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An Analogue Computer for Convective Heating Problems

By

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An analogue computer for convective heating problems

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H. G. R. Robinson Wh.Sc. B.Sc. A.C.G.I. A.M.I.E.E.

SUMMARY

The computer was designed to assist in the solutions of kinetic heating problems which occur in guided weapon design. Its flexibility allows it to be used for a number of problems involving one dimensional convective heating in a wall of finite thickness, providing the basic data such as the source temperature of the convective medium and the heat transfer coefficient in the region of interest are known. Completely arbitrary time varying functions of source temperatures and heat transfer coefficient are acceptable. This note describes the design and construction of the computer.

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LIST OF SYMBOLS

T	absolute temperature
H	heat
t	time
x	distance from outer surface along a normal to surface
d	thickness
K	thermal conductivity
ρ	density
c	specific heat
h	heat transfer coefficient
N	number of cells
j	cell number
V	electrical potential
I	" current
R	" resistance
C	" capacity
m, n, r	scale factors
w	suffix referring to wall outer surface ($x = 0$)

1 Introduction

For most convective heat flow problems where unsteady heat transfer conditions exist, the engineer cannot employ an analytic method, but has to resort to step by step solutions. An electrical analogue of the type described considerably reduces the computing effort and time to produce a solution.

The computer was primarily designed to reduce the effort in completing a large programme of kinetic heating investigations covering a wide range of flight programmes and conditions.

Interesting points in its design are the almost complete absence of electronic amplifiers and the simplificity of the function generators for the arbitrary time varying quantities.

2 The basis of the analogue[1,2]

The flow of heat between the convective medium boundary layer, at a temperature T_r, and a wall having distributed conductivity and thermal capacity, is simplified by reducing the problem to three elements, the heat source, a transfer impedance, and the heat sink.

2.1 The wall

The equation of heat flow in one dimension is:

$$\frac{\partial}{\partial x}\left\{K(x)\,\frac{\partial T}{\partial x}\right\} = \rho c\,(x)\,\frac{\partial T}{\partial t} \qquad (1)$$

where $K(x)$ is the conductivity

$\rho c\,(x)$ is the heat capacity, or "water equivalent".

For a generalised material, both K and ρc are functions of x. Practical examples are sandwich construction and insulating material with graded conductivity. The change in thermal properties with temperature is not considered here.

The ratio $\frac{K}{\rho c}\,(x)$ is generally known as the "diffusivity" of the material.

To obtain an exact electrical analogue, distributed electrical parameters analogous to the thermal conductivity and capacity are required. Since such a model is not readily realisable, an electrical model using lumped circuit elements is used, the wall being represented by a ladder network.

Considering the n^{th} node of a ladder network, with the interval between each cell of the ladder, assuming uniform grading, to be Δx and defining

$$x_{n+\frac{1}{2}} = x_n + \frac{\Delta x}{2}$$

etc.,

$$x_{n-\frac{1}{2}} = x_n - \frac{\Delta x}{2}$$

equation (1) in terms of finite differences becomes

$$K \frac{(x_{n+\frac{1}{2}})}{\Delta x} \left[T_{n+1} - T_n \right] + K \frac{(x_{n-\frac{1}{2}})}{\Delta x} \left[T_{n-1} - T_n \right] = \Delta x f(x_n) \frac{dT_n}{dt} \qquad (2)$$

where $f(x)$ is written for $\rho c(x)$.

The R.H.S. of this equation has the same form as the electric current flowing into a capacitor of value $\Delta x f(x_n)$ at a potential $T_n(t)$. Similarly the L.H.S. terms represent currents flowing in resistors. Equation (2) is thus seen to represent a ladder network of the type shown in Figure 1a. For a uniform medium divided into N equal cells and having a thickness d, $\Delta x = \frac{d}{N}$, and the resistive arms of the ladder network have a value $R = \frac{d}{NK}$ and the capacitative arms a value $C = \frac{d}{N.\rho c}$. The cells may be represented as T or π elements as shown in Figures 1b and 1c.

2.2 Boundary conditions

(i) At the heated surface (x = o)

The flow of heat into the wall is

$$\frac{dH}{dt} = h\,(T_r - T_w)$$

where T_r is the "recovery or, adiabatic wall temperature", T_w the true wall temperature at the surface and h the heat transfer coefficient of the boundary layer. T_r is the effective "source" temperature of the convective medium, and for the case of kinetic heating, generally taken as the stagnation or total temperature modified by the "recovery" factor.

i.e. $T_r = \beta T_t$ where T_t is the stagnation or total temperature.

At the surface also, the heat conduction is given by

$$\frac{dH}{dt} = \left[- K \frac{\partial T}{\partial x} \right]_{x=0}$$

so that the boundary conditions at $x = 0$ is given by

$$h \left[T_r - T_w \right] = \left[- K \frac{\partial T}{\partial x} \right]_{x=0} \qquad (3)$$

In finite differences the equation becomes

$$h\left[T_r - T_w\right] = -K_{x=0}\left[\frac{T_1 - T_w}{\Delta\frac{x}{2}}\right] \qquad (4)$$

where T_1 is the temperature of the centre point of the equivalent "T" cell.

Equation (4) represents the current equation for the loop network shown in Figure 2a.

(ii) At the inner surface of the wall ($x = d$)

For a wall not cooled or warmed at the inner surface, the temperature gradient at $x = d$ is zero and the boundary conditions are fulfilled by leaving the network open as in Figure 2b. If convection to or from the surface occurs, due to an internal heat source or cooling plant, similar boundary conditions to those pertaining to the heated outer surface apply and are represented by a similar analogue.

2.3 The complete analogue

Figure 3 shows the complete electrical model for a wall warmed on one surface only, with zero heat transfer at the inner surface. Table I below shows the corresponding thermal and electrical quantities.

TABLE I

Thermal quantity	Electrical quantity
Temperature T	Potential t
Heat flow $\frac{dH}{dt}$	Current I $(= \frac{dQ}{dt})$
Heat H	Charge Q
Heat transfer coefficient h	Conductance $\frac{1}{R_h}$
Heat capacity per cell at $x = j$ $\Delta x\,(\rho c)_j$	Capacity C_j
Conductivity per cell at $x = j$ $\frac{K_j}{\Delta x}$	Conductance $\frac{1}{R_j}$
	$\left[\text{Resistance per arm of a "T" cell} = \frac{R_j}{2} = \frac{\Delta x}{2K_j}\right]$
Time t	Time t

3 Units and scale factors

To convert the thermal quantities into analogous electrical parameters, it is necessary to fix scale factors. The three basic conversion factors chosen here relate true and model time, temperature and electrical potential, and heat and charge. These scale factors must be chosen by the operator setting up the problem on the machine to produce readily realisable electrical circuit elements such as resistance and capacity, and a time scale suited to the arbitrary function generators for input temperature and heat transfer coefficient.

3.1 The scale factors are derived as follows:

(i) Time

$$\text{Time scale} = \frac{\text{Heat problem time}}{\text{Analogue time}} = \frac{t'}{t} = h$$

(ii) Potential

$$\text{Potential scale} = \frac{\text{Temperature}}{\text{Electrical potential}} = \frac{T}{V} = m$$

(iii) Quantity

$$\text{Quantity scale} = \frac{\text{Heat}}{\text{Charge}} = \frac{H}{Q} = r$$

From these may be derived the model parameters, as below.

