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Some Proposals Regarding the Definitions of Terms Relating to Various Flow Régimes of a Gas

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1. Introduction

In recent years there has been a rapid expansion of the ranges of speed and altitude that are of practical interest to aircraft and missile designers. In consequence, several new terms have been introduced to describe the new flow régimes that require to be considered, and, at the same time the meanings of some of the older terms have undergone a measure of modification. The meanings attached to these terms by different workers have not always been consistent, differences have sometimes arisen from the differences of outlook between theoretical and experimental workers or between those concerned primarily with the motion of a fluid and those concerned primarily with the forces on a body in the fluid. To avoid the confusion that may readily arise if such differences become too well established it was felt desirable at this stage to attempt the formulation of unambiguous definitions of the more important terms in current use.

A source of difficulty in this task that must be noted is the fact that some of the terms have been adopted for reasons other than their etymological aptness to the flow régime or characteristic considered. For example, 'transonic speed' suggests a speed increasing or decreasing through the speed of sound. Yet it has come to denote amongst other things the speed of an aircraft not very different from the speed of sound in the undisturbed air at the altitude of flight. The reason for this will be clear from the discussion that follows. Nevertheless the term 'near-sonic' would clearly be more apt than 'transonic' in this context. Where, however, a term such as the latter has become widely accepted it was felt that it would be unwise to reject it on etymological grounds provided a clear unambiguous meaning could be assigned to it.

The definitions are developed and enlarged upon in Section 3, and the more important are summarised, along with related terms for completeness, in the Appendix.

2. List of Symbols

a	speed of sound
ℓ	a typical linear dimension
M	Mach number
K	Knudsen number (λ/ℓ)
R	Reynolds number
λ	Mean force path of molecules
ν	Kinematic viscosity

3./

3. Discussion

An essential preliminary is to define the speed of sound at a point in moving fluid. It is the speed with which small disturbances are propagated in the neighbourhood of the point measured relative to axes moving with the velocity of the fluid there. We then define a Mach number in general as the ratio of a speed to a related speed of sound. This general concept makes it easy to adopt subsequent and more specific definitions to either a frame of reference associated with a moving element of fluid or one associated with the aircraft or model past which the fluid is moving. Definitions of such particular Mach numbers readily follow and these then lead naturally to definitions of subsonic and supersonic speeds associated with the relevant Mach number being less than or greater than unity, respectively. From this follow the definitions of subsonic and supersonic flows, which are described as flows for which, in the regions considered, the speed is everywhere subsonic or supersonic respectively.

We come next to transonic flow which is defined as a region of flow in which there are conterminous smaller regions of subsonic and supersonic flow, each of significant extent for the problem considered. Across the common boundaries of these sub-regions of flow the flow Mach number passes through the value unity. Such mixed flows present special theoretical and experimental difficulties because of the very different manner in which boundary conditions impress themselves on the flow at subsonic and supersonic speeds, and in particular linearised small perturbation theory loses its validity for such flows. The term transonic flow is frequently confined to regions where the flow Mach number is everywhere close to unity, and the so-called transonic flow theory is a non-linear small perturbation theory specially developed for such cases; nevertheless, it was felt that the term need not in general be so restricted.

As already remarked the term transonic speed has become so widely accepted to mean an aircraft or stream speed for which the Mach number is close to unity, in the range from about 0.8 - 1.2, that it was clearly desirable to retain this definition. At such speeds the flow past bodies of practical interest is generally transonic in the sense defined above, although this will depend on the body thickness and shape. The thinner or more slender a configuration is, and the smaller is its incidence, the nearer the stream or aircraft Mach number must be to unity for the flow engendered to be transonic.

We consider next the term hypersonic flow. When an analysis is made of the flow about a body at Mach numbers large compared with unity it becomes evident that the flow characteristics are such as to merit a special name, and the name hypersonic was introduced. As in the case of transonic flow, linearised small perturbation theory loses its validity for the flow of an inviscid fluid at high Mach numbers unless the body concerned is of extreme slenderness. This is because the disturbances in flow speed then introduced by the body cannot in general be regarded as small compared with the speed of sound, and any valid simplifications of the basic flow equations leave them essentially non-linear. A characteristic feature of hypersonic flow is that the overall inclination of the body nose shock is of the same order of magnitude as the mean body slope so that the region of flow near the body influenced by it is a relatively narrow one between the shock and body surface. The flow of a perfect fluid is then characterised by the fact that an increase of Mach number of the undisturbed stream is accompanied by little change in the velocity distribution, the accompanying changes are essentially in the temperature distribution. The analysis further demonstrates the importance of the so-called hypersonic similarity parameter, formed by the product of the stream Mach number and

a/

a representative body slope measured relative to the undisturbed stream direction. It can be shown that the flows of a perfect fluid about bodies of geometrical similarity at high Mach numbers are similar for the same value of this parameter, and the breakdown of linearised small perturbation theory is associated with the value of this parameter being of the order of unity or higher.

