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A Note on Flutter of Asymmetric Controls

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Summary. This Paper attempts to give some guidance on the question of how best to approximate to an aircraft with an asymmetric elevator control for the purpose of flutter calculations. Two examples of asymmetric controls that are common in practice are (1) the design case of a partitioned elevator each part having its own separate power unit and with one such unit having failed, and (2) the asymmetric arrangement of tabs on the elevator. Several instances of flutter involving asymmetric tab arrangements are known to have occurred in practice, but for the purpose of the calculations reported in this Paper the elevator itself is made asymmetric; this was done to reduce the work to manageable proportions. It is concluded that the more stable side of the aircraft can be replaced by a mirror image of the less stable side without serious error, at least for calculations of the type described. This appears to give a rather better approximation than taking the asymmetric aircraft and constraining it to vibrate in either symmetric or antisymmetric modes.

1. *Introduction.* It is a common practice to fit an elevator with tabs which are not arranged symmetrically. For example, a spring-tab might be fitted to one side and a trim-tab (of quite different frequency) to the other side of the elevator, which will then have a stiff interconnection through the fuselage. This type of arrangement has led to three or four flutter incidents in the past, and one of the difficulties met with is that the analytical treatment becomes more complicated. With a symmetric aircraft it is possible to split the calculation into two parts, a symmetric calculation and an antisymmetric calculation, but with the asymmetric aircraft the flutter motion will, in general, be neither symmetric nor antisymmetric. In order to represent this motion analytically both symmetric and antisymmetric modes must be included together, and this leads to a single calculation with nearly twice as many degrees of freedom as for either calculation on the symmetric aircraft.

Even today, however, the number of degrees of freedom that can be included in a routine calculation is limited to about six which is sufficient for a symmetric calculation or an antisymmetric calculation but not both together. The practice has therefore arisen of making four calculations for an asymmetric tail using modes as follows,

(1) symmetric modes;

(2) antisymmetric modes;

(3) the same modes as (1) but for only half the aircraft;†

(4) the same modes as (2) for only half the aircraft.

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* Previously issued as R.A.E. Tech. Note No. Structures 278—A.R.C. 22,084.

† It is convenient to refer to calculations of this type as being for the half aircraft. In reality what is done is to replace the more stable half of the aircraft (*e.g.*, the side without spring-tab) by a mirror image of the less stable half and then make the calculations for this hypothetical symmetric aircraft.

The combination of (1) and (2) therefore gives an estimate of the complete effect but is always optimistic (*i.e.*, errs on the danger side). The combination of (3) and (4) errs on the safe side. The present Paper compares these two methods with the full calculation (in an example) and gives examples of the relative magnitude of the different errors. In order to reduce the full calculation to manageable proportions it is assumed that the elevator itself is on one side only and that there are no tabs. A problem closely similar to that considered in the present Paper can arise in the design of an aircraft fitted with powered flying controls. The elevator on such an aircraft may be split into two halves and each half driven by its own separate power unit. If the possibility of a partial failure is now considered in which one power unit continues to apply a very stiff restraint to its half-elevator such that effectively on that side of the aircraft the tailplane and elevator are inexorably linked together, whereas on the other side of the aircraft much of the stiffness has been lost, then the flutter problem reduces to the kind considered in the present Paper. With regard to asymmetric tab flutter as discussed above, it is hoped that the results obtained may be typical in a comparative sense; to have included tab freedoms as well as those of the fuselage, tailplane and elevator would have made the full calculation too large to put on the Royal Aircraft Establishment flutter simulator.