3.11 Heat transfer coefficient

$$R_h \sim \frac{1}{h}$$

But $$R_h = \frac{V}{\frac{dQ}{dt}} = \frac{T/m}{\frac{dH/r}{dt'/n}} = \frac{r}{m.n} \cdot \frac{T}{\frac{dH}{dt'}} = \frac{r}{m.n} \cdot \frac{1}{h}$$

$$\therefore \qquad \underline{R_h = \frac{r}{m.n} \cdot \frac{1}{h}}$$

3.12 Cell resistance

Conductivity per cell at $x = j^{th}$ node is $\frac{K_j}{\Delta x}$

$$\therefore \qquad \underline{R_j = \frac{r}{m.n} \cdot \frac{\Delta x}{K_j}}$$

For a uniform wall and uniform grading,

$$R = \frac{r}{m.n} \cdot \frac{d}{NK} \qquad \text{where } N \text{ is the number of cells}$$

This resistance is the total resistance per cell and for a "T" cell, each arm will be half this value.

3.13 Cell capacities

Capacity per cell at $x = j^{th}$ node is $\Delta x(\rho c)_j = C_j$

$$\text{But} \qquad C_j = \frac{Q}{V} = \frac{H/r}{T/m} = \frac{H}{T} \cdot \frac{m}{r} = \frac{m}{r} \cdot \Delta x(\rho c)_j$$

$$\therefore \qquad C_j = \frac{m}{r} \cdot (\rho c)_j \cdot \Delta x$$

For a uniform wall and uniform grading

$$C = \frac{m}{r} \cdot \rho c \frac{d}{N}$$

3.2 System of units

The system employed in the computer is given in Table II below.

TABLE II

Symbol	Unit	Meaning
d	feet	Wall thickness
T	°K	Temperature
K	C.H.U./ft sec °K	Thermal conductivity
c	C.H.U./lb °K	Specific heat
h	C.H.U. ft²/sec °K	Heat transfer coeff.
ρ	lb/ft³	Density
N	-	Number of cells
V	volts	Potential
I	Amperès	Current
Q	Coulombs	Charge
H	C.H.U.	Heat
C	Farads	Electrical capacity
R	ohms	Resistance

4 Modifications to the analogue

In addition to problems of the type described in para. 2.3, variations of the problem requiring very small modifications and additions to the basic circuit of Figure 3 may be studied. One such variant, in which other conditions hold than zero heat transfer at the inner surface has been mentioned in para. 2.2. Convection from the surface, radiation, or cooling to a fixed temperature (refrigeration), are easily represented. Several such problems, those of thermal radiation, sandwich construction, and the solution of problems in which the heat transfer coefficient is dependent on wall temperature are described below.

4.1 Thermal radiation

The radiation from the heated inner or outer surface of the wall becomes important at elevated temperatures.

The heat flux is given by the well known relation

$$\frac{dH_r}{dt} = \varepsilon\left(T_w^{\,4} - T_o^{\,4}\right) \tag{5}$$

where T_o is the ambient temperature. For significant radiation $T_w^{\,4} >> T_o^{\,4}$ and equation (5) is usually written

$$\frac{dH_r}{dt} = \varepsilon\, T_w^{\,4} \tag{5'}$$

The electrical analogue is

$$\frac{dq_r}{dt} = I_r = K \cdot V_w^{\,4} \qquad \text{where} \quad K \text{ is a constant.}$$

To simulate this a non linear resistive device is required. For the problems so far presented to the machine, radiation losses have constituted 5% or less of the total heat flux, and the fourth power law has been represented by a simple two line curve, as in Figure 4(a). Such a current/voltage curve is easily obtained by the biased diode circuit of 4(b), where the current flowing in R_1 follows the "two straight line law" OB-BA [3]. Resistors R_2 and R_3 are made small with respect to R_1, so that the biasing voltage V_b ($\sim T_b$) is given by

$$V_b = \frac{R_2}{R_2 + R_3} \cdot V_S$$

and is readily adjustable. The analogue scaled slope of tangent line BA is adjusted by altering R_1.

$$R_1 = \frac{V_{BC}}{I_{AC}} \sim \frac{T_{BC}}{\left(\frac{dH}{dt}\right)_{AC}}$$

The circuit can be readily elaborated to obtain a closer approximation to the radiation curve using a number of tangent lines, as in Figure 4(c), should this prove necessary.

4.2 Sandwich construction

Walls consisting of several sandwiched materials may be readily represented by groups of ladder network cells connected in series, each group having resistance and capacitative values analogous to the corresponding wall layer. "Semi-infinite" heat sinks connected to wall inner surfaces may be represented by single large capacitances.

4.3 Iterative solutions

In the simple relation for heat transfer given by

$$\frac{dH}{dt} = h(t)\left[T_r(t) - T_w(t)\right]$$

the assumption that $h(t)$ is independent of $T_w(t)$ can lead to error. If some knowledge of the dependence of $h(t)$ on $T_w(t)$ is available, the equation

$$\frac{dH}{dt} = h(t,T_w)\left[T_r(t) - T_w(t)\right]$$

may be solved by the electrical analogy in two ways

(i) A feedback loop from the ladder network input node to the "h" function generator may be used to modify "h" according to the required law. This is a complicated device and not readily adaptable to this machine.

(ii) The second solution, used in this machine, is not so elegant, but has been found to yield satisfactory results without increasing the problem solution time excessively.

An iterative method is employed which is found to converge rapidly in the problems so far undertaken. An approximation to $h(t)$ independent of T_w is calculated and the problem run through. From the solution, an $h(t)$ dependent on $\left[T_w(t)\right]$, is calculated, and the problem is again run through, obtaining a new solution $\left[T_w(t)\right]_2$. A second iteration may prove necessary if $\left[T_w(t)\right]_1$ and $\left[T_w(t)\right]_2$ differ appreciably. The only dependent function needing alteration in the machine is $h(t)$ and the small alterations necessary are usually easily accomplished on the continuously variable resistors comprising one element of the "h" function resistors (see para. 5.1).

5 The design of the computer

The basic circuit of the machine is shown in Figure 3. To enable the widest possible range of problems to be solved, the voltage generator representing T_r and the conductance representing boundary layer heat transfer coefficient h were designed to allow these quantities to be completely arbitrary time varying values. For the free flight kinetic heating case, T_r and h are functions of speed and altitude.

The thermal properties of most wall materials are to some extent functions of the temperature of the material, especially when temperatures are very high. No provision was made for this, though this is planned in a later model.

For walls at very high temperatures, too, loss of heat by radiation becomes an important factor. Provision was made to represent this, as described in para. 4.1, using a very simple non linear resistive device.

5.1 The arbitrary function generators[1,2,4,5]

A variety of ways exist of generating $T_r(t)$ and $h(t)$. These include manual or automatic curve following, electronic function generators or sequential superposition of step functions. After examining the possible alternatives it was decided that the cheapest, most flexible and most reliable method of generating the variable voltage and resistance functions would be the step by step method, using mechanical switching. Using a Post Office uniselector, a function consisting of 48 discrete changes in level could be built up, the complete function being traced through in a minimum of two seconds.

This was considered to be completely adequate for the type of culminations envisaged, and standard 25 way, multibank uniselectors with make-before-break contacts were used for both input temperature and transfer coefficient functions. The rate at which the uniselectors are driven is determined by the time scale employed, and the duration of the heating of the wall.

Analogue time is derived from a standard continuously variable audio frequency oscillator. This triggers a "driver unit" which in turn operates a "dividing switch". Pulses from this switch are taken to the main arbitrary function uniselector motor.

