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By

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Summary.—The design of a turbine stage is considered on the basis of free vortex flow from the nozzles and blades and some of the factors which limit the design of an efficient turbine stage are discussed.

As the flow conditions at the root of the blades are of greater importance in limiting the design than those at the mean diameter, calculations of the stage performance are made for various values of nozzle angle, reaction and exhaust swirl at the inner diameter of the nozzles and blades. The results of these calculations are presented in the form of a series of curves which show how the design conditions, such as mass flow per unit annulus area, rim speed and Mach numbers relative to the blades, vary with work output from the turbine stage. These curves enable a quick estimate to be made of a suitable turbine stage design to meet given requirements of mass flow and work output, and an example is given showing their application.

Introduction.—In most text books on steam turbine design the flow relative to the nozzles and moving blades is estimated on the assumption of a uniform radial pressure distribution at outlet from the nozzles and blades. This enables turbines to be classified as :—

(i) Impulse turbines—when the pressure at inlet to the blades is equal to the pressure aft of the blades and the velocity through the blading is almost constant.

(ii) Reaction turbines—when the pressure at inlet to the blades is greater than the pressure aft of the blades and the flow is accelerated through the blading.

When the leaving velocity aft of the blades is axial the chief factor in determining whether a turbine works as an impulse turbine or reaction turbine is the ratio of peripheral blade speed to gas speed from the nozzles. With the assumption of uniform pressure distribution this ratio is usually determined from the flow conditions at the mean diameter.

It is now well known that the centrifugal forces due to the swirl component of velocity from the nozzles give rise to a radial pressure gradient across the nozzle annulus. With a uniform static pressure distribution at outlet from the blades (this is approximately true if there is little swirl in the exhaust) the pressure drop across the blading is greater at the tip than at the root. The old distinction between reaction and impulse turbines ceases to apply, and turbines operating with a high degree of reaction at the blade tip may have impulse conditions at the blade root. This also means that the gas inlet angle to the blade varies considerably from tip to root. Moreover for impulse conditions at the mean diameter the turbine would have a pressure rise across the blade roots (negative reaction) and with highly cambered blades this unfavourable pressure gradient may cause high blade losses.

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To avoid high losses due to the turbine working with an unfavourable pressure gradient across the blade roots and incorrect blade inlet angles, any basis of turbine design should take into consideration the radial pressure distribution at outlet from the nozzles. One such method frequently used, although strictly it is only correct for zero nozzle and blade loss, is to design for free vortex flow from the nozzles (and blades if there is any exit swirl); that is with constant axial velocity and with the swirl velocity in any annulus inversely proportional to the diameter of the annulus. Unfortunately this adds somewhat to the labour involved in determining the turbine design.

Some curves have therefore been prepared to enable a quick estimate to be made of the performance and size of turbine required for a given work output. The design is based on free vortex flow from the nozzles and blades.

2. *Method of Design.*—At outlet from the nozzles the radial pressure distribution for any form of nozzle is determined from the vortex conditions for radial equilibrium[1]. When the nozzle angle is fixed these conditions give the variation in swirl velocity with radius. In the free vortex design considered here, the law of variation of swirl velocity is given and this imposes a certain change of nozzle angle with radius. Neglecting losses* the variation in swirl velocity is given by the free vortex conditions:—

$$\begin{aligned} \text{swirl velocity} \times \text{radius} &= \text{constant} \\ \text{axial velocity} &= \text{constant} \end{aligned}$$

and the nozzle efflux angle increases with the radius ($\tan \alpha_1$ is proportional to r).

Since the density of the gas at outlet from the nozzles is greater at the tip than at the root (this follows from the radial pressure distribution at the nozzle outlet) and the axial velocity is constant over the whole nozzle annulus, then the mass flow per unit area is greater at the tip than at the root. Thus, for constant axial leaving velocity from the blades, the stream tubes for constant mass flow take the form shown in Fig. 1, and the gas entering the blade at a radius r_1 may leave the blade at some other radius r_2. The assumption that the gas enters and leaves the blades at the same diameter, an assumption frequently used in turbine design to obtain the velocity diagrams, is therefore not strictly correct except in special circumstances.

Using the notation given in Fig. 1, the relation between available, *i.e.* isentropic, heat drop across the stage (from total head to static pressure aft of the blades), work output, losses and leaving energy for any stream tube is given by

$$\dagger \Delta H = \underbrace{u_1 c_1 \cos \alpha_1 + u_2 c_2 \cos \alpha_2}_{\text{work done when reheat is neglected}} + \underbrace{\lambda_N \tfrac{1}{2} c_1^2 + \lambda_B \tfrac{1}{2} v_2^2}_{\text{losses}} + \underbrace{\tfrac{1}{2} c_2^2}_{\text{leaving energy}} .$$

With free vortex flow design

$$\begin{aligned} u_1 c_1 \cos \alpha_1 &= \text{const} , \\ u_2 c_2 \cos \alpha_2 &= \text{const} , \end{aligned}$$

and, therefore, the work done per lb of gas is constant over the whole blade annulus.

Although the free vortex flow conditions do impose some twist on the stationary nozzles they are convenient to take as a basis of design because of this constancy of work done. With other forms of vortex flow it is generally necessary to integrate over the nozzle and blade outlet areas to obtain the work done.

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* The effect of nozzle losses are considered in Ref. 1. For reasonably efficient nozzles the effect of nozzle loss on the radial variation of swirl can be neglected.

† The losses are slightly less than shown when reheat is taken into account.

It is readily seen that with zero swirl in the exhaust the pressure is uniform aft of the blades; this implies constant heat drop over the stage, and therefore with uniform leaving energy the stage losses must be constant over the whole annulus, *i.e.*

$$*\underbrace{\Delta H}_{\substack{\text{Heat drop}\\ =\text{ constant}}} = \underbrace{u_1 c_1 \cos \alpha_1}_{\substack{\text{Work done}\\ =\text{ constant}}} + \underbrace{\lambda_N \tfrac{1}{2} c_1^2 + \lambda_B \tfrac{1}{2} v_2^2}_{\substack{\text{Losses, assumed}\\ \text{constant}}} + \tfrac{1}{2} c_2^2 .$$

If the stage losses are not constant the leaving velocity is not uniform and an integration becomes necessary to obtain the exhaust leaving energy.

In these calculations the estimate of the turbine performance was made on the basis of constant stage losses over the whole of the nozzle and blade annulus. For convenience in calculation the nozzle-loss coefficient λ_N was taken to be $0 \cdot 1$ over the whole nozzle height and the turbine performance estimated on the basis of a blade-loss coefficient $\lambda_B = 0 \cdot 1$ at the blade root. With little reaction at the blade root this implies that the blade-loss coefficient increases slightly towards the tip of the blade in order to maintain constant stage losses.

The design considered here is, in effect, based on root values of nozzle loss, blade loss, nozzle angle and reaction. This was done in preference to designing on the mean diameter, which would have given a slightly better estimate of turbine performance based on the same nozzle and blade-loss coefficients, since the root conditions are, in general, of greater importance in limiting the design than those at the mean diameter. The lowest reaction and highest inlet Mach number relative to the blading occur at the root section. Also the root section is the most important in determining the stiffness of the blade section, and a rough estimate of the section modulus can be made once the gas angles at the root are known.[2]

3. *Reaction.*—In the design curves the reaction is given at the root radius and the percentage reaction is defined as:—

$$\text{Reaction} = 100 \left[1 - \frac{\text{Isentropic heat drop across nozzles at root radius}}{\text{Isentropic heat drop across the stage from total pressure before the nozzles to static pressure aft of the blades}} \right].$$

This definition means that a 50 per cent reaction turbine has approximately equal heat drop across nozzles and blades.

With free vortex flow, the estimation of the ratio of rim speed to gas speed from the nozzles to give a certain reaction at the blade root is complicated by the radial inflow through the blades. A brief outline of the method used for determining the reaction is therefore given below, and it should be noted that in the design considered it is assumed that there is no "flare" through the blades, *i.e.* the tip and root diameters of the blade at inlet are equal to those at outlet.

When the blade losses are known the amount of reaction determines the relationship between the inlet and exit velocities relative to the blade. For no reaction this relationship at the root of the blade is

$$v_1^2 = v_2^2 (1 + \lambda_B) .$$

From the velocity triangles at the root of the blade (with the assumed conditions of no flare through the blading the gas at the root enters and leaves the blade at the same radius) this gives for no reaction,

$$c_{1r}^2 \{1 + \xi^2 - 2\xi \cos \alpha_{1r}\} = (1 + \lambda_B) c_{1r}^2 \left\{ \frac{c_2^2}{c_{1r}^2} + \xi^2 + 2\xi \frac{c_2}{c_{1r}} \cos \alpha_{2r} \right\},$$

or

$$\xi = \frac{1 - (1 + \lambda_B) \dfrac{c_2^2}{c_{1r}^2} - \lambda_B \xi^2}{2 \cos \alpha_{1r} + (1 + \lambda_B) \dfrac{c_2}{c_{1r}} \cos \alpha_{2r}}, \qquad \dots \quad (1)$$

where

$$\xi = \frac{u}{c_{1r}} = \frac{\text{Rim speed}}{\text{Gas speed from root of nozzles}}$$

* The losses are slighty less than shown when reheat is taken into account.