2. *Details of the Calculation.* If an aircraft is fully symmetric about a vertical plane through its centre-line, the flutter calculations can be split into two parts which include only the symmetric modes on the one hand and only the antisymmetric modes on the other; this is because there is no coupling between any symmetric and any antisymmetric mode. In many aircraft however some of the minor features are not symmetric, and a particular case which often causes difficulties in flutter work is that of tabs on an elevator. For example, an elevator may be fitted with a spring-tab on one side of the fuselage and a trim-tab on the other side of the fuselage, the two halves of the elevator being connected by a torque tube. The two tabs will in general be of different sizes and of very different frequencies and in these circumstances elevator-tab flutter, if it occurs, will be asymmetric. In a similar way, if a control surface is partitioned and the power supply to any one segment fails the resulting increase in flexibility will be asymmetric, and if flutter subsequently occurs it too will be asymmetric. This means that if the motion is to be described in terms of symmetric and antisymmetric modes of the structure, some of each will be required. Thus the flutter calculation cannot be separated into its symmetric and antisymmetric parts and hence will require nearly twice as many degrees of freedom as for either calculation on a symmetric aircraft. For instance if n_1 symmetric modes need to be considered and n_2 antisymmetric modes as well as the two tabs or elevator segments, it will be necessary to carry out a single calculation with $n_1 + n_2 + 2$ degrees of freedom instead of two separate calculations of $n_1 + 1$ and $n_2 + 1$ degrees of freedom which would have been possible had the two tabs been identical. However this would lead to a total number of degrees of freedom greater than could be solved on a typical flutter simulator; so that in practice this complete solution is generally avoided and instead the four smaller calculations are carried out using modes as follows:

(1) symmetric modes for the whole aircraft;
(2) symmetric modes for half the aircraft;
(3) antisymmetric modes for the whole aircraft;
(4) antisymmetric modes for half the aircraft.

In these calculations it is assumed that the configuration on one side of the centre-line of the aircraft, say the starboard side, is more prone to flutter than on the other (port) side. Thus in the flutter motion, energy will be extracted from the airstream on the starboard side and given back on the port side, so the balance is zero. For this reason the amplitudes on the starboard are likely to be greater than those of the port side, which is being damped, hence calculations 1 and 3 above are likely to be optimistic. It is for this reason that calculations 2 and 4 (which assume that the port side is a mirror image of the starboard side) are carried out to obtain a conservative result. Unfortunately the difference between the two results is often so great that the practical value of this procedure is small.

The calculations in the present Paper give a comparison between a set similar to the four given above, and the complete calculation in which both symmetric and antisymmetric modes are included simultaneously. However, to restrict the calculation to a practical size a hypothetical system has been considered, consisting of a fuselage and a tailplane with an elevator on one side only. This is equivalent to the design case mentioned in the introduction in which the elevator circuit stiffness has been reduced on one side of the aircraft, *e.g.*, by a partial power failure, whereas on the other side it remains effectively infinite. The alternative problem involving tabs would have brought in too many degrees of freedom for a complete solution to be obtained.

2.1. *Geometry and Modes of Deformation.* A plan view of the tailplane is given in Fig. 1. The leading dimensions are:

Tailplane chord	c
Tailplane span	$2s = 3c$
Elevator chord	$c_\beta = 0 \cdot 25s$
Elevator span	$s = 1 \cdot 5c$

This represents a fairly small elevator, being one-eighth of the tailplane area, and in an extreme case tabs on an elevator could be asymmetric in about the same proportion.

The assumed modes of deformation are

(1) parabolic bending of the fuselage (symmetric);

(2) linear torsion of the fuselage (antisymmetric);

(3) starboard tailplane rotation about the quarter-chord;

(4) port tailplane rotation about the quarter-chord;

(5) elevator rotation.

Mode 1 is a symmetric mode, the fuselage assumed clamped at a point 2c ahead of the tailplane quarter-chord. The tailplane is attached to the fuselage only at its pivot point (*see* modes 3 and 4) at the quarter-chord. Mode 2 is the antisymmetric mode. The five calculations are then made up as follows:

(1) asymmetric quinary—Modes 1, 2, 3, 4, 5;

(2) symmetric ternary for the whole aircraft—Modes 1, (3+4), 5;

(3) symmetric ternary for the half aircraft—Modes 1, 3, 5;

(4) antisymmetric ternary for the whole aircraft—Modes 2, (3−4), 5;

(5) antisymmetric ternary for the half aircraft—Modes 2, 3, 5.

The flutter coefficients for the calculations can be obtained in the usual way by applying Lagrange's equations. For solution on the R.A.E. flutter simulator[2] the equations are written in the form:

$$[a\ddot{q} + bv\dot{q} + cv^2q + d\dot{q} + eq] = 0$$

where $v = V/V_0$ and V_0 is the chosen reference speed

a	Square inertia matrix
b	Aerodynamic damping matrix
c	Aerodynamic stiffness matrix
d	Structural damping matrix
e	Structural stiffness matrix
q	Column of generalised co-ordinates q_r.

