R. & M. No. 3299

MINISTRY OF AVIATION

AERONAUTICAL RESEARCH COUNCIL
REPORTS AND MEMORANDA

The Aerodynamic Forces on an Oscillating Two-Dimensional Wing in Accelerated Supersonic Flight

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LONDON: HER MAJESTY'S STATIONERY OFFICE
1962
FIVE SHILLINGS NET

The Aerodynamic Forces on an Oscillating Two-Dimensional Wing in Accelerated Supersonic Flight

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Communicated by the Deputy Controller Aircraft (Research and Development), Ministry of Aviation

*Reports and Memoranda No. 3299**
March, 1961

Summary. The lift distribution on a two-dimensional wing oscillating in accelerated flight is obtained by solving the linearised partial differential equation for the velocity potential in the fluid flow about the wing and applying the linearised Bernoulli equation. Expressions for the total lift and moment are obtained as integrals involving the upwash on the wing. Numerical values for the lift and moment on a heaving wing and on a pitching wing have been obtained when the forward acceleration of the wing is uniform.

1. *Introduction.* It is important to investigate whether acceleration will have an appreciable effect on the airforces acting on the lifting surfaces of certain missiles which have moderate acceleration in order to determine whether this effect need be taken into account in stability and flutter calculations.

For a wing travelling at supersonic speed any disturbances emanating at the wing surface move backwards relative to the wing and after a short interval of time do not affect the lift distribution over the wing. If its acceleration is small then the lift distribution over the wing is affected only by disturbances which emanated from the wing surface when the speed was not much different from its actual speed, and it is therefore to be expected that the lift distribution is very nearly the same as if the wing had been travelling at uniform speed. However, in order to have a quantitative estimate of the effect of acceleration, airforces on an oscillating two-dimensional wing in uniformly-accelerated supersonic flight have been evaluated. The wing is assumed to be oscillating with constant amplitude and frequency.

Values of lift and moment on a heaving and on a pitching two-dimensional wing in uniform acceleration are given at different speeds in a supersonic speed range. For comparison the steady-state values of lift and moment are given at each of these speeds.

2. *Basic Equations.* The two-dimensional wing will be assumed thin and nearly plane and to be executing oscillations of small amplitude only so that linearised theory may be used. As far as the effect of wing oscillations is concerned it is consistent with the accuracy of linearised theory to replace the wing by an indefinitely thin plate oscillating about a mean plane and to take the airforces on the wing to be the same as those on the plate. The projection of the wing onto the mean plane

* Previously issued as R.A.E. Report No. Structures 262—A.R.C. 23,019.

is assumed to move with velocity V, dependent on time, in the direction normal to the leading edge of the projection and parallel to the mean plane. The fluid medium through which the wing moves is assumed to be at rest at infinity.

Two systems of co-ordinates will be taken. The origin of the (x, y, z) system is stationary with respect to the fluid medium at infinity. The positive x-axis is in the direction opposite to that of the wing velocity V, the positive z-axis is vertically upwards (i.e., normal to the mean plane) and the positive y-axis is mutually at right angles to form a right-handed system. The distances of the point (x, y, z) from the co-ordinate planes are defined by cx, cy, cz where c is the wing chord. The origin of the (ξ, η, ζ) system moves with velocity V in the same direction as the wing so that it is at rest relative to the projection of the wing on its mean plane. The positive axes of ξ, η and ζ are in the same directions as those of x, y, z respectively and the distances of the point (ξ, η, ζ) from the co-ordinate planes are $c\xi$, $c\eta$ and $c\zeta$. The origin of the (ξ, η, ζ) system of co-ordinates is taken at the mean position of the leading edge of the wing and the origins of the two systems are taken to be coincident at time $t = 0$.

It is convenient to define a non-dimensional time by means of the equation:

$$\tau = \frac{a}{c} t \tag{1}$$

where a is the speed of sound in the fluid medium, c is the chord of the wing, and t the ordinary time. The velocity of the wing may then be taken as a function of τ.

$$V = V(\tau). \tag{2}$$

The velocity potential $\phi(x, y, z, \tau)$ of the gaseous medium satisfies the linearised partial differential equation

$$\frac{1}{c^2}\left(\frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial y^2} + \frac{\partial^2\phi}{\partial z^2}\right) = \frac{1}{a^2}\frac{\partial^2\phi}{\partial t^2}$$

or

$$\frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial y^2} + \frac{\partial^2\phi}{\partial z^2} = \frac{\partial^2\phi}{\partial \tau^2}. \tag{3}$$

The wing is assumed to distort so that the vertical upward displacement of the point $(\xi, \eta, 0)$ on the wing may be given by the formula

$$z = c\,Z(\xi, \tau) \tag{4}$$

where $Z(\xi, \tau)$ is a given function.