With no radial flow the ratio c_2/c_{1r} is obtained from the continuity equation for equal mass flow per unit area at inlet and exit from the blades; *i.e.*

$$\rho_{1r} c_{1r} \sin \alpha_{1r} = \rho_2 c_2 \sin \alpha_{2r} \,, \qquad \qquad (2)$$

or

$$\frac{c_2}{c_{1r}} = \frac{\rho_{1r} \sin \alpha_{1r}}{\rho_2 \sin \alpha_{2r}} \,.$$

With zero blade loss ($\rho_{1r} = \rho_2$) and axial leaving velocity ($\alpha_{2r} = 90$ deg.) $c_2/c_{1r} = \sin \alpha_{1r}$. Substituting in equation (1) gives the well known ratio for impulse conditions and no swirl, namely,

$$\frac{u}{c_{1r}} = \frac{\cos \alpha_{1r}}{2} \,.$$

Owing to the radial inflow through the blading with free vortex flow the mass flow per unit area at entry to the blade root is not equal to the mass flow per unit area at exit from the blades and the continuity conditions expressed by equation (2) no longer hold. The ratio c_2/c_{1r} at the blade root has now to be determined by integrating over the nozzle annulus and equating the total mass flow entering the blades to the total mass flow leaving the blades.

With axial leaving velocity this gives

$$\pi \{r_t^2 - r_r^2\} \rho_2 c_2 = \int \rho_1 c_1 \sin \alpha_1 \, 2\pi r \, dr \,,$$

and therefore, for free vortex conditions

$$\frac{c_2}{c_{1r}} = \frac{\sin \alpha_{1r} \int \rho_1 \, 2\pi r \, dr}{\pi \, [r_t^2 - r_r^2] \, \rho_2} \,.$$

With reaction the same method is applied for obtaining the speed ratio ξ, but the equation giving the value of ξ now involves a reaction term and becomes

$$\xi = \frac{\left\{1 + \dfrac{R\,(1 + \lambda_N)\,(1 + k)}{1 - R} - (1 + \lambda_B)\,\dfrac{c_2^2}{c_{1r}^2}\right\} - \lambda_B\,\xi^2}{2\left\{\cos \alpha_{1r} + (1 + \lambda_B)\,\dfrac{c_2}{c_{1r}} \cos \alpha_{2r}\right\}} \,,$$

where k is a reheat factor which depends on the nozzle loss, reaction and pressure ratio across the stage ($k = 0$ for $\lambda_N = 0$),

and

$$R = \frac{\text{percentage reaction}}{100} \,.$$

4. *Gas Angles Relative to Blade.*—The gas angles relative to the blades are obtained from the velocity triangles (Fig. 1). At the root

$$\cot \beta_{1r} = \frac{\cos \alpha_{1r} - \xi}{\sin \alpha_{1r}} \,, \qquad \cot \beta_{2r} = \frac{\xi + \dfrac{c_2}{c_{1r}} \cos \alpha_{2r}}{\dfrac{c_2}{c_{1r}} \sin \alpha_{2r}} \,.$$

5. *Efficiency.*—The turbine efficiency has been determined from the root conditions with the assumed values of nozzle and blade-loss coefficients $\lambda_N = 0 \cdot 1$ and $\lambda_B = 0 \cdot 1$. The curves, however, are plotted so that the general design and efficiency can be estimated to sufficient accuracy for values of the nozzle-loss coefficient λ_N within the range $\lambda_N = 0 \cdot 05$ to $0 \cdot 15$. Unfortunately no simple correction is possible to allow for other values of the blade-loss coefficient.

The actual values of the blade and nozzle losses depend on several factors, such as the geometry of the blades and nozzles, pitch/chord ratio, aspect ratio, degree of reaction, deflection, incidence relative to the blade leading edge, Mach number and Reynolds number. Ultimately it is hoped that wind-tunnel tests will provide a basis for the estimation of these losses and so give some idea of the attainable turbine efficiencies. So far only a few results are readily available, and these indicate that the mean losses for the aspect ratios normally used (about 2·0) lie roughly between 5 and 10 per cent. (λ_N and λ_B between 0·05 and 0·1). Kearton gives[3] a curve, deduced from a few test results, showing the variation in velocity coefficient with nozzle efflux angle, but it is not known under what conditions of aspect ratio, Reynolds number etc. this applies. The curve is shown in Fig. 2 plotted on the basis of λ_N against nozzle angle and indicates a rapid increase in loss with decrease in nozzle angle below about 15 deg.

Due to gland leakage and tip-clearance losses the actual turbine efficiency will, in general, be below that estimated on the basis of wind-tunnel tests alone. On this account, therefore, it is safer to base the estimate of turbine efficiency on slightly pessimistic values of blade and nozzle loss.

On the curves two values of turbine efficiency are given:—

(1) Static efficiency η_S defined as

$$\frac{\text{work done} + \text{outlet kinetic energy}}{\text{isentropic heat drop from inlet total-head pressure to outlet static pressure}}\,.$$

(2) Total-head efficiency η_T defined as

$$\frac{\text{work done}}{\text{isentropic heat drop from inlet total-head pressure to outlet total-head pressure}}\,.$$

For arithmetical convenience an approximate method was used for calculating total-head efficiency which assumed that the isentropic heat drop from inlet total-head pressure to outlet total-head pressure was equal to the isentropic heat drop from the inlet total-head to outlet static pressure less the heat equivalent of the leaving energy.

The error involved, although generally small, gives a slightly optimistic value of turbine efficiency for the large nozzle angles at the higher pressure ratios.

6. *Results* of *Calculations*.—The calculations were made for a range of nozzle angles from 15 to 40 deg. and for ratios of tip radius to root radius of 1·2 and 1·4. The value of specific heat was taken as 0·274; this gives $\gamma/(\gamma - 1) = 4{\cdot}0$. No great error in design is introduced if substantially different values of c_p are taken.

The results of the calculation are shown plotted in a non-dimensional form in nine sets of curves. These cover the following range of reaction and outlet swirl.

(1) No reaction at root ..	0 deg. swirl at root, Fig. 3.
	10 deg. swirl at root, Fig. 4.
	20 deg. swirl at root, Fig. 5.
(2) 10 per cent. reaction at root	0 deg. swirl at root, Fig. 6.
	10 deg. swirl at root, Fig. 7.
	20 deg. swirl at root, Fig. 8.
(3) 20 per cent. reaction at root	0 deg. swirl at root, Fig. 9.
	10 deg. swirl at root, Fig. 10.
	20 deg. swirl at root, Fig. 11.

Each figure is divided into two parts; one part gives the total-head efficiency, rim speed, mass flow per unit area and Mach numbers plotted against work done; the other gives the static efficiency, work ratio and the gas angles relative to the blades at the root plotted against work done. The first part is the more important for determining the turbine scantlings. The second enables an estimate to be made of the blade-root camber; this is useful in estimating the stiffness of the blade section. The work ratio was used in the intermediate calculations and gives the ratio of

$$\frac{\text{work output from turbine}}{\text{available work for the heat drop from inlet total-head to outlet static pressure}}.$$

The constants to be used in conjunction with the curves are given in Table 1.

Once the conditions at the blade root are known the conditions at any other radius can be readily determined. The relationship between root values of nozzle angle, blade inlet and outlet angles and those for any other radius are given in Table 2. It should be noted that the blade inlet angle need not necessarily be made equal to the gas angle, since wind tunnel tests show that there is usually little change in blade loss over an incidence range + 5 to − 15 deg.

7. *Use of Curves*.—7.1. *Design of Turbine Stage*.—Considering a turbine driving a compressor the known quantities are

ΔT_c = temperature rise per lb. of air through the compressor.

Q_c = mass flow through the compressor (lb/sec).

Q = mass flow through the turbine (lb /sec).

T_0 = total-head temperature at inlet to turbine.

P_0 = total-head pressure at inlet to turbine.

c_{pc} = mean specific heat at constant pressure, during compression.

c_{pT} = 0·274, mean specific heat at constant pressure during expansion through the turbine.

($c_{pT} = 0{\cdot}274$ should always be taken in using the given curves. The estimated mean c_p over the expansion will generally be within ∓ 5 per cent. of this value and differences substantially greater than this have little effect on the design).

The temperature drop per lb of air through the turbine is

$$\Delta T_w = \frac{c_{pc}}{0{\cdot}274} \cdot \frac{Q_c}{Q} \cdot \frac{1}{\eta_m} \times \Delta T_c ,$$

where η_m is the mechanical efficiency.

With the value of $\Delta T_w/T_0$ an estimate of the rim speed, annular flow area, Mach number relative to the blade inlet and absolute Mach number in the exhaust can be obtained from the curves for any root nozzle angle and for various values of reaction and exit swirl. At first it will be necessary to assume a value of the ratio of tip radius to root radius, but this can easily be corrected once an estimate of the required flow area has been obtained. The above important factors governing the turbine design are derived from the curves as follows :—

(1) Rim speed (ft/sec) = $157{\cdot}2\sqrt{T_0}$ × value of $u/\sqrt{T_0}$ obtained from curve.

(2) Mass flow per unit annulus area (lb/sec/sq in).

$$= 1{\cdot}64 \times \frac{P_0}{\sqrt{T_0}} \times \text{value of } \frac{Q}{S}\sqrt{\left(\frac{T_0}{P_0}\right)} \text{ obtained from curve.}$$

(3) Annular flow area (sq in.) $= \dfrac{Q}{\text{Mass flow per unit area}}$.

(4) Centrifugal-force stress for untapered blade = $0{\cdot}000605\,(\text{rpm}/1000)^2$ × flow area in sq in. (tons/sq in. for blade material with a density 0·3 lb/cu in.).