Numerical values of the coefficients are given in Table 2 at the end of the Paper. In addition each mode was given structural damping of $\frac{1}{2}$ per cent critical; *i.e.*, $d_{rr} = 0 \cdot 01 \sqrt{(a_{rr}\ e_{rr})}$ and $d_{rs} = 0$ for $r \neq s$.

The ternary calculations for the whole aircraft (numbers 2 and 4 above) each require the combination of two of the five degrees of freedom into a single mode. For the symmetric calculation for example, the tailplane rotation can be represented by a single co-ordinate in which the displacement on each side of the aircraft has the same magnitude and sign. Numerically, therefore, the coefficients for the new second mode can be obtained by first adding the fourth row to the third row to reduce the matrices from order 5×5 to order 5×4, and then adding the 4th column to the third column to give a set of 4×4 matrices. This process preserves the Lagrangian form of the matrices so that, for example, the inertia matrix will remain symmetric and positive definite. For the ternary calculation the second row and column are discarded in the usual way. To obtain the anti-symmetric ternary (2, $3-4$, 5) a similar procedure is followed, but the fourth row and column are successively subtracted from the third row and column.

2.2. *Mass Distribution.* A typical tailplane mass distribution was assumed and the appropriate structural inertia coefficients were calculated. In the fuselage modes there will be additional inertia arising from the rear fuselage masses, and this was assumed to increase a_{11} by 25 per cent and a_{22} by 10 per cent. In a control-surface flutter calculation it is the inertia distribution of the control surface which is most important. In the present example the mass per unit area over the elevator was assumed constant but its value was varied in a preliminary investigation. The reason for this is that a very heavy elevator shows severe flutter, but is insensitive to typical parameters, and a very light elevator gives no flutter at all. It was important for the present example to have a degree of control-surface flutter for the unbalanced elevator that was fairly mild and therefore sensitive to the differences between the various types of flutter under investigation. Having settled a suitable value of elevator mass the principal variable in the flutter calculations was taken to be elevator mass-balance which was assumed to be a concentrated mass at the elevator mid-span on an arm length $0 \cdot 1$ of the wing chord.

2.3. *Structural Stiffnesses.* The structural stiffnesses in the calculations are prescribed by assuming values for the uncoupled frequency ratios of the different degrees of freedom. The frequency ratios were varied to some extent, but for the purpose of the investigation the effect of the variations was found to be relatively unimportant and results are presented only for a single set of frequency ratios. Table 1 below gives the frequency ratios for which the results are presented and also the range of variations covered in the calculations. The values of frequency, ω_r, refer to the condition of zero mass-balance, and as mass-balance is increased these frequencies will change slightly although not to an important extent.

TABLE 1

	Calculation	ω_2/ω_1	ω_3/ω_1	
1	Asymmetric quinary	$\sqrt{2}$*, 1, $1/\sqrt{2}$	$2\sqrt{2}$*, 2, $\sqrt{2}$	All combinations covered
2	Symmetric ternary for whole aircraft	—	$2\sqrt{2}$*, 2, $\sqrt{2}$	—
3	Symmetric ternary for half aircraft	—	$2\sqrt{2}$*, 2, $\sqrt{2}$	—
4	Antisymmetric ternary for whole aircraft	$\sqrt{2}$*, 1, $1/\sqrt{2}$	$2\sqrt{2}$*, 2, $\sqrt{2}$	All combinations covered
5	Antisymmetric ternary for half aircraft	$\sqrt{2}$*, 1, $1/\sqrt{2}$	$2\sqrt{2}$*, 2, $\sqrt{2}$	Checks for all combinations

* Results presented in Figs. 2 and 3.

2.4. *Aerodynamic Assumptions.* For simplicity the aerodynamic derivatives are assumed to be constant with frequency parameter and have been evaluated by use of the Minhinnick Rules[1].