The velocity potential must then satisfy the following linearised boundary condition on the plane $z = 0$ over the wing chord:

$$\begin{aligned} \frac{1}{c}\left(\frac{\partial\phi}{\partial z}\right)_{z=0} &= V(\tau)\frac{\partial Z}{\partial \xi} + c\frac{\partial Z}{\partial t} \\ &= a\left[M(\tau)\frac{\partial Z}{\partial \xi} + \frac{\partial Z}{\partial \tau}\right] \\ &= aw(\xi, \tau) \end{aligned} \tag{5}$$

where

$$M(\tau) = \frac{1}{a} V(\tau) \tag{6}$$

is the instantaneous Mach number and $w(\xi, \tau)$ is the upwash at $z = 0$.

The distance between the origins of the two systems of co-ordinates at the time τ is $cs(\tau)$ where

$$s(\tau) = \frac{1}{a}\int_0^\tau V(u)du \tag{7}$$

The following relationships then exist between the co-ordinates in the two systems:

$$\begin{aligned} \xi &= x + s(\tau) \\ \eta &= y \\ \zeta &= z. \end{aligned} \tag{8}$$

3. *The Derivation of the Velocity Potential.* The partial differential equation (3) is satisfied by the function

$$\phi(x, y, z, \tau) = \frac{1}{2\pi}\iint\limits_{\substack{z=0 \\ \text{plane}}} m(x_0, y_0, \tau - r)\,\frac{dx_0 dy_0}{r} \tag{9}$$

where

$$r = \sqrt{\{(x-x_0)^2 + (y-y_0)^2 + z^2\}} \tag{10}$$

and $m(x_0, y_0, \tau)$ is an arbitrary function which is twice differentiable with respect to τ.

This solution of the differential equation corresponds to the velocity potential about a layer of sources in the $z = 0$ plane.

Differentiating equation (9) with respect to z, and proceeding to the limit $z = +0$, we obtain

$$\lim_{z \to +0}\left(\frac{\partial \phi}{\partial z}\right) = m(x, y, \tau) \tag{11}$$

when $m(x, y, \tau)$ satisfies some elementary conditions of continuity and boundedness.

If we proceed to the limit $z = -0$ we obtain

$$\lim_{z \to -0}\left(\frac{\partial \phi}{\partial z}\right) = -m(x, y, \tau). \tag{12}$$

There is therefore a discontinuity in $(\partial\phi/\partial z)$ in passing through the layer of sources. Since there is no discontinuity of $(\partial\phi/\partial z)$ in passing through the wing it is not possible to replace the wing by a layer of sources. However it is possible to do this as far as the flow either above the wing or below the wing is concerned for these two flows are independent of each other and of conditions behind the wing in supersonic flow. For flow above the wing we take

$$m(x, y, \tau) = caw\{x + s(\tau), \tau\} \tag{13}$$

with $w(\xi, \tau)$ defined by equation (5).

The velocity potential in the flow above the wing is therefore given by

$$\phi(x, y, z, \tau) = \frac{ca}{2\pi}\iint\limits_{\substack{z=0 \\ \text{plane}}} w\{x_0 + s(\tau - r), \tau - r\}\,\frac{dx_0 dy_0}{r}. \tag{14}$$

The velocity potential in the flow below the wing is given by the same formula (14) only that the sign of the right-hand side is changed. The velocity potential is then seen to be antisymmetric about the plane $z = 0$.

The wing will be assumed to oscillate harmonically with circular frequency ω. Since the problem is linear it is possible to express the displacement as the real part of

$$
\begin{aligned}
z &= cZ(\xi, \tau) \\
&= c\bar{Z}(\xi)e^{i\nu\tau}
\end{aligned}
\tag{15}
$$

where

$$\nu = \frac{\omega c}{a} \tag{16}$$

is the frequency parameter. The physical quantity corresponding to any complex number or function will always be taken as the real part of the complex number or function.

