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Flutter Prediction in Practice

By

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Flutter Prediction in Practice

by

E. G. Broadbent

SUMMARY

A brief summary is given of the resources available for flutter prediction, both experimental and theoretical. Specific flutter investigations are described for four separate flutter incidents which have occurred in Great Britain during the past few years. In all four examples good agreement is eventually obtained between the calculations and full scale experience and it is thought that useful lessons can be learned from the steps which proved necessary to get this agreement. Two of the examples relate to classical flutter, in which compressibility is unimportant, but in the other two examples, although compressibility is important, it would have been wrong to conclude that the incidents were only due to negative aerodynamic damping in one degree of freedom. Conclusions for future guidance are drawn from all the examples, with particular reference to the aerodynamic treatment. The paper concludes with a few points from a more general statistical survey of recent incidents.

This paper is to be presented at the meeting of the Structures and Materials Panel, Advisory Group for Aeronautical Research and Development, NATO, to be held at Washington, U.S.A., April, 1956.

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1 Introduction

Flutter prediction in recent years has been the subject of considerable and increasing effort in industry. Considerable use is now made of aerodynamic models in wind tunnels and free flight, and techniques of ground resonance testing and flight flutter testing are continually being improved. Perhaps the greatest development, however, has been in the field of high speed computing equipment which has enabled lengthy calculations to be carried out as a matter of routine, where they could not have been attempted a few years ago. With these advances the accuracy of flutter prediction is greatly improved, and some of the flutter incidents described later in the paper would probably have been prevented had sufficient resources been available during the design period. On the other hand the uncertainties associated with transonic speeds, low aspect ratios and higher frequency modes, involving chordwise distortion for example, are also increasing. The biggest single problem of the last few years has been that of predicting and countering control surface flutter in one degree of freedom. It is, however, wrong to think of all Mach number sensitive oscillations as being examples of this type of flutter, as can be seen from the examples described in section 4 of this paper. The intention here is to give a brief description of the resources available for flutter prediction and then to examine the histories of chosen incidents in some detail to show what steps were necessary in order to obtain good agreement between calculations and full scale. A number of points of interest can be noted from these examples and some further evidence is given from a survey of a larger number of incidents of recent times. In the four examples quoted in detail it is interesting that the final solution is always more complicated than appeared likely at an earlier stage in the investigation.

2 Experimental resources available

The prediction of an aircraft flutter speed will generally make use of results obtained from experimental research programmes, but here we are concerned with experimental work related specifically to the aircraft itself. Such work falls into two classes; work on models, which is carried out in the early design stages, and work on the complete aircraft, which is carried out immediately before and during the flight testing of the first prototype. It is not intended here to give a detailed description of these branches of flutter prediction, but a general appreciation of their form and value is attempted.

2.1 Models

The main purpose of wind tunnel and rocket models is to investigate the aerodynamic forces that enter the flutter problems. Two sorts of model can be used: a rigid model on which selected degrees of freedom are permitted for the measurement of derivatives, and a flexible model designed to give the flutter speeds directly. Derivative models can be used to measure overall wing derivatives or control surface derivatives, and these measured values may then be used in flutter calculations. They can be designed either for wind tunnel testing or for ground launched rocket tests, in which case three models are needed pitching about different axes and with different control surface gearings in order to get a complete set of derivatives. The results cannot always be used directly in flutter calculations because of the effect of the modal shape; instead they can be used to check or modify the theory which is then applied to the modes of distortion assumed in the flutter calculation.

Flutter models can also be designed either for testing in a wind tunnel or by ground launched rocket. In either case the model represents the aircraft plan form, inertia distribution and stiffness distribution to a suitable scale. The wind tunnel model offers scope for detailed examination of the modes of flutter and for considerable variation of stiffnesses and masses; on the other hand the rocket model provides a vehicle for investigating transonic effects. Generally the models will be built to represent a calculated stiffness distribution, but occasionally the full scale structure may be copied, particularly at any complicated junctions where calculation is unreliable.

Some advantages and disadvantages of models are compared in the following two tables. Table I compares rocket flutter models with wind tunnel flutter models and Table II compares flutter models with derivative models.

TABLE I

	rocket models	wind-tunnel models
construction	flexible-probably confined to the wing or wing and aileron	flexible-whole aircraft may be represented.
Mach number	transonic and supersonic	almost certainly subsonic
testing	one test per model with telemetry	several tests (usually hundreds) for varying stiffnesses
analysis	limited to the telemetry records. Often vibrations occur of uncertain origin.	close examination of the flutter mode is possible, and the start of true flutter can be reliably established.

TABLE II

	flutter models	derivative models
construction	as above - a fairly long and elaborate process	relatively simple - e.g. solid steel
testing	flutter speeds measured - the testing itself being very simple. Measurement of modes is more difficult. Stiffness and resonance tests are generally needed.	derivatives measured. A reliable technique and a suitable rig are needed. When these are available the testing need not be very elaborate.
interpretation of results	representation of full scale must be checked and any deficiencies investigated by calculation	the accuracy of the measurements must be demonstrated: the results are used in flutter calculations.
special merits	where the structure can be modelled directly and calculation would be doubtful the model is most helpful.	derivatives which are difficult to calculate may be measured e.g. the effects of wing body interference, and tip effects on a wing of moderate to low aspect ratio at low supersonic speeds.

2.2 Ground Resonance Tests

These tests are ideally carried out on the first prototype immediately before its first flight. A principal value of the tests is that they give a complete check of the structural data used in all the work on flutter prediction. They may be designed to measure the normal modes of the aircraft, in which case the technique for excitation, support and measurement will need to be well developed. If sufficient accuracy can be obtained, however, the results are extremely valuable as they can be compared directly with the calculated normal modes. Any discrepancies can be investigated and the data used in the flutter calculation can then be modified accordingly. Alternatively the measured normal modes can be used directly as a basis for new flutter calculations, but in general a better continuity of flutter prediction is obtained if they are used to check the data as first described. It is, perhaps, unsafe to rely on the measured modes as being faultless, without checking them against corresponding calculations. This is a strong argument for carrying out the normal mode calculations, and it is important that the tests and calculations should relate to the same conditions of mass distribution, in one case at least; the tests (and, of course, the calculations) may also be carried out for different mass conditions that will occur in practice.

Because of the difficulty of measuring normal modes accurately, it may be preferred in some cases to measure resonance modes as excited by single, or paired, exciters near the relevant control surfaces. This would enable the modes to be checked for control surface flutter one by one, but calculations in which more than one resonance mode was included would be difficult. This technique has certain advantages over the measurement of normal modes, but among other things, the comparison with calculations is more difficult. Some attempt at comparison should be made, however, probably by estimating the structural damping and calculating resonance modes for the same exciter positions as will be used in the ground resonance tests.

2.3 Flight flutter tests

Flight flutter tests represent the only way of obtaining information on flutter by direct measurement in flight. In principle there are two methods of doing this both of which are regularly used in practice. One method, the forced oscillation technique, applies a sinusoidal exciting force to the aircraft and records the response as the frequency is slowly increased through the important range. The resonances are picked off, as in a ground resonance test, and the peak amplitudes plotted against airspeed; an approach to flutter is indicated by a sudden increase in peak amplitude. The alternative method, of decaying oscillations, is to set the aircraft vibrating at a resonant frequency, either by an impulse or an oscillating force as before, and then to allow the vibrations to die away naturally, the rate of decay being proportional to the net damping, and this is then plotted against airspeed.

Flight flutter testing, backed up by calculations and sometimes by model tests, certainly offers the best chance of avoiding flutter of the aircraft in practice. The main objection to flight flutter tests is the length of time required to complete them, for during this time little other flight research can be carried out because of the rather bulky equipment that must be carried. The theoretical backing is necessary in case some forms of flutter are approached so suddenly that there would be virtually no warning; in any case it is not easy even if it is possible to watch all frequencies and all parts of the aircraft at the same time, and flutter calculations enable the more dangerous possibilities to be seen in advance and investigated.

3 Theoretical work

Flutter calculations are usually made in different forms as the aircraft design progresses. In the project stage simple formulae for the flutter speed are sufficient, and in fact essential for preliminary weight estimates when alternative designs are being compared. Later simple flutter calculations will be made for the design chosen, and finally when the detailed design is complete normal mode calculations and full flutter calculations are undertaken.

3.1 Design criteria

As far as flutter is concerned design criteria at present generally take the form of simple flutter speed formulae. A set of three critoria used for comparing the wing stiffness requirements of flutter, aileron reversal and divergence are given below to indicate the typical form. These criteria are obtained partly from theoretical and partly from experimental results, and they are clearly most accurate when applied to the same type of wing as was used in obtaining them. They should not be applied to unorthodox wings for which the critical speed may depend on a parameter not included, e.g. the aileron reversal speed may be raised by using an inboard aileron not allowed for in the criterion, and the flutter speed may be greatly affected by a large concentrated mass. Safety margins are included.

Subsonic and transonic flutter

The value of U is given by

$$U \geq 0.0035\, s \cos \Omega \left\{ \frac{c\, Vg}{\left(0.77 + \frac{0.1}{r}\right)\left(1 - 0.166\, M \cos \Omega_{LE}\right)} \right\}^2 \times \cos^2(\Omega_{LE} - 11) \Big/ \left\{ 0.9\left(1 + \frac{0.8}{A}\right) \right\}^2$$

where U lb ft/radian is the strain energy for a torsional deflection about the centre-line of the box varying linearly from zero at the wing centre-line to one radian at the wing tip

V is the maximum forward speed of the aircraft in knots E.A.S. and M the corresponding Mach number ($M \cos \Omega_{LE} < 1$)

s ft is the distance from wing root to tip measured normal to the centre line of the aircraft

Ω angle of sweepback of centre-line of box

c ft is the chord parallel to centre line of aircraft at 0.7s from the root for $1 > \lambda_r > 0.4$, and at $(0.65 + 0.125\,\lambda_r)$s from the root for $\lambda_r < 0.4$

λ_r taper ratio $= \frac{\text{tip chord}}{\text{root chord}}$

g average distance of inertia axis aft of leading edge ÷ wing chord

r stiffness ratio $= \dfrac{\ell\phi \; c_m^2}{0.81 \; m_\theta \; s^2}$

An initial estimate is required of the value of r. It is unlikely to be less than 0.5 (the minimum value for which the formula should be used) for box wings of aspect ratio eight, and increases rapidly as aspect ratio is reduced.

$\ell\phi$ lb ft/radian is the wing flexural stiffness measured at 0.7s

m_θ lb ft/radian is the wing torsional stiffness measured at 0.7s

c_m ft is the mean chord of the wing outboard of the root

Ω_{LE} degrees is the angle of sweepback of leading edge.

Supersonic flutter

No criterion for flutter at supersonic speeds is available but it is probable that if the transonic criterion is satisfied at a given altitude, flutter is not likely to occur at that altitude for Mach numbers up to about 2.5 provided that the aerodynamic axis is within $\pm 5\%$ chord of the inertial and flexural axes.

Aileron reversal (conventional flap-type ailerons on the trailing edge of the outer wing)

The value of U is given by

$$U > 0.00048 \frac{V^2 \, c^2 \, b}{\cos^{\frac{1}{2}} \Omega_{\frac{1}{4}}} \left(\frac{a_1 \, m}{a_2}\right)$$

where V knots E.A.S. is the maximum forward speed and M is the corresponding Mach number

c as defined above

b ft is the wing span normal to centre-line of aircraft

$\Omega_{\frac{1}{4}}$ angle of sweepback of $\frac{1}{4}$ chord line

a_1 slope of curve of lift coefficient against incidence

a_2 slope of curve of lift coefficient against aileron angle

m slope of curve of nose down moment coefficient against aileron angle at constant incidence

The values of a_1, a_2 and m should correspond to the value of M.

Wing divergence

The value of U is given by

$$U > 0.00056 \frac{V^2 \, c^2 \, b}{\cos^{\frac{1}{2}} \Omega_{\frac{1}{4}}} a_1 \, e$$

where e is the average distance of the flexural axis aft of the aerodynamic axis expressed as a fraction of the chord.

This equation is the same as that for aileron reversal except that e is substituted for $\frac{m}{a_2}$, and the safety margin increased.

This formula should only be applied to wings with little or no sweepback but it is unlikely that divergence will be critical for highly swept wings.