For the $T_r(t)$ function, the voltage steps approximating the continuous function are derived from a potential divider connected to a 210 volts direct current mains supply. The divider has 100 tapping points, so that voltages in 2 volt steps from 0 to 200V are available. To maintain a source impedance low compared with possible values of R_h, the total divider resistance was kept to about 200 ohms. To prevent large circulating currents flowing during the brief period of shorting of adjacent contacts - the wiper is a make-before-break one - resistors are inserted between divider tapping points and the uniselector contacts. In setting up R_h values these resistances should be allowed for, though for all practical purposes, the value of these resistors, 100 ohms, is negligible compared with even the lowest values of R_h usually required.

The tappings from the divider are taken to a plugboard, as shown in Figure 5. Forty eight plugleads are connected to the uniselector contacts, each contact position representing a time interval. By plugging the leads into the appropriate divider taps, the desired approximation to the T_r input function is rapidly set up.

The variation in R_h between the first and last steps is very large in many problems, and to obtain the greatest flexibility in values of R_h, each step of R_h was designed to be a separate resistance. Each resistance arm is composed of two "plug-in" fixed resistors and a "plug-in" variable resistor. By suitable choice of these three elements, and final adjustment of the variable resistor, a wide range of R_h values is readily set up. To assist in this, a built-in components bridge has been installed to enable adjustment to be made in situ.

5.2 The time base and driver unit

The arbitrary function driver unit was briefly mentioned in para. 4.1. Figure 6 shows the arrangement in more detail. Basic time is derived from a stable variable frequency oscillator operating a thyratron pulse generating unit which drives an intermediate uniselector. This device enables the switch to be driven at speeds up to 50 steps per second. This uniselector is used as a pulse dividing device, the division factor being selected by the position of a five-way twelve pole Yaxley switch.

The main uniselector, driven by pulses from the divider, generates functions T_r and R_h on banks I and II and III and IV. In addition the uniselector supplies a time base voltage. This is derived via the "horizontal deflection" network of Figure 6, and consists of a chain of resistors connected across a D.C. supply, and from which a stepped voltage is tapped by banks V, VI, VII and VII of the uniselector.

5.3 The recorder

The temperature at any point in the wall, at the inner or outer surfaces, or within the material, is given by the voltage at the corresponding point in the analogous R-C ladder network. This voltage is tapped by a crocodile clip connected directly to the recorder. To determine heat flow, the voltage difference across a known conductance, either R_h, or one of the cell resistance elements, is determined, rather than a direct measurement of the analogous quantity, electric current.

The recorder used is a cathode ray tube, chosen for its very high input impedance, a very important factor in the analogues of some problems on the computer where an input impedance less than several megohms would result in a severe loss of charge from the ladder network capacitances and very serious errors in the solutions.

Originally the output from the wall was connected directly to the Y_1 plate, Y_2 being earthed, but the asymmetrical arrangement resulted in severe defocussing and consequent loss in accuracy of reading. A simple phase-splitting valve circuit giving a balanced input to the tube solved this problem - see Figure 7.

The X_1 X_2 plates are connected to the time base resistance network described in para. 5.2. The time base voltage output is also a balanced one to avoid defocussing of the image.

The C.R.T. face is photographed on 5" roll film. Figure 10 shows a print of a typical record. Since the time base in stepped, and the temperature at any point in the wall will vary with time, the trace drawn during the time interval when the spot is stationary horizontally results in a vertical line, one end of which represents temperature at the beginning of the time interval and the other the temperature at the end.

5.4 Calibration

In order to read the film records directly, calibrating voltages are fed into the recorder at the end of each run. These allow a check to be made on the voltage scale factor and the linearity of the recording tube. At the end of a run, a cam attached to the main uniselector operates a switch which starts the calibration network uniselector. This uniselector is connected to a resistance chain inserted across the D.C. 210V supply - see Figure 8. The input to the Y plates of the C.R.O. is changed over to the stepped voltage output from this selector. A 50 c/s alternating voltage is applied to the X plates. The resultant picture is a series of horizontal voltage or temperature calibration lines. These lines are photographed on to the film which has just previously recorded the temperature/time solution. By brightness modulation, and altering the trace duration, these calibration lines may be graded in density for easier reading.

5.5 Analogue reset

To examine the temperature gradient through a thick skin, and to avoid duplication of recording devices, the problem is re-run through completely for each node point measurement required. At the end of each run the wall is in its "heated" condition and must be "cooled" to the initial conditions. This is done by discharging the condensers to the common earth bus-bar. This operation may be carried out manually, but for speed of operation, discharging relays have been fitted which are automatically closed at the termination of a sequence.

5.6 Radiation term analogue

The circuit used in the machine is identical to that described in para. 4.1. Thermionic diodes were found to be preferable to other rectifiers in most cases.

5.7 Layout of the computer

The machine has been built on what is virtually a "breadboard" construction. The various units, such as plug boards, driver unit, uniselector banks, oscillator and power packs, are placed on shelves as shown in Figure 9. They are very accessible for servicing and easily removed for modification. This construction affords the maximum flexibility and ease of modification as well as keeping cost and manufacturing time to a minimum. Oscillator, power packs and component bridge are all proprietary items.

5.8 Circuit elements

Standard good quality radio components are used in the wall ladder networks and "R_h" function generator. High stability carbon resistors and paper dielectric condensers are used, their values being chosen by selection to be within 1% of the required analogue value. The condensers are tested carefully for leakage. In some problems, leakages of several megohms can result in appreciable errors. (Where condensers are required to have unrealisably large leakage resistance - capacity time constants, a new time scale factor should be chosen).

6 Accuracy

For the solution of engineering problems of the heat transfer type, excessive accuracy is not warranted, since the physical constants employed and the theoretical laws on which the problems are based cannot be guaranteed to very high accuracy. A solution giving, as an arbitrary figure, an accuracy to within 2 to 3% of the rigorously computed value is admissible.

Errors arise in the machine due to

(i) Time inaccuracy.

(ii) Component value inaccuracies in ladder network, $T_r(t)$ and $h(t)$ analogues.

(iii) Reading errors from the film record.

(iv) The representation of $T_r(t)$ and $h(t)$ by superimposed step functions.

(v) The representation of the wall by a multi-cell ladder network.

Errors arising from (i) and (ii) can be made as small as one pleases within the limits of measuring equipment and component stability. Such errors are estimated to be within 1% on this machine.

Error due to film reading is difficult to assess. It is known that such errors can be large, up to 5% when readings are a small fraction of full scale deflection. Errors largely depend upon the clarity of the spot and the skill of the reader. Great care has been taken in obtaining a well defined cathode ray spot image.

Also difficult to assess are errors due to (iv) and (v). In simple cases these errors can be assessed analytically, but in actual problems, determination of error is a formidable task.

The error diminishes with the increase in the number of steps making up the time dependent functions and the number of cells making up the wall. The diminution in error is rapid, especially with increase of the number of cells in the ladder network. As simply realisable values, 48 steps for "T_r" and "h", and 10 cells for the wall were taken. Overall computer error has been determined by an empirical method, comparing computer solutions with step by step hand worked solutions. Differences of up to 3% have been found, a generally acceptable measure of agreement.

7 Conclusions

The machine described affords a ready means of computing certain types of heat transfer problems by making a simple electrical model of the heat flow system from readily obtained standard electrical components. This approach yields a computer far less complicated and costly than one using the differential analyser method, and eliminates reliance on valve and amplifier characteristics.

The analogue used has been described many times, the literature extends back to the beginning of the century. The references in para. 8 are but a fraction of those readily available.

The computer was built in a few weeks by one and sometimes two people, and after over a year of continuous work, has already saved a considerable amount of computing effort.

Being an analogue device, exact solutions are not obtainable, but errors in problems so far worked appear to be much less significant than the uncertainties in the input data.

A serious limitation in the machine is its present inability to deal with walls of temperature dependent physical properties. It is hoped to incorporate provision for this, in certain specific problems, at a later date.