(5) Rim diameter in inches $= \dfrac{\text{Rim speed (ft/sec)}}{(\text{rpm}/1000)} \times 0{\cdot}229$

(6) Radius ratio $(r_t/r_r) = \left[\dfrac{\text{Annular flow area}}{\frac{\pi}{4} \times (\text{rim diameter})^2} + 1\right]^{1/2}$.

(7) Estimates of Mach number are obtained directly from the curves.

In general the turbine r.p.m. will be fixed by the compressor design, and then it will often be found that the design of turbine is fixed by the following limiting conditions:—

(1) Disc stress.

(2) Blade centrifugal-force stress.

(3) Mach number relative to blade inlet or in exhaust.

(4) Gas bending stress.

(5) Radius ratio.

In some instances these limiting conditions may also fix the permissible r.p.m. When no limiting factors are involved there would be some latitude in the turbine design. If high efficiency were desired the turbine would be designed for a good aspect-ratio blade (2 to 4), a fairly high degree of reaction at the blade root and no outlet swirl. On the other hand if low weight is the chief requirement the turbine would be designed with low reaction and exit swirl.

An example showing the use of the design curves is given in the Appendix.

7.2. *Limiting Factors in Turbine Design.*—(1) *Disc Stress.*—This fixes the maximum value of $u/\sqrt{T_0}$. The actual permissible value depends on the type and size of disc, *i.e.* whether the disc is bored or solid, single or multi-stage, and on the relative depth of blade roots. From present experience a satisfactory disc design can be obtained with the following rim speeds and maximum centrifugal stress.

(a) 900 ft/sec and 15 t/sq in. for a single-stage unbored disc.

(b) 800 ft/sec and 20 t/sq in. at the bore for a single-stage bored disc with ratio of bore-to-rim diameter 0·2.

(*c*) 750 ft/sec 12 t/sq in. for a two-stage unbored disc.

The value of disc stress for a given rim speed does, of course, depend on the blade and blade-root loadings, and with lightly loaded discs, rim speeds greater than the above values may be possible for the same stresses.

The maximum permissible disc centrifugal-force stress depends on the temperature and material of the disc. For existing gas turbines the centrifugal-force stress is generally about 15 t/sq in. for a single-stage disc and slightly lower for a multi-stage disc having a large disc head.

(2) *Centrifugal Blade Stress.*—For a given rotational speed and mass flow this limits the blade annulus flow area and so fixes the nozzle efflux angle. Conditions of work output and mass flow can occur when the centrifugal-force blade stress actually imposes a limit on the rotational speed.

In existing gas turbines using nimonic 80 blade material the centrifugal-force stress in the blade at maximum r.p.m. is often about 11 t/sq in. The blades have a high degree of taper and for a parallel-sided blade the corresponding stress would be about 20 t/sq in.

(3) *Mach Number.*—Although no information is at present available to show what limits should be placed on entry Mach number relative to the blades, entry Mach numbers greater than 0·75 should be avoided as far as possible.

Similarly the Mach number of the leaving velocity in the turbine exhaust should be kept as low as possible.

(4) *Gas Bending Stress in the Blading.*—This is often a limiting factor but is one which can only be taken into consideration in conjunction with turbine weight and efficiency. It is generally possible to keep the gas bending stress low by increasing the blade chord, and the limit of increase in blade chord is largely determined by the increase in weight of disc and decrease in blade aspect ratio which the increase in blade chord entails.

(5) *Blade Height and Radius Ratio.*—Even though the blade stresses may not be unduly high, values of the ratio tip-to-root radius greater than 1·5 are generally undesirable since they give a highly twisted blade. Difficulties of manufacture are thereby increased.

Also relatively long blades (high aspect ratio) are liable to give greater trouble due to vibration than shorter blades, and long blades are usually associated with high values of tip radius/root radius.

7.3. *Two-stage Turbine.*—The design of a two-stage turbine follows from the design of a single stage. The leaving conditions from the first stage give the entry conditions to the second stage, that is,

$$\text{total-head pressure} = P_0\left[1 - \frac{\Delta T_{w1}}{\eta_T T_0}\right]^4,$$

$$\text{total-head temperature} = T_0 - \Delta T_{w1}\,,$$

where ΔT_{w1} is the work output from the first stage.

The work output required from the second stage is ΔT_{w2} where $\Delta T_{w2} = \Delta T_w - \Delta T_{w1}$. In general a satisfactory design will be given with ΔT_{w1} approximately equal to ΔT_{w2}.

It will usually be found that any conditions limiting the turbine design occur in the last stage and that the first stage design can be chosen to suit the dimensions fixed by the last stage.

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3	Kearton	Steam Turbine Theory and Practice.

APPENDIX

Example of Use of Design Curves

As an example consider the turbine design required for the following conditions:—

$$Q_c = Q = 50 \text{ lb/sec},$$
$$\Delta T_c = 175 \text{ deg. C},$$
$$T_0 = 800 \text{ deg. C} = 1073 \text{ deg K},$$
$$P_0 = 57 \text{ lb/in.}^2 \text{ (abs)},$$
$$\eta_m = 0 \cdot 99\,,$$
$$\text{r.p.m.} = 12{,}000.$$

Centrifugal force stresses in the disc not to be greater than 15 t/sq in. Centrifugal force stress in an untapered blade not to be greater than 17 t/sq in. Mach number at entry to the blade not to be greater than 0·75.

The stress conditions give a limiting rim speed of 900 ft/sec and a maximum flow area of 195 sq in. These give limiting values of $u/\sqrt{T_0} = 0 \cdot 175$ (maximum) and $\frac{Q}{S}\frac{\sqrt{T_0}}{P_0} = 0 \cdot 09$ (minimum).

From the given conditions $\frac{\Delta T_w}{T_0} = 0 \cdot 145$ and assuming $\lambda_N = \lambda_B = 0 \cdot 1$ the turbine design for 0 deg. of swirl, impulse at root, 10 deg. swirl, impulse at root and 10 deg. swirl, 10 per cent. reaction at root can be obtained as shown in the following table.

From the following table it is apparent that with impulse no-swirl conditions at the blade root any design satisfying the stress requirements gives a high-entry Mach number to the blades. With swirl the disc stress conditions are eased somewhat since the work output can be obtained at lower rim speeds, but the blade stress requirement can only be obtained for nozzle angles greater than 20 deg. when the inlet Mach number is between 0·77 and 0·95. For 10 per cent. reaction and 10 deg. swirl the results are shown plotted in Fig. 12 and a satisfactory design is possible for nozzle angles between 20½ and 22½ deg. In this particular example the limitations imposed by the disc stresses are roughly the same as those imposed by the blade stresses, but this would not necessarily hold in other instances.

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Design Particulars obtained from the Curves

Conditions	Nozzle angle	Assumed $\frac{r_t}{r_r}$	Calculated for assumed r_t/r_r			Calculated for corrected r_t/r_r				Remarks
			$\frac{u}{\sqrt{T_0}}$	$\frac{Q}{S}\frac{\sqrt{T_0}}{P_0}$	$\frac{r_t}{r_r}$	$\frac{u}{\sqrt{T_0}}$	$\frac{Q}{S}\frac{\sqrt{T_0}}{P_0}$	M_{1R}	η_T	
Impulse, no swirl ..	15	1·35	0·184	0·0672	1·41	0·1832	0·0685	0·63	87	Blade stress, Rim speed } too high
	20	1·35	0·181	0·0875	1·34	0·181	0·0865	0·72	85·5	Blade stress, Rim speed } too high
	25	1·35	0·173	0·105	1·31	0·1745	0·1035	0·85	83·1	High inlet Mach number
Impulse, 10 deg swirl	15	1·35	0·173	0·0672	1·45	0·1725	0·070	0·67	86·5	Blade stress } too high
	20	1·35	0·167	0·0875	1·40	0·1655	0·089	0·77	84·8	Blade stress } too high
	25	1·35	0·155	0·104	1·39	0·153	0·1055	0·95	81·5	High inlet Mach number
10 per cent reaction, 10 deg swirl.	15	1·4	0·184	0·068	1·415	—	—	0·55	87·8	Blade stress, Rim speed } too high
	20	1·35	0·176	0·088	1·36	—	—	0·66	86·1	Blade stress, Rim speed } too high
	25	1·35	0·163	0·106	1·35	—	—	0·83	83·0	High inlet Mach number

TABLE 1

Constants and Definitions Required for Use of Curves

P_0 total-head pressure at entry to stage in lb/sq in. absolute

T_0 total-head temperature at entry to stage in deg. C absolute

ΔT_w $\dfrac{\text{heat equivalent of work output from the turbine}}{\text{mass flow through turbine} \times 0{\cdot}274}$, or for turbine driving a compressor

temperature rise of cold air through compressor $\times \dfrac{0{\cdot}241}{0{\cdot}274} \cdot \dfrac{Q_c}{Q \cdot \eta_m}$

Q_c mass flow through compressor in lb/sec

Q mass flow through turbine in lb/sec

η_m mechanical efficiency of drive

λ_N nozzle-loss coefficient (defined by nozzle loss $= \lambda_N \frac{1}{2} c_1^2$ where c_1 is the leaving velocity from the nozzles)

λ_B blade-loss coefficient (defined by blade loss $= \lambda_B \frac{1}{2} v_2^2$ where v_2 is the leaving velocity relative to the blade))

u rim speed (ft/sec) $= 157{\cdot}2\sqrt{T_0} \times$ value of $u/\sqrt{T_0}$ obtained from curve

Q/S mass flow lb/sec/sq in. annulus area) $= 1{\cdot}64 \dfrac{P_0}{\sqrt{T_0}} \times$ value of $\dfrac{Q}{S} \cdot \dfrac{\sqrt{T_0}}{P_0}$ obtained from curve

c_{1r} velocity of gas from root of nozzles (ft/sec) $= 157{\cdot}2\sqrt{\left\{\dfrac{(1-R)}{(1+\lambda_N)} \cdot \dfrac{\Delta T_w}{K}\right\}}$

c_2 velocity of gas leaving the stage (ft/sec) $= 157{\cdot}2\sqrt{\left\{\Delta T_w \left[\dfrac{\eta_T - K}{\eta_T \cdot K}\right]\right\}}$

η_T total-head efficiency of turbine—obtained from curve

η_s static turbine efficiency

K work ratio obtained from curve

Centrifugal stress for untapered blade $= 0{\cdot}000605 \left(\dfrac{\text{r.p.m.}}{1000}\right)^2 \times S$ (t/sq in. for blade material with density $0{\cdot}3$ lb/cu in.)