In more complicated cases where the simple formula does not apply, an approximate flutter speed can usually be obtained by consulting the results of experimental research work. For example the effect of a heavy mass on wing flutter can be determined and if the position of the mass can be varied, the most favourable can be selected. In this way simple design criteria help in the choice of an optimum design.

3.2 Flutter calculations

The structural assumptions made in the design flutter calculations have already been mentioned; probably in the early stages arbitrary modes will be used and in the later stages the results will be checked using calculated normal modes. On the aerodynamic side it is difficult to find a satisfactory middle course between two extremes.* On the one hand are available approximate mathematical solutions to the integral equations for the velocity potential at various conditions of Mach number. The numerical work involved in solutions of this kind is very considerable, but the preparation of numerical tables and the use of high speed digital computers will probably lead to the accurate solutions of this type becoming a routine process. The other extreme is to use strip theory; in this case the procedure is either to estimate by rough rules aerodynamic strip derivatives appropriate to the aspect ratio, and possibly the Mach number also, which is being considered, or to use two-dimensional incompressible derivatives and to apply empirical corrections to the flutter speed for the aspect ratio and Mach number. Of these two simple methods the latter is perhaps best for main surface flutter and the former for control surface flutter. These simple methods can be made to give good agreement with the flutter speeds measured on simple models, but it is not necessarily true that they will give equally accurate results for an aircraft wing.

3.3 Correlation between calculations and flight measurements

So many doubtful quantities occur in the coefficients of the flutter equations that it is important to try to obtain agreement between theory and experiment in as many ways as possible. Thus calculations should be carried out on any flutter models that are built and also on the aircraft in flight for comparison with any flight measurements that are intended. Nor is it satisfactory to rely on experiment alone: a wind-tunnel (or rocket) model is unlikely to represent the aircraft with sufficient accuracy to be relied on without calculation, and the flight tests might be dangerous if they are not preceded by thorough calculations. Another important advantage gained by matching calculations with experiment is that the experience can be used in future calculations on similar problems.

This purpose is illustrated by the examples discussed in the next section. Four flutter incidents which have occurred in Great Britain in the past few years are discussed in some detail in relation to the

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* One possibility, however, is discussed in section 4.4.

analytical and experimental investigations that were carried out to provide in each case an explanation and cure of the incident. Studies of this kind should be of value in avoiding similar occurrences in future.

4 Specific examples of flutter incidents and associated investigations

The examples have been chosen to give a fair cross-section of the work of flutter incident investigation in recent times. The first, a form of tab flutter, starts with a fatal accident and no direct information from flight work. The second, elevator flutter, also starts with an accident in which the elevator was lost although the pilot safely landed the aircraft; a record showing the frequency and growth of the oscillation was preserved. In these two cases compressibility was unimportant, but in the third and fourth derivative changes at high subsonic Mach number were at least partly responsible; the incidents were relatively mild and occurred more than once.

4.1 Combined effect of two tabs

The tail configuration of a twin boom aircraft is shown in Fig.1. The shaded tabs, bounded by dotted lines, were fitted to the previous Mark of the aircraft, but the larger tabs, bounded by full lines, were fitted to the aircraft which crashed. The reason for the modification was to offset the effect of other changes. The important structural modes were symmetric boom bending at 8 c.p.s.* and symmetric tail bending (with maximum amplitude on the centre-line and greater at the trailing edge than the leading edge) at 25 c.p.s.

The accident investigation soon established that the tail broke up first and evidence of violent repeated loading was found. The trim-tab and part of the spring-tab broke away from the aircraft in flight well before the main wreckage, and the underslung elevator massbalance weights came away and were not recovered. One explanation put forward was that an elevator balance weight had partly failed during catapult launching and had later been lost thus causing elevator flutter. The evidence of the early tab failures however rather conflicted with this view as elevator flutter would not be expected to cause high local stresses. Flutter calculations were started using as degrees of freedom, the two modes mentioned above, a third resonance mode later found to be unimportant, elevator rotation, spring-tab rotation and trim-tab rotation. Violent tab flutter at 24 c.p.s. was found from these calculations, even with full elevator massbalance. The result of the calculations is shown in Fig.2 where flutter speed is plotted against structural damping in the motion of the tabs. The quantities involved are considerably greater than critical damping of the tabs (the scale of Fig.2 is arbitrary) which indicates the violence of the flutter, and an important point is that the two tabs together have a much stronger tendency to flutter than either tab alone.

The reason for the lack of prediction was that the tabs were designed and fitted on the basis of massbalance criteria (as was customary at the time in Great Britain), and the small tabs proved satisfactory on the previous Mark. The trim-tab, which was probably the principal cause of the accident, also satisfied the frequency relations

$$\frac{\omega_\gamma}{\omega_\beta} > 2; \quad \frac{\omega_\gamma}{\omega_z} > 1.5 \qquad (1)$$

* Numbers quoted here and elsewhere are approximate only.

where ω_γ is the tab frequency

ω_β is the control surface frequency

ω_z is the main surface frequency

as long as the main surface frequency is assumed to be the fundamental, in this case 8 c.p.s. The tab frequencies were

$$\left.\begin{array}{ll} \text{trim-tab frequency} & = 16 \text{ c.p.s.} \\ \text{spring-tab frequency} & = 6 \text{ c.p.s.} \end{array}\right\} \quad (2)$$

Although the spring-tab frequency is low however, it is well damped. Finally the criteria do not take account of the cumulative effect of the two tabs.

It is apparent from the flutter calculations that the two tabs will combine to promote flutter much more strongly than either alone, (see Fig.2) particularly if the flutter frequency is higher than both their natural frequencies.* Conclusions on the use of criteria can therefore be summarised as

(i) The frequency requirements (1) are just as important as the massbalance requirements.

(ii) Consideration of the fundamental main surface mode only is unsafe - probably frequencies at least up to the main surface torsion mode should be considered, and this may be quite high on a tail surface.

(iii) The combined effect of tabs should be considered.

A combination of four modifications was found to provide a satisfactory cure. These were

(a) trim-tab chord reduced to standard - the improvement due to this alone is shown in Fig.2.

(b) trim-tab inertia reduced in addition to (a)

(c) trim-tab statically balanced

(d) spring-tab statically balanced

These modifications were proved in flight, but it was later found satisfactory to return to the smaller tabs of the previous Mark of the aircraft.

A further general conclusion can be made from the work, namely

(iv) The modifications to an aircraft should be scrutinised as carefully as the original design.

There is no suggestion that this was not done in this case, as the modification was in fact treated in the same way as the original, and at the time lengthy flutter calculations were very rarely undertaken in Great Britain because of the lack of high speed computational aids; but it could well be imagined that a similar modification could be made on a different aircraft without the flutter possibilities being considered.

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* This qualitative result was investigated by varying the trim-tab circuit stiffness.

In general it is a fair assumption that any aerodynamic modification should be analysed for the possibility of flutter.

Finally the estimation of the aerodynamic derivatives in the calculations is of interest. Some rules suggested by Minhinnick were used, and they are given below.

Rule 1 Assume all the vertical displacement derivatives, denoted by the suffix z, to be zero.

Rule 2 Assume all the other displacement derivatives to have the same value as the corresponding static derivatives ($\nu \to 0$), as estimated for stability purposes.

Rule 3 Assume all the $\dot{z}$-derivatives equal to the corresponding α-derivatives.

Rule 4 Derive the other damping derivatives from the displacement derivatives of Rule 2 and the maximum and minimum values of the two-dimensional derivatives with respect to frequency parameter, thus

$$(\ell_{\dot{\alpha}})_{3D} = (\ell_{\dot{\alpha}})_{2D} \times \frac{(\ell_{\alpha})_{3D}}{(\ell_{\alpha})_{2D}}$$

Rule 5 Assume the virtual inertia derivatives have their two-dimensional values.

Here ν is the frequency parameter $= {}^{\omega c}/_{V}$

Suffix 3D refers to the three-dimensional derivative, i.e. the derivative actually used in the calculations.

Suffix 2D refers to the two-dimensional derivatives, and for the dampings the <u>maximum</u> value is used, for the displacement derivatives the <u>minimum</u> value (with respect to ν) is used.*

These rules are intended for aspect ratios of about 2 to 4; they are thus suitable for most control surface flutter calculations. Without simple rules such as these the calculation of tab flutter becomes very difficult since the two-dimensional derivatives differ greatly from the very few measured values available.

It was thought that these rules might have been pessimistic in this case, i.e. might have led to a greater degree of instability than occurred in practice. A similar calculation was therefore carried out on the configuration of the previous Mark of aircraft as this was known to be free from flutter in practice; the calculation however was also stable. On the other hand the calculated flutter speed for the accident condition was definitely too low since the pilot was estimated to be flying at about 700 to 750 ft/sec. We may therefore conclude

(v) the Minhinnick rules gave good qualitative agreement in this instance, although they underestimated the flutter speed. Compressibility effects were unimportant.

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* The general scheme of the notation for derivatives is indicated at the end of the paper.

4.2 Elevator flutter followed by flight flutter tests

This was another case of flutter at relatively high indicated speed and low altitude, where compressibility effects were unimportant. Records from vibrographs in the fin (see Fig.3) and from an elevator circuit force recorder were fortunately preserved and the flutter frequency of 22 c.p.s. was therefore known. The elevator was power boosted and at first instability of the booster was suspected, but this was shown to be impossible at the frequency involved so that flutter calculations were undertaken. Previous analysis had only covered the lower frequency modes, again because the lack of computing aids prevented fuller treatment.

The first calculation after the accident was a binary between a tailplane bending mode at 22 c.p.s. and elevator rotation, with the result shown in Fig.4 where flutter speed is plotted against elevator massbalance and the flight incident is marked. Two-dimensional derivatives were used at this stage and it can be seen that the agreement was poor. Furthermore, inclusion of the wing motion measured in the mode at 22 c.p.s. made the discrepancy greater still, and the wing terms continued to be ignored until the final calculations.

The next step was to repeat the ground resonance tests paying particular attention to the mode at 22 c.p.s. It was found that an inadequate number of recording points had been used in the previous tests with the results that the mode of distortion as drawn was inaccurate, and the corrected mode was found to have a node that crossed the leading edge some distance from the root instead of at the root. The new mode gave a less favourable product of inertia with the elevator. Flutter calculations were now repeated with the new mode, and at the same time the aerodynamic derivatives were changed to conform to the Minhinnick rules quoted in the last section. Each of these factors extended the nose of the flutter curve so that the combined effect (still neglecting wing terms) was as shown in Fig.5. It was now felt that the agreement was good enough to indicate the cause of the flutter and the cure, which was clearly to increase the massbalance. That the massbalance was not defective was proved by recovery of the elevator. The port elevator was checked by cutting it in half and measuring the c.g. of the outer half and inner half separately; for the outer half the c.g. was on the hinge line and for the inner half was appreciably ahead of it. In spite of this the massbalance was less than dynamic because of the position of the nodal line, which was such that the massbalance near the tip was ineffective; this result is quite contrary to the initial fear that as the massbalance stopped a few inches short of the tip the outer half of the elevator might have been underbalanced.

The final stage of the calculations was to include the degree of freedom tailplane rotation. The tailplane incidence was adjustable in flight for trimming purposes, and this rotational freedom had a natural frequency of about 40 c.p.s. The calculation now agreed with the accident very well, and a slight adjustment to the aerodynamic hinge-moment derivatives made the agreement exact as shown in Fig.6. The necessary adjustments were

$$-h_{\dot{\beta}} \text{ multiplied by } 0.9$$

$$-h_{\beta} \text{ multiplied by } 1.1$$

In this calculation the wing terms were fully included. This question of what account to take of the wing in tail flutter calculations is not easy to answer with certainty. To ignore the effect of the wing altogether must lead to errors although they will usually be errors on the safe side;

as a particular example flutter calculations on the Spitfire always gave low elevator flutter speeds until the effect of the wing was included in the calculations. The difficulty is that including the wing terms often seems to lead to errors on the danger side. One possible reason for this is that the wing amplitude in the normal modes which go into the calculation, assuming normal modes of the whole aircraft are used, is probably overestimated, because in the aeroplane structural damping will reduce the wing-to-tail amplitude ratio under tail excitation as compared with the normal modes. One solution is to carry out flutter calculations using branch modes of the wing, fuselage and tail with structural damping included where appropriate. In the present example, however, it appeared simply to be necessary to include the right degrees of freedom in order to get good agreement.