8 Acknowledgment

The author wishes to thank Mr. R.H. Heald, late of this Department for his enthusiastic assistance, and Mr. A.H. Stevens for his untiring work in assembling the computer in so short a time.

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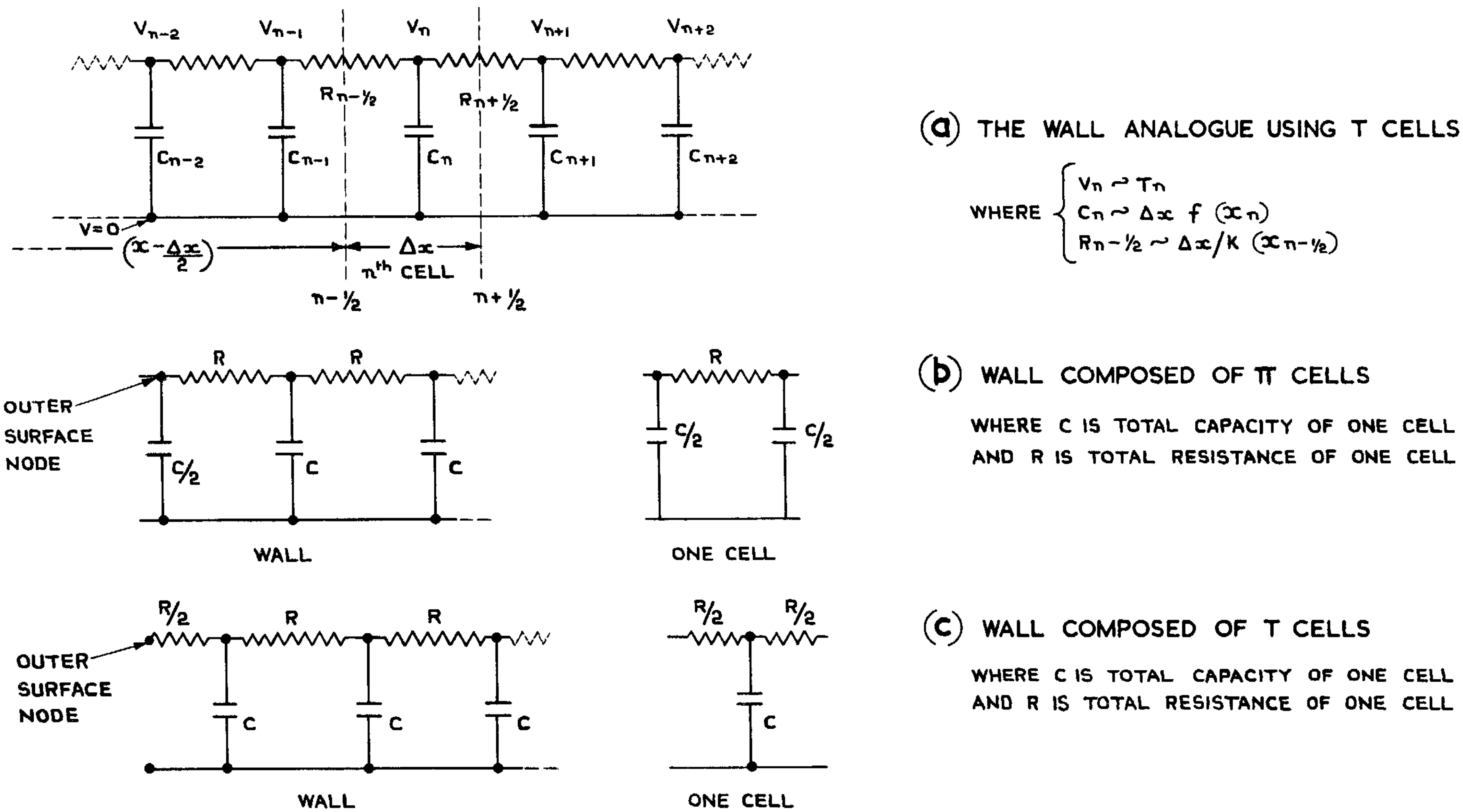
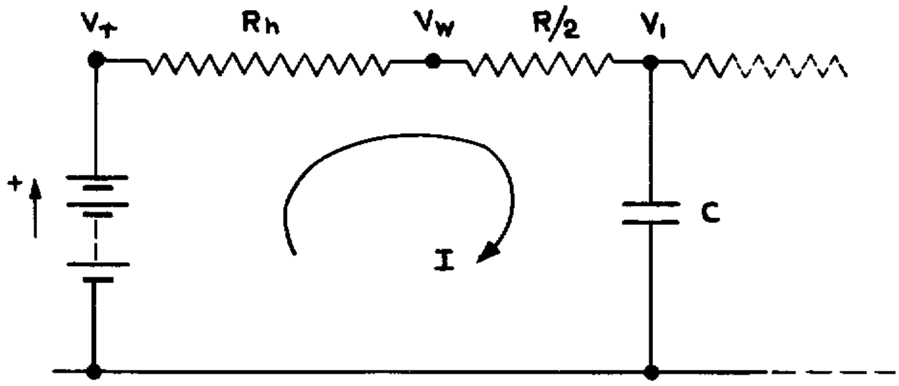


FIG. 1. (a—c) THE ELECTRICAL ANALOGY FOR ONE DIMENSIONAL FLOW IN THE THICK WALL.



$$I = \frac{V_+ - V_w}{R_h} = -\frac{V_1 - V_w}{R/2}$$

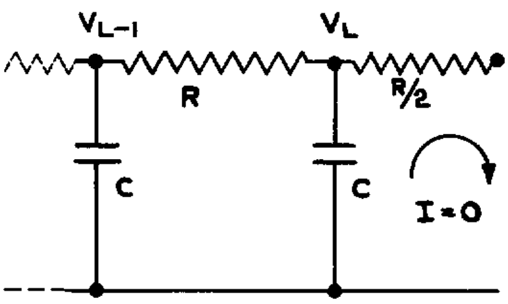
ANALOGOUS TO

$$\frac{dH}{dt} = h\left(T_+ - T_w\right) = -\underset{x=0}{K}\left(\frac{T_1 - T_w}{\Delta x/2}\right)$$

WHERE

- $R/2$ IS RESISTANCE OF ARM OF T CELL AT WALL SURFACE $\left[R/2 \sim \dfrac{\Delta x/2}{K_{x=0}}\right]$
- R_h IS RESISTANCE ANALOGUE OF TRANSFER COEFF. h $\left[R_h \sim 1/h\right]$
- $V \sim T$

(a) OUTER SURFACE



FOR NO PASSAGE OF HEAT THRO' THE INNER SURFACE

$$\frac{dH}{dt} = 0 \sim I = 0$$

(b) INNER SURFACE

FIG. 2. (a & b) WALL BOUNDARY CONDITIONS.

