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CONTENTS

THE DEVELOPMENT OF TURBULENT BOUNDARY LAYERS

By D. A. SPENCE

OF THE CAMBRIDGE UNIVERSITY AERONAUTICS LABORATORY

Published in full in the *Journal of the Aeronautical Sciences*, Vol. 23, No. 1, January, 1956

The structure of several recently measured sets of velocity profiles in quasi-two-dimensional turbulent boundary layers is examined. It is shown that the inner one-fifth of each profile is well represented, independently of pressure gradient, by the universal logarithmic law that was observed by Ludwieg and Tillman. An adequate expression for the velocity in the outer four-fifths of the layer is provided by the well-known power law $u/U = (y/\delta)^n$. The significance of the composite type of profile obtained from the two laws is discussed in the light of Townsend's work on the eddy structure of the boundary layer on a flat plate.

The logarithmic expression for velocity is substituted into the equation of motion at height $y = \theta$ from the surface, at which height it is still valid. (θ = momentum thickness.) With an approximation suggested by Schubauer and Klebanoff's measurements for the term that involves shear stress, this leads to an equation for the variation of the form parameter $\gamma = (u/U)_{y=\theta}$. γ is a simple function of the usual parameter $H = \delta^*/\theta$, and the equation could be put into the form

$$\theta(dH/dx) = [-(\theta/U)(dH/dx)]\phi(H) + c_f 3/2\psi(H)$$

(c_f = skin-friction coefficient). This is rather similar to the form assumed by von Doenhoff and Tetervin in their semi-empirical investigation in 1943, but c_f occupies a different position.

When used with the momentum equation and a skin-friction relation, the form parameter equation enables one to calculate θ, H, c_f, and the detailed profile shape. Predictions have been found to agree well with experiment. A simple method of step-by-step integration which does not require knowledge of velocity derivatives is outlined, and the effect of variations in initial conditions can be examined rapidly. The ultimate separation point appears to depend rather sensitively on the value of H at transition, but, broadly speaking, the equation predicts separation when the external velocity has fallen to between one-half and two-thirds of its value at transition.

UNSTEADY TWO-DIMENSIONAL FLOWS WITH FREE BOUNDARIES

PART I. GENERAL THEORY

By N. CURLE, M.Sc.

OF THE AERODYNAMICS DIVISION, N.P.L.

Published in full in the *Proceedings of the Royal Society*, Series A. Vol. 235, p.375, May, 1956

By expanding in ascending powers of $e^{-\lambda t}$ for large time t, it is possible to find analytical solutions for the immediate history of steady incompressible jet and cavity flows in two dimensions. Special attention is given to the exact boundary conditions at the free surfaces.

UNSTEADY TWO-DIMENSIONAL FLOWS WITH FREE BOUNDARIES
PART II. THE INCOMPRESSIBLE INVISCID JET

By N. CURLE, M.SC.
OF THE AERODYNAMICS DIVISION, N.P.L.

Published in full in the *Proceedings of the Royal Society*, Series A, Vol. 235, p. 382, May, 1956

The paper deals with the flow when a slit is suddenly opened in a large container and a jet of fluid issues into free space. In the early stages of the flow it is suggested that the jet may be M-shaped. The flow after large finite time is calculated by the general theory of Part I.

SOLUTION OF AERO-ELASTIC PROBLEMS BY MEANS OF INFLUENCE COEFFICIENTS

By D. WILLIAMS.

Published in full in the *Journal of the Royal Aeronautical Society*, Vol. 61, No. 556, pp. 247–251, April, 1957

The paper describes a method for solving non-oscillatory aero-elastic problems by matrix manipulation of the elastic and aerodynamic coefficients that define respectively the elastic and aerodynamic forces in terms of wing (or other lifting surface) deformation. The principle of the method is described and illustrated by application to particular aero-elastic problems, but, in view of the number of stations required adequately to represent the wing displacements, a digital computer is indispensable for obtaining numerical answers.