When we come to consider a real fluid in hypersonic flow it will be clear that since the region of influence of the body is a narrow one the distinction between boundary layer and external flow becomes increasingly difficult to draw with increase of stream Mach number. Problems of skin friction, heat transfer, etc., therefore have stimulated the development of hypersonic flow boundary layer theories that attempt to take account of this fact. The picture is made more complex by the fact that at very high Mach numbers the temperature changes at the nose shock may be such as to cause dissociation and ionisation, and these effects may in turn modify the flow and, in particular, the development of the boundary layer.

Bearing in mind these different aspects of flow at high Mach numbers that have at various times been associated with the term 'hypersonic', it was felt that the best definition of hypersonic flow would be the simplest and most general. With configurations of practical interest the important characteristics of hypersonic flow are in evidence at Mach numbers of the order of 5 or more. Hypersonic flow is therefore here described simply as a region of flow in which such Mach numbers occur.

The definition of hypersonic speed readily follows as that of a stream or aircraft for which the Mach number is of the order of 5 or more.

It will be noted that 'supersonic' includes 'hypersonic' and overlaps 'transonic', but by implication it will usually be used to denote the intermediate régime where these other terms do not apply.

Reference is sometimes made in the literature of flow at high Mach numbers to Newtonian flow. This is because at such Mach numbers the flow bears a rough overall resemblance to the flow of a fluid postulated by Newton in which all changes produced by a body occurred at the surface of the body and were essentially changes of momentum necessary to change the flow from its undisturbed stream direction to the local surface direction. Thus the zone of influence of the body was confined to its surface, a state of affairs that can be regarded as occurring in the limit of hypersonic flow of an ideal fluid as the stream Mach number tends to infinity. Likewise some qualitative resemblance can readily be demonstrated between formulae for the aerodynamic characteristics of simple shapes at high Mach numbers based in hypersonic flow theory and the corresponding formulae based on the so-called Newtonian flow theory, and by improvements in the latter differences between the two sets of formulae can be readily reduced.

We come now to consider continuum flow and in contrast slip flow. All gases consist of molecules in random motion relative to the ordered mean motion. This random motion is characterised by a length called the mean free path which is the average distance traversed by molecules between collisions. If this mean free path, denoted here by λ, is very small compared with a typical linear dimension of the body or its boundary layer then for most purposes the discrete molecular structure of the gas can be ignored and the gas can be regarded as physically continuous. The gas is then referred to as a continuum, and the flow of the gas is a continuum flow. The ratio of λ to the typical linear dimension, ℓ, is here referred to as the Knudsen number (K). The length ℓ at body

Reynolds/

Reynolds numbers where one can reasonably refer to a boundary layer (i.e., $R > 10^2$) may be pertinently be taken to be the boundary layer thickness at some datum point, at lower Reynolds numbers a purely geometric dimension of the body is appropriate. Broadly speaking continuum flow will apply for values of K of the order of 10^{-2} or less. It is an essential characteristic of continuum flow that at a solid surface the adjacent fluid can be regarded as at rest relative to the surface, this characteristic is sometimes referred to as the condition of no-slip. Since the vast majority of practical problems concern flows which can be regarded as continuum flows they are not usually referred to as such, and in general a flow may be taken as a continuum flow unless it is specifically indicated to be otherwise.

Where K is small but not insignificant by comparison with unity (i.e., of order 10^{-1}), the discrete molecular structure of the fluid plays an appreciable part in determining the character of the flow, particularly near the boundary where the condition of no-slip no longer applies. Such a flow is therefore referred to as slip flow. In cases where K is of the order of unity or larger the characteristics of the flow are dominated by the molecular structure of the fluid and collisions of the molecules with the boundary are more important than collisions between molecules. Such a flow is referred to as free molecule flow. It will be clear that K increases with decrease of density.

