MINISTRY OF AVIATION

AERONAUTICAL RESEARCH COUNCIL
REPORTS AND MEMORANDA

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COMMUNICATED BY THE DEPUTY CONTROLLER AIRCRAFT (RESEARCH AND DEVELOPMENT)
MINISTRY OF SUPPLY

Reports and Memoranda No. 3198*

April, 1959

Summary.—An analysis is made of the general buckling of a pressurised stiffened cylinder under axial load. Torsional buckling of the complete cylinder, local buckling of the stringers and pure compressive failure of the stringers are also considered, and the design of stiffened pressurised cylinders is discussed, with an example.

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1. *Introduction.*—This report is concerned with the determination of the critical axial load for buckling of a long pressurised cylinder stiffened by stringers only. The possibility that the cylinder may be so long that buckling as a column might occur is not considered. Timoshenko[1] gives an analysis of the buckling of an unpressurised stiffened cylinder and shows that if the stiffeners lie in one direction only (whether frames or stringers) the critical axial load is less than it would be if the stiffening material were added to the thickness of the skin. When the cylinder is pressurised, however, this result is no longer true and this paper shows that the critical axial load increases with the hoop tension due to internal pressure. In the analysis the axial load is considered in two parts, the load carried by the action of the internal pressure on the ends of the cylinder and the net load carried by the cylinder itself. The critical value of the net axial load is found to reach a theoretical maximum at some value of the hoop tension, although in practice the cylinder would usually fail in hoop tension before this maximum could be reached.

An additional complication is introduced by the local buckling of the skin between the stringers. If the skin is sufficiently thin the panels between stringers buckle at approximately the same stress as a complete unstiffened cylinder of the same skin-thickness and radius. It follows that if the stiffened cylinder is to be more stable than an unstiffened cylinder of equal weight, the panels between stringers will be in the post-buckled state. An analysis of the post-buckling behaviour of curved panels under the action of axial load and lateral pressure would be extremely difficult,[2] and has not been attempted here. However, since the critical axial load for general buckling of the stiffened cylinder is usually much greater than that for local buckling of the skin, it is reasonably accurate to assume that the skin carries a negligible direct axial load. The effective shear stiffness of the buckled skin is more difficult to estimate and remains arbitrary in the analysis.

From the design point of view there are three main factors to be considered:

(i) Pure compressive failure of the stringers

(ii) General buckling

(iii) Local buckling of the stringers.

Although local buckling of the stringers does not in itself cause immediate failure, it does lead to local crippling of the stringers and also to a decreased load for general buckling. It has not, however, been considered worthwhile to consider these effects in detail because it is possible to ensure that local buckling of the stringers does not occur for a load smaller than that for general buckling.

2. *The Analysis of General Buckling.*—2.1 *Assumptions.*—The following assumptions are made in the analysis:

(i) The displacements of the middle surface of the skin are small compared with the height of the centroid of a stringer above the middle surface.

(ii) The thickness of the skin is small compared with the height of the centre of gravity of a stringer above the middle surface (which is itself small compared to the radius of the cylinder).

(iii) Elements normal to the unstrained middle surface of the skin remain normal to the strained middle surface.

(iv) There is no direct stress normal to the middle surface of the skin.

(v) Because of local buckling the skin carries a negligible load, and its shear modulus is reduced.

(vi) Local buckling of the stringers does not occur.

(vii) The effect of constraints on the ends of the cylinder is ignored.

(viii) The wavelength of the buckles in the axial direction is small compared with the circumference of the shell.

(ix) The wavelength of the buckles in the circumferential direction is large compared with the distance between stringers.

2.2. *Method of Analysis.*—Expressions are derived for the average forces and couples acting on the stringer-skin combination shown in Fig. 1. These forces are taken to act in the middle surface of the skin, and the couples adjusted accordingly. The expressions so obtained are then substituted into the equations of equilibrium for a cylinder under the action of internal pressure and axial load.[1] In this way, three independent equations for the displacements u, v and w are derived.

Suitable expressions for the displacements, each containing an arbitrary multiplying constant, are next substituted into these equations and the determinant of the coefficients of the multiplying constants equated to zero, to give the condition for general buckling.

The buckling load is minimised keeping the internal pressure constant, first with respect to the axial wavelength, keeping the buckle shape constant, and then with respect to the buckle shape. In this way, the following implicit relationships between the parameters Ψ_{cr} and Φ are obtained:

$$\Phi = \frac{1}{\left(1 + \frac{2\mu(1+\nu)}{g} + \frac{\mu^2}{S}\right)^2}\left[1 - \frac{\mu^2}{S} + \frac{\chi(1+\nu)}{g} + \frac{\mu\left\{K - 2 + \frac{(2K-3)(1+\nu)\mu}{g} + \frac{K\mu^2}{S}\right\}}{\chi}\right], \qquad (1)$$

$$\Psi_{cr} = \mu\Phi + \frac{\chi - \mu}{1 + \frac{2\mu(1+\nu)}{g} + \frac{\mu^2}{S}}. \qquad (2)$$

The analysis is given in full in Appendix I.

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2.3. *The Range of the Parameters.*—Ψ_{cr} reaches a theoretical maximum Ψ_M at a value Φ_M of Φ, where

$$\Psi_M = \sqrt{\{S(K-1)\}}, \tag{3}$$

$$\Phi_M = 1 + \frac{1+\nu}{g}\sqrt{\{S(K-1)\}}. \tag{4}$$

It is convenient, when presenting the results graphically, to introduce the parameters

$$\hat{\psi} = \frac{\Psi_{cr}}{\Psi_M}, \tag{5}$$

$$\hat{\phi} = \frac{\Phi}{\Phi_M}, \tag{6}$$

both of which vary between 0 and 1.