S annular flow area in sq in.

$$\text{Rim diameter in inches} = \frac{\text{Rim speed (ft/sec)}}{(\text{r.p.m.}/1000)} \times 0{\cdot}229$$

$$\text{Radius ratio}\left(\frac{r_{\text{tip}}}{r_{\text{root}}}\right) = \left[\frac{S}{\frac{\pi}{4} \times (\text{rim diameter})^2} + 1\right]^{1/2}$$

$$\left(\frac{r_{\text{tip}}}{r_{\text{root}}}\right)^2 = 1 + 0{\cdot}000598 \frac{Q}{P_0\sqrt{T_0}} \cdot \frac{(\text{r.p.m.}/1000)^2}{\left[\text{Value of } \frac{Q}{S}\frac{\sqrt{T_0}}{P_0}\right]\left[\text{Value of } \frac{u}{\sqrt{T_0}}\right]^2}$$

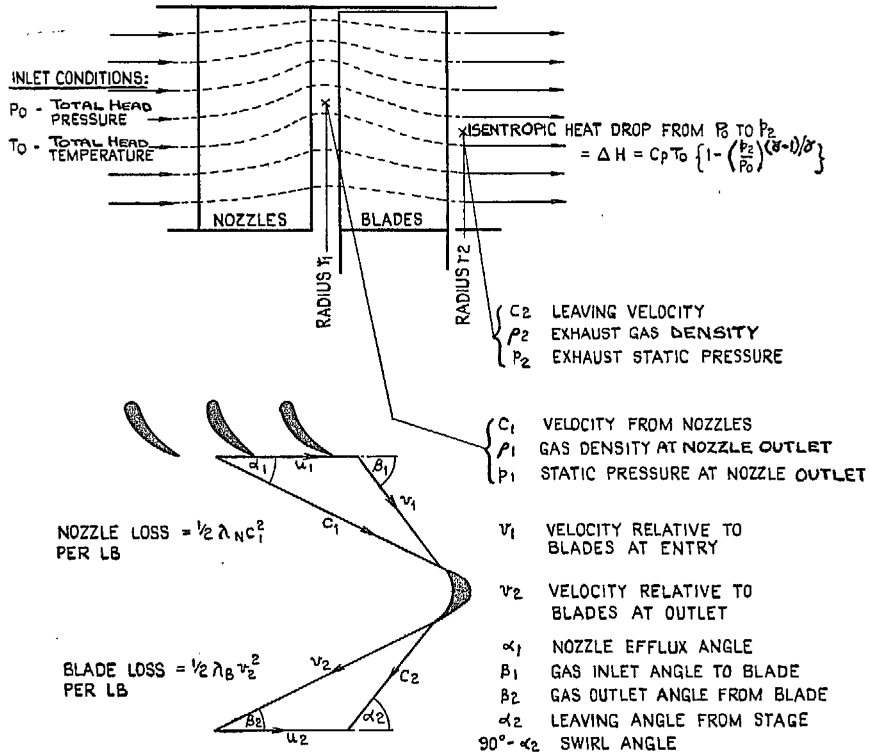
$$\text{Total-head pressure at outlet from stage} = P_0\left[1 - \frac{\Delta T_w}{\eta_T T_0}\right]^4$$

$$\text{Total-head temperature at outlet from stage} = T_0 - \Delta T_w$$

TABLE 2

Relation between Root Values and Values at Any Other Radius

	Root value at radius r_r	Value at any radius r is given by
Gas efflux angle from nozzle	α_{1r}	$\tan \alpha_1 = \frac{r}{r_r} \tan \alpha_1$
Gas entry angle to blade	β_{1r}	$\cot \beta_1 = \frac{r}{r_r} \cot \beta_{1r} - \left(\frac{r}{r_r} - \frac{r_r}{r}\right) \times \cot \alpha_{1r}$
Gas efflux angle from blade	β_{2r}	$\cot \beta_2 = \frac{r}{r_r} \cot \beta_{2r} - \left(\frac{r}{r_r} - \frac{r_r}{r}\right) \times \cot \alpha_{2r}$
Leaving angle from stage = 90° — swirl angle	α_{2r}	$\tan \alpha_2 = \frac{r}{r_r} \tan \alpha_{2r}$
Velocity from nozzle	c_{1r}	$c_1^2 = c_{1r}^2 \left\{ \sin^2\alpha_{1r} + \frac{r_r^2}{r^2} \cos^2\alpha_{1r} \right\}$
Leaving velocity from stage	c_{2r}	$c_2^2 = c_{2r}^2 \left\{ \sin^2\alpha_{2r} + \frac{r_r^2}{r^2} \cos^2\alpha_{2r} \right\}$
Static pressure at outlet to nozzles	$p_{1r} = P_0 \left\{1 - \frac{(1-R)}{K} \frac{\Delta T_w}{T_0}\right\}^{[4]}$	$p_1 = P_0 \left\{1 - \frac{(1-R)}{K} \frac{\Delta T_w}{T_0} \cdot \frac{c_1^2}{c_{1r}^2}\right\}^4$
Degree of reaction	R_R	$R = 1 - (1 - R_R) \left\{\sin^2\alpha_{1R} + \frac{r_R^2}{r^2} \cos^2\alpha_{1R}\right\}$



SUFFIX r IS GENERALLY USED TO DENOTE ROOT VALUES

Fig 1. Diagram showing notation used in text.

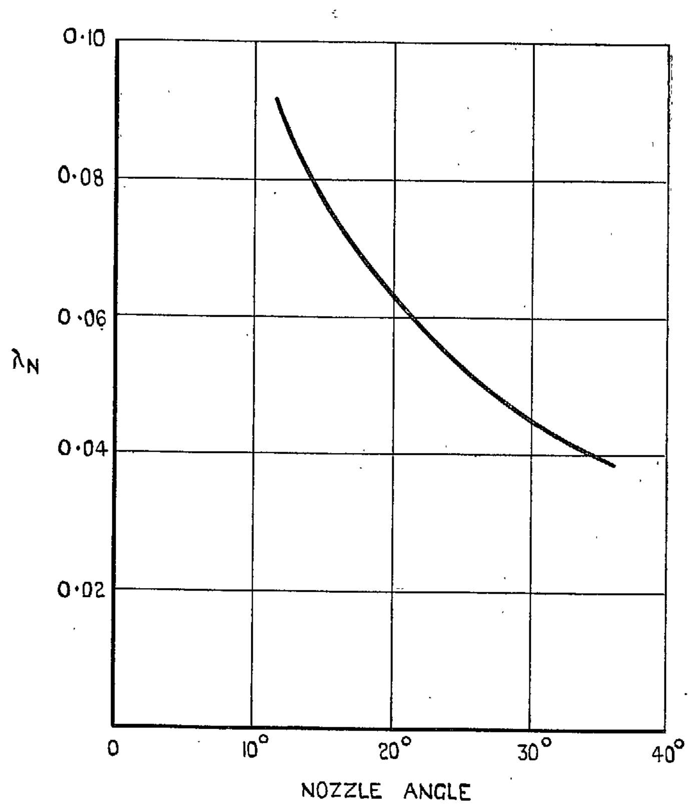


Fig. 2. Variation of nozzle loss coefficient with nozzle angle (taken from Kearton's *Steam Turbine Theory and Practice*).