The calculations described above did not follow each other immediately, nor were the three degrees of freedom mentioned the only ones considered. Several calculations in up to six degrees of freedom were carried out, in which it was shown, for example, that elevator torsion, fuselage bending and wing bending were all unimportant. Before the final calculation the elevator massbalance was increased and flight flutter tests, using a single inertia vibrator in the nose of the aircraft, were begun. During these tests traces of high frequency were observed in flight without the vibrator running, and it was this which led to the investigation of tailplane torsion and rotation, and to the inclusion of these modes in the flutter calculations.

The flight flutter tests themselves did not proceed smoothly, because they had to be interrupted more than once to take account of modifications to the aircraft which affected the flutter position although they were made for other reasons. These included stiffening the elevator circuit and the elevator in torsion and changing the wing planform. The general basis of comparison between theory and experiment was in terms of a variable degree of massbalance. Thus peak amplitude response was calculated for various forward speeds, and the results plotted for different values of massbalance. The experimental curve was then fitted onto this background, as shown in Fig.7. Qualitatively the agreement is seen to be very good, and although the individual flight points are not shown, the consistency of the flight results was found to be remarkably good also. The results given in Fig.7 are for the configuration tested after the accident with increased massbalance but without the other modifications; it will be noted that the calculations (this was in the intermediate stage) overestimated the beneficial effect of the massbalance. After the other modifications, of which the circuit stiffening was probably the most important, a second series of flight flutter tests was carried out with the results shown in Fig.8; it can be seen that the modifications were favourable.

Conclusions drawn from this investigation are

(i) In ground resonance tests a high standard of accuracy is essential.

(ii) High speed computing equipment should be used to give a routine check with a large number of modes

(iii) Distributed massbalance, even though more than static, does not guarantee freedom from flutter.

(iv) On aerodynamic derivatives, the Minhinnick rules gave very good agreement with flight in this case. Mach number effects were again unimportant.

(v) Flight flutter tests backed up by flutter calculations add greatly to one's confidence in flutter prediction, at least for classical flutter (i.e. shock wave effects unimportant). On the other hand they involve a good deal of time and effort.

4.3 Aileron flutter at high subsonic Mach number

The aileron is a conventional wing tip aileron of about 25% chord ratio fitted on a wing of moderate sweepback. In the first incident the aileron (only) was instrumented and clear sinusoidal oscillations were recorded; the frequency will be referred to as unity and all other frequencies will be quoted relative to this flutter frequency. The Mach number of the incident was about 0.93 and the height about 40,000 ft. The high Mach number of the incident coupled with the fact that no trouble had occurred up to much higher indicated speeds at lower Mach numbers (and therefore lower altitudes), suggested that the incident was caused by shock-wave effects. Against this, however, was the fact that some damage occurred to the undercarriage uplock, not inconsistent with wing torsion, whereas aileron buzz is usually confined to the aileron itself and only local damage would be expected. To check the wing amplitude a similar aircraft was flown to the same conditions with more complete instrumentation and the records showed considerable wing motion associated with the aileron rotation at the same frequency as before; no damage was done on this occasion.

At this stage the flutter calculations were reviewed. Again the original calculations for flight clearance were restricted in scope by the lack of high speed computing equipment at the time, but in this case the binary which later became so important was not overlooked. The flutter speed was, however, shown to be very high on the basis of zero circuit stiffness - an assumption which might be thought pessimistic but which turned out not to be so. The aileron is power operated and has a frequency of 0.8. After the incident the calculations took into account several degrees of freedom, but it was found that a binary between wing torsion at a frequency of 1.1 and aileron rotation was sufficient, and the results quoted here relate to this type of binary. For aerodynamic derivatives the Minhinnick rules were applied, but some uncertainty existed for the aileron derivatives because the aileron has a sealed aerodynamic balance. In practice this balance area was assumed fully effective in the displacement derivatives and completely ineffective in the damping derivatives. The latter effect was checked by wind-tunnel measurements at low speed for the direct derivative ($-h_{\dot{\beta}}$), and although some favourable effect of the beak was found the net value of ($-h_{\dot{\beta}}$) as measured was if anything a little below the calculated value. The massbalance fitted was distributed along the beak and amounted to a good deal more than static balance.

Two important general effects were found, and these are shown in Figs.9 and 10. Fig.9 shows the effect of aileron natural frequency on the flutter speed (the circuit stiffness being varied) and it can be seen that a considerable fall in speed occurs up to the nose of the curve just below the wing torsional frequency. This curve relates to a height of 40,000 feet. Fig.10 shows the importance of height and below about 25,000 feet (or even higher if structural damping is allowed for) the band of flutter has disappeared altogether. These results showed in a general way that the incident could be explained as classical flutter and that it was not inconsistent with freedom from flutter at higher indicated airspeed but lower altitude. If this explanation is correct, the fact that the incident occurred at high Mach number is incidental. The calculations showed that an increase in massbalance should provide a curve.

The aircraft was therefore modified by increasing the massbalance and flown again to a Mach number of 0.93 at 40,000 feet. The aircraft felt smoother to the pilot until a violent oscillation occurred which caused a failure of the control circuit. Examination of the record showed a smooth growing oscillation of the aileron up to the point of failure at the same frequency as before (unity) followed by a further oscillation of the free aileron at a frequency of 0.7 which persisted until speed had been reduced. The wing amplitude was very small in contrast with the previous incident. The small wing amplitude implied that this incident was a form of aileron buzz directly dependent on Mach number. The origin of aileron buzz is thought to be that shock waves induce boundary layer separation, which in turn leads to negative aerodynamic damping of the aileron if the separation at the aileron is strong enough. There was no reason to doubt this explanation of the second incident except that the two incidents now seemed inconsistent with each other; if the aileron damping falls to zero (or nearly so) at a Mach number of 0.93, why had coupled flutter rather than buzz occurred in the first instance? Three further calculations, leading to Figs.11, 12 and 13 seem to account for this and to provide a complete explanation of the observed facts.

In all three Figures the flutter speed is plotted against a factor k which is applied to the direct aerodynamic damping of the aileron, it being assumed that at Mach numbers near 0.93 the value of k falls almost to zero. With the original massbalance (Fig.11) the flutter speed changes little, the change being in the sense of an increase in speed with reduction in k. The agreement in speed and frequency between this calculated curve and the flight incident is good, and the amplitude ratio, wing to aileron, is also of the right order. Fig.12 shows the corresponding diagram to Fig.11, for the condition of extra massbalance. Flutter is absent at a value of k of unity but is introduced when k falls to about 0.75. The band of flutter increases as k is reduced and if the damping is reduced to zero the region of flutter is actually greater than for the lower degree of massbalance. It seems, therefore, that the inertia coupling with the wing for the original massbalance has a stabilising effect at low values of k, although the coupling remains positive, i.e. the aileron is dynamically underbalanced, until well beyond static balance, as is usual with a wing torsion mode. It is noticeable from Fig.12 how the amplitude ratio falls much more rapidly than in Fig.11, and for $k = 0$ the flutter is almost entirely confined to the aileron. The flutter frequency is lower than in Fig.11, but not to a very great extent. Fig.13 shows what happens when the circuit has failed, and the elastic restraint on the aileron has disappeared. The results are similar to those of Fig.12 except that the frequency has fallen still further as it was found to do in flight. More precise quantitative agreement can hardly be expected when it is remembered that Figs.11 to 13 are drawn for variation in one derivative ($-h_{\dot{\beta}}$) only; in practice the associated derivatives, $\ell_{\dot{\beta}}$ and $-m_{\dot{\beta}}$, must change in a similar manner, and it is likely that the displacement derivatives will be affected also.

The cure which has been adopted is to fit vortex generators ahead of the aileron to reduce the area of separated flow and hence avoid the drop in aerodynamic damping and the circuit strength and stiffness has been increased. At the same time the extra massbalance is retained. An alternative to the use of vortex generators would be to restore the lost aerodynamic damping by the provision of artificial damping, e.g. hydraulically.

Conclusions from the investigation may be summarised as

(i) Control surface flutter calculations should always cover variations in the circuit stiffness; zero stiffness is not necessarily the worst condition.

(ii) As in the previous example, distributed massbalance, even beyond static, does not ensure freedom from coupled flutter.

(iii) Even though flutter only occurs at high Mach numbers, it may not be a compressibility phenomenon but rather a coupled flutter which is only possible at an adverse combination of height and equivalent airspeed.

(iv) The effect of a serious loss in aerodynamic damping on a control surface is not necessarily to promote pure buzz of the control surface alone. Simple flutter calculations in this case gave good agreement with flight.

(v) The Minhinnick rules led to good agreement with flight as regards coupled flutter, but did not, of course, predict the loss of aerodynamic damping.

4.4 Rudder flutter at high subsonic Mach number

The rudder is a fairly narrow chord rudder with a small horn balance fitted on a delta shaped fin. Massbalance weights are carried at the bottom and in the horn. The possibility of transonic flutter was envisaged before flight and the flutter calculations included a variation of the direct aerodynamic damping on the rudder, but it was decided to fly the aircraft with instrumentation and by careful examination of the records it was hoped to pick out any tendency to flutter before it became dangerous. In fact mild flutter occurred at a Mach number of 0.93. The flutter was considered to have been caused by a change in sign of the direct aerodynamic damping, but paradoxically the cure proposed was to increase the horn massbalance.

The reason for this proposal is shown in Fig. 14 which gives the results from flutter calculations involving flexibilities of the fin and rudder. The natural frequencies involved, referred to a flutter frequency of unity as in example 4.3, are

fin fundamental bending	0.6
fin torsion	1.6
fin overtone bending	2.1
rear fuselage torsion	0.8
rudder fundamental	0.76
tailplane bending	0.8

The rudder fundamental mode is almost pure torsion as the rudder is operated by a stiff powered control from its base; its frequency is unexpectedly low because of the length of the rudder and the heavy massbalance at the top. The curve of Fig. 14 did not give very good agreement with the frequency of the flutter incident (the calculated values being rather too low), but as the aerodynamic stiffness on the rudder was uncertain this discrepancy was not thought to be vital: moreover the high frequency modes were not included in the calculation. The effect of an increase in massbalance is shown to be favourable, the mechanism being that it causes more fin amplitude in the combined motion, and the fin bending is positively damped aerodynamically. To increase the horn massbalance was an easy modification to carry out in practice, and so it was tried. Flutter again occurred, however, in roughly the same conditions as before.

The flutter calculations were now re-examined and a more methodical variation of the derivatives was carried out. A few of the results are plotted in Fig.15 which also differs from Fig.14 in that the fin torsion mode is included. In Fig.15 the effect of the aerodynamic couplings is investigated. The centre of pressure under rudder rotation ($^{-m}\beta/\ell_\beta$) is assumed to move back between Mach numbers of 0.85 and 0.95 from 0.48 of the fin chord to 0.77, and a similar movement is assumed for the out-of-phase centre of pressure, $^{-m}\dot{\beta}/\ell_{\dot{\beta}}$. It can be seen that both these changes have an adverse effect, and especially the former. By comparison the effect of a change in sign of the direct damping coefficient is small. The curves as drawn relate to the configuration with the increased mass-balance, and the agreement with the corresponding flutter incident is good for a value of k of -0.5. In this case the amplitude and phase were obtained from different pickups in flight, and these values are compared with the calculated values in Table III.

TABLE III

Pickup Position	Amplitude		Phase	
	Flight	Calculated	Flight	Calculated
Fin	1.0	1.0	0	0
Tailplane	0.16	0.24	-150°	-180°
Rudder horn	0.25	0.26	30°	46°

For the first incident the instrumentation was not so complete, but a value of k of -0.5 was again shown to give good agreement with flutter speed and frequency. It will be seen from Fig.15 that the tendency is for flutter speeds to rise again as the aerodynamic centre on the fin moves aft, which it does at a slightly higher Mach number. It is therefore suggested that the curve of flutter speed against Mach number passes through a minimum at about M = 0.95 as shown in Fig.16, and in fact enough evidence was obtained from full scale to show that the true curve for the configuration flown must be of the form of Fig.16.