For most practical stringers, the parameters S and K lie in the ranges

$$\tfrac{1}{4} \leqslant S \leqslant 4, \tag{7}$$

$$1\cdot5 \leqslant K \leqslant 2. \tag{8}$$

Poisson's ratio is taken to lie in the range

$$\tfrac{1}{4} \leqslant \nu \leqslant \tfrac{1}{3}. \tag{9}$$

2.4. *Results.*—Using equations (1), (2), (3) and (4), Figs. 2, 3 and 4 are plotted showing the variation of $\hat{\psi}$ with $\hat{\phi}$ for a number of sets of values of the parameters S, K and ν. Fig. 2 shows the effect of varying ν from $\tfrac{1}{4}$ to $\tfrac{1}{3}$, Fig. 3 the effect of varying K from 1·5 to 2, and Fig. 4 the effect of varying S from $\tfrac{1}{4}$ to 4. From these Figures it can be seen that the only important parameter governing the relationship of $\hat{\psi}$ with $\hat{\phi}$ is S. In any particular case, therefore, the critical axial load can be determined using equations (3) and (4) and Fig. 4. Although ν and K have negligible effects on the relationship of $\hat{\psi}$ with $\hat{\phi}$, they have a somewhat greater effect on the buckling load through their influence on the values of Ψ_M and Φ_M.

In all these calculations it was assumed that $g = 1$. Some estimate of the errors introduced by this assumption may be obtained from Fig. 8 which shows for a particular example, the way in which the buckling load is affected by a decrease in the value of g. These errors will, in any event, be partly compensated for by the errors due to the assumption that the skin carries no direct load.

2.5. *The Total Load.*—The total axial load carried by the cylinder is given by

$$L = 2\pi a(P + \tfrac{1}{2}qa).$$

Now

$$P_{cr} = 2\Psi_M \hat{\psi} EhH$$

and

$$\frac{qa}{2} = \Phi EhH,$$

therefore

$$L_{cr} = 2\pi ahEH(2\Psi_M \hat{\psi} + \Phi). \tag{10}$$

3. *Other Possible Modes of Failure.*—In this Section some possible modes of failure which are not allowed for in the analysis of general buckling are briefly considered. Local buckling of the stringers is included since, although it does not itself lead to immediate failure, it has a considerable influence on the critical load for general buckling.

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3.1. Local Buckling of the Stringers.

The stress in the stringers is given by

$$\sigma'_x = \frac{E'P}{ESH}.$$
(11)

If b is the width of the side of the stringer most likely to buckle locally, the critical stress for local buckling of a top-hat stringer is given by

$$\sigma'_{cr} = \frac{4\pi^2 E'}{12(1-\nu'^2)}\left(\frac{h'}{b}\right)^2$$
(12)

if both edges are assumed to be effectively simply supported, giving for the total load

$$L_{cr} = 20 \cdot 67 \frac{ahES}{1-\nu'^2}\left(\frac{h'}{b}\right)^2 + \pi q a^2.$$
(13)

Top-hat stringers would appear to be the most useful in the present case because other types, such as Z-section stringers, become liable to torsional buckling after the skin has buckled locally. A more detailed discussion of stringer buckling is given in Ref. 3.

3.2. Pure Compressive Failure of the Stringers.

This occurs when the stress in the stringers reaches the ultimate stress σ'_{ult}. The corresponding total load is given by

$$L_{cr} = 2\pi a h \frac{ES}{E'}\sigma'_{ult} + \pi q a^2.$$
(14)

3.3. Failure due to Torsional Buckling of the Complete Cylinder.

If, after local buckling of the skin, the effective shear modulus drops below a certain critical value, the cylinder will fail in torsion. If the value of the effective shear modulus is taken to be $\bar{G}$, the critical load on the cylinder for torsional buckling is:

$$L_{cr} = 2\pi a h \bar{G} + \pi q a^2.$$
(15)

For this load to be equal to that for pure compressive failure of the stringers,

$$\bar{G} = \frac{ES}{E'}\sigma'_{ult}.$$
(16)

Therefore

$$\frac{\bar{G}}{G} = 2(1+\nu) S \frac{\sigma'_{ult}}{E'}.$$
(17)

For a typical metal, this equation becomes:

$$\frac{\bar{G}}{G} \simeq 0 \cdot 01 S.$$
(18)

Therefore the shear modulus would have to drop to about 5 per cent of its original value before there was any danger of this type of failure.

4. Design Considerations.

In the design of a pressurised stiffened cylinder to withstand axial load, there are three main factors to be considered; general buckling of the cylinder, local buckling of the stringers and pure compressive failure of the stringers. Provided the skin is thin compared with the distance between stringers, the skin buckles locally at approximately the same stress as a complete unstiffened cylinder. In practice, therefore, local buckling of the skin will always occur before general buckling.

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The shape and height to thickness ratio of the stringers should be made such as to give the maximum moment of inertia about the middle surface of the skin, while ensuring that local buckling of the stringers does not occur at a smaller load than general buckling.

The stringers should be sufficiently numerous to satisfy assumption (9) of Section 2, and the material should be distributed between skin and stringers in such a way as to give as great a load for general buckling as possible. For a given axial load and hoop stress, a design for minimum weight, assuming failure occurs due to general buckling, would have the material about equally distributed between the skin and the stringers (the precise optimum depending on the shape and height to thickness ratio of the stringers). The possibility of pure compressive failure of the stringers, however, may necessitate having more material in the stringers than in the skin.

