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The Interference on a Three-Dimensional Jet-Flap Wing in a Closed Wind Tunnel

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Summary. The classical theory of wind-tunnel interference is extended to cover interference on the effectiveness of a full-span jet flap issuing from the trailing edge of a high aspect ratio unswept wing. It is shown that, for small constraint, corrections ΔC_J and $\Delta\alpha$ must be added to the observed jet momentum coefficient and wing incidence, respectively. These corrections are derived, together with the corresponding corrections to the observed lift and thrust coefficients.

Corrections to the observed downwash field over a limited interval downstream of the trailing edge of the wing are also derived. These lead to a corrected jet path and a downward displacement of the downwash pattern, in addition to the direct increment to the observed downwash. Corresponding corrections to tail height and setting are also given.

1. *Introduction.* There is reason to suppose that the wind-tunnel corrections appropriate to a jet-flap wing of finite span may differ significantly from those derived for conventional wings. An attempt is therefore made below to re-interpret the results of classical interference theory (as given, for example, by Glauert[1]), in the light of a recent theory of the jet-flap wing[2]. Attention is confined to the so-called lift constraint in a wind tunnel of closed working section; and the dimensions of the model under test are assumed to be small compared with the cross-sectional dimensions of the air-stream.

2. *The Mathematical Model.* 2.1. *Unlimited Stream.* A thin three-dimensional wing with a full-span jet flap can be represented by a vortex sheet which, if its slope is small everywhere, may be taken to lie in a horizontal plane $z = 0$ at which the boundary conditions are satisfied. The strength of the vortex sheet is defined by a function $\gamma(x, y)$, where γ is the strength of the bound or spanwise vortex element at the point $(x, y, 0)$, and where x is the streamwise and y the spanwise co-ordinate. $\gamma(x, y)$ is related to the downwash velocity $w(x, y)$ in the plane $z = 0$ by the usual downwash equation of lifting surface theory; but only the boundary conditions satisfied at the sheet will be required here. They are[2]

$$0 \leqslant x \leqslant c; \qquad w = U_\infty \alpha \tag{1}$$

$$x = c + 0; \qquad w = U_\infty(\tau + \alpha) \tag{2}$$

$$c < x < \infty; \qquad (\bar{c}/2) C_J w'(x) = -\gamma(x) \tag{3}$$

$$x = \infty; \qquad w = U_\infty \epsilon(\infty) \tag{4}$$

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where similarity in the flow conditions at all spanwise stations is assumed; that is to say, it is assumed that w and γ are functions of (x/c) only (where $c(y)$ is the chord length). In these circumstances the downwash angle $\epsilon(\infty)$ far downstream of the wing is independent of y; and the lift coefficient, like the momentum coefficient C_J, is the same for the whole wing as for a section. It follows, also, that the chord length c is distributed elliptically along the span.

The Expression (3) indicates the essential difference between the jet sheet and the trailing vortex sheet of classical aerofoil theory. Bound vortex elements are no longer confined to the wing, their strength in the jet being proportional to the momentum coefficient C_J and to the local curvature of the sheet.

2.2. *Limited Stream.* If the wing is now considered to be in a wind-tunnel stream, a system of images must be introduced to represent the constraint due to the tunnel walls. They ensure that the appropriate boundary condition is satisfied at the walls, and they give rise to a downwash distribution $-\Delta w(x, y)$ in the plane $z = 0$, which will be assumed small compared with the total downwash w. The vortex distribution $\gamma(x, y)$ now contributes a downwash $w + \Delta w$ in the plane $z = 0$. But on the assumption that w and γ are again functions of (x/c) only, the boundary conditions (1) to (4) remain unchanged.

In this case, however, the downwash w is not generated solely by the vortex distribution γ. Hence if the given wind-tunnel conditions are to be related to a corresponding unlimited flow, the boundary conditions must be recast in terms of γ and $w + \Delta w$. This can be done most easily if it is first assumed that Δw is also a function of (x/c) only, for the boundary conditions can then be written

$$\begin{aligned}
0 \leqslant x \leqslant c; &\qquad w + \Delta w = U_\infty\{\alpha + \Delta\epsilon(x)\} \\
x = c + 0; &\qquad w + \Delta w = U_\infty\{\tau + \alpha + \Delta\epsilon(c)\} \\
c < x < \infty; &\qquad (c/2)\{C_J + \Delta C_J(x)\}\{w'(x) + \Delta w'(x)\} = -\gamma(x) \\
x = \infty; &\qquad w + \Delta w = U_\infty\{\epsilon(\infty) + \Delta\epsilon(\infty)\}
\end{aligned}$$

