R. & M. No. 3253

MINISTRY OF AVIATION

AERONAUTICAL RESEARCH COUNCIL
REPORTS AND MEMORANDA

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By D. S. WHITEHEAD, M.A., Ph.D., A.M.I.Mech., A.F.R.Ae.S.,
CAMBRIDGE UNIVERSITY ENGINEERING LABORATORY

LONDON: HER MAJESTY'S STATIONERY OFFICE
1962
SEVEN SHILLINGS NET

The Analysis of Blade Vibration due to Random Excitation

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CAMBRIDGE UNIVERSITY ENGINEERING LABORATORY

*Reports and Memoranda No. 3253**
August, 1960

Summary. This Report gives an account of the experimental measurement of the power spectra of waveforms derived from vibrating blades in axial compressors under running conditions. This enables the damping factor of the blades to be found, and these damping factors agree reasonably well with theoretical estimates of the aerodynamic damping. There is no significant decrease in the damping factor when the blades stall. The Report includes theoretical estimations of the length of data which has to be examined to get accurate measurements of the R.M.S. amplitude and the power spectrum. The amount of data required is much greater for the power spectrum, but even for the R.M.S. amplitude it is surprisingly long.

The Report also includes a theoretical prediction of the effect of variations of air density and blade material density under otherwise similar conditions.

1. *Introduction.* One of the most troublesome problems that has arisen in the development of gas-turbine engines has been the vibration of axial compressor and turbine blades. The trouble arises particularly on the front stages of axial compressors, since these stages necessarily run stalled when the compressor is operating below design speed. Detailed examination has shown that the vibration can be divided into four types, all of which can be very dangerous. The first type is vibration of rotor blades excited by fixed obstructions in the flow, such as struts supporting the main bearings. This type of vibration is characterized by the vibration frequency, which is always an integral multiple of the frequency of rotation of the machine. The second type is vibration due to cells of stalled flow which are often found when a compressor is stalled, and which usually rotate at about 40 per cent of the speed of rotation of the machine. The third type is stalled flutter, which is rather rare, but is characterized by its waveform which is very nearly a pure sine wave with a frequency which is not an integral multiple of the frequency of rotation of the machine. This Report is concerned solely with the fourth type of vibration which is random vibration excited by the turbulence of the flow over the blades, and it is presumed that vibration of the first three types is not present.

A typical waveform for this type of vibration is shown in Fig. 1. It can be seen that it consists of an oscillation with a frequency which corresponds to the natural frequency of the blade, but the amplitude of the oscillation varies in a random manner with time.

It is found that when a blade row becomes stalled, the amplitude of the random vibration increases very greatly. This could be accounted for in two ways. It could either be due to an increase in the

* Previously issued as A.R.C. 22,119.

level of turbulence of the flow, or it could be due to a decrease in the amount of aerodynamic damping. A measurement of the total damping factor for the vibration would be very valuable, since it would show the relative importance of these two effects, and this would enable an assessment of the probable effect of building in mechanical damping to be made. It would also enable the relative magnitudes of the aerodynamic and mechanical damping to be inferred from an experiment in which the aerodynamic conditions were altered while the machine was running at constant speed. This would give information on the effects to be expected from changes of air density and blade material.

The objective of the present investigation is therefore to analyse the observed waveforms so as to obtain the damping factor for the blade.

The method used is based on the hypothesis that the excitation is purely random and does not contain any special frequencies, at least over a range of frequency near the blade natural frequency. The power spectrum of the exciting force, G_i, is therefore flat, as shown plotted against frequency ω in Fig. 2. If the blade is assumed to behave as a simple oscillator, then the power spectrum G_0 of the blade displacement is a function of frequency of the form also shown in Fig. 2. It has exactly the form that would be expected from simple resonance theory, assuming that the blade is subject to a large number of small sinusoidal exciting forces, each of the same amplitude, but differing in frequency and equally distributed over the frequency band.

If, therefore, the power spectrum of the signal from a strain gauge stuck onto a blade could be measured, the result would be expected to be of the form shown in Fig. 2. From this the damping factor of the blade can be easily determined. This will be specified by the logarithmic decrement δ, which is given by

$$\delta = \pi\Delta\omega/\omega_0 \tag{1}$$

where $\Delta\omega$ is the difference in frequency between the points at which the amplitude of the power spectrum is half the maximum value.

It should be noted that the assumption that the input power spectrum is constant is only required to hold over a frequency range of about $\Delta\omega$ on each side of the blade natural frequency. For blades with low damping this is only a very narrow frequency range, and, provided there are no wakes or eddies with a regular repetition rate present, it should be a very good approximation.

2. *Experimental Procedure.* In order to measure a power spectrum such as that shown in Fig. 2 experimentally, it is necessary to use an analyser with a frequency pass band which is much smaller than the frequency scale of the spectrum required, *i.e.*, a pass band of not more than (say) $\Delta\omega/10$. If the logarithmic decrement of the blade is $0 \cdot 03$ this means that the analyser pass band ($\Delta\omega'$) must be not more than about $\omega_0/1{,}000$. This demands a standard of resolution of an order of magnitude higher than that obtainable from any commercially built wave analyser, except for those using magnetostrictive bars as filters. A special analyser was therefore built for this investigation, and this is described in a separate paper (Ref. 8).