4.1. *An Example.*—It is required to design a cylinder of radius 60 in. to withstand an axial load of 3,000 lb per inch of circumference. The material to be used is a steel for which

$$E = 27 \cdot 5 \times 10^6 \text{ lb/in.}^2,$$

$$\nu = \tfrac{1}{4},$$

$$\sigma_{\text{ult}} = 150,000 \text{ lb/in.}^2.$$

The first step is to choose a value for the hoop stress. This value should be as large as possible, bearing in mind that the hoop stress in part of the cylinder may be greater than this specified value owing to the hydrostatic pressure of fuel carried in the cylinder. In this example the hoop stress was chosen to be 50,000 lb/in.^{2}.

For the reasons given in Section 3.1, top-hat stringers are used, and the particular cross-section chosen was that shown in Fig. 1c. This gives the following expressions for the 'height' and 'shape' parameters:

$$\left. \begin{aligned} H &= \frac{b+h+h'}{2a}, \\ K &= 1 + \frac{2}{3}\left[1 - \frac{2(h+h')}{b}\right] \end{aligned} \right\}. \tag{19}$$

The next step is to decide upon the number of stringers. This is governed by the circumferential wavelength of the buckled form, which cannot be estimated accurately at this stage. In this example it was decided to use 50 stringers.

If local buckling of the stringers is specified to take place at 3,000 lb/in. total load, equation (13) and the fact that $A = 4bh'$, can be used to obtain expressions for the height of the stringers and thickness of stringer material in inches:

$$\left. \begin{aligned} h'^2 &= 1 \cdot 0513 \times 10^{-2} \sqrt{\{hS(1 - 8 \cdot 333h)\}} \\ b &= 1 \cdot 885 \frac{hS}{h'} \end{aligned} \right\}. \tag{20}$$

Using these equations and taking $S = 1$, values of the critical axial load were computed for a series of values of the total cross-sectional area of skin and stringers. The results are plotted in Fig. 6, from which it can be seen that the minimum cross-sectional area to withstand 3,000 lb/in. axial load is 0·036 sq in./in. circumference. The critical axial load was then computed for this cross-sectional area and a series of values of S. These results are shown in Fig. 7, together with the

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values of the stringer stress at buckling. It can be seen that although from the point of view of general buckling $S = 1$ is very near to the optimum design, when the possibility of pure compressive failure in the stringers is considered, it is better to take $S = 1.5$.

Fig. 5 shows the values of μ plotted against $\hat{\phi}$ for various values of S. Using this graph and equation (30) in Appendix I, it is found that when $S = 1.5$,

$$\mu = 1.68,$$

$$\lambda^2 = 37.2,$$

$$n = 7.9.$$

Therefore the ratio of the wavelength of buckles in the circumferential direction to the distance between stringers is 6.3. This shows that the choice of 50 stringers was in reasonable agreement with assumption (ix) of Section 2.

Therefore the cylinder of optimum design to withstand 3,000 lb/in. axial load when subjected to 50,000 lb/in.² hoop stress has the following specification:

$$h = 0.0144 \text{ in.},$$

$$N = 50,$$

$$h' = 0.0381 \text{ in.},$$

$$b = 1.069 \text{ in.},$$

$$q = 12 \text{ lb/in.}^2.$$

5. *Conclusions.*—An analysis has been made of the general buckling of a circular cylinder under the action of internal pressure and axial load. The design of stiffened cylinders is discussed with the conclusion that from the point of view of stability it is best to have the material about equally divided between skin and stringers. An example of an optimum design to withstand a given axial load is presented.

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LIST OF SYMBOLS

l		Length of cylinder
a		Radius of cylinder
h		Thickness of skin
h'		Thickness of stringer material
N		Number of stringers
A		Cross-sectional area of a stringer
I		Moment of inertia of a stringer about the middle surface of the skin
b		Width of side of stringer most likely to buckle locally
E		Young's modulus for skin material
E'		Young's modulus for stringer material
ν		Poisson's ratio for skin material
ν'		Poisson's ratio for stringer material
S	$= \dfrac{E'NA}{E2\pi ah} =$	'weight' parameter of stringers
H	$=$	(height of centroid of stringer above middle surface of skin)$/a$
	$=$	'height' parameter of stringers
K	$= \dfrac{I}{a^2 H^2 A} =$	'shape' parameter of stringers
x, θ, z		Cylindrical coordinates shown in Fig. 1a
y	$= a\theta$	
$\sigma_x, \sigma_y, \tau_{xy}$		Stress components in the skin averaged over a local buckle
σ'_x		Stress in stringers
ϵ_y, γ		Strain components in the skin averaged over a local buckle
ϵ'_x		Strain in stringers
N_x, N_y, N_{xy}		Average forces in middle surface of the skin as shown in Fig. 1b
M_x, M_y, M_{xy}		Average couples
e_1, e_2, γ_0		Strains of middle surface of skin averaged over a local buckle
κ		Curvature of middle surface of skin in axial direction
u, v, w		Components in x, θ and z directions respectively of the displacement of the middle surface of the skin from uniformly expanded state, averaged over a local buckle
ξ, η, ζ		Arbitrary constants in expressions for displacements
m		Number of half-waves in the axial direction
n		Number of waves in the circumferential direction
λ	$= \dfrac{m\pi a}{l}$	
μ	$= \dfrac{n^2}{\lambda^2}$	
P		Net axial load on the cylinder per unit length of circumference
P_{cr}		Critical value of P
L		Total axial load on cylinder
	$= 2\pi a(P + \frac{1}{2}qa)$	
q		Internal pressure

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L_{cr} Critical value of L

$$\psi = \frac{P}{Eh}$$

$$\phi = \frac{qa}{EH}$$

$$\Psi = \frac{P}{2EhH}$$

$$\Phi = \frac{qa}{2EhH}$$

$$\Psi_{cr} = \frac{P_{cr}}{2EhH}$$

Ψ_M Maximum value of Ψ_{cr}

Φ_M Value of Φ corresponding to Ψ_M

$$\hat{\psi} = \frac{\Psi_{cr}}{\Psi_M}$$

$$\hat{\phi} = \frac{\Phi}{\Phi_M}$$

σ_{ult} Ultimate stress of stringer material

σ'_{cr} Critical stress in stringers (for local buckling)