It may be well to mention here that it is usual in problems on oscillating wings having constant forward speed to define the frequency parameter $\nu = \omega c/V$. In problems involving variable forward speed of the wing this definition of ν is unsuitable, so a definition such as that given in equation (16) is introduced.

The upwash is now given by

$$w(\xi, \tau) = W(\xi, \tau)e^{i\nu\tau} \tag{17}$$

where

$$W(\xi, \tau) = M(\tau)\frac{\partial \bar{Z}}{\partial \xi} + i\nu\bar{Z}. \tag{18}$$

In order to obtain the lift distribution it is necessary to know the velocity potential for $z = +0$ and $z = -0$. Since the potential is independent of y we shall henceforth drop this co-ordinate where it is appropriate to do so. The velocity potential $\phi(x, +0, \tau)$ at $z = +0$ is then given by

$$\phi(x, +0, \tau) = \frac{ca}{2\pi}\iint\limits_{\substack{z=0\\ \text{plane}}} W\{x_0 + s(\tau - r_0), \tau - r_0\}e^{i\nu(\tau - r_0)}\frac{dx_0 dy_0}{r_0} \tag{19}$$

where

$$r_0 = \sqrt{\{(x - x_0)^2 + y_0^2\}}. \tag{20}$$

If we write

$$\phi(x, +0, \tau) = \bar{\phi}(\xi, \tau)e^{i\nu\tau} \tag{21}$$

we obtain from (19)

$$\bar{\phi}(\xi, \tau) = \frac{ca}{2\pi}\iint\limits_{\substack{z=0\\ \text{plane}}} W\{x_0 + s(\tau - r_0), \tau - r_0\}e^{-i\nu r_0}\frac{dx_0 dy_0}{r_0} \tag{22}$$

where

$$r_0 = \sqrt{[\{\xi - x_0 - s(\tau)\}^2 + y_0^2]}. \tag{23}$$

The polar co-ordinates (r_0, θ) are introduced through the equations

$$
\left.
\begin{aligned}
x_0 &= r_0 \cos\theta + \xi - s(\tau) \\
y_0 &= r_0 \sin\theta
\end{aligned}
\right\}
\tag{24}
$$

so that the expression for $\bar{\phi}(\xi, \tau)$ may be written

$$
\begin{aligned}
\bar{\phi}(\xi, \tau) &= \frac{ca}{2\pi}\int_0^{2\pi} d\theta \int_0^{\infty} W\{\xi + r_0\cos\theta - s(\tau) + s(\tau - r_0), \tau - r_0\}e^{-i\nu r_0}dr_0 \\
&= \frac{ca}{\pi}\int_0^{\pi} d\theta \int_0^{\infty} W\{\xi + r_0\cos\theta - s(\tau) + s(\tau - r_0), \tau - r_0\}e^{-i\nu r_0}dr_0.
\end{aligned}
\tag{25}
$$

Since there is no disturbance propagated forwards from the wing in supersonic flow the function $W(\xi, \tau)$ is to be taken zero for $\xi < 0$. This will make the effective area of integration in (25) a finite one.

The double integral in equation (25) may be evaluated numerically for given values of ξ in (0, 1) and of τ. The procedure is to evaluate the integral as a repeated integral in the order indicated in equation (25). Since

$$\xi + r_0 \cos \theta - s(\tau) + s(\tau - r_0) \tag{26}$$

is a monotonically decreasing function of r_0 when the forward speed of the wing is always supersonic the upper limit of the inner integral in (25) may be replaced by the finite value of r_0 at which the expression (26) becomes zero for all the values of θ.

A number of integration points θ are taken in the interval $(0, \pi)$ and the inner integral is evaluated numerically for each of the integration points θ. The outer integration with respect to θ may then be easily evaluated numerically. Gaussian quadrature methods are most suitable for performing these numerical integrations.