For the purpose of deriving the elastic stiffness equations the wing (or other surface) is represented by a fairly large number of discrete points or stations arranged in a square network. By solving these, *i.e.*, by 'inverting' the corresponding stiffness matrix, the elastic characteristics of the wing are expressed by a flexibility matrix, that gives directly the deflection at any one station in terms of any conceivable distribution of load over the wing.

The aerodynamic data needs to be presented in the form of an aerodynamic stiffnesses matrix, that gives the aerodynamic load, or lift, at any one station in terms of the incidences (or downwash) at the several aerodynamic stations (or pivotal points).

The aerodynamic stations are distributed over the wing to a different pattern from the square network of stations used for the flexibility matrix, but the data presented by the latter can easily be converted (as shown in the paper) to give a flexibility matrix expressed in terms of incidences at the aerodynamic stations. In this way the elastic deflections (or incidences) due to any set of loads and the aerodynamic loads due to any set of incidences are related to the same set of stations. It is on the availability of the two corresponding matrices that the method described in the paper is based.

Methods for deriving the structural flexibility matrix for a wing (or other surface) are described in Refs. 1, 2 and 3, and methods for deriving the aerodynamic loads in terms of a set of incidences are described in Ref. 4 by J. R. Richardson based on the work of Multhopp (Ref. 5) and Garner (Ref. 6).

An Electronic Instrument for the Accurate Measurement of the Frequency of Structural Oscillations

By W. D. T. Hicks

Published in full in the *Journal of the Royal Aeronautical Society*, Vol. 61, p. 126, February, 1957

An instrument is described which measures the time duration of a specified number of complete cycles of an oscillation over the frequency range 0·1 cycle per second to 500 cycles per second. The accuracy of the reading is dependent on the frequency measured and the number of cycles taken; but, in most cases, an accuracy better than 0·1 per cent is obtainable. The construction of the instrument is described and circuit diagrams given.

Hydrodynamic Stability in Unlimited Fields of Viscous Flow

By N. Curle, M.Sc.
of the Aerodynamics Division, N.P.L.

Published in full in the *Proceedings of the Royal Society*, Vol. 238, No. 1215, pp. 489–501, January, 1957

This paper considers the hydrodynamic stability of flows in which there are no solid boundaries in the field of flow. The method used is an extension of that initiated by McKoen (1957), in which the fourth derivative, ϕ'''', is assumed to be significant only near to the singular layer, but otherwise the complete fourth order Orr-Sommerfeld equation is considered.

An alternative derivation is given for McKoen's integral form of the boundary condition for an antisymmetrical perturbation. In this integral it is necessary to approximate for ϕ but not for any of its derivatives.

It is shown that the present method will always lead to a neutral stability curve of wave number against Reynolds number, having two branches as $R\infty$. and hence a least critical R.

The case of the plane laminar jet is considered, and a critical Reynolds number of 4 is obtained, which does not compare unreasonably with experiment in which unsteadiness is first detected at a Reynolds number of about 10. The lower branch of the neutral curve is found to be almost coincident with the R-axis.

Approximate Method for Predicting Separation Properties of Laminar Boundary Layers

By N. Curle, M.Sc., and S. W. Skan
of the Aerodynamics Division, N.P.L.

Published in full in the *Aeronautical Quarterly*, Vol. VIII, *Part* 3, August, 1957

Some new solutions for steady incompressible laminar boundary layer flow, obtained by Görtler, have been used to test the accuracy of two methods which are commonly used to predict separation. A modification of Stratford's criterion for separation is given in this paper, and is probably the most accurate and the simplest of all methods at present in use. Modified numerical functions are also given for Thwaites' method of predicting the main characteristics of the boundary layer over the whole surface, which improve the accuracy of the method.

Hydrodynamic Stability of Laminar Wakes

By N. Curle, M.Sc.
of the Aerodynamics Division, N.P.L.