G Shear modulus of skin material

$\bar{G}$ Decreased value of G after local buckling of the skin in compression has taken place

$$g = \frac{\bar{G}}{G}$$

$$\chi = \sqrt{\left[S(K-1)\left\{ 1 + \frac{2\mu(1+\nu)}{g} \right\} + K\mu^2 \right]}$$

LIST OF REFERENCES

No.	Author	Title, etc.
1	S. Timoshenko	*Theory of Elastic Stability.* McGraw-Hill. 1936.
2	H. L. Cox and E. Pribram	The elements of the buckling of curved plates. *J. R. Ae. Soc.* Vol. 52. p. 551. 1948.
3	—	Royal Aeronautical Society Structures Data Sheets Series 02.01.

APPENDIX I

The Analysis of the Critical Axial Load

The first step is to formulate stress-strain relations for the skin which are consistent with the presence of local buckles. This is done by using components of stress and strain averaged over each local buckle. These average stresses and strains can be used with reasonable accuracy provided that the local buckles are small compared with the general buckles (a consequence of assumption (ix) of Section 2).

Thus, using assumption (v) of Section 2, the stress-strain relations for the skin are derived as

$$\left.\begin{aligned} \sigma_x &= 0, \\ \sigma_y &= E\epsilon_y, \\ \tau_{xy} &= \frac{Eg}{2(1+\nu)}\gamma, \end{aligned}\right\} \tag{21}$$

while for the stringers $\qquad\qquad \sigma'_x = E'\epsilon'_x.$

Thus the average forces and couples acting on the stringer-skin combination, taking the forces to act in the middle surface of the skin, are given by:

$$\left.\begin{aligned} N_x &= Eh(Se_1 - aHS\kappa), \\ N_y &= Ehe_2, \\ N_{xy} &= Eh\frac{g}{2(1+\nu)}\gamma_0, \\ M_x &= Eh(SHae_1 - a^2H^2SK\kappa), \end{aligned}\right\} \tag{22}$$

and $\qquad\qquad\qquad M_y = 0 = M_{xy},$

h^2 being neglected compared with a^2H^2.

The equations of equilibrium for a cylindrical shell under axial load and lateral pressure are given by Timoshenko[1] as

$$\left.\begin{aligned} a\frac{\partial N_x}{\partial x} + \frac{\partial N_{yx}}{\partial \theta} - qa\left(\frac{\partial^2 v}{\partial x\,\partial\theta} - \frac{\partial w}{\partial x}\right) &= 0, \\ \frac{\partial N_y}{\partial\theta} + a\frac{\partial N_{yx}}{\partial x} - aP\frac{\partial^2 v}{\partial x^2} + \frac{\partial M_{xy}}{\partial x} - \frac{1}{a}\frac{\partial M_y}{\partial\theta} &= 0, \\ a\frac{\partial^2 M_x}{\partial x^2} - 2\frac{\partial^2 M_{xy}}{\partial x\,\partial\theta} + \frac{1}{a}\frac{\partial^2 M_y}{\partial\theta^2} + N_y - aP\frac{\partial^2 w}{\partial x^2} + q\left(w + \frac{\partial^2 w}{\partial\theta^2}\right) &= 0, \end{aligned}\right\} \tag{23}$$

where displacements from the uniformly expanded state are considered.

Writing

$$e_1 = \frac{\partial u}{\partial x}, \quad e_2 = \frac{1}{a}\left(\frac{\partial v}{\partial \theta} - w\right), \quad \gamma_0 = \frac{1}{a}\left(\frac{\partial u}{\partial \theta} + a\frac{\partial v}{\partial x}\right), \tag{24}$$

$$\kappa = \frac{\partial^2 w}{\partial x^2},$$

and defining

$$\phi = \frac{qa}{Eh},$$

$$\psi = \frac{P}{Eh},$$

substitution from equations (20) to equations (21) gives:

$$aS\frac{\partial^2 u}{\partial x^2} + \frac{g}{2a(1+\nu)}\frac{\partial^2 u}{\partial \theta^2} + \frac{\partial^2 v}{\partial x\partial \theta}\left[\frac{g}{2(1+\nu)} - \phi\right] - a^2 HS\frac{\partial^3 w}{\partial x^3} + \phi\frac{\partial w}{\partial x} = 0,$$

$$\frac{g}{2(1+\nu)}\frac{\partial^2 u}{\partial x\partial \theta} + a\left[\frac{g}{2(1+\nu)} - \psi\right]\frac{\partial^2 v}{\partial x^2} + \frac{1}{a}\frac{\partial^2 v}{\partial \theta^2} - \frac{1}{a}\frac{\partial w}{\partial \theta} = 0, \tag{25}$$

and

$$HSa^2\frac{\partial^3 u}{\partial x^3} + \frac{1}{a}\frac{\partial v}{\partial \theta} - a^3 H^2 SK\frac{\partial^4 w}{\partial x^4} - \psi\frac{\partial^2 w}{\partial x^2} + \frac{\phi}{a}\left(w + \frac{\partial^2 w}{\partial \theta^2}\right) - \frac{w}{a} = 0.$$

Substituting for u, v, and w the expressions

$$\left.\begin{array}{l} u = \xi \sin n\theta \, \cos \lambda x/a \\ v = \eta \cos n\theta \, \sin \lambda x/a \\ w = \zeta \sin n\theta \, \sin \lambda x/a \end{array}\right\}, \tag{26}$$

and equating the determinant of the coefficients of ξ, η and ζ to zero, the buckling condition

$$\begin{vmatrix} S\lambda^2 + \dfrac{g}{2(1+\nu)}n^2, & \left[\dfrac{g}{2(1+\nu)} - \phi\right]\lambda n, & -\lambda(\phi + HS\lambda^2) \\[3mm] \dfrac{g}{2(1+\nu)}\lambda n, & \left[\dfrac{g}{2(1+\nu)} - \psi\right]\lambda^2 + n^2, & n \\[3mm] -HS\lambda^3, & n, & \begin{array}{c} H^2 SK\lambda^4 + 1 - \psi\lambda^2 \\ + \phi(n^2 - 1) \end{array} \end{vmatrix} = 0 \tag{27}$$

is obtained.