where

$$\Delta w(x) = U_\infty \Delta\epsilon(x)$$

and

$$\Delta C_J(x)/C_J = -\Delta w'(x)/\{w'(x) + \Delta w'(x)\}.$$

The boundary conditions can now be recovered in their original form if it is assumed, firstly, that $\Delta\epsilon$ is independent of x (and equal to $\Delta\alpha$) in the interval $0 \leqslant x \leqslant c$ and, secondly, that ΔC_J is independent of x in the interval $c < x < \infty$. They then become

$$0 \leqslant x \leqslant c; \qquad w + \Delta w = U_\infty(\alpha + \Delta\alpha) \tag{5}$$

$$x = c + 0; \qquad w + \Delta w = U_\infty(\tau + \alpha + \Delta\alpha) \tag{6}$$

$$c < x < \infty; \qquad (c/2)(C_J + \Delta C_J)(w' + \Delta w') = -\gamma \tag{7}$$

$$x = \infty; \qquad w + \Delta w = U_\infty\{\epsilon(\infty) + \Delta\epsilon(\infty)\} \tag{8}$$

which are the boundary conditions appropriate to an unlimited flow about the same wing, with the same jet deflexion angle τ, but with the incidence increased by $\Delta\alpha$ and the momentum coefficient of the jet increased by ΔC_J.

2.3. *Discussion of the Assumptions.* There is a close analogy between the technique adopted here to deal with the interference downwash field, and that previously used to handle the induced downwash in the absence of interference[2]. For example, by taking ΔC_J to be independent of x, it is assumed, in effect, that Δw and w are related linearly, in the manner shown in Fig. 1. The precise relation taken is

$$\Delta w(x) - \Delta w(\infty) = \frac{\{\Delta\alpha - \Delta\epsilon(\infty)\}\{w(x) - w(\infty)\}}{\tau + \alpha - \epsilon(\infty)} \tag{9}$$

so that

$$\frac{\Delta C_J}{C_J} = \frac{\Delta\epsilon(\infty) - \Delta\alpha}{\tau + \alpha + \Delta\alpha - \epsilon(\infty) - \Delta\epsilon(\infty)}\,. \tag{10}$$

But since the interference downwash field is assumed small compared with the total downwash field, $\Delta\alpha$ and $\Delta\epsilon(\infty)$ can be neglected compared with $\tau + \alpha$ and $\epsilon(\infty)$ respectively. The relation for ΔC_J can then be simplified to

$$\frac{\Delta C_J}{C_J} = \frac{\Delta\epsilon(\infty) - \Delta\alpha}{\tau + \alpha - \epsilon(\infty)} \tag{11}$$

an expression which resembles closely the relation for the effective loss of jet momentum obtained in Ref. 2 with the so-called 'second interpolation for induced downwash', to which the Expression (9) is analogous.

The choice of the function $\Delta w(x)$ is evidently arbitrary. However, it will appear later that the terminal values $\Delta w(c)$ and $\Delta w(\infty)$ can be considered known, so that the Expression (9) simply amounts to a convenient interpolation between them. Its validity might therefore be expected to depend upon the magnitude of the interference downwash field Δw. Provided that this is small compared with the total downwash w, it seems reasonable to suppose that almost any monotonic interpolation between the known end values might suffice. But since a rigorous discussion of this point cannot be given, it may be illuminating to consider some alternative expressions of the assumptions underlying the relations (9) and (10).

It may be noted, firstly, that the condition (3) leads to an expression for the circulation Γ about a streamwise section of the jet in terms of the terminal values $w(c)$ and $w(\infty)$ of the downwash, giving $\Gamma = (cC_J/2)\{w(c) - w(\infty)\}$. It is easy to see that the Expressions (7), (9) and (10) lead to the same value of Γ, so that the approximations amount to the assumption that the corresponding unlimited flow has the same jet circulation Γ as the wind-tunnel flow. Alternatively, the increment ΔC_J in the jet momentum may be interpreted as a suitably weighted mean value of the correct x-dependent function $\Delta C_J(x)$. For

$$\Delta C_J = \int_c^\infty \Delta C_J(x)(w' + \Delta w')\,dx \Big/ \int_c^\infty (w' + \Delta w')\,dx\,.$$

Accepting these approximations, provided that the interference is small, it appears from Equation (11) that $\Delta C_J/C_J$ is $O\{\Delta\alpha/(\tau+\alpha)\}$. Whence it follows that if the effective increment $\Delta\alpha$ is non-negligible, so also is the effective increment ΔC_J.