It will be noticed that the adverse c.p. shift has reduced the flutter speed considerably even for full damping values (see Fig.15). In this condition (k = 1.0) the flutter is approaching flexure torsion of the fin, and this fact complicated the possibility of finding a cure, since the obvious cures of fitting a hydraulic damper or a second power jack some way up the fin lead to a serious reduction in the fin flexure torsion flutter speed purely through the adverse mass effect. It was therefore decided to increase this flutter speed first by stiffening up the fin in torsion. The calculated curve for the stiffened fin is shown in Fig.17, and it can be seen that the increase in flutter speed is sensitive to the value of k which applies in the range k =-0.4 to -0.6. It was therefore decided to fly the aircraft and approach the dangerous speeds by explosive charge excitation in a flight flutter test. Unfortunately only one pickup worked satisfactorily, and that was near a node, so the technique was unsuccessful although it was not tried very fairly.

The effect of the stiffened fin was to raise the flutter speed by about 30 knots E.A.S. at the same Mach number and frequency as before. This result was investigated by new calculations based on the measured resonance modes of the stiffened fin and the curves of Fig.18 were obtained. Good agreement was obtained with the incident by assuming the factor k falls to -0.25 with $^{-h}\beta/E_{\beta}{}^{3}$ =-0.15, and it was decided that these derivatives could give good agreement with the earlier incidents if slight improvement were made in the structural assumptions.

The conclusion to this series of transonic incidents was that the size and inertia of the rudder were reduced, thereby increasing its torsional frequency to 2 units or more, a large contribution in this increase being due to the deletion of the horn massbalance weight. In addition a hydraulic damper was fitted, and no further flutter incident has occurred. Many more calculations were carried out to show the final configuration to be satisfactory, but as they are only supported by flight results in a negative way, they are not quoted here.

The general scheme for evaluating basic aerodynamic derivatives in this work was more complicated than the application of the Minhinnick rules. The stiffness and damping derivatives were both calculated for the condition $\nu \to 0$ using three dimensional theory (e.g. Multhopp-Garner) but for rigid modes. The effect of the distortion modes was introduced by assuming that the flutter coefficients can be calculated as though they consist of two parts which are directly additive; one part depends on the product of the mode and the rigid body loading (the usual strip-theory approach) and the other part depends on the rigid body loading only. In this way some allowance is made for induced aerodynamic effects, while the labour of calculating the coefficients is very much less than if the full three-dimensional theory were used.

The following conclusions may be drawn

(i) where possible a power operated control surface on a transonic aircraft should be operated from two points or more; if only one point is used the middle is better than one end.

(ii) Massbalance and stiffness do not combine well in the prevention of flutter.

(iii) The aft shift in control surface centre of pressure at transonic speeds can lead to flutter between main surface torsion and control surface rotation (or torsion). The flutter is sensitive both to Mach number and equivalent airspeed.

(iv) At transonic speeds the β-derivatives are likely to go negative, in this case reaching values of about $-\frac{1}{4}$ of their maximum positive values.

(v) The change in sign of the cross-damping derivatives may be more important than the change in sign of the direct hinge moment damping. Although this appears to be true for a fin and rudder, however, it is unlikely to be true for a wing and aileron where the aileron forces on the wing are relatively so much less.

(vi) The effect of modal shape on three-dimensional derivatives can be obtained with fair accuracy by combining the rigid body loading with the rigid body loading times the mode in suitable proportions. In this case the agreement appeared to be good (some of the derivatives were checked by wind-tunnel measurement) but the changes that were made for transonic conditions were purely empirical.

(vii) Although the use of explosive charges for excitation in a flight flutter test cannot be condemned from this investigation, it is apparent that a continuous excitation technique (see section 4.2) offers a more positive method of exciting the dangerous frequency.

5 Statistical evidence

In the last section four of the more informative of recent flutter incidents have been discussed in some detail and a number of points have been made for the avoidance of similar incidents in future. In this section a far greater number of incidents have been analysed, but the analysis is much more rudimentary.

One point of interest is the trend with time of the character of the incidents. Mach numbers have increased and so has the prominence of flutter in one degree of freedom, both as would be expected. Another trend has been for higher frequency modes to lead to flutter. This is illustrated in Fig.19 where the flutter frequency divided by the symmetric wing fundamental bending frequency is plotted against the year of the incident. There were two exceptional incidents in 1944, but apart from those the widening spread of the flutter frequencies is clearly shown. This is to be expected with the reduction in wing thickness for high Mach number aircraft and other allied changes all of which have tended to reduce frequencies. There is an apparent increase in number of incidents with time in Fig.19, but this is occasioned mainly by the greater number which are fully recorded and to which a frequency can be assigned with confidence; in the earlier years many incidents could not be plotted.

An attempt has also been made to analyse the more recent incidents for reasons as to why they were not predicted. Three main possibilities are considered and the number of incidents which fall under each heading is

(a)	Inadequate aerodynamic information	11
(b)	Calculations with the wrong (or too few) degrees of freedom	10
(c)	Inadequate structural information	7

Some incidents occur under more than one heading: that of section 4.2, for example, is counted under all three because (a) the aerodynamic derivatives in use before the incident and in the first calculations afterwards gave too small a region of flutter, (b) the mode which ultimately led to flutter was not included in the pre-flight calculations, and (c) the resonance mode was inadequately measured. The importance of (a) is expected, particularly as it contains all the cases of aileron buzz, but it is perhaps surprising to find that (b) and (c) are not far behind (a). Lines of attack are however well established for dealing with these difficulties; they may be summarised

(a) (i) Good transonic design

(ii) Use of three-dimensional subsonic and supersonic derivative theories - supported by the use of high speed computing equipment

(iii) Use of derivative and flutter models.

(b) (iv) Development and use of high speed computing equipment

(c) (v) Improvement in techniques for ground resonance testing

(vi) Improvement in techniques for influence coefficient measurement

(vii) Use of high speed computing equipment in making normal mode, and other, structural calculations,

and, in addition

(viii) The use of flight flutter tests.

6 Conclusions

The survey of recent incidents, both in detail and in general has led to conclusions which have been stated in the paper and are not reproduced here. The main broad trend in flutter prediction, however, may be summarised as the correlation to a steadily increasing degree between calculations and measurements. Normal modes are calculated and compared with measured modes; flutter models are built and the test results compared with calculations; aerodynamic derivatives are calculated and selected ones are measured; flight flutter tests are carried out and compared with calculations. All these stages are not yet covered on every aircraft, but recent progress has been considerable and it must lead to a greater accuracy in flutter prediction for the future. The importance of high speed computing equipment is evident.

One omission in the paper may excite comment. There is no incident quoted in which the powered flying control circuit has been a fundamental cause of flutter. Power control instabilities have certainly occurred, but they have usually been detected and cured on the ground. Powered control instability has been suspected as a contributary cause in some of the flutter incidents, but it has always been shown (for flutter incidents which have occurred in Great Britain) that the powered control could not supply energy to the motion at the known flutter frequency. At the same time no attempt has been made in Great Britain to use structural feed-back through a power circuit to stabilise potential flutter modes; in many cases inertia couplings which have existed have been eliminated by massbalancing the circuit. Flutter prediction in the future will, however, have to take increasing account of the effects of powered flying controls and auto-pilots; in this field the use of electronic simulators may be particularly valuable.

Perhaps the most important design recommendation to come out of the analysis of the present paper, is that control surfaces on transonic (and supersonic) aircraft which are power controlled should be designed without massbalance and operated from two or more spanwise positions. Backlash should be kept to a minimum and the dynamic stiffness as high as possible. A damper installation should be designed as an emergency in case of flutter in a single degree of freedom.

Acknowledgement

The author wishes to acknowledge the generous help given to him by his colleagues in the industry without which the accounts given in section 4 would have been impossible. With the exception of most of the calculations described in section 4.1 all the investigations were carried out by the design firm concerned.

Notation Used

The derivative notation used is as follows:-

$$dL = \rho V^2 c dy \{(-\nu^2 \ell_{\ddot{z}} + i\nu \ell_{\dot{z}} + \ell_z)^z/c + (\ldots \ell_\alpha)\alpha + (\ldots \ell_\beta)\beta + (\ldots \ell_\gamma)\gamma\}$$

$$dM = \rho V^2 c^2 dy \{(-\nu^2 m_{\ddot{z}} + i\nu m_{\dot{z}} + m_z)^z/c + (\ldots m_\alpha)\alpha + (\ldots m_\beta)\beta + (\ldots m_\gamma)\gamma\}$$

$$dH = \rho V^2 c^2 dy \{(-\nu^2 h_{\ddot{z}} + i\nu h_{\dot{z}} + h_z)^z/c + (\ldots h_\alpha)\alpha + (\ldots h_\beta)\beta + (\ldots h_\gamma)\gamma\}$$

$$dT = \rho V^2 c^2 dy \{(-\nu^2 t_{\ddot{z}} + i\nu t_{\dot{z}} + t_z)^z/c + (\ldots t_\alpha)\alpha + (\ldots t_\beta)\beta + (\ldots t_\gamma)\gamma\}$$

where dL is the lift force on a strip dy

dM is the nose up pitching moment about the leading edge on a strip dy

dH is the nose up hinge moment on a strip dy

dT is the nose up tab hinge moment on a strip dy

z is the down displacement of the leading edge of the strip

α is the nose up rotation of the strip

β is the down control surface angle of the strip

γ is the down tab angle of the strip

ρ is the air density

V is the forward speed

c is the strip chord

ν is the frequency parameter $^{\omega c}/V$ where ω is the frequency

$\ell_{\ddot{z}} \ldots t_{\ddot{z}}$ are the aerodynamic inertia derivatives

$\ell_{\dot{z}} \ldots t_{\dot{z}}$ are the aerodynamic damping derivatives

$\ell_z \ldots t_z$ are the aerodynamic stiffness derivatives

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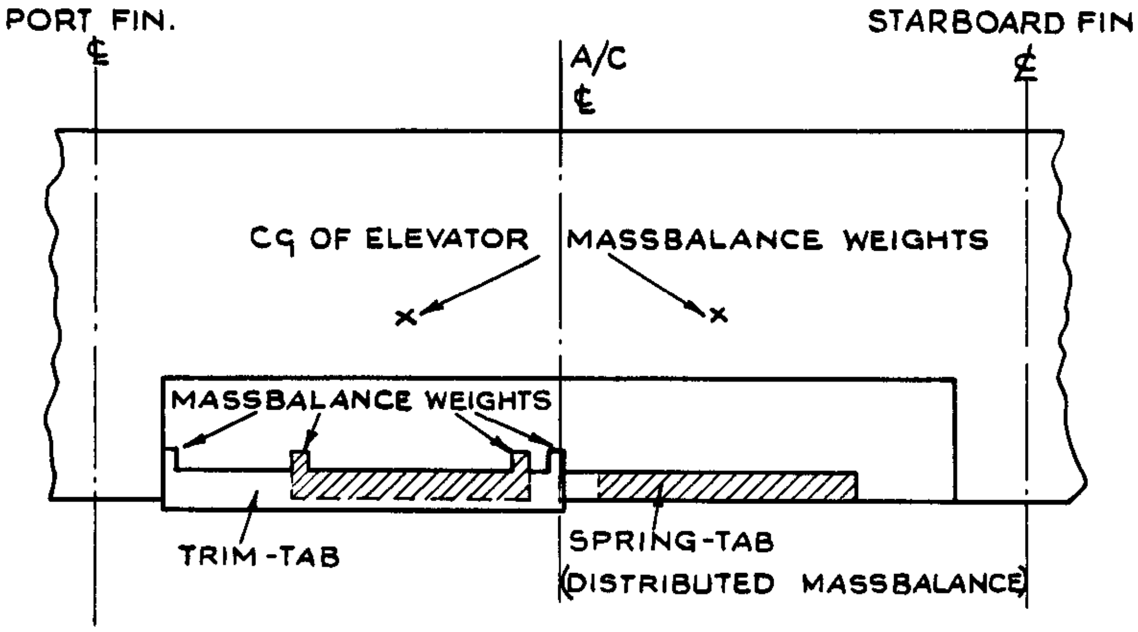

FIG.1. TAILPLANE & ELEVATOR WITH TWO TABS.