4. *The Airforces on the Wing.* The lift distribution $l(\xi, \tau)$ on the wing is obtained by using the linearised Bernoulli equation, and observing the fact that the velocity potential is antisymmetric about the $z = 0$ plane. The lift distribution is given by

$$\begin{aligned} l(\xi, \tau) &= 2\rho \left[\frac{V(\tau)}{c} \frac{\partial \phi(x, +0, \tau)}{\partial \xi} + \frac{\partial \phi(x, +0, \tau)}{\partial t} \right] \\ &= 2\rho \frac{a}{c} \left[M(\tau) \frac{\partial \bar{\phi}(\xi, \tau)}{\partial \xi} + i\nu \bar{\phi}(\xi, \tau) \right] e^{i\nu\tau} . \end{aligned} \tag{27}$$

Generalised airforces may now be obtained by multiplying the lift distribution by the expressions for the shapes of the modes of oscillation and integrating over the wing chord. For example the lift per unit span is

$$\begin{aligned} L(\tau) &= c \int_0^1 l(\xi, \tau) d\xi \\ &= 2\rho a \left[M(\tau) \bar{\phi}(1, \tau) + i\nu \int_0^1 \bar{\phi}(\xi, \tau) d\xi \right] e^{i\nu\tau} \end{aligned} \tag{28}$$

since $\bar{\phi}(0, \tau) = 0$; and the moment per unit span positive nose up is

$$\begin{aligned} N(\tau) &= -c^2 \int_0^1 \xi l(\xi, \tau) d\xi \\ &= -2\rho a c \left[M(\tau) \bar{\phi}(1, \tau) - M(\tau) \int_0^1 \bar{\phi}(\xi, \tau) d\xi + i\nu \int_0^1 \xi \bar{\phi}(\xi, \tau) d\xi \right] e^{i\nu\tau} . \end{aligned} \tag{29}$$

The integrals in (28) and (29) may be evaluated for given τ once $\bar{\phi}(\xi, \tau)$ has been evaluated at a set of points ξ in (0, 1). The function $\bar{\phi}(\xi, \tau)$ is well behaved and has no singularities for ξ in (0, 1).

5. *Supersonic Wing in Uniformly-Accelerated Flight.* The wing is assumed to be travelling always at supersonic speed. At time $\tau = 0$ its Mach number is M_0 and from time $\tau = 0$ onwards it is assumed to have uniform acceleration b. Its Mach number at time $\tau > 0$ is then

$$M(\tau) = M_0 + p\tau \tag{30}$$

where p is a coefficient given by

$$p = bc/a^2 . \tag{31}$$

The airforces on the wing at time τ will be influenced by disturbances generated by the wing at earlier times.

The disturbance generated at the leading edge of the wing at time $\tau' < \tau$ will at time τ be confined to the surface of a circular cylinder of radius $a(\tau - \tau')$ with axis fixed at the line coincident with the leading edge at time τ'. If τ' is near to τ this circular cylinder will intersect the wing surface at two chordwise positions and, the earlier τ' is, the further away from the leading edge will these two positions be. The foremost of these intersections will be at the trailing edge for a disturbance generated at a certain time $\tau' = \tau_1$ and the disturbances generated at times earlier than τ_1 will not influence the airforces on the wing at time τ.

Since distance travelled by the wing is a quadratic function of time, and distance travelled by a disturbance is a linear function of time, the value of τ_1, is found by solving a quadratic equation and is given by

$$\tau_1 = \tau - \frac{1}{p}\{M(\tau) - 1\} \pm \frac{1}{p}\sqrt{[\{M(\tau) - 1\}^2 - 2p]}. \tag{32}$$

The Mach number at time τ_1 is then found from (30) to be

$$M(\tau_1) = 1 \pm \sqrt{[\{M(\tau) - 1\}^2 - 2p]}. \tag{33}$$

The negative signs in (32) and (33) are inadmissible for they correspond to a negative value of τ_1 and a subsonic $M(\tau_1)$. The positive sign in (33) does give a supersonic $M(\tau_1)$. If then

$$M(\tau_1) > M_0, \tag{34}$$

the only disturbances reaching the wing at time τ will be those generated at the times when the wing was already travelling with uniform acceleration.

We assume

$$p < \tfrac{1}{2}\{M(\tau) - 1\}^2 \tag{35}$$

and this obviously excludes only irrelevant cases when either $M(\tau)$ exceeds 1 by very little only or p is unduly large.