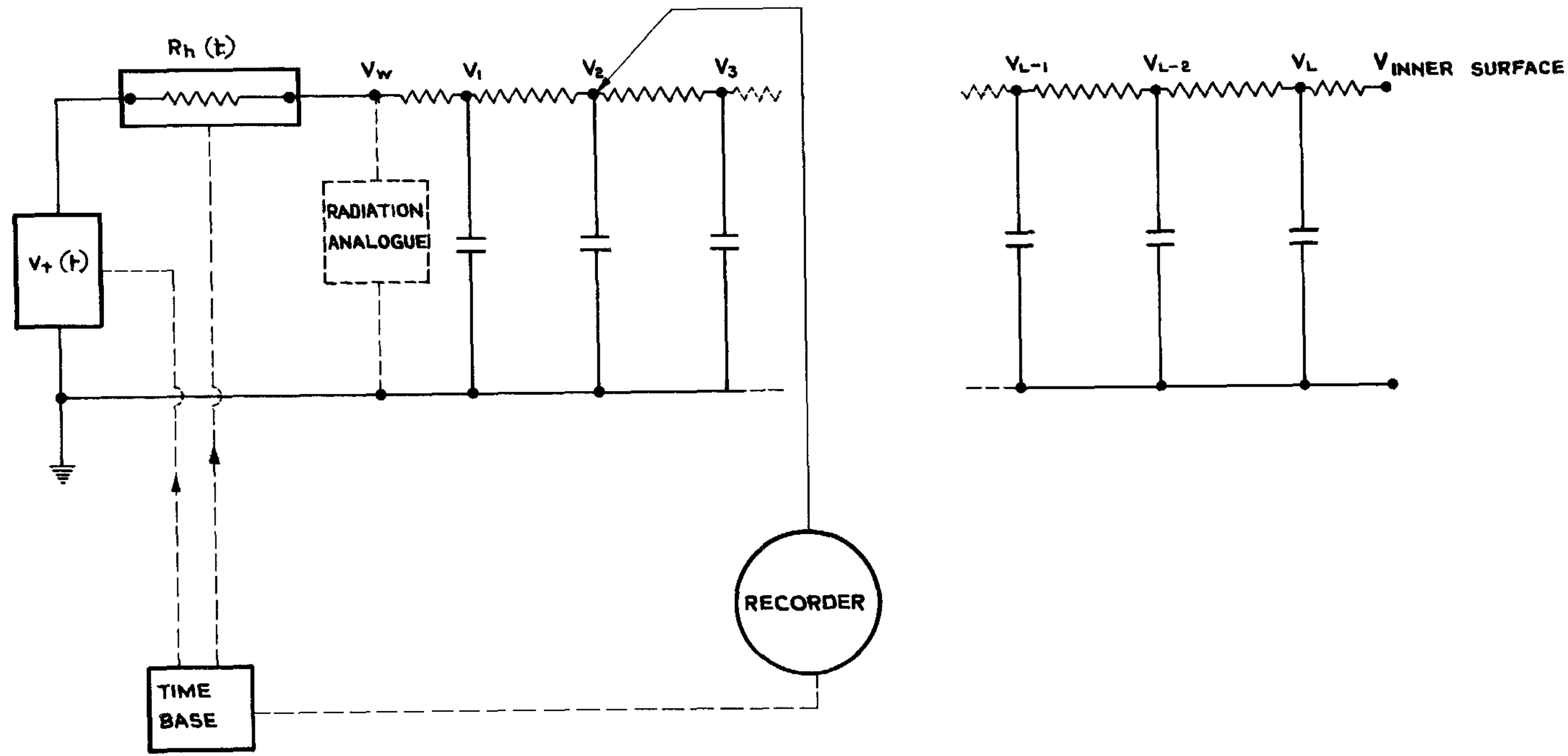
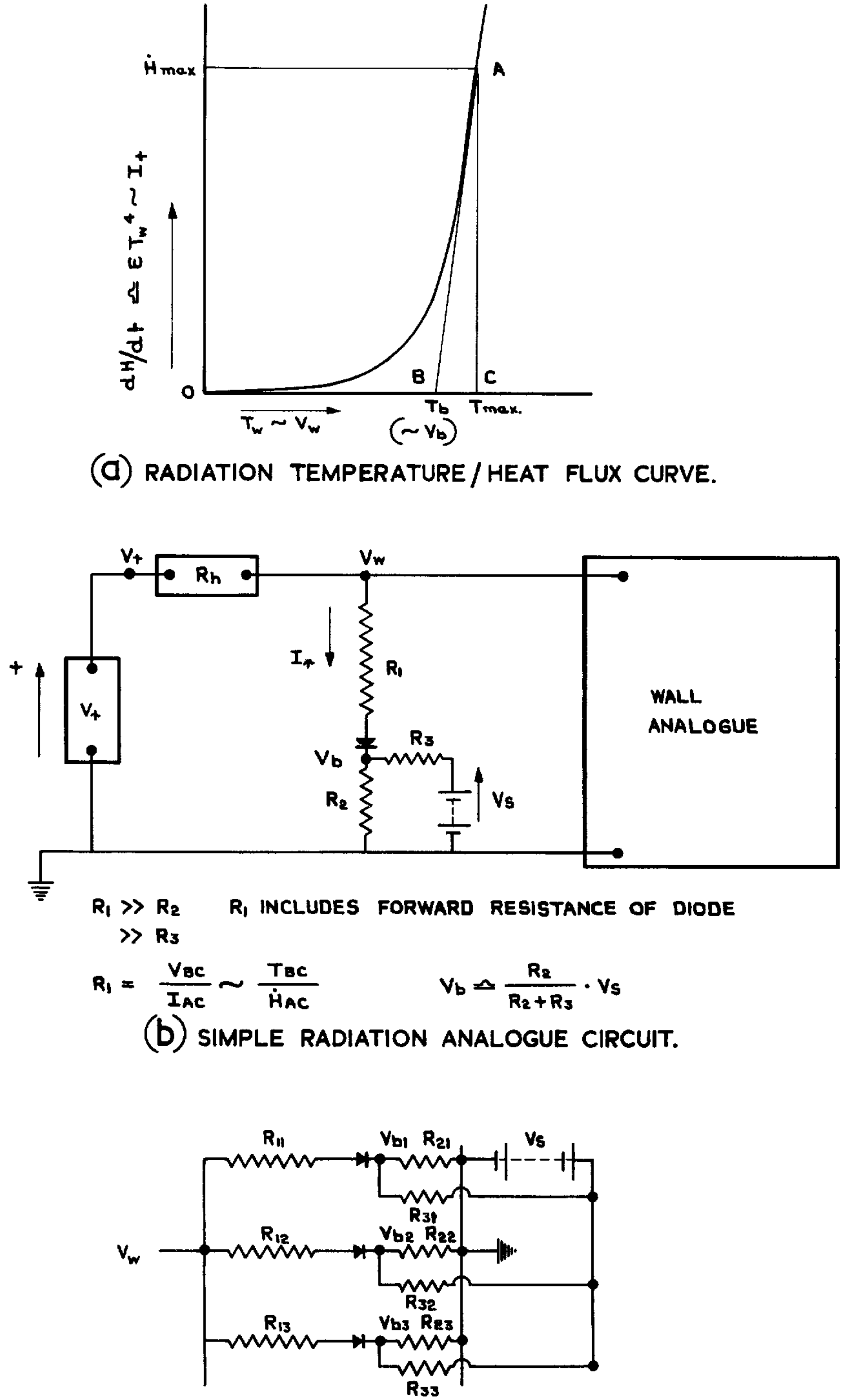


FIG. 3. SCHEMATIC OF COMPLETE ELECTRICAL ANALOGUE.



(a) RADIATION TEMPERATURE / HEAT FLUX CURVE.

(b) SIMPLE RADIATION ANALOGUE CIRCUIT.

(c) CIRCUIT FOR SEVERAL TANGENT LINES.

FIG. 4. (a–c) RADIATION ANALOGUE.