3. *Results and Conclusions.* The results are presented in graphic form of velocity (v) against mass-balance weight factor, m, for frequency ratios $\omega_2/\omega_1 = \sqrt{2}$ and $\omega_3/\omega_1 = 2\sqrt{2}$ only. Fig. 2 gives the results obtained for the four ternary calculations separately, and Fig. 3 the comparison between the combined symmetric and antisymmetric ternaries for the whole aircraft, for the half aircraft, and the asymmetric quinary.

The combined flutter boundaries from the ternary calculations for the whole aircraft give a greater flutter-free area than does that for the quinary, whereas the combined boundary for the half aircraft gives a pessimistic result with a smaller flutter-free area. These results take the form one would expect on physical grounds, but on the whole the calculations for the half aircraft give rather better agreement with the true asymmetric results than do those for the whole aircraft, as well as erring on the safe side. It is concluded that on an aircraft with an asymmetric elevator control, *e.g.*, with a spring tab on one side only, the flutter calculations should assume that the more stable side is a mirror image of the less stable side unless a truly asymmetric calculation is made. If this leads to adverse flutter characteristics the asymmetric calculation may have to be tackled in full.

LIST OF SYMBOLS

a		Inertia matrix
b		Aerodynamic damping matrix
c		Aerodynamic stiffness matrix
d		Structural damping matrix
e		Structural stiffness matrix
q		Column matrix of generalised co-ordinates
V		Airspeed
V_0		Chosen reference speed
v	=	V/V_0
c		Wing chord
s		Elevator span
ω_r		Natural frequency of mode r taken alone with the condition $m = 0$
m		Mass-balance weight factor

REFERENCES

No.	*Author*	*Title, etc.*
1	E. G. Broadbent	Flutter prediction in practice. A.R.C. C.P. 373. February, 1956.
2	F. Smith and W. D. T. Hicks ..	The R.A.E. electronic simulator for flutter investigations in six degrees of freedom or less. A.R.C. R. & M. 3101. September, 1953.

TABLE 2

Asymmetric Quinary

	Modes $r = 1$	2	3	4	5
a_{1r}	$800 \cdot 3 + 28 \cdot 16m$	$-57 \cdot 64 + 26 \cdot 4m$	$320 \cdot 7 + 50 \cdot 28m$	$320 \cdot 7$	$27 \cdot 98 - 16 \cdot 76m$
b_{1r}	$188 \cdot 0$	0	$262 \cdot 4$	$262 \cdot 4$	$62 \cdot 67$
c_{1r}	$208 \cdot 0$	0	650	650	$628 \cdot 3$
e_{11}	160				
a_{2r}	$-57 \cdot 64 + 26 \cdot 4m$	$1057 \cdot 5 + 24 \cdot 75m$	$276 \cdot 2 + 47 \cdot 14m$	$-276 \cdot 2$	$22 \cdot 40 - 15 \cdot 71m$
b_{2r}	0	$238 \cdot 9$	$267 \cdot 2$	$-267 \cdot 2$	$47 \cdot 67$
c_{2r}	0	0	$853 \cdot 1$	$-853 \cdot 1$	$659 \cdot 7$
e_{22}		$422 \cdot 9$			
a_{3r}	$320 \cdot 7 + 50 \cdot 28m$	$276 \cdot 2 + 47 \cdot 14m$	$689 \cdot 0 + 89 \cdot 79m$	0	$68 \cdot 23 - 29 \cdot 93m$
b_{3r}	$58 \cdot 9$	0	$368 \cdot 0$	0	$164 \cdot 5$
c_{3r}	0	0	0	0	$785 \cdot 4$
e_{33}			$1102 \cdot 1$		
a_{4r}	$320 \cdot 7$	$-276 \cdot 2$	0	$689 \cdot 0$	0
b_{4r}	$58 \cdot 9$	0	0	$368 \cdot 0$	0
c_{4r}	0	0	0	0	0
e_{44}				$1102 \cdot 1$	
a_{5r}	$27 \cdot 98 - 16 \cdot 76m$	$22 \cdot 40 - 15 \cdot 71m$	$68 \cdot 23 - 29 \cdot 93m$	0	$21 \cdot 96 + 9 \cdot 977m$
b_{5r}	$3 \cdot 56$	$1 \cdot 368$	$15 \cdot 76$	0	$26 \cdot 03$
c_{5r}	$2 \cdot 08$	0	$13 \cdot 02$	0	$81 \cdot 8$
e_{55}					0