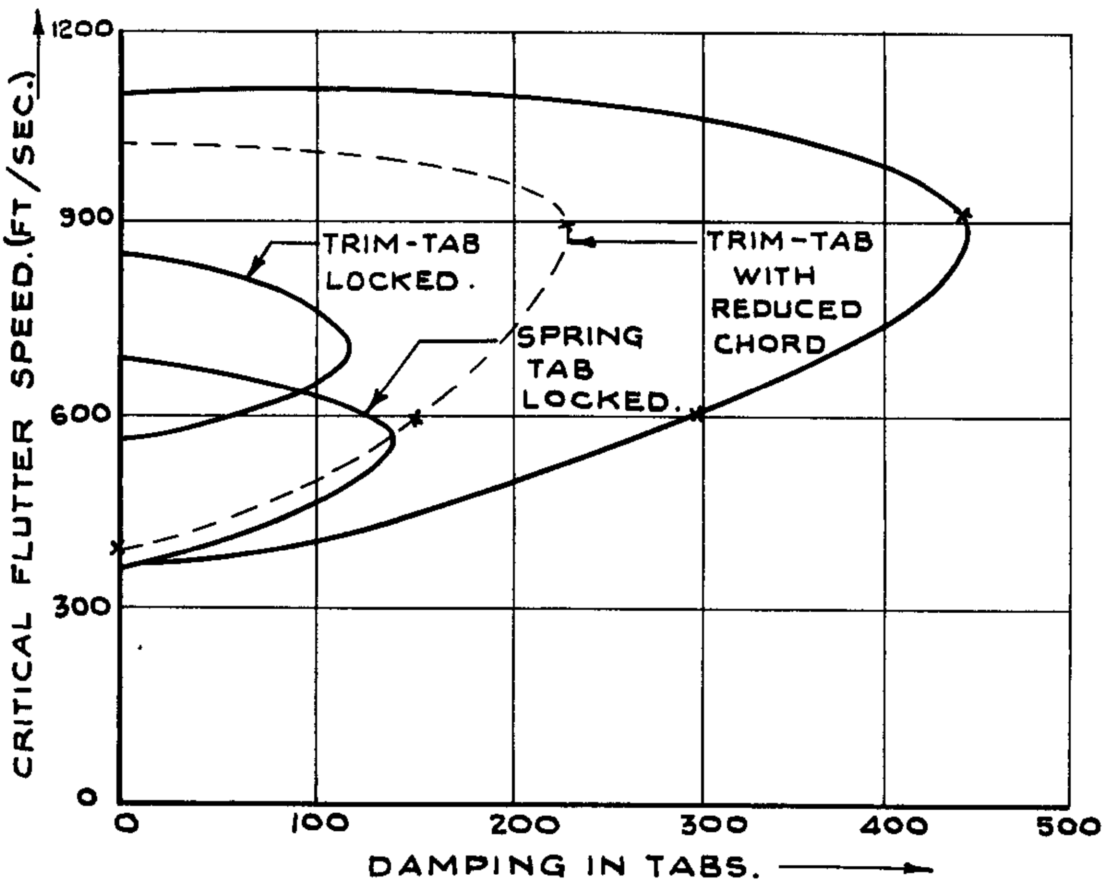

FIG.2. FLUTTER SPEED v TAB DAMPING.

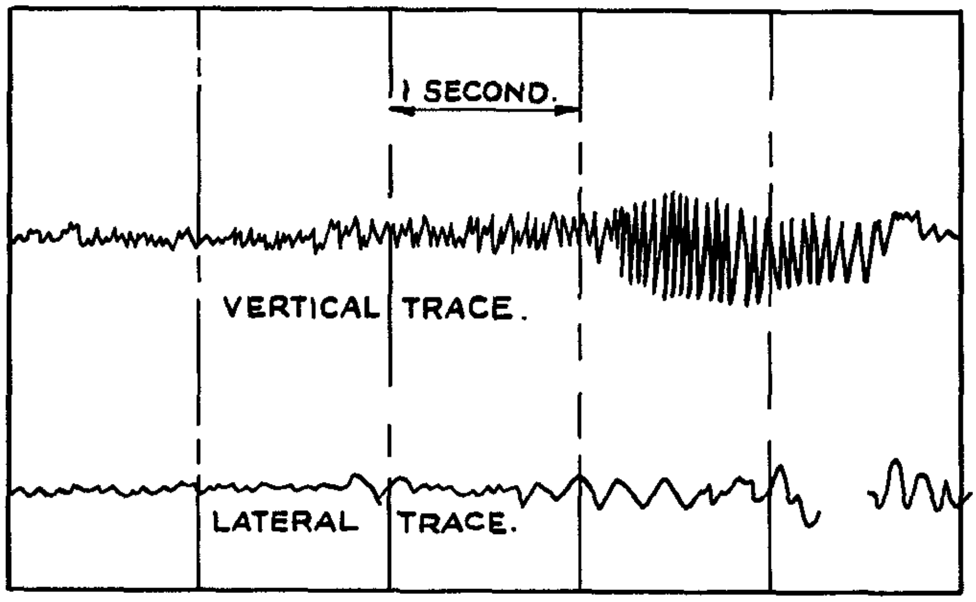

FIG.3. RECORD OF ELEVATOR FLUTTER.

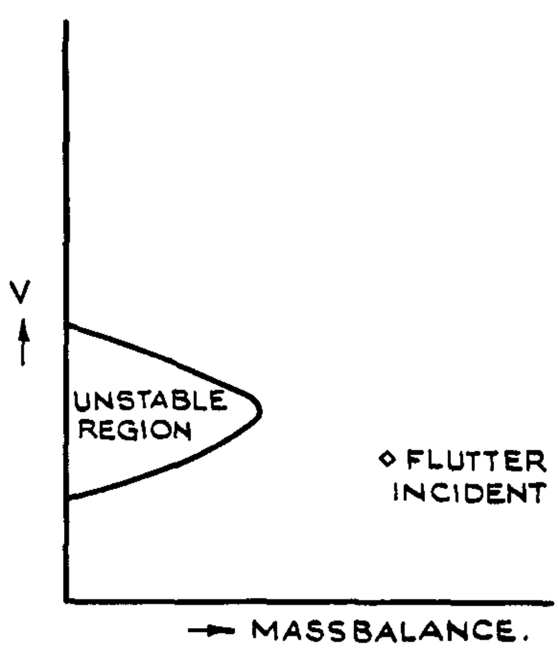

FIG.4. EARLY BINARY CALCULATION OF ELEVATOR FLUTTER.

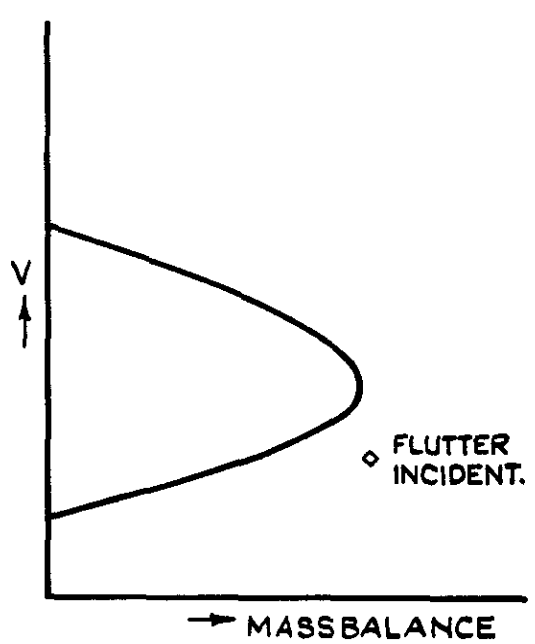

FIG.5. BINARY CALCULATION WITH CORRECTED MODE + NEW DERIVATIVES.

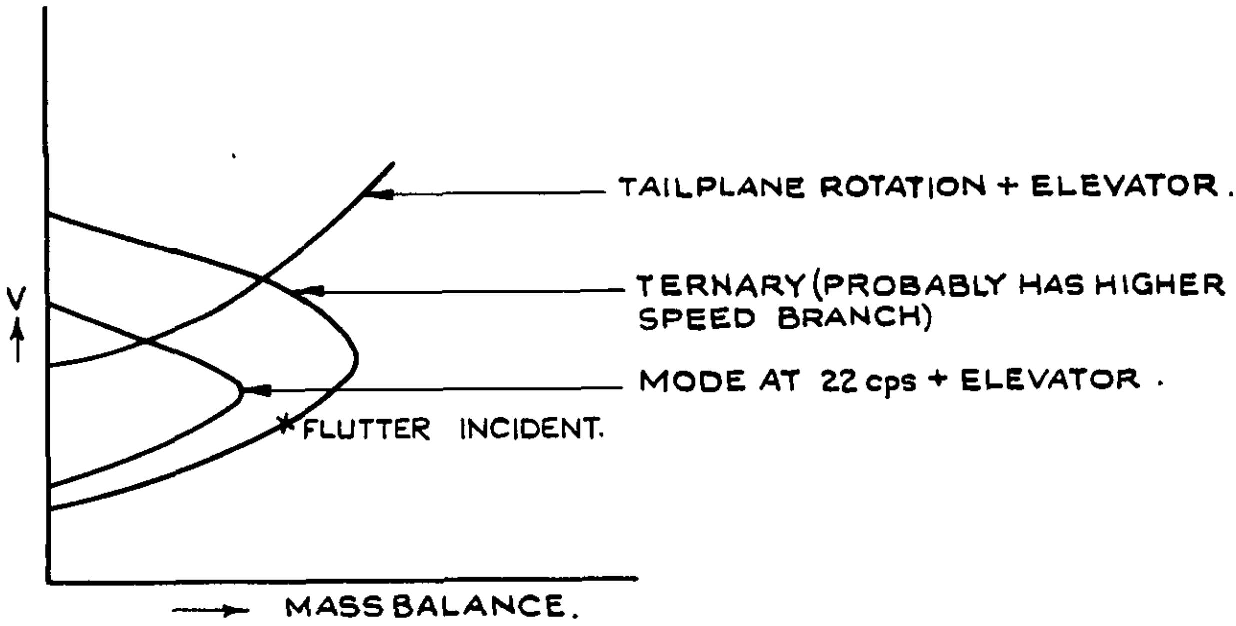

FIG.6. TERNARY CALCULATION INCLUDING TAILPLANE ROTATION.

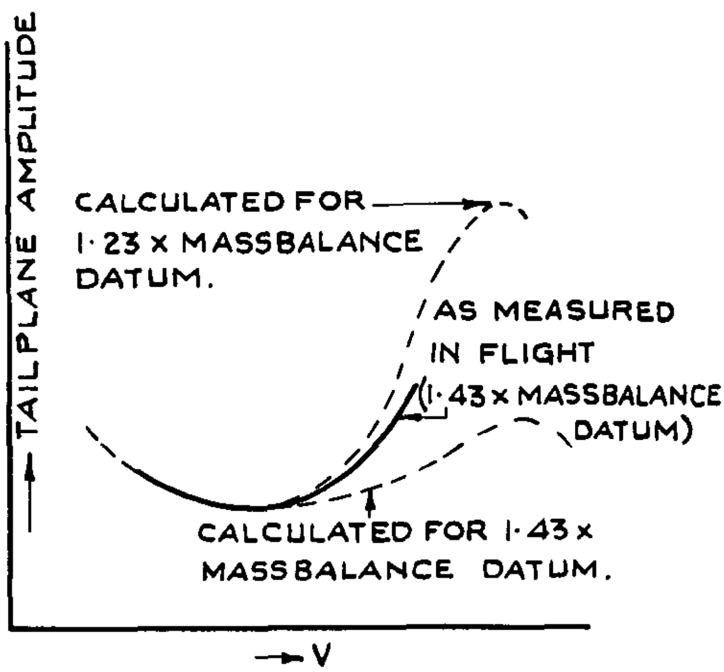

FIG.7. FLIGHT FLUTTER TESTS-COMPARISON OF CALCULATED AMPLITUDES WITH THOSE MEASURED.