The standard deviation of a reading obtained from a spectral analyser operating on a random input depends on the frequency pass band of the analyser and the length of the sample analysed. For the analyser used here, which behaves like two simple tuned circuits in series and gives a reading proportional to the square root of the power spectrum, the standard deviation is given by

$$\sigma_n = \sqrt{\left(\frac{1 \cdot 61}{T\Delta\omega'}\right)}$$

where $\Delta\omega'$ is the frequency difference between the points at which the power is down by a factor of 2, T is the time over which the reading is taken, and σ_n is the standard deviation expressed as a fraction of the mean reading.

For many of the results reported herein, $\Delta\omega'$ was $5 \cdot 9$ rad/sec and T was 180 sec. For this case, therefore, $\sigma_n = 0 \cdot 039$, *i.e.*, nearly 4 per cent standard deviation. This shows that in order to get accurate results a very long sample of data is required, and that 3 min is undesirably short for this filter setting.

This requirement for long data samples means that it is impractical to analyse the waveform direct from a machine as it is running, and it is essential to make a tape recording of the waveform which can subsequently be analysed to obtain the power spectrum at a number of different frequencies.

The requirement for long data samples also severely limited the choice of data. It was not desired to run an engine or a large compressor rig especially for this purpose, but opportunities were taken to make a few long recordings during tests organized for other purposes. This means that no systematic investigation has been possible. Two series of analyses have been carried out on records from compressors chosen because they do not show any evidence of wake resonances, rotating stall or flutter.

The first series of analyses was carried out on recordings made during a test of a Bristol Olympus 104 engine on a test bed. Some rotor blades in the first two stages were strain gauged. (These are known as the 0 and 1 stages.) These blades are in aluminium alloy and have fir-tree root fixings, the strain gauges being placed near the root of the blade at about mid-chord. The analysis usually carried out on this type of recording has been reported by Armstrong and Stevenson[1]. In this case the vibration is mainly in the two bending modes with lowest frequency. The tape recordings used for this analysis were made from the original recordings, by re-recording through 1/3 octave filters, in order to make sure that the unwanted modes did not interfere with the analysis.

The second series of analyses was carried out on recordings made during a test of a 6 stage Rolls-Royce experimental compressor on a compressor test rig. Some rotor blades in the third, fourth, and fifth stages were strain gauged. These blades are in aluminium alloy and have pin root fixings. In this case the vibration is mainly in the first torsional mode and the strain gauges were stuck on to the concave side of the blade near the trailing edge at the root, a position sensitive to torsion. In this case no preliminary filtering was done, except for that included in the analyser itself.

3. *Experimental Results.* The results of the analysis are shown in Figs. 3, 4 and 5. In these figures the reading of the analyser is plotted directly against frequency. This reading is proportional to the square root of the power spectrum at each frequency, and can be loosely thought of as an 'amplitude spectrum'. The amplitude scales are arbitrary and the relative amplitudes of different curves have no significance.

For all the spectra shown in Figs. 3 and 4 the filter bandwidth ($\Delta\omega'/2\pi$) was $0 \cdot 94$ c.p.s. with an integration time of 3 min, so that the theoretical standard deviation is $3 \cdot 9$ per cent. For the spectra shown in Fig. 5 the filter bandwidth was $4 \cdot 7$ c.p.s. with an integration time of 2 to 4 min, so that the maximum theoretical standard deviation is $2 \cdot 1$ per cent. The results show scatter roughly corresponding to these figures.

Some trouble was experienced due to fluctuations in the speed of the tape recorder during play-back. Clearly a change in speed will shift the whole curve horizontally, with a proportional change of frequency equal to the proportional change in tape speed. The tape recorder used was an ordinary commercial instrument, in which the tape speed is synchronized to the mains frequency, and it is thought that the results contain some error due to variation in the mains frequency over periods of a few minutes.

The amplitude spectra are mainly of the expected shape, and the corresponding logarithmic decrements have been determined from Equation (1) and are shown in Tables 1 and 2. The results shown at the bottom of Fig. 4 for the second bending mode at 6,420 r.p.m. are, however, considerably distorted and the interpretation for this case is doubtful. This case corresponds to a very low level of vibration, and channel noise may be appreciable.

The double peak shown in Fig. 3 for the 0 stage blades at 6,420 r.p.m. calls for special comment. It appears that the peak at 249 c.p.s. is the genuine effect of random excitation on the blades. The peak at 255 c.p.s. is a 'spike' corresponding to a specific sinusoidal component of the waveform with this frequency, but it appears blurred due to the finite bandwidth of the analyser. This was examined in more detail by reducing the bandwidth of the analyser, when this peak at 255 c.p.s. appeared very much higher and narrower. Its effect has been ignored in estimating the damping factor for the blades in this case. The cause of the 'spike' is not known. It is not an engine order and it does not appear on the stage 1 blades.

These results will be discussed more generally after some of the theoretical aspects involved have been considered.