Another approximation employed in Section 2.2 which requires special mention is the assumption that the interference upwash Δw is a function of (x/c) only. This is not altogether consistent with the basic assumption of small constraint. For if the images which represent the wall effects are all remote from the model under test, the upwash Δw that they induce would be expected to be approximately independent of y. And exact dependence upon (x/c) would follow only if the local chord c were independent of y. In fact, the plan-form has already been taken to be elliptic.

However, if the aspect ratio of the wing is sufficiently large, the variation of c with y becomes appreciable only near the wing tips, where the contribution to the total load is least. An appreciable departure from the correct variation of Δw in these regions need not, therefore, greatly affect the overall results. It seems unlikely that the particular approximation used would lead to serious error.

The remaining approximations involved in the foregoing derivation of the mathematical model are common to classical interference theory and to the existing jet-flap theories. Nevertheless, the representation of the wing and jet as a planar vortex sheet may appear to require special justification in the present case. This is because the displacement of the jet below the centre-line of the wind tunnel would be accompanied, in practice, by an increased distortion of the sheet, leading ultimately to zero total downwash at infinity downstream (if the transverse distortion of the sheet is, unrealistically, ignored). Now although the bound vorticity γ is distributed throughout the interval $0 \leqslant x < \infty$, the bulk of it is confined in an interval of order s (where s is the semi-span of the wing) for a wing of sufficiently large aspect ratio[3]. No appreciable additional distortion would be expected, therefore, if the jet displacement in this interval remains small compared with the distance to the nearest image. The precise physical conditions far downstream are unimportant so far as the field in the immediate neighbourhood of the wing is concerned.

A similar argument justifies the assumption that $\Delta\epsilon(x)$ can be taken constant in the interval $0 \leqslant x \leqslant c$. With the bulk of the bound vorticity confined in an interval of $O(s)$, and with s small compared with the distance to the nearest image, the order of magnitude of the interference upwash on the wing can be found by replacing the images by horseshoe vortices of span $2s$ and strength K (where K is the total circulation about the centre section of the wing and jet). Then, if $\Delta\epsilon_1$ and $\Delta\epsilon_1 + \Delta\epsilon_2$ are the values of the interference upwash angle at points in the chord separated by a distance x, the standard analysis[1] indicates that the ratio $\Delta\epsilon_2/\Delta\epsilon_1$ is $O(2x/h)$, where h is the height of the wind-tunnel section. It follows that $\Delta\epsilon_2$ is small compared with $\Delta\epsilon_1$ provided that the chord length c is small compared with h. If $\Delta\alpha$ is the effective increase of incidence due to the interference, the corresponding effective camber—which is ignored—is $O(c\Delta\alpha/4h)$.

A variation of the same order in the downwash over the jet cannot necessarily be neglected. On the jet it is the derivative of the downwash which leads to modification of the flow pattern. Thus it is the ratio of $d(\Delta\epsilon)/dx$ to $d\epsilon/dx$ which determines whether the interference effects on the jet are significant.

3. *The Corrections* $\Delta\alpha$ *and* ΔC_J. The interference effects have been reduced, in so far as the pressure distribution over the wing is concerned, to effective increments $\Delta\alpha$ and ΔC_J in the incidence α and momentum coefficient C_J. The magnitudes of these increments will now be derived from the standard results given, for example, by Glauert[1].

It has already been argued that each image can be replaced, to the degree of approximation considered, by a single horseshoe vortex of strength K. The only remaining obstacle to the estimation of $\Delta\alpha$ is, therefore, that the position appropriate to the single bound vortex within the interval of $O(s)$ is unknown. In general, however, the total circulation K about a section of the wing and jet is significantly greater than twice the circulation Γ about the same section of the jet. And since the vortex distribution in the jet is concentrated mostly in a region close behind the trailing edge of the wing[3], the effective concentrated bound vortex K is likely to lie within the wing chord, though possibly fairly close to the trailing edge. But it has been shown that with c/h small the variation in upwash over a distance of $O(c)$ is negligible. It is, accordingly, immaterial where the effective concentrated vortex is situated within the wing chord, or where the upwash due to it is calculated, provided only that this point also is within the wing chord. It is natural, therefore, to calculate the interference upwash at the value of x corresponding to the position of the effective concentrated bound vortex, whence it follows[1] that

$$\Delta\alpha = \delta \frac{4sK}{CU_\infty} \tag{12}$$

where C is the cross-sectional area of the wind tunnel, and δ is a factor dependent upon the shape and dimensions of the wind-tunnel section. δ is precisely the same factor δ defined by Glauert, and determined by him for both circular and rectangular section shapes.