Assuming

$$\lambda^2 = O\!\left(\frac{1}{H}\right)$$

and neglecting small terms, the determinant can be reduced to the equation

$$\psi = \phi\mu + \frac{H^2\lambda^2\left[S(K-1)\left\{1 + \dfrac{2\mu(1+\nu)}{g}\right\} + K\mu^2\right] - 2H\mu + \dfrac{1}{\lambda^2}}{1 + \dfrac{\mu(1+\nu)}{g} + \dfrac{\mu^2}{S}}, \tag{28}$$

where

$$\mu = \frac{n^2}{\lambda^2}.$$

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Keeping ϕ (the internal pressure parameter) constant, ψ (the axial-load parameter) can be minimised with respect to λ^2, regarding μ as a constant. This gives

$$\Psi = \Phi\mu + \frac{\chi - \mu}{1 + \dfrac{2\mu(1+\nu)}{g} + \dfrac{\mu^2}{S}}, \tag{29}$$

where

$$\chi = \sqrt{\left[S(K-1)\left\{1 + \frac{2\mu(1+\nu)}{g}\right\} + K\mu^2 \right]}$$

when

$$\lambda^2 = \frac{1}{H\chi}. \tag{30}$$

Now minimising Ψ with respect to μ, keeping Φ constant gives

$$\Phi = \frac{1}{\left\{1 + \dfrac{2\mu(1+\nu)}{g} + \dfrac{\mu^2}{S}\right\}^2} \left[1 - \frac{\mu^2}{S} + \frac{\chi(1+\nu)}{g} + \frac{\mu}{\chi}\left\{K - 2 + \frac{(2K-3)(1+\nu)\mu}{g} + \frac{K\mu^2}{S}\right\} \right] \tag{31}$$

$$\Psi_{cr} = \Phi\mu + \frac{\chi - \mu}{1 + \dfrac{2\mu(1+\nu)}{g} + \dfrac{\mu^2}{S}}.$$

When μ is zero (the case of waves in the axial direction only), the value of Ψ_{cr} becomes a maximum

$$\Psi_M = \sqrt{\{S(K-1)\}} \tag{32}$$

and the corresponding value of Φ is given by

$$\Phi_M = 1 + \frac{1+\nu}{g}\sqrt{\{S(K-1)\}}. \tag{33}$$

For the convenience of graphical presentation the parameters

$$\left. \begin{array}{l} \hat{\phi} = \dfrac{\Phi}{\Phi_M} \\[3mm] \hat{\Psi} = \dfrac{\Psi}{\Psi_M} \end{array} \right\} \tag{34}$$
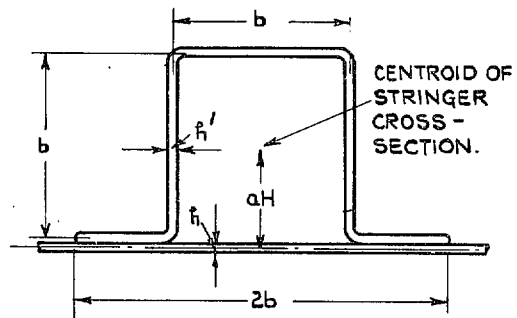
are introduced, both of which always lie between 0 and 1.
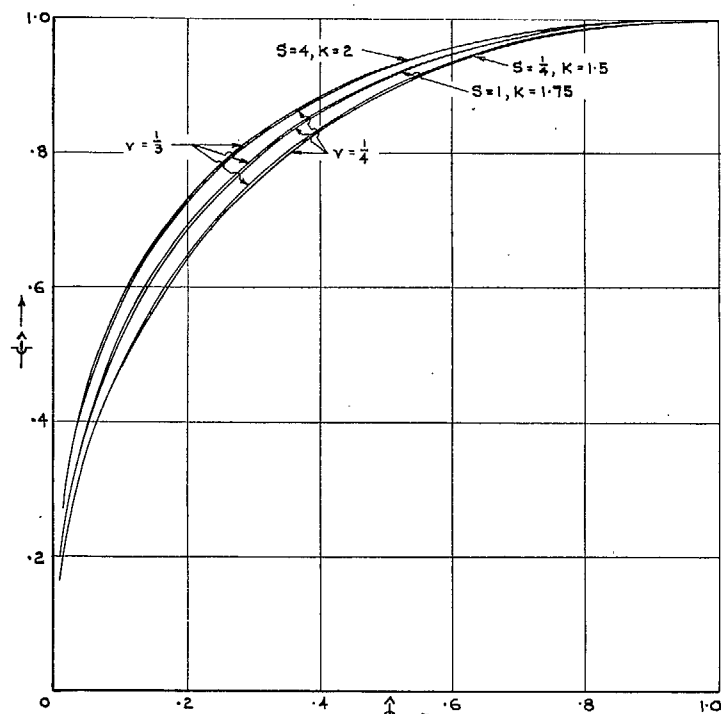
11
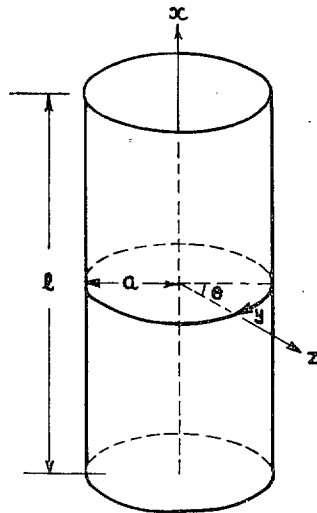
(a) THE COORDINATE SYSTEM
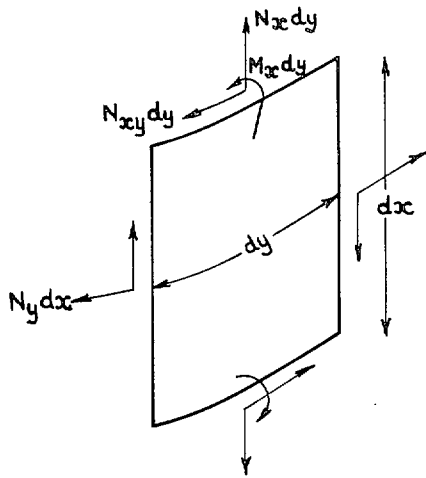
(b) FORCES AND COUPLES
ACTING ON ELEMENT.