4. *Theoretical Considerations.* 4.1. *Calculation of Aerodynamic Damping.* It will be supposed initially that only one blade in the cascade is vibrating, leaving the effect of aerodynamic coupling between blades in the same row for later consideration. Then the aerodynamic force per unit length $Fe^{i\omega t}$ acting on the blade due to its vibrational velocity $qe^{i\omega t}$ can be expressed in terms of a non-dimensional complex force coefficient, as follows

$$F = \pi\rho U c q C_{Fq}. \tag{2}$$

In addition to this, the blade is subject to an exciting force $Xe^{i\omega t}$ per unit length. It will be supposed that a reasonable estimate of the blade vibration induced can be obtained by considering only a typical two-dimensional section, chosen arbitrarily somewhere near the tip of the blade. The equation of motion for unit length of blade at this section is

$$m\frac{d^2}{dt^2}(xe^{i\omega t}) + \frac{m\omega_0\delta_M}{\pi}\frac{d}{dt}(xe^{i\omega t}) + m\omega_0{}^2xe^{i\omega t} = (F+X)e^{i\omega t}$$

where the terms on the left-hand side give the inertia, mechanical damping, and spring forces respectively.

Using (2) this gives

$$m\left\{-\omega^2 + \frac{i\omega\omega_0\delta_M}{\pi} + \omega_0{}^2 - \frac{i\omega\pi\rho Uc}{m}C_{Fq}\right\}x = X.$$

If R and I are the real and imaginary parts of C_{Fq} then this can be written

$$\left\{\left(-\omega^2 + \omega_0{}^2 + \frac{\omega^2 I}{\lambda\chi}\right) + \frac{i\omega\omega_0}{\pi}\left(\delta_M - \frac{\pi R}{\lambda\chi}\frac{\omega}{\omega_0}\right)\right\}x = X/m$$

where $\lambda = \omega c/U$ is the frequency parameter and $\chi = m/\pi\rho c^2$ is the mass-ratio parameter. χ is always rather large, and it will also be assumed that the mechanical damping is small. The vibration is then only considerable when ω is near ω_0 and is given to a good approximation by

$$x = \frac{X/m}{\left(-\omega^2 + \omega_0^2 + \frac{\omega_0^2 I}{\lambda\chi}\right) + \frac{i\omega_0^2}{\pi}\left(\delta_M - \frac{\pi R}{\lambda\chi}\right)}. \tag{3}$$

This is a normal resonance curve, since λ, R, and I can be considered invariant over a small range of frequency near resonance. The effect of I is to modify the resonant frequency to ω_0' where

$$\omega_0'^2 = \omega_0^2\left(1 + \frac{I}{\lambda\chi}\right). \tag{4}$$

The effect of R (which is normally negative) is to provide aerodynamic damping, the corresponding logarithmic decrement being given by

$$\delta_A = -\frac{\pi R}{\lambda\chi}. \tag{5}$$

The values of R and I have been calculated for two-dimensional incompressible inviscid unstalled flow, assuming that the blades behave like flat plates (Ref. 7). In these calculations it is assumed that all blades in the cascade vibrate with the same amplitude, but with a constant phase angle, β, between one blade and the next. If the corresponding values of the force coefficient are given by R_β and I_β, then the values of R and I for the case when only one blade vibrates are given by (Ref. 6):

$$R = \frac{1}{2\pi}\int_0^{2\pi} R_\beta d\beta \qquad I = \frac{1}{2\pi}\int_0^{2\pi} I_\beta d\beta. \tag{6}$$

Then from Equations (5) and (6), the aerodynamic damping is given by

$$\delta_A = -\frac{1}{2\lambda\chi}\int_0^{2\pi} R_\beta d\beta. \tag{7}$$

This result has been evaluated for the Bristol Olympus compressor, using published tables of the force coefficients (Ref. 7), and the results are shown in Table 1. Since the blades are in fact tapered and twisted, and the air velocities and angles are not accurately known, the calculation involves many arbitrary assumptions. The air velocities have been calculated at the blade mid-height, assuming the design air angle out of the previous stator and a ratio of axial velocity to mean blade speed calculated from interstage static-pressure tappings. The mass-ratio parameter χ has been evaluated at a point three quarters of the way up the blade, and is 29. The aerodynamic coefficients have been taken as corresponding to a space/chord ratio of 1·0 and a stagger angle of 45 deg. On account of these arbitrary features, the calculation can only be regarded as a rough estimate of the aerodynamic damping. The results will be compared with the measured results in Section 5.

4.2. *Random Vibration Theory.* The theory of random vibration is well established, and accounts have been given by Crandall[2] and by Thompson and Barton[5].

If the excitation X is random, with a power spectrum G_i then the power spectrum of the vibration (G_0) can be deduced from Equation (3)

$$G_0 = \frac{1}{m^2}\frac{1}{\left(-\omega^2 + \omega_0^2 + \frac{\omega_0^2 I}{\lambda\chi}\right)^2 + \frac{\omega_0^4}{\pi^2}\left(\delta_M - \frac{\pi R}{\lambda\chi}\right)^2} G_i. \tag{8}$$

If G_i is constant, this gives the form shown in Fig. 2 for G_0. If δ is the logarithmic decrement due to the total damping, this is

$$G_0 = \frac{1}{m^2} \frac{1}{(\omega_0'^2 - \omega^2)^2 + \omega_0^4 \delta^2/\pi^2} G_i . \tag{9}$$

The autocorrelation function of the vibration (ϕ) is given by the Fourier transform of the power spectrum (Laning and Battin[3], Section 3.6). Hence

$$\begin{aligned} \phi(\tau) &= \overline{x(t)x(t+\tau)} \\ &= \tfrac{1}{2} \int_{-\infty}^{+\infty} G_0 e^{i\omega\tau}\, d\omega \\ &= \frac{\pi^2 G_i}{2m^2 \omega_0^3 \delta} \exp(-\omega_0 \tau \delta / 2\pi) \cos \omega_0 \tau \end{aligned} \tag{10}$$

where it has been assumed that δ is small. The autocorrelation function has exactly the same form as a decaying free vibration of the system.