The Formula (12) is the standard formula provided that the latter is expressed in terms of circulation rather than lift coefficient. But K is not related to C_L in the standard manner. Since there is a downwash angle $\epsilon(\infty)$ far behind a wing of finite span, there is a contribution $C_J\epsilon(\infty)$ (assuming $\epsilon(\infty)$ to be small and independent of y) to the flux of downward momentum crossing the Trefftz plane. The vortex sheet which represents the wing and jet must therefore support a lift of coefficient $[C_L - C_J\epsilon(\infty)]$, where C_L is the total lift coefficient of the wing (including the direct lift due to the deflected jet). It is to this reduced lift that the circulation K corresponds, so that

$$K = \frac{SU_\infty}{4s}\{C_L - C_J\epsilon(\infty)\}\,. \tag{13}$$

The downwash angle $\epsilon(\infty)$ is presumably obtained with sufficient accuracy by assuming that it is independent of y and that the interference term $\Delta\epsilon(\infty)$ can be ignored. It then follows that[2]

$$\epsilon(\infty) = \frac{2C_L}{\pi A + 2C_J} \tag{14}$$

and, from (12), (13) and (14), that

$$\Delta\,\alpha = \frac{1}{1 + \dfrac{2C_J}{\pi A}}\,\delta\,\frac{S}{C}\,C_L \tag{15}$$

ΔC_J can then be found in terms of $\Delta\alpha$ from Equation (11). For, to the approximation adopted above, $\Delta\epsilon(\infty)$ is equal to $2\Delta\alpha$. Thus

$$\frac{\Delta C_J}{C_J} = \frac{\Delta\alpha}{\tau + \alpha - \epsilon(\infty)}\,. \tag{16}$$

4. *Application of the Corrections* $\Delta\alpha$ *and* ΔC_J. Some care is required in the application of $\Delta\alpha$ and ΔC_J to the correction of experimental results. Incidence α is measured relative to the centre-line of a wind tunnel, and coefficients of lift C_L and thrust C_T are defined accordingly. The measured value of α must evidently be corrected by addition of the increment $\Delta\alpha$. At the same time, however, the forces sustained by the wing must be expressed relative to the effective stream direction instead of to the centre-line of the tunnel. And in doing this it must be remembered that the pressure distribution over the wing corresponds to a momentum coefficient which is increased by ΔC_J, so that the components of the forces resulting from the direct reaction of the jet are changed not only by the rotation of the axes through the angle $\Delta\alpha$ but also as a direct result of the increment ΔC_J.

Resolving the forces normal and parallel to the effective stream direction, and taking $\Delta\alpha$ and ΔC_J to be small, it is evident that the pressure forces on the external surfaces of the wing lead to increments in the lift and thrust coefficients of $\{C_T - C_J \cos(\tau+\alpha)\}\, \Delta\alpha$ and $-\{C_L - C_J \sin(\tau+\alpha)\}\, \Delta\alpha$, respectively. To these must be added increments in the corresponding components of the direct thrust, respectively equal to

$$(C_J+\Delta C_J)\sin(\tau+\alpha+\Delta\alpha) - C_J \sin(\tau+\alpha) \doteqdot C_J \cos(\tau+\alpha)\Delta\alpha + \Delta C_J \sin(\tau+\alpha)$$

and

$$(C_J+\Delta C_J)\cos(\tau+\alpha+\Delta\alpha) - C_J \cos(\tau+\alpha) \doteqdot -C_J \sin(\tau+\alpha)\Delta\alpha + \Delta C_J \cos(\tau+\alpha)$$

so that the corrections which must be applied to the wind-tunnel observations (in addition to the corrections $\Delta\alpha$ and ΔC_J to the observed incidence and jet momentum) are

$$\Delta C_L = C_T\Delta\alpha + \Delta C_J \sin(\tau+\alpha)$$

and

$$\Delta C_T = -C_L\Delta\alpha + \Delta C_J \cos(\tau+\alpha)\,.$$

Using Equation (16), these become

$$\left.\begin{aligned} \Delta C_L &= \left(C_T + \frac{C_J \sin(\tau+\alpha)}{\tau+\alpha-\epsilon(\infty)}\right)\Delta\alpha \\ \Delta C_T &= -\left(C_L - \frac{C_J \cos(\tau+\alpha)}{\tau+\alpha-\epsilon(\infty)}\right)\Delta\alpha \end{aligned}\right\}. \tag{17}$$