(c) CROSS-SECTION OF
TYPICAL STRINGER.

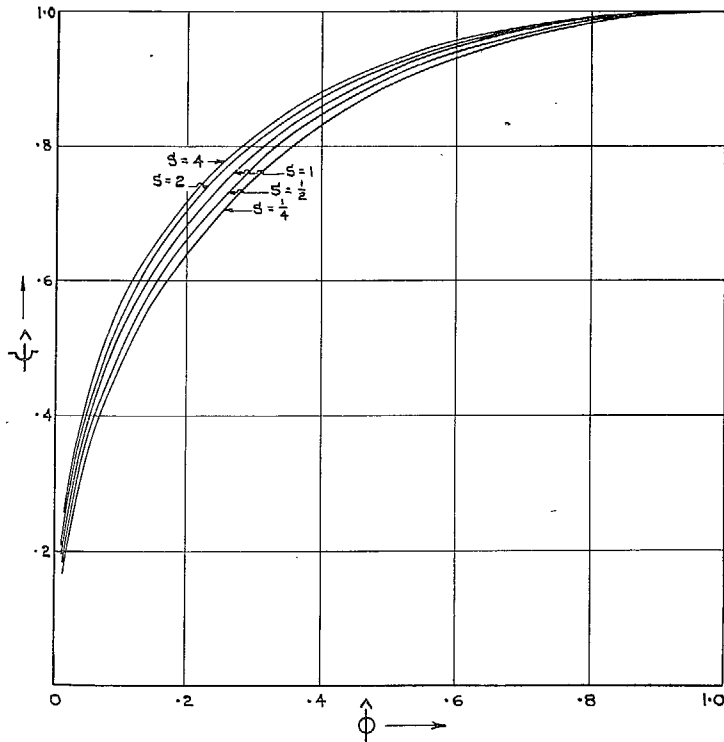FIGS. 1a to 1c.   Diagrams showing co-ordinate system and various parameters.

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$\hat{\psi}$ = PARAMETER FOR CRITICAL NET AXIAL LOAD

$\hat{\phi}$ = PARAMETER FOR INTERNAL PRESSURE
(BOTH SCALED TO LIE BETWEEN O AND I)

S = RATIO OF STRINGER MATERIAL TO SKIN MATERIAL.

K = PARAMETER FOR SHAPE OF STRINGER CROSS-SECTION.

ν = POISSON'S RATIO.

g = FACTOR BY WHICH THE SHEAR MODULUS OF THE SKIN
IS DECREASED AFTER LOCAL BUCKLING.

Fig. 2.   Graphs of $\hat{\psi}$ against $\hat{\phi}$ showing the effect of variation
of ν for different values of S and K ($g = 1$).
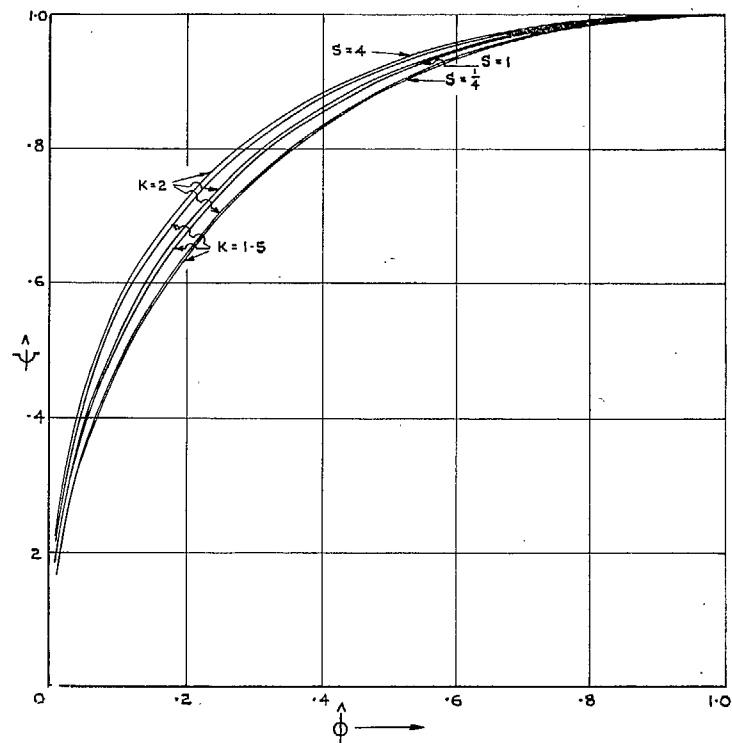
$\hat{\psi}$ = PARAMETER FOR CRITICAL NET AXIAL LOAD

$\hat{\phi}$ = PARAMETER FOR INTERNAL PRESSURE
(BOTH SCALED TO LIE BETWEEN O AND I)

S = RATIO OF STRINGER MATERIAL TO SKIN MATERIAL.