Fig. 1 has been prepared to illustrate in a broad qualitative way the boundaries between these various flow régimes in terms of Reynolds number and Mach number.

We come finally to the various types of wind tunnels that have been developed for producing these régimes of flow for test purposes. Their definitions present no difficulty once the régimes themselves have been defined. It is a matter of some practical interest to note, however, that at the high working section Mach numbers appropriate to a hypersonic wind tunnel the ambient temperature is considerably below the stagnation temperature, so much so that if air is used as the working fluid it is liable to liquefy unless it is preheated. This necessity for preheating when air is used is sometimes regarded as an essential characteristic distinguishing supersonic from hypersonic wind tunnels. Another point of practical interest concerns the tunnel designed for investigating slip flow or free molecule flow. Here the stagnation densities and pressures are generally required to be so low that the latter are most conveniently quoted in microns of mercury or millionths of an atmosphere.

APPENDIX/

APPENDIX

Summary of Proposed Definitions

Speed of sound	The speed with which small disturbances are propagated in the neighbourhood of a point measured relative to axes moving with the velocity of the fluid at that point.
Mach number	The ratio of a speed to a related speed of sound.
Flow Mach number	The ratio of the speed of an element of fluid relative to a solid boundary (e.g., aircraft or wind tunnel) to the local speed of sound.
Stream Mach number	The flow Mach number at points where the flow can be regarded as uniform.
Aircraft or flight Mach number	The ratio of the speed of an aircraft relative to the undisturbed air to the speed of sound in the undisturbed air at the altitude of flight. (This is the same as the stream Mach number of the undisturbed air relative to axes fixed in the aircraft.)
Subsonic speed	A flow, stream or aircraft speed for which the flow, stream or aircraft Mach number is less than unity.
Supersonic speed	A flow, stream or aircraft speed for which the flow, stream or aircraft Mach number is greater than unity.
Subsonic flow	A flow in which, in the region considered, the speed is everywhere subsonic.
Supersonic flow	A flow in which, in the region considered, the speed is everywhere supersonic.
Transonic flow	A region of flow in which both subsonic and supersonic speeds occur, so that fluid particles are accelerated or decelerated through the local speed of sound across the common boundaries of the sub-regions of subsonic and supersonic flow. (The term is frequently confined to regions where the flow Mach number is everywhere close to unity.)
Transonic speed	A stream or aircraft speed for which the stream or aircraft Mach number is near unity, usually between 0.8 and 1.2. (At such speeds the flow near the aircraft or model concerned is generally transonic.)
Hypersonic flow	A flow in which, in the region considered, Mach numbers greater than about 5 occur.
Hypersonic speed	A stream or aircraft speed for which the stream or aircraft Mach number is greater than about 5.