The function $s(\tau)$ for the uniformly-accelerating wing is given by

$$s(\tau) = M_0\tau + \tfrac{1}{2}p\tau^2. \tag{36}$$

Then

$$s(\tau) - s(\tau - r_0) = r_0 M(\tau) - \frac{p}{2}r_0^2. \tag{37}$$

Let us now consider a heaving and a pitching wing.

For a heaving wing we may write for $\bar{Z}(\xi)$ in equation (15)

$$\bar{Z}(\xi) = \delta_H \tag{38}$$

so that

$$W(\xi, \tau) = i\nu\delta_H. \tag{39}$$

Expressions for L_H and N_H, the lift and moment, per unit length, of the airforces acting on the heaving wing may be written as

$$L_H = \rho c a^2 \delta_H [l_H'\{p, M(\tau), \nu\} + i l_H''\{p, M(\tau), \nu\}] e^{i\nu\tau} \tag{40}$$

$$N_H = \rho c^2 a^2 \delta_H [m_H'\{p, M(\tau), \nu\} + i m_H''\{p, M(\tau), \nu\}] e^{i\nu\tau}. \tag{41}$$

For a wing pitching about its leading edge we may write for $\bar{Z}(\xi)$ in equation (15)

$$\bar{Z}(\xi) = \delta_P \xi \tag{42}$$

so that

$$W(\xi, \tau) = \delta_P [M(\tau) + i\nu\xi] . \tag{43}$$

Expressions for L_P and N_P, the lift and moment, per unit length, of the airforces acting on the pitching wing may be written as

$$L_P = \rho c a^2 \delta_P [l_P'\{p, M(\tau), \nu\} + i l_P''\{p, M(\tau), \nu\}] e^{i\nu\tau} \tag{44}$$

$$N_P = \rho c^2 a^2 \delta_P [m_P'\{p, M(\tau), \nu\} + i m_P''\{p, M(\tau), \nu\}] e^{i\nu\tau} . \tag{45}$$

The functions $l_H'\{p, M(\tau), \nu\}$, $l_H''\{p, M(\tau), \nu\}$, etc., are all real functions and their values may be determined by using the formulae of this paper. In order to obtain these values for a number of different combinations of the values p, $M(\tau)$, ν a large number of integrals have to be evaluated. These integrals are all of a similar nature and can be suitably dealt with on an electronic digital computer.

Some results which have been obtained are given in Section 6. For comparison the values of $l_H'(0, M(\tau), \nu)$, $l_H''(0, M(\tau), \nu)$, etc., which are the steady-state values are given as well. The difference between the corresponding values is a measure of the effect of acceleration.

6. *Results.* The results below are for different combinations of acceleration parameter p and frequency parameter ν for a series of Mach numbers $M(\tau)$ occuring during a uniformly-accelerated motion. When the acceleration parameter p is zero the aerodynamic force coefficients are the steady-state values but it must be remembered that the definition of frequency parameter usual with steady-state values is $\omega c/V$ while for these values it is $\omega c/a$.

The values of p taken are all small numbers but in fact they correspond to enormous values of acceleration b in cases of practical significance.

e.g., if we take

$$c = 4 \text{ ft}, \; a = 1000 \text{ ft/sec}, \; p = 0 \cdot 01 ,$$

then

$$b = \frac{a^2 p}{c} = 2500 \text{ ft/sec}^2$$

$$\simeq 80g$$

(i) $p = 0$, $\nu = 1$, heaving wing

$M(\tau)$	$l_H'(p, M(\tau), \nu)$	$l_H''(p, M(\tau), \nu)$	$m_H'(p, M(\tau), \nu)$	$m_H''(p, M(\tau), \nu)$
2	0·17773	2·2480	−0·11655	−1·1089
3	0·04314	2·1131	−0·02862	−1·0545
4	0·01716	2·0633	−0·01131	−1·0311
5	0·00844	2·0404	−0·00562	−1·0200

(ii) $p = 0{\cdot}01$, $\nu = 1$, heaving wing

$M(\tau)$	$l_H'(p, M(\tau), \nu)$	$l_H''(p, M(\tau), \nu)$	$m_H'(p, M(\tau), \nu)$	$m_H''(p, M(\tau), \nu)$
2	0·18113	2·2525	−0·11907	−1·1117
3	0·04361	2·1146	−0·02897	−1·0555
4	0·01716	2·0641	−0·01142	−1·0316
5	0·00851	2·0408	−0·00567	−1·0203