The mean square value of x is obtained by putting $\tau = 0$ in Equation (10)

$$\bar{x}^2 = \frac{\pi^2 G_i}{2m^2 \omega_0^3 \delta} . \tag{11}$$

4.3. *Effect of Blade and Air Density.* The power spectrum of the excitation G_i must be a function of the blade chord c, the air velocity U, the air density ρ, the frequency ω_0, and the blade geometry and a measure of the proportion of turbulence in the air stream. If these last two factors are constant, then by dimensional analysis

$$\frac{G_i}{\rho^2 U^3 c^3} = f(\lambda) \tag{12}$$

where f is an unknown function of the frequency parameter λ based on the blade natural frequency ω_0. From Equations (11) and (12), and separating δ into its mechanical and aerodynamic components:

$$\bar{x}^2 = \frac{\pi^2 \rho^2 U^3 c^3 f(\lambda)}{2m^2 \omega_0^3 \{\delta_M - (\pi R/\lambda\chi)\}} . \tag{13}$$

This may be written non-dimensionally

$$\frac{\sqrt{(\bar{x}^2)}}{c} = \frac{1}{\lambda\chi} \{\delta_M - (\pi R/\lambda\chi)\}^{-1/2} \{f(\lambda)/2\lambda\}^{1/2} . \tag{14}$$

This does not allow any prediction to be made about the effect of λ, but it does give the effect of changes in χ. In particular, if the damping is all mechanical, then the R.M.S. amplitude is proportional to $1/\chi$ or ρ/ν, where ν is the density of the blade material. Alternatively, if the damping is all of aerodynamic origin, then the amplitude is proportional to $1/\sqrt{\chi}$ or $\sqrt{(\rho/\nu)}$.

4.4. *Standard Deviation of Measurement of R.M.S. Amplitude.* Consider a measurement of the R.M.S. amplitude of vibration, obtained by integrating the square of the signal for a time T. Assuming that the process is Gaussian, Laning and Battin[3] (Section 4.2) have shown that the variance σ^2 of the mean square reading $\bar{x}^2$ is approximately given by

$$\sigma^2 = \frac{4}{T} \int_0^T \phi^2(\tau) d\tau .$$

Hence

$$\frac{\sigma^2}{(\overline{x^2})^2} = \frac{4}{T}\int_0^T \left\{\frac{\phi(\tau)}{\phi(0)}\right\}^2 d\tau = \frac{2\pi}{\omega_0 T\delta} \tag{15}$$

from Equation (10), where the integral has been evaluated approximately, assuming that δ is small and that $\omega_0 T\delta$ is large.

When the square root is taken to get the R.M.S. value, the standard deviation is halved. Hence the standard deviation of a measurement of the R.M.S. amplitude is given by

$$\sqrt{(\pi/2\omega_0 T\delta)}\,. \tag{16}$$

The presence of δ in this result means that in order to get accurate results a surprisingly long sample of data must be analysed, although it is still much less than that required for a spectral analysis. For instance, if $\delta = 0\cdot 1$ and $\omega_0/2\pi = 250$ c.p.s. (typical values for the first bending mode of vibration analysed in this Paper), then the analysis time required to give 2 per cent standard deviation is

$$T = \frac{\pi}{2\,.\,2\pi\,.\,250\,.\,0\cdot 1\,.\,0\cdot 02^2} = 25 \text{ sec}.$$

In normal vibration testing it is not usual to spend as long as this over a reading, and the results obtained always show a correspondingly large scatter.

4.5. *The Effect of Aerodynamic Coupling between Blades.* In the above analysis it has been assumed that only one blade in the machine is vibrating. In fact, all blades will vibrate, and there will be some mutual interaction between them. This will be considered for the ideal case in which all blades are identical and there is no mechanical coupling between blades through their roots. It will also be assumed that there is no mechanical damping.

If there are N blades in a given row, then corresponding to each mode of a blade considered as an individual, there will be N modes of the whole row considered as a system. Each of these N modes can be taken as a motion in which each blade has the same amplitude, but with a phase angle β between one blade and the next. The values of β will be given by $2\pi n/N$, where n is an integer $(1 \leqslant n \leqslant N)$.

Owing to the aerodynamic forces, each of these modes will have a slightly different natural frequency and a different damping factor. The approach will be to analyse the excitation in terms of these normal modes, consider the effect due to each, and then superimpose the results.