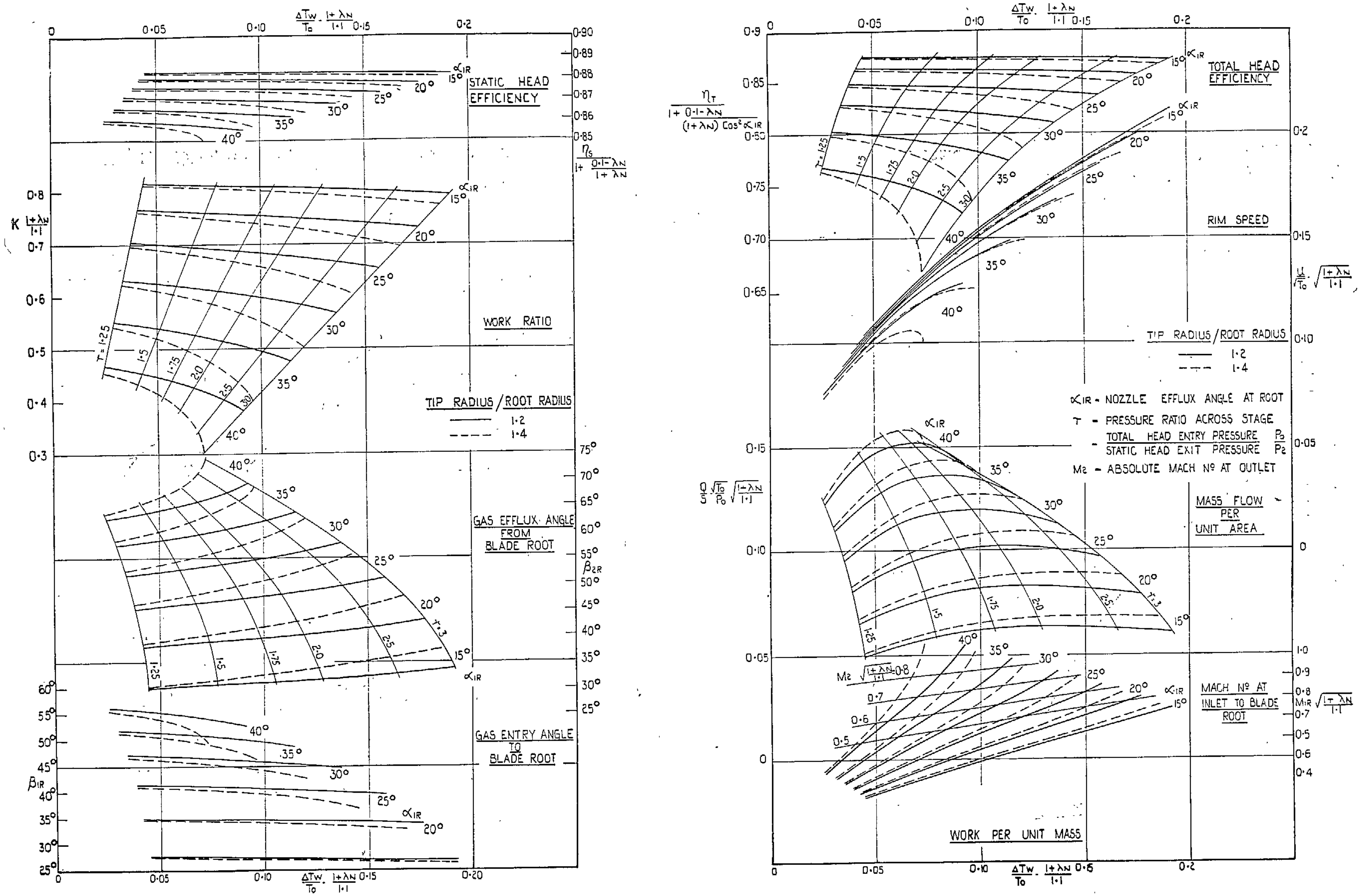


Fig. 3. Turbine designs impulse at root. No swirl. Calculated for $\lambda_N = \lambda_B = 10$ per cent, $c_p = 0{\cdot}274$, $\gamma = 1{\cdot}333$.

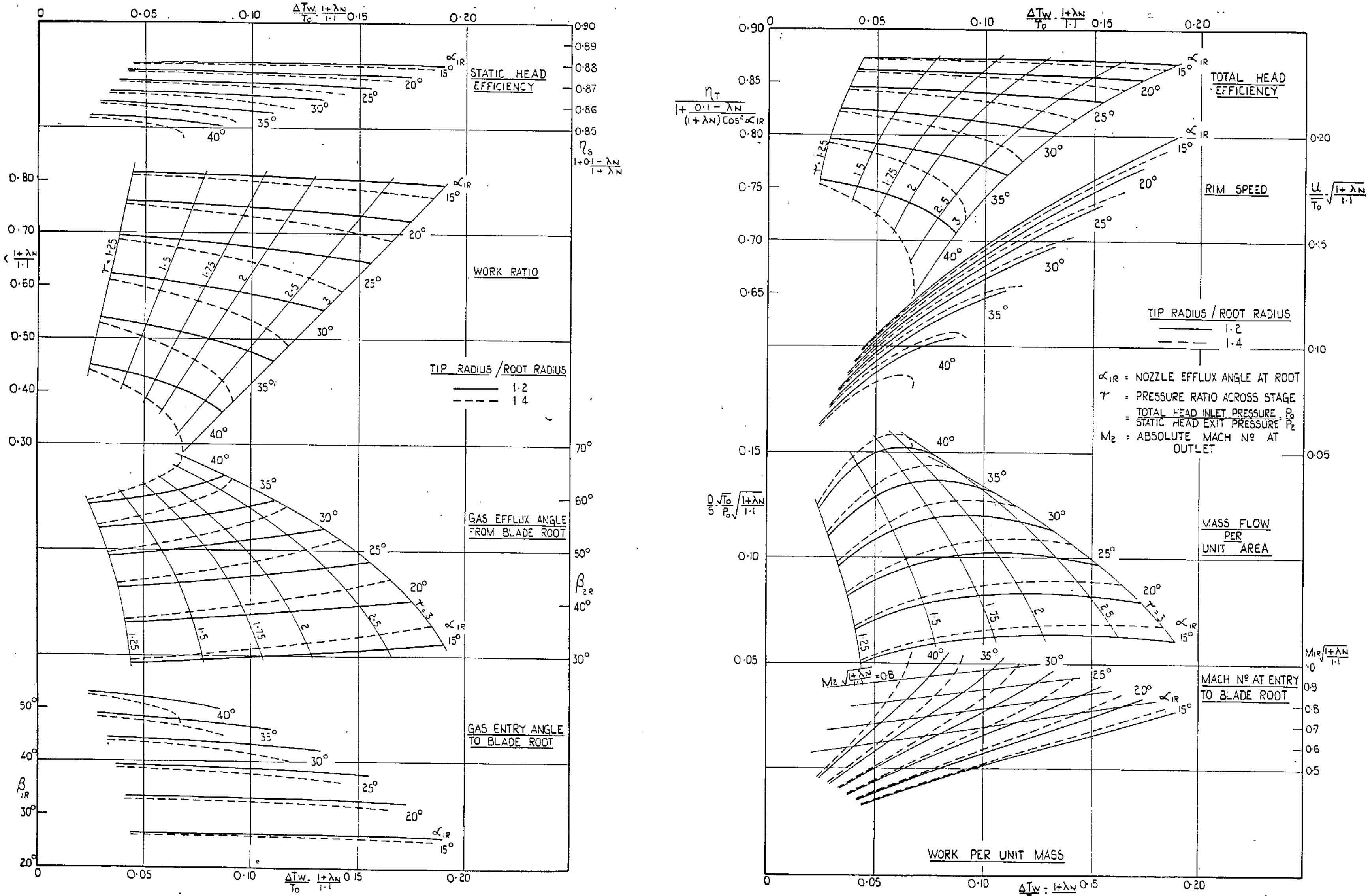


Fig. 4. Turbine design impulse at root. 10 deg. swirl. Calculated for $\lambda_N = \lambda_B = 10$ per cent, $c_p = 0{\cdot}274$, $\gamma = 1{\cdot}333$.

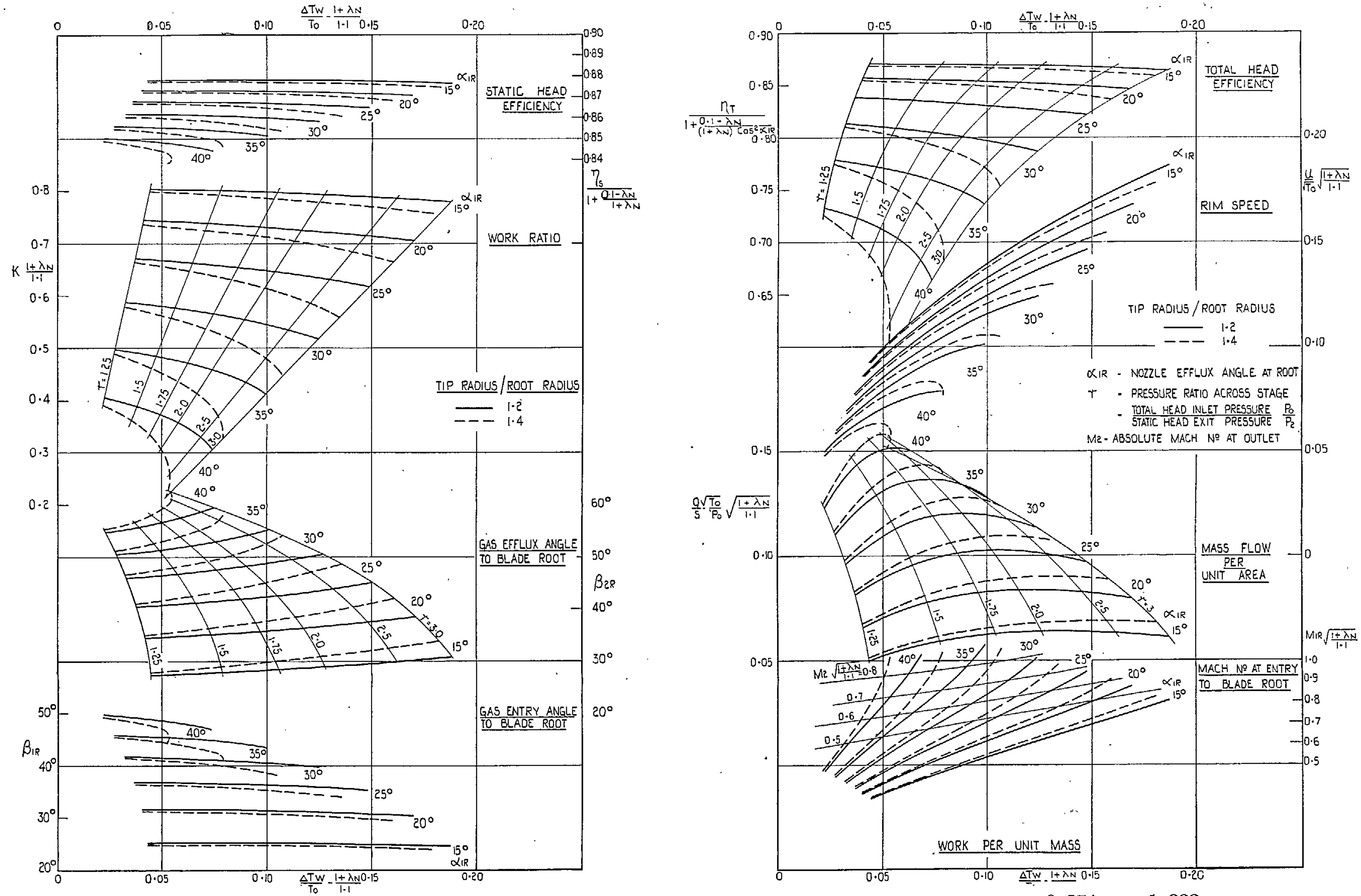


Fig. 5. Turbine design impulse at root. 20 deg. swirl. Calculated for $\lambda_N = \lambda_B = 10$ per cent, $c_p = 0\cdot274$, $\gamma = 1\cdot333$.