And if a drag coefficient is defined according to

$$C_D = C_J - C_T$$

then

$$\Delta C_D = \Delta C_J - \Delta C_T$$

so that the corrections can also be expressed in the form

$$\left.\begin{aligned} \Delta C_L &= \left\{C_J\left(1 + \frac{\sin(\tau+\alpha)}{\tau+\alpha-\epsilon(\infty)}\right) - C_D\right\}\Delta\alpha \\ \Delta C_D &= \left\{C_J\left(\frac{1-\cos(\tau+\alpha)}{\tau+\alpha-\epsilon(\infty)}\right) + C_L\right\}\Delta\alpha \end{aligned}\right\}. \tag{18}$$

It should be noted that the angles τ and α have not been assumed small in this section, although the underlying theory of the jet-flap wing is a small-angle one. This is because the theory agrees well with experiment[2, 3] for angles that are not small. So to make a small-angle approximation in the above resolution of forces might be unnecessarily restrictive.

5. *The Upwash Angle* $\Delta\epsilon$. For the correction of tests on complete models it is also necessary to consider the effects of constraint on the performance of the tailplane. This means that an estimate must be made of the function $\Delta\epsilon(x)$ over an interval of $O(s)$ downstream of the trailing edge of the wing, so that the observed downwash $\epsilon(x)$ can be corrected.

The estimate of $\Delta\epsilon$ must plainly be consistent with the corrections $\Delta\alpha$ and ΔC_J already derived. But this requirement reveals a certain ambiguity in the foregoing analysis, in the course of which two distinct approximations to $\Delta\epsilon(x)$ have been employed. Firstly, the interpolation (9) has been used in the estimation of ΔC_J and, secondly, it has been argued that the images can be replaced by horseshoe vortices in the estimation of the downwash variation near the wing. The latter approximation has been used, so far, only to justify neglect of the effective camber induced on the wing. But if it is valid in this respect, it might be expected to lead, also, to a valid estimate of the downwash variation over the required interval downstream of the trailing edge.

The two available approximations to $\Delta\epsilon(x)$ have been derived independently. But it now appears that the entire analysis is greatly dependent upon their compatibility. For the interpolation (9) can only be expected to lead to an adequate interpretation of the effects of the interference on the jet, if it already provides a satisfactory estimate of the interference upwash over that part of the jet where the vortex strength is greatest and most able to induce additional circulation about the wing. And so it must be regarded as an equally valid alternative to the linear variation of upwash obtained, in the usual way, from the doubly-infinite series of horseshoe vortices. It is necessary, therefore, to compare the two relations, in order both to shed light upon the validity of the approximations used hitherto, and to indicate the form of the correction which might be applied to the observed downwash.

Continuing the analysis of Section 3, in which the images were replaced by horseshoe vortices, it is found (directly from Glauert[1]) that the interference upwash on the centre-line of the wind tunnel varies with x according to

$$\Delta\epsilon = \Delta\alpha + \frac{\delta'}{\delta}\frac{x}{h}\Delta\alpha \tag{19}$$

where δ', like δ, is a factor depending only on the shape of the cross-section of the wind tunnel, and is $O(2\delta)$ for most rectangular sections in common use. The interpolation (9), on the other hand, leads to

$$\Delta\epsilon = \frac{\epsilon(x) - \epsilon(\infty)}{\tau + \alpha - \epsilon(\infty)}\Delta\alpha + \frac{\tau + \alpha - \epsilon(x)}{\tau + \alpha - \epsilon(\infty)}\Delta\epsilon(\infty)\,. \tag{20}$$

Both approximations to $\Delta\epsilon$ are monotonically increasing functions of x which lead to $\Delta\epsilon = \Delta\alpha$ at $x = 0$ (implying that the effective concentrated bound vortex is taken to lie at the trailing edge of the wing*). They are most easily compared by considering the point at which they again give the same value of $\Delta\epsilon$. To do this, $\Delta\epsilon(\infty)$ must first be replaced by $2\Delta\alpha$ in the approximation (20), since this is consistent with a concentrated bound vortex at the trailing edge. It then follows that the two approximations give the same value of $\Delta\epsilon$ at the value of x (measured from the trailing edge) satisfying

$$\frac{\delta'}{\delta}\frac{x}{h} = \frac{\tau + \alpha - \epsilon(x)}{\tau + \alpha - \epsilon(\infty)}\,. \tag{21}$$

Since $\epsilon(x) > \epsilon(\infty)$ it is clear that $x < \frac{\delta h}{\delta'}$, *i.e.*, that $x < 0\left(\frac{h}{2}\right)$.