K = PARAMETER FOR SHAPE OF STRINGER CROSS-SECTION

ν = POISSON'S RATIO.

g = FACTOR BY WHICH THE SHEAR MODULUS OF THE SKIN
IS DECREASED AFTER LOCAL BUCKLING.

Fig. 3.   Graphs of $\hat{\psi}$ against $\hat{\phi}$ showing the effect of variation
of K for different values of S ($\nu = \frac{1}{4}$, $g = 1$).

$\hat{\psi}$ = PARAMETER FOR CRITICAL NET AXIAL LOAD.

$\hat{\phi}$ = PARAMETER FOR INTERNAL PRESSURE
(BOTH SCALED TO LIE BETWEEN 0 AND 1)

S = RATIO OF STRINGER MATERIAL TO SKIN MATERIAL.

K = PARAMETER FOR SHAPE OF STRINGER CROSS-SECTION.

ν = POISSON'S RATIO

g = FACTOR BY WHICH THE SHEAR MODULUS OF THE SKIN IS DECREASED AFTER LOCAL BUCKLING.

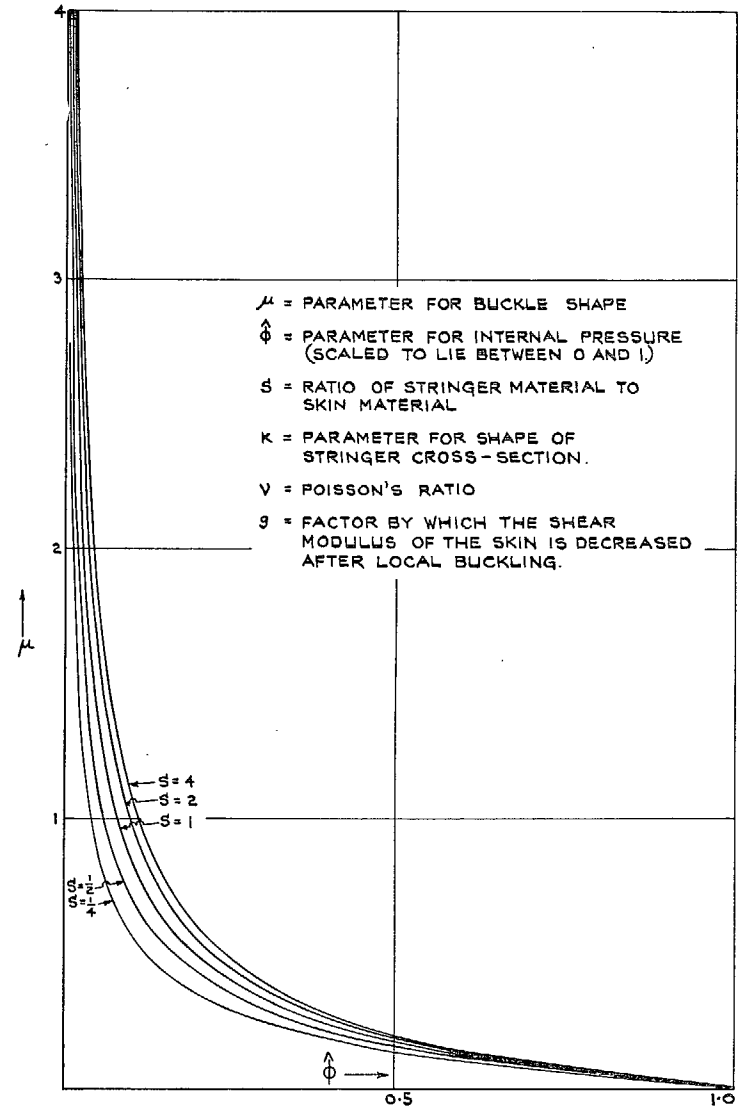FIG. 4. Graphs of $\hat{\psi}$ against $\hat{\phi}$ for different values of S ($K = 1.75$, $\nu = \frac{1}{4}$, $g = 1$).

μ = PARAMETER FOR BUCKLE SHAPE

$\hat{\phi}$ = PARAMETER FOR INTERNAL PRESSURE
(SCALED TO LIE BETWEEN 0 AND 1.)

S = RATIO OF STRINGER MATERIAL TO SKIN MATERIAL

K = PARAMETER FOR SHAPE OF STRINGER CROSS-SECTION.

ν = POISSON'S RATIO

g = FACTOR BY WHICH THE SHEAR MODULUS OF THE SKIN IS DECREASED AFTER LOCAL BUCKLING.