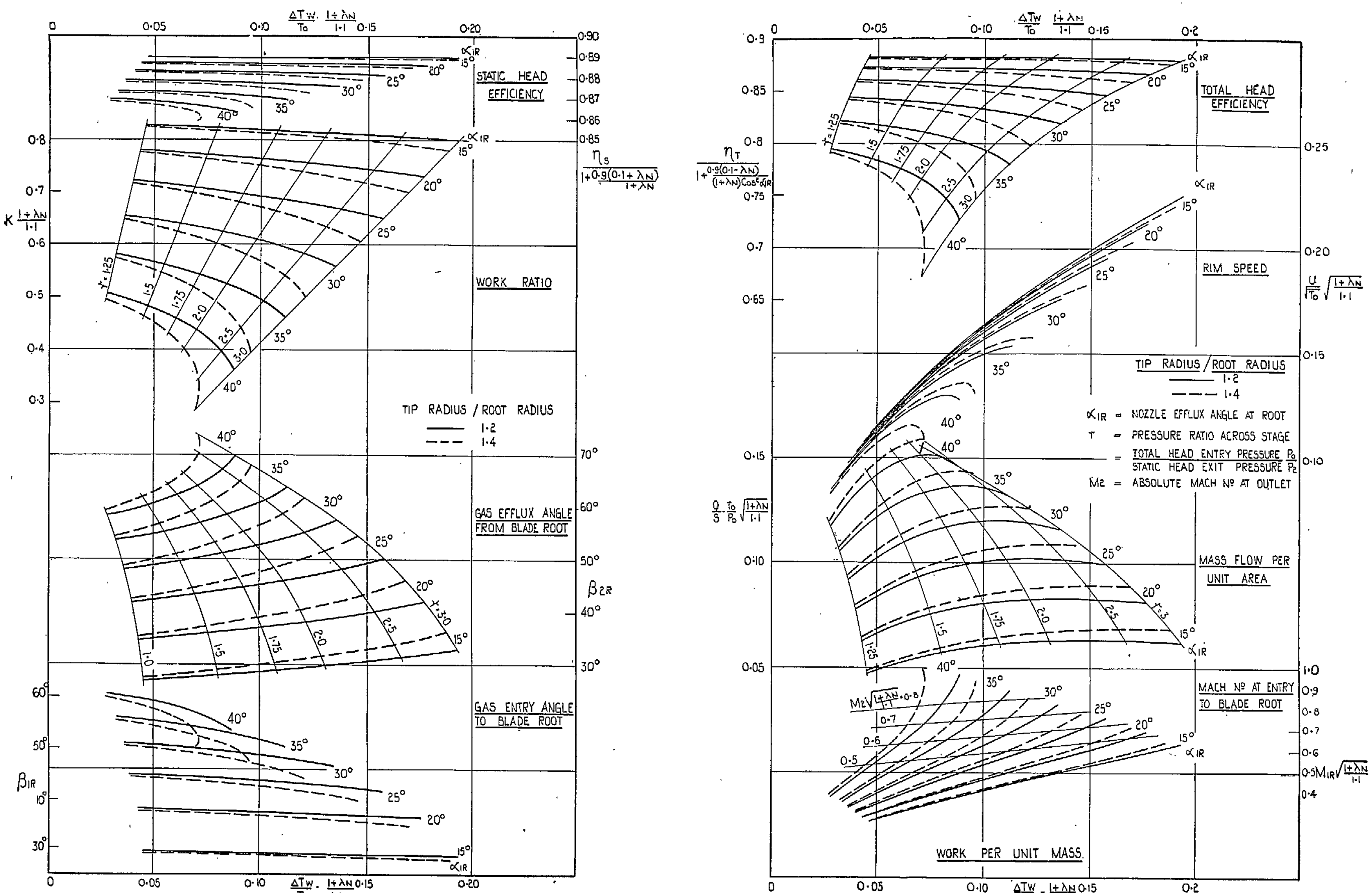


Fig. 6. Turbine design 10 per cent reaction at root. No swirl. Calculated for $\lambda_N = \lambda_B = 10$ per cent, $c_p = 0 \cdot 274$, $\gamma = 1 \cdot 333$.

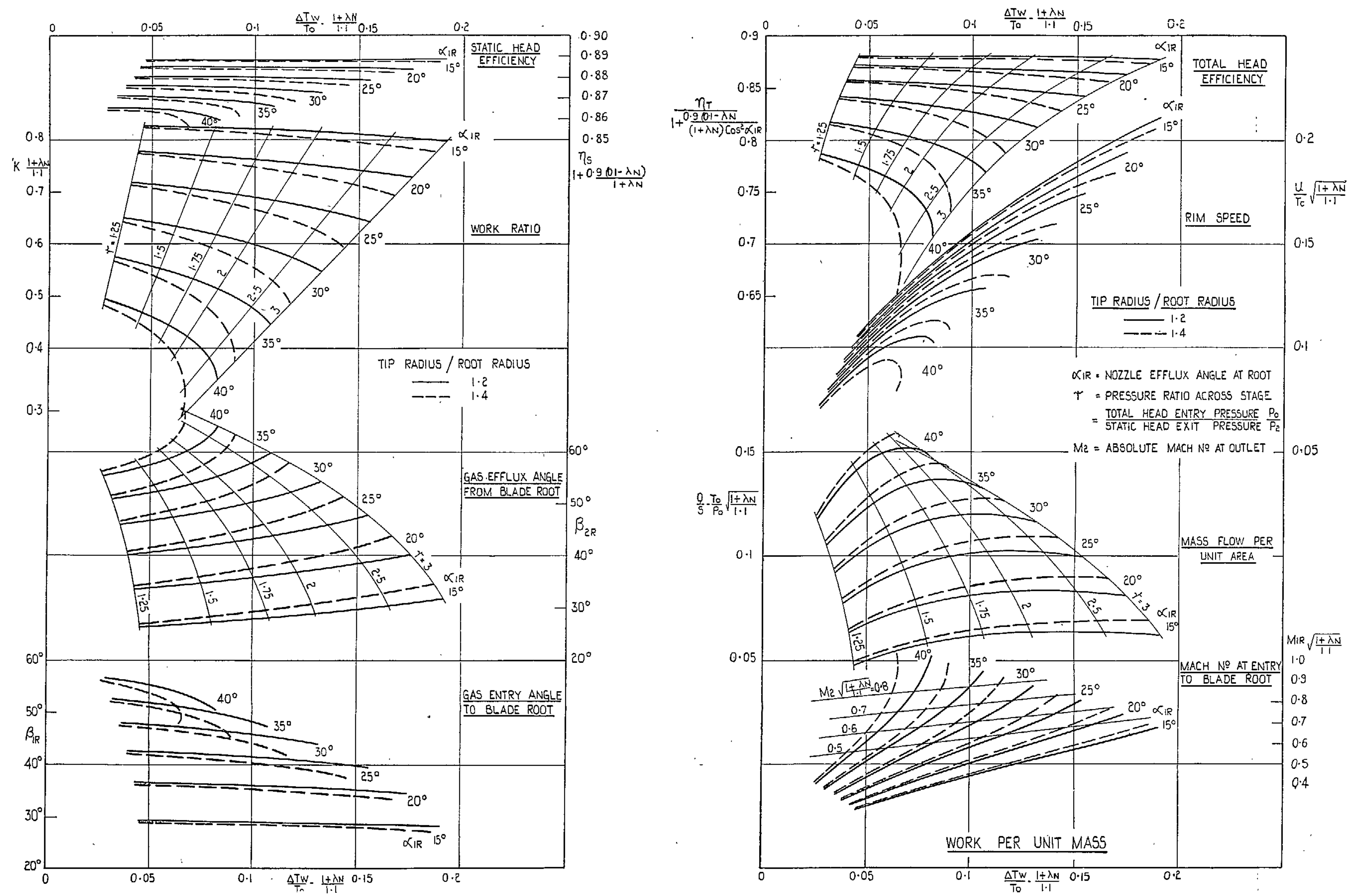


Fig. 7. Turbine design 10 per cent reaction at root. 10 deg. swirl. Calculated for $\lambda_N = \lambda_B = 10$ per cent, $c_p = 0 \cdot 274$, $\gamma = 1 \cdot 333$