The above coefficients are appropriate to the frequency ratios

$$\frac{\omega_2}{\omega_1} = \sqrt{2}\,; \quad \frac{\omega_3}{\omega_1} = 2\sqrt{2}$$

for the condition $m = 0$. The coefficients for which no numerical values are given in Table 2, *i.e.*, the coefficients e_{rs} where $r \neq s$, are all identically zero. Structural damping coefficients, d_{rs}, were assumed as stated in Section 2.1.

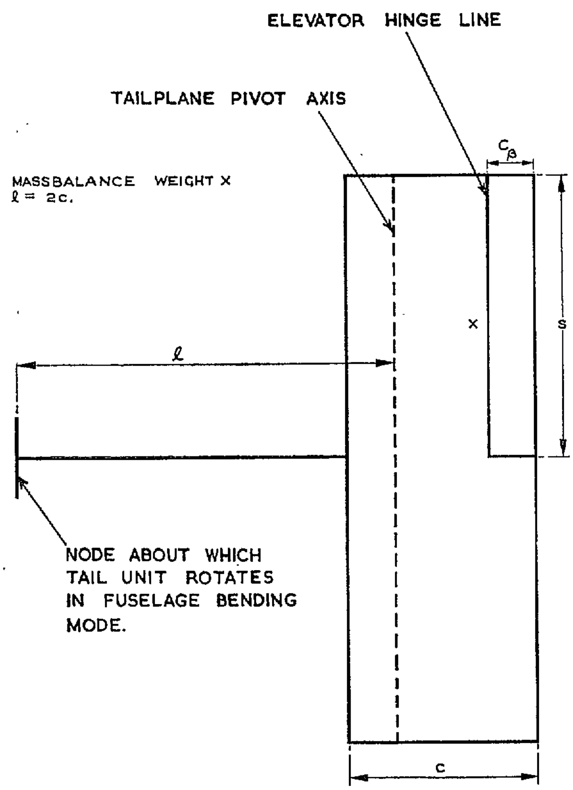

FIG. 1. Planform of asymmetric tailplane.