FIG. 5. Graphs of μ against $\hat{\phi}$ for different values of S ($K = 1.75$, $\nu = \frac{1}{4}$, $g = 1$).
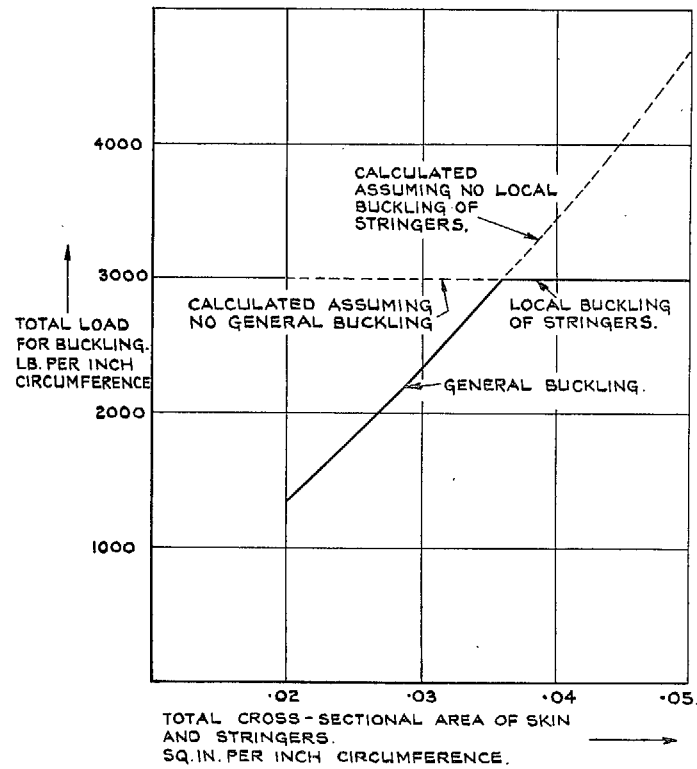
FIG. 6. Buckling load against total cross-sectional area of skin and stringers for the example of Section 4.1 ($S = 1$).
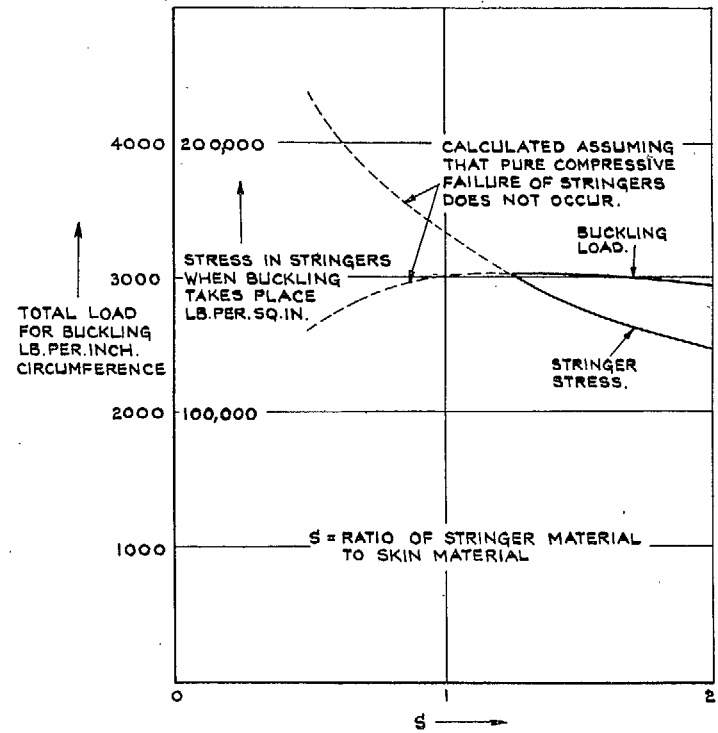

FIG. 7. Buckling load and the stress in the stringers at buckling against S for the example of Section 4.1. Total cross-sectional area of skin and stringers is 0·036 sq in. per in. circumference.
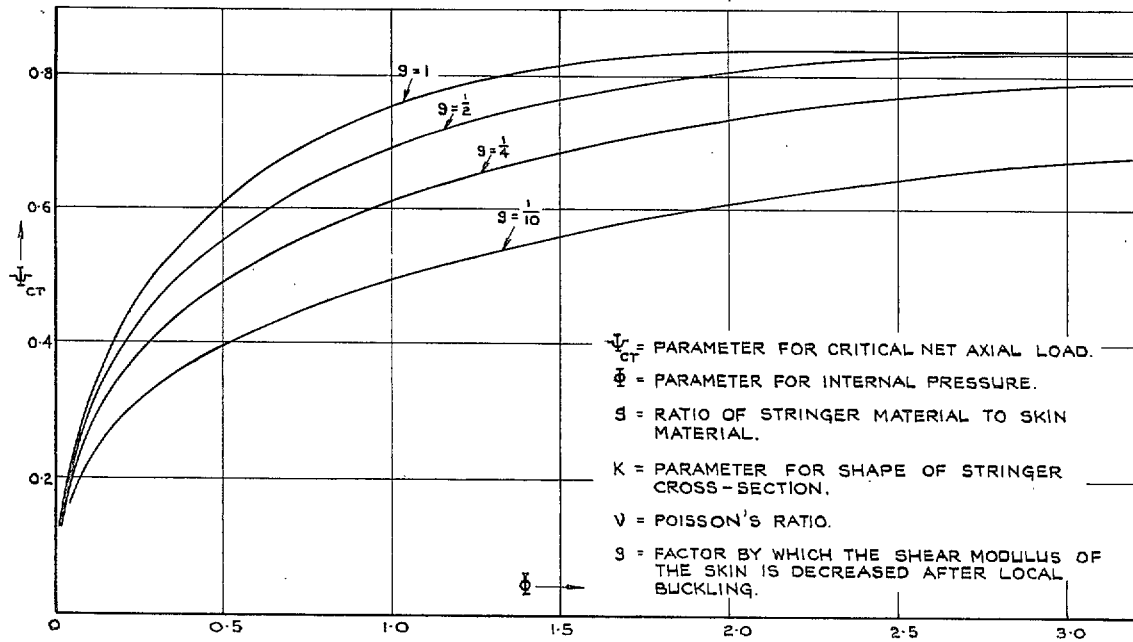
FIG. 8.   Graphs of Ψ against Φ for $S = 1$, $K = 1\cdot75$, $\nu = \frac{1}{4}$ and different values of the shear stiffness parameter g.

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