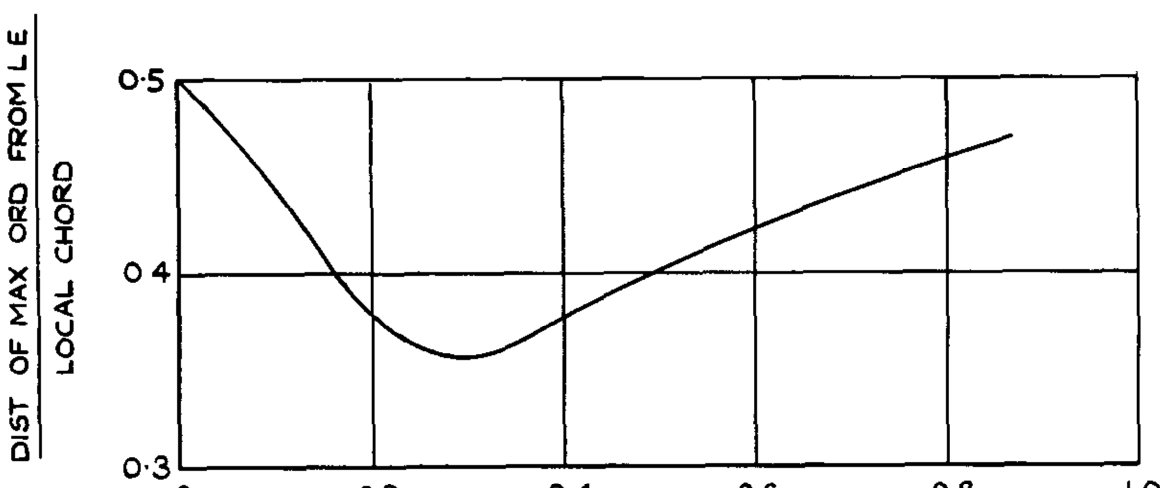


FIG. 12. SPANWISE VARIATION OF MAXIMUM THICKNESS POSITION ON WING OF FIG. 10.

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