(iii) $p = 0{\cdot}04$, $\nu = 1$, heaving wing

$M(\tau)$	$l_H'(p, M(\tau), \nu)$	$l_H''(p, M(\tau), \nu)$	$m_H'(p, M(\tau), \nu)$	$m_H''(p, M(\tau), \nu)$
2	0·19173	2·2661	−0·12695	−1·1202
3	0·04502	2·1191	−0·03003	−1·0585
4	0·01761	2·0663	−0·01176	−1·0331
5	0·00871	2·0422	−0·00582	−1·0212

(iv) $p = 0$, $\nu = 1$, pitching wing

$M(\tau)$	$l_P'(p, M(\tau), \nu)$	$l_P''(p, M(\tau), \nu)$	$m_P'(p, M(\tau), \nu)$	$m_P''(p, M(\tau), \nu)$
2	4·5572	0·78366	−2·2634	−0·52426
3	6·3539	0·92919	−3·1745	−0·61961
4	8·2589	0·96420	−4·1286	−0·64283
5	10·2046	0·97818	−5·1019	−0·65213

(v) $p = 0{\cdot}01$, $\nu = 1$, pitching wing

$M(\tau)$	$l_P'(p, M(\tau), \nu)$	$l_P''(p, M(\tau), \nu)$	$m_P'(p, M(\tau), \nu)$	$m_P''(p, M(\tau), \nu)$
2	4·5599	0·78098	−2·2651	−0·52228
3	6·3546	0·92890	−3·1749	−0·61939
4	8·25927	0·96412	−4·1288	−0·64277
5	10·2048	0·97815	−5·1021	−0·65210

(vi) $p = 0 \cdot 04$, $\nu = 1$, pitching wing

$M(\tau)$	$l_P'(p, M(\tau), \nu)$	$l_P''(p, M(\tau), \nu)$	$m_P'(p, M(\tau), \nu)$	$m_P''(p, M(\tau), \nu)$
2	4·5683	0·77256	−2·2707	−0·51603
3	6·3567	0·92801	−3·1763	−0·61872
4	8·2600	0·96390	−4·1293	−0·64261
5	10·2051	0·97807	−5·1023	−0·65205

7. *Conclusions.* Rather simple physical reasons suggest that for practical dimensions and accelerations the values of the airforces for accelerated flight are only little different from the steady-state values. This is borne out in the two-dimensional case by the results given in the tables of Section 6, where the force coefficients for accelerated flow are seen to be very little different from the steady-state values.

NOTATION

x, y, z		Non-dimensional co-ordinates in a system with origin fixed relative to the fluid medium at infinity
ξ, η, ζ		Non-dimensional co-ordinates in a system with origin fixed relative to the wing
c		Wing chord
a		Speed of sound in the fluid medium
t		Time
τ	$=$	$\frac{a}{c}t$, Non-dimensional time
V	$=$	$V(\tau)$, Forward velocity of wing
$M(\tau)$	$=$	$\frac{V(\tau)}{a}$, Mach number
$cZ(\xi, \tau)$	$=$	$c\bar{Z}(\xi)e^{i\nu\tau}$, Vertical displacement of a point on wing surface at time τ
$w(\xi, t)$		Upwash
$\phi(x, y, z, \tau)$		Velocity potential in the gaseous medium
$cs(\tau)$		Distance between origins of the two co-ordinate systems at time τ
r_0		*See* equation (20)
ω		Circular frequency
ν	$=$	$\frac{\omega c}{a}$, Frequency parameter
$l(\xi, \tau)$		Lift distribution
$L(\tau)$		Lift per unit span
$N(\tau)$		Moment per unit span, positive nose up
b		Acceleration of wing
p	$=$	$\frac{bc}{a^2}$, Acceleration parameter
l_H', l_H'', m_H', m_H'', etc.		Coefficients defined in Section 5
δ_H		Parameter determining amplitude of heaving oscillation {*see* equation (39)}
δ_P		Parameter determining amplitude of pitching oscillation {*see* equation (43)}

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(85554) Wt. 64/1857 K.5 11/62 Hw.

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S.O. Code No. 23-3299