Considering first the excitation, it will be supposed that the random forces acting on each blade are uncorrelated. Then, if $G_{i\beta}$ is the power spectrum of the component of the exciting force corresponding to the mode with a phase angle β, it follows that $G_{i\beta}$ is the same for all values of β. Also, since the excitation in all the modes is uncorrelated, the power spectrum of the total force acting on any one blade is the sum of the power spectra of the components. Hence

$$G_{i\beta} = \frac{1}{N} G_i\,. \tag{17}$$

For each of these modes, an expression of the form of Equation (8) will apply. Neglecting the mechanical damping this is

$$G_{0\beta} = \frac{G_{i\beta}}{m^2} \frac{1}{\left(-\omega^2 + \omega_0{}^2 + \dfrac{\omega_0{}^2 I_\beta}{\lambda\chi}\right)^2 + \left(\dfrac{\omega_0{}^2 R_\beta}{\lambda\chi}\right)^2} \tag{18}$$

where R_β and I_β are functions of β. Putting $y = \lambda\chi\{(\omega/\omega_0)^2 - 1\}$, a measure of the deviation from the blade natural frequency, and using Equation (17), gives

$$G_{0\beta} = \frac{\lambda^2\chi^2 G_i}{m^2\omega_0^4 N}\frac{1}{(I_\beta - y)^2 + R_\beta^2}. \tag{19}$$

Since the vibration in all modes is uncorrelated, the power spectrum of the motion of any one blade is the sum of the power spectra of the motion in each mode. Hence

$$G_0 = \frac{\lambda^2\chi^2 G_i}{m^2\omega_0^4 N}\sum_{n=1}^{N}\frac{1}{(I_\beta - y)^2 + R_\beta^2}. \tag{20}$$

If the number of blades is very large, this sum may be replaced by an integral, with $d\beta = 2\pi/N$. Then

$$G_0 = \frac{\lambda^2\chi^2 G_i}{m^2\omega_0^4}H(y) \tag{21}$$

where

$$H(y) = \frac{1}{2\pi}\int_0^{2\pi}\frac{d\beta}{(I_\beta - y)^2 + R_\beta^2}. \tag{22}$$

This result is illustrated in Fig. 6, which shows $\sqrt{H}$ plotted against y. This is therefore in exactly the same form as the form in which the results of the experimental analysis have been presented, except for scale factors and a shift of frequency origin. This example has been calculated from the theoretical values of R_β and I_β for a space/chord ratio of 1, a stagger angle of 45 deg, and a frequency parameter of 1.

Also shown in Fig. 6 is the value of H corresponding to the case when only one blade vibrates. This is

$$H = \frac{1}{(I - y)^2 + R^2}$$

where R and I are given by Equations (6).

Fig. 6 shows that the effect of the aerodynamic coupling is to make the curve slightly more 'peaky'. In this case the value of R_β varies from $-0\cdot35$ to $-0\cdot88$. For a case in which the value of R_β varied over a wider range, the effect of the aerodynamic coupling would be greater.

5. *Discussion of Results and Conclusions.* The technique described in this Paper is successful in determining the damping factor of blades running in an engine by analysing the random vibration. The accuracy attained is reasonable, and could be improved by using longer samples and a tape recorder with better speed stability. The aerodynamic coupling between blades has been shown to be a minor effect. Since no systematic investigation has been carried out, the conclusions drawn from the results must be highly tentative.

The most obvious conclusion that can be drawn from Table 1 is that the damping factor for the second bending mode is very considerably lower than for the first bending mode. This trend is also shown by the theoretical calculation of the aerodynamic damping, and is due to the increased frequency parameter. The damping factors shown in Table 2 for the torsional vibration are still lower, but since the position of the torsional axis is unknown in this case no theoretical calculation has been possible.

In the Olympus compressor the first two stages would be expected to be at least partially stalled at the speeds of 4,000 r.p.m. and 4,500 r.p.m., but unstalled at 6,420 r.p.m. This is borne out by the amplitude of vibration which is also shown in the Table as *af* (amplitude × frequency), and is much greater at speeds of 4,000 and 4,500 r.p.m. than it is at 6,420 r.p.m. However there is no significant change in damping factor with speed, which shows that in this case the increase of vibration when the compressor stalls is due to an increased level of turbulence, and not to a reduction in damping. In the Rolls-Royce compressor all the results are taken with the compressor unstalled.

Bearing in mind the large number of arbitrary assumptions involved in the calculation, the calculated theoretical aerodynamic damping factor shown in Table 1 agrees reasonably well with the measured total damping factor. The calculated aerodynamic damping is in fact generally the greater of the two. This may be an indication that the damping is mainly aerodynamic, as has long been believed for resonant vibration.

It has been shown that if the damping is all aerodynamic, then the amplitude of vibration is proportional to $\sqrt{(\rho/\nu)}$ (where ν is the material density). Hence in a jet engine the amplitude will increase with forward speed and decrease with increasing altitude, and a change from aluminium alloy to steel blading will have a beneficial effect. If the damping is all mechanical, however, the amplitude is directly proportional to (ρ/ν), and these effects are more powerful.

Acknowledgement. Grateful acknowledgement is made to Bristol Siddeley Engines Ltd., and Rolls-Royce Ltd., who made the recordings analysed herein available to the author, and also gave him much assistance. The author is also indebted to them for permission to publish this Paper.

NOTATION

c		Blade chord
i		Indicates component leading 90 deg in phase
	$=$	$\sqrt{(-1)}$
m		Mass per unit length of blade
n		Integer
$qe^{i\omega t}$		Translational velocity of blade due to vibration
t		Time
$xe^{i\omega t}$		Displacement of blade due to vibration
C_{Fq}		Aerodynamic coefficient
	$=$	$R + iI$
$Fe^{i\omega t}$		Aerodynamic force per unit length on blade due to its motion
G_i		Power spectrum of exciting force
G_0		Power spectrum of blade displacement
I		Imaginary part of C_{Fq}
N		Number of blades in wheel
R		Real part of C_{Fq}
T		Time taken over a reading
U		Air inlet velocity relative to blade
$Xe^{i\omega t}$		Aerodynamic force per unit length on blade due to turbulence
β		Phase angle between one blade and the next
δ_A		Logarithmic decrement of blade due to aerodynamic damping
δ_M		Logarithmic decrement of blade due to mechanical damping
δ		Logarithmic decrement of blade due to total damping
	$=$	$\delta_A + \delta_M$
λ		Frequency parameter
	$=$	$\omega c/U$
ν		Density of blade material
ρ		Density of air
σ		Standard deviation
τ		Time interval
ϕ		Autocorrelation function of blade displacement
χ		Mass ratio parameter
	$=$	$m/\pi\rho c^2$
ω		Angular frequency
ω_0		Natural angular frequency of blade *in vacuo*

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TABLE 1

Bristol Olympus Compressor

First Bending Mode

Stage	Blade No.	r.p.m.	Frequency c.p.s.	δ	δ_A	af ft./sec.	λ
0	413	4000	205	0·125	0·135	0·6	0·60
0	415	4000	202	0·087		0·5	
0	413	4500	214	0·103	0·147	0·6	0·56
0	415	4500	213	0·068		0·3	
0	413	6420	249	0·098	0·193	0·3	0·44
0	415	6420	249	0·082		0·2	
1	419	4000	225	0·077	0·126	0·7	0·63
1	420	4000	225	0·067		0·3	
1	419	4500	233	0·063	0·141	0·7	0·58
1	420	4500	234	0·094		0·4	
1	419	6420	268	0·114	0·190	0·2	0·45
1	420	6420	268	0·088		0·1	

Second Bending Mode

Stage	Blade No.	r.p.m.	Frequency c.p.s.	δ	δ_A	af ft/sec	λ
0	413	4000	614	0·036	0·034	1·0	1·80
0	415	4000	615	0·023		0·8	
0	413	4500	644	0·032	0·037	0·5	1·67
0	415	4500	640	0·027		0·4	
0	413	6420	655	?	0·057	0·1	1·18
0	415	6420	670	0·070?		0·1	
1	419	4000	705	0·045	0·031	0·5	1·97
1	420	4000	705	0·032		0·5	
1	419	4500	719	0·066	0·034	0·4	1·78
1	420	4500	727	0·073		0·4	
1	419	6420	738	0·026?	0·059	0·1	1·23
1	420	6420	738	0·012?		0·1	

TABLE 2

Rolls-Royce Compressor

Torsional Mode

Stage	Blade No.	r.p.m.	Frequency c.p.s.	δ	Stress p.s.i.
3	1	11820	1969	0·028	860
3	2	11820	1965	0·021	790
3	1	13260	1995	0·020	585
3	2	13260	1988	0·020	929
3	1	14050	2120	0·045	654
4	6	14050	2592	0·023	638
5	10	11820	2704	0·020	1730
5	10	13260	2655	0·031	1810
5	10	14050	2586	0·023	433

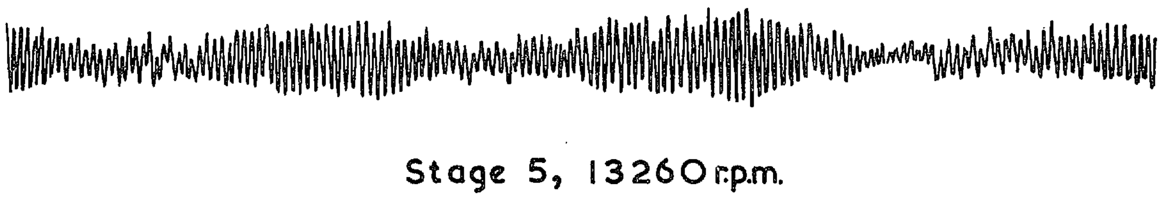

FIG. 1. Example of random vibration waveform.

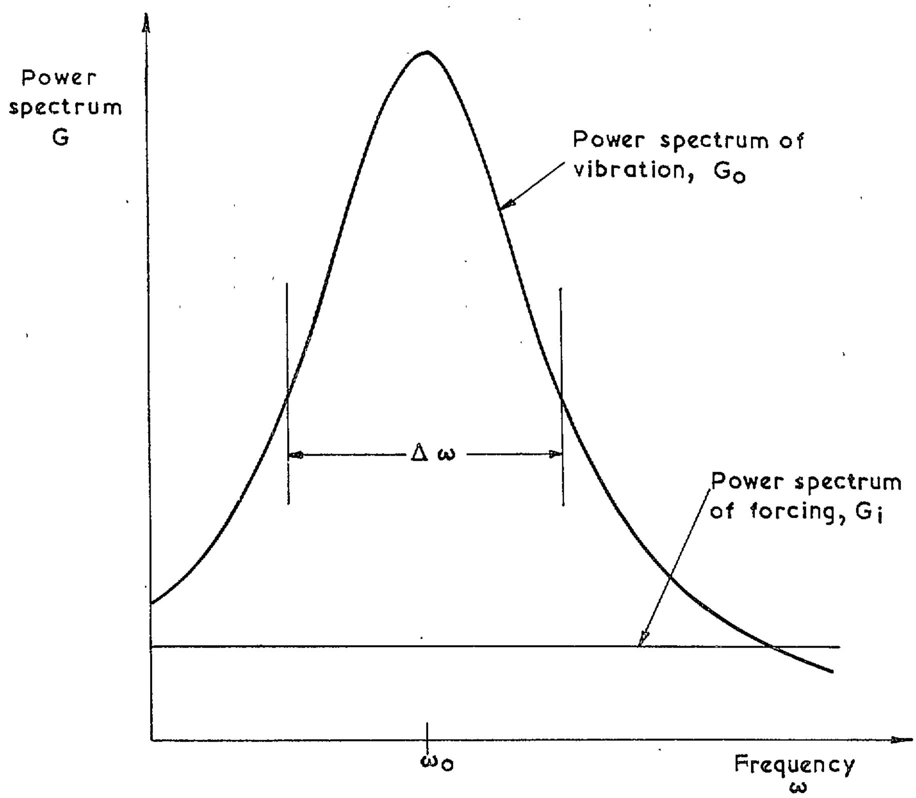

FIG. 2. Power spectra of forcing and vibration waveforms.

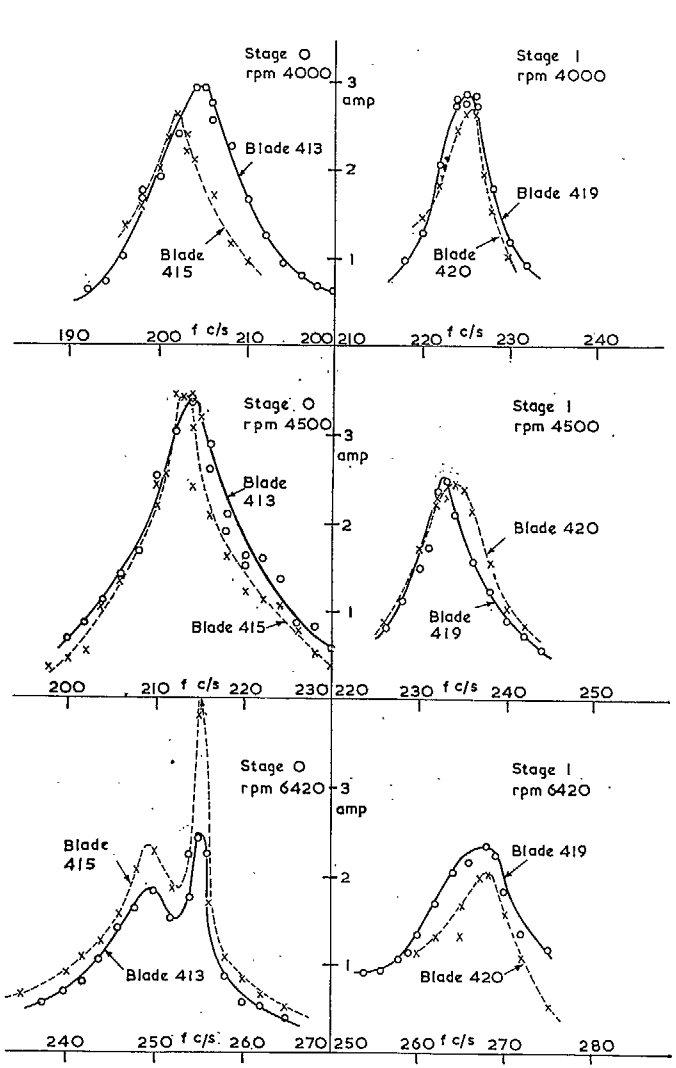

FIG. 3. First bending mode.
(Bristol Olympus compressor)

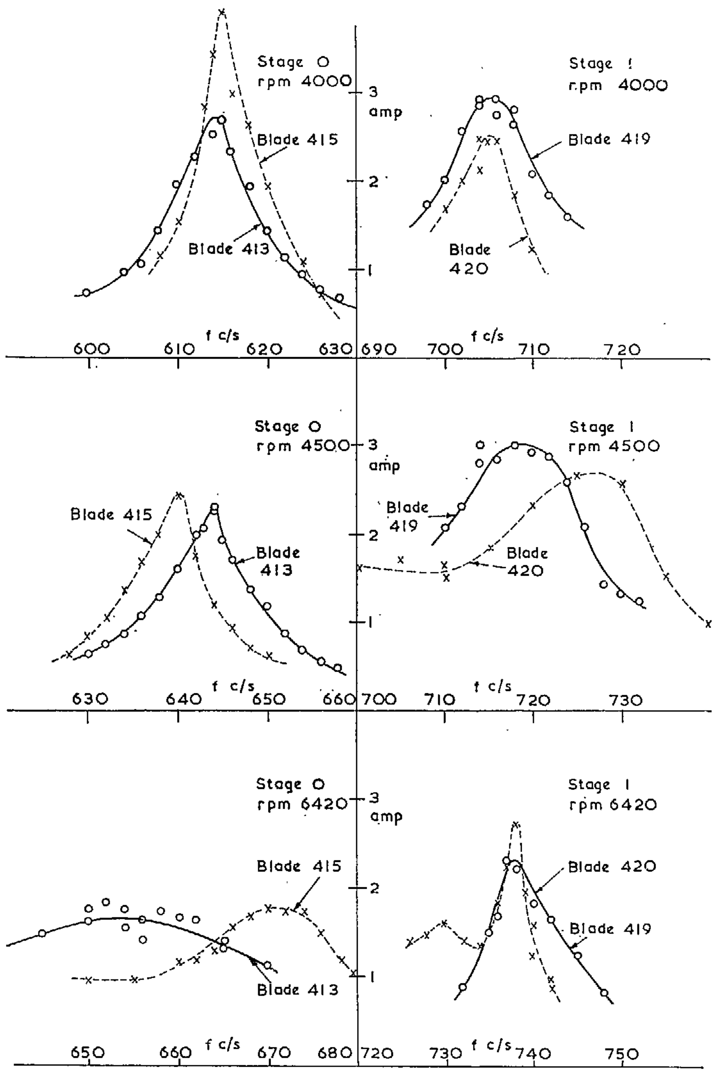

FIG. 4. Second bending mode.
(Bristol Olympus compressor)

(83535) Wt. 67/1876 K.5 2/62 Hw.

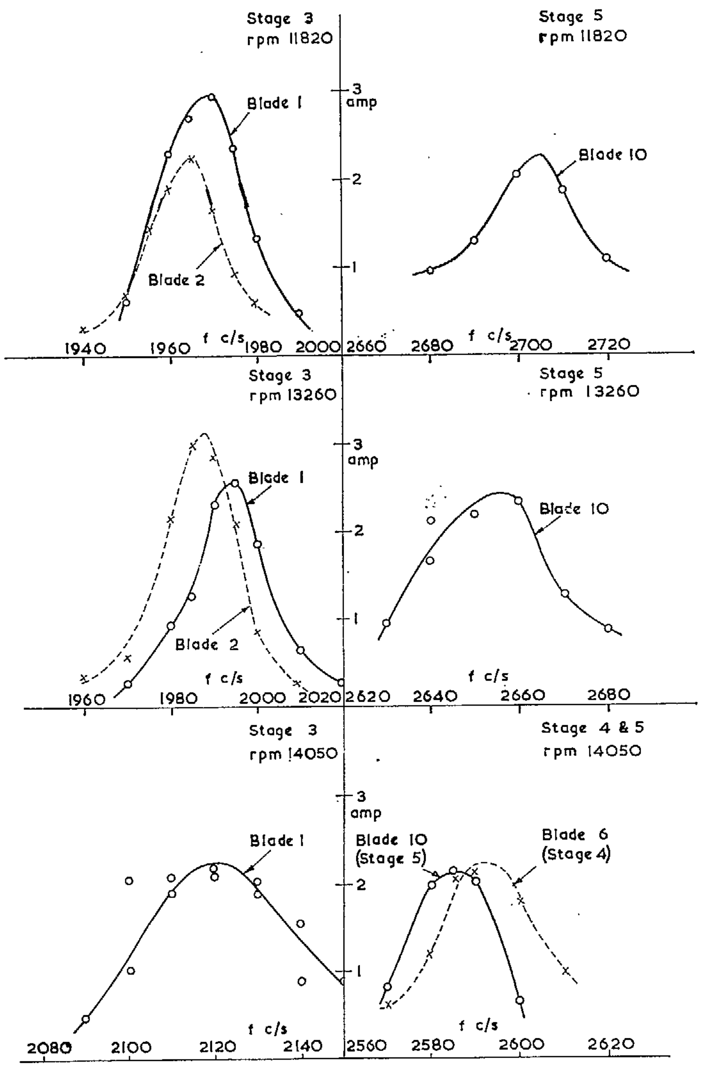

FIG. 5. Torsional mode. (Rolls-Royce compressor)

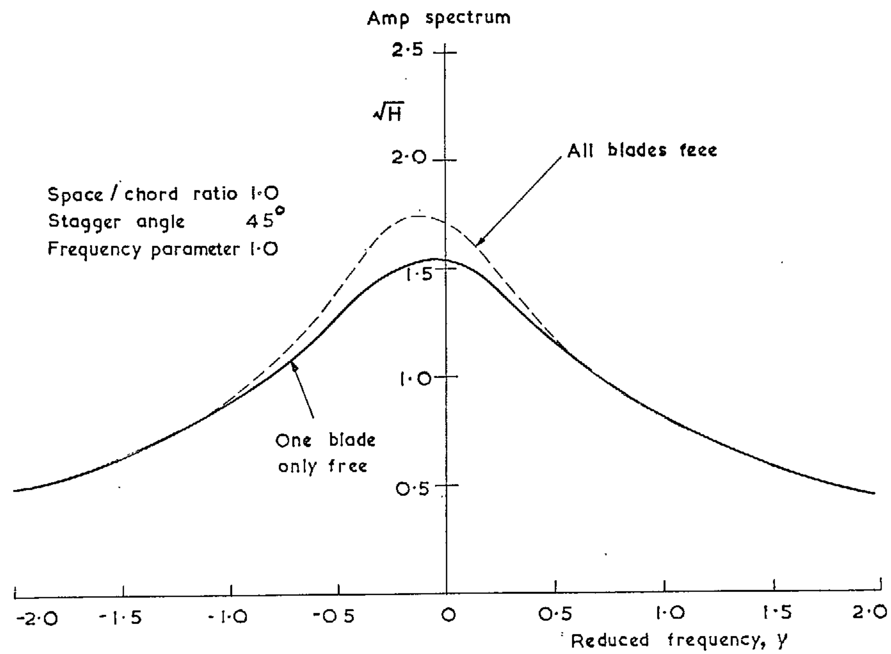

FIG. 6. Effect of aerodynamic coupling.

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