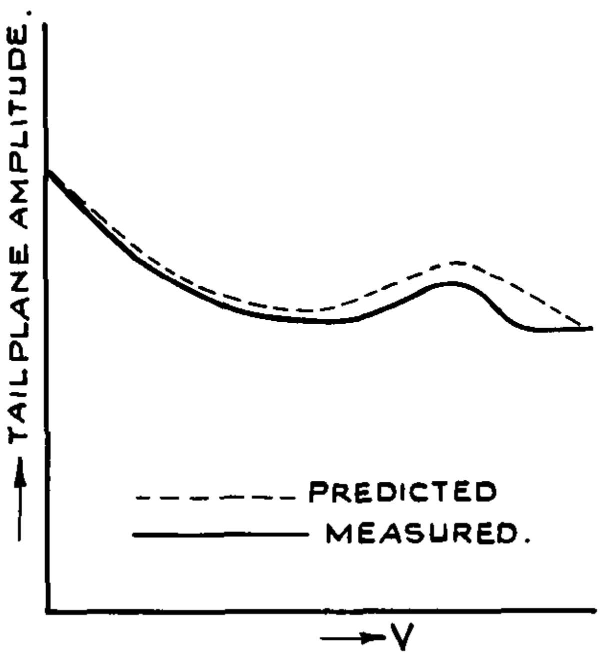

FIG.8. AMPLITUDES FROM SECOND SERIES OF FLIGHT FLUTTER TESTS.

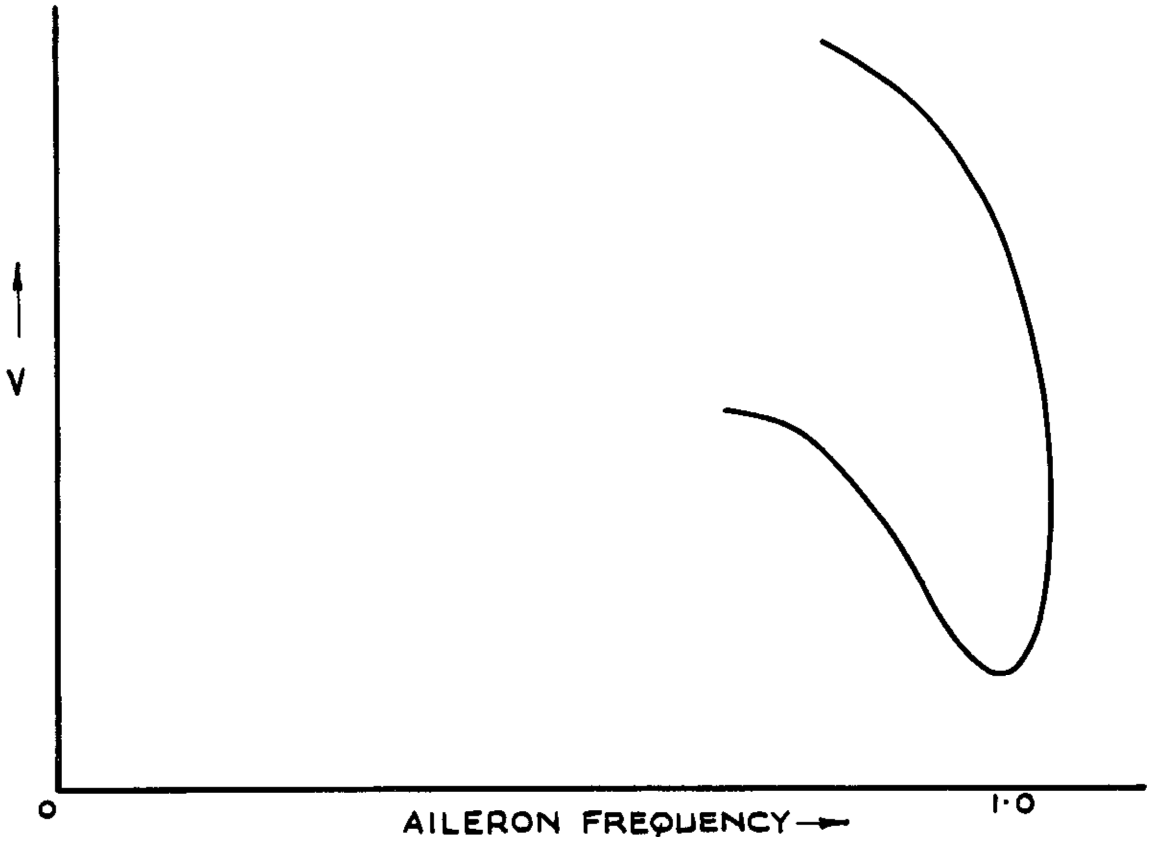

FIG.9. EFFECT OF AILERON NATURAL FREQUENCY ON FLUTTER SPEED.

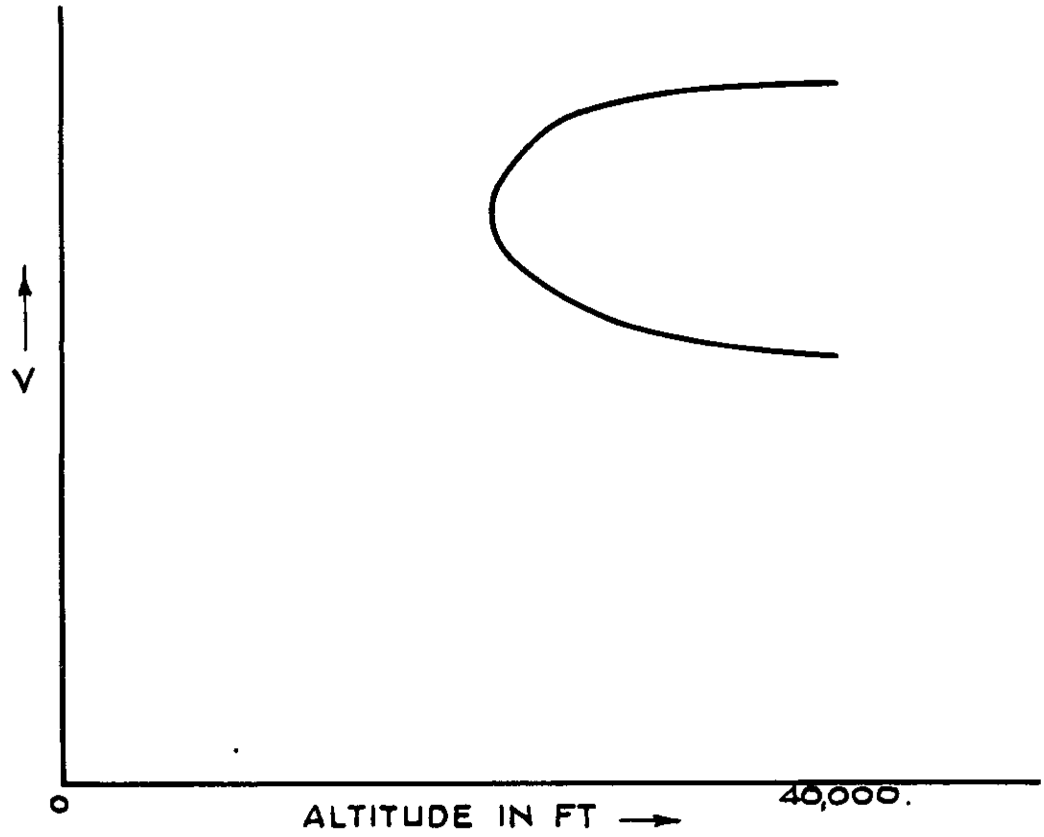

FIG.10. EFFECT OF HEIGHT ON FLUTTER SPEED.

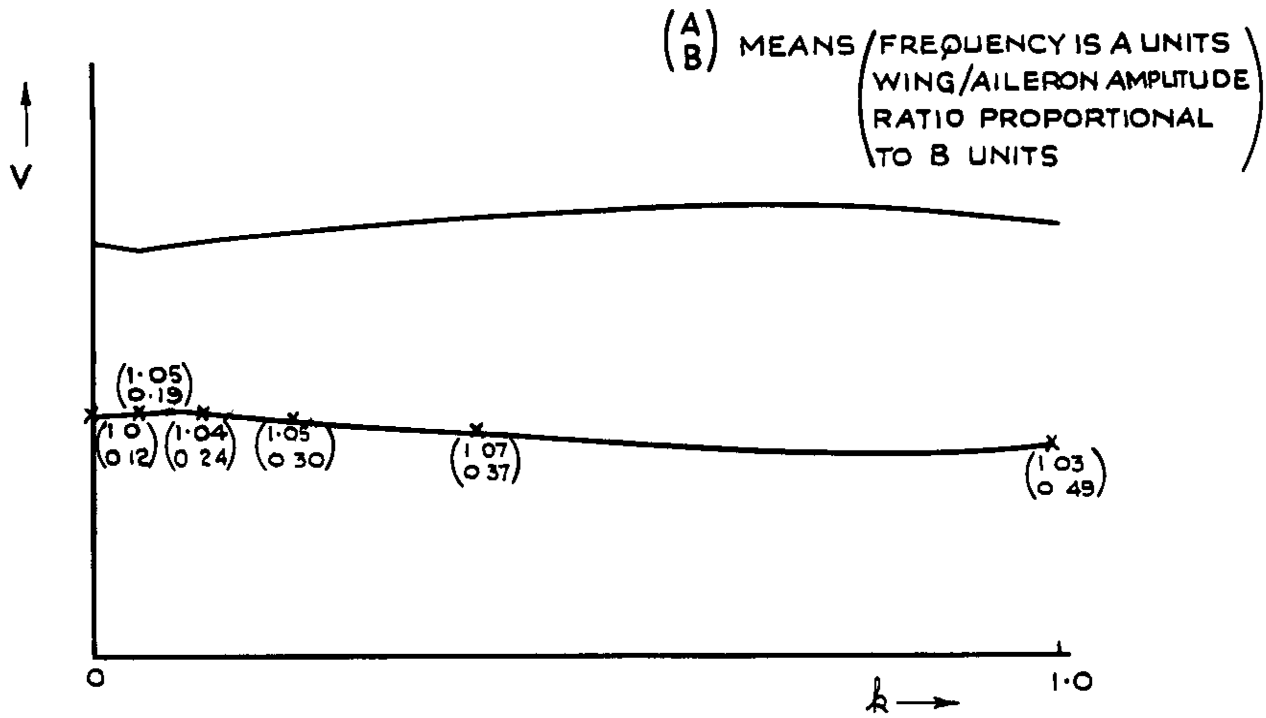

FIG.11. EFFECT OF LOSS OF AERODYNAMIC DAMPING (ORIGINAL MASSBALANCE.)

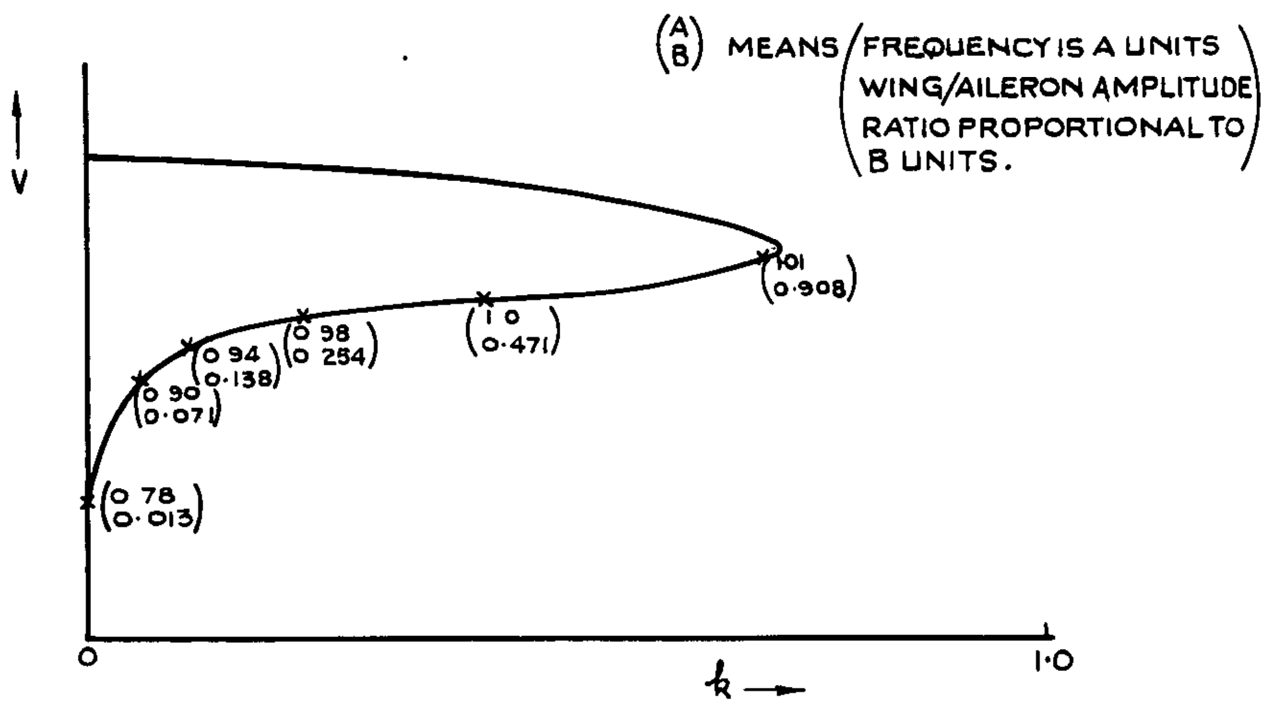

FIG.12. EFFECT OF LOSS OF AERODYNAMIC DAMPING (EXTRA MASSBALANCE.)