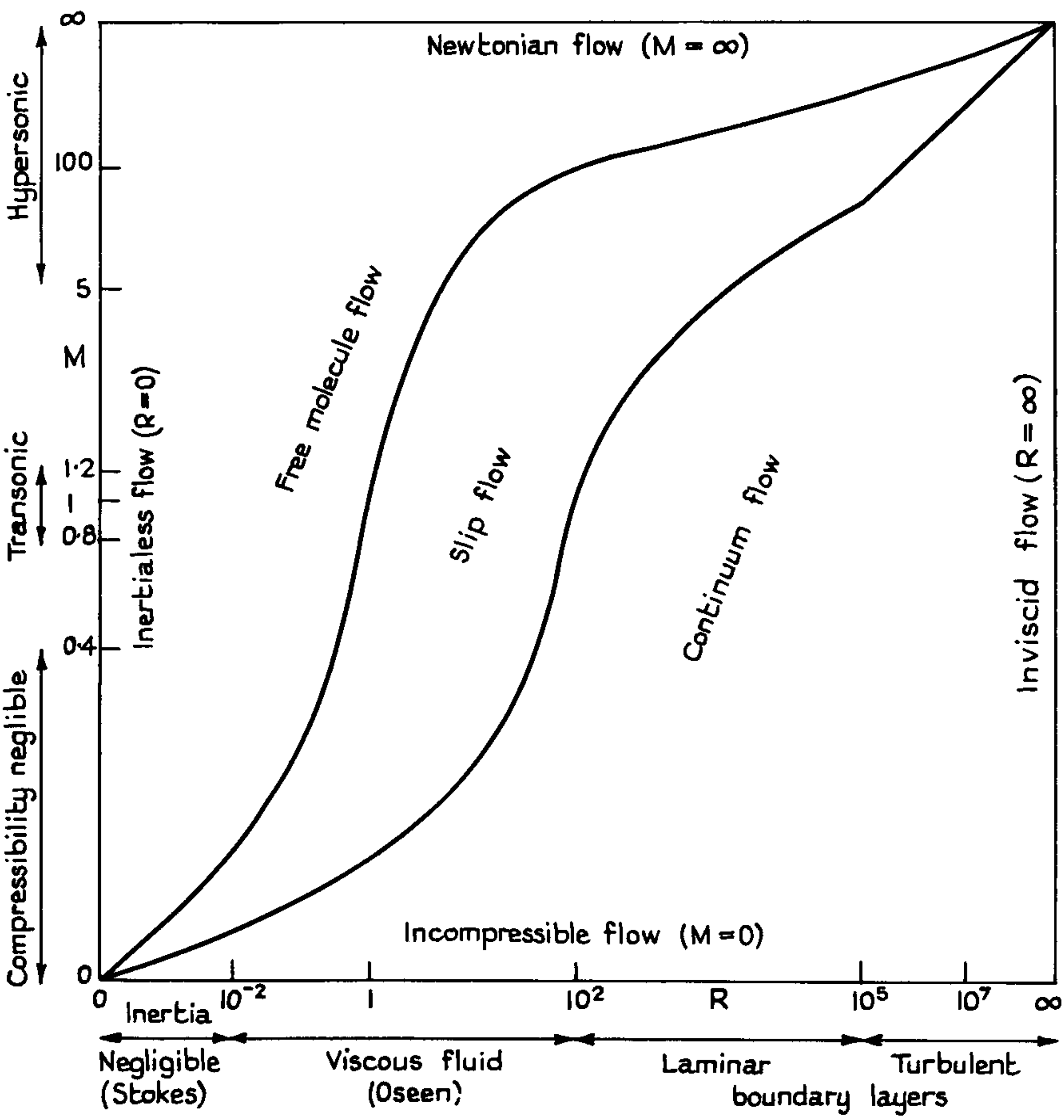
Continuum/

Continuum flow	Flow of a fluid under conditions for which the density is sufficiently high that the molecular structure of the fluid can be ignored and the fluid can be considered as physically continuous. These conditions imply that the ratio of the mean free path of the molecules to a typical linear dimension of the body or its boundary layer (the so-called Knudsen number) is very small, and fluid adjacent to a solid surface can be taken as at rest relative to the surface.
Slip flow	Flow where the mean free path of the molecules is a significant fraction of a typical linear dimension of the body, and fluid adjacent to a solid surface is not necessarily at rest relative to the surface.
Free molecule flow	Flow where the mean free path of the molecules is comparable with or larger than a typical linear dimension of the body.
Subsonic wind tunnel	A wind tunnel capable of running only at subsonic speeds.
Supersonic wind tunnel	A wind tunnel capable of running at supersonic speeds.
Transonic wind tunnel	A wind tunnel capable of running at transonic speeds.
Hypersonic wind tunnel	A wind tunnel capable of running at hypersonic speeds. When air is the medium preheating is usually required to avoid liquefaction in the working section.
Low density wind tunnel	A wind tunnel capable of producing slip flow or free molecule flow, characterised by a low stagnation density and stagnation pressure, the latter being of the order of one millionth of an atmosphere.

FIG. 1.

The boundaries between continuum flow and slip flow and between slip flow and free molecule flow occur roughly at Knudsen numbers of 0·01 and 1 respectively. For Reynolds numbers less than 100 it is pertinent to base the Knudsen number, K, on the typical length, l, but when boundary layers exist it is more relevant to base it on the boundary layer thickness, δ_L (if laminar) or δ_T (if turbulent).

Assuming that $\nu = \lambda a$, where λ is the mean free path
$\delta_L / l = 10/R^{1/2}$ for laminar boundary layers
$\delta_T / l = 1/10^{1/2} R^{1/5}$ for turbulent boundary layers
then the boundaries are given by $M/R = K_l$ for $0 < R < 10^2$
$M/R^{1/2} = 10 K_{\delta_L}$ for $10^2 < R < 10^5$; $M/R^{4/5} = K_{\delta_T} / 10^{1/2}$ for $10^5 < R < \infty$



Flow regimes for a gas

Ignoring liquefaction, dissociation and ionisation.

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