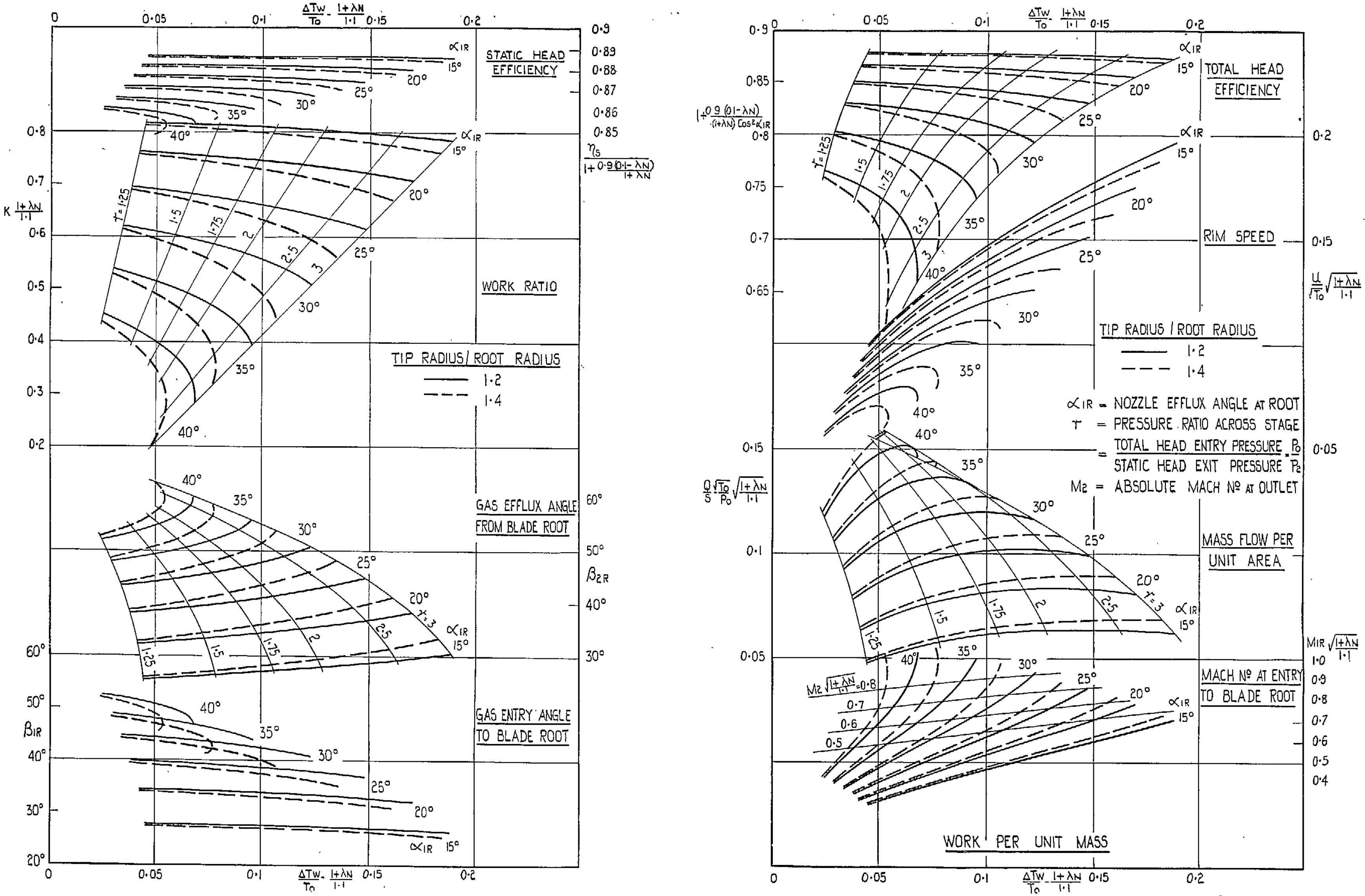


Fig. 8. Turbine design 10 per cent reaction at root. 20 deg. swirl. Calculated for $\lambda_N = \lambda_B = 10$ per cent, $c = 0{\cdot}274$, $\gamma = 1{\cdot}333$

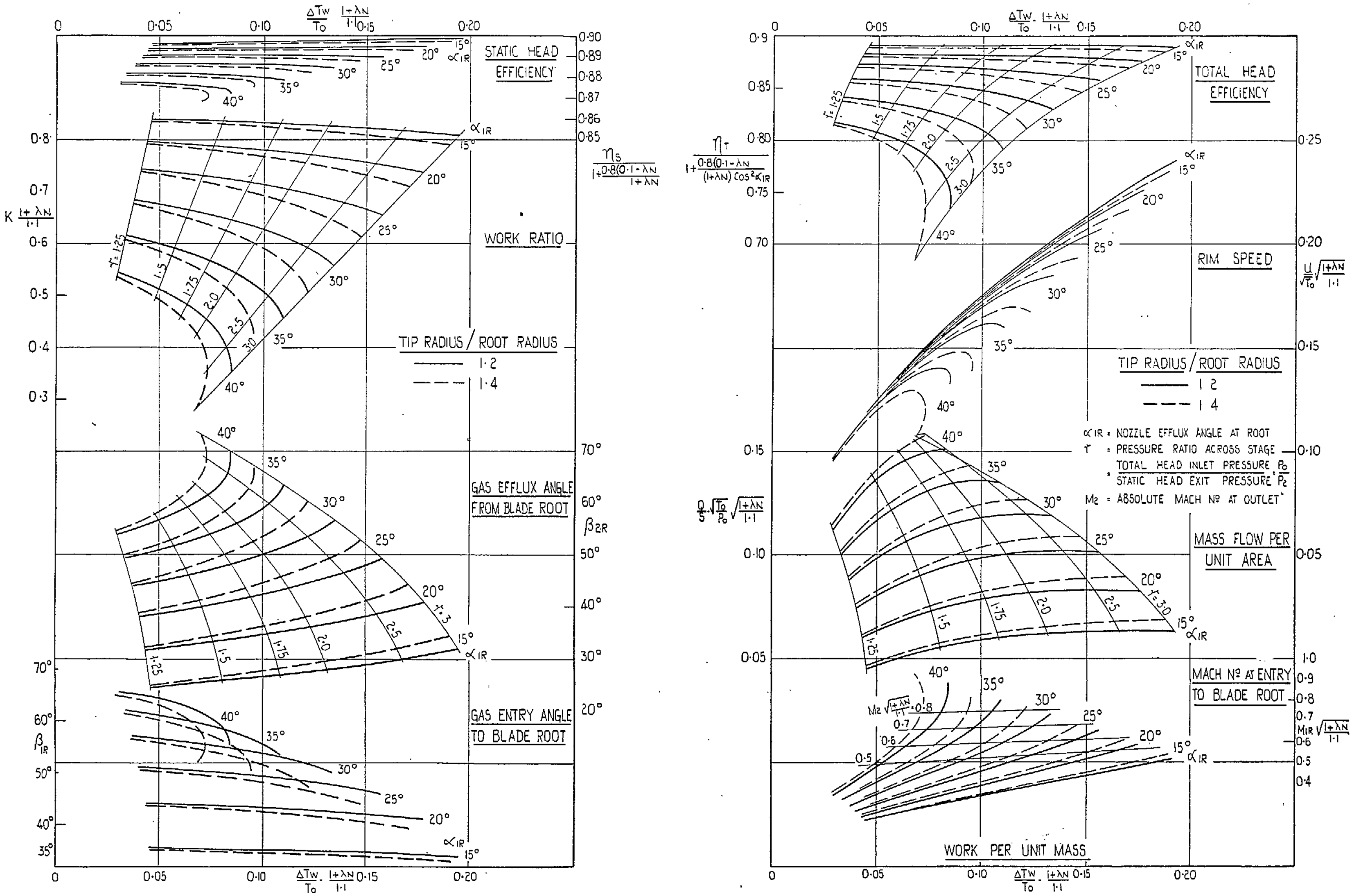


Fig. 9. Turbine design 20 per cent reaction at root. No swirl. Calculated for $\lambda_N = \lambda_B = 10$ per cent, $c_p = 0{\cdot}274$, $\gamma = 1{\cdot}333$.