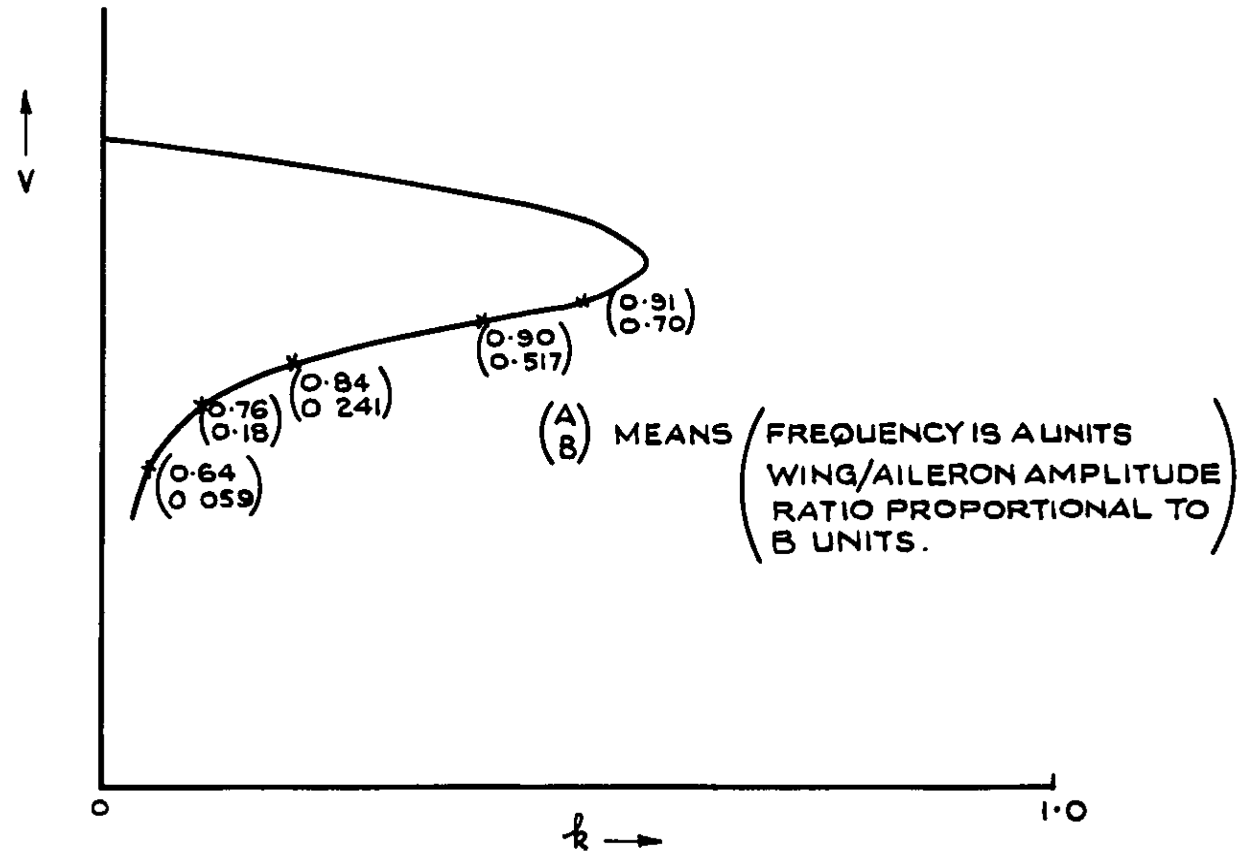

FIG.13. LOSS OF DAMPING WITH CIRCUIT CUT (EXTRA MASSBALANCE)

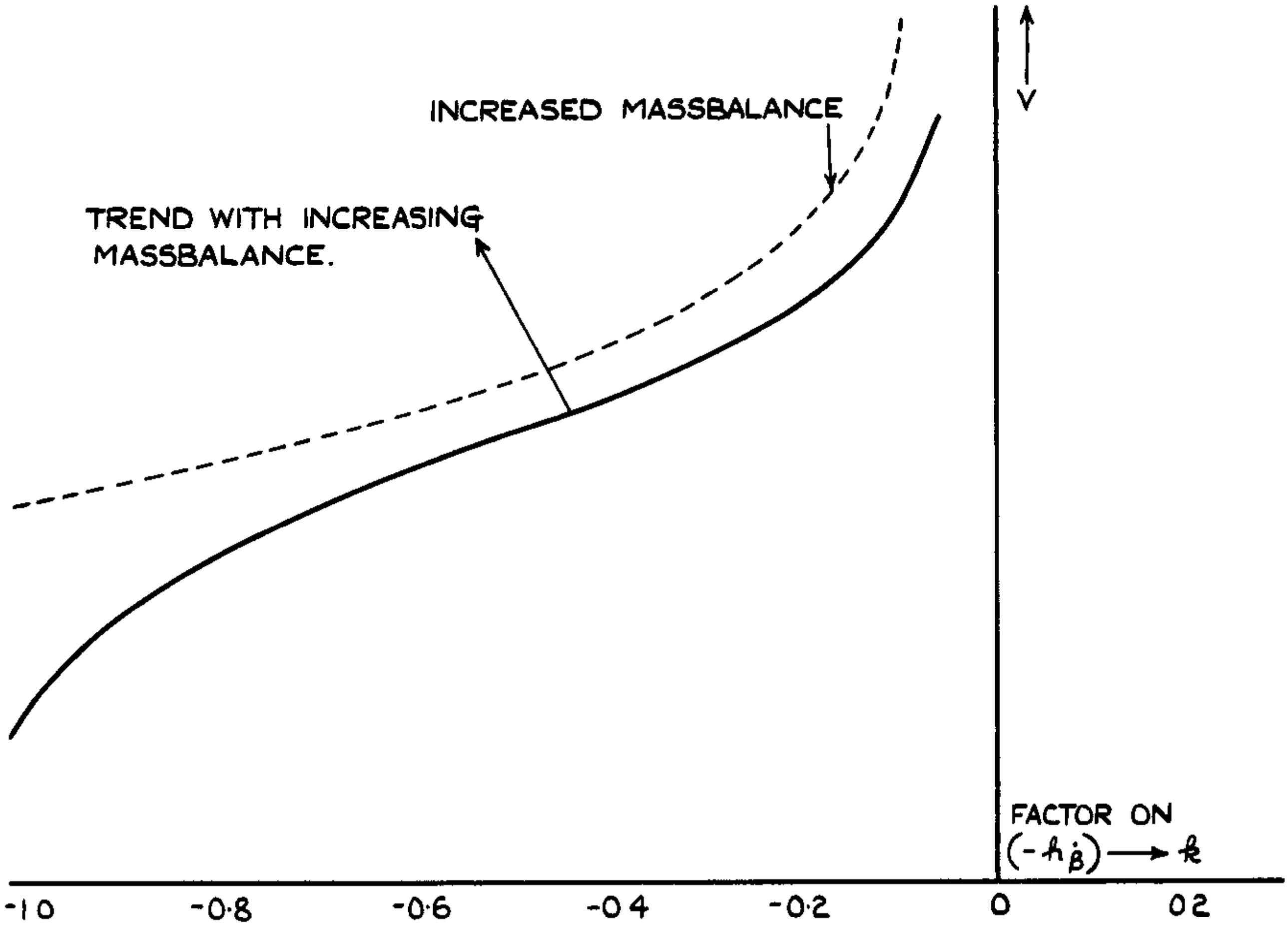

FIG. 14. LOSS OF RUDDER DAMPING FOR DIFFERENT MASSBALANCE WEIGHTS.

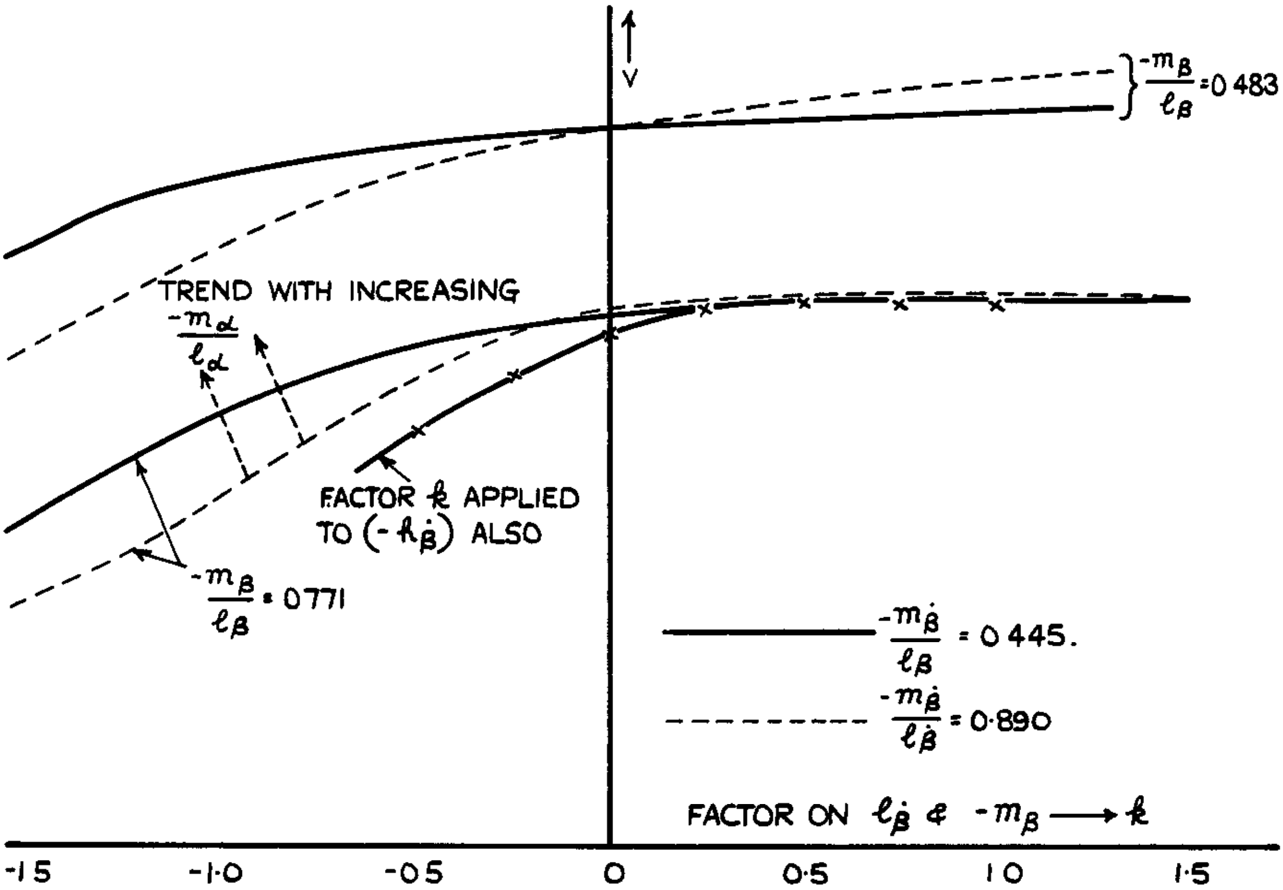

FIG. 15. EFFECT OF VARIATION IN AERO DYNAMIC CROSS DAMPINGS.