Taking a specific example, it appears[3] that the downwash angle ϵ at two chords length behind the trailing edge of a two-dimensional wing at zero incidence is approximately $0{\cdot}38\tau$ for a C_J of 5. Assuming that this indicates the order of the downwash behind a high aspect ratio wing at the same C_J, it follows from Equation (21) that the two approximations to $\Delta\epsilon$ would coincide at $x \doteqdot 2c$,

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* According to the argument of Section 3 it is permissible to concentrate the vortex distribution of an image into a single line anywhere within the wing chord. The same argument suggests that it should also be permissible to adjust the distribution so that it leads to the upwash $\Delta\epsilon$ given by Equation (20), provided that the bulk of the adjusted vortex distribution is still confined within the appropriate interval of $O(s)$.

in this case, if $h/c = 2\delta'/0\cdot62\delta$; i.e., if $h/c \doteqdot 6\cdot4$. Coincidence would occur closer to the wing for smaller values of h/c (it occurs at $x \doteqdot c$ when $h/c \doteqdot 4$). But if C_J is reduced the point of coincidence moves further aft (occurring at $x \doteqdot 2c$ for $h/c \doteqdot 5\cdot3$ when $C_J = 2$, and for $h/c \doteqdot 4\cdot5$ when $C_J = 0\cdot5$).

For most practical cases, therefore, the relation (19) is likely to be an adequate linear approximation to the relation (20) over an interval exceeding twice the chord of the wing. And it seems reasonable to infer that the corrections derived in the foregoing analysis are likely to be valid first approximations, and that either of the relations (19) or (20) might be used to correct the downwash field. The approach adopted does not permit the expression of a preference for either; in particular, it cannot be assumed that because the relation (20) is non-linear in x it is necessarily superior to the relation (19).

6. *Application of the Correction* $\Delta\epsilon$. Correction of the observed downwash is quite straightforward, provided that it is remembered that the path of the jet will also require correction. With the origin of co-ordinates at the trailing edge of the wing, x measured positive downstream, and z positive downwards, suppose that $z = Z(x)$ is the observed path of the jet and $\epsilon(x, z)$ is the observed distribution of downwash angle. In the corresponding unlimited flow the path of the jet will become $z = Z + \Delta Z$, where

$$\Delta Z = \int_0^x \Delta\epsilon \, dx \qquad (22)$$

and not only will the downwash angle in a plane $x =$ constant be increased uniformly by $\Delta\epsilon$, but the downwash pattern will also be displaced downwards by ΔZ. In mathematical terms, the observed downwash angle ϵ at the point (x, z) becomes the corrected downwash angle $\epsilon + \Delta\epsilon$ at the point $(x, z + \Delta Z)$.

In terms of the linear approximation (19), ΔZ becomes

$$\Delta Z = x\left(1 + \frac{\delta' x}{2\delta h}\right)\Delta\alpha\,. \qquad (23)$$

A similar formula cannot be derived using the approximation (20). But if $\epsilon(x)$ is measured, ΔZ could be derived from Equation (22) without difficulty. For this purpose, a slightly more convenient form of Equation (20) might be

$$\Delta\epsilon = 2\Delta\alpha - \frac{\Delta C_J}{C_J}\{\epsilon(x) - \epsilon(\infty)\} \qquad (24)$$

since, apart from the measured distribution of downwash angle $\epsilon(x)$, this involves only terms already calculated for the other corrections.

If the observed tail setting to trim the aeroplane in a wind tunnel is α_T, for a tailplane at height t above the chordal plane of the wings (*see* Fig. 2), the above corrections imply that trim would also be obtained in free air if the tail height is reduced to $t - \Delta Z$ and the tail setting increased by $\Delta\alpha_T = \Delta\epsilon - \Delta\alpha$.

7. *Conclusions.* Corrections have been derived to account for the effects of the vortex systems of the images of a three-dimensional jet-flap wing in the walls of a wind tunnel with a closed working section. They correspond to the first order corrections usually applied to experimental data, and have been obtained by means of a simple extension to conventional interference theory.