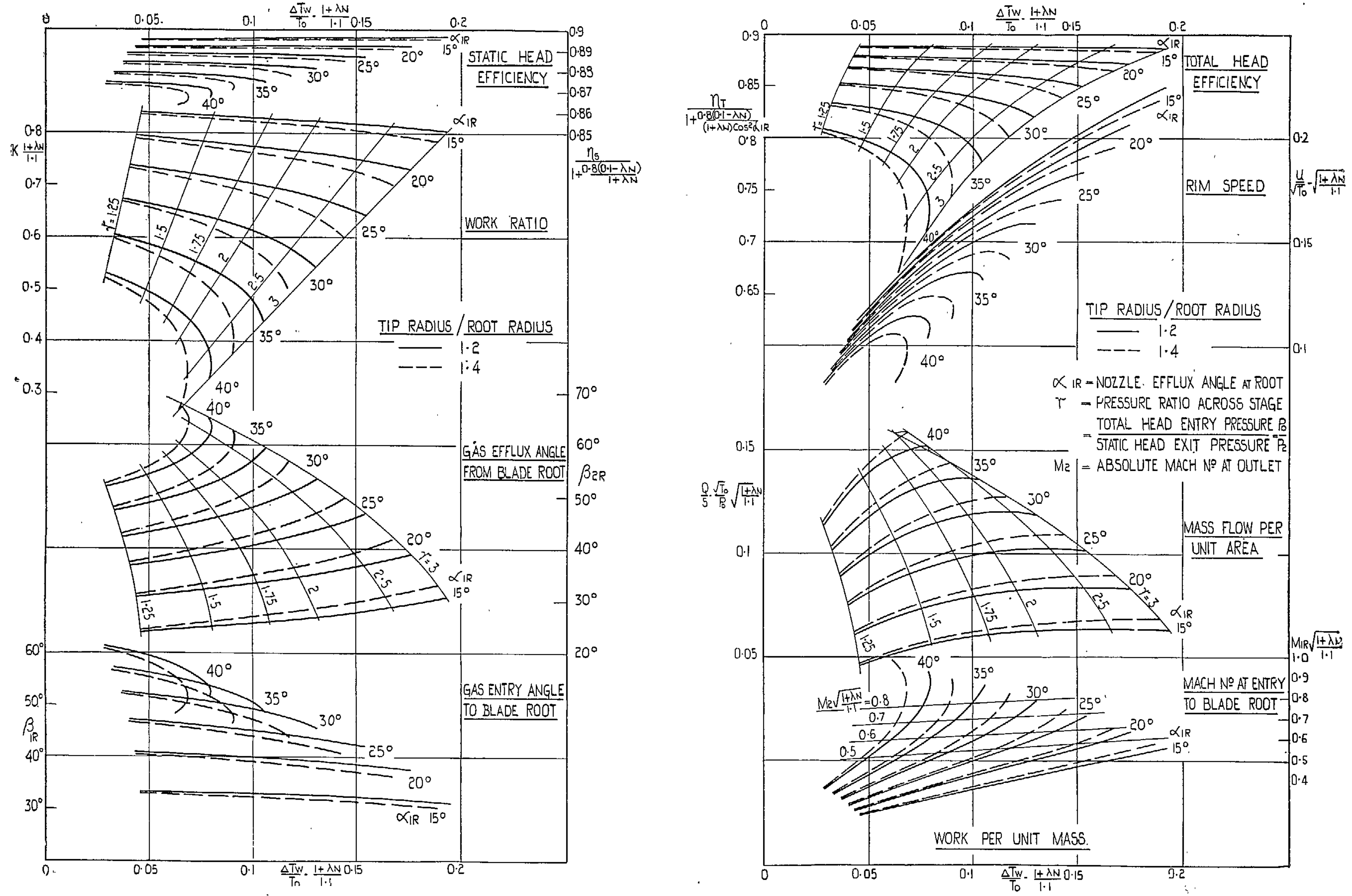


Fig. 10. Turbine design 20 per cent reaction at root. 10 deg. swirl. Calculated for $\lambda_N = \lambda_B = 10$ per cent, $c_p = 0 \cdot 274$, $\gamma = 1 \cdot 333$.

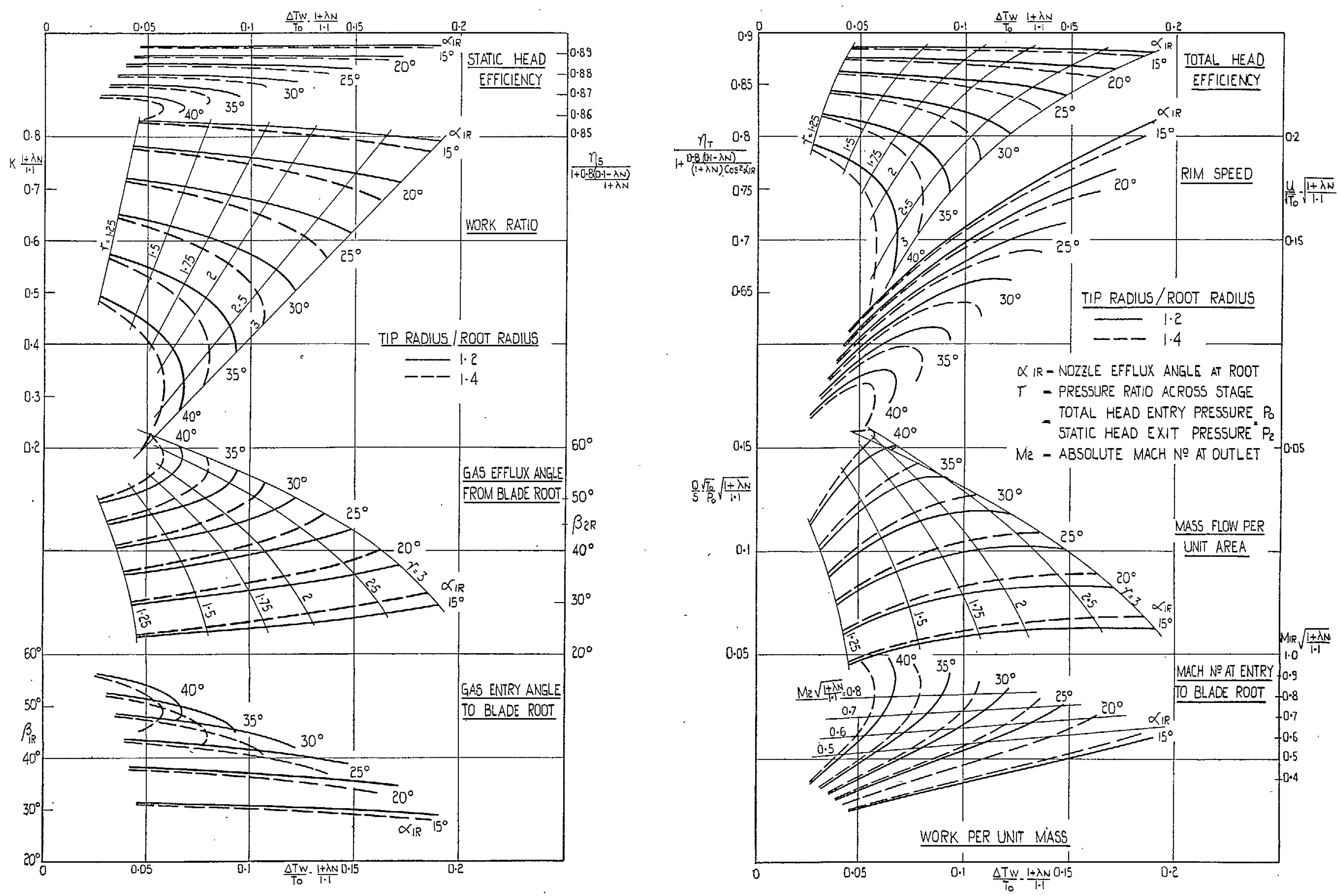


Fig. 11. Turbine design 20 per cent reaction at root. 20 deg. swirl. Calculated for $\lambda_N = \lambda_B = 10$ per cent, $c_p = 0{\cdot}274$, $\gamma = 1{\cdot}333$.

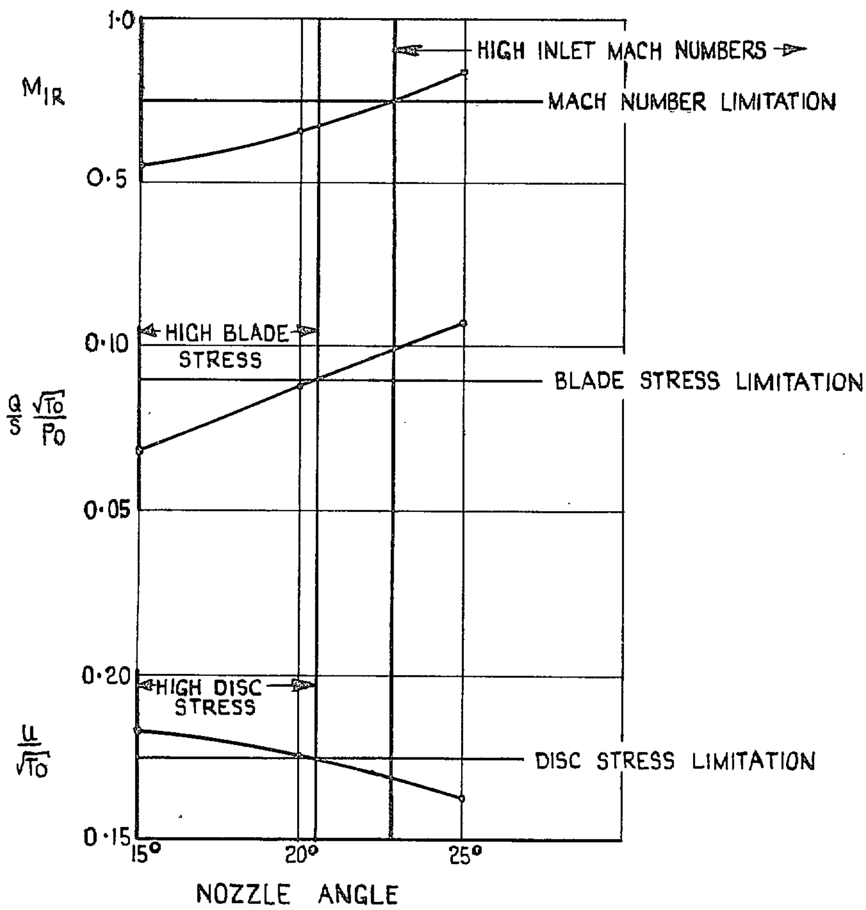


Fig. 12. Turbine design with 10 per cent reaction and 10 deg. swirl at blade root.

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