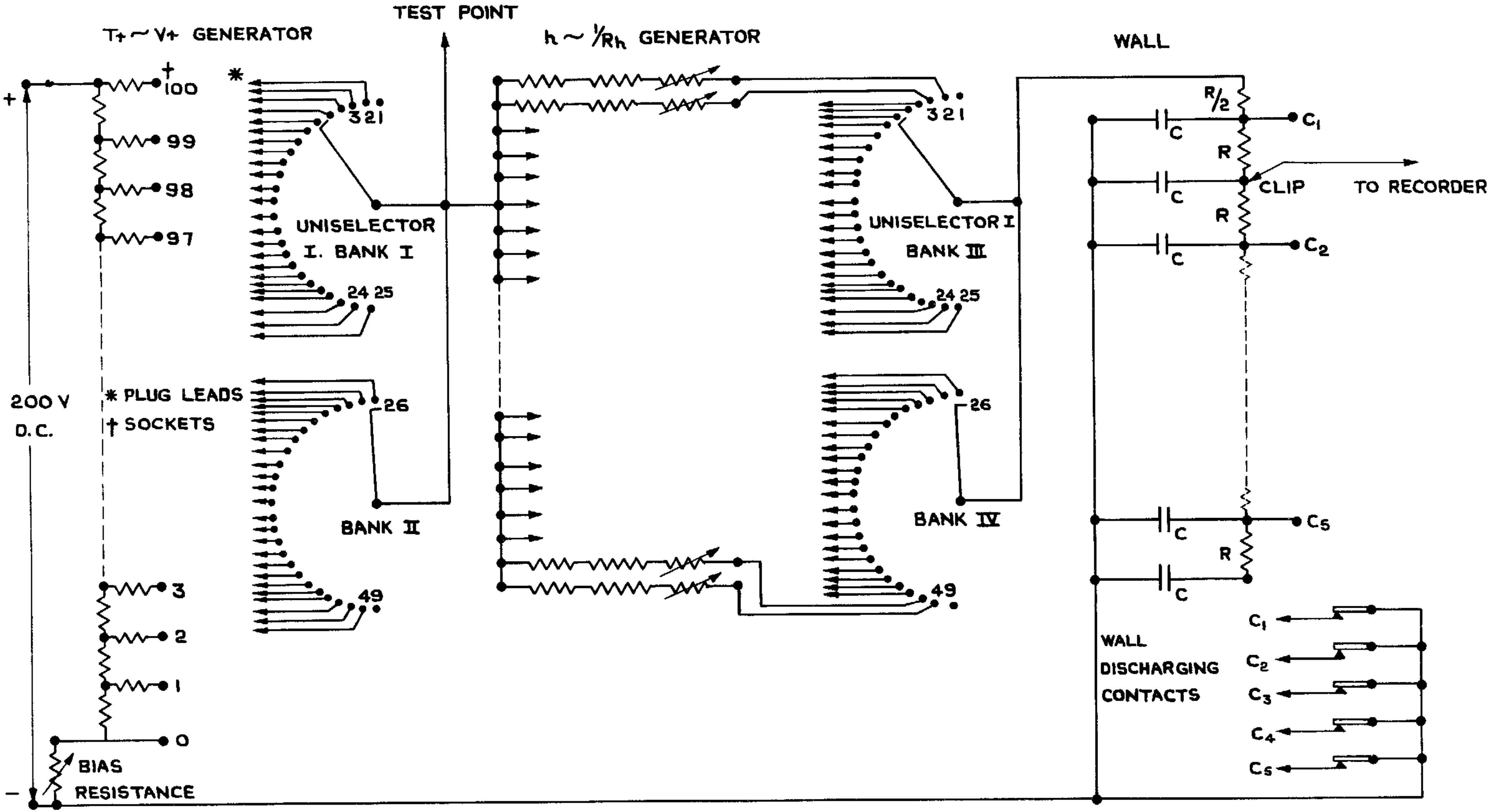
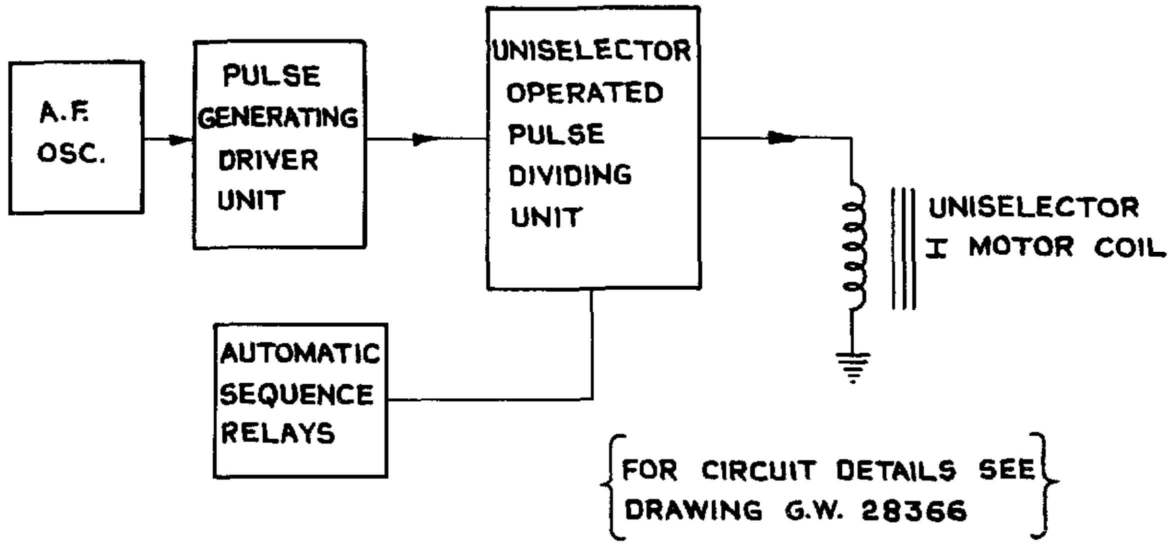
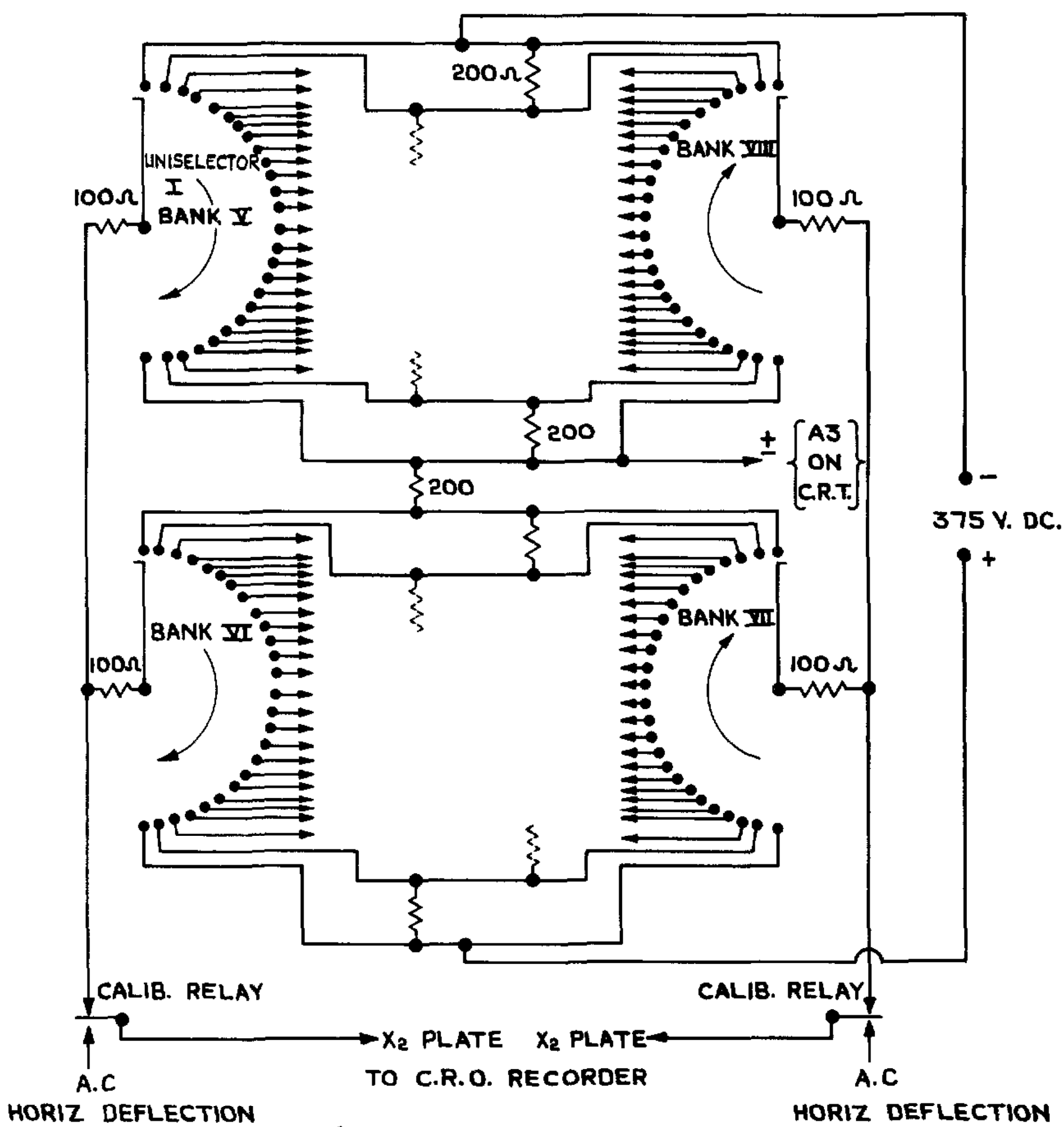