The corrections may be summarised as follows. The observed incidence α and jet momentum coefficient C_J must be increased by $\Delta\alpha$ and ΔC_J, respectively, where

$$\Delta\alpha = \frac{1}{1 + 2C_J/\pi A} \delta \frac{S}{C} C_L \tag{15}$$

and

$$\Delta C_J = \frac{C_J \Delta\alpha}{\tau + \alpha - \epsilon(\infty)} \tag{16}$$

taking $\epsilon(\infty)$ to be given by

$$\epsilon(\infty) = \frac{2C_L}{\pi A + 2C_J}\,. \tag{14}$$

The measured force coefficients C_L and C_T (T being the measured thrust force on the wing) must then be increased by ΔC_L and ΔC_T, respectively, where

$$\left.\begin{aligned} \Delta C_L &= \left(C_T + \frac{C_J \sin(\tau+\alpha)}{\tau + \alpha - \epsilon(\infty)}\right)\Delta\alpha \\ \Delta C_T &= -\left(C_L - \frac{C_J \cos(\tau+\alpha)}{\tau + \alpha - \epsilon(\infty)}\right)\Delta\alpha \end{aligned}\right\}. \tag{17}$$

Finally, the observed path of the jet $z = Z(x)$ downstream of the trailing edge of the wing, and the observed distribution of downwash angle $\epsilon(x, z)$, must be adjusted to $z + \Delta Z$ and $\epsilon(x, z + \Delta Z) + \Delta\epsilon$, respectively, where

$$\Delta Z = \int_0^x \Delta\epsilon \, dx \tag{22}$$

and $\Delta\epsilon$ may be taken as either of the two approximations

$$\Delta\epsilon = \left(1 + \frac{\delta'}{\delta}\frac{x}{h}\right)\Delta\alpha \tag{19}$$

or

$$\Delta\epsilon = 2\Delta\alpha - \frac{\Delta C_J}{C_J}\{\epsilon(x) - \epsilon(\infty)\}\,. \tag{24}$$

These corrections imply that if the tail height and tail setting to trim in a wind tunnel are t and α_T, respectively, the equivalent free-air tail setting is $\alpha_T + \Delta\alpha_T$ (where $\Delta\alpha_T = \Delta\epsilon - \Delta\alpha$) for a tail height $t - \Delta Z$.

The factors δ and δ' are functions solely of the cross-sectional shape of the wind-tunnel working section. They are given by Glauert[1], for rectangular tunnels, but may be found in any standard work on the subject (*e.g.*, Pankhurst and Holder[4], where the δ' of Glauert and the present paper is denoted by $\delta\eta$).

NOTATION

x, y, z	Rectangular Cartesian co-ordinates, with x measured positive downstream and z positive downwards
$Z(x)$	Ordinate to the jet in the plane $y = 0$
U_∞	Undisturbed stream velocity
α	Incidence of wing
τ	Angle of deflexion of the jet to the chord-line of the wing at the trailing edge
$c(y)$	Chord of the wing
$2s$	Span of the wing
S	Plan area of the wing
A	Aspect ratio of the wing ($= 4s^2/S$)
h	Height of wind-tunnel working section
C	Cross-sectional area of wind-tunnel working section
$\gamma(x, y)$	Bound vortex distribution in the plane $z = 0$, representing the wing and jet
Γ	Circulation about a streamwise section of the jet
K	Circulation about a streamwise section of the wing and jet
$w(x, y)$	Downwash distribution in the plane $z = 0$
$\epsilon(x)$	Downwash angle along $z = 0$, $y =$ constant
C_J	Momentum coefficient of the jet ($= J/\frac{1}{2}\rho U_\infty^2 S$)
C_L	Lift coefficient ($= L/\frac{1}{2}\rho U_\infty^2 S$)
C_T	Thrust coefficient ($= T/\frac{1}{2}\rho U_\infty^2 S$)
C_D	Drag coefficient ($= C_J - C_T$)
$\Delta w(x, y)$	Upwash distribution on $z = 0$ due to constraint
$\Delta\epsilon(x)$	Upwash angle on $z = 0$, $y =$ constant due to constraint
$\Delta C_J(x)$	Function defining increase in jet effectiveness due to constraint
$\Delta\alpha$	Correction to the wing incidence
ΔC_J	Correction to the momentum coefficient
ΔC_L	Correction to the lift coefficient
ΔC_D	Correction to the drag coefficient
ΔZ	Correction to the ordinate to the jet
δ, δ'	Factors dependent upon the shape of the wind-tunnel cross-section (occurring in Equations (15), (19) and (23))
t	Height of tailplane above chordal plane of wing
α_T	Tail setting to trim