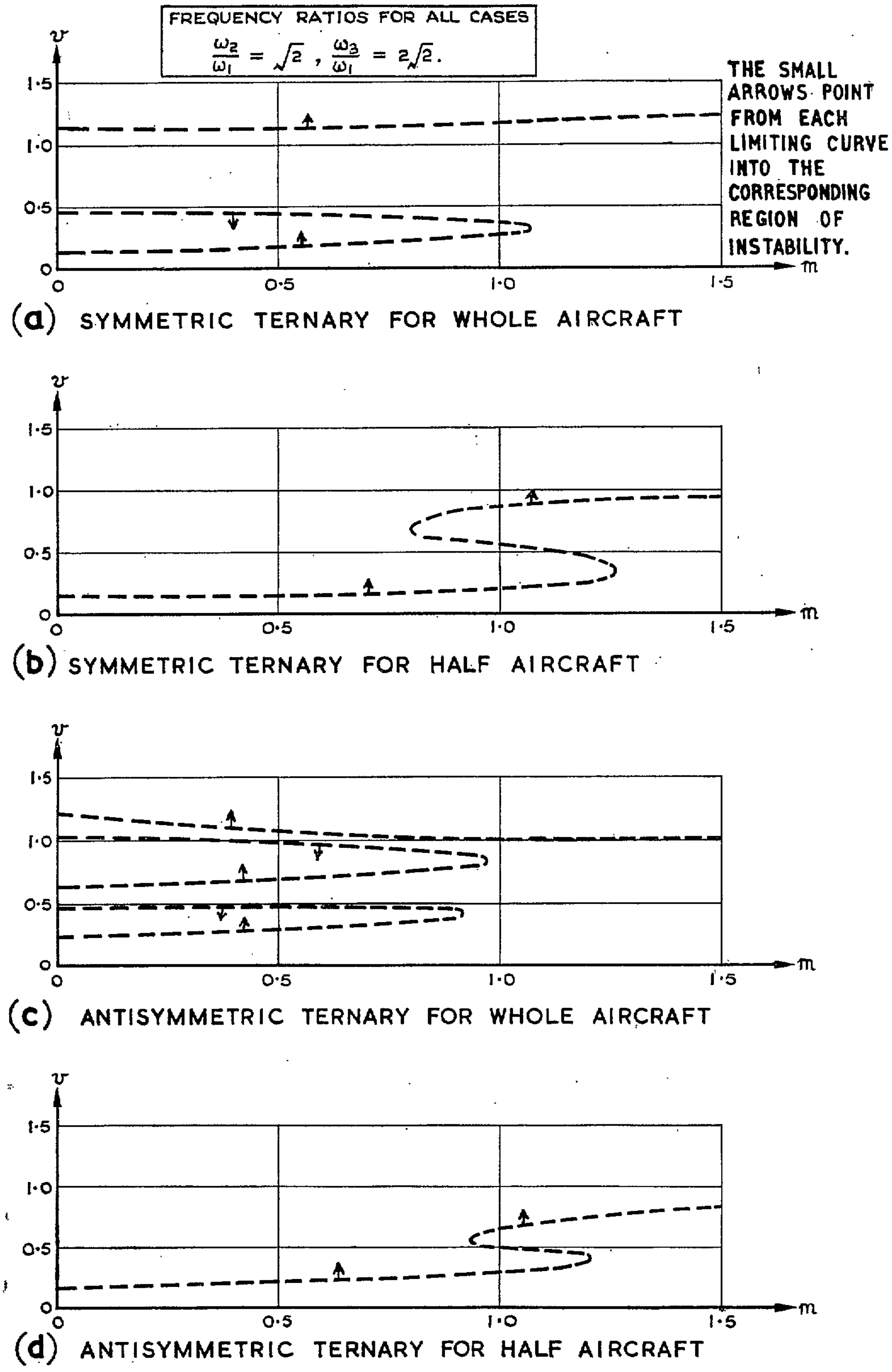

FIG. 2a to d. Results for ternary calculations.

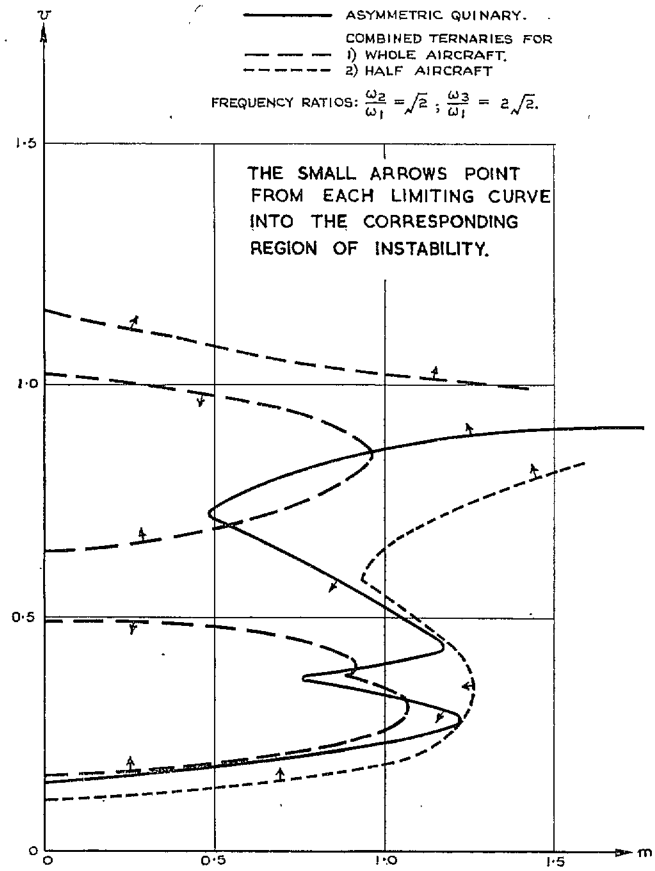

FIG. 3. Comparison of the two approximate methods of calculation with the exact quinary result.

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