FIG. 5. FUNCTION GENERATORS.



(a) SCHEME OF DRIVER UNIT.



(b) TIME BASE CIRCUIT.

FIG. 6. (a & b) DRIVER UNIT AND TIME BASE.

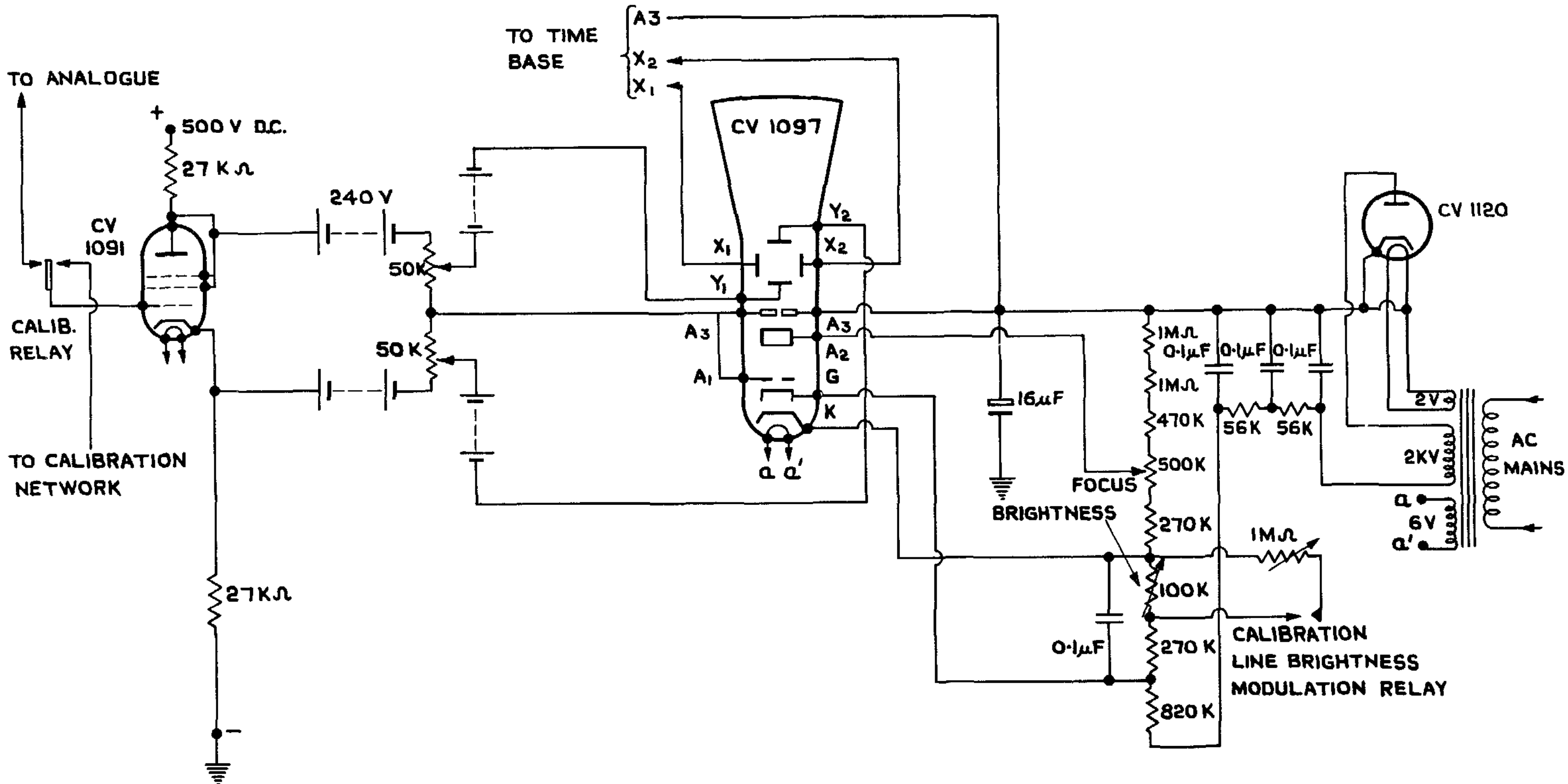


FIG. 7. C.R.O. RECORDER.