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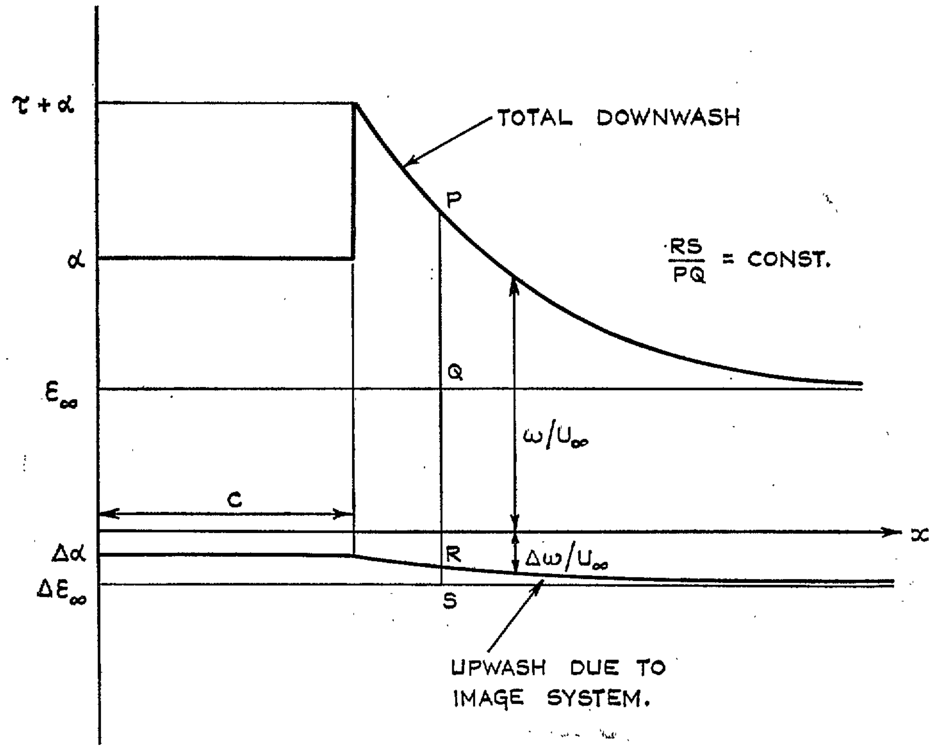

FIG. 1. Assumed variation of downwash.

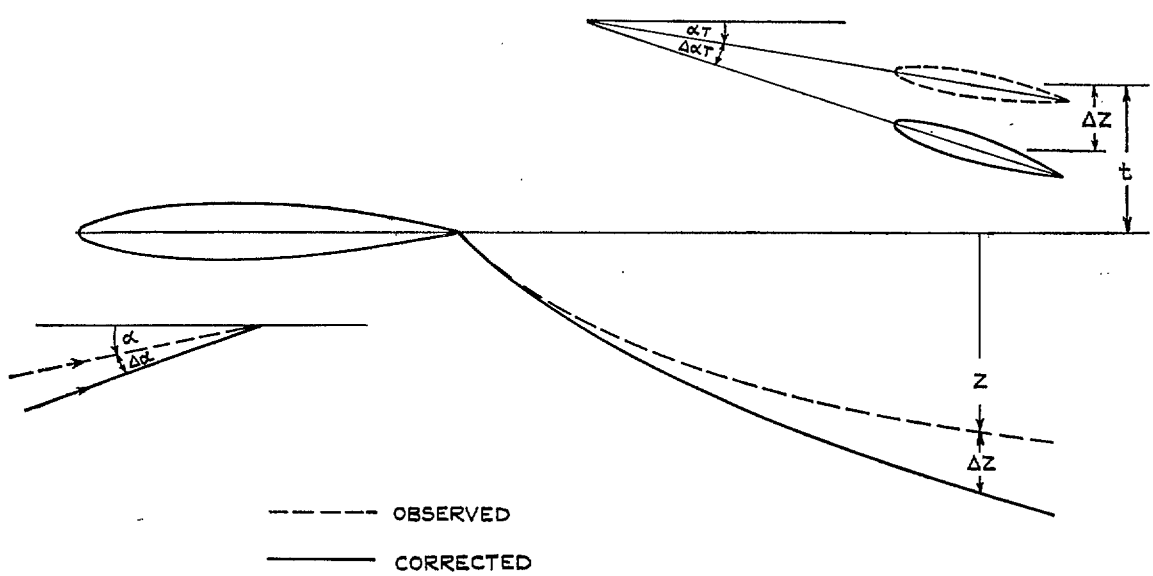

FIG. 2. Application of downwash corrections to tailplane position and setting.

(82478) Wt. 66/991 K.5 8/61 Hw.

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