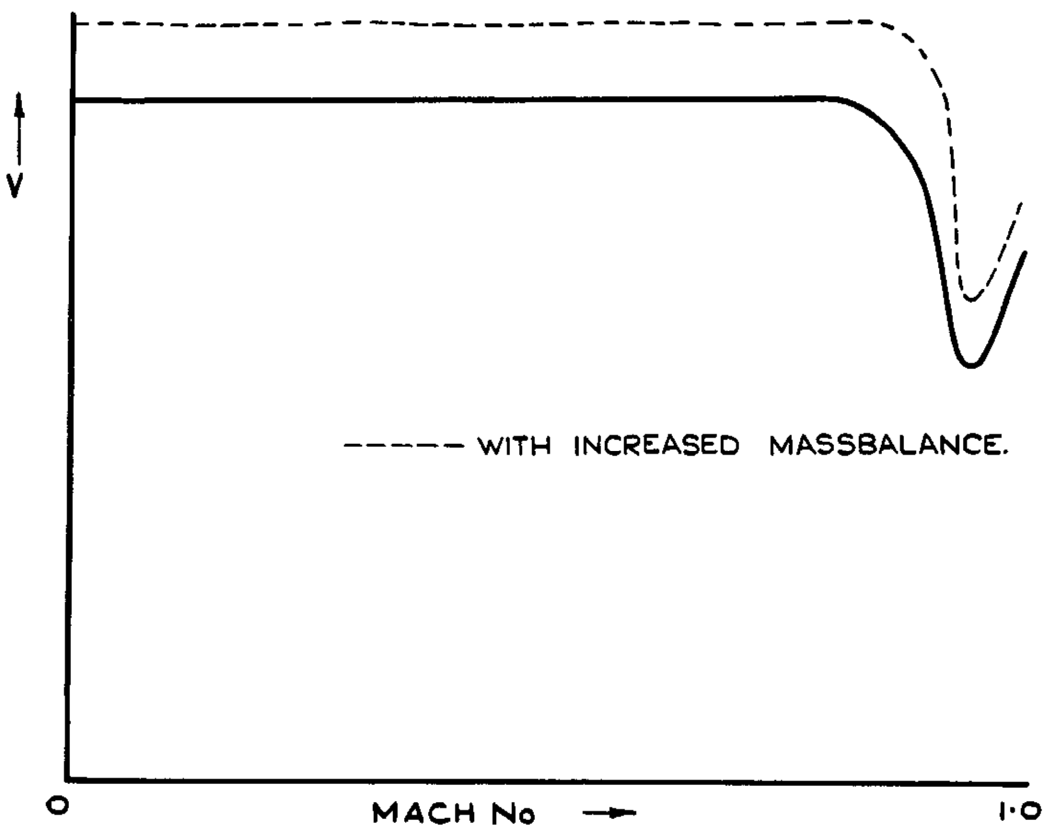

FIG.16. APPROXIMATE VARIATION OF FLUTTER SPEED WITH MACH. NUMBER.

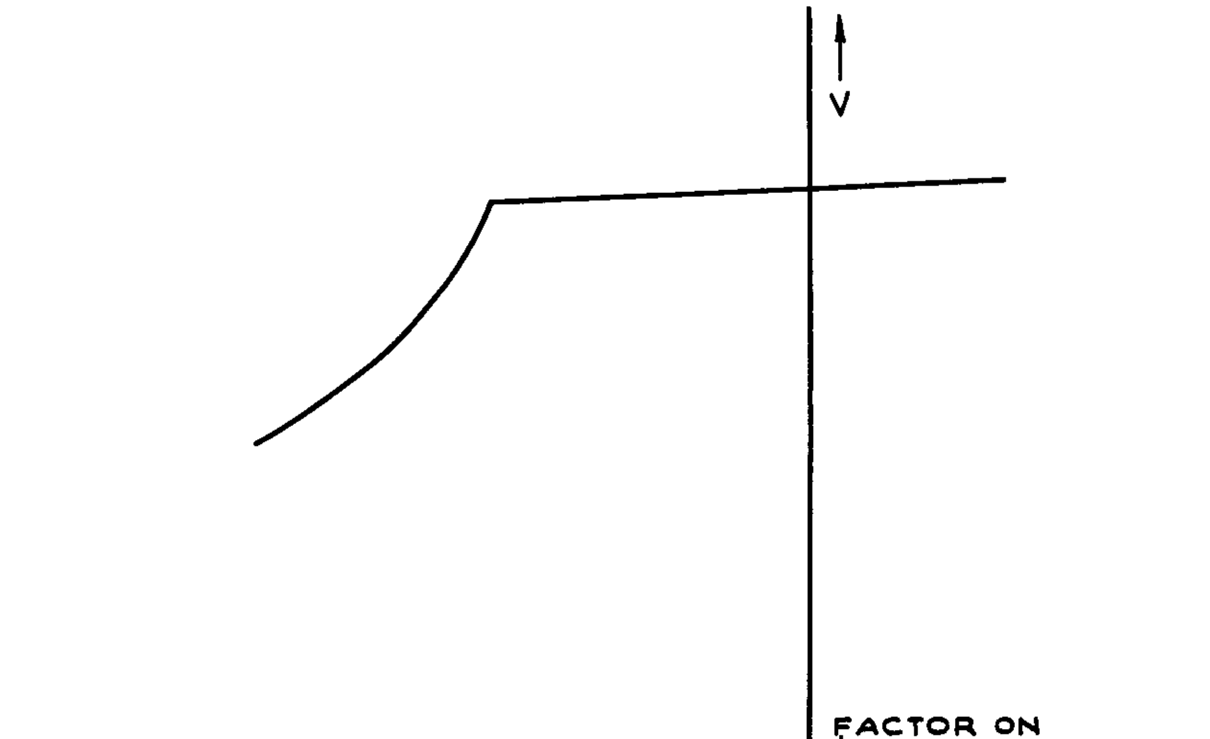

FIG.17. EFFECT OF VARIATION IN $\dot{\beta}$-DERIVATIVES FOR STIFFENED FIN.

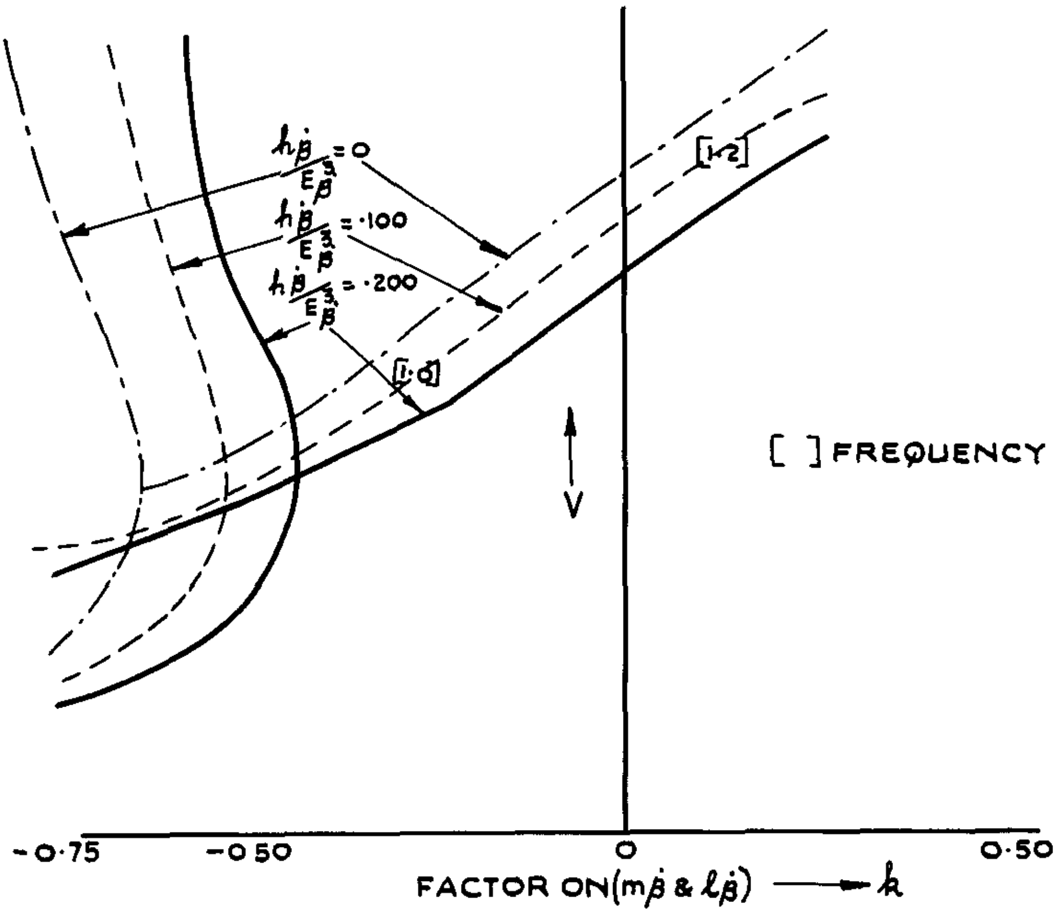

FIG.18. STIFFENED FIN CALCULATION BASED ON MEASURED RESONANCE MODES.

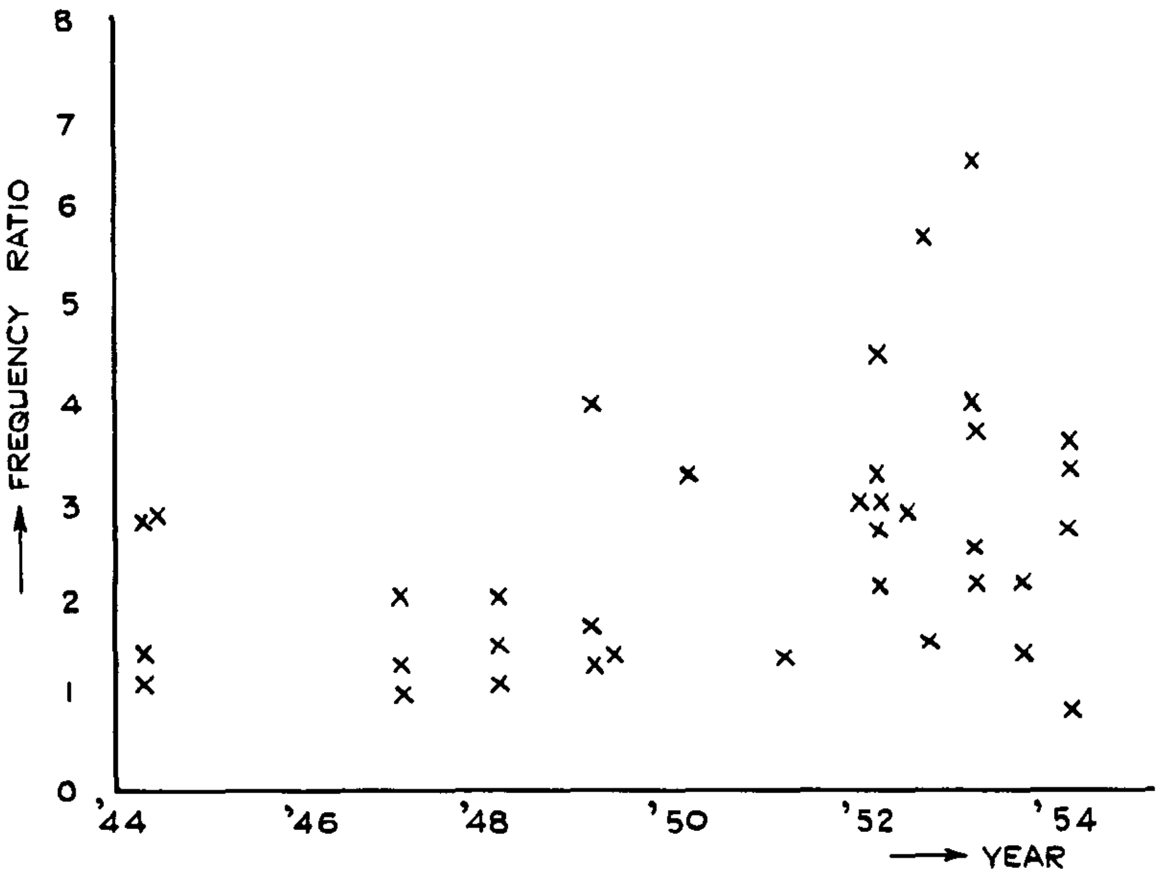

FIG.19. TREND OF FLUTTER FREQUENCIES OVER A TEN YEAR PERIOD.

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