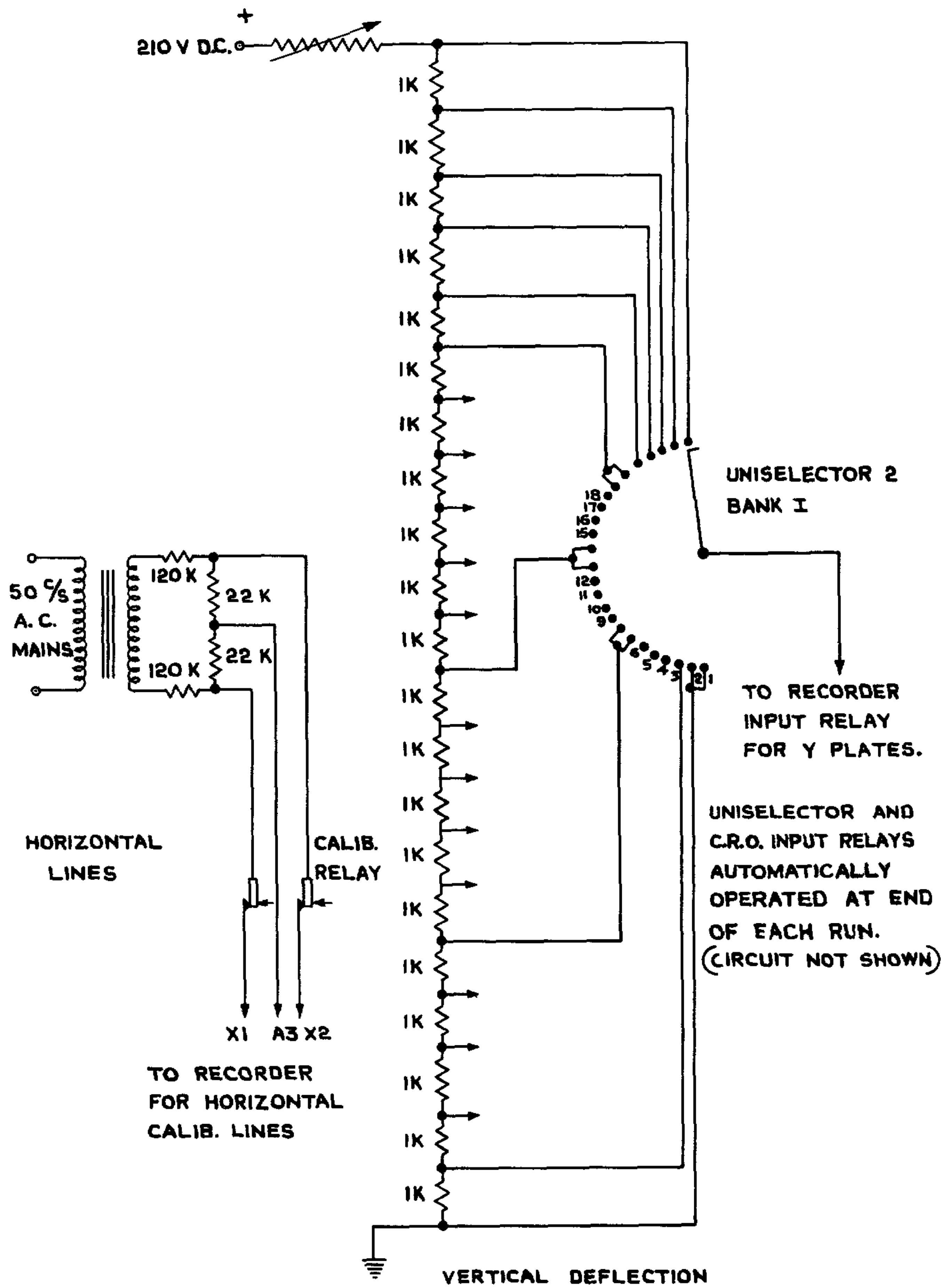


FIG. 8. CALIBRATION CIRCUIT.

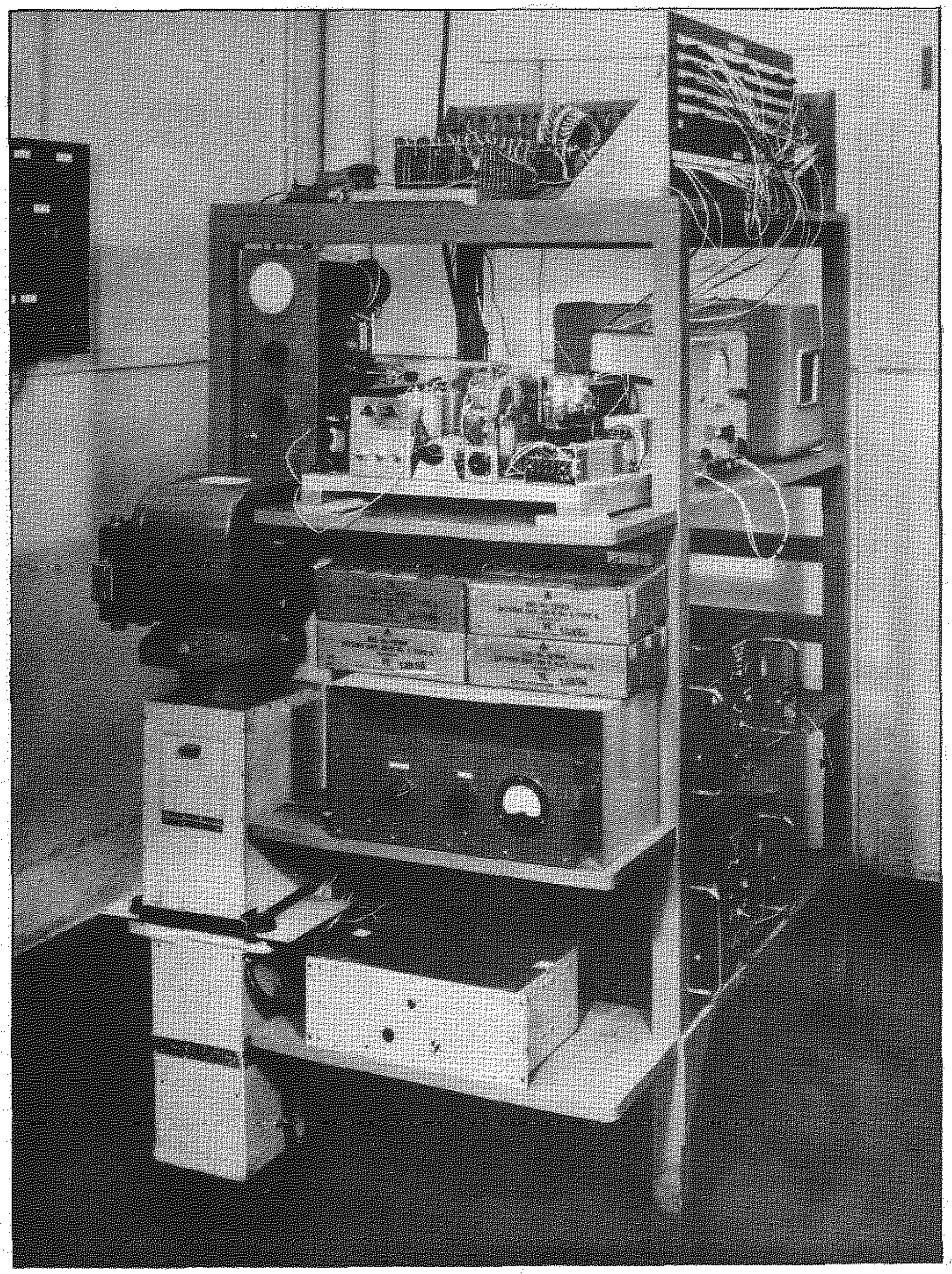


FIG.9. GENERAL VIEW OF COMPUTER

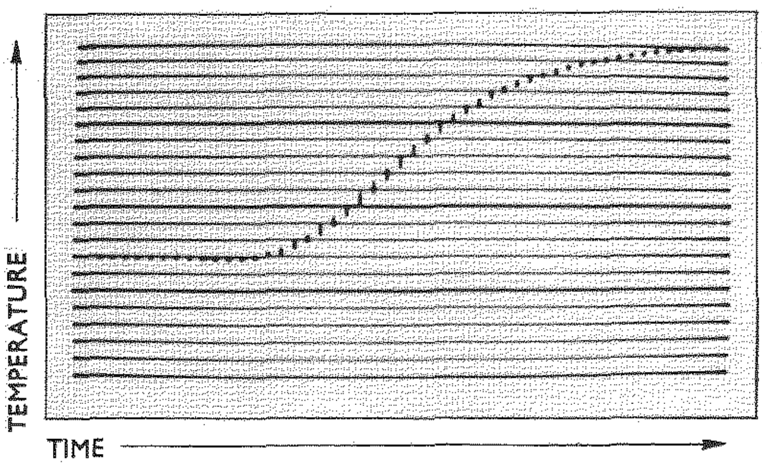


FIG.10. TYPICAL RECORD FROM COMPUTER (ACTUAL SIZE)

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