Published in full in the *Physics of Fluids*, Vol. I, p. 159, March-April, 1958

The theoretical results obtained for the stability of a laminar jet (Curle, Proc. Roy. Soc. A 238, p. 489) are used to obtain a theoretical criterion for the stability of a laminar wake. When the experimental critical Reynolds number for the jet is used to yield an empirical correction, it is found that the critical Reynolds number for the wake is given by

$$R_c = 57 C_D^{-1}$$

where C_D is the drag coefficient of the body producing the wake.

Cavitating Flow about a Wedge at Incidence

By A. D. Cox

Published in full in the *Journal of Fluid Mechanics*, Vol. 3, Part 6, p. 615, March, 1958

A mathematical model is constructed for cavitating flow past a wedge with sides of equal length but with its axis of symmetry placed at an angle to the incident stream. The model involves a subsidiary cavity with a re-entrant jet at the vertex. Only the case of zero cavitation number is considered. The flow field is worked out in some detail for small angles of incidence, and the lift,

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drag and moment coefficients are calculated as far as first-order terms in the angle of incidence. It is shown that the effect of the rate of loss of momentum in the re-entrant jet on these force coefficients is negligible to this order.

Experimentally, it is shown that the secondary cavity does exist under suitable conditions, and the force coefficients obtained agree with the theory.

Propellers in the Wake of an Axisymmetric Body

By R. Hickling

Published in full in the *Transactions of the Institute of Naval Architects*, Vol. 99, pp. 601–617, October, 1957

The paper develops a unified theory of the flow due to an axisymmetric propeller-body system, the propeller being downstream of the body. Viscous effects are allowed for by assuming a turbulent boundary layer inflow to the propeller and by satisfying boundary conditions on the surface of a body which is the original body with the displacement thickness added. The theory applies to lifting-line propellers with a finite number of blades and an arbitrary distribution of circulation and includes a method of determining an optimum propeller design—it being intended that the necessary calculations should be carried out on a digital computor. The paper is followed by a written discussion.

A Theory of Slender Delta Wings with Leading Edge Separation

By K. W. Mangler, and J. H. B. Smith

Published in full in the *Proceedings of the Royal Society*, Series A, Vol. 251, p. 200, May, 1959

The flow at incidence to a slender delta wing with a sharp leading edge usually separates along this edge, i.e., a vortex layer extends from the edge into the main flow. This layer rolls up above and inboard of the leading edge to form a region of high vorticity which strongly influences the flow pattern.

This paper gives a theoretical treatment, more complete than those hitherto available, of this type of flow. A potential flow model is constructed, in which the vortex layer is replaced by a vortex sheet of spiral form, and the problem is then reduced to a two-dimensional one by the use of slender-body theory and the assumption of a conical velocity field. The boundary conditions expressing that the vortex sheet is a stream surface and sustains no pressure difference determine, in principle, its shape and strength. In practice, the inner part of the spiral and the finite, outer part of the spiral, which joins the inner part to the leading edge, are treated separately. The inner part is regarded as small and a solution is given for it which is asymptotically correct as the centre of the spiral is

approached. The outer part is replaced by a sheet whose shape and strength depend on a finite number of parameters; these parameters are determined by applying the boundary conditions at isolated points.

Results are given for the shape of the sheet and the pressures, loadings and forces on the wing, as functions of the ratio of the incidence to the aspect ratio.

THE MINIMUM REYNOLDS NUMBER FOR A TURBULENT BOUNDARY LAYER AND THE SELECTION OF A TRANSITION DEVICE

By J. H. PRESTON, M.A., PH.D., A.F.R.AE.S.

OF THE DEPARTMENT OF FLUID MECHANICS, UNIVERSITY OF LIVERPOOL

Published in full in the *Journal of Fluid Mechanics*, Vol. 3, Part 4, p. 373, January, 1958

In the case of turbulent flow in a pipe there is a lower experimental limit to the Reynolds number for which fully developed turbulent flow occurs. From the similarity and close agreement of the curves showing the coefficient of skin friction c_f as a function of the Reynolds number R_θ (based on the momentum thickness θ) for the circular pipe and flat plate, it is suggested that there should be a lower limit to R_θ for fully developed turbulent flow on a flat plate. Rather limited experimental data confirm this and place the lower limit at $R_\theta = 320$. The choice and size of transition device is examined in relation to this minimum R_θ and an approximate theory leads to a 'wire' Reynolds number in fair agreement with experience.

PREDICTION OF TRANSITION IN THE BOUNDARY LAYER OF AN AEROFOIL

By L. F. CRABTREE

Published in full in the *Journal of the Royal Aeronautical Society*, Vol. 62, No. 571, pp. 525–528, July, 1958

Transition to turbulence in the boundary layer of an aerofoil may occur in several different ways, not the least important of which is by bubble separation of the laminar layer. Thus in establishing an empirical criterion for the limit of existence of a laminar boundary layer the necessary parameters must be able to describe separation and instability, as well as being physically reasonable and simple to calculate and use.

It is shown that only two parameters are necessary to fulfil these conditions, and that a universal curve may be constructed of boundary layer Reynolds number $R_{\delta 2}$ against pressure gradient parameter m at transition.

This curve may be used to predict transition, and an assessment of its accuracy in comparison with previous methods is given.

Theoretically the method can be extended to the limited case of bodies of revolution at zero incidence.

A NOTE ON THE WAVE DRAG OF 'EXPOSED' RECTANGULAR WINGS

By L. M. SHEPPARD

Published in full in the *Journal of the Royal Aeronautical Society*, Vol. 62, pp. 306–307, April, 1958

This note investigates the relationship between the zero-lift wave drag of the exposed wing of a rectangular wing and body combination and the wave drag of the portion of the gross wing blanketed by the body, both evaluated as if isolated (*i.e.*, ignoring the interference of other parts of the combination). It is shown that the isolated exposed wing wave drag is equal to the isolated blanketed wing wave drag added to the difference between the exposed and blanketed wing wave drags, as calculated by a two-dimensional strip method. This result is valid only when the tip Mach cones of the wing lie downstream of the blanketed wing.

THE TWO-DIMENSIONAL LAMINAR FLOW NEAR THE STAGNATION POINT OF A CYLINDER WHICH HAS AN ARBITRARY TRANSVERSE MOTION

By J. WATSON

OF THE AERODYNAMICS DIVISION, N.P.L.

Published in full in the *Quarterly Journal of Mechanics and Applied Mathematics*, 12, p. 175, May, 1959

Exact solutions of the Navier-Stokes equations are derived for two-dimensional flow against an infinite flat plate normal to the stream and performing an arbitrary transverse motion. This generalizes Glauert's solution for an oscillating transverse motion.

After a brief approximate investigation of the reaction of an arbitrary boundary layer to arbitrary changes in the free-stream velocity, the problem described above is considered. When the wall moves from rest, then, by means of a Laplace-transform technique, expansions of the velocity distribution for small and large times are given in terms of the velocity of the wall. Further, it is indicated how a Pohlhausen type of method may always be used to obtain an approximate solution for the purpose of linking up these expansions across the range of times for which neither is valid. The method may also be used for motions other than those starting from rest. In uniform motion started impulsively from rest, the expansions overlap and the approximate solution agrees well with them.

Definitions and Terms Relating to Aircraft Control Systems and Components

By H. R. Hopkins, and H. H. B. M. Thomas

Published in full in the Journal of the *Royal Aeronautical Society*, Vol. 63, No. 586, pp. 572–576, October, 1959

Recent developments in the control systems of aeroplanes have been such as to underline the need for new terms, and the need to remove any ambiguity relating to terms already in existence. The question has also been raised by the British Standards Institution in relation to the revision of the Glossary of Aeronautical Terms. The present paper examines the problem in the light of possible future developments, lists proposed terms with their definitions, and gives an outline of the underlying arguments.

It is recommended that when terms are first sought for new developments these should be such as to embrace the most general concepts involved in these developments, and it is noted that in complex control systems it is difficult to classify in a unique manner the elements